A potential-flow model of viscous dissipation for the oscillating wave surge converter

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Summary A mathematical model of an oscillating wave surge converter is developed to study the effect that viscous dissipation has on the behaviour of the device. Recent theoretical and experimental testing have suggested that the standard treatment of viscous drag (e.g., Morison's equation) may not be suitable when the effects of diffraction dominate the wave torque on the device. In this paper, a new model of viscous dissipation is presented and explored within the framework of linear potential flow theory, and application of Green's theorem yields a hypersingular integral equation for the velocity visco-potential in the fluid domain. The hydrodynamic coefficients in the device's equation of motion are then calculated, and used to examine the effect of dissipation on the device's performance. A special focus is given to the effects of dissipation on the performance of a device that is tuned to resonate with the incoming waves.

INTRODUCTION

The oscillating wave surge converter (OWSC) belongs to a family of wave energy converters (WECs) known as 'flap-type' WECs [6]. In its simplest form, the OWSC comprises a buoyant flap that is hinged to the seabed, and typically operates in the nearshore environment where its pitching motion couples with the surge component of the incident waves [9]. Recent theoretical and experimental studies have shown that OWSCs can achieve high levels of power capture in commonly occurring seas [9]. This has prompted researchers to develop new mathematical models of the OWSC in order to better understand its hydrodynamic behaviour. To date, considerable understanding has been achieved primarily through the use of semi-analytical models to examine the hydrodynamic performance of the OWSC [4, 5], an array of OWSCs [3], and a farm of OWSCs [7].

In each of these studies, the fluid is deemed to be inviscid, and hence the effects of viscous dissipation are neglected. However, recent experimental wave tank tests and computational fluid dynamics simulations have found that flow separation occurs at the flap's tips [8]. In this paper, we assess the effect that viscous dissipation has on an OWSC by modifying the semi-analytical theory of [4] to include the effects of viscous dissipation near the flap's edge. We achieve this by applying an effective pressure discharge in the vicinity of the flap's tips. The equation of motion of the flap is solved in the frequency domain, and the solution to which is used to quantify the effect of viscous dissipation on the device's performance.

MATHEMATICAL MODEL

Consider an OWSC in an open ocean of constant water depth h', where the prime indicates a quantity with a physical dimension. The OWSC is represented as a buoyant box-shaped flap of width w' and thickness $2a'$, hinged at a depth d' on a rigid platform of height $h' - d'$ as shown in figure 1. Monochromatic waves of amplitude A'_I and period T' are normally incident upon the flap, which set the device oscillating about its hinge line. Let $\theta'(t')$ be the pitching amplitude of the device and let t' denote time. We define the reference system of coordinates $O'(x', y', z')$ with x' pointing in the opposite direction to the incoming waves, let the y'-axis lie along the width of the device and the z'-axis points upwards from the still water level. The origin O' is located in the middle of the device at the still water level.

The fluid is deemed to be inviscid and incompressible and the flow irrotational. Hence, there exists a scalar potential $\Phi'(x', y', z', t')$ for the velocity field $\boldsymbol{v}' = \nabla'\Phi'$ that satisfies the following equations (and invoking $2a' \ll w'$ [4, 5, 7]):

$$\nabla'^2\Phi' = 0, \quad (x', y', z') \in V', \tag{1a}$$

$$\Phi'_{,t't'} + g\Phi'_{,z'} = 0 \text{ at } z' = 0 \quad \text{and} \quad \Phi'_{,z'} = 0 \text{ at } z' = -h', \tag{1b}$$

$$\Phi'_{,x'} = -\theta'_{,t'}(t')(z' + d') \,\text{Heaviside}(z' + d'), \quad x' = \pm 0, \quad |y'| < \tfrac{1}{2}w', \tag{1c}$$

where V' is the fluid domain, g the acceleration due to gravity, and we have assumed that the behaviour of the system is linear [4] – a subscripted comma indicates differentiation. In order to incorporate the dissipative effects due to viscosity, we introduce a 'dissipative surface' $\mathcal{D}'$ close to the flap's edge, as shown in figure 1(b). Across $\mathcal{D}'$, we impose a pressure discharge $\Delta P'$, which is a function of the local velocity component ($v'_n = \Phi'_{,n}$) along the outward unit normal vector $\boldsymbol{n}$ to $\mathcal{D}'$

$$\Delta P' = f\left(\Phi'_{,n}\right), \quad x = \pm 0, \quad \tfrac{1}{2}w' < |y'| < \tfrac{1}{2}\left(w' + \ell'\right); \quad \Delta P'(y', z', t') = P'(-0, y', z', t') - P'(+0, y', z', t'), \tag{2}$$

where P' is the hydrodynamic pressure in the fluid. The relationship (2) represents the energy loss due to vortex shedding near the flap's edge [1], previously reported in simulations and laboratory tests of the OWSC [8]. The function f typically takes the form of a linear [1] or quadratic function [2] of the flow velocity. We consider a linear law, with $\bar{\epsilon}$ the linear coefficient measuring the strength of viscous dissipation; setting $\bar{\epsilon} = 0$ gives equivalence to existing inviscid models of the OWSC [5].

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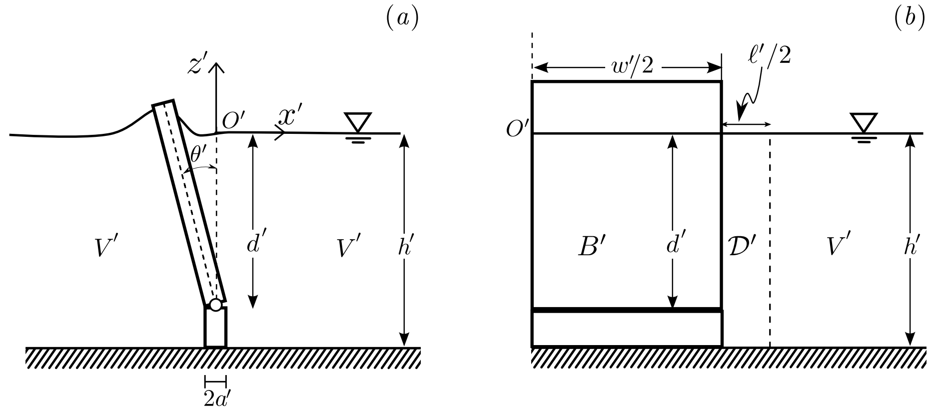

Figure 1: The geometry of the surface-piercing OWSC with dissipative surface $\mathcal{D}'$: (a) section (b) front view.

THE RESULTS

Using the model formulated in [5], we find that the capture factor (the ratio of captured to available wave power) curve contains a spike near the flap's resonant period; however, including a small amount of dissipation removes this spike. Figure 2 reveals the effect that increasing dissipation has on the resonant peak ($T' \approx 9$ s). We find that increasing $\bar{\epsilon}$ to 0.01 is sufficient to reduce the resonant peak to below the level of the peak corresponding to maximum exciting torque ($T' \approx 2.2$ s).

The capture factor curve of an OWSC contains an unphysical spike when standard inviscid linear potential theory is used. We show that the inclusion of dissipation in the form of a pressure discharge in the vicinity of the flap's tips is sufficient to eliminate this spurious behaviour, and give a more realistic prediction of the hydrodynamic performance of the OWSC.

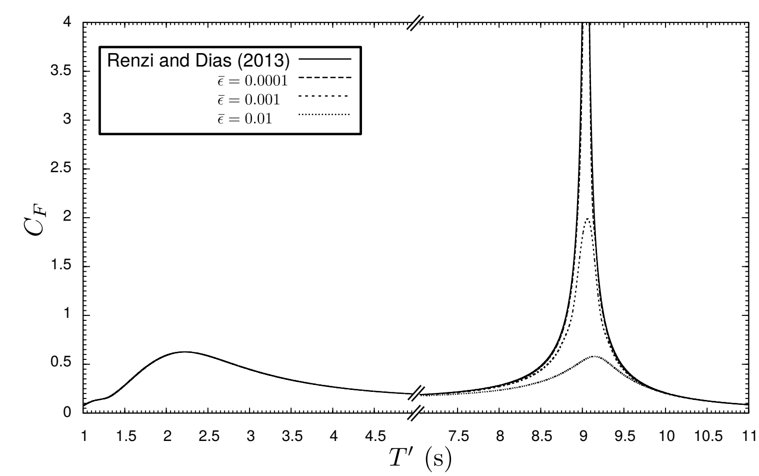

Figure 2: The influence of dissipation on the capture factor of an OWSC with spike at the resonant wave period at $T' \approx 9$ s.

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