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Optimal extended warranty strategy: Offering trade-in service or not?

By

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Abstract

Retailers and manufacturers are increasingly selling extended warranties to obtain high profitability. In addition to the traditional extended warranty (EWR), that offers a free repair and replacement service, a new extended warranty (EWT) comes to the market, under which an additional trade-in service is provided during the warranty coverage. The service provider faces three important decisions: (1) Whether to offer EWR or EWT? (2) How to set the optimal selling prices of EWR and EWT? (3) When choosing to sell EWT, how to determine the optimal trade-in price? To address such challenging issues, we first develop two theoretical models regarding EWR and EWT for a retailer, and further consider the cases where a manufacturer sells the extended warranties and the upgraded product has different failure probabilities. The results show that EWT should never be offered at a higher price than EWR, and when the handling cost for used products is relatively low, EWT will outperform EWR. While the optimal EWR and EWT selling prices increase with the product failure probability, the optimal trade-in price decreases with it. Interestingly, the optimal trade-in discount is not always increasing or decreasing with the failure probability. Moreover, an earlier trade-in time is usually better for the service provider. Compared with the retailer, the manufacturer will always set a lower warranty selling price, but neither of them will always offer a lower trade-in price or discount. We also find that the upgraded product's failure probability will affect the retailer's optimal warranty strategy and profit.

Keywords: Supply chain management; extended warranty; trade-in service; warranty selling price; trade-in price

1. Introduction

Warranty is actually a typical service under which the providers promise to repair, replace or maintain products freely for consumers over a certain time period. A product is generally sold accompanied by a manufacturer's base warranty. In addition to this warranty bundled with the product, manufacturers, retailers or even third-party providers sell extended warranties in the market. An extended warranty is an optional service plan that provides consumers an additional coverage and is often sold separately from the products (Desai and Padmanabhan, 2004; Li et al., 2012). In today's fiercely competitive marketplace, as product profit margins on durable products such as home appliances, consumer electronics and PC are decreasing rapidly, high-margin post-sales services like extended warranties are increasingly important to the profitability of manufacturers and retailers (Gallego et al., 2014). Moore (2002) finds that extended warranty services for new vehicles increase the profit of the automobile industry by 100%. A recent survey shows that roughly 60-70% of a retailer's margin is generated from selling extended warranties (Hsiao et al., 2010).

In addition to the profitability, extended warranties have turned out to be a useful strategy that can help extend the useful life of products, and thus improve consumer loyalty, brand image and brand equity (Shahanagki et al., 2013; Gallego et al., 2014). In view of these merits, extended warranties are offered on almost all durable products, ranging from bicycles to wedding jewelry, home appliances, consumer electronics and automobiles. To better segment consumers and achieve competitive advantages, manufacturers and retailers are increasingly launching various types of new extended warranties, such as price-discrimination extended warranties that are sold with different prices according to purchase time points, i.e., at the time of product sales or at the end of base warranty, or indeed at any time (Warranty Week, 2007; Hartman and Laksana, 2009; Zheng et al., 2018), flexible-duration extended warranties that are sold monthly (Gallego et al., 2014), residual value extended warranties under which residual values are refunded when their coverage expires (Gallego et al., 2015) and bundled extended warranties that are combined with the base warranties (Bian et al., 2015).

Compared to traditional extended warranties, the aforementioned new extended warranties can provide more choices or additional benefits for consumers. More recently, a new typical extended warranty has arisen in the market under which consumers can trade in used products for new or new-generation products with lower prices at a particular time within the warranty duration. The trade-in service is offered in addition to the traditional repair and replacement service. Generally, such extended warranty services can be offered by a manufacturer or a retailer. For example, Apple Inc. has launched a typical extended warranty service "Apple Care+" in recent years (Apple, 2018). This service provides an opportunity for consumers to replace old iPhones with new ones (or upgraded ones) at discounted prices 10-13 months after the purchase of the service in China or 12 months after the purchase of the service in U.S. Notably, the old iPhones are required to meet the trade-in condition criteria provided by Apple Inc. Similarly, as a manufacturer, EVGA.com provides customers a 3-year extended warranty. During the warranty coverage, customers are allowed to exchange their used products with original

factory condition and free from physical modification or damage per the terms of the warranty for similar types of new products (EVGA, 2018). As a retailer, T-Mobile.com launched an extended warranty with trade-in service “JUMP!” that allows consumers to upgrade their used flagship devices like Samsung Galaxy, iPhone and other smartphones during the 6th month of the warranty coverage (Beren, 2013).

The above-mentioned evidence shows that, the typical extended warranty allows consumers to trade in their used products at a particular time over the warranty coverage under certain conditions. Common wisdom suggests that, any extended warranty service will incur substantial warranty costs during the warranty duration. In addition to the repair costs, an extended warranty with trade-in service may lead to more costs including handling cost, shipping cost, packaging cost and inventory cost with regard to used products. These costs may directly affect the decisions on whether to offer such service, the service price and new product trade-in price.

These typical considerations raise three questions for the service providers: (1) Whether to offer such extended warranty with trade-in service or not? Under what conditions, can this typical service benefit the providers? (2) How to price the typical extended warranty service? (3) How to determine the new product trade-in price?

Despite the importance of the decisions regarding the extended warranty with trade-in service, prior studies have not documented the above considered issues well. To fill this gap, we first consider a retailer who sells a type of durable product bundled with a manufacturer’s base warranty to a group of consumers. In addition to the base warranty, the retailer may choose one of two distinct extended warranties to sell in the market: a traditional extended warranty with free repair and replacement service or a new extended warranty with an additional trade-in service. Without loss of generality, we assume that each consumer will purchase at most one unit of new product. To investigate the optimal warranty strategy and the associated pricing decisions, we first consider the case where the trade-in service is for the same product or an upgraded version of the used product and consider two scenarios, i.e., offering a traditional extended warranty (EWR) and a new extended warranty with trade-in service (EWT), respectively. We then consider the case where a manufacturer offers extended warranties, and then examine the corresponding warranty strategy and associated pricing decisions. Finally, we further consider the case where the original product and the upgraded product have different failure probabilities. Our analysis yields the following three important findings. *First*, EWT is not always superior to EWR. Specifically, EWT is more beneficial to the service provider if the handling cost is relatively low; otherwise, both warranties are equivalent. Furthermore, when the warranty coverage is relatively short and the product failure probability is relatively low, the retailer is more willing to provide EWT. In contrast, when the coverage is relatively long, the retailer will benefit from offering EWT if trade-in time is relatively early or the product failure probability is relatively low. Unexpectedly, while EWT offers more service to consumers, its optimal selling price is no more than that of EWR. *Second*, while the optimal EWR and EWT selling prices increase with the product failure probability, the optimal trade-in price decreases with it. Interestingly, the optimal trade-in discount is not always increasing or

decreasing with the product failure probability. Moreover, trade-in time significantly affects the optimal pricing decisions and an earlier trade-in time is usually better for the service provider. Notably, though the retailer may raise the selling price against the increasing cost, its trade-in price decreases in the warranty cost. However, when the manufacturer offers EWT, the trade-in price may increase in the warranty cost when trade-in transaction occurs relatively late. *Third*, interestingly, we find that, whether a manufacturer or a retailer can offer a lower warranty selling price regarding EWR and EWT, as well as trade-in price or trade-in discount under EWT, completely depends on their warranty cost-efficiencies. Though the manufacturer may set a lower warranty selling price, neither of them always offer a lower trade-in price. In addition, we further find that the failure probability of the upgraded product has certain effects on the retailer's optimal warranty strategy and profit.

The remainder of this paper is organized as follows. In the next section, we review the most relevant literature. In Section 3, we present our theoretical models and results. In Section 4, the optimal extended warranty strategy and the corresponding pricing strategy are examined. In Section 5, we extend the base model to consider the cases where a manufacturer sells the extended warranty and the upgraded products have different failure probabilities. Conclusions appear in Section 6. All proofs are offered in Appendix.

2. Literature Review

Since our work is primarily related to extended warranty strategy and trade-in pricing strategy, we review these most relevant studies in this section.

2.1 Extended warranty strategy

In recent years, extended warranty strategy has drawn extensive concerns in the literature. Padmanabhan and Rao (1993) empirically reveal that a suitable menu of warranty coverages and associated pricing strategies can effectively help segment the market in the presence of consumer heterogeneity regarding risk preference and moral hazard. Padmanabhan (1995) further shows that manufacturers can provide warranty service for various market segments by designing a menu of extended warranty options when facing consumer moral hazard and usage heterogeneity. Lam and Lam (2001) investigate how to determine the optimal extended warranty coverage at the end of base warranty by considering consumer heterogeneity with respect to warranty renewing. These studies focus on examining extended warranty duration and pricing strategies by considering consumer heterogeneities.

Unlike the above-mentioned studies, some recent work has explored warranty service distribution strategies and related warranty coverage and pricing decisions. Desai and Padmanabhan (2004) is the first to examine extended warranty distribution strategy in a supply chain setting. They show that distribution strategy depends on double marginalization and complementary goods effect, and the manufacturer benefits from selling extended warranty in a dual distribution channel, i.e., the manufacturer's direct channel and a retailer's channel. Li et al. (2012) show that, the retailer can offer longer warranty coverage than the manufacturer, and hence a higher supply chain profit can be achieved.

By examining the interaction between the manufacturer's base warranty and the retailer's extended warranty, Jiang and Zhang (2011) find that the duration of base warranty is negatively affected by the retailer's extended warranty coverage when consumers can estimate product quality. A similar finding is also presented in Heese (2012). Dai et al. (2012) investigate the impact of product quality on warranty decisions in a supply chain, and show that both players are better off when the warranty coverage is determined by the player who undertakes a larger proportion of warranty cost. In a competitive market, Chen et al. (2012) examine the impact of the manufacturer's pricing strategy on supply chain decisions and performance when the warranties are offered by two competing retailers.

An increasing number of studies have examined the optimal design and pricing strategies regarding emerging extended warranties in recent years. Jack and Murthy (2007) examine how the manufacturer determines the pricing strategy of a type of flexible extended warranty in which consumers determine when to start the extended warranty. Hartman and Laksana (2009) explore the optimal coverage menus and associated pricing strategies of a number of extended warranty contracts including renewable warranties and restricted warranties with restrictions on deferrals. Tong et al. (2014) examine the optimal design and pricing decisions of two-dimensional extended warranties (i.e., duration and usage) that can be purchased at the point of product sale or at the end of base warranty. By considering consumer product quality learning effects, Gallego et al. (2014) explore the optimal pricing strategy of a flexible-duration extended warranty with month-by-month commitments. Similarly, Lei et al. (2017) investigate dynamic pricing strategies of both product and warranty in a multi-period setting where consumers can learn quality. Zheng et al. (2018) examine the optimal pricing decisions on a new flexible extended warranty that can be sold at the sales of the product or at the end of the base warranty. By considering consumer strategic claim behaviors, Gallego et al. (2015) examine dynamic pricing strategies for a typical residual value of extended warranty where residual values are refunded to consumers when the warranty coverage expires.

Note that, the aforementioned studies have not addressed the issue when extended warranty allows consumers to trade in their products during the warranty duration. In the literature, a number of studies have examined the optimal warranty coverage and pricing strategy considering replacement service during the base warranty duration, e.g., Faridimehr and Niaki (2012) and Park et al. (2014). However, trade-in or replacement service offered by extended warranty has not received much attention in the extant studies. One exception is Wu and Longhurst (2011). In their work, it is assumed that, upon minor failures, the product will be corrected with a minimal repair, whereas upon catastrophic failures, the product will be replaced for free. Under these assumptions, they examine the optimal product lifetime and extended warranty duration by minimizing total expected cost. Note that, the extended warranty considered in their study is a traditional one, and the warranty selling pricing is not examined.

2.2 Trade-in pricing strategy

An increasing number of studies explore trade-in pricing strategies in the literature. These studies

can be grouped into two streams. The first stream focuses on examining the optimal trade-in rebates and product prices. Van Ackere and Reyniers (1995) show that offering trade-in rebate can increase consumer purchase frequency, and examine the optimal product price and trade-in rebate under a two-period framework. In the first period, the optimal price is determined; and in the second period, firms determine the optimal trade-in rebate for repeat purchases, or discounts for replacing old products. Rao et al. (2009) theoretically and empirically examine the motivation of implementing trade-ins, and find that trade-in service can effectively increase firms' profitability. Subsequently, Yin and Tang (2014) examine the optimal trade-in rebate and product price under the situation when the firm offers a trade-in service that requires customers to pay an up-front fee, and find that a firm is always better off offering such service. By considering strategic consumers, Yin et al. (2015) find that these consumers are willing to pay higher prices than their product valuations in order to obtain the trade-in privilege. Zhu et al. (2016) apply a two-period model to examine the optimal trade-in rebates and product prices for two competing firms. Unlike these studies that examine the optimal trade-in rebate in dynamic settings, Ray et al. (2005) assume that the technology regarding a durable product is relatively stable, and examine the optimal trade-in rebate and product price at the time when offering trade-in service. In a closed-loop supply chain setting, Miao et al. (2017) examine the optimal trade-in rebate and product wholesale price. Cao et al. (2019) explore the optimal trade-in strategy and associated pricing decisions of business-to-consumer platform with dual-format retailing model.

Note that, trade-in consumers can gain trade-in rebates by returning used products, which can be regarded as price discrimination for further purchases. The second stream of related literature considers this issue. Kwon et al. (2015) empirically investigate the effect of trade-ins on pricing durable goods, and find that a higher price is charged for consumers who have traded-in used vehicles than those who have not. Chen (2015) examines the impact of price discrimination on the choice of trade-in strategies in the presence of strategic consumers. To achieve price discrimination, Chen and Hsu (2015) investigate when and how a firm provides a trade-in rebate, and show that the rebate magnitude decreases with customers' willingness for trade-in rebate but increases with product deterioration rate. Notably, this stream has also not conducted analysis on trade-in service offered by extended warranties.

This work attempts to identify the optimal extended warranty strategy, the associated warranty price and the new product trade-in price for an extended warranty offering a trade-in service. To our best knowledge, this is the first study to address these issues, and some new findings are obtained.

3. Theoretical models

Consider a retailer who sells a type of new durable product bundled with a manufacturer's base warranty to a group of consumers. The retailer may choose traditional extended warranty with free repair and replacement service (EWR) or extended warranty with trade-in service (EWT) including the traditional free repair and replacement service to sell in the market. We assume that each consumer will purchase no more than one unit of new product. Then, consumers may decide whether to buy EWR or

EWT. If a consumer chooses to purchase EWT, he may face a decision on whether to trade in his used product for a new one in a particular time period during the warranty coverage. Notably, the new product provided via the trade-in service can either be the same as the original product or a newer version of the original product. Many firms such as Apple Inc., JD.com and T-Mobile provide trade-in services for the same product version or upgraded products. For instance, the AppleCare+ plans allow consumers who buy the service to trade in the used products (the Apple Watch, iphone, ipad, etc.), and state that “all replacement products provided under this Plan will at a minimum be functionally equivalent to the original product”. In related literature, some studies also assume that used products can be exchanged for the same version of new products, e.g., Rao et al. (2009) and Zhu et al. (2016). Accordingly, the retailer makes the following important decisions, i.e., new product selling price, whether to sell EWR or EWT, both warranty prices and new product trade-in price when consumers conduct trade-in transactions. To examine the optimal extended warranty strategy and associated prices, we assume that the retailer is rational and self-interested. Main notations used in this work are summarized in Table 1.

In this study, extended warranty prices (i.e., p_e^r and p_e^t), new product selling prices (p_0^r and p_0^t) and new product trade-in price (p_n^t) are decision variables, whereas all other variables such as base warranty coverage ω and extended warranty coverage T are exogenous. We denote the variable c_0 as the price for a retailer to purchase a product from a manufacturer. Note that, T is assumed to be total coverage of base warranty and extended warranty regardless of whether under EWR or EWT, i.e., extended warranty coverage begins at the moment of product purchase rather than the expiration of the base warranty, and thus $T \geq \omega$. This assumption is consistent with prevalent industry practice and is widely used in related studies (Jiang and Zhang, 2011; Heese, 2012). Consequently, there is an initial period of duplicate duration ω of base warranty and extended warranty, in which the base warranty cost is undertaken by the manufacturer. Notably, the extended warranty takes effect after the expiration of the base warranty, i.e., the effective extended warranty length is $[\omega, T]$. We also assume that both ω and T are fixed, and the maximum lifetime of the product is 1. These assumptions allow us to focus on the main considered decisions while retaining analytical tractability. Actually, this assumption is also consistent with the practice. For example, most electronic products sold in the market in the real world carry a bundled and free one-year limited base warranty, and their extended warranty lengths are usually set as two years. For simplicity, we further assume that when consumers trade in their products, the remaining extended warranty coverage $(1-\alpha)T$ terminates. This assumption coincides with the emerging real-world practices. For example, Apple Inc. allows consumers to trade in their iPhones 10-13 months (or 12 months in U.S.) after purchasing the typical service “Apple Care+” in China. Thus, new traded-in product is only bundled with a manufacturer’s base warranty.

Table 1. Summary of notations

| Notation | Interpretation |
|----------|----------------|
|----------|----------------|

| | |
|------------------------|---|
| v | Net product value when the product works well |
| ρ | Product failure probability, and $\rho \in (0,1]$ |
| ω | Base warranty coverage and $\omega \in (0,1)$ |
| T | Extended warranty coverage and $T \in [\omega,1)$ |
| λ | Coefficient of the new product value for trade-in service relative to the original product and $\lambda \geq 1$ |
| β | Discount factor and $\beta \in (0,1)$ |
| $p_0^j (p_e^j)$ | New product (extended warranty) price, where $j = r$ or t for EWR and EWT, respectively |
| p_n^t | Trade-in price of new product under EWT offered by the retailer |
| c_0 | New product wholesale price |
| c_r | Warranty cost-efficiency coefficient of the retailer |
| c_i | Extended warranty cost, where $i = e$ or $i = et$ for EWR and EWT, respectively |
| θ | Consumer's coefficient of absolute risk aversion |
| α | Trade-in time parameter and $\alpha \in (\omega/T, 1]$. Note the trade-in time is given by αT . |
| δ | The rate of change in the residual value with the remaining warranty coverage |
| c_h | Unit used product handling cost including shipping, repackaging, inventory and other related costs |
| s | Unit used product residual value |
| $u_p^j (u_e^j)$ | Consumer utility function regarding purchasing new product (both product and extended warranty), where $j = r$ or t for EWR and EWT, respectively |
| $u_e^{tt} (u_e^{tk})$ | Consumer utility function regarding trade-in transactions (keeping the used product) |
| $D_0^j (D_n^j, D_e^j)$ | New product (trade-in, warranty) demand function, where $j = r$ or t for EWR and EWT, respectively |
| D_e^{tt} | Trade-in product demand |
| C_j | Total cost, where $j = r$ or t for EWR and EWT, respectively |
| π_j | The retailer's profit function, where $j = r$ or t for EWR and EWT, respectively |

Furthermore, we assume that the product is subject to a certain nontrivial probability of failure ρ , which is known to all, and thus $1 - \rho$ denotes the probability that the product works well during the extended warranty period. Similar to Mai et al. (2017), ρ refers to failure probability rather than failure rate, and does not change over time. The manufacturer will bear the warranty cost during the base warranty coverage ω , while the extended warranty provider undertakes the costs during $[\omega, T]$. Such warranty cost is caused by repair service in the presence of product failure during the corresponding warranty coverage. Generally, the repair costs may differ from failure to failure. Similar to Li et al. (2012), we use the average of these repair costs in this study. As Jiang and Zhang (2011) suggest, warranty cost function has the properties that it is more costly to offer a service to a low-quality

product than to a high-quality product, and a longer warranty coverage costs more than a shorter one. Hence, warranty cost is dependent on product failure probability and warranty coverage. To capture these properties, we take a quadratic form to formulate the extended warranty cost function, i.e., $c_e = c_r \rho (T^2 - \omega^2)$. Such cost form is widely used in related studies, e.g., Jiang and Zhang (2011), Li et al. (2012) and Heese (2012). Note that, c_r is the cost coefficient of the service provider, which can be estimated from total warranty cost and the number of warranty claims per year in practice. A larger c_r indicates a lower cost-efficiency, and vice versa. We next present theoretical models regarding EWR and EWT, respectively.

3.1 Traditional Extended Warranty (EWR)

We assume that the product offers consumers a net value (or monetary transfer) v when it works well, and zero when it fails. Similar assumptions are found in Desai and Padmanabhan (2004), Jiang and Zhang (2011) and Mai et al. (2017). Accordingly, if consumers do not purchase the extended warranty, consumers will receive a monetary transfer ωv from the manufacturer when the product breaks. Following Desai and Padmanabhan (2004) and Jiang and Zhang (2011), we assume that the consumer's utility function exhibits constant absolute risk aversion, thus a consumer will derive a utility of u_p from buying the product without purchasing an extended warranty. The consumer's utility function takes the following form:

$$u_p = v(1 - \rho) + v\rho\omega - \theta v^2 \rho(1 - \rho)(1 - \omega)^2 - p_0^r. \quad (1)$$

In Eq.(1), $v(1 - \rho) + v\rho\omega - p_0^r$ is the expected utility for a risk-neutral consumer who purchases a product with a selling price of p_0^r , and $\theta v^2 \rho(1 - \rho)(1 - \omega)^2$ is the disutility due to the risk. Notably, $v^2 \rho(1 - \rho)(1 - \omega)^2$ represents the variance of the "lottery", and θ is the consumer's coefficient of absolute risk aversion (Jiang and Zhang, 2011; Mai et al., 2017). We assume that consumers are heterogeneous in their risk aversion degree, and θ is uniformly distributed over $[0,1]$. Such assumption is reasonable because a consumer who has a high risk aversion may not purchase the product.

When consumers buy an extended warranty with coverage T , similar to Jiang and Zhang (2011), they will receive a total monetary transfer Tv when the product breaks during $[\omega, T]$. Since both base warranty and extended warranty are effective from the product purchase time, and the extended warranty does not take effect if $T \leq \omega$. From the consumer's viewpoint, total warranty coverage is determined by the extended warranty coverage if they buy the service. Hence, the consumer's expected utility function derived from purchasing both a product and an extended warranty is formulated as

$$u_e^r = v(1 - \rho) + v\rho T - \theta v^2 \rho(1 - \rho)(1 - T)^2 - p_e^r - p_0^r. \quad (2)$$

It is easy to derive the indifference point between purchasing both the product and the extended

warranty and buying nothing at all, i.e., $\bar{\theta} = \frac{v(1-\rho) + v\rho T - p_0^r - p_e^r}{v^2\rho(1-\rho)(1-T)^2}$. Accordingly, new product

demand can be obtained, i.e., $D_n^r = \bar{\theta}$. Consumers may purchase the extended warranty if they can obtain a higher utility than not purchasing the service. Thus, it is easy to get the indifference point

between buying and not buying the service, i.e., $\bar{\bar{\theta}} = \frac{p_e^r - v\rho(T - \omega)}{v^2\rho(1-\rho)(2-T-\omega)(T-\omega)}$. Accordingly, the

extended warranty demand is $D_e^r = \bar{\theta} - \bar{\bar{\theta}}$. Thus the retailer's profit function regarding selling EWR is expressed as

$$\pi_r = D_n^r(p_0^r - c_0) + D_e^r(p_e^r - c_e). \quad (3)$$

Note that, the first term represents the profit derived from new product sales, and the second term denotes the profit obtained from the extended warranty sales. The retailer aims to maximize his profit by determining the optimal extended warranty selling price and the new product retailing price, which are summarized in Theorem 1.

Theorem 1. The retailer's optimal warranty selling price and product selling price are $p_e^{r*} = \frac{1}{2}\rho((T-\omega)v + C_{er})$ and $p_0^{r*} = \frac{1}{2}(c_0 + V_p)$, respectively, where $C_{er} = c_r\rho(T-\omega)(T+\omega)$ and $V_p = v(1-\rho) + \rho\omega v$.

Theorem 1 shows that p_e^{r*} decreases with ω but increase with c_r and ρ . However, p_0^{r*} increases with ω and decreases with ρ . These findings are intuitive in that, as the base warranty coverage ω increases, the effective coverage of the corresponding extended warranty decreases, and thus the incurred warranty cost goes down. Similar results can be found in Jiang and Zhang (2011) and Li et al. (2012). Thus, the retailer will have more incentive to lower the warranty selling price to boost the EWR sales. Meanwhile, the retailer will undertake more possible base warranty costs and will increase the product selling price to cover such costs. This is also applicable to c_r . As for the product failure probability, a larger ρ means that it is more urgent for consumers to purchase EWR to fight against the failure risk, and thus leads to more EWR demand. Notably, retailer's warranty cost will also increase with ρ , which further leads to a higher EWR selling price in order to compensate for the possible loss. Such a finding is also presented by Ding et al. (2014) and Xie et al. (2014). In such a circumstance, the retailer may have more motivation to reduce the product price to boost the product demand and thus the EWR demand.

3.2 Extended Warranty with Trade-in Service (EWT)

In this scenario, if consumers purchase EWT with a price p_e^t , they are allowed to trade in their used products at sometime during the warranty coverage. In particular, we assume that consumers conduct trade-in transactions at the time αT . Consumers first return their used products to the retailer,

and then pay p_n^t (trade-in price) for the new products. The new product can be the same as the original version or an upgrade one. If the trade-in product is of an upgraded version, it generates a relatively higher value than the original version. We use λv ($\lambda \geq 1$) to denote the new trade-in product value. In general, λ may represent some new functions introduced in the new version of the product. Similar assumptions on the upgraded product value are also found in Liang et al. (2014) and Liu et al. (2018). For simplicity, we assume that the unit production cost is the same for both versions, which is a commonly used assumption for high-tech products (Ray et al., 2005; Liang et al., 2014) and is also adopted by Liu et al. (2018). Without loss of generality, following Zhu et al. (2016), we further assume that the two versions have the same failure probability. We further extend the analysis to the case where the two versions have different failure probabilities in sub-section 5.2.

The trade-in process is summarized as follows. After purchasing the service, consumers will decide whether to conduct trade-in transactions at αT . At this time, if consumers do not trade in their products, they will continue to use their old products, and the extended warranties continue to be effective during the remaining warranty coverage $[\alpha T, T]$. In such a case, consumers will receive an expected value (monetary benefit) $v(1-\rho)(1-\alpha T)$ from holding the old products, and also gain a monetary transfer $v(T-\alpha T)$ from the rest of EWT if the used products break. Thus, the consumer's utility function regarding keeping the used products during the remaining lifetime (i.e., $[\alpha T, 1]$) is formulated as

$$\begin{aligned} u_e^{tk} &= v(1-\rho) + v\rho T - \theta v^2 \rho(1-\rho)(1-T)^2 - \alpha T v(1-\rho) - \alpha T v \rho \\ &= v(1-\rho)(1-\alpha T) + v\rho(T-\alpha T) - \theta v^2 \rho(1-\rho)(1-T)^2 \end{aligned} \quad (4)$$

If consumers choose to trade in their products, they first return their used products, and pay a trade-in price p_n^t to get new products with a bundled base warranty provided by the manufacturer. In this circumstance, consumers will lose the used products and the remaining extended warranty service. Therefore, consumer utility function regarding trade-in transactions is derived as

$$u_e^{tt} = \lambda v(1-\rho) + \lambda v \rho \omega - \theta(\lambda v)^2 \rho(1-\rho)(1-\omega)^2 - p_n^t - u_e^{tk}. \quad (5)$$

At the time αT , a consumer will choose to trade in the product if the utility regarding trade-in product is larger than zero. The indifference point between conducting trade-in service or not can be

obtained, i.e., $\bar{\theta}_3 = \frac{v(T\alpha + \rho(1-T) + \lambda(1-\rho(1-\omega)) - 1) - p_n^t}{v^2(1-\rho)\rho(\lambda^2(1-\omega)^2 - (1-T)^2)}$.

Unlike EWR, consumer utility of purchasing EWT is affected by the trade-in service. That is, consumers will have more incentive to buy the EWT service if they can benefit from the trade-in transaction. In this regard, we introduce a discount factor to characterize the value of trade-in service (Besanko and Winston, 1990; Loewenstein et al., 2002). Hence, the consumer's utility function derived from purchasing the new product and EWT is formulated as

$$u_e^t = v(1-\rho) + v\rho T - \theta v^2 \rho(1-\rho)(1-T)^2 - p_0^t - p_e^t + \beta \max\{u_e^{tt}, 0\}, \quad (6)$$

where β ($0 < \beta \leq 1$) is the discount factor.

Similar to EWR, a consumer with non-negative utility will decide to buy only the new product or both the new product and the EWT service. Hence, the indifference point between purchasing both the new product and the extended warranty and buying nothing is $\bar{\theta}_1 = \frac{v(1-\rho) + v\rho T - p_0^t - p_e^t}{v^2\rho(1-\rho)(1-T)^2}$. Then, the indifference point between purchasing EWT or not can be derived, i.e., $\bar{\theta}_2 = \frac{p_e^t + p_n^t\beta + v(\beta(1-\lambda - T\alpha - \rho + \lambda\rho + T\rho - \lambda\rho\omega) - \rho(T-\omega))}{v^2(1-\rho)\rho(T^2(1-\beta) + \beta(\lambda^2(1-\omega)^2 - 1) + (2-\omega)\omega - 2T(1-\beta))}$. The demand function with respect to EWT can be directly obtained, i.e.,

$$D_e^t = \bar{\theta}_1 - \bar{\theta}_2 = \frac{v(1-\rho) + v\rho T - p_0^t - p_e^t}{v^2\rho(1-\rho)(1-T)^2} - \frac{p_e^t + p_n^t\beta + v(\beta(1-\lambda - T\alpha - \rho + \lambda\rho + T\rho - \lambda\rho\omega) - \rho(T-\omega))}{v^2(1-\rho)\rho(T^2(1-\beta) + \beta(\lambda^2(1-\omega)^2 - 1) + (2-\omega)\omega - 2T(1-\beta))} \quad (7)$$

Consequently, we can directly derive the demand functions regarding trade-in products and new products, i.e., $D_e^u = \bar{\theta}_3 - \bar{\theta}_2$ and $D_n^t = \bar{\theta}_1$, respectively.

Fig.1 provides a better illustration for new product, the EWT and trade-in service demands. Specifically, consumers with $\theta \in [0, \bar{\theta}_2]$ buy the product without the extended warranty service; consumers with $\theta \in (\bar{\theta}_2, \bar{\theta}_3]$ buy the extended warranty service and trade in their products; consumers with $\theta \in (\bar{\theta}_3, \bar{\theta}_1]$ buy the extended warranty service but do not trade in their products; consumers with $\theta \in (\bar{\theta}_1, 1]$ whose risk aversion is relatively high will always not purchase the product.

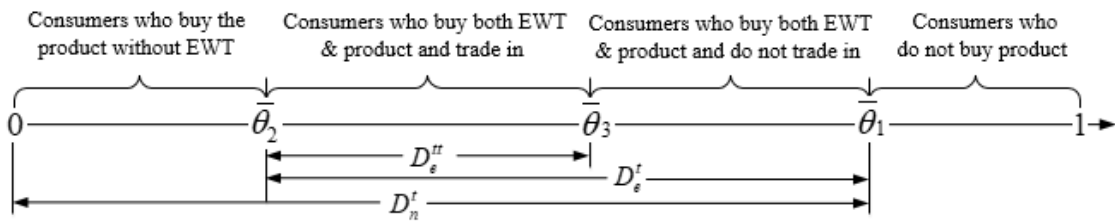


Fig.1. New product, warranty and trade-in demands.

Through trade-in transactions, the retailer will charge a trade-in price for providing a new product and call back the used product. Suppose that the trade-in used product has a sufficient salvage value for recycling. Following Van Ackere and Reyniers (1995) and Ray et al. (2005), we assume that the used product's residual value is linearly related to the product age, i.e., $s = \delta(1 - \alpha T)$. The term " $\delta(1 - \alpha T)$ " represents the perceived residual value, and δ is the rate at which the residual value changes with the remaining extended warranty coverage. A higher δ means a larger residual value of the used products,

and vice versa. In this sense, δ is a measure of the durability of the product (Ray et al., 2005). Notably, if consumers trade in their products, the repair and replacement component of the warranty service is only effective during $[\omega, \alpha T]$. Thus, the corresponding warranty cost is $c_{et} = c_r \rho (\alpha^2 T^2 - \omega^2)$.

The retailer's profit is sourced from three parts: profit derived from new product sales, profit obtained from selling the extended warranty to those consumers keeping the used products and profit generated from those consumers who accept trade-in service. The retailer's profit is formulated as

$$\pi_t = D'_n(p'_0 - c_0) + (D'_e - D''_e)(p'_e - c_e) + D''_e(p'_e - c_{et} + p'_n - c_h + s - c_0). \quad (8)$$

The retailer aims to determine the optimal new product selling price, the warranty selling price and the trade-in price in order to maximize his profit. In order to avoid the trivial cases where the retailer's trade-in demand increases with the handling cost, we assume that $\frac{\partial(\bar{\theta}_3 - \bar{\theta}_2)}{\partial c_h} \leq 0$ always holds, then

we can obtain the upper threshold of λ , which equals to $\lambda_u = \frac{\sqrt{1 + 2T - T^2 + \beta - 2T\beta + T^2\beta - 4\omega + 2\omega^2}}{\sqrt{(1 + \beta)(1 - \omega)^2}}$. Hence we confine our analysis to $1 \leq \lambda \leq \lambda_u$. This is

reasonable in that, in practice, product upgrade is usually an incremental improvement rather than radical innovation, and thus the upgraded product value will not increase too much (Galbreth et al., 2013). In this regard, λ will not be too large. Proposition 1 shows that the retailer's optimal decisions under EWT can be achieved.

Proposition 1. There exists a unique set of optimal pricing decisions $(p_e^{t*}, p_0^{t*}$ and $p_n^{t*})$ for the retailer under EWT when the condition $1 \leq \lambda \leq \lambda_u$ holds.

Since it is difficult to explicitly identify the retailer's optimal pricing decisions under EWT, Lemma 1 shows the retailer's optimal pricing strategies in the special case when $\lambda = 1$.

Lemma 1. When $\lambda = 1$, the retailer's optimal decisions under EWT $(p_e^{t*}, p_0^{t*}$ and $p_n^{t*})$ are

$$p_e^{t*} = \frac{\rho(v(T - \omega) + C_{er})}{2} - \frac{\Delta \bar{c}_{h2r}}{3 + \beta}, \quad p_0^{t*} = \frac{c_0 + V_p}{2} + \frac{\Delta \bar{c}_{h2r}}{3 + \beta}, \quad \text{and} \quad p_n^{t*} = \frac{2\alpha T v - C_{er} - (T - \omega)v\rho}{2} - \frac{\Delta \bar{c}_{h2r}}{3 + \beta},$$

where $\Delta \bar{c}_{h2r} = s + \alpha T v + \frac{(c_r T^2 (1 - 2\alpha^2 - \beta)\rho + c_r (1 + \beta)\omega^2 \rho - (1 - \beta)(T - \omega)\rho v)}{2} - c_h - c_0$,

$C_{er} = c_r \rho (T - \omega)(T + \omega)$, and $V_p = v(1 - \rho) + \rho \omega v$.

4. Analysis

To identify the optimal extended warranty strategy and associated pricing strategy, we first compare the retailer's optimal decisions and profits under the services EWR and EWT. If the trade-in price is sufficiently high under EWT such that no consumer will ever use the trade-in service, EWT

would be equivalent to EWR. In this sense, EWR can be regarded as a special case of EWT. Consequently, the profit when the retailer offers EWT will always be at least as high as that when offering EWR. These issues are formally stated in the following proposition.

Proposition 2. (a) When $c_h \leq \bar{c}_{h1}$, $D_e'' = D_e'$; otherwise $D_e'' < D_e'$;

(b) When $c_h \leq \bar{c}_{h2}$, $D_e' \geq D_e^r$, EWT outperforms EWR and $p_e^{t*} \leq p_e^{r*}$;

(c) When $c_h > \bar{c}_{h2}$, $D_e'' = 0$ and EWT and EWR are equivalent so that $D_e' = D_e^r$, $p_e^{t*} = p_e^{r*}$;

where \bar{c}_{h1} and \bar{c}_{h2} are the closed-form solutions of the equations $\bar{\theta}_3 - \bar{\theta}_1 = 0$ and $\bar{\theta}_3 - \bar{\theta}_2 = 0$ regarding c_h , respectively.

Proposition 2(a) shows that, when the handling cost for the used product is relatively low, i.e., $c_h \leq \bar{c}_{h1}$, all consumers who buy EWT will trade in their used products during the warranty coverage. Otherwise, the trade-in demand is less than the demand of EWT. The reason is that, when $c_h \leq \bar{c}_{h1}$, the retailer will set a relatively low trade-in price, and all consumers are enticed to conduct trade-in transactions; otherwise, the retailer may set a relatively high trade-in price. Accordingly, consumers may have less incentive to trade in their used products.

Proposition 2(b) and proposition 2(c) indicate that when the handling cost for a used product is relatively small, i.e., $c_h \leq \bar{c}_{h2}$, the retailer may set a relatively low trade-in price (i.e., $p_n^{t*} \leq \bar{p}_n^t$), and thus a lower selling price of EWT than EWR, i.e., $p_e^{t*} \leq p_e^{r*}$. This further leads to a higher demand of EWT than EWR, i.e., $D_e' \geq D_e^r$. In contrast, when $c_h > \bar{c}_{h2}$, trade-in demand will be zero, and thus EWT is reduced to EWR. Particularly, when $\lambda = 1$, the condition $c_h > \bar{c}_{h2}$ is equivalent to $p_n^{t*} > \bar{p}_n^t$, where $\bar{p}_n^t = \frac{(2\alpha T v - C_{er} - (T - \omega)v\rho)}{2}$, which means that the retailer will set a sufficiently large trade-in price such that no consumers will conduct trade-in transactions.

It can be observed that whether the retailer should offer the EWT or EWR is heavily dependent on the handling cost. When $c_h \leq \bar{c}_{h2}$, the retailer is better off offering EWT; otherwise, EWR is preferable. Intuitively, if the handling cost is sufficiently high (i.e., $c_h > \bar{c}_{h2}$), the retailer's profit generated from trade-in service will be very low or even incur a loss. In such a case, the retailer will have less incentive to provide a lower trade-in price for trade-ins. As a result, EWT and EWR are equivalent. This finding coincides with Chen and Hsu (2015), who show that the decision on whether offering a trade-in service or not is related to the recovery costs for collected products. This finding can explain why Apple Inc. provides EWT for iPhone, but EWR for iPad and Mac instead.

Notably, $c_h \leq \bar{c}_{h2}$ can also be transformed into its equivalent form $\delta \geq \bar{\delta}$. For instance, when $\lambda = 1$, $\bar{\delta} = (c_0 + c_h - \alpha T v - \frac{(c_r T^2 \rho (1 - 2\alpha^2 - \beta) + c_r \omega^2 \rho (1 + \beta) - (1 - \beta)(T - \omega)\rho v)}{2}) / (1 - \alpha T)$. As

stated earlier, a greater δ means a higher residual value per unit of used product. Accordingly, when δ is relatively large, the retailer may gain high profit from the recycled used product. In this case, the retailer will have more motivation to offer EWT. By contrast, the retailer will have less incentive to offer a higher trade-in price, and thus no consumers will trade in their products.

To better illustrate Proposition 2(b) and Proposition 2(c), we apply a numerical example by setting $\lambda = 1.1$, $\nu = 4$, $\beta = 0.4$, $\rho = 0.2$, $c_r = 15$, $\omega = 0.2$, $T = 0.5$, $\alpha = 0.8$, $c_0 = 0.3$ and $\delta = 0.2$, and then let c_h increase from 0.2 to 2. The optimal selling prices of EWR and EWT are depicted in Fig.2. We can observe that when $c_h < \bar{c}_{h2} = 1.4599$, the optimal selling price under EWT is lower than that under EWR; otherwise, the optimal warranty selling prices of both EWR and EWT are equal.

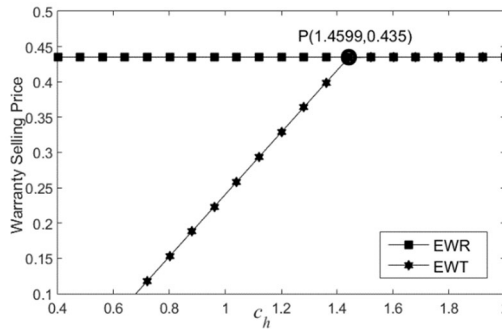


Fig.2. The optimal selling prices of EWR and EWT regarding c_h .

As \bar{c}_h is closely related to the trade-in time and used product residual value, we present the following conclusion to illustrate the impacts of these parameters on the threshold.

Corollary 1. The impacts of δ and α on the threshold \bar{c}_{h2} are characterized as follows:

- (a) \bar{c}_{h2} is increasing in δ ;
- (b) if $\alpha \in (\frac{\omega}{T}, \frac{\nu - \delta}{2c_r \rho T})$, \bar{c}_{h2} is increasing in α ; otherwise, \bar{c}_{h2} is decreasing in α .

Corollary 1(a) can be interpreted as follows. A used product with a higher residual value means a higher recovery profit, and thus the retailer may have more incentive to accept a higher handling cost. Miao et al. (2017) present a similar finding that a firm is more likely to adopt a trade-in program for high residual value products, and undertake all associated costs such as transportation costs and reprocessing costs. A similar finding is also provided in Genc and Giovanni (2016).

Corollary 1(b) indicates that \bar{c}_{h2} increases in α when α is relatively small, whereas it decreases in α when α is relatively large. This is because a relatively small α means a high residual value. In such a case, consumers have less incentive to trade in their used products. By delaying the trade-in time, consumers may have higher motivation to conduct trade-in transactions. This generates more trade-in demand and hence the retailer has more incentive to bear the handling cost. In contrast, when α is relatively large, the residual value of used product is relatively low, thus consumers will have more incentive to trade in their products. This in turn incurs a higher handling cost

due to additional inspection of used product residual value (Klausner and Hendrickson, 2000) and thus leads a lower profitability to the retailer. Also, when α is relatively large, the retailer has already incurred heavy warranty cost. In such context, offering trade-in service cannot effectively reduce the warranty cost. This suggests that, the retailer should set a suitable time during the warranty coverage to allow consumers to trade in their products. This finding can explain why Apple Inc. requires that consumers who buy the service “Apple Care+” in China trade in their products 10-13 months after purchasing the service (Apple, 2017).

As shown in Proposition 2, whether the retailer chooses EWT or not primarily relies on c_h . Generally, such handling cost threshold is significantly affected by the product failure probability ρ . Since it is hard to theoretically address this issue when $\lambda > 1$, we take the special case when $\lambda = 1$ to examine the influence of ρ on the retailer’s optimal warranty strategy, which is formally stated by the following corollary.

Corollary 2. When the trade-in service is for the same version of the original product, i.e., when $\lambda = 1$,

(a) if $T \leq \frac{v}{c_r} - \omega$, or $T > \frac{v}{c_r} - \omega$ and $\alpha \in (\alpha_0, 1)$, there exists a $\bar{\rho} \in (0, 1)$ such that if $\rho \leq \bar{\rho}$, the

retailer is more willing to provide EWT, where $\alpha_0 = \frac{\sqrt{T(c_r T - v)(1 - \beta) + v(1 - \beta)\omega + c_r(1 + \beta)\omega^2}}{T\sqrt{2c_r}}$

and $\bar{\rho} = \frac{2(c_0 + c_h - \delta - T\alpha(v - \delta))}{c_r T^2(1 - 2\alpha^2 - \beta) - v(1 - \beta)(T - \omega) + c_r(1 + \beta)\omega^2}$;

(b) if $T > \frac{v}{c_r} - \omega$ and $\alpha \in (\frac{\omega}{T}, \alpha_0)$, the retailer always benefits from offering EWT regardless of the value of ρ .

Corollary 2(a) shows that, when T is relatively small (i.e., $T \leq \frac{v}{c_r} - \omega$), if $\rho \leq \bar{\rho}$, the retailer is more likely to offer EWT. This finding is intuitive. When both conditions (i.e., $T \leq \frac{v}{c_r} - \omega$ and $\rho \leq \bar{\rho}$) hold, the handling cost threshold \bar{c}_{h2} will be relatively high, and the condition $c_h \leq \bar{c}_{h2}$ in Proposition 2(b) will easily hold. Thus, the retailer will have more incentive to provide EWT. In contrast, if ρ is sufficiently large (i.e., $\rho > \bar{\rho}$), even though when $T \leq \frac{v}{c_r} - \omega$, the retailer may incur a substantially large handling cost, and thus offering EWT will not lead to more benefit for the retailer. For a larger T , a similar finding holds provided the trade-in time is relatively late.

Corollary 2(b) suggests that, when $T > \frac{v}{c_r} - \omega$ and the trade-in time is relatively early, the retailer will always benefit from providing EWT regardless of whether ρ is low or high. This is because when the two conditions hold, the retailer’s incurred handling cost would be relatively low, and thus the condition

$c_h \leq \bar{c}_{h2}$ can hold. In this context, the retailer will be better off providing EWT.

It is difficult to theoretically examine the influences of ρ on the retailer's warranty strategy, and thus we here simply employ two numerical examples by setting $\alpha=0.5$ and 0.8 , respectively. The values of other parameters are based on the same data used in Proposition 2 to investigate the impacts of ρ on the retailer's warranty strategy. Specifically, we consider the extended warranty coverage is relatively long, i.e., $T=0.5$. Fig.3 displays that, consistent with Corollary 2(b), when α is relatively small (e.g., $\alpha=0.5$) and T is relatively high, the retailer always benefits more from offering EWT regardless of the failure probability. However, when α is relatively large (e.g., $\alpha=0.8$) and T is relatively high, the retailer is more willing to provide EWT if the failure probability is not extremely high (e.g. $\rho \leq \bar{\rho}=0.858$).

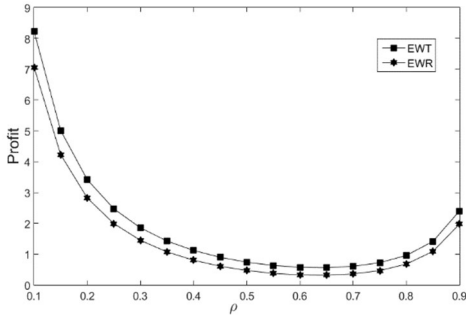


Fig. 3.1. $\alpha = 0.5$

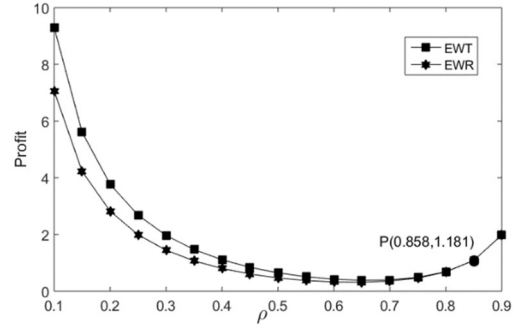


Fig. 3.2. $\alpha = 0.8$

Fig. 3. The optimal profits under EWT and EWR regarding ρ .

We next further investigate the impacts of parameters related to trade-in service (i.e., δ , α , c_h and c_r) on the optimal warranty selling price and trade-in price under EWT, and have the following proposition.

Proposition 3. (a) Both p_e^{t*} and p_n^{t*} decrease with δ .

(b) If $\alpha \leq (v-\delta)/(2c_r\rho T)$, p_e^{t*} decreases with α , but p_n^{t*} increases with α ; otherwise, both p_e^{t*} and p_n^{t*} increase with α .

(c) Both p_e^{t*} and p_n^{t*} increase with c_h .

(d) p_e^{t*} increase with c_r , whereas p_n^{t*} decreases with c_r .

Proposition 3(a) shows that, both p_e^{t*} and p_n^{t*} are decreasing in δ . Generally, for each used product, a larger δ means a higher residual value, and thus a larger marginal profit from trade-in transactions. Thus, the retailer may be willing to offer a higher trade-in rebate to consumers, and thus a lower trade-in price p_n^{t*} . Similar findings are presented in Ray et al. (2005) and Yin et al. (2015). Furthermore, as δ increases, the retailer may have more motivation to reduce the warranty selling price p_e^{t*} to boost EWT demand, and hence trade-in demand. This leads to a higher profit obtained

from both warranty selling and trade-in transactions.

Proposition 3(b) provides a counter-intuitive finding. One might expect that p_e^{t*} increases with α due to the increase of warranty cost before trade-ins. However, Proposition 3(b) indicates p_e^{t*} is convex in α . This is because, when α is relatively small, the retailer can considerably reduce the warranty cost from consumers who choose to trade in. Hence, the retailer may have an incentive to decrease the EWT selling price since this will boost the warranty sales and associated trade-in demand. However, when α is sufficiently large, the retailer will incur substantial warranty cost before the trade-in time. Though consumers may have more incentive to purchase a new product, it is not beneficial for the retailer to continually increase the total demand by sacrificing the EWT selling price. Furthermore, to cover the incurred warranty cost, the retailer will increase the warranty selling price accordingly. Interestingly, regardless of whether α is large or small, the optimal trade-in price is always increasing in α . As α increases, consumers have more motivation to trade-in their products, and thus leads to a higher trade-in profit. This phenomenon is also presented in Ray et al. (2005).

Proposition 3(c) shows that, as c_h increases, the retailer may set a higher trade-in price p_n^{t*} to cover the associated handling cost. This finding can be illustrated from the opposite case. As c_h decreases, the retailer may reduce trade-in price p_n^{t*} to boost the trade-in demand. This leads to an increasing warranty demand and thus trade-in demand, which can generate more profit for the retailer.

Proposition 3(d) shows that the optimal warranty selling price is increasing in the warranty cost-efficiency coefficient c_r , but the optimal trade-in price decreases with c_r . This is because, as c_r increases, the warranty cost goes up. Thus, on one hand, the retailer will increase EWT selling price to cover such cost. On the other hand, there is an urgent need for the retailer to attract consumers to trade in their used products to reduce this warranty cost. As a consequence, the retailer has more incentive to reduce the trade-in price to entice more consumers to conduct trade-in transactions. This may lead to a lower profit from trade-in transactions, but it will also reduce the warranty cost substantially.

Notably, the optimal warranty selling price and trade-in price are also closely related to ω and ρ , and we then attempt to examine the impacts of these two parameters on the p_e^{t*} and p_n^{t*} . We first take the special case when $\lambda = 1$ to illustrate these impacts, which are summarized as follows.

Proposition 4. When $\lambda = 1$, p_e^{t*} decreases with ω , but increases with ρ ; in contrast, p_n^{t*} increases with ω but decreases with ρ .

Proposition 4 shows that, similar to EWR, EWT selling price is decreasing in ω , but increasing in ρ . By contrast, unlike the warranty selling prices, the optimal trade-in price is increasing in ω , but decreasing in ρ . This is because, a higher ω means less risk that consumers will suffer from a failure of a new product (Lei et al., 2017). As ω increases, consumers are more willing to trade in their products. In this regard, the retailer may increase trade-in price to gain more profit from trade-in

transactions. With regard to the product failure probability, as ρ increases, to reduce warranty costs, the retailer has more incentive to reduce the trade-in price to encourage more consumers to trade in products.

Notably, when $\lambda > 1$, it is difficult to theoretically examine the impacts of ω and ρ on p_e^{t*} and p_n^{t*} , we apply two numerical examples based on the data used in Proposition 2 to illustrate these impacts (i.e., $\lambda = 1.1$). Particularly, we further set $c_n = 0.1$, and let ω increase from 0.2 to 0.4, and ρ increase from 0.2 to 0.4, respectively. Fig.4.1 and Fig.4.2 display that, consistent with the case when $\lambda = 1$ as shown in Proposition 4, the retailer's EWT selling price (p_e^{t*}) is still decreasing in ω but increasing in ρ , and the trade-in price (p_n^{t*}) always increases with ω but decreases with ρ .

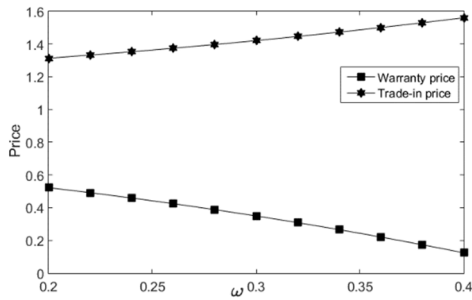


Fig.4.1. Optimal prices regarding ω .

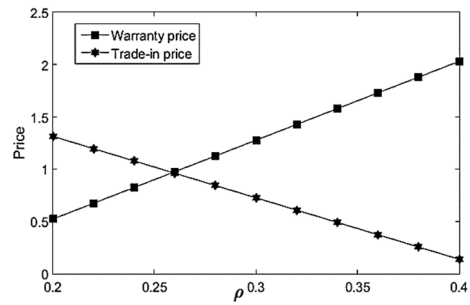


Fig. 4.2. Optimal prices regarding ρ .

Note that, the difference between new product selling price and the trade-in price can be seen as trade-in discount, also called trade-in rebate. For simplicity, set $R^{t*} = p_0^{t*} - p_n^{t*}$. To examine the effects of main market parameters on this discount, we take the case when $\lambda = 1$ to illustrate the impacts and achieve the following findings:

Proposition 5. When $\lambda = 1$,

(a) R^{t*} increases with c_r and δ , but decreases with α .

(b) When $\omega \leq \frac{v}{c_r}$, R^{t*} increases with ω ; otherwise, R^{t*} decreases with ω .

(c) When $\alpha \leq \bar{\alpha}$, R^{t*} increases with ρ ; otherwise, R^{t*} decreases with ρ , where

$$\bar{\alpha} = \frac{\sqrt{c_r T^2 (5 - \beta) - c_r (1 - \beta) \omega^2 + v(-3 + T - \beta + 3T\beta + 2\omega - 2\beta\omega)}}{2\sqrt{c_r} T}.$$

Proposition 5(a) shows that, the optimal trade-in discount is always increasing in c_r and δ , but decreasing in α . As c_r increases, the warranty cost goes up. The retailer may have more incentive to provide a higher trade-in discount (i.e., a lower trade-in price) to consumers. This can help boost trade-in transactions, and thus reduces the warranty cost. This is similar to that of the trade-in price in Proposition 3(d). The rationales for the monotonicity of trade-in discount regarding δ and α are similar to the trade-in price in Proposition 3(a) and Proposition 3(b).

Proposition 5(b) shows that the optimal trade-in discount is concave in the base warranty coverage.

This is counter-intuitive. As shown in Proposition 4, p_n^{t*} increases with ω . This intuitively indicates that the trade-in discount will decrease with ω . However, Proposition 5 (b) shows that, when $\omega \leq \frac{v}{c_r}$, this conclusion does not hold. This is because, when ω is relatively small, the retailer will incur more warranty cost. Hence, the retailer have more motivation to provide a higher discount to entice more consumers to trade in their products.

Proposition 5(c) shows that the optimal trade-in discount increases with the product failure probability when α is relatively small, but decreasing otherwise. When α is large, the handling costs due to trade-ins are high and the residual value of used products is low. Increasing ρ might motivate more consumers to trade in their products which might not be the retailer's interest. To reduce the number of trade-ins and increase the profit from trade-ins, the retailer might set a higher trade-in price accordingly.

It is noteworthy that, we have further used some numerical examples to examine the impacts of the parameters in Proposition 5 on the trade-in discount in the case when $\lambda > 1$. Fortunately, we find that the results are consistent with those in the case when $\lambda = 1$.

According to the above mentioned findings, the retailer's optimal decisions significantly affect the warranty costs and associated handling cost of trade-in used products. We next investigate which warranty can lead a lower total cost to the retailer. For ease of notation, let C_r and C_t denote the total costs regarding EWR and EWT, respectively. For simplicity, we take the case when $\lambda = 1$ to illustrate this issue. By comparing the total costs under these two warranties, we have the following finding:

Proposition 6. When $\lambda = 1$, if $\alpha \leq \bar{\alpha}_1$, $C_r \geq C_t$; otherwise, $C_r < C_t$, where

$$\bar{\alpha}_1 = \frac{\sqrt{(c_r \rho ((1 - \beta) T^2 + (1 + \beta) \omega^2) - 2c_h)^+}}{\sqrt{2c_r \rho T^2}}.$$

Proposition 6 shows that, when the trade-in time parameter is relatively low, the trade-in service may help reduce total cost for the retailer, and thus we have $C_r \geq C_t$. In contrast, when α is relatively large, i.e., $\alpha > \bar{\alpha}_1$, the total cost under EWT is larger than that under EWR due to the incurred handling cost regarding used products. This suggests that the retailer should set a suitable trade-in time in order to reduce the total cost. This explains why Apple Inc. and T-Mobile set strict time restrictions for consumer trade-in service during the warranty coverage (Apple, 2017; Beren, 2013).

Note that, $\alpha \leq \bar{\alpha}_1$ can be equivalently transformed into $c_h \leq \frac{c_r \rho ((1 + \beta) \omega^2 + T^2 (1 - \beta - 2\alpha^2))}{2}$.

This indicates that, when the handling cost c_h is relatively low, we have $C_r \geq C_t$; otherwise, we have $C_r < C_t$. Note that, the threshold $\frac{c_r \rho ((1 + \beta) \omega^2 + T^2 (1 - \beta - 2\alpha^2))}{2}$ is increasing in ρ when

$\alpha^2 \leq \frac{(1+\beta)\omega^2 + T^2(1-\beta)}{2T^2}$. This indicates that, a higher product failure probability leads to a higher

handling cost threshold when the retailer sets an early trade-in time, and thus the condition is more easily satisfied. This suggests that, when selling a product with a higher failure probability, the total cost under EWR is usually higher than that under EWT. This further means that, EWT can help reduce total cost for the retailer, especially when the product failure probability is relatively high. In the 3rd quarter of 2017, Apple Inc. reported \$1.022 billion in claims paid, up from \$932 million in the second calendar quarter but down from \$1.156 billion in the 3rd quarter of 2016. It was also down significantly from the three-out-of-the-last nine quarters in which the company had spent more than \$1.2 billion on warranty claims. Note that, the Upgrade program was implemented in the second half of 2016 (Warranty Week, 2017). This evidence partly supports our finding that trade-ins help reduce the cost.

Since it is hard to theoretically investigate the impact of α on the comparisons between the total costs under the two warranties when $\lambda > 1$, we apply a numerical example with the same data used in Proposition 2 and set $c_h = 0.1$ to illustrate the impact. To this end, we let α vary from 0 to 0.5. Fig.5 shows that, when $\alpha \leq \bar{\alpha} = 0.275$, the total cost of EWT is less than that of EWR; otherwise, the total cost of EWT is larger than that of EWR. Such results are also consistent with those in Proposition 6.

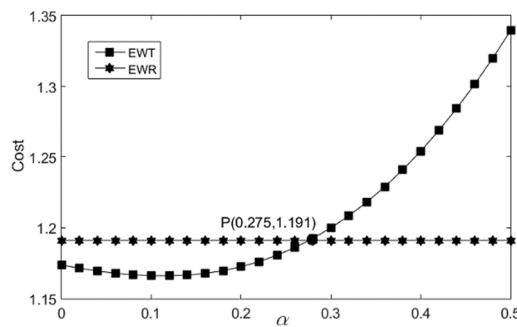


Fig. 5. The total costs of EWT and EWR regarding α .

According to the above mentioned results, we find that trade-in time has significant effect on the optimal decisions. We next explore the effect of this parameter on the retailer's optimal profit, and have the following finding.

Proposition 7. When $\alpha \leq (v - \delta)/(2c, \rho T)$, the retailer's optimal profit under EWT increases with α ; otherwise, it decreases with α .

Proposition 7 characterizes the effect of α on the retailer's optimal profit. The findings are intuitive, and the reasons are similar to those of Proposition 3(b). This further suggests that it is beneficial for a retailer with low warranty cost efficiency to shift to the trade-in transaction.

Since it is hard to theoretically examine the impacts of λ , ρ and T on the retailer's optimal profits under both EWR and EWT, we use three numerical examples based on the data used in

Proposition 2 to illustrate them. First, we set $c_h = 0.3$, $\beta = 0.5$, $\lambda = 1$ and 1.1, respectively; and further increase ρ from 0.1 to 0.9. Fig.6.1 and Fig.6.2 show that the retailer's optimal profit under either warranty is decreasing in ρ when ρ is relatively low, but increasing in ρ when ρ is relatively high. This is intuitive in that, when ρ is relatively low, as ρ increases, the warranty cost goes up, and thus the profit goes down. However, when ρ is sufficiently high, consumers purchasing the product will have more incentive to buy the corresponding extended warranty as well. This will lead to more extended warranty demand, and thus a higher profit.

Clearly, the retailer's optimal profit under EWT is no less than that under EWR, and the profit difference shrinks with ρ . In particular, when $\lambda = 1$ and $\rho \geq \bar{\rho} = 0.853$, $\lambda = 1.1$ and $\rho \geq \bar{\rho} = 0.711$, the retailer gains the same profit under both warranties. Corollary 2(a) partly indicates that, when ρ is relatively small, EWT outperforms EWR. In contrast, when ρ is sufficiently large, the advantage of EWT disappears, and thus is reduced to EWR. The reason is that, as ρ increases, the failure risk of trade-in new products also increases, which reduces consumers' willingness to trade-in their products.

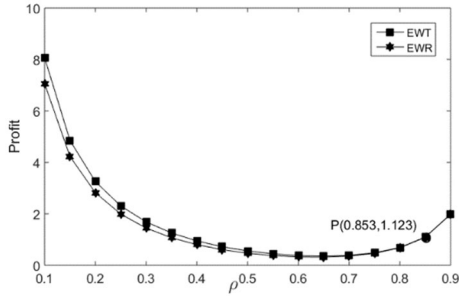


Fig. 6.1. $\lambda = 1$

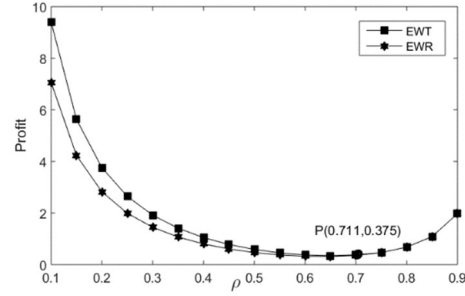


Fig. 6.2. $\lambda = 1.1$

Fig.6. The retailer's optimal profits under EWR and EWT regarding ρ

We next investigate the interactive impacts of T and λ on the retailer's optimal profit by setting $\lambda = 1, 1.1$ and 1.2, respectively; and varying T from 0.5 to 1. Typically, we set $c_h = 0.1$ and $c_r = 25$, while keep other parameters unchanged. The results are shown in Fig.7.

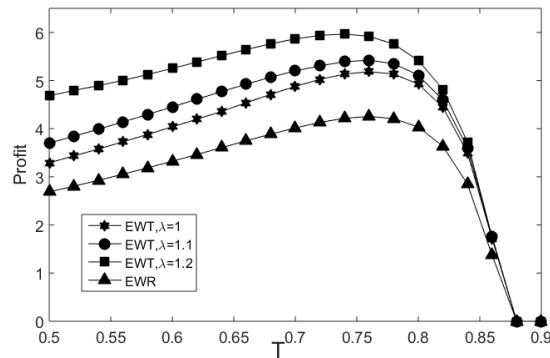


Fig.7. The retailer's optimal profit under EWR and EWT regarding T .

Fig.7 shows the interactive impacts of T and λ on the retailer's optimal profit. Specifically, the

retailer's optimal profit is concave in T regardless of λ . When T is relatively small, a larger warranty coverage will lead to more warrant sales, and thus more profit. Surprisingly, when T is sufficiently large, the interactive impact decreases, and the retailer's profit decreases with T . This is because, when T is sufficiently large, a larger warranty length helps boost warranty sales but also reduces consumers' willingness to conduct trade-in transactions. Meanwhile, such a long warranty length will also greatly increase warranty cost. In addition, Fig.7 also shows that, a higher λ always helps increase the retailer's optimal profit, since it can provide consumers with greater utility. However, when T is sufficiently large, there is no difference between providing an upgraded product and the same version of the original product due to a sufficiently low trade-in demand.

5. Extensions

In this section, we first extend the base model to consider the case where a manufacturer sells extended warranties and then the case where the upgraded product has different failure probabilities from the original product.

5.1 Who should provide the extended warranty: the retailer or the manufacturer?

In practice, extended warranties can also be offered by manufacturers such as Apple Inc. In this sub-section, we extend our models from a retailer to a manufacturer. In such a case, we consider that the manufacturer sells a product directly to consumers, and then determines whether to sell EWR or EWT. The main difference between the retailer's warranty and the manufacturer's warranty is that the manufacturer should undertake the warranty cost regarding the base warranty while the retailer does not. Therefore, all the consumer's utility, warranty demand and trade-in demand functions under a manufacturer's extended warranty are the same as those of the retailer's warranty.

Notably, we follow the assumptions that all consumers will purchase no more than one new product, and then will make a decision on whether to buy an extended warranty, i.e., EWR or EWT. If they buy EWT, they will determine whether to trade in their products. Note that, under the manufacturer's EWT, $c_{et} = c_m \rho (\alpha^2 T^2 - \omega^2) + c_m \rho \omega^2 = c_m \rho \alpha^2 T^2$ and the base warranty cost is $c_b = c_m \rho \omega^2$. Note that, parameter c_m refers to the manufacturer's warranty cost-efficiency coefficient. We also assume that the retailer's wholesale price c_0 in EWR-R and EWT-R will be no less than the base warranty cost undertaken by a manufacturer, i.e., $c_m \rho \omega^2 \leq c_0$; otherwise no manufacturer is willing to offer the product. Notably, the manufacturer's warranty cost-efficiency coefficient c_m is usually lower than that of the retailer in practice. This is because, in real world cases, without research and development, established production facilities and a significant amount of experience, retailers are unlikely to be more efficient in providing a warranty than manufacturers (Jiang and Zhang, 2011). Without loss of generality, we assume that $c_r \geq c_m$.

Denote by p_0^{mr} , p_e^{mr} , p_0^{mt} , p_e^{mt} and p_n^{mt} the new product selling price under EWR, EWR selling price, new product price under EWT, EWT selling price and trade-in price of the manufacturer, respectively; and π_{mr} and π_{mt} as the manufacturer's profit regarding EWR and EWT, respectively. These two profit functions are expressed as

$$\pi_{mr} = D_n^r(p_0^{mr} - c_b) + D_e^r(p_e^{mr} - c_e), \quad (9)$$

$$\pi_{mt} = D_n^t(p_0^{mt} - c_b) + (D_e^t - D_e^u)(p_e^{mt} - c_e) + D_e^u(p_e^{mt} - c_{et} + p_n^{mt} - c_h + s). \quad (10)$$

Note that, these two profit functions are similar to those of the retailers. In order to explicitly examine the optimal decisions and profits of the manufacturer, we confine our analysis to the case when $\lambda=1$ for simplicity, and the optimal decisions are then obtained, i.e., $p_e^{mr*} = \frac{1}{2}\rho(v(T-\omega) + C_{em})$

and $p_0^{mr*} = \frac{1}{2}(c_m\rho\omega^2 + V_p)$ under EWR, and $p_e^{mt*} = \frac{\rho(v(T-\omega) + C_{em})}{2} - \frac{\Delta\bar{c}_{h2m}}{3+\beta}$,

$p_0^{mt*} = \frac{(c_m\rho\omega^2 + V_p)}{2} + \frac{\Delta\bar{c}_{h2m}}{3+\beta}$ and $p_n^{mt*} = \frac{2\alpha T v - C_{em} - (T-\omega)v\rho}{2} - \frac{\Delta\bar{c}_{h2m}}{3+\beta}$ under EWT, respectively,

where $\Delta\bar{c}_{h2m} = s + \alpha T v - \alpha^2 c_m T^2 \rho + \frac{(1-\beta)(C_{em} - (T-\omega)\rho v)}{2} - c_h$, $V_p = v(1-\rho) + \rho\omega v$ and

$$C_{em} = c_m\rho(T-\omega)(T+\omega).$$

By examining the optimal decisions and profits under the manufacturer's extended warranties, we find that our major findings and insights remain unchanged. Nevertheless, there exist some impacts of c_m on the optimal trade-in price, and we have the following finding.

Proposition 8. p_n^{mt*} decreases with c_m when $\alpha \leq \frac{\sqrt{2(T^2 - \omega^2)}}{T}$, but increases with c_m otherwise.

Proposition 8 shows that, when the trade-in time parameter α is relatively small, p_n^{mt*} decreases with c_m . This is because, as c_m goes up, the manufacturer will undertake more warranty cost. In this case, the manufacturer has more incentive to reduce the trade-in price to entice more consumers to conduct trade-in transactions in order to reduce total cost. When α is relatively large, the manufacturer has already incurred heavy warranty cost. In such context, offering trade-in service cannot effectively reduce the warranty cost, and will incur extra cost from trade-in service. In this case, as c_m increases, the manufacturer has less incentive to reduce the trade-in price, and might increase the trade-in price instead.

Since extended warranties can be sold by a retailer or a manufacturer, who can offer a lower warranty selling price and trade-in price associated with EWT? The following conclusion will answer this question formally.

Proposition 9. Given that $c_r \geq c_m$, the relationships between the optimal prices under EWR and EWT

are characterized as follows:

(a) The optimal warranty selling prices under the manufacturer's EWR and EWT are always lower than those under the retailer's, respectively;

(b) When $c_r \leq c_m + \frac{c_0 - c_m \rho \omega^2}{\rho(T^2(2 - \alpha^2) - \omega^2)}$, the optimal trade-in price under the manufacturer's EWT is less than that under the retailer's.

(c) When $c_r \leq c_m + \frac{(1 - \beta)(c_0 - c_m \rho \omega^2)}{T^2(5 - 4\alpha^2 - \beta)\rho - (1 - \beta)\rho \omega^2}$, the optimal trade-in discount under the manufacturer's EWT is more than that under the retailer's.

Proposition 9(a) shows that, when the manufacturer has greater warranty cost-efficiency, the selling prices of EWR and EWT offered by the manufacturer are always lower than those of the retailer, respectively. The rationale is that the retailer may set a higher warranty price to cover a higher warranty cost. The condition $c_r \geq c_m$ in turn suggests that, the manufacturer's warranty selling price is generally lower than that of a retailer in practice. According to Business Week (Armstrong, 2004), electronics retailer CompUSA charges \$369.99 for 3-year traditional extended warranty on a Toshiba Satellite laptop computer, while Toshiba Corp. charges only \$199 for the warranty. This evidence directly supports this finding.

Proposition 9(b) shows that, if $c_r \leq c_m + \frac{c_0 - c_m \rho \omega^2}{\rho(T^2(2 - \alpha^2) - \omega^2)}$, the manufacturer will set a lower trade-in price than that of the retailer under EWT; otherwise, the manufacturer may set a higher trade-in price. As shown in Proposition 3(d), p_n^* decreases with the retailer's warranty cost-efficiency coefficient. In such a case, we can conclude that, the manufacturer will set a lower trade-in price than that of the retailer. Specifically, when $c_r \leq c_m + \frac{c_0 - c_m \rho \omega^2}{\rho(T^2(2 - \alpha^2) - \omega^2)}$, the manufacturer will set a lower trade-in price than that of the retailer, and vice versa.

Proposition 9(c) shows that, if $c_r \leq c_m + \frac{(1 - \beta)(c_0 - c_m \rho \omega^2)}{T^2(5 - 4\alpha^2 - \beta)\rho - (1 - \beta)\rho \omega^2}$, the manufacturer may offer a higher trade-in discount to consumers than the retailer. Otherwise, the retailer will provide a higher trade-in discount to consumers. This finding is similar to Proposition 5(a), that the higher the retailer's warranty cost-efficiency coefficient is, the greater the trade-in discount will be.

To better illustrate Proposition 9, we use an example based on the same data used earlier. In particular, we set $\lambda = 1$, $c_m = 14$, $c_h = 0.1$ and increase c_r from 10 to 20. The optimal warranty selling prices regarding EWR and EWT of both the manufacturer and the retailer are shown in Fig.8.1

and Fig.8.2, respectively; and the corresponding optimal trade-in prices and trade-in discounts are depicted in Fig.9.1 and Fig.9.2, respectively.

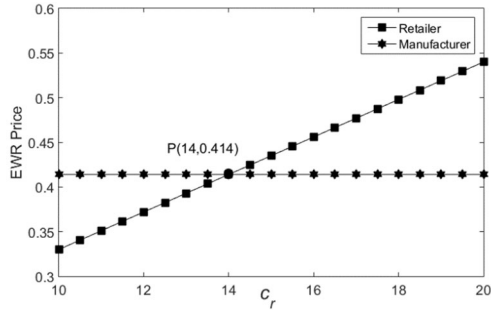


Fig.8.1 EWR selling prices of both providers.

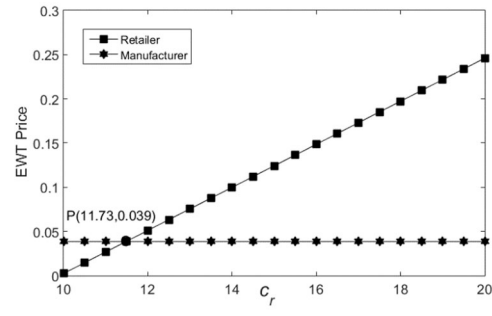


Fig.8.2 EWT selling prices of both providers.

Fig.8.1 shows that, when $c_r \geq c_m = 14$, the selling price of the retailer's EWR is higher than that of the manufacturer. Fig.8.2 shows that, when $c_r \geq c_m = 11.73$, the retailer's EWT selling price is higher than that of the manufacturer.

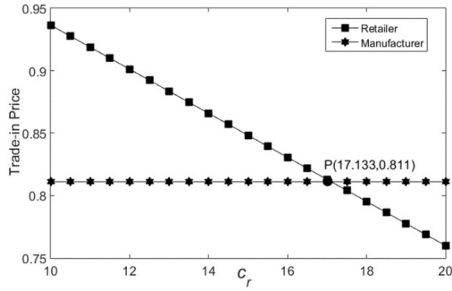


Fig.9.1. Trade-in prices of both providers

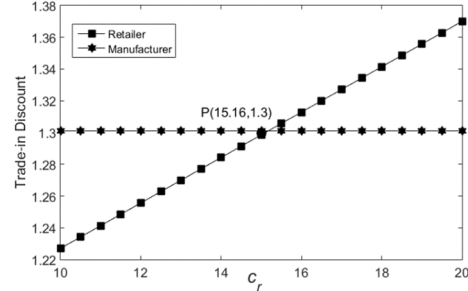


Fig.9.2. Trade-in discount of both providers

Fig.9.1 shows that, the manufacturer's trade-in price is less than that of the retailer when $c_r \leq c_m + \frac{c_0 - c_m \rho \omega^2}{\rho(T^2(2 - \alpha^2) - \omega^2)} = 17.133$, whereas is higher than that of the retailer otherwise. Fig.9.2

shows that, the manufacturer's trade-in discount is larger than that of the retailer when

$c_r \leq c_m + \frac{(1 - \beta)(c_0 - c_m \rho \omega^2)}{T^2(5 - 4\alpha^2 - \beta)\rho - (1 - \beta)\rho \omega^2} = 15.16$, whereas is lower than that of the retailer otherwise.

5.2 Impact of the product failure probability

In the base model, we assume that the failure probability of the new upgraded product is the same as the original product. In practice, the upgraded product may have improved quality and thus have a relatively lower failure probability (Xiong et al., 2016), or have some new, complex and even some unstable functions and thus may have a relatively higher failure probability. In this sub-section, we extend our analysis to further examine the effects of the upgraded product's failure probability on the retailer's warranty decisions and profit. To be specific, we assume that the failure probability of the

upgraded product is $\gamma\rho$, where $\gamma > 0$. In this case, the consumer's utility function of keeping the used products during the remaining lifetime (i.e., $[\alpha T, 1]$) is the same as that defined in Eq.(4), i.e., $u_{e,\gamma}^{tk} = u_e^{tk}$. If consumers choose to trade-in their used products, consumer utility function regarding trade-in service is derived as

$$u_{e,\gamma}'' = \lambda v(1 - \gamma\rho) + \lambda v\gamma\rho\omega - p_0 - p_e' - \theta\lambda^2 v^2 \gamma\rho(1 - \gamma\rho)(1 - \omega)(1 - \omega) - p_n'. \quad (11)$$

Then, the indifference point between conducting trade-in service or not can be obtained, i.e.,

$$\bar{\theta}_{3,\gamma} = \frac{v(-1 + T\alpha + \rho - T\rho + \lambda(1 - \gamma\rho(1 - \omega))) - p_n'}{v^2\rho(2T(1 - \rho) - T^2(1 - \rho) + \rho + \lambda^2\gamma(1 - \gamma\rho)(1 - \omega)^2 - 1)}. \quad (12)$$

derived from purchasing the new product and EWT is formulated as

$$u_{e,\gamma}' = v(1 - \rho) + v\rho T - \theta v^2\rho(1 - \rho)(1 - T)^2 - p_0' - p_e' + \beta \max\{u_{e,\gamma}'', 0\}. \quad (12)$$

Similar to the base model, the indifference point between purchasing both the new product and the

extended warranty and buying nothing is $\bar{\theta}_{1,\gamma} = \frac{v(1 - \rho) + v\rho T - p_0' - p_e'}{v^2\rho(1 - \rho)(1 - T)^2}$. Then, the indifference point

between purchasing EWT or not can be derived, i.e.,

$$\bar{\theta}_{2,\gamma} = \frac{p_e' + p_n'\beta + v(\beta(1 - T\alpha - \rho + T\rho + \lambda(1 - \gamma\rho(1 - \omega))) - \rho(T - \omega))}{v^2\rho(2T(1 - \beta - \rho + \beta\rho) + \beta(1 - \rho - \lambda^2\gamma(1 - \gamma\rho)(1 - \omega)^2) - (1 - \rho)(2 - \omega)\omega - T^2(1 - \beta)(1 - \rho)}.$$

The demand functions with respect to EWT, trade-in new product and the original product can be directly obtained, i.e., $D_{e,\gamma}' = \bar{\theta}_{1,\gamma} - \bar{\theta}_{2,\gamma}$, $D_{e,\gamma}'' = \bar{\theta}_{3,\gamma} - \bar{\theta}_{2,\gamma}$ and $D_{n,\gamma}' = \bar{\theta}_{1,\gamma}$, respectively.

According to the above demand functions, the retailer's profit is formulated as

$$\pi_{1,\gamma} = D_{n,\gamma}'(p_0' - c_0) + (D_{e,\gamma}' - D_{e,\gamma}'')(p_e' - c_e) + D_{e,\gamma}''(p_e' - c_{et} + p_n' - c_h + s - c_0). \quad (13)$$

We can easily obtain the optimal decisions by solving the retailer's problem. Unfortunately, it is difficult to theoretically explore the impact of γ and its interactions between other parameters on the retailer's warranty choice and profit, we use a numerical example to address these issues. We adopt the same data as those in Proposition 2 in this example. In particular, we set $c_h = 1.2$, $\lambda = 1.1$, respectively and let γ increase from 0.6 to 1.2. The results are depicted in Fig.10.1 and Fig.10.2, respectively.

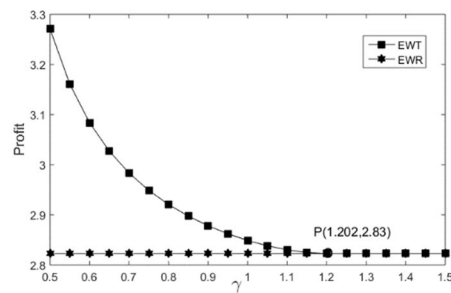
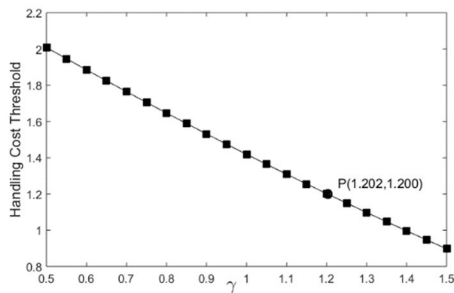


Fig.10.1. The handling cost threshold \bar{c}_{h2} regarding γ .

Fig.10.2 The retailer's optimal profit of EWT regarding γ .

Fig.10.1 and Fig.10.2 show that, the handling cost thresholds \bar{c}_{h2} and the retailer's profit decrease with γ . In general, when γ is relatively low, the upgraded product failure probability is relatively low, consumers may have more willingness to conduct trade-in transactions, and thus the retailer can obtain more profit from such trade-in service. In this case, the retailer is more willing to bear a high used product handling cost. As such, the condition $c_h \leq \bar{c}_{h2}$ as shown in Proposition 2(b) can easily hold, and thus the retailer will benefit more from providing EWT accordingly. In contrast, when γ is sufficiently high, the condition $c_h \leq \bar{c}_{h2}$ may not hold. This may lead the retailer to choose EWR instead. This can be confirmed by Fig.10.2 that, with the increase of γ , a relatively high failure probability of the upgraded product will reduce consumers' willingness to trade-in their products, which results in a reduction in the retailer's profit (e.g., when $\gamma \geq 1.202$, the profit of EWT is equivalent to that of EWR).

5. Conclusions

Retailers and manufacturers are increasingly selling extended warranties to obtain higher profits due to the decreasing profit margins of most durable products. In addition to traditional extended warranties (EWR) that offer free repair or replacement service in the presence of product failures, in recent years, retailers and manufacturers have launched a new type of extended warranty (EWT), under which an additional trade-in service is offered. EWT requires that the used products to be traded in work well, and consumers should pay for new products with the same version as the original product or an upgraded version through trade-in transactions. In such a circumstance, it is important for retailers and manufacturers to determine whether or not to offer EWT in the market, and to determine associated pricing decisions. To address such issues, we first consider a retailer who sells a typical durable product to consumers. The retailer may choose to sell EWR or EWT. Regardless of whether selling EWR or EWT, the retailer will set the selling prices for new product and the extended warranty. When selling EWT, the retailer will further determine the optimal trade-in price for new products through trade-in transactions. We develop theoretical models to examine the optimal warranty strategy and associated pricing decisions for the retailer. We then further consider the case where the extended warranties are offered by a manufacturer and the upgraded product has different failure probabilities. Some important findings and insights are summarized as follows.

- The optimal extended warranty choice depends on the unit used product handling cost. When the handling cost is relatively low, i.e., less than a specified threshold, the service provider is better off selling EWT; otherwise, it is interesting that, EWT is reduced to EWR. It is noteworthy that, the specified threshold is significantly affected by some important market parameters, e.g., used product residual value, trade-in time and product failure probability. Specifically, when the extended warranty coverage is sufficiently short, if the product failure probability is relatively low, the retailer is more

willing to provide EWT. In contrast, when the coverage is relatively long, the retailer always benefits from offering EWT if trade-in time is relatively early; otherwise, if the product failure probability is relatively low, the retailer will have more incentive to provide EWT. These findings importantly suggest that the provider can always choose a preferred extended warranty to sell according to market parameters.

- The optimal warranty selling price under EWT is never more than that under EWR. In particular, when unit used product handling cost is larger than a particular threshold, both warranties have the same selling price. Surprisingly, we find that, while the optimal EWR and EWT selling prices may increase with product failure probability, the optimal trade-in price may decrease in it. It is interesting that, the optimal trade-in discount is not always increasing or decreasing in the product failure probability.

- Trade-in time is a typically important market parameter, which affects the optimal decisions on EWT selling price, trade-in price and the optimal profit as well as total cost. Generally speaking, the service provider's optimal profit is concave in the trade-in time. In particular, when trade-in time is relatively early, total cost under EWT is lower than that under EWR, and the optimal profit increases with the time delay. This directly suggests that the service provider should set a suitable trade-in time during the warranty coverage for trade-in transactions.

- Whether a manufacturer or a retailer can offer a lower warranty selling price depends on their warranty cost-efficiencies. The manufacturer may offer a lower warranty selling price regardless of whether choosing to sell EWR or EWT, and may offer a lower trade-in price or trade-in discount under certain conditions, and so might the retailer.

- The failure probability of the upgraded product significantly affects consumers' willingness to conduct trade-in transactions. When the failure probability is relatively low, the retailer will benefit more from providing EWT; otherwise, EWT tends to be equivalent to EWR instead.

This study identifies some key findings that shed light on the optimal decisions on whether a retailer or a manufacturer should sell a traditional extended warranty or a typical extended warranty with trade-in service and the associated pricing strategies. Nevertheless, there are some related issues that should be further examined. First, we only consider one retailer (or manufacturer) in this study. When two or more service providers are considered, competition occurs, which may lead to different results. Second, our work only considers that a retailer (or manufacturer) offers the warranty service rather than in a supply chain setting. In a supply chain, the warranty service can be offered by a manufacturer, a retailer or both, or even provided by a manufacturer or retailer but sold through a retailer. This leads to the following questions. Whose offering can lead to a higher profit for the supply chain and both players? How to determine the optimal decisions on warranty selling prices and associated trade-in prices? These issues are related to our work, but also different from our work. Therefore, these issues are left as interesting topics for future research.

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