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Cubic Bézier curve approximation for the accurate estimation of perivascular spaces morphometrics in brain MRI*

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Abstract. The morphometrics of perivascular spaces (PVS) such as diameter and length have been associated with stroke and hypertension. While the conventional ellipsoid approximation used for measuring diameter and length holds for straight PVS, significant inaccuracies occur for curved PVS. We propose a model based on cubic Bézier curves to cope with this limitation. On curved digital reference objects, we show that our proposal outperforms the conventional method.

Keywords: Perivascular spaces · cubic Bézier curve · morphometrics

1 Introduction

The computational quantification of MRI-visible perivascular spaces (PVS) in the brain has increasingly gained attention because of its importance in validating PVS as a biomarker of brain health function. PVS morphometrics can provide useful information to complement volume and count. Median PVS diameter and length have been associated with stroke and hypertension [1]. The use of digital reference objects (DRO) has been instrumental in establishing the limits of validity of the PVS segmentation methods [2]. It has shown inaccuracies in the assessment of the diameter and length of curved PVS if the ellipsoid approximation model, implemented by the function regionprops3 in MATLAB [4], is used. Here we propose the use of a cubic Bézier curve approximation model and compare the two methods using a curved DRO.

2 Methods

In this section, we describe the construction of our curved DRO and the proposed method for measuring the diameter and length of the simulated PVS.

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Algorithm 1 Cubic Bézier approximation

Input: Skeleton of the image $S(n)$ with N voxels, initial control points $\boldsymbol{b}_0^{(0)}, \boldsymbol{b}_0^{(1)}, \boldsymbol{b}_0^{(2)}, \boldsymbol{b}_0^{(3)}$
Optimise control points by alternating minimisation
  for $i = 0, 1, \ldots$ **do**
    Estimated cubic Bézier curve
      $T^{(i)}(t) = \left[\boldsymbol{b}_0^{(i)}\ \boldsymbol{b}_1^{(i)}\ \boldsymbol{b}_2^{(i)}\ \boldsymbol{b}_3^{(i)}\right] \boldsymbol{M} f(t)$
    Update the skeleton t values using the t value of the curve closest point
      for $n = 1, \ldots, N$ **do**
        $\boldsymbol{t}^{(i)}(n) = \underset{0 \leq t \leq 1}{\operatorname{argmin}} \left(S_p(n) - T^{(i)}(t)\right)^\intercal \left(S_p(n) - T^{(i)}(t)\right)$
      end for
    Update control points using least square error
      $\boldsymbol{T} = \left[T^{(i)}(\boldsymbol{t}^{(i)}(1))\ T^{(i)}(\boldsymbol{t}^{(i)}(2))\ \ldots\ T^{(i)}(\boldsymbol{t}^{(i)}(N))\right]$
      $\boldsymbol{C} = \boldsymbol{T}\boldsymbol{M}^\intercal = \left[\boldsymbol{c}_0\ \boldsymbol{c}_1\ \boldsymbol{c}_2\ \boldsymbol{c}_3\right]^\intercal$
      $\boldsymbol{S} = \left[S_p^{(1)}\ S_p^{(2)}\ \ldots\ S_p^{(N)}\right]$
      $\boldsymbol{E} = \boldsymbol{S} - \boldsymbol{b}_0\boldsymbol{c}_0 - \boldsymbol{b}_3\boldsymbol{c}_3$
        $\left[\boldsymbol{b}_1\ \boldsymbol{b}_2\right] = \left(\left[\boldsymbol{c}_1\ \boldsymbol{c}_2\right]^\intercal \left[\boldsymbol{c}_1\ \boldsymbol{c}_2\right]\right)^{-1} \left[\boldsymbol{c}_1\ \boldsymbol{c}_2\right]^\intercal \boldsymbol{E}$
  end for

2.1 Curved DRO

We constructed our curved PVS-DRO using the following parametric equations:

$$p(t) = \left[x(t)\ y(t)\ z(t)\right]^\intercal = \left[0\ \cos(t/2)\ t\right]^\intercal, \tag{1}$$

where $t_1 < t < t_2$ (see Figure 1 (Left)), the PVS-DRO model is constructed from all the points such that the shortest distance to the line described in (1) is smaller than the radius $d/2$. The length of the line is given by:

$$l = d + \int_{t_1}^{t_2} \sqrt{\left(\frac{\partial x}{\partial t}\right)^2 + \left(\frac{\partial y}{\partial t}\right)^2 + \left(\frac{\partial z}{\partial t}\right)^2}. \tag{2}$$

For our simulation, we set the $t_1 = -6$ and $t_2 = 6$ and we vary the diameter of the PVS-DRO from 0.3 to 3.0 millimetres. Please note the choice of the curve for the DRO was arbitrary and any smooth continuous line can be used.

2.2 Proposed Cubic Bézier Curve Approximation

Cubic Bézier curves can be described with the following equation [3]:

$$p(t) = \boldsymbol{B}\boldsymbol{M}f(t) = \left[\boldsymbol{b}_0\ \boldsymbol{b}_1\ \boldsymbol{b}_2\ \boldsymbol{b}_3\right] \begin{bmatrix} 1 & -3 & 3 & -1 \\ 0 & 3 & -6 & 3 \\ 0 & 0 & 3 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ t \\ t^2 \\ t^3 \end{bmatrix}, \tag{3}$$

where the vectors conforming $\boldsymbol{B}$ are control points. This curve is defined in the interval of $0 \leq t \leq 1$ (see Figure 1 (Right)). The $\boldsymbol{b}_0$ and $\boldsymbol{b}_3$ are the initial ($t = 0$) and the end ($t = 1$) points, respectively. From the skeleton ordered voxel list of $S_p(n)$ we approximate the cubic Bézier curve using our proposed algorithm 1.

3 Results and Conclusions

Our PVS-DRO uses a voxel size of $0.35 \times 0.35 \times 0.35$ mm^3, and the binary mask considers voxels containing any proportion of the PVS-DRO. Figure 2 shows the measurements of the length and diameter using the ellipsoid (red) and the cubic Bézier curve (magenta) models. The Bézier curve approximation performs better than the ellipsoid, being closer to the ideal measurements.

Our proposed method creates a DRO for curved tubular objects (e.g., PVS) by giving the parametric equation of the curve. Our optimisation method approximates a Bézier curve allowing us to accurately measure the diameter and length of curved objects representing PVS in brain MRI scans. It outperforms the ellipsoid approximation for measuring the length and diameter of curved objects.

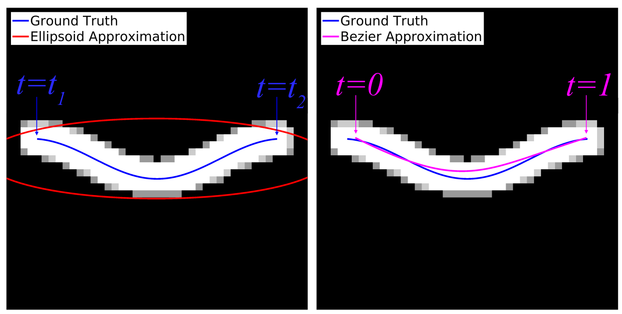

Fig. 1. Example of the approximation. (Left) Ellipsoid (Right) Cubic Bézier curve.

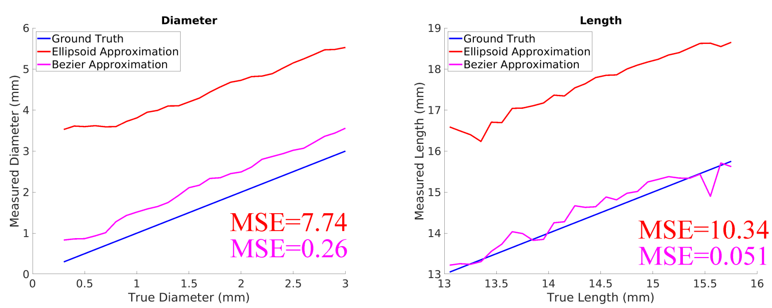

Fig. 2. Measurements from the Ellipsoid and the cubic Bézier curve methods. (Left) Diameter results (Right) length results.

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