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Citation for published version:

Richmond, A 2013, 'Hilbert's Inferno: Time Travel for the Damned', *Ratio*, vol. 26, no. 3, pp. 233-249.
<https://doi.org/10.1111/rati.12012>

Digital Object Identifier (DOI):

[10.1111/rati.12012](https://doi.org/10.1111/rati.12012)

Link:

[Link to publication record in Edinburgh Research Explorer](#)

Document Version:

Peer reviewed version

Published In:

Ratio

Publisher Rights Statement:

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HILBERT'S INFERNO: TIME TRAVEL FOR THE DAMNED.*

Abstract:

Combining time travel with certain kinds of supertask, this paper proposes a novel model for Hell. Temporally-closed spacetimes allow otherwise impossible opportunities for material kinds of damnation and reveal surprising limitations on metaphysical objections to Hell. *Prima facie*, eternal damnation requires either infinite amounts of time or time for the damned to speed-up arbitrarily. However, spatiotemporally finite 'time travel' universes can host unending personal torment for infinitely many physical beings, while keeping fixed finite limits on rates of temporal passage. Such 'Hilbert's Inferno' spacetimes suggest neither materialism nor the finitude of time and space need forbid Hell. A material Hell can be spatiotemporally finite yet eternal for its inhabitants. Hilbert's Inferno also sheds light on Hell's location and accessibility, and shows that some spacetimes are intrinsically better suited to punishment than reward.

Key words: time travel, supertasks, Zeno, damnation.

I) Introduction

While the impact of Big Bang cosmology on First Cause arguments has been much debated,¹ discussions that otherwise combine philosophy of religion and philosophy of space and time are rather rare.² This is unfortunate, because eschatology and philosophy of space and time can be mutually illuminating. For example, living forever appears impossible if time is finite. However, spacetime eschatology can offer counter-examples to many normally compelling generalisations. In particular, the feasibility and location of Hell receive unexpected twists if physical eschatology includes spacetimes with unusual causal structures. Hell could be purely material, with finite spatiotemporal extent and a fixed rate of temporal passage, yet still house

* Many thanks to Hud Hudson, Roy Sorensen and an anonymous referee for *Ratio* for comments on earlier versions of this paper. A preliminary version of this paper was produced during a year-long sabbatical which was supported by a research award from the Arts and Humanities Research Council, to whom I owe many thanks.

¹See (e.g.) Quentin Smith, 'Stephen Hawking's Cosmology and Theism', *Analysis*, Vol. 54, 1994, pp. 236-43, John Earman, *Bangs, Crunches, Whimpers and Shrieks: Singularities and Acausalities in Relativistic Spacetimes*, Oxford, Oxford University Press, 1995, Ch. 7, Section 4, pp. 207-10, and William Lane Craig, 'Hartle-Hawking Cosmology and Atheism', *Analysis* Vol. 57, 1997, pp. 291-95.

²Rare, but not unknown - see for example Hud Hudson's excellent *The Metaphysics of Hyperspace*, Oxford, Oxford University Press, 2005, especially chapters 7 and 8, pp. 163-204.

infinitely many damned for an eternity of their time. Metaphysical arguments against damnation must do more than establish that time and space are finite.

Can Hell be a physical place even if space and time are finite? If Hell was a physical place, would it be detectable? How might the spaces and times of the damned relate to those of the saved? The following (admittedly highly) speculative exercise in physical eschatology explores these questions by considering a temporally-closed ('time travel') spacetime as a model Hell. Time-travel spacetimes widen the conceptual space for physical Hells and allow otherwise impossible outcomes. Our model Hell requires two counterfactual conditions: i) time is closed, and ii) space and matter are arbitrarily divisible along one dimension. Given these conditions, spatiotemporally finite Hells can hold all the (personally) infinite futures of infinitely many material damned beings. Indeed, such Hells can do so even if time consists of indivisible atomic units and passes at the same rate for all observers.

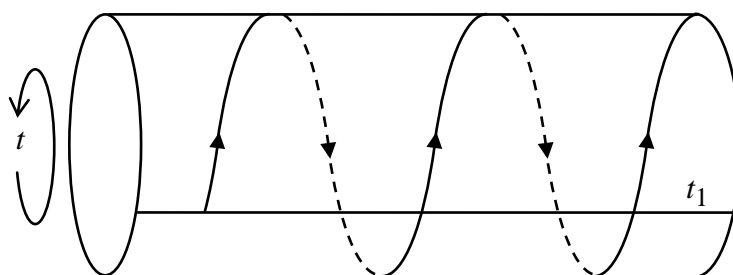
II) The Structure of Hilbert's Inferno

To dramatise time travel's eschatological possibilities, imagine you are a demon tasked with tormenting legions of the damned forever. Your task faces certain restrictions. Firstly, the damned are extended material beings. Secondly, you must accept infinitely many damned. Thirdly, your inferno is spatiotemporally finite. Fourthly, rates of temporal passage are either constant for all observers or at most can only diverge within finite bounds, (e.g. so you can't give some observers an infinite duration in a finite interval of other observers' time simply by making time pass ever-faster for the former observers).³ Can an inferno meeting these desiderata

³Like Roy Sorensen's 'pseudo-immortals', whose personal rate of temporal passage comes to diverge exponentially from that of external time so they live an infinite personal duration in a finite external time. See 'The Cheated God: Death and Personal Time', *Analysis*, Vol. 65, 2005, pp. 119-125. More on personal time and its relationship to time travel in section III.

accommodate infinitely many physical damned forever? Yes, provided i) space and matter are arbitrarily divisible in one dimension, and ii) time itself is closed.

Our model Hell is a time travel spacetime. The easiest time travel spacetime to picture (although not the most physically realistic) is an otherwise ordinary (e.g. Minkowski) spacetime wherein two complete timeslices are identified. Such identification gives spacetime a closed time co-ordinate. Our closed-time universe has three spatial dimensions and one time dimension but it can be pictured as the two-dimensional surface of a cylinder, where space is the linear axis and time the closed axis. As an object persists over time, it progresses its way around the 'cylinder', (figure after Earman 1995, p. 214, fig. 7.1):



An object that moved only in time (i.e. suffered no spatial displacement) would eventually persist its way around the cylinder's (finite) circumference and right back into becoming its own earlier self. Any physically realistic self-becoming 'object loop' would present a host of difficulties concerning (e.g.) apparent reversals of entropy, restoration of lost parts, undoing of aging effects, etc.⁴ However, our Hell requires no self-becoming objects and not every object in temporally-closed spacetimes must turn back into its earlier self. A traveller can avoid object-looping simply by moving correctly. Indeed, a moving object in a cylindrical universe can recede to arbitrarily-great spatial distances from its starting point, encountering every

⁴For more on object loops, see Richard Hanley, 'No End In Sight: Causal Loops in Philosophy, Physics and Fiction', *Synthese*, Vol. 141, 2004, pp. 123-152.

global timeslice (e.g. t_1) arbitrarily many times *en route*. So spacetime paths of infinite length (and hence duration) are possible even if time itself is finite and closed.

Granted, objects can follow spacetime paths (or 'worldlines') of infinite length in temporally finite (i.e. closed-time) worlds. However, objects with fixed non-zero velocity can only follow infinite worldlines in spatially infinite cylindrical universes. Still, an object's prospects for eternal survival in finite cylindrical spacetimes improve if its velocity *and length* can both approach arbitrarily closely to zero. For example, imagine an object's length and velocity both halve at regular intervals. Objects which contract and slow in such a fixed Zeno-style geometric series can follow infinite worldlines in cylindrical universes of overall spatiotemporally finite dimensions.

Suppose a damned person ('Faustus-1') starts off from point q on timeslice t_1 in our cylindrical spacetime. Faustus-1 leaves q with finite velocity⁵ and a finite length equal to d (measured along Faustus-1's direction of travel). Ordinary persistence will bring Faustus-1 round to t_1 again. While winding around the cylinder, Faustus-1 contracts along the direction of motion (although in no other direction), getting arbitrarily shorter as the number of circuits increases. Faustus-1's velocity and linear extension shrink by half during the first circuit around the cylinder, so Faustus-1 re-encounters t_1 with length $d/2$. As circuit #2 ends, Faustus-1 is $d/4$ long, and so on. Faustus-1's length progressively diminishes, equalling $d/2^n$ as circuit number n ends. This ever-slower and ever-narrower winding means each of Faustus-1's circuits occupies progressively smaller spatial distance. Thus, Faustus-1's front asymptotically approaches a point $2d$ away from q , while its front and back

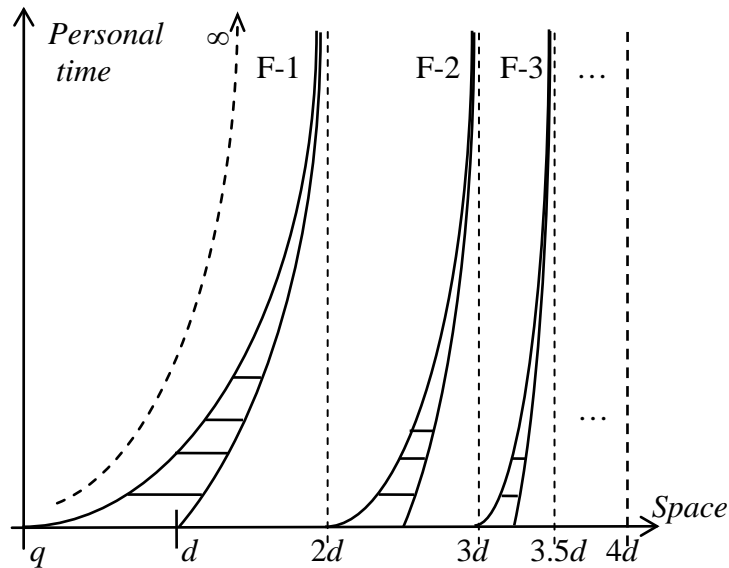
⁵A Faustus that began with, or accelerated to, unbounded velocity could escape 'to infinity' without ever intersecting its starting timeslice again. See John Earman on classical 'escape particles' that disappear from space in finite time, (*A Primer on Determinism*, Dordrecht, D. Reidel, 1986, pp. 34-5).

extremities draw asymptotically closer together. Like loops of thread wound around a bobbin, all Faustus-1's (infinitely many) stages can be physically contiguous with their immediate successors. Given correct contraction and deceleration, Faustus-1's worldline can be infinitely long yet fit into a spatial interval no more than $2d$ across.

Our next damned being, Faustus-2, starts off $2d$ away from q with the same finite velocity as Faustus-1 but with length $d/2$, (i.e. half that of Faustus-1). Faustus-2's length and velocity again diminish Zeno-style, but this time, the total infinite worldline fits into a spatial interval no longer than d . Faustus-3 starts off $3d$ away from q with length $d/4$ but otherwise proceeds as above, so has a total worldline occupying an interval no more than $d/2$. And so on, ad infinitum. Measured along the direction of travel, a spatial interval no longer than $4d$ can hold infinitely many damned. The total spatiotemporal volume swept out by Faustus-1 is topologically open towards the direction of motion, i.e. there is no last circuit.⁶ Just as each Faustus worldline makes a topologically open spacetime object, so does the total set of all the Faustus family. Each Faustus has infinitely many successors and each stage of each Faustus has infinitely many successors. (So there is no last Faustus.) Call the Faustus family's spacetime 'Hilbert's Inferno' for short. Just as Hilbert's Hotel holds (countably) infinitely many guests, Hilbert's Inferno can hold (countably) infinitely many damned.⁷ Below, a schematic of Hilbert's Inferno in simply connected (i.e. 'unwound' or unrolled) form, plotting personal time against spatial distance travelled:

⁶Each Faustus worldline forms a spacetime version of what Simon Prosser calls a 'Zeno object', i.e. an object of finite overall dimensions which lacks at least one boundary - see 'Zeno Objects and Supervenience', *Analysis*, Vol. 69, 2009, pp.18-26.

⁷Cf. Rudy Rucker's variant of Hilbert's Hotel, which packs infinitely many floors into a total height of thirty feet. In Rucker's Hotel, the ground floor is ten feet tall and every subsequent floor is two-thirds as tall as the floor below. See *Infinity and the Mind*, Harmondsworth, Penguin, 1997, p. 73.



While personal time increases along Faustus-1's worldline without limit, this worldline nonetheless tends to a definite *spatial* limit, i.e. a distance $2d$ from q . Likewise, Faustus-2 asymptotically approaches a distance $3d$ from q as personal time blows up, and so on – each of the damned has its own unique distance from q to which it tends in the limit as personal time tends to infinity. None of the vertical dotted lines above marks an achievable destination for any Faustus – rather than marking an end-point to any worldline, each such line instead marks a limit towards which a given damned being's worldline will tend as arbitrarily great personal time elapses. Hence $2d$ really marks the first point *beyond* Faustus-1's worldline rather than any point thereon. Faustus-1's worldline has no stage at $2d$, and Faustus-2's worldline never intersects $3d$, etc. Hence the half-open interval $[q, 2d)$ contains all of Faustus-1's worldline, Faustus-2's worldline will fit entirely into $[2d, 3d)$ and all infinitely many damned in the Faustus series fit into $[q, 4d)$. No Faustus worldline actually reaches the suppositional limit at $4d$. (Any Faustus that occupied the $4d$ line would form a finite object loop.) So Hilbert's Inferno holds infinitely many worldlines, each infinitely long, and yet has only finite spatiotemporal volume.

All our damned only diminish to arbitrarily small length after arbitrarily great personal time has elapsed – after no finite interval of personal time do any damned reach literally zero length. Also, our damned contract only along their direction of travel – their extension in other spatial dimensions is unaffected. If Faustus-1 starts out spatially three-dimensional, it would take infinite elapsed personal time for Faustus-1 to become literally two-dimensional. Just as the career of any Faustus has no end, so the length of any Faustus, howsoever small initially, never becomes nil.

Just as achieved zero length is always an infinite number of iterations away in damned personal time, at no point can any of our damned look back upon a literal eternity of elapsed personal time. Measured in damned personal time, Hilbert's Inferno does not allow the completion of a traditional supertask, i.e. at no point in personal time can any Faustus look back over a completed infinity of moments or sub-tasks. Rather, points along the career of a Faustus are only surveyable in finite time for external observers – for the damned, their careers are strictly and literally endless. Rather than unbounded velocities or accelerations, Hilbert's Inferno requires unlimited compression and progressively slower motion instead. (While quantum physics forbids arbitrarily fine-grained physical differences and relativity forbids faster-than-light transmission of mass-energy, neither forbids arbitrarily slow motion.) Again, the damned tend only asymptotically to literally zero velocity – it would take infinite personal time for any Faustus to come to a complete standstill.

Prima facie, each Faustus approaches literal rest and zero length as Zeno's Achilles approaches his quarry, i.e. via a geometrical series that takes infinitely many steps to complete. However, the two cases differ significantly: where Achilles supposedly packs infinitely many sub-runs into a finite time *simpliciter* (i.e. measured by Achilles or anybody else), the stages in the shortening and slowing of a Faustus

require infinite (personal) time. If Achilles' run really comprised infinitely many sub-runs, his catching his quarry in finite time would be a supertask in any frame of reference – the task's duration is finite for Achilles or anybody else. Hence Achilles' supertask (if such it be) is observer-independent. However, any Faustus can perform only a perspective-relative 'bifurcated' supertask.⁸

Non-bifurcated supertasks involve completing infinitely many sub-tasks in a finite interval of *anybody's* time (observer or performer), whereas with bifurcated supertasks, infinitely many sub-tasks occupy finite time for an observer but infinite time for their performers. Suppose Faustus-1 is told to count all the even integers without enumerating them progressively faster – because of the unusual structure of Hilbert's Inferno, such a task could fit into finite external time but would take infinite time for Faustus-1. Hence any worries about what state Faustus-1 is in after the task are misplaced: there is an 'outside' the task but there is no 'after'. For its hapless performer, there is nothing super about a bifurcated supertask, since it takes a literal eternity of the performer's time. There is no last circuit of Hilbert's Inferno for any of our damned and no terminus for their finitely bounded but unending worldlines. While any Faustus worldline has sharply differing durations measured externally or by a Faustus, not all properties of Hilbert's Inferno are perspectival or observer-relative – notably, all Faustus worldlines are genuinely endless for damned and external observers alike.

III) Three Definitions of Time Travel

Assuming for now our Faustus series are eternally damned, are they really time travellers? Philosophers and physicists offer three major definitions of time travel, which invoke a) durational discrepancies, b) closed timelike curves and/or c) multiple

⁸See John Earman and John Norton, 'Infinite Pains: the Trouble with Supertasks', *Benacerraf and His Critics*, A. Morton and S. Stich (eds.), Oxford, Blackwell, 1996, pp. 231-61, especially pp. 232 ff.

simultaneous locations of the traveller. First, consider David Lewis's 'durational discrepancy' definition, part of Lewis's thoroughgoing defence of the logical possibility of time travel and perhaps the most useful definition to date:

What is time travel? Inevitably, it involves discrepancy between time and time. Any traveler departs and then arrives at his destination; the time elapsed from departure to arrival (positive, or perhaps zero) is the duration of the journey. But if he is a time traveler, the separation in time between departure and arrival does not equal the duration of the journey.⁹

Hence, by Lewis's (1976) definition, a time traveller is one whose journey has different durations in personal and external time. 'Personal' time is time as registered by a travelling object, whereas external time is time as it elapses in the outside world: "External time is simply time itself. Personal time is [...] the way in which time is registered by a given object: a heart beating, hair growing, a minute hand moving, a candle burning".¹⁰ Clearly, points more than one circuit apart on Faustus's worldline are separated by greater personal than external duration, and on each Faustus worldline, there are stretches where personally later events correspond to externally earlier times. (In Hilbert's Inferno, personal time can be open and infinite but external time is finite and closed.) Hence each Faustus exhibits the personal/external duration discrepancy Lewis held definitive of time travel. Indeed, Faustus worldlines represent a logical extremity of personal/external time discrepancy, since points thereon which are simultaneous in external time (e.g. along t_1) can be arbitrarily far apart in personal time. While the duration of a Faustus-style journey differs from the perspective of a Faustus or an external observer, there is no reason in principle why the rate of temporal passage should differ between observers – Hilbert's Inferno does not require time to elapse at different rates for different observers (whether via time

⁹'The Paradoxes of Time Travel', *American Philosophical Quarterly*, Vol. 13, 1976, pp. 145-152, p. 145.

¹⁰Robin Le Poidevin, 'The Cheshire Cat Problem and Other Spatial Obstacles to Backwards Time Travel', *The Monist*, Vol. 88, 2005, (pp. 336-352), p. 339.

dilation or otherwise), and the requisite time travel is achieved purely through geometric identification of distinct timeslices. While Hilbert's Inferno allows one and the same journey to have divergent durations in personal or external time, it can still function as a model Hell even if time elapses at the same rate for all observers.

Second, consider David Malament's geometrical 'closed timelike curve' definition:

I take 'time travel' to be nothing more, and nothing less, than the act of starting at a particular point in spacetime, taking an otherwise conventional trip, and somehow returning to (or close to) that very point. ... In geometric terms, my time traveler is simply one whose worldline is closed (or almost closed).¹¹

'Otherwise conventional' here means always keeping below the (local) speed of light. Every Faustus begins with finite velocity and thereafter progressively slows down. Hilbert's Inferno has closed timelike curves throughout its extent and it lets each successor-stage of Faustus-1 (and each successor-stage of every succeeding Faustus) approach its immediate predecessor as closely as physical contiguity permits.

Thirdly, perhaps objects time-travel if different temporal stages of them can be present at the same global time. Only time travel seems to allow the same concrete object to have distinct temporal stages simultaneously present. Such a 'Multiple location' definition of backward time travel can be derived from David Deutsch: "In past-directed time travel the traveller experiences the same instant, as defined by external clocks and calendars, more than once in subjective succession".¹² Thus, by Deutsch's (1997) criterion, a time-traveller is a concrete object entirely present in multiple instantiations on a single global timeslice. Hilbert's Inferno offers 'multiple location' *deluxe*, since therein infinitely many persons are wholly present infinitely many times on every global timeslice.

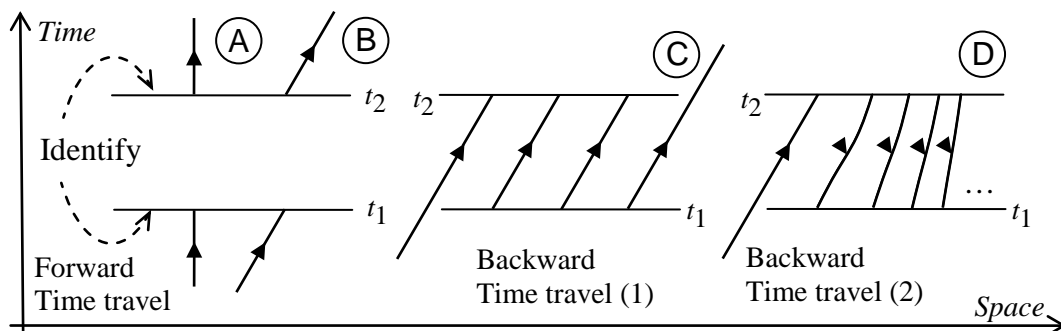
¹¹David Malament, "'Time Travel' in the Gödel Universe", *PSA: Proceedings of the Philosophy of Science Association*, Vol. 2, 1984, pp. 91-100, p. 91.

¹²David Deutsch, *The Fabric of Reality*, Harmondsworth, Penguin, 1997, p. 319.

These three definitions clearly differ. The Lewis (1976) definition is the most inclusive, not least because it covers forward and backward time travel equally, and neither Malament's (1984) nor Deutsch's (1997) definitions readily lend themselves to forward time travel. However these differences don't matter greatly here because our damned count as time-travellers by all three definitions: i) exhibiting personal/external durational discrepancies, ii) traversing (nearly) closed timelike curves and iii) registering multiply on global timeslices.

IV) Cylindrical and Local Forms of Hilbert's Inferno

In the original cylindrical model spacetime above, it was assumed that complete timeslices were identified, i.e. that the whole spacetime was temporally closed. One might worry that it was only by fiat that objects in our original cylindrical spacetime were assumed to maintain sufficient velocity that they didn't form object loops (i.e. become their earlier selves). After all, leave a stationary object long enough in a temporally rolled-up spacetime and it must form an object-loop. One might also worry about how any temporally-closed 'cylindrical' spacetime can connect to a causally normal universe. Both worries can be reduced if our temporally-closed Hell is simply a region formed by localised identification between spatially limited regions of two timeslices but residing in an otherwise causally normal universe:



Two localised regions of timeslices t_1 and t_2 are identified. These regions look from outside as if they reside on separate timeslices but they are identical and will not

appear separate to travellers who encounter them. Consider four travellers A, B, C and D. Travellers A and B have respectively zero and fixed finite velocity. A and B encounter t_1 from 'below' and exit instantaneously from t_2 . Such travellers reach future external destinations in shorter external than personal time, hence travel forward in time by Lewis's (1976) definition. (Their worldlines only appear discontinuous from outside – the travellers experience no discontinuity.) However, traveller C encounters t_2 from below and exits instantaneously from t_1 at an earlier external time. (Indeed, C makes three such backward jumps in time.) Traveller D begins with the same initial velocity as C but subsequently D's velocity (and length if D is an extended object) halve at each transit of the time travel region in the familiar Faustus-style Zeno sequence. Again, given correct deceleration and contraction, D can enter the time travel region but never thereafter emerge, i.e. D follows an infinite worldline in finite spatiotemporal volume. (In a two- or three-dimensional space, travellers can enter the closed-time region 'from the side'.)

In a local Hilbert's Inferno, all of the infinitely many damned have infinite personal futures and yet face a definite finite limit to their travel into the external future – each Faustus will encounter timeslice t_2 infinitely many times but no Faustus ever reaches an external time later than t_2 . All the damned disappear from external history as of t_2 , and while they survive personally forever, there will be external times (perhaps distributed across an infinite external future) which they will never see.

There are physical advantages in Hilbert's Inferno being a local time travel region in an otherwise normal universe: firstly, any non-moving traveller (like A) who encounters the local time travel region from outside through mere persistence cannot form an object loop. (Indeed, no traveller who originates outside the time travel

region can form an object loop.) Secondly, the damned may enter a localised Hilbert's Inferno without needing to leave their native spacetime, (e.g. via a wormhole or shortcut through higher-dimensional space). If Hilbert's Inferno resided in an otherwise normal spacetime, mere spatial movement might be all it took to enter Hell.

V) Hilbert's Inferno as Hell

Given the Faustus family qualify as time travellers, little further argument seems needed to establish that their fate is genuinely hellish. All the damned in Hilbert's Inferno face an infinitely long (personal) future of being compressed, ever more tightly, into a single contiguous mass with other stages of themselves. Hilbert's Inferno also has the dismaying feature that if the damned are at liberty to look ahead along their direction of travel, they can witness the succeeding stages in their own damnation and thus be continually presented with a reminder of the impossibility of escape – a refinement no causally normal Hell can seemingly offer. Perhaps part of damnation for the Faustus family is that once damned, their free will is thereafter rescinded, since they are always given a view (not merely a vision) of their own irrevocable future. The contiguity of each succeeding stage of each Faustus also presents the dismaying feature that any physical shock or impact (cries, lamentations etc.) felt/heard by any individual stage can potentially be transmitted to arbitrarily great personal distances. All the Faustus family might spend eternity listening to a cacophony of their own making, (perhaps multiplied indefinitely in recollection too).

However, while each Faustus can be observed from without and each Faustus is brought into dismaying proximity with stages of itself, it is not clear how far any of the Faustus family can communicate with its successors or predecessors. Each Faustus faces a definite spatial limit beyond which it will not travel. Also, it is not at all clear what Faustus-1's successors would see if they looked *back* at their

predecessors, since each predecessor has no last circuit of the spacetime in external or personal time. Consider Faustus-2 looking back toward Faustus-1. No stage of Faustus-1 ever reaches a point located beyond $2d$ from q but the worldline Faustus-1 follows can be assumed continuous and hence should present a barrier to signals. However, there is no unique 'last' stage of Faustus-1 and hence no unique stage for any of the later damned to see. So what, if anything, later members of the Faustus series could see if looking back at their predecessors is undetermined.¹³

Just as Hilbert's Inferno offers endless multiplication of shocks and noises, it also permits a unique economy of effort for any attendant demons. Suppose you wish to add to the torments of the damned by goading them with regular reminders of their plight. Suppose a denunciation is affixed to a billboard and displayed for a short interval after t_1 . Since t_1 (like all global timeslices in Hilbert's Inferno) will recur infinitely many times in the personal time of each and every Faustus, this single admonitory message can be viewed infinitely many times by each and every one of the infinitely many damned. Of course, each of the damned will receive only a diminishing exposure to any light reflected from this denunciation. But then again, the damned's sense-organs presumably contract along the direction of travel too so their visual acuity may improve proportionally as they progress.

Any demons supervising a local Hilbert's Inferno could have finite personal futures that nonetheless take them to external times after the eternal damnation of all their charges - a demon could 'shadow' its charges throughout their eternal damnation without spending infinite demonic time so doing, since in principle any point on an infinite Faustus worldline could be reached from outside in a finite amount of an

¹³For similar compositional problems with infinite objects, see John Hawthorne, 'Before-Effect and Zeno Causality', *Noûs*, Vol. 34, 2000, pp. 622-33.

external observer's time. Thus, one more dismaying feature to being in the Faustus series: one's attendant demons could be present throughout one's eternal damnation without being eternally damned themselves. Being reminded that one's attendant demon would be with one only for an eternity of one's own time (i.e. not of the demon's time) might be very disheartening. Curiously, a local Hilbert's Inferno embedded in a normal spacetime would allow demons to bestow eternal torments on the damned and yet leave room for said demons to be themselves redeemable.

VI) Conclusions

Hilbert's Inferno seems a possible world and so logic permits (personally) eternal torment for each of infinitely many fully-physical damned in a finite spacetime wherein divergences in temporal passage remain within finite bounds. If Hilbert's Inferno is a possibility, what might this suggest about eschatological metaphysics? *Prima facie*, unending personal futures are hard to reconcile with metaphysical finitism.¹⁴ However, metaphysical finitism can take different forms. If micro-finitism and macro-finitism respectively reject infinite divisibility and actually infinite quantities, then Hilbert's Inferno is not only compatible with macro-finitism about space, time and matter but with micro-finitism about time. Thus, Hilbert's Inferno is still possible even if time itself is only finitely divisible. Living an eternal life within a finite spacetime wherein time consists of finitely-sized atomic units (or 'chronons') seems highly counter-intuitive but Hilbert's Inferno is chronon-compatible.¹⁵ However, Hilbert's Inferno is not compatible with micro-finitism about space or matter (i.e. with space or matter made of irreducible 'atoms' of fixed finite size). So Hilbert's Inferno is a possible world even if infinitely divisible time is forbidden and

¹⁴A sustained argument for metaphysical finitism is the centrepiece of A. W. Moore's *The Infinite*, (London, Routledge, second edition 2001).

¹⁵For chronons as the hypothetical indivisible minimum unit of time, see e.g. W. Newton-Smith, *The Structure of Time*, (London, Routledge, 1984), p. 115.

so are infinite quantities of matter, space and time. Only micro-finitism about space seems to forbid Hilbert's Inferno. However, if a local Hilbert's Inferno has arbitrarily great extent in one spatial dimension, the damned need not contract as they travel and their Hell could be both temporally localised and compatible with spatial micro-finitism. However while such a revised Hilbert's Inferno could be temporally finite, it requires infinite spatial extent in at least one direction. No Hilbert's Inferno is compatible with both macro- and micro-finitism about space and time.

Following Lewis (1976), this discussion has taken the logical possibility of time travel as given and focussed on metaphysical and eschatological consequences of time travel. Throughout, Hilbert's Inferno has been considered as a possible material world. But in what sense is Hilbert's Inferno a 'material' world? Hilbert's Inferno is a highly idealised model, and only physically possible in a rather attenuated sense. In particular, Hilbert's Inferno is not compatible with our best physical theories in at least two respects, even given the significant concession that temporally closed spacetimes are physically possible.¹⁶ Firstly, Hilbert's Inferno is not compatible with quantum minima for meaningful spatial intervals – which alone presumably stops Hilbert's Inferno being governed by the same physical laws as we think govern our world.¹⁷ Secondly, Hilbert's Inferno is not compatible with General Relativity either, in so far as General Relativity assumes that matter and spacetime interact dynamically. The spacetime structure of Hilbert's Inferno must be non-dynamic, in

¹⁶For a thorough analysis of the physical possibility of time travel, see Douglas N. Kutach, 'Time Travel and Consistency Constraints', *Philosophy of Science*, Vol. 70 Supplement, 2003, pp. 1098-1113

¹⁷As an anonymous referee for this journal helpfully pointed out, a local Hilbert's Inferno joined to an otherwise normal spacetime poses problems in material constitution, identity-conditions and accessibility. Certainly the human constitution would not survive arbitrary diminishment by halving even in one direction. It would diminish any interest this model might possess if a literal miracle (*qua* suspension of the laws of nature) were required to ensure that any organism structured like ourselves could make the transition into a Hilbert's Inferno region. Such a need for external intervention would also suggest that even if a Hilbert's Inferno region were physically accessible from a world like ours, it might not be nomologically so accessible.

that its spacetime background must stay fixed and not change as the distribution of matter changes. Allowing the spacetime background to respond to changes in matter-distribution would be fatal to Hilbert's Inferno, since each and every member of the Faustus family would then pack an (effectively) infinite mass into a finite spatiotemporal volume and behave like a black hole. So Hilbert's Inferno may not be a Hell acceptable to physicalists in that it seems to conflict with both of our best-confirmed physical theories. However, Hilbert's Inferno might be acceptable to materialists in so far as all its constituent states of affairs supervene on physical facts, i.e. it requires no non-physical 'Cartesian' components.

Damnation might be rejected for ethical/theological reasons. One might believe in universal salvation, whereby all are ultimately saved, or in conditional immortality, whereby only the saved inherit eternal life and the lost cease to exist at death.¹⁸ Likewise, physical eschatology is presumably a branch of speculative metaphysics and one might reject damnation for non-finitist metaphysical reasons, e.g. thinking no realistic physical vehicle for consciousness seems plausibly immortal. However, Hilbert's Inferno shows that purely metaphysical arguments against damnation need more resources than either macro- or micro-finitism can offer independently. Hilbert's Inferno is compatible with i) finite space and time (i.e. spatiotemporal macro-finitism), ii) fixed finite upper bounds on the rate of temporal passage and iii) fixed finite minimal temporal intervals (i.e. temporal micro-finitism).

Perhaps eschatological states are necessarily non-physical and hence physical eschatology is at best otiose. Then again, perhaps physical eschatology is theoretically acceptable but difficult to reconcile with observation or materialism.

¹⁸Hudson (2005 p. 184) suspects fewer theists believe in Hell than Heaven, citing "moral problems endangering the hypothesis of Hell that do not have counterparts (or else have less pressing counterparts) to threaten the hypothesis of Heaven".

Hilbert's Inferno at least suggests a purely material Hell can be eternal for the damned and yet finite overall. Hilbert's Inferno might also breathe new life into allegedly outmoded 'regional' or 'receptacle' conceptions of Hell whereby such eschatological states are physical locales or containers. Hudson (2005, pp. 184-8) argues for regarding Heaven and Hell as spatiotemporal regions and not merely states of affairs or relations between minds. However, 'regional' views seem hard to reconcile with the apparent absence of eschatological spaces from our well-observed physical neighbourhood, e.g. "the Hubble telescope has yet to photograph any pearly gates", (Hudson, 2005, p. 186).

Hudson (2005) favours regarding eschatological places as real physical locations but ones which are separated from our local space by an extra spatial dimension. While Hudson's (2005) hyperspace regional solution has much to commend it, Hilbert's Inferno offers another kind of regional explanation as to whereabouts in space and whenabouts in time endless suffering might occur - since both Hilbert's Inferno itself and any entrance to it could be arbitrarily small. Likewise, if Hell-mouth can be arbitrarily small, epistemic access to it is that bit easier to control. If the damned are confined to a local Hilbert's Inferno, their infinitely long personal futures will have finite bounds in external time, hence if the saved enjoy eternal futures in an infinite causally normal spacetime surrounding Hilbert's Inferno, the eternal careers of damned and saved need not occupy co-extensive times. In this case, the 'falling away' of the lost would involve a literal divergence from the careers of the saved, and the eternity of the blessed could extend infinitely after that of the damned. Hence t_2 above would be an external temporal horizon to damnation and mean that the saved could not indefinitely contemplate the condition of the lost, nor vice versa – perhaps a merciful provision.

Finally, Hilbert's Inferno suggests an unsuspected eschatological asymmetry, namely that some spacetimes are intrinsically better-suited to supporting some eschatological states than others. Whereas a spatiotemporally infinite universe seems equally amenable to hosting infinite bliss or infinite suffering, Hilbert's Inferno seems vastly better suited to the latter. Because the only infinite worldlines in Hilbert's Inferno commit travellers to ever-greater compression, this spacetime seems intrinsically infernal, i.e. its very nature is better fitted to punishment than reward. Above, we imagined our damned as subject to the ministrations of demons but, provided correct contraction and slowing are maintained, the Faustus series could be literally self-tormenting. Where Sartre's Garcin and Marlowe's Mephistopheles aver (respectively) that "L'Enfer, c'est les Autres" and "Hell hath no limits, nor is circumscrib'd / in one self-place"¹⁹, the hellishness of Hilbert's Inferno consists precisely in the eternal persistence, ever more tightly circumscribed, of one's self.

¹⁹Jean Paul Sartre, *Huis Clos: Pièce En Un Acte*, scène v, (London, Horizon, 1945, p. 59); Christopher Marlowe, *The Tragical History of Doctor Faustus*, (The B-Text, 1616), Act One, Scene 5, lines 117-8, (as in *Christopher Marlowe – The Plays*, edited by Emma Smith, Ware, Wordsworth, 2000, p. 223).