Entailment for Structured Specifications

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$$\frac{SP \vdash \varphi_1 \quad \cdots \quad SP \vdash \varphi_n \qquad \{\varphi_1,\ldots,\varphi_n\} \vdash_{Sig[SP]} \varphi}{SP \vdash \varphi}$$

$$\frac{}{\langle \Sigma, \Phi\rangle \vdash \varphi}\ \varphi \in \Phi$$

$$\frac{SP_1 \vdash \varphi}{SP_1 \cup SP_2 \vdash \varphi} \qquad \frac{SP_2 \vdash \varphi}{SP_1 \cup SP_2 \vdash \varphi}$$

$$\frac{SP \vdash \varphi}{SP \textbf{ with } \sigma \vdash \sigma(\varphi)} \qquad \frac{SP \vdash \sigma(\varphi)}{SP \textbf{ hide via } \sigma \vdash \varphi}$$

Clarifications: INS = $\langle$**Sign**, **Sen**: **Sign** $\to$ **Set**, **Mod**: **Sign**$^{op} \to$ **Cat**, $\langle \models_\Sigma \subseteq |\mathbf{Mod}(\Sigma)| \times \mathbf{Sen}(\Sigma)\rangle_{\Sigma \in |\mathbf{Sign}|}\rangle$ is an institution that defines the logical system used for specifications, SP, SP_1 and SP_2 are structured Σ-specifications over **INS**, where Σ is a signature in the category **Sign**, $\varphi, \varphi_1, \ldots, \varphi_n$ are Σ-sentences, i.e. elements in $\mathbf{Sen}(\Sigma)$, Φ is a set of Σ-sentences, and $\sigma(\varphi)$ denotes $\mathbf{Sen}(\sigma)(\varphi)$, the translation of the sentence φ along $\sigma\colon \Sigma \to \Sigma'$. Structured specifications in **INS** are built from basic specifications $\langle \Sigma, \Phi\rangle$, the union of Σ-specifications $SP_1 \cup SP_2$, the translation "SP **with** σ" of SP along a signature morphism $\sigma\colon \Sigma \to \Sigma'$, and hiding "$SP$ **hide via** σ" for hiding the symbols in SP not occurring in the image of $\sigma\colon \Sigma' \to \Sigma$. $Sig[SP]$ is the signature of SP. Translations of Σ-sentences and Σ'-models along $\sigma\colon \Sigma \to \Sigma'$ are required to preserve satisfaction: for any $\varphi \in \mathbf{Sen}(\Sigma)$ and $M' \in |\mathbf{Mod}(\Sigma')|$, $M' \models_{\Sigma'} \mathbf{Sen}(\sigma)(\varphi) \Leftrightarrow \mathbf{Mod}(\sigma)(M') \models_\Sigma \varphi$. Finally, $\langle \vdash_\Sigma \subseteq Pow(\mathbf{Sen}(\Sigma)) \times \mathbf{Sen}(\Sigma)\rangle_{\Sigma \in |\mathbf{Sign}|}$ is a sound entailment relation for the satisfaction relation $\langle \models_\Sigma\rangle_{\Sigma \in |\mathbf{Sign}|}$.

The judgement $SP \vdash \varphi$ is meant to capture the property that φ is satisfied in all models of SP.

History: The first systems for proving entailment in structured specifications were given by Sannella and Burstall [1], Sannella and Tarlecki [2], and Wirsing [3]. The above presentation can be found in [6], Sect. 9.2.

Remarks: The system is sound; completeness is shown in [3] for the first-order logic instance and in [5, 6] for an institution **INS** which is finitely exact, admits propositional operators, satisfies Craig interpolation, and has a complete entailment relation $\langle \vdash_\Sigma\rangle_{\Sigma \in |\mathbf{Sign}|}$. [7] shows that this is the most powerful sound proof system that is compositional in the structure of specifications. [4] provides additional rules for observability operators.

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[3] Martin Wirsing. "Structured Specifications: Syntax, Semantics and Proof Calculus". In: *Logic and Algebra of Specification, NATO Advanced Institute, 1991*. Vol. 94. Springer, 1993, pp. 411–442.

[4] Rolf Hennicker. *Structured Specifications with Behavioural Operators: Semantics, Proof Methods and Applications*. Habilitation thesis. LMU Munich, 1997.

[5] Tomasz Borzyszkowski. "Logical systems for structured specifications". In: *Theoretical Computer Science* 286.2 (2002), pp. 197–245.

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[7] Donald Sannella and Andrzej Tarlecki. "Property-oriented semantics of structured specifications". In: *Mathematical Structures in Computer Science* 24.2 (2014), e240205.

Entry 22 by: Rolf Hennicker, Donald Sannella , Andrzej Tarlecki , Martin Wirsing