Entailment for Structured Specifications

Citation for published version:

Link:
Link to publication record in Edinburgh Research Explorer

Document Version:
Peer reviewed version

Published In:
Towards an Encyclopaedia of Proof Systems

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Entailment for Structured Specifications (1988)

\[
\begin{array}{c}
SP \vdash \varphi_1 \quad \cdots \quad SP \vdash \varphi_n \quad (\varphi_1, \ldots, \varphi_n) \vdash_{\text{Sig}(SP)} \varphi \\
\hline
SP \vdash \varphi
\end{array}
\]

\[
\begin{array}{c}
(\Sigma, \Phi) \vdash \varphi \\
\hline
SP \vdash \varphi
\end{array}
\]

Clarifications: INS = \langle \text{Sign}, \text{Sen} : \text{Sign} \rightarrow \text{Set}, \text{Mod} : \text{Sign}^{op} \rightarrow \text{Cat} \rangle, \langle \models_{\Sigma} \subseteq [\text{Mod}(\Sigma)] \times \text{Sen}(\Sigma) \rangle_{\Sigma \in \text{Sign}} \rangle is an institution that defines the logical system used for specifications. SP, SP_1 and SP_2 are structured \Sigma-specifications over INS, where \Sigma is a signature in the category Sign, \varphi, \varphi_1, \ldots, \varphi_n are \Sigma-sentences, i.e. elements in Sen(\Sigma). \Phi is a set of \Sigma-sentences, and \sigma(\varphi) denotes Sen(\sigma)(\varphi), the translation of the sentence \varphi along \sigma : \Sigma \rightarrow \Sigma'. Structured specifications in INS are built from basic specifications (\Sigma, \Phi), the union of \Sigma-specifications SP_1 \cup SP_2, the translation “SP with \sigma” of SP along a signature morphism \sigma : \Sigma \rightarrow \Sigma', and hiding “SP hide via \sigma” for hiding the symbols in SP not occurring in the image of \sigma : \Sigma' \rightarrow \Sigma. Sig(SP) is the signature of SP. Translations of \Sigma-sentences and \Sigma'-models along \sigma : \Sigma \rightarrow \Sigma' are required to preserve satisfaction: for any \varphi \in \text{Sen}(\Sigma) and M' \in [\text{Mod}(\Sigma')], M' \models_{\Sigma'} \text{Sen}(\sigma)(\varphi) \leftrightarrow \text{Mod}(\sigma)(M') \models_{\Sigma} \varphi. Finally, \models_{\Sigma} \subseteq \text{Pow}(\text{Sen}(\Sigma)) \times \text{Sen}(\Sigma) \rangle_{\Sigma \in \text{Sign}} \rangle is a sound entailment relation for the satisfaction relation \models_{\Sigma}. The judgement \models_{\Sigma} \vdash \varphi is meant to capture the property that \varphi is satisfied in all models of SP.

\textbf{History:} The first systems for proving entailment in structured specifications were given by Sannella and Burstall \[1\], Sannella and Tarlecki \[2\], and Wirsing \[3\]. The above presentation can be found in \[6\], Sect. 9.2.

\textbf{Remarks:} The system is sound; completeness is shown in \[3\] for the first-order logic instance and in \[5\] for an institution INS which is finitely exact, admits propositional operators, satisfies Craig interpolation, and has a complete entailment relation \models_{\Sigma} \vdash \varphi \models_{\Sigma} \models_{\Sigma} \models_{\Sigma}. \[7\] shows that this is the most powerful sound proof system that is compositional in the structure of specifications. \[4\] provides additional rules for observability operators.

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