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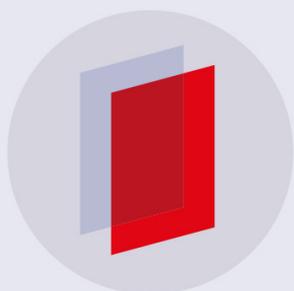
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Implementation of problem-based learning in geometry lessons

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Abstract. The aim of this study is twofold. Firstly, it aims to examine the effects of the Problem-Based Learning (PBL) approach on students' performance in the learning of geometry. Secondly, it seeks to gain insights from the students regarding the implementation of PBL in geometry lessons. The participants were 22 students from one Year 10 class in a co-educational secondary school in Brunei Darussalam. A mixed method design was employed with data collected from the pre-, post- and retention tests, and interviews. The findings from this study revealed positive influences on students' performance in learning geometry as gain and retention of knowledge was observed. Meanwhile, mixed responses from the interviews implied that in terms of 1) learning attitudes, students favoured the idea of independent learning but some critiqued that the process of PBL might be time-consuming; 2) learning difficulties, some students struggled in assimilating information leading to poor decision-making; and 3) knowledge and skills, some students believed to have nurtured some skills such as communication and research skills.

1. Introduction

The development of Problem-Based Learning (PBL) started at McMaster University in Canada in the 1960s to facilitate medical students in their learning [1]. PBL is generally described as a student-centred learning environment where students construct their own knowledge and work collaboratively to solve problems, which drive the learning. The teacher facilitates learning with him or her acknowledging the students' efforts, ideas and prior knowledge [2-6]. In PBL, there are expectations for students to be engaged in collaborative activities to solve problems [7]. PBL has been widely adapted in many research studies worldwide and invited discussions among educators, psychologists and researchers. According to Barrows and Tamblyn [1], the fundamental aspect of PBL is based on social constructivism. Since its implementation, several PBL models for student-centred, problem-based, collaborative learning have been developed widely. Considering much empirical evidence showing the advantages of PBL over traditional teacher-centred teaching, there has been adaptation phenomenon of PBL across many levels and subject areas.

1.1. The benefits of PBL

PBL as a student-centred learning strategy has been implemented and developed in many different educational levels [8]. One of which relates to the effectiveness of PBL approach in learning,



specifically in the increase of information or knowledge retention [9]. In comparison to traditional teaching methods, PBL allows students to work collaboratively and construct their knowledge through social interaction [10-12]. Group composition is one of the essential features in PBL that may affect learning outcomes and motivation [13]. Small group structure in PBL lesson helps distribute the cognitive tasks among group members [14]. It was also suggested that small group discussions in PBL lessons improves problem solving and higher order thinking, and promote shared knowledge construction [15]. According to Hmelo-Silver [16], critical reflection can provide students with feedback that can be utilised to make improvements during the collaboration process. It was also noted that if the students struggle reflecting on their own learning hence a teacher would be required to facilitate the students in the reflection process.

Previous studies have highlighted the benefits of PBL in developing the students' mathematical knowledge [17-20]. In a recent report by Botty et al. [19], they found improvements on students' performances in learning mathematics based on their pre- and post-test results. This shows that PBL implementation could positively impact the students' progress in the areas relating to real-life and syllabus oriented knowledge. PBL could also promote the learners' gain in the content of the subject and in solving real-life problems [3]. Furthermore, a study on PBL approach in developing Taiwanese students' understanding of fractions also observed improvements in students' learning performance, which implies that the PBL intervention may be effective in developing students' understanding of fractions [21]. In learning using PBL, students were more engaged in learning and had positive attitude towards their learning [22]. With these PBL strengths, it has been used in multiple domains that include mathematics education [23]. Schettino [4] conducted a study based on her strong belief in PBL curriculum and observed higher learning enthusiasm as students were not restricted to create their own solutions to a problem, and also through collaborative works, they get to exchange ideas and feedback which further improves the quality of learning. Furthermore, the use of PBL could be more effective with lower ability students [24].

1.2. The learning of geometry

In geometry lesson, the teachers tend to explain to students the properties associated with geometrical shapes and the properties, and subsequently, requiring the students to attempt the given exercises to indicate whether they have understood the topic or simply responding from memorising the facts [25]. There were few made attempts in encouraging the students to explain their reasoning and to make connections logically. Developing the students' thinking and geometrical senses is important and they will require this by possessing a solid understanding of the facts in geometry. This problem can be tackled by varying teaching approaches such as PBL that can support students in making the necessary connections using the various depictions of geometrical ideas and the interdisciplinary concepts in other area of mathematics [4, 26]. This is to ensure that students gain a full understanding of the concepts and the processes within and not by just learning the rules. This will enable students to retain knowledge and skills and provide confidence when solving new mathematical problems.

2. Methodology

This present study utilises an action research framework involving mixed method approaches of data gathering. An action research approach, using prominent stages such as problem identification, action planning, implementation, evaluation, and reflection, is well suited to the need of the study as it enables a systematic collection of data that provides the basis for social change [27] to seek improvement on students' learning. Additionally, all the data collected were triangulated [28] to provide a better idea on the effects of PBL on students' performance in learning geometry and their perceptions towards the PBL approach.

The aim of this study is twofold; firstly, it aims to examine the effects of PBL approach on students' performance in the learning of geometry, and secondly, it seeks to gain insights from the students regarding the implementation of PBL in geometry lessons. This study was conducted in a co-educational secondary school situated in Bandar Seri Begawan, the capital city of Brunei Darussalam.

The sample consisted of 22 students from a Year 10 class. According to Mason [29], a sample size of 20 to 30 is feasible in studies involving action research. The sampled students were collected by means of random sampling. All the relevant permissions from the school, parents and students were appropriately obtained before the start of the study.

2.1. Design of intervention

The topic of the intervention lessons (PBL lessons) was on symmetrical properties of circles, which was included in the Year 10 geometry syllabus in Brunei. The PBL lessons were conducted over a period of two weeks, and met three times a week. The first author, who implemented all the PBL lessons during the course of the intervention, also designed the PBL lessons. The PBL model by Lee and Bae [30] was adapted to design the PBL lessons (see Figure 1).

<p>Lesson 1: Introduction, Understand the Problem, and Searching for Information (on-going)</p> <ul style="list-style-type: none"> → Students are introduced to understand and analyse the given problem. → Students make inquires and perform searches individually or collaboratively to gain understanding of the problem. → Facilitator monitors students' progress and make sure that they are on the right track. <p>Lesson 2: Construct and Gather Solution</p> <ul style="list-style-type: none"> → Students gather all the necessary information, and discuss to produce a draft of their possible solutions to solve the given problem. <p>Lesson 3: Presentation and Reflection</p> <ul style="list-style-type: none"> → Students prepare for a 5-7 minutes presentation to share their findings in front of the whole class. → Facilitator checks that students have completed their work and asks students additional questions upon closure of PBL session.
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Figure 1. PBL lesson intervention design.

<p style="text-align: center;"><u>How to Fix a Broken Piece Ceramic Plate?</u></p> <p>During a geography field trip, Sarah found a broken piece of an antique ceramic plate and she strongly believes that the plate is in a circular shape. In order to make a replica of the entire plate for her geography project, Sarah knows that she needs to find the radius of the broken plate but she has no idea how to find it. Can you and your team help Sarah identify the step-by-step ways on how to find the radius of the broken plate?</p> <p>Your task is to come up with convincing strategies (step-by-step ways) on how to find the radius of the broken plate.</p> <p>As a reminder, you are not required to find the exact value (length of the radius) for this task. You are expected to do a presentation to discuss your findings with justifications.</p>
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Figure 2. The PBL problem.

The PBL intervention allowed students to work in groups of 4 or 5 students in solving the presented PBL problem [14]. There were 5 groups of students in this study and each group had to nominate a leader, a writer and a presenter to ensure that the tasks were evenly distributed among the group members. A PBL Activity Booklet was given to each group. This booklet consisted of a cover page, the PBL model [30], a designed problem, a PBL Facts List and blank A4 papers to record the findings. The PBL problem (see Figure 2) given during the lessons presented students with real-life situation and had typically not been covered in class. Specifically, the PBL problem aims to allow students to solve it in several manners. As recommended by Li [31], the content was carefully constructed as problem. Furthermore, the PBL problem was designed based on three important aspects of a good PBL problem, which are ill structured, contextualised and related to real-life situations [32].

The students were encouraged to perform active interactions within their group members where students explained their reasoning when solving the problem. Wandering facilitation model was performed as a facilitation strategy according to Hmelo-Silver [16] to successfully facilitate multiple groups. Memo pads were also given to each group so that students could record all their queries to be addressed later during facilitation process. Once the students understood the context of the problem, they were asked to identify the statement of the problem such as, what information is given, and what information is needed. These were then recorded in the adapted PBL Facts List. The searching for information phase took a full 30-minute lesson in which students searched for relevant resources in finding the solutions. They were also to access and assess multiple resources such as textbook and notebook. This was carried out as an on-going process where students could perform this task at home after school. In the next lesson, the students worked collaboratively to analyse all the gained information and produce a draft on their approach and methods in solving the problem. At this stage, the students also evaluated their group members' findings and decided the best possible approach and methods to solve the given problem. The presentation was conducted when all group members had completed their proposal of findings. A presenter was appointed from each group to present their group's proposal for about 5 to 7 minutes. A reflection sheet was also given to each student to evaluate his or her peer's overall presentation. A whole-class discussion was conducted after each presentation to allow interactions in giving constructive feedback followed by a wrapping up session [19]. The entire PBL lessons were video-recorded for analysis and to examine the process of PBL implementation in the classroom.

2.2. Data collection instruments and analysis

2.2.1. Pre-test, post-test and retention test. The purpose of the pre- and post-tests were to examine whether there were any differences observed in the students' academic achievement after the conduct of the PBL lessons. The post-test was given to the students about a week after the pre-test was administered. Following a gap of about more than four weeks, the same students were again given the retention test questions. The main aim of the retention test was to examine how much knowledge from the intervention lessons the students have retained. There was 7 items in all the three test instruments. The test items consisted of questions that examined the students' prior-knowledge on basic geometrical terms of a circle, their geometrical representation skills and procedural skills in finding unknown angles and sides in a circle. Scores were allocated for each question that weighted 11 marks in total, and the time given was 25 minutes to complete the test.

To validate the instruments, two experienced mathematics teachers with more than eight years of teaching experiences were invited to review the test questions and face validation was appropriately done. A pilot study was also conducted which involved a different set of students at the same school. This was done to approximate the length of time required to answering the tests and the appropriateness of the questions with the students' ability. The reliability of the test items was measured using Cronbach Alpha and it yielded a correlation coefficient of 0.628, which suggested acceptable reliability [33]. After the pilot study, the test questions were revised and improved in order to fine-tune the test instrument.

The number of pre-tests collected was 21 out of 22 students; hence one data was missing due to an absentee. However, the number of post-tests collected was 22 with no missing data. In order to do comparison, the post-test from that particular student who was absent during the pre-test had to be discarded. Hence the number of each pre-tests and post-tests was left with 21. As for the retention test, there were only 15 students who took the test. To examine knowledge retained, scores from the retention test were compared with the scores from the post-test. Therefore, the post-tests from 15 students who were present during the retention test were only used for comparison and analysis. For each correct answer, 1 mark was awarded and for each wrong answer or incomplete answer, it was awarded 0 mark. However, there was a case where the student got the correct answer but the working was incorrect. This was given 0 mark because the student portrayed a misconception. The difference

scores between the pre- and post-tests, and the differences between the post- and retention tests were calculated using the SPSS statistical software.

2.2.2. Semi-structured interviews. Generally, the questions in semi-structured interviews were aimed to gain students' perspectives that analysed their understanding, learning attitudes, and learning difficulties [34]. There were three individual interviews and three group interviews with 5, 5 and 2 students in each group. Each interview lasted for about less than 10 minutes. All the interviews were conducted with confidentiality and appropriate interview etiquettes. In reporting the results, pseudonyms were used to replace the students' names.

All the interviews were audio-recorded and transcribed for analysis. The analysis of the video recorded lessons was conducted using the seven steps process [35] to support and gain a sense of the students' perceptions on PBL experience obtained during the interviews. The video was viewed and some prominent excerpts of interactions during the lessons were transcribed. The students' sampled work from the PBL facts list were also examined to understand the extent of group efforts and participation and how well they organised their work in a problem-solving task.

3. Results

3.1. The effects of the PBL approach on students' performance in the learning of geometry

Assumptions of independence and normality were examined using SPSS statistical software [36] as illustrated in Table 1. It shows that the data was not normal ($p < 0.05$) hence the assumptions of normality was violated. Thus, non-parametric tests were utilised using the Wilcoxon Signed Rank Test [37] to examine whether there were statistically significant differences in the mean scores between the pre- and post-tests, and between the post- and retention tests.

Table 1. Test of normality for the pre- and post-test scores.

	Shapiro-Wilk		
	Statistic	df	Sig (<i>p</i>)
Difference	.719	21	.000

Following the comparisons shown in Table 2, there was a slight increase in the average mean scores of the students in the post-test. Similarly, an increase was observed during the retention test.

Table 2. Mean scores for the pre-, post- and retention tests.

	Pre-test (<i>N</i> =21)	Post-test (<i>N</i> =21)	Retention test (<i>N</i> =15)
Average mean scores	8.81	9.19	10.6

Further analysis using Wilcoxon Signed Ranks Test was done to examine the significant difference of mean scores between the pre- and post-tests since assumption of normality was violated. Table 3 shows that after the intervention, four students performed worse while five students showed improvements, and 12 students did not show any changes in academic performance. The analysis did not confirm any significant differences between the pre- and post-tests.

Table 3. Analysis using Wilcoxon Signed Ranks test (*N*=21).

	Negative ranks	Positive ranks	Ties	Sig. (<i>p</i>)
Post-test – Pre-test	4	5	12	0.298

Results from the Wilcoxon test (see Table 4) shows that during the retention test, none of the students have performed badly. While seven students performed better and eight maintained their scores. Moreover, the test revealed a statistically significant difference in the mean scores between the post-test and retention test ($z = 2.43$, $p = 0.015$), with a medium effect size ($r = .44$). According to Pallant [36], for Wilcoxon test, the most appropriate method to calculate the effect size is by dividing the critical- z value by the square root of N , such that N is the number of samples over the two time points $r = \frac{z}{\sqrt{N}}$.

Table 4. Analysis using Wilcoxon Signed Ranks test ($N=15$)

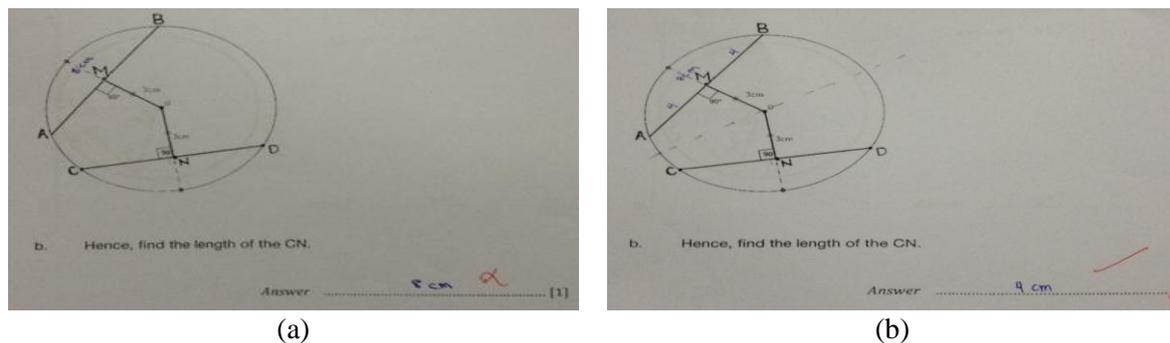
	Negative Ranks	Positive Ranks	Ties	Sig. (p)
Retention test – Post-test	0	7	8	0.015

Among the seven questions in the test instrument, Questions 1 and 2(a) examined the students' ability to identify the line of symmetry and the key terms such as diameter, radius, chord and segment (major and minor), Questions 2(b) and 5(a) were concerned about the students' ability to recognise the geometric properties of circles, and Questions 3, 4 and 5(b) were associated with the students' geometrical procedural skills in calculating unknown sides and angles. The percentages of correct responses in each question were compared as illustrated in Table 5.

Table 5. Comparisons of percentages of correct responses in the pre- and post-tests.

Areas	Questions	Percentage of correct responses (%)	
		Pre-test	Post-test
Prior knowledge	1 and 2(a)	77.4	78.6
Geometrical representations	2(b) and 5(a)	85.7	95.3
Procedural skills	3, 4 and 5(b)	59.5	67.5

It is worth noting that all questions observed an increase in correct percentage response: 95.3% of the students correctly responded to Questions 2(b) and 5(a) in the post-test, a 9.6% increase when compared with the results in the pre-test. This indicated that the students had improved in their geometrical representation ability. Examining the students' answers for Question 2(b), during the pre-test, majority of the students did not provide any answers and for those who provided answers, the responses were incorrect. However, there were more students with correct answers in the post-test for Question 2(b). The increase in the mean percentage of Question 2(b) appeared to be highly associated with students' ability to recognise the geometrical representation particularly on the first symmetrical property of a circle which is equal chords are equidistant from the centre. Figure 3(a) shows a sample of a student's work on Question 2(b) who incorrectly solved this question in the pre-test. Initially, she thought that the length of CN was equal to the length of AB, which is 8 cm. However, in the post-test she successfully realised that the length of CN was actually halves the length of AB as shown on the correct labeling in Figure 3(b).



(a) (b)
Figure 3. Samples of a student's work for Question 2(b).

In this present study, the intervention had a positive impact on student's performance in terms of acquisition of knowledge and knowledge retention in learning geometry. A significant improvement in students' achievement was seen during the retention test, which implies that PBL intervention could have a positive influence on students' performance particularly in developing long-term knowledge retention in learning geometry. This finding supports a review reported by Norman and Schmidt [38] as they noted that an in-depth process of PBL information seems to promote greater retention knowledge if done over a substantial period. Similarly, Robbs and Meredith [9] associated effectiveness of PBL with greater retention of information.

The students collaboratively worked in small groups of four or five, thus allowing for a fair distribution of task and participation during the activity. The students shared facts and thoughts through discussions and responses to questions. Based on their conversations during the discussion, it was seen that sometimes they repeatedly discuss similar ideas. The following excerpts exemplify this example, where the students discussed about diameter and the shape of circle repetitively. This subsequent recall strengthens understanding of the concept, making it easier to remember.

[Start of conversation]

Fay: *That's a sector... How do you find a diameter from a sector?*

Des: *What's the point of finding the missing piece?*

Aldo: *It does not even look like a circle [laughs]*

Des: *A quar...a quarter of a circle.*

Fay: *It's a plate! Not a quarter of circle.*

Des: *Oh! A quarter of a plate.*

[Middle of conversation]

Aldo: *What do we need to know?*

Kayla: *The diameter is... from one end to one end right?*

Des: *Yes.*

Aldo: *Diameter... yeah.*

Kayla: *So for me I think we need to find the diameter.*

Aldo: *Why do keep saying...*

3.2. Students' insights on the use of PBL in geometry lessons

During the interviews, students shared their personal insights on PBL experience in terms of learning attitudes, learning difficulties, and knowledge and skills. Data from video-recordings of classroom observations and the teacher's notes were also used to support the findings obtained from the interviews with the students.

3.2.1. Learning attitudes. The students were asked if they preferred the PBL method or the traditional method in general in order to seek their preferences of the two learning styles. From the students' interviews, there were mixed responses observed. Some students favoured PBL generally because they believed that PBL would provide them with some valuable skills and also greater level of knowledge acquisition. For instance, as shown in the following excerpt, Starla considered that the PBL method was supposedly a relevant learning approach that she should practice because it would give her the opportunity to take control of her own learning. She felt that it would be necessary gain such skills that will help her in future learning.

Well...maybe for this time, I think the...method we do just now is supposed to be the method we should do because its more...us doing the work, so we know what to do, because in the other time not everyone will

tell us what to do, we need to be independent on finding our own information. So starting from now I guess... it is better so that we know how we will do it for the other time... so yeah. (Starla)

Similarly, Samsun claimed that he learned better in PBL mainly because he gained better understanding of the content knowledge as he searched the information by himself. He also described that in the traditional learning, he mostly listened to the teacher's explanation and sometimes he merely understood the knowledge if he was not fully attentive to the teacher. The following excerpt illustrates his perception of PBL.

Umm...I think this method (PBL) is better because... like from the traditional method you just listen and then sometimes you are like... when you don't give a 100% attention... you just like... not really understand. But when you do it yourself, you understand more because you are the one that searching for the information... so its not spoon-fed or anything. (Samsun)

Based on the findings, the students felt that PBL allowed them to be more independent in their learning. For Starla, PBL method would give her the opportunity to be more independent in her learning, and she also believed that the skills to search for information effectively would foster a greater learning competency in her future learning experiences. Relatively, Samsun felt more attentive in PBL because it has given him a deeper understanding as he searched and learnt the information by himself. These findings support the result by Huang [39] that reported 70.6% of the students had positive perceptions by learning independently in PBL.

In contrast, some students preferred the traditional approach to PBL because of two aspects, direct instructions and shorter time duration. For example, Wani and Gaby claimed that it was much easier for them to understand the concept (refer to the following excerpts).

Wani: Traditional... it's umm... a more easier to understand and you can solve the problems... like different situations... [one concept]

Gaby: We understand more in traditional... yeah.

Laila: Well... the... the old one, because this one may take too long.

This may imply that students gained better understanding if the teacher directly provided the answers or explanations. Having said that, this study did not obtain further explanation from the students as evidence to support the assumption. This may neither ensure a much deeper understanding of the knowledge content nor the development of the students' thought. It is vital for the students to acquire in-depth understanding in the subject content so as to master the necessary skills to apply knowledge to other situations in real-life context. Here, it is the responsibility of the teacher who should try to provide authentic and active learning environment that enhance deeper understanding of content knowledge and consequently develop other valuable lifelong learning skills [40].

In terms of implementation, as shown above, Laila argued that too much time might be needed for PBL as compared to the traditional approach. As stated by Mansor et al. [41], large amount of time spent in PBL may be observed particularly during teacher supervision and giving advices to groups. In this present study however, based on the teacher's notes and classroom observations, wandering facilitation strategy [16] was performed, and this enabled the teacher to facilitate the students effectively. Hence, it may be inferred that there might be other reasons that contributed to the student's concerns in terms of time allocation in PBL. Perhaps this warrants further investigation in the future.

3.2.2. Learning difficulties. From the interviews, it seemed that some students encountered difficulties during the PBL process. Some of the issues given by students from the interviews were, difficulty in processing information and struggling to generate ideas at the beginning of the activity. For instance, Jenny and Laila who were both in the same group claimed that there was a lot of information as each group members shared different ideas. So they struggled to process all the information that was acquired during the discussions in order to select the relevant and accurate information. Jenny also claimed that this created some disagreements among the group members. The following excerpts illustrate these examples:

There is one problem... during the activity... in the group work... there is five of us in the group... and each of us have different ideas of how to explain... the activity at that time. There is so much information... so we don't know which one is correct... so we always arguing about it. (Jenny)

And it was a bit hard during the discussion... because everybody has their own... ideas... so it's hard to choose which one. (Laila)

Referring to the students' conversations during the group discussions prompts such as "why" or "can you explain further" were barely observed during the conversations. The students were mostly extracting their prior knowledge and ideas without providing any profound reasons. The following excerpts exemplify this example.

Nancy: If you want to find it... We need to find the "thing" and then we times it by four. [Silent pause]

Fez: That's nice!

Kayla: For me for the next question... what do we need to know... that ... [pointing to the diagram]

This may imply that most students showed poor process skills such as reasoning skills, critical thinking skills, and argument skills that could be one of the causes of the students' difficulties. Moreover, referring to the previous excerpts, Fez purely accepted Nancy's suggestion based on its logic nature without a deep notion of the idea. This shows that the students lack the ability to make judgment (evaluation), which could also be the reason why the students argued during the discussion. Nevertheless, argument skills can be used for learning when we treat reasoning as a process of argumentation [42]. Moreover, a study by Nussbaum [43] reported that argumentation might promote the retention of course content, which supported the quantitative finding in this present study. Importantly, critical thinking and argument skills are the significant elements in decision-making [44]. Thus, poor thinking skills and argument skills may lead to bad decision-making. Therefore, it is necessary for the teacher to put adequate emphasis on the students in developing and practicing relevant process skills in order to maximise the students' learning. Then again, Zabit [45] argued that it is rather challenging to develop the students' such skills especially when the teachers may not have developed the skills fully themselves. Consequently, it is also important that teachers should also constantly try to develop these important skills to be able to give full support in the student's learning.

The second learning difficulty was that some students struggled to generate ideas at the beginning of the activity. As shown in the excerpt below, Samsun noted that he did not have any ideas on how to start solving the problem. From the teacher's notes, at first, the student was seen to be timid.

Samsun: Well... umm... at the start of the... the... problem, we were clueless on how we are going to start.

Other than that... umm... we have no problem during the discussion... a lot of ideas but it was okay.

It is also worth noting that the students had no previous experience with PBL. So, it seems that the student needed more guidance and greater facilitation from the teacher even though the full implementation of the PBL lessons by the teacher was clearly conducted. Initially, the teacher let them discuss among the group to understand the problem first before she assisted the students. This could be the cause for the difficulty. Moreover, the teacher also noted that initially, none of the students were observed using their notes or textbooks to search for information. Thus, this could be another cause for the difficulty. The teacher managed to assist the students by asking open-ended questions such as "what do you understand about the problem?" and she also used probing questions to encourage students' active participation in discussion. Furthermore, she encouraged her students in searching for answers with the use of textbooks and notes. In doing so, the teacher observed gradual group collaboration. The classroom environment steadily changed as students started talking about the given problem with each other among their respective groups. The teacher also noted that the students were seen making some progress as they managed to record some of their thoughts on the columns provided in the PBL Fact List. Appropriate inquiry techniques are crucial in an inquiry-based classroom especially at the early stages [46]. Expertise in facilitation skills is essential in attaining effective PBL learning. Hence, proper training workshops may be required to develop a clear understanding of effective facilitation strategies such as role-playing, in order to facilitate a better PBL experience.

3.2.3. Knowledge and skills. Evidence from the presentations suggests the students' ability in explaining the procedures to solve the problem correctly using the appropriate strategies. However, it was less likely they gave explicit justifications and reasoning to their procedures. The following excerpts reveal these examples.

To find the centre of the circle... we used the two chords to find the perpendicular bisector... the perpendicular bisectors must pass through the centre which gives us the centre of the circle... and the radius must be the same. (Group 3)

First of all just draw a long chord and then find... like you did in maths... find the arcs like this... and then... and then if you check it will be 90 degrees because it is a perpendicular bisector... and draw the lines and that will make a triangle. After that draw another arc on both lines and just joint them up. And all the perpendicular bisectors will touch the centre... they all intersect at a point which indicates the centre of the circle. (Group 4)

It can be seen that most students were able to explain the steps to find the radius of the circle. For instance, they were able to note that they first needed to find the centre of the circle in order to find the radius. This was followed by an explanation on the ways to find the centre of the circle that eventually derived the radius of a circle. In addition, Group 4 also portrayed some understanding on the concept of perpendicular bisector as they justified that the angle between the chord and perpendicular bisector equals 90 degrees. Hence, it was observed that the students gained procedural skills. It was also observed that fewer students validated their strategy, as they did not explain which symmetrical properties were being applied to find the centre of a circle. For instance, Group 5 did not give any reasons for using two chords to find the centre, as shown in the following excerpt.

To find the radius we need to find the centre of a circle. To find the centre of a circle, we need to find the perpendicular bisector of chord. We have to use two chords... so we just choose anywhere from the broken plate which is like half of them... and then after we draw the chord, we use a compass to find the arc. The arc is to find the perpendicular bisector of chords. So this line here... from the centre to this line is the radius. (Group 5)

In this case, the students were expected to mention that both property 1 and property 2, which are equal chords, are equidistance from the centre and the perpendicular bisector of a chord passes through the centre respectively, could be applied to find the centre of a circle, but they failed to do so. This may imply that the students did not seem to notice that some of the procedures are the symmetrical properties of a circle. The teacher also observed that initially most groups only constructed a single chord with its perpendicular bisector. At this point, they got stuck and clearly had no idea how to proceed to find the centre of a circle. The teacher encouraged the students to search for more information. However, the students still did not manage to relate both property 1 and property 2 to apply into the problem. Thus, the teacher had to prompt the students by questioning them “what do you think will happen if you use more than one chords?” in a way to assist the students to perform trial and error and make comparisons. The students were then seen to construct the perpendicular bisector of another chord and they eventually noticed that the intersection point of two perpendicular bisectors was indeed the centre of a circle. Apparently, further questionings from the teacher were not facilitated. Thus this could be a possible for the insufficient understanding of the concepts. However, no further evidence was observed associating the students’ lack of conceptual understanding.

Conceptual understanding is equally as essential as procedural understanding to allow students to develop proficiency in mathematics. This includes strong understanding of mathematical concepts, operations and relations [47]. According to the National Research Council [48], students who own conceptual understanding in mathematics are less likely to make critical errors particularly during problem solving. Teachers must give the students opportunities to communicate their conceptual understanding [47]. Therefore, it is necessary for the teacher to fully acquire the student’ thoughts. This may involve using several strategic questionings or promptings [49] to avoid giving students direct instructions to proceed. It is also equally important to guide students to reason critically and also to assess their knowledge.

In relation to skills, from the interviews with the students, they all stated that they have nurtured some learning skills after the PBL intervention. The most mentioned learning skill was communication skills. Some students also mentioned other skills, for instance leadership skills, critical-thinking skills and information skills, which they have developed throughout the PBL intervention. For example, Laila mentioned that she has improved her research skills and communication skills. However, she

also emphasised that during collaborative discussion, sometimes their conversation was off topic and the group tend to lose focus. The following excerpts illustrates this example:

Well... searching... for information and for communication skills... well... the problem about our group is they talked a lot not about the thing... not about the question and they get distracted... too distracted.

(Laila)

This may imply that students have developed some communication skills through the PBL experience. In terms of communication skills, good problems in PBL foster the development of communication skills when the students presented their findings to their peers. Based on the teacher's notes, during the presentation stage, the students were able to deliver various findings confidently in front of the audiences, listen to their peers effectively and most importantly they were able to gain audiences' attention by making eye contact. The students were very fluent in their command of the English language during the delivery of their presentations. These reflect the students' good communication competency. However, the only limitation in doing presentation is that in general, the students who volunteered to be the presenter are among the extrovert students. Hence, communication competency may not apply to the quiet students. Moreover, during the presentation, the students were observed using only the whiteboard to present their findings. The teacher should have encouraged her students to perform their presentations using technological tools found on Internet or even other presentation software [50].

Analysing the case for Laila, the intervention from the teacher may be necessary. Hence, teachers should guide the students to be better communicators that will help them to be more productive and effective in communicating [51]. Furthermore, teachers should also consider giving students feedback on their communication skills during the presentations in addition to giving feedback on the content. Apart from communication skills, some students also believed to have developed research skills as they search for the information on their own [52, 53]. During the PBL lessons, the students used different resources in searching for information. Out of five groups, only Group 4 utilised the use of Internet to search for information and others, such as Group 2 only used the textbook and notebook. It was evident that Group 4 utilising the use of Internet were able to gain broader information. They also proposed several solutions that were related to real-life applications. This is in contrast to Group 2, which displayed solutions that were mostly subject-based. Thus, this may imply that the use of Internet [54, 55] may enhance the integration of knowledge.

4. Conclusion

This study reported evidence on how students' learning and their insights on PBL were positively influenced with regards to the implementation of PBL in the context of learning geometry. The results from Wilcoxon test show a significant increase in students' mean scores during the retention test. From the interviews, the students noted some challenges during learning process, which consisted of difficulties in decision-making and in generating ideas at the start of problem solving. The mixed responses from the interviews implied that in terms of learning attitudes, students favoured the idea of learning on their own but some critiqued that the process of PBL might be time-consuming. In terms of learning difficulties, some students struggled in assimilating information, which leads to poor decision-making. Nevertheless, the PBL was well perceived by students mainly due to the experience of independent learning and the development of communication and research skills, which was observed during the learning process leading up to the presentations of the group work.

Some limitations of this study were observed in the intervention lessons. Firstly, there was a lack of proper introduction at the beginning of the study. Due to time constraints and lack of technical support, the students were instructed to watch two educational videos on PBL at home prior to first lesson. These two videos provide students an introduction to PBL. However, only few students watched the videos, as little response from students was observed during questionings in the first lesson. Therefore regardless of any limiting factors, a proper introduction in class should be implemented to ensure better understanding of PBL practice in classroom [56]. Secondly, the post-test results did not show significant improvements directly after the intervention lessons because the test

was administered to the students only about a week after they were given the pre-test, and due to time constraints, only one cycle of the action research was conducted. In relation to time factors, it is recommended that for future research should implement PBL in a longer time period. This will allow teachers and students to familiarise and adapt with the PBL environment effectively. Besides, more support is necessary to help teacher to handle any challenges and limitations to improve teachers' knowledge and ability to implement a successful PBL lesson [6]. The implementation of PBL in the context of teaching and learning of geometry is still relatively sparse in the Brunei secondary schools. There is limited amount of resources on PBL problems especially in geometry topics and designing one was rather difficult for the teachers. This present study only offered one PBL problem for the students to work on during the activity, which may reflect inadequate gain of content knowledge. Hence, further support from other on-going research studies is needed in guiding mathematics teachers to design relevant PBL problems particularly in geometry topics.

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