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We present determinations of the B meson decay constant f_B and of the ratio f_{B_s}/f_B using the MILC collaboration unquenched gauge configurations which include three flavors of light sea quarks. The mass of one of the sea quarks is kept around the *strange* quark mass, and we explore a range in masses for the two lighter sea quarks down to $m_s/8$. The heavy b quark is simulated using Nonrelativistic QCD, and both the valence and sea light quarks are represented by the highly improved (AsqTad) staggered quark action. The good chiral properties of the latter action allow for a much smoother chiral extrapolation to physical *up* and *down* quarks than has been possible in the past. We find $f_B = 216(9)(19)(4)(6)$ MeV and $f_{B_s}/f_B = 1.20(3)(1)$.

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Accurate determination of the CKM matrix of the Standard Model and tests of its consistency and unitarity constitute an important part of current research in experimental and theoretical particle physics. Experimental studies of neutral $B_d - \overline{B}_d$ mixing, carried out as part of this program, are now well established and the mass difference ΔM_d is known with high precision [1]. Uncertainty in our present knowledge of the CKM matrix element $|V_{td}|$ is hence dominated by theoretical uncertainties, the most important of which are errors in $f_B\sqrt{B_B}$, where f_B is the B meson decay constant and B_B its bag parameter. Lattice QCD allows for first principles calculation of the hadronic matrix elements that lead to f_B and $f_B\sqrt{B_B}$ and in recent years the onus of reducing theoretical errors in determinations of $|V_{td}|$ has been on the Lattice QCD community. In this article we address and significantly improve upon two of the errors that have plagued f_B calculations on the lattice in the past, namely uncertainties due to lack of correct vacuum polarization in the simulations and errors due to chiral extrapolations to physical *up* and *down* quarks. The generation of unquenched gauge configurations by the MILC collaboration [2], which include effects of vacuum polarization from the *strange* plus two lighter dynamical quarks, has led to successful and realistic full QCD calculations of a variety of quantities involving both heavy and light quarks [3, 4, 5, 6, 7, 8, 9]. Here we also take advantage of these well tested configurations. Another innovation in recent years has been to use the same improved staggered light quark action [10], which is being employed for sea light quarks and for light hadron physics, also for the valence light quarks inside heavy-light mesons [11]. This has been crucial for allowing heavy-light simulations close to the real world. Chiral extrapolation requirements are now much milder than in the past thus reducing effects coming from this source of uncertainty.

In this study we work mainly with four of the "coarse" MILC ensembles with lattice spacing a around 0.12fm. These have dynamical light quark masses (in units of the *strange* quark mass) of $m_f/m_s = 0.125, 0.175, 0.25$ and 0.5. We have also accumulated results on two of MILC's "fine" lattices with $a \sim 0.087$fm. On the fine lattices we use staggered valence light propagators created by the Fermilab collaboration. The heavy b quark is simulated using the same NRQCD action employed in recent studies of the Υ system [9]. For many of the ensembles the lattice spacing was determined from the Υ $2S-1S$ splitting. For two ensembles where Υ results are not available, we used the heavy quark potential variable r_1 measured by the MILC collaboration [3, 9]. The bare s and b quark masses have been fixed by the Kaon and Υ masses, respectively [4, 9] and based on studies of light quark masses in [6] we take as the physical chiral limit the point $m_s/m_q = 27.4$.

The basic quantity that needs to be calculated in decay constant determinations is the matrix element of the heavy-light axial vector current between the B meson state and the hadronic vacuum. Taking, as is customary, the temporal component of the axial current, in Euclidean space and in the B rest frame one has

$$\langle 0|\, A_0 \,|B\rangle = M_B\, f_B. \tag{1}$$

In the last couple of years we have made considerable progress in reducing statistical errors in numerical determinations of this matrix element. We have developed better operators to create the B meson state on the lattice and fit to a matrix of correlators with different smearings. Details of smearings and matrix fits are similar to those in the Υ spectroscopy studies of reference [9] and will not be repeated here.

Table I summarizes results for the quantity $\Phi_q \equiv$

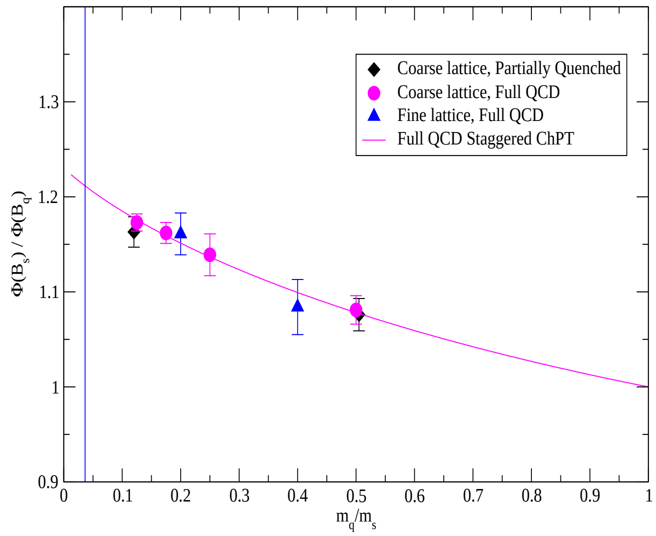

FIG. 1: The ratio $\xi_\Phi = \Phi_s/\Phi_q$ versus m_q/m_s. The full line through the data shows a fit to full QCD Staggered χPT (see text). Errors are statistical errors only. The fine lattice points were not included in the fit. The vertical line at $m_q/m_s = 1/27.4$ denotes the physical chiral limit.

$f_{B_q}\sqrt{M_{B_q}}$, where B_q denotes a $''B''$ meson with a light valence quark of mass m_q. In the third column we show $a^{3/2}\,\Phi_q^{(0)}$, the result for Φ_q in lattice units when only the zeroth order lattice heavy-light current $J^{(0)} = \overline{\Psi}_q\,\gamma_5\gamma_0\,\Psi_Q$ is used. The next column shows $a^{3/2}\,\Phi_q$, our results after one-loop matching and inclusion of $1/M$ currents. All corrections to the heavy-light current at $\mathcal{O}(\Lambda_{QCD}/M)$, $\mathcal{O}(\alpha_s)$, $\mathcal{O}(a\,\alpha_s)$, $\mathcal{O}(\alpha_s/(aM))$ and $\mathcal{O}(\alpha_s\,\Lambda_{QCD}/M)$ have been included. The dimension 4 current corrections that enter into the matching at this order have been discussed in [12]. The one-loop perturbative matching coefficients specific to the actions used in this study are given in [13]. One sees that the difference between $\Phi_q^{(0)}$ and Φ_q is small, about $2 \sim 4\%$ on the coarse lattices and $\sim 7\%$ on the fine lattices. The very small change on the coarse lattices may be partially accidental. There is cancellation between the $\mathcal{O}(\alpha_s)$ correction to the zeroth order current and the $1/M$ corrections. The coefficient of the $\mathcal{O}(\alpha_s)$ term switches sign as one goes from a bare b quark mass of $aM_0 = 2.8$ on the coarse lattices to $aM_0 = 1.95$ on the fine lattices, so that the cancellation does not occur on the latter. In the last column of Table I we give results for Φ_q in $\mathrm{GeV}^{3/2}$. The first errors are statistical and the second come from lattice spacing uncertainties. One sees that for most ensembles scale uncertainties dominate over statistical errors. The scales, a^{-1}, employed here are, in order of the most chiral to the least chiral ensembles, 1.623(32)GeV, 1.622(32)GeV, 1.596(30)GeV and 1.605(29)GeV, respectively on the four coarse lattices and 2.258(32)GeV and 2.312(31)GeV on the two fine lattices.

TABLE I: Simulation results for $\Phi_q \equiv f_{B_q}\sqrt{M_{B_q}}$. Sea (valence) quark masses are denoted by m_f (m_q) and $u_0 = [plaq]^{1/4}$ is the link variable used by the MILC collaboration in their normalisation of quark masses. See text for definitions of the last three columns. The second error in the last column comes from uncertainties in the scale $a^{-3/2}$.

$u_0\,am_f$	$u_0\,am_q$	$a^{3/2}\Phi_q^{(0)}$	$a^{3/2}\Phi_q$	Φ_q (GeV)$^{3/2}$
Coarse				
0.005	0.005	0.2579(26)	0.2494(26)	0.516(5)(15)
	0.040	0.3024(15)	0.2926(17)	0.605(4)(18)
0.007	0.007	0.2571(27)	0.2512(26)	0.519(5)(15)
	0.040	0.2993(20)	0.2917(20)	0.603(4)(18)
0.010	0.005	0.2571(23)	0.2507(24)	0.506(5)(14)
	0.010	0.2622(28)	0.2562(38)	0.517(8)(15)
	0.020	0.2767(27)	0.2710(27)	0.547(5)(15)
	0.040	0.3000(32)	0.2917(38)	0.588(8)(17)
0.020	0.020	0.2751(22)	0.2658(23)	0.540(5)(15)
	0.040	0.2988(24)	0.2873(28)	0.586(6)(16)
Fine				
0.0062	0.0062	0.1550(17)	0.1443(22)	0.490(7)(10)
	0.031	0.1804(15)	0.1676(16)	0.569(5)(12)
0.0124	0.0124	0.1583(39)	0.1474(42)	0.519(15)(10)
	0.031	0.1718(45)	0.1584(54)	0.557(19)(11)

TABLE II: Simulation results for $\xi_\Phi \equiv \Phi_s/\Phi_q$ without and with $1/M$ plus one-loop corrections.

$u_0\,am_f$	$u_0\,am_q$	$\Phi_s^{(0)}/\Phi_q^{(0)}$	Φ_s/Φ_q
Coarse			
0.005	0.005	1.173(7)	1.173(9)
0.007	0.007	1.164(11)	1.162(11)
0.010	0.005	1.166(15)	1.163(16)
	0.010	1.144(17)	1.139(22)
	0.020	1.085(15)	1.076(17)
0.020	0.020	1.086(13)	1.081(15)
Fine			
0.0062	0.0062	1.164(17)	1.161(22)
0.0124	0.0124	1.092(19)	1.084(29)

Table II shows results for the ratio $\xi_\Phi \equiv \Phi_s/\Phi_q$. This quantity, unlike Φ_q itself, is not affected directly by errors in the lattice spacing. Several other systematic errors inherent in f_B determinations, that will be discussed in more detail below, are also cancelled to a large extent in the ratio. For instance, one sees that going from ratios of $\Phi^{(0)}$ to ratios of Φ's that include $1/M$ and one-loop matching corrections, produces almost no change at all. The data for ξ_Φ are plotted in Fig.1 as a function of m_q/m_s. The full curve comes from a fit to formulas of staggered chiral perturbation theory ($S\chi PT$) [14, 15, 16] and represents the prediction for full QCD. The vertical line at small m_q corresponds to the physical chiral limit $m_q/m_s = 1/27.4$.

$S\chi PT$ for heavy-light decay constants has been developed by Aubin & Bernard in reference [16]. For Φ_q their formula reads,

$$\Phi_q = c_0\,(1\;+\;\Delta_q\;+\;analytic). \tag{2}$$

The term encompassing the chiral logarithms, $\Delta_q \equiv \delta f_{Bq}/(16\pi^2 f^2)$, is given in [16] and includes $\mathcal{O}(a^2)$ effects coming from taste symmetry breaking, both in the mass splittings among light-light pseudoscalars and in lattice artifact hairpin diagrams. For the ratio ξ_Φ we use the ansatz,

$$\xi_\Phi = 1 + (\Delta_s - \Delta_q) + \sum_k^{N_k} c_k\,(am_q - am_s)^k. \tag{3}$$

N_k was increased until $\xi_\Phi^{(phys.)}$, the fit result for ξ_Φ at $m_q/m_s = 1/27.4$, and its error had stabilized (in practice $N_k = 2$ was sufficient). Other ansaetze such as the direct ratio, $\frac{1+\Delta_s + c_1\,(2m_f + m_{sd}) + c_2\,m_s}{1+\Delta_q + c_1\,(2m_f+m_{sd}) + c_2\,m_q}$ (m_{sd} is the sea *strange* quark mass which, on the coarse lattices, is slightly larger than the true *strange* quark mass m_s we use for valence *strange* quarks) or simple linear fits without any chiral logarithms were also tried as were fits with all the $\mathcal{O}(a^2)$ taste breaking terms turned off. All these different chiral extrapolations lead to values for $\xi_\Phi^{(phys.)}$ that differ at most by 3%. We fit simultaneously to the six coarse lattice points, 4 full QCD and 2 partially quenched (PQQCD) points, using full QCD and PQQCD $S\chi PT$ formulas respectively. Fig.1 shows just the full QCD curve.

The terms Δ_q involve the $BB^*\pi$ coupling $g_{B\pi}$ which is not known experimentally. We have carried out fits at several fixed values for $g^2_{B\pi}$ between $g^2_{B\pi} = 0$ and $g^2_{B\pi} = 0.75$. Good fits were obtained ($\chi^2/dof \approx 1$ or less) for $g^2_{B\pi} < 0.5$ with $\xi_\Phi^{(phys.)}$ differing again by less than 3% in the range $\xi_\Phi^{(phys.)} = 1.21 \sim 1.24$. We have also let $g_{B\pi}$ float as one of the fit parameters and find $g^2_{B\pi} = 0.0(2)$ together with $\xi_\Phi^{(phys.)} = 1.21(2)$. This fit result for $g^2_{B\pi}$ with the large uncertainty of $\Delta g^2_{B\pi} = 0.2$ shows that our data is not able to determine $g^2_{B\pi}$ with any accuracy, the same message we get from the fixed $g_{B\pi}$ fits, where a range of $g^2_{B\pi}$ between zero and $\sim 2\times\Delta g^2_{B\pi}$ all give acceptable fits. Fortunately, within this range $\xi_\Phi^{(phys.)}$ is not very sensitive to $g^2_{B\pi}$. We take as our central value for $\xi_\Phi^{(phys.)}$ the result from the floating $g_{B\pi}$ fit, which we consider the least biased fit. This fit gives the curve shown on Fig.1. We then take ± 0.03 as the error due to statistics and chiral extrapolation uncertainties, and which also covers the spread we observe upon trying different ansaetze and different ways of handling $g^2_{B\pi}$. Remaining errors such

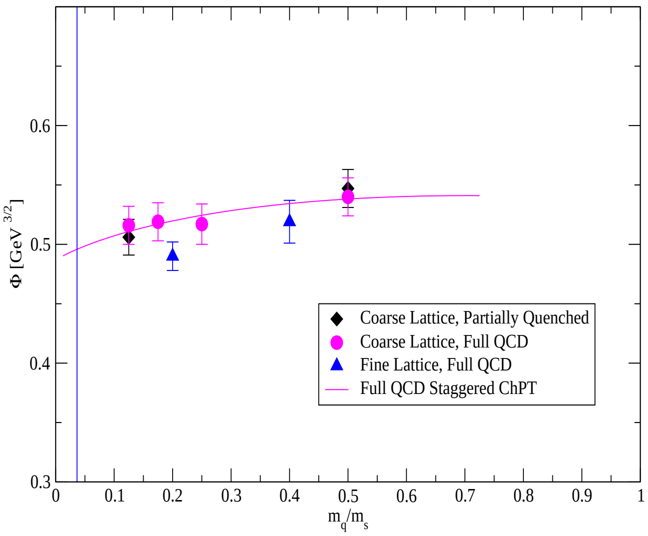

FIG. 2: Φ_q versus m_q/m_s. Errors include both statistical and scale uncertainty errors. The fine lattice points were not included in the fit.

as those due to discretization and relativistic corrections and higher order operator matchings not yet included, will affect f_B and f_{B_s} in similar ways and largely cancel in the ratio. One expects their effects to come in at the level of the corresponding error in Φ_q times $a(m_s - m_q)$. We have already seen that $1/M$ and one-loop matching corrections cancel almost completely in ξ_Φ. Furthermore the two full QCD fine lattice points in Fig.1 fall nicely on the full QCD $S\chi PT$ curve fixed by the coarse lattice points indicating that any residual discretization errors in ξ_Φ are smaller than the current statistical errors. Taking all these arguments into account, we estimate a $\sim 1\%$ further uncertainty in ξ_Φ from these other sources. Our final result for $f_{B_s}/f_B = \xi_\Phi\sqrt{\frac{M_B}{M_{B_s}}}$ is then

$$f_{B_s}/f_B = 1.20(3)(1). \tag{4}$$

We emphasize that the reason the chiral extrapolation errors are small here is because the light quark action employed in this study allowed us to go down as low as $m_s/8$ and only a modest extrapolation to the physical chiral limit was required. This differs from the case with Wilson type light quarks, where simulations have typically been restricted to $m_q/m_s > 0.5$, i.e. to the region to the right of the heaviest data point in Fig.1.

Fig.2 shows the data points for Φ_q itself for $m_q/m_s \leq 0.5$ together with a full QCD $S\chi PT$ fit curve. For chiral extrapolation of Φ_q we use directly eq.(2) with analytic terms $c_1(2m_f + m_{sd}) + c_2 m_q$. We again carry out simultaneous fits to the coarse lattice full QCD and PQQCD points. Fits with the coupling $g^2_{B\pi}$ held fixed between 0.0 and 0.6 all lead to good fits with $\Phi^{(phys.)}$ varying by 4%. Allowing this coupling to float gives $g^2_{B\pi} = 0.1(5)$, which is consistent with the fixed $g_{B\pi}$ fit results, and

$\Phi^{(phys.)} = 0.496(20)$ GeV$^{3/2}$ with again a 4% error. We take the 4% to be our best estimate for the combined error from statistics, chiral extrapolation and determination of a^{-1}. The full QCD $S\chi PT$ curve in Fig.2 comes from the floating $g^2_{B\pi}$ fit. We turn next to estimates of the other systematic errors in $\Phi^{(phys.)}$.

A major source of systematic error in $\Phi^{(phys.)}$ is higher order matching of the heavy-light current. Although the one-loop contributions turned out to be small (as described above), in fact much smaller than a naive estimate of $\mathcal{O}(\alpha_s) \sim 30\%$, we have no argument guaranteeing this to be true at higher orders. Hence we allow for an $\mathcal{O}(\alpha_s^2) \approx 9\%$ systematic matching error. This will be the dominant systematic error in our decay constant determination. Another source of systematic error comes from discretization effects. The fine lattice points in Fig.2 lie about $3 \sim 5\%$ lower than those from the coarse lattices. Since the statistical plus scale uncertainty errors on all our points range between $2 \sim 3\%$, it is not obvious how much of this difference comes from discretization effects. The size of fluctuations between independent coarse ensembles is comparable to this difference. It should also be noted that the difference between the coarse and fine lattice data would disappear if it were not for the one-loop matching corrections (recall the $2 \sim 4\%$ corrections on the coarse lattices versus the $\sim 7\%$ corrections on the fine lattices giving a $3 \sim 5\%$ difference in the radiative corrections on the two lattices). In other words it is difficult to disentangle discretization errors from radiative corrections. One could quote a combined discretization and higher order matching error again at the $\sim 9\%$ level. We opt instead to keep the 9% as the pure (and dominating) $\mathcal{O}(\alpha_s^2)$ error and use a conventional naive estimate of $\mathcal{O}(a^2\alpha_s) \approx 2\%$ for discretization errors. As the last nontrivial systematic error we estimate uncertainties from relativistic corrections and tuning of the b quark mass [9] to be at the $\sim 3\%$ level. Putting all this together we obtain $\Phi^{(phys.)} = 0.496(20)(45)(10)(15)\,\mathrm{GeV}^{3/2}$. This leads to our result for the B meson decay constant of

$$f_B = 0.216(9)(19)(4)(6)\ \mathrm{GeV}. \tag{5}$$

The errors, from left to right, come from statistics plus scale plus chiral extrapolations, higher order matching, discretization, and relativistic corrections plus m_b tuning respectively. Combining this result with our result for f_{B_s}/f_B, eq.(4), one finds $f_{B_s} = 0.259(32)$ GeV. This is very consistent with the direct calculation of f_{B_s} published earlier in [5] where we quote a value of 0.260(29) GeV.

To summarize, we have completed a determination of the B meson decay constant in full (unquenched) QCD. Our main results are given in eqns. (4) and (5). The use of a highly improved light quark action has led to good control over the chiral extrapolation to physical *up* and *down* quarks. Better smearings have significantly reduced statistical errors. For the ratio f_{B_s}/f_B these improvements translate into an accurate final result with errors at the $\sim 3\%$ level. For f_B itself other systematic errors not yet addressed in the present study dominate and the current total error is at the $\sim 10\%$ level. The main remaining source of uncertainty comes from higher order operator matching. More studies should also be carried out on the fine lattices and on even finer lattices currently being created by the MILC collaboration, to reduce discretization uncertainties. Errors in the scale a^{-1} need to come down for all the ensembles. Improvements on all these fronts are underway. Calculations of the bag parameter B_B have also been initiated.

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