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# Mixing Metaphors: Actors as Channels and Channels as Actors\*

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## Abstract

Channel- and actor-based programming languages are both used in practice, but the two are often confused. Languages such as Go provide anonymous processes which communicate using buffers or rendezvous points—known as channels—while languages such as Erlang provide addressable processes—known as actors—each with a single incoming message queue. The lack of a common representation makes it difficult to reason about translations that exist in the folklore. We define a calculus  $\lambda_{\text{ch}}$  for typed asynchronous channels, and a calculus  $\lambda_{\text{act}}$  for typed actors. We define translations from  $\lambda_{\text{act}}$  into  $\lambda_{\text{ch}}$  and  $\lambda_{\text{ch}}$  into  $\lambda_{\text{act}}$  and prove that both are type- and semantics-preserving. We show that our approach accounts for synchronisation and selective receive in actor systems and discuss future extensions to support guarded choice and behavioural types.

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**Keywords and phrases** Actors, Channels, Communication-centric Programming Languages

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## 1 Introduction

When comparing channels (as used by Go) and actors (as used by Erlang), one runs into an immediate mixing of metaphors. The words themselves do not refer to comparable entities!

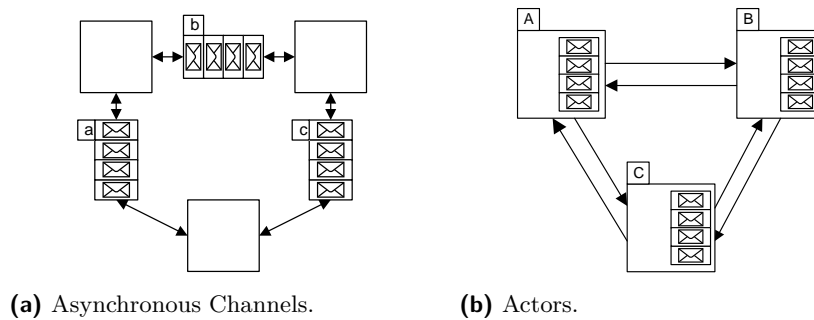
In languages such as Go, anonymous processes pass messages via named channels, whereas in languages such as Erlang, named processes accept messages from an associated mailbox. A channel is either a named rendezvous point or buffer, whereas an actor is a process. We should really be comparing named processes (actors) with anonymous processes, and buffers tied to a particular process (mailboxes) with buffers that can link any process to any process (channels). Nonetheless, we will stick with the popular names, even if it is as inapposite as comparing TV channels with TV actors.

Figure 1 compares asynchronous channels with actors. On the left, three anonymous processes communicate via channels named  $a, b, c$ . On the right, three processes named  $A, B, C$  send messages to each others' associated mailboxes. Actors are necessarily asynchronous, allowing non-blocking sends and buffering of received values, whereas channels can either be asynchronous or synchronous (rendezvous-based). Indeed, Go provides both synchronous *and* asynchronous channels, and libraries such as `core.async` [24] provide library support

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■ **Figure 1** Channels and Actors.

for asynchronous channels. However, this is not the only difference: each actor has a single buffer which only it can read—its *mailbox*—whereas asynchronous channels are free-floating buffers that can be read by any process with a reference to the channel.

Channel-based languages such as Go enjoy a firm basis in process calculi such as CSP [25] and the  $\pi$ -calculus [38]. It is easy to type channels, either with simple types (see [46], p. 231) or more complex systems such as session types [26, 27, 17]. Actor-based languages such as Erlang are seen by many as the "gold standard" for distributed computing due to their support for fault tolerance through supervision hierarchies [6, 7].

Both models are popular with developers, with channel-based languages and frameworks such as Go, Concurrent ML [45], and Hopac [28]; and actor-based languages and frameworks such as Erlang, Elixir, and Akka.

## 1.1 Motivation

This paper provides a formal account of actors and channels as implemented in programming languages. Our motivation for a formal account is threefold: it helps clear up confusion; it clarifies results that have been described informally by putting practice into theory; and it provides a foundation for future research.

**Confusion.** There is often confusion over the differences between channels and actors. For example, the following questions appear on StackOverflow and Quora respectively:

“If I wanted to port a Go library that uses Goroutines, would Scala be a good choice because its inbox/[A]kka framework is similar in nature to coroutines?” [31], and

“I don’t know anything about [the] actor pattern however I do know goroutines and channels in Go. How are [the] two related to each other?” [29]

In academic circles, the term *actor* is often used imprecisely. For instance, Albert et al. [5] refer to Go as an actor language. Similarly, Harvey [21] refers to his language Ensemble as actor-based. Ensemble is a language specialised for writing distributed applications running on heterogeneous platforms. It is actor-based to the extent that it has lightweight, addressable, single-threaded processes, and forbids co-ordination via shared memory. However, Ensemble communicates using channels as opposed to mailboxes so we would argue that it is channel-based (with actor-like features) rather than actor-based.

**Putting practice into theory.** The success of actor-based languages is largely due to their support for *supervision*. A popular pattern for writing actor-based applications is to arrange processes in *supervision hierarchies* [6], where *supervisor* processes restart child processes should they fail. Projects such as Proto.Actor [44] emulate actor-style programming in a channel-based language in an attempt to gain some of the benefits, by associating queues with processes. Hopac [28] is a channel-based library for F#, based on Concurrent ML [45]. The documentation [1] contains a comparison with actors, including an implementation of a simple actor-based communication model using Hopac-style channels, as well as an implementation of Hopac-style channels using an actor-based communication model. By comparing the two, this paper provides a formal model for the underlying techniques, and studies properties arising from the translations.

**A foundation for future research.** Traditionally, actor-based languages have had untyped mailboxes. More recent advancements such as TAkka [22], Akka Typed [4], and Typed Actors [47] have added types to mailboxes in order to gain additional safety guarantees. Our formal model provides a foundation for these innovations, characterises why naïvely adding types to mailboxes is problematic, and provides a core language for future experimentation.

## 1.2 Our approach

We define two concurrent  $\lambda$ -calculi, describing *asynchronous* channels and type-parameterised actors, define translations between them, and then discuss various extensions.

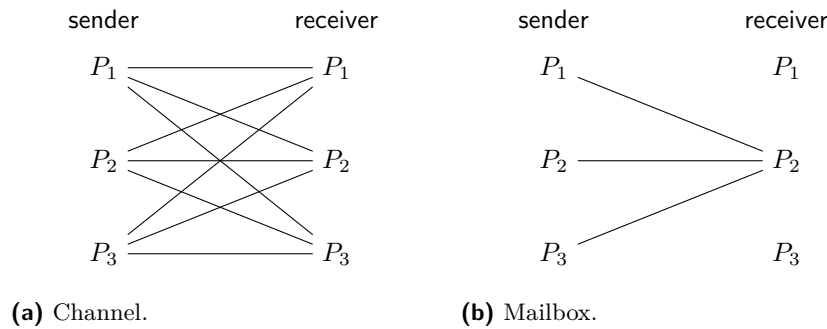
**Why the  $\lambda$  calculus?** Our common framework is that of a simply-typed concurrent  $\lambda$ -calculus: that is, a  $\lambda$ -calculus equipping a term language with primitives for communication and concurrency, as well as a language of *configurations* to model concurrent behaviour. We work with the  $\lambda$ -calculus rather than a process calculus for two reasons: firstly, the simply-typed  $\lambda$ -calculus has a well-behaved core with a strong metatheory (for example, confluent reduction and strong normalisation), as well as a direct propositions-as-types correspondence with logic. We can therefore modularly extend the language, knowing which properties remain; typed process calculi typically do not have such a well-behaved core.

Secondly, we are ultimately interested in functional programming languages; the  $\lambda$  calculus is the canonical choice for studying such extensions.

**Why asynchronous channels?** While actor-based languages must be asynchronous by design, channels may be either synchronous (requiring a rendezvous between sender and receiver) or asynchronous (where sending happens immediately). In this paper, we consider asynchronous channels since actors must be asynchronous, and it is possible to emulate asynchronous channels using synchronous channels [45]. We could adopt synchronous channels, use these to encode asynchronous channels, and then do the translations. We elect not to since it complicates the translations, and we argue that the distinction between synchronous and asynchronous communication is not *the* defining difference between the two models.

## 1.3 Summary of results

We identify four key differences between the models, which are exemplified by the formalisms and the translations: process addressability, the restrictiveness of communication patterns, the granularity of typing, and the ability to control the order in which messages are processed.



■ **Figure 2** Mailboxes as pinned channels.

**Process addressability.** In channel-based systems, processes are *anonymous*, whereas channels are named. In contrast, in actor-based systems, processes are named.

**Restrictiveness of communication patterns.** Communication over full-duplex channels is more liberal than communication via mailboxes, as shown in Figure 2. Figure 2a shows the communication patterns allowed by a single channel: each process  $P_i$  can use the channel to communicate with every other process. Conversely, Figure 2b shows the communication patterns allowed by a mailbox associated with process  $P_2$ : while any process can send to the mailbox, only  $P_2$  can read from it. Viewed this way, it is apparent that the restrictions imposed on the communication behaviour of actors are exactly those captured by Merro and Sangiorgi’s localised  $\pi$ -calculus [37].

Readers familiar with actor-based programming may be wondering whether such a characterisation is too crude, as it does not account for processing messages out-of-order. Fear not—we show in Section 7 that our minimal actor calculus can simulate this functionality.

Restrictiveness of communication patterns is not necessarily a bad thing; while it is easy to distribute actors, *delegation* of asynchronous channels is more involved, requiring a distributed algorithm [30]. Associating mailboxes with addressable processes also helps with structuring applications for reliability [7].

**Granularity of typing.** As a result of the fact that each process has a single incoming message queue, mailbox types tend to be less precise; in particular, they are most commonly variant types detailing all of the messages that can be received. Naïvely implemented, this gives rise to the *type pollution problem*, which we describe further in Section 2.

**Message ordering.** Channels and mailboxes are ordered message queues, but there is no inherent ordering between messages on two different channels. Channel-based languages allow a user to specify from which channel a message should be received, whereas processing messages out-of-order can be achieved in actor languages using selective receive.

The remainder of the paper captures these differences both in the design of the formalisms, and the techniques used in the encodings and extensions.

## 1.4 Contributions and paper outline

This paper makes five main contributions:

1. A calculus  $\lambda_{\text{ch}}$  with typed asynchronous channels (Section 3), and a calculus  $\lambda_{\text{act}}$  with type-parameterised actors (Section 4), based on the  $\lambda$ -calculus extended with communication

<pre> <b>chanStack</b>(<i>ch</i>) <math>\triangleq</math> <i>rec loop</i>(<i>st</i>).   let <i>cmd</i> <math>\leftarrow</math> <b>take</b> <i>ch</i> in   case <i>cmd</i> {     <b>Push</b>(<i>v</i>) <math>\mapsto</math> <i>loop</i>(<i>v</i> :: <i>st</i>)     <b>Pop</b>(<i>resCh</i>) <math>\mapsto</math>       case <i>st</i> {         [ ] <math>\mapsto</math> <b>give</b> (<b>None</b>) <i>resCh</i>;           loop [ ]         <i>x</i> :: <i>xs</i> <math>\mapsto</math> <b>give</b> (<b>Some</b>(<i>x</i>)) <i>resCh</i>;           loop <i>xs</i> }       } </pre>	<pre> <b>actorStack</b> <math>\triangleq</math> <i>rec loop</i>(<i>st</i>).   let <i>cmd</i> <math>\leftarrow</math> <b>receive</b> in   case <i>cmd</i> {     <b>Push</b>(<i>v</i>) <math>\mapsto</math> <i>loop</i>(<i>v</i> :: <i>st</i>)     <b>Pop</b>(<i>resPid</i>) <math>\mapsto</math>       case <i>st</i> {         [ ] <math>\mapsto</math> <b>send</b> (<b>None</b>) <i>resPid</i>;           loop [ ]         <i>x</i> :: <i>xs</i> <math>\mapsto</math> <b>send</b> (<b>Some</b>(<i>x</i>)) <i>resPid</i>;           loop <i>xs</i> }       } </pre>
<pre> <b>chanClient</b>(<i>stackCh</i>) <math>\triangleq</math>   <b>give</b> (<b>Push</b>(5)) <i>stackCh</i>;   let <i>resCh</i> <math>\leftarrow</math> <b>newCh</b> in   <b>give</b> (<b>Pop</b>(<i>resCh</i>)) <i>stackCh</i>;   <b>take</b> <i>resCh</i> </pre>	<pre> <b>actorClient</b>(<i>stackPid</i>) <math>\triangleq</math>   <b>send</b> (<b>Push</b>(5)) <i>stackPid</i>;   let <i>selfPid</i> <math>\leftarrow</math> <b>self</b> in   <b>send</b> (<b>Pop</b>(<i>selfPid</i>)) <i>stackPid</i>;   <b>receive</b> </pre>
<pre> <b>chanMain</b> <math>\triangleq</math>   let <i>stackCh</i> <math>\leftarrow</math> <b>newCh</b> in   <b>fork</b> (<b>chanStack</b>(<i>stackCh</i>) [ ]);   <b>chanClient</b>(<i>stackCh</i>) </pre>	<pre> <b>actorMain</b> <math>\triangleq</math>   let <i>stackPid</i> <math>\leftarrow</math> <b>spawn</b> (<b>actorStack</b> [ ]) in   <b>actorClient</b>(<i>stackPid</i>) </pre>

(a) Channel-based stack. (b) Actor-based stack.

■ **Figure 3** Concurrent stacks using channels and actors.

- primitives specialised to each model. We give a type system and operational semantics for each calculus, and precisely characterise the notion of progress that each calculus enjoys.
2. A simple translation from  $\lambda_{\text{act}}$  into  $\lambda_{\text{ch}}$  (Section 5), and a more involved translation from  $\lambda_{\text{ch}}$  into  $\lambda_{\text{act}}$  (Section 6), with proofs that both translations are type- and semantics-preserving. While the former translation is straightforward, it is *global*, in the sense of Felleisen [12]. While the latter is more involved, it is in fact *local*. Our initial translation from  $\lambda_{\text{ch}}$  to  $\lambda_{\text{act}}$  sidesteps type pollution by assigning the same type to each channel in the system.
  3. An extension of  $\lambda_{\text{act}}$  to support synchronous calls, showing how this can alleviate type pollution and simplify the translation from  $\lambda_{\text{ch}}$  into  $\lambda_{\text{act}}$  (Section 7.1).
  4. An extension of  $\lambda_{\text{act}}$  to support Erlang-style selective receive, a translation from  $\lambda_{\text{act}}$  with selective receive into plain  $\lambda_{\text{act}}$ , and proofs that the translation is type- and semantics-preserving (Section 7.2).
  5. An extension of  $\lambda_{\text{ch}}$  with input-guarded choice (Section 7.3) and an outline of how  $\lambda_{\text{act}}$  might be extended with behavioural types (Section 7.4).

The rest of the paper is organised as follows: Section 2 displays side-by-side two implementations of a concurrent stack, one using channels and the other using actors; Section 3–7 presents the main technical content; Section 8 discusses related work; and Section 9 concludes.

## 2 Channels and actors side-by-side

Let us consider the example of a concurrent stack. A concurrent stack carrying values of type  $A$  can receive a command to push a value onto the top of the stack, or to pop a value and return

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```

chanClient2(intStackCh, stringStackCh)  $\triangleq$ 
  let intResCh  $\leftarrow$  newCh in
  let strResCh  $\leftarrow$  newCh in
  give (Pop(intResCh)) intStackCh;
  let res1  $\leftarrow$  take intResCh in
  give (Pop(strResCh)) stringStackCh;
  let res2  $\leftarrow$  take strResCh in
  (res1, res2)

actorClient2(intStackPid, stringStackPid)  $\triangleq$ 
  let selfPid  $\leftarrow$  self in
  send (Pop(selfPid)) intStackPid;
  let res1  $\leftarrow$  receive in
  send (Pop(selfPid)) stringStackPid;
  let res2  $\leftarrow$  receive in
  (res1, res2)

```

■ **Figure 4** Clients interacting with multiple stacks.

it to the process making the request. Assuming a standard encoding of algebraic datatypes, we define a type  $\text{Operation}(A) = \text{Push}(A) \mid \text{Pop}(B)$  (where  $B = \text{ChanRef}(A)$  for channels, and  $\text{ActorRef}(A)$  for actors) to describe operations on the stack, and  $\text{Option}(A) = \text{Some}(A) \mid \text{None}$  to handle the possibility of popping from an empty stack.

Figure 3 shows the stack implemented using channels (Figure 3a) and using actors (Figure 3b). Each implementation uses a common core language based on the simply-typed  $\lambda$ -calculus extended with recursion, lists, and sums.

At first glance, the two stack implementations seem remarkably similar. Each:

1. Waits for a command
2. Case splits on the command, and either:
  - Pushes a value onto the top of the stack, or;
  - Takes the value from the head of the stack and returns it in a response message
3. Loops with an updated state.

The main difference is that **chanStack** is parameterised over a channel  $ch$ , and retrieves a value from the channel using **take**  $ch$ . Conversely, **actorStack** retrieves a value from its mailbox using the nullary primitive **receive**.

Let us now consider functions which interact with the stacks. The **chanClient** function sends commands over the  $stackCh$  channel, and begins by pushing 5 onto the stack. Next, it creates a channel  $resCh$  to be used to receive the result and sends this in a request, before retrieving the result from the result channel using **take**. In contrast, **actorClient** performs a similar set of steps, but sends its process ID (retrieved using **self**) in the request instead of creating a new channel; the result is then retrieved from the mailbox using **receive**.

**Type pollution.** The differences become more prominent when considering clients which interact with multiple stacks of different types, as shown in Figure 4. Here, **chanClient2** creates new result channels for integers and strings, sends requests for the results, and creates a pair of type  $(\text{Option}(\text{Int}) \times \text{Option}(\text{String}))$ . The **actorClient2** function attempts to do something similar, but cannot create separate result channels. Consequently, the actor must be able to handle messages either of type  $\text{Option}(\text{Int})$  or type  $\text{Option}(\text{String})$ , meaning that the final pair has type  $(\text{Option}(\text{Int}) + \text{Option}(\text{String})) \times (\text{Option}(\text{Int}) + \text{Option}(\text{String}))$ .

Additionally, it is necessary to modify **actorStack** to use the correct injection into the actor type when sending the result; for example an integer stack would have to send a value  $\text{inl}(\text{Some}(5))$  instead of simply  $\text{Some}(5)$ . This *type pollution* problem can be addressed through the use of subtyping [22], or synchronisation abstractions such as futures [10].

## Syntax

Types	$A, B ::= \mathbf{1} \mid A \rightarrow B \mid \text{ChanRef}(A)$
Variables and names	$\alpha ::= x \mid a$
Values	$V, W ::= \alpha \mid \lambda x.M \mid ()$
Computations	$L, M, N ::= V W$ $\mid \text{let } x \leftarrow M \text{ in } N \mid \text{return } V$ $\mid \text{fork } M \mid \text{give } V W \mid \text{take } V \mid \text{newCh}$

## Value typing rules

$\frac{\text{VAR} \quad \alpha : A \in \Gamma}{\Gamma \vdash \alpha : A}$	$\frac{\text{ABS} \quad \Gamma, x : A \vdash M : B}{\Gamma \vdash \lambda x.M : A \rightarrow B}$	$\frac{\text{UNIT}}{\Gamma \vdash () : \mathbf{1}}$
---------------------------------------------------------------------------	---------------------------------------------------------------------------------------------------	-----------------------------------------------------

$$\boxed{\Gamma \vdash V : A}$$

## Computation typing rules

$\frac{\text{APP} \quad \Gamma \vdash V : A \rightarrow B \quad \Gamma \vdash W : A}{\Gamma \vdash V W : B}$	$\frac{\text{EFFLET} \quad \Gamma \vdash M : A \quad \Gamma, x : A \vdash N : B}{\Gamma \vdash \text{let } x \leftarrow M \text{ in } N : B}$	$\frac{\text{RETURN} \quad \Gamma \vdash V : A}{\Gamma \vdash \text{return } V : A}$
$\frac{\text{GIVE} \quad \Gamma \vdash V : A}{\Gamma \vdash \text{give } V W : \mathbf{1}}$	$\frac{\text{TAKE} \quad \Gamma \vdash V : \text{ChanRef}(A)}{\Gamma \vdash \text{take } V : A}$	$\frac{\text{FORK} \quad \Gamma \vdash M : \mathbf{1}}{\Gamma \vdash \text{fork } M : \mathbf{1}}$
		$\frac{\text{NEWCH}}{\Gamma \vdash \text{newCh} : \text{ChanRef}(A)}$

$$\boxed{\Gamma \vdash M : A}$$

■ **Figure 5** Syntax and typing rules for  $\lambda_{\text{ch}}$  terms and values.

### 3 $\lambda_{\text{ch}}$ : A concurrent $\lambda$ -calculus for channels

In this section we introduce  $\lambda_{\text{ch}}$ , a concurrent  $\lambda$ -calculus extended with asynchronous channels. To concentrate on the core differences between channel- and actor-style communication, we begin with minimal calculi; note that these do not contain all features (such as lists, sums, and recursion) needed to express the examples in Section 2.

#### 3.1 Syntax and typing of terms

Figure 5 gives the syntax and typing rules of  $\lambda_{\text{ch}}$ , a  $\lambda$ -calculus based on fine-grain call-by-value [34]: terms are partitioned into values and computations. Key to this formulation are two constructs:  $\text{return } V$  represents a computation that has completed, whereas  $\text{let } x \leftarrow M \text{ in } N$  evaluates  $M$  to  $\text{return } V$ , substituting  $V$  for  $x$  in  $N$ . Fine-grain call-by-value is convenient since it makes evaluation order explicit and, unlike A-normal form [13], is closed under reduction.

Types consist of the unit type  $\mathbf{1}$ , function types  $A \rightarrow B$ , and channel reference types  $\text{ChanRef}(A)$  which can be used to communicate along a channel of type  $A$ . We let  $\alpha$  range over variables  $x$  and runtime names  $a$ . We write  $\text{let } x = V \text{ in } M$  for  $(\lambda x.M) V$  and  $M; N$  for  $\text{let } x \leftarrow M \text{ in } N$ , where  $x$  is fresh.

**Communication and concurrency for channels.** The  $\text{give } V W$  operation sends value  $V$  along channel  $W$ , while  $\text{take } V$  retrieves a value from a channel  $V$ . Assuming an extension of the language with integers and arithmetic operators, we can define a function  $\text{neg}(c)$  which receives a number  $n$  along channel  $c$  and replies with the negation of  $n$  as follows:

$$\text{neg}(c) \triangleq \text{let } n \leftarrow \text{take } c \text{ in let } \text{neg}N \leftarrow (-n) \text{ in give } \text{neg}N c$$



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Syntax of evaluation contexts and configurations

$$\begin{array}{ll}
\text{Evaluation contexts} & E ::= [] \mid \text{let } x \leftarrow E \text{ in } M \\
\text{Configurations} & \mathcal{C}, \mathcal{D}, \mathcal{E} ::= \mathcal{C} \parallel \mathcal{D} \mid (\nu a)\mathcal{C} \mid a(\vec{V}) \mid M \\
\text{Configuration contexts} & G ::= [] \mid G \parallel \mathcal{C} \mid (\nu a)G
\end{array}$$

Typing rules for configurations

$$\begin{array}{c}
\boxed{\Gamma; \Delta \vdash \mathcal{C}} \\
\text{PAR} \quad \frac{\Gamma; \Delta_1 \vdash \mathcal{C}_1 \quad \Gamma; \Delta_2 \vdash \mathcal{C}_2}{\Gamma; \Delta_1, \Delta_2 \vdash \mathcal{C}_1 \parallel \mathcal{C}_2} \quad \text{CHAN} \quad \frac{\Gamma, a : \text{ChanRef}(A); \Delta, a : A \vdash \mathcal{C}}{\Gamma; \Delta \vdash (\nu a)\mathcal{C}} \quad \text{BUF} \quad \frac{(\Gamma \vdash V_i : A)_i}{\Gamma; a : A \vdash a(\vec{V})} \quad \text{TERM} \quad \frac{\Gamma \vdash M : \mathbf{1}}{\Gamma; \cdot \vdash M}
\end{array}$$

■ **Figure 6**  $\lambda_{\text{ch}}$  configurations and evaluation contexts.

The **fork**  $M$  operation spawns a new process to evaluate term  $M$ . The operation returns the unit value, and therefore it is not possible to interact with the process directly. The **newCh** operation creates a new channel. Note that channel creation is decoupled from process creation, meaning that a process can have access to multiple channels.

### 3.2 Operational semantics

**Configurations.** The concurrent behaviour of  $\lambda_{\text{ch}}$  is given by a nondeterministic reduction relation on *configurations* (Figure 6). Configurations consist of parallel composition ( $\mathcal{C} \parallel \mathcal{D}$ ), restrictions ( $(\nu a)\mathcal{C}$ ), computations ( $M$ ), and buffers ( $a(\vec{V})$ , where  $\vec{V} = V_1 \cdot \dots \cdot V_n$ ).

**Evaluation contexts.** Reduction is defined in terms of evaluation contexts  $E$ , which are simplified due to fine-grain call-by-value. We also define configuration contexts, allowing reduction modulo parallel composition and name restriction.

**Reduction.** Figure 7 shows the reduction rules for  $\lambda_{\text{ch}}$ . Reduction is defined as a deterministic reduction on terms ( $\longrightarrow_{\text{M}}$ ) and a nondeterministic reduction relation on configurations ( $\longrightarrow$ ). Reduction on configurations is defined modulo structural congruence rules which capture scope extrusion and the commutativity and associativity of parallel composition.

**Typing of configurations.** To ensure that buffers are well-scoped and contain values of the correct type, we define typing rules on configurations (Figure 6). The judgement  $\Gamma; \Delta \vdash \mathcal{C}$  states that under environments  $\Gamma$  and  $\Delta$ ,  $\mathcal{C}$  is well-typed.  $\Gamma$  is a typing environment for terms, whereas  $\Delta$  is a linear typing environment for configurations, mapping names  $a$  to channel types  $A$ . Linearity in  $\Delta$  ensures that a configuration  $\mathcal{C}$  under a name restriction  $(\nu a)\mathcal{C}$  contains exactly one buffer with name  $a$ . Note that **CHAN** extends both  $\Gamma$  and  $\Delta$ , adding an (unrestricted) *reference* into  $\Gamma$  and the *capability* to type a buffer into  $\Delta$ . **PAR** states that  $\mathcal{C}_1 \parallel \mathcal{C}_2$  is typeable if  $\mathcal{C}_1$  and  $\mathcal{C}_2$  are typeable under disjoint linear environments, and **BUF** states that under a term environment  $\Gamma$  and a singleton linear environment  $a : A$ , it is possible to type a buffer  $a(\vec{V})$  if  $\Gamma \vdash V_i : A$  for all  $V_i \in \vec{V}$ . As an example,  $(\nu a)(a(\vec{V}))$  is well-typed, but  $(\nu a)(a(\vec{V}) \parallel a(\vec{W}))$  and  $(\nu a)(\text{return } ())$  are not.

**Relation notation.** Given a relation  $R$ , we write  $R^+$  for its transitive closure, and  $R^*$  for its reflexive, transitive closure.

Reduction on terms

$$(\lambda x.M)V \longrightarrow_M M\{V/x\} \quad \text{let } x \leftarrow \text{return } V \text{ in } M \longrightarrow_M M\{V/x\} \quad E[M_1] \longrightarrow_M E[M_2] \\ (\text{if } M_1 \longrightarrow_M M_2)$$

Structural congruence

$$\mathcal{C} \parallel \mathcal{D} \equiv \mathcal{D} \parallel \mathcal{C} \quad \mathcal{C} \parallel (\mathcal{D} \parallel \mathcal{E}) \equiv (\mathcal{C} \parallel \mathcal{D}) \parallel \mathcal{E} \quad \mathcal{C} \parallel (\nu a)\mathcal{D} \equiv (\nu a)(\mathcal{C} \parallel \mathcal{D}) \text{ if } a \notin \text{fv}(\mathcal{C}) \\ G[\mathcal{C}] \equiv G[\mathcal{D}] \text{ if } \mathcal{C} \equiv \mathcal{D}$$

Reduction on configurations

$$\begin{array}{llll} \text{GIVE} & E[\text{give } W a] \parallel a(\vec{V}) & \longrightarrow & E[\text{return } ()] \parallel a(\vec{V} \cdot W) \\ \text{TAKE} & E[\text{take } a] \parallel a(W \cdot \vec{V}) & \longrightarrow & E[\text{return } W] \parallel a(\vec{V}) \\ \text{FORK} & E[\text{fork } M] & \longrightarrow & E[\text{return } ()] \parallel M \\ \text{NEWCH} & E[\text{newCh}] & \longrightarrow & (\nu a)(E[\text{return } a] \parallel a(\epsilon)) \quad (a \text{ is a fresh name}) \\ \text{LIFTM} & G[M_1] & \longrightarrow & G[M_2] \quad (\text{if } M_1 \longrightarrow_M M_2) \\ \text{LIFT} & G[\mathcal{C}_1] & \longrightarrow & G[\mathcal{C}_2] \quad (\text{if } \mathcal{C}_1 \longrightarrow \mathcal{C}_2) \end{array}$$

■ **Figure 7** Reduction on  $\lambda_{\text{ch}}$  terms and configurations.

**Properties of the term language.** Reduction on terms preserves typing, and pure terms enjoy progress. We omit most proofs in the body of the paper which are mainly straightforward inductions; selected full proofs can be found in the extended version [15].

► **Lemma 1** (Preservation ( $\lambda_{\text{ch}}$  terms)). *If  $\Gamma \vdash M : A$  and  $M \longrightarrow_M M'$ , then  $\Gamma \vdash M' : A$ .*

► **Lemma 2** (Progress ( $\lambda_{\text{ch}}$  terms)). *Assume  $\Gamma$  is empty or only contains channel references  $a_i : \text{ChanRef}(A_i)$ . If  $\Gamma \vdash M : A$ , then either:*

1.  $M = \text{return } V$  for some value  $V$ , or
2.  $M$  can be written  $E[M']$ , where  $M'$  is a communication or concurrency primitive (i.e.,  $\text{give } V W$ ,  $\text{take } V$ ,  $\text{fork } M$ , or  $\text{newCh}$ ), or
3. There exists some  $M'$  such that  $M \longrightarrow_M M'$ .

**Reduction on configurations.** Concurrency and communication is captured by reduction on configurations. Reduction is defined modulo structural congruence rules, which capture the associativity and commutativity of parallel composition, as well as the usual scope extrusion rule. The GIVE rule reduces  $\text{give } W a$  in parallel with a buffer  $a(\vec{V})$  by adding the value  $W$  onto the end of the buffer. The TAKE rule reduces  $\text{take } a$  in parallel with a non-empty buffer by returning the first value in the buffer. The FORK rule reduces  $\text{fork } M$  by spawning a new thread  $M$  in parallel with the parent process. The NEWCH rule reduces  $\text{newCh}$  by creating an empty buffer and returning a fresh name for that buffer.

Structural congruence and reduction preserve the typeability of configurations.

► **Lemma 3.** *If  $\Gamma; \Delta \vdash \mathcal{C}$  and  $\mathcal{C} \equiv \mathcal{D}$  for some configuration  $\mathcal{D}$ , then  $\Gamma; \Delta \vdash \mathcal{D}$ .*

► **Theorem 4** (Preservation ( $\lambda_{\text{ch}}$  configurations)). *If  $\Gamma; \Delta \vdash \mathcal{C}_1$  and  $\mathcal{C}_1 \longrightarrow \mathcal{C}_2$  then  $\Gamma; \Delta \vdash \mathcal{C}_2$ .*

### 3.3 Progress and canonical forms

While it is possible to prove deadlock-freedom in systems with more discerning type systems based on linear logic [48, 35] or those using channel priorities [41], more liberal calculi such

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as  $\lambda_{\text{ch}}$  and  $\lambda_{\text{act}}$  allow deadlocked configurations. We thus define a form of progress which does not preclude deadlock; to help with proving a progress result, it is useful to consider the notion of a *canonical form* in order to allow us to reason about the configuration as a whole.

► **Definition 5** (Canonical form ( $\lambda_{\text{ch}}$ )). A configuration  $\mathcal{C}$  is in *canonical form* if it can be written  $(\nu a_1) \dots (\nu a_n)(M_1 \parallel \dots \parallel M_m \parallel a_1(\vec{V}_1) \parallel \dots \parallel a_n(\vec{V}_n))$ .

Well-typed open configurations can be written in a form similar to canonical form, but without bindings for names already in the environment. An immediate corollary is that well-typed closed configurations can always be written in a canonical form.

► **Lemma 6.** *If  $\Gamma; \Delta \vdash \mathcal{C}$  with  $\Delta = a_1 : A_1, \dots, a_k : A_k$ , then there exists a  $\mathcal{C}' \equiv \mathcal{C}$  such that  $\mathcal{C}' = (\nu a_{k+1}) \dots (\nu a_n)(M_1 \parallel \dots \parallel M_m \parallel a_1(\vec{V}_1) \parallel \dots \parallel a_n(\vec{V}_n))$ .*

► **Corollary 7.** *If  $;\cdot \vdash \mathcal{C}$ , then there exists some  $\mathcal{C}' \equiv \mathcal{C}$  such that  $\mathcal{C}'$  is in canonical form.*

Armed with a canonical form, we can now state that the only situation in which a well-typed closed configuration cannot reduce further is if all threads are either blocked or fully evaluated. Let a *leaf configuration* be a configuration without subconfigurations, i.e., a term or a buffer.

► **Theorem 8** (Weak progress ( $\lambda_{\text{ch}}$  configurations)).

*Let  $;\cdot \vdash \mathcal{C}$ ,  $\mathcal{C} \not\rightarrow$ , and let  $\mathcal{C}' = (\nu a_1) \dots (\nu a_n)(M_1 \parallel \dots \parallel M_m \parallel a_1(\vec{V}_1) \parallel \dots \parallel a_n(\vec{V}_n))$  be a canonical form of  $\mathcal{C}$ . Then every leaf of  $\mathcal{C}$  is either:*

1. A buffer  $a_i(\vec{V}_i)$ ;
2. A fully-reduced term of the form `return V`, or;
3. A term of the form  $E[\text{take } a_i]$ , where  $\vec{V}_i = \epsilon$ .

**Proof.** By Lemma 2, we know each  $M_i$  is either of the form `return V`, or can be written  $E[M']$  where  $M'$  is a communication or concurrency primitive. It cannot be the case that  $M' = \text{fork } N$  or  $M' = \text{newCh}$ , since both can reduce. Let us now consider `give` and `take`, blocked on a variable  $\alpha$ . As we are considering closed configurations, a blocked term must be blocked on a  $\nu$ -bound name  $a_i$ , and as per the canonical form, we have that there exists some buffer  $a_i(\vec{V}_i)$ . Consequently, `give V a_i` can always reduce via GIVE. A term `take a_i` can reduce by TAKE if  $\vec{V}_i = W \cdot \vec{V}'_i$ ; the only remaining case is where  $\vec{V}_i = \epsilon$ , satisfying (3). ◀

### 4 $\lambda_{\text{act}}$ : A concurrent $\lambda$ -calculus for actors

In this section, we introduce  $\lambda_{\text{act}}$ , a core language describing actor-based concurrency. There are many variations of actor-based languages (by the taxonomy of De Koster et al; [11],  $\lambda_{\text{act}}$  is *process-based*), but each have named processes associated with a mailbox.

Typed channels are well-established, whereas typed actors are less so, partly due to the type pollution problem. Nonetheless, Akka Typed [4] aims to replace untyped Akka actors, so studying a typed actor calculus is of practical relevance.

Following Erlang, we provide an explicit `receive` operation to allow an actor to retrieve a message from its mailbox: unlike `take` in  $\lambda_{\text{ch}}$ , `receive` takes no arguments, so it is necessary to use a simple *type-and-effect system* [18]. We treat mailboxes as a FIFO queues to keep  $\lambda_{\text{act}}$  as minimal as possible, as opposed to considering behaviours or selective receive. This is orthogonal to the core model of communication, as we show in Section 7.2.

## Syntax

Types	$A, B, C ::= \mathbf{1} \mid A \rightarrow^C B \mid \text{ActorRef}(A)$
Variables and names	$\alpha ::= x \mid a$
Values	$V, W ::= \alpha \mid \lambda x. M \mid ()$
Computations	$L, M, N ::= V W$ $\mid \text{let } x \leftarrow M \text{ in } N \mid \text{return } V$ $\mid \text{spawn } M \mid \text{send } V W \mid \text{receive} \mid \text{self}$

## Value typing rules

$\text{VAR}$	$\text{ABS}$	$\text{UNIT}$
$\frac{\alpha : A \in \Gamma}{\Gamma \vdash \alpha : A}$	$\frac{\Gamma, x : A \mid C \vdash M : B}{\Gamma \vdash \lambda x. M : A \rightarrow^C B}$	$\frac{}{\Gamma \vdash () : \mathbf{1}}$

$$\boxed{\Gamma \vdash V : A}$$

## Computation typing rules

$\text{APP}$	$\text{EFFLET}$	$\text{EFFRETURN}$	$\text{SEND}$
$\frac{\Gamma \vdash V : A \rightarrow^C B \quad \Gamma \vdash W : A}{\Gamma \mid C \vdash V W : B}$	$\frac{\Gamma \mid C \vdash M : A \quad \Gamma, x : A \mid C \vdash N : B}{\Gamma \mid C \vdash \text{let } x \leftarrow M \text{ in } N : B}$	$\frac{\Gamma \vdash V : A}{\Gamma \mid C \vdash \text{return } V : A}$	$\frac{\Gamma \vdash V : A}{\Gamma \mid C \vdash \text{send } V W : \mathbf{1}}$
$\text{RECV}$	$\text{SPAWN}$	$\text{SELF}$	
$\frac{}{\Gamma \mid A \vdash \text{receive} : A}$	$\frac{\Gamma \mid A \vdash M : \mathbf{1}}{\Gamma \mid C \vdash \text{spawn } M : \text{ActorRef}(A)}$	$\frac{}{\Gamma \mid A \vdash \text{self} : \text{ActorRef}(A)}$	

$$\boxed{\Gamma \mid B \vdash M : A}$$

■ **Figure 8** Syntax and typing rules for  $\lambda_{\text{act}}$ .

## 4.1 Syntax and typing of terms

Figure 8 shows the syntax and typing rules for  $\lambda_{\text{act}}$ . As with  $\lambda_{\text{ch}}$ ,  $\alpha$  ranges over variables and names.  $\text{ActorRef}(A)$  is an *actor reference* or process ID, and allows messages to be sent to an actor. As for communication and concurrency primitives, **spawn**  $M$  spawns a new actor to evaluate a computation  $M$ ; **send**  $V W$  sends a value  $V$  to an actor referred to by reference  $W$ ; **receive** receives a value from the actor's mailbox; and **self** returns an actor's own process ID.

Function arrows  $A \rightarrow^C B$  are annotated with a type  $C$  which denotes the type of the mailbox of the actor evaluating the term. As an example, consider a function which receives an integer and converts it to a string (assuming a function **intToString**):

$$\text{recvAndShow} \triangleq \lambda(). \text{let } x \leftarrow \text{receive in intToString}(x)$$

Such a function would have type  $\mathbf{1} \rightarrow^{\text{Int}} \text{String}$ , and as an example would not be typeable for an actor that could only receive booleans. Again, we work in the setting of fine-grain call-by-value; the distinction between values and computations is helpful when reasoning about the metatheory. We have two typing judgements: the standard judgement on values  $\Gamma \vdash V : A$ , and a judgement  $\Gamma \mid B \vdash M : A$  which states that a term  $M$  has type  $A$  under typing context  $\Gamma$ , and can receive values of type  $B$ . The typing of **receive** and **self** depends on the type of the actor's mailbox.

## 4.2 Operational semantics

Figure 9 shows the syntax of  $\lambda_{\text{act}}$  evaluation contexts, as well as the syntax and typing rules of  $\lambda_{\text{act}}$  configurations. Evaluation contexts for terms and configurations are similar to  $\lambda_{\text{ch}}$ . The primary difference from  $\lambda_{\text{ch}}$  is the actor configuration  $\langle a, M, \vec{V} \rangle$ , which can be read as

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Syntax of evaluation contexts and configurations

$$\begin{array}{ll}
\text{Evaluation contexts} & E ::= [] \mid \text{let } x \leftarrow E \text{ in } M \\
\text{Configurations} & \mathcal{C}, \mathcal{D}, \mathcal{E} ::= \mathcal{C} \parallel \mathcal{D} \mid (\nu a)\mathcal{C} \mid \langle a, M, \vec{V} \rangle \\
\text{Configuration contexts} & G ::= [] \mid G \parallel \mathcal{C} \mid (\nu a)G
\end{array}$$

Typing rules for configurations

$$\begin{array}{c}
\boxed{\Gamma; \Delta \vdash \mathcal{C}} \\
\text{PAR} \quad \frac{\Gamma; \Delta_1 \vdash \mathcal{C}_1 \quad \Gamma; \Delta_2 \vdash \mathcal{C}_2}{\Gamma; \Delta_1, \Delta_2 \vdash \mathcal{C}_1 \parallel \mathcal{C}_2} \quad \text{PID} \quad \frac{\Gamma, a : \text{ActorRef}(A); \Delta, a : A \vdash \mathcal{C}}{\Gamma; \Delta \vdash (\nu a)\mathcal{C}} \quad \text{ACTOR} \quad \frac{\Gamma, a : \text{ActorRef}(A) \mid A \vdash M : \mathbf{1} \quad (\Gamma, a : \text{ActorRef}(A) \vdash V_i : A)_i}{\Gamma, a : \text{ActorRef}(A); a : A \vdash \langle a, M, \vec{V} \rangle}
\end{array}$$

■ **Figure 9**  $\lambda_{\text{act}}$  evaluation contexts and configurations.

“an actor with name  $a$  evaluating term  $M$ , with a mailbox consisting of values  $\vec{V}$ ”. Whereas a term  $M$  is itself a configuration in  $\lambda_{\text{ch}}$ , a term in  $\lambda_{\text{act}}$  must be evaluated as part of an actor configuration in order to support context-sensitive operations such as receiving from the mailbox. We again stratify the reduction rules into functional reduction on terms, and reduction on configurations. The typing rules for  $\lambda_{\text{act}}$  configurations ensure that all values contained in an actor mailbox are well-typed with respect to the mailbox type, and that a configuration  $\mathcal{C}$  under a name restriction  $(\nu a)\mathcal{C}$  contains an actor with name  $a$ . Figure 10 shows the reduction rules for  $\lambda_{\text{act}}$ . Again, reduction on terms preserves typing, and the functional fragment of  $\lambda_{\text{act}}$  enjoys progress.

► **Lemma 9** (Preservation ( $\lambda_{\text{act}}$  terms)). *If  $\Gamma \vdash M : A$  and  $M \longrightarrow_M M'$ , then  $\Gamma \vdash M' : A$ .*

► **Lemma 10** (Progress ( $\lambda_{\text{act}}$  terms)). *Assume  $\Gamma$  is either empty or only contains entries of the form  $a_i : \text{ActorRef}(A_i)$ . If  $\Gamma \mid B \vdash M : A$ , then either:*

1.  $M = \text{return } V$  for some value  $V$ , or
2.  $M$  can be written as  $E[M']$ , where  $M'$  is a communication or concurrency primitive (i.e. `spawn`  $N$ , `send`  $VW$ , `receive`, or `self`), or
3. There exists some  $M'$  such that  $M \longrightarrow_M M'$ .

**Reduction on configurations.** While  $\lambda_{\text{ch}}$  makes use of separate constructs to create new processes and channels,  $\lambda_{\text{act}}$  uses a single construct `spawn`  $M$  to spawn a new actor with an empty mailbox to evaluate term  $M$ . Communication happens directly between actors instead of through an intermediate entity: as a result of evaluating `send`  $Va$ , the value  $V$  will be appended directly to the end of the mailbox of actor  $a$ . `SENDSSELF` allows reflexive sending; an alternative would be to decouple mailboxes from the definition of actors, but this complicates both the configuration typing rules and the intuition. `SELF` returns the name of the current process, and `RECEIVE` retrieves the head value of a non-empty mailbox.

As before, typing is preserved modulo structural congruence and under reduction.

► **Lemma 11.** *If  $\Gamma; \Delta \vdash \mathcal{C}$  and  $\mathcal{C} \equiv \mathcal{D}$  for some  $\mathcal{D}$ , then  $\Gamma; \Delta \vdash \mathcal{D}$ .*

► **Theorem 12** (Preservation ( $\lambda_{\text{act}}$  configurations)). *If  $\Gamma; \Delta \vdash \mathcal{C}_1$  and  $\mathcal{C}_1 \longrightarrow \mathcal{C}_2$ , then  $\Gamma; \Delta \vdash \mathcal{C}_2$ .*

### 4.3 Progress and canonical forms

Again, we cannot guarantee deadlock-freedom for  $\lambda_{\text{act}}$ . Instead, we proceed by defining a canonical form, and characterising the form of progress that  $\lambda_{\text{act}}$  enjoys. The technical development follows that of  $\lambda_{\text{ch}}$ .

Reduction on terms

$$(\lambda x.M)V \longrightarrow_M M\{V/x\} \quad \text{let } x \leftarrow \text{return } V \text{ in } M \longrightarrow_M M\{V/x\} \quad E[M] \longrightarrow_M E[M'] \\ \text{(if } M \longrightarrow_M M')$$

Structural congruence

$$\mathcal{C} \parallel \mathcal{D} \equiv \mathcal{D} \parallel \mathcal{C} \quad \mathcal{C} \parallel (\mathcal{D} \parallel \mathcal{E}) \equiv (\mathcal{C} \parallel \mathcal{D}) \parallel \mathcal{E} \quad \mathcal{C} \parallel (\nu a)\mathcal{D} \equiv (\nu a)(\mathcal{C} \parallel \mathcal{D}) \text{ if } a \notin \text{fv}(\mathcal{C}) \\ G[\mathcal{C}] \equiv G[\mathcal{D}] \text{ if } \mathcal{C} \equiv \mathcal{D}$$

Reduction on configurations

SPAWN	$\langle a, E[\text{spawn } M], \vec{V} \rangle$	$\longrightarrow$	$(\nu b)(\langle a, E[\text{return } b], \vec{V} \rangle \parallel \langle b, M, \epsilon \rangle)$ ( <i>b is fresh</i> )
SEND	$\langle a, E[\text{send } V' b], \vec{V} \rangle \parallel \langle b, M, \vec{W} \rangle$	$\longrightarrow$	$\langle a, E[\text{return } ()], \vec{V} \rangle \parallel \langle b, M, \vec{W} \cdot V' \rangle$
SENDSSELF	$\langle a, E[\text{send } V' a], \vec{V} \rangle$	$\longrightarrow$	$\langle a, E[\text{return } ()], \vec{V} \cdot V' \rangle$
SELF	$\langle a, E[\text{self}], \vec{V} \rangle$	$\longrightarrow$	$\langle a, E[\text{return } a], \vec{V} \rangle$
RECEIVE	$\langle a, E[\text{receive}], W \cdot \vec{V} \rangle$	$\longrightarrow$	$\langle a, E[\text{return } W], \vec{V} \rangle$
LIFT		$\longrightarrow$	$G[\mathcal{C}_1] \longrightarrow G[\mathcal{C}_2] \text{ (if } \mathcal{C}_1 \longrightarrow \mathcal{C}_2)$
LIFTM	$\langle a, M_1, \vec{V} \rangle$	$\longrightarrow$	$\langle a, M_2, \vec{V} \rangle \text{ (if } M_1 \longrightarrow_M M_2)$

■ **Figure 10** Reduction on  $\lambda_{\text{act}}$  terms and configurations.

► **Definition 13** (Canonical form ( $\lambda_{\text{act}}$ )). A  $\lambda_{\text{act}}$  configuration  $\mathcal{C}$  is in *canonical form* if  $\mathcal{C}$  can be written  $(\nu a_1) \dots (\nu a_n)(\langle a_1, M_1, \vec{V}_1 \rangle \parallel \dots \parallel \langle a_n, M_n, \vec{V}_n \rangle)$ .

► **Lemma 14.** *If  $\Gamma; \Delta \vdash \mathcal{C}$  and  $\Delta = a_1 : A_1, \dots, a_k : A_k$ , then there exists  $\mathcal{C}' \equiv \mathcal{C}$  such that  $\mathcal{C}' = (\nu a_{k+1}) \dots (\nu a_n)(\langle a_1, M_1, \vec{V}_1 \rangle \parallel \dots \parallel \langle a_n, M_n, \vec{V}_n \rangle)$ .*

As before, it follows as a corollary of Lemma 14 that closed configurations can be written in canonical form. We can therefore classify the notion of progress enjoyed by  $\lambda_{\text{act}}$ .

► **Corollary 15.** *If  $\cdot; \cdot \vdash \mathcal{C}$ , then there exists some  $\mathcal{C}' \equiv \mathcal{C}$  such that  $\mathcal{C}'$  is in canonical form.*

► **Theorem 16** (Weak progress ( $\lambda_{\text{act}}$  configurations)).

*Let  $\cdot; \cdot \vdash \mathcal{C}$ ,  $\mathcal{C} \not\rightarrow$ , and let  $\mathcal{C}' = (\nu a_1) \dots (\nu a_n)(\langle a_1, M_1, \vec{V}_1 \rangle \parallel \dots \parallel \langle a_n, M_n, \vec{V}_n \rangle)$  be a canonical form of  $\mathcal{C}$ . Each actor with name  $a_i$  is either of the form  $\langle a_i, \text{return } W, \vec{V}_i \rangle$  for some value  $W$ , or  $\langle a_i, E[\text{receive}], \epsilon \rangle$ .*

## 5 From $\lambda_{\text{act}}$ to $\lambda_{\text{ch}}$

With both calculi in place, we can define the translation from  $\lambda_{\text{act}}$  into  $\lambda_{\text{ch}}$ . The key idea is to emulate a mailbox using a channel, and to pass the channel as an argument to each function. The translation on terms is parameterised over the channel name, which is used to implement context-dependent operations (i.e., `receive` and `self`). Consider again `recvAndShow`.

$$\text{recvAndShow} \triangleq \lambda(). \text{let } x \leftarrow \text{receive in intToString}(x)$$

A possible configuration would be an actor evaluating `recvAndShow()`, with some name  $a$  and mailbox with values  $\vec{V}$ , under a name restriction for  $a$ .

$$(\nu a)(\langle a, \text{recvAndShow } (), \vec{V} \rangle)$$

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Translation on types

$$\llbracket \text{ActorRef}(A) \rrbracket = \text{ChanRef}(\llbracket A \rrbracket) \quad \llbracket A \rightarrow^C B \rrbracket = \llbracket A \rrbracket \rightarrow \text{ChanRef}(\llbracket C \rrbracket) \rightarrow \llbracket B \rrbracket \quad \llbracket \mathbf{1} \rrbracket = \mathbf{1}$$

Translation on values

$$\llbracket x \rrbracket = x \quad \llbracket a \rrbracket = a \quad \llbracket \lambda x.M \rrbracket = \lambda x.\lambda ch.(\llbracket M \rrbracket ch) \quad \llbracket () \rrbracket = ()$$

Translation on computation terms

$$\begin{aligned} \llbracket \text{let } x \leftarrow M \text{ in } N \rrbracket ch &= \text{let } x \leftarrow (\llbracket M \rrbracket ch) \text{ in } \llbracket N \rrbracket ch \\ \llbracket V W \rrbracket ch &= \text{let } f \leftarrow (\llbracket V \rrbracket \llbracket W \rrbracket) \text{ in } f ch & \llbracket \text{spawn } M \rrbracket ch &= \text{let } chMb \leftarrow \text{newCh in} \\ \llbracket \text{return } V \rrbracket ch &= \text{return } \llbracket V \rrbracket & & \text{fork } (\llbracket M \rrbracket chMb); \\ \llbracket \text{self} \rrbracket ch &= \text{return } ch & & \text{return } chMb \\ \llbracket \text{receive} \rrbracket ch &= \text{take } ch & \llbracket \text{send } V W \rrbracket ch &= \text{give } (\llbracket V \rrbracket) (\llbracket W \rrbracket) \end{aligned}$$

Translation on configurations

$$\llbracket \mathcal{C}_1 \parallel \mathcal{C}_2 \rrbracket = \llbracket \mathcal{C}_1 \rrbracket \parallel \llbracket \mathcal{C}_2 \rrbracket \quad \llbracket (\nu a)\mathcal{C} \rrbracket = (\nu a) \llbracket \mathcal{C} \rrbracket \quad \llbracket \langle a, M, \vec{V} \rangle \rrbracket = a(\llbracket \vec{V} \rrbracket) \parallel (\llbracket M \rrbracket a)$$

■ **Figure 11** Translation from  $\lambda_{\text{act}}$  into  $\lambda_{\text{ch}}$ .

The translation on terms takes a channel name  $ch$  as a parameter. As a result of the translation, we have that:

$$\llbracket \text{recvAndShow}() \rrbracket ch = \text{let } x \leftarrow \text{take } ch \text{ in } \text{intToString}(x)$$

with the corresponding configuration  $(\nu a)(a(\llbracket \vec{V} \rrbracket) \parallel \llbracket \text{recvAndShow}() \rrbracket a)$ . The values from the mailbox are translated pointwise and form the contents of a buffer with name  $a$ . The translation of `recvAndShow` is provided with the name  $a$  which is used to emulate `receive`.

### 5.1 Translation ( $\lambda_{\text{act}}$ to $\lambda_{\text{ch}}$ )

Figure 11 shows the formal translation from  $\lambda_{\text{act}}$  into  $\lambda_{\text{ch}}$ . Of particular note is the translation on terms:  $\llbracket - \rrbracket ch$  translates a  $\lambda_{\text{act}}$  term into a  $\lambda_{\text{ch}}$  term using a channel with name  $ch$  to emulate a mailbox. An actor reference is represented as a channel reference in  $\lambda_{\text{ch}}$ ; we emulate sending a message to another actor by writing to the channel emulating the recipient's mailbox. Key to translating  $\lambda_{\text{act}}$  into  $\lambda_{\text{ch}}$  is the translation of function arrows  $A \rightarrow^C B$ ; the effect annotation  $C$  is replaced by a second parameter  $\text{ChanRef}(C)$ , which is used to emulate the mailbox of the actor. Values translate to themselves, with the exception of  $\lambda$  abstractions, whose translation takes an additional parameter denoting the channel used to emulate operations on a mailbox. Given parameter  $ch$ , the translation function for terms emulates `receive` by taking a value from  $ch$ , and emulates `self` by returning  $ch$ .

Though the translation is straightforward, it is a *global* translation [12], as all functions must be modified in order to take the mailbox channel as an additional parameter.

### 5.2 Properties of the translation

The translation on terms and values preserves typing. We extend the translation function pointwise to typing environments:  $\llbracket \alpha_1 : A_1, \dots, \alpha_n : A_n \rrbracket = \alpha_1 : \llbracket A_1 \rrbracket, \dots, \alpha_n : \llbracket A_n \rrbracket$ .

► **Lemma 17** ( $\llbracket - \rrbracket$  preserves typing (terms and values)).

1. If  $\Gamma \vdash V : A$  in  $\lambda_{\text{act}}$ , then  $\llbracket \Gamma \rrbracket \vdash \llbracket V \rrbracket : \llbracket A \rrbracket$  in  $\lambda_{\text{ch}}$ .

2. If  $\Gamma \mid B \vdash M : A$  in  $\lambda_{act}$ , then  $\llbracket \Gamma \rrbracket, \alpha : ChanRef(\llbracket B \rrbracket) \vdash \llbracket M \rrbracket \alpha : \llbracket A \rrbracket$  in  $\lambda_{ch}$ .

The proof is by simultaneous induction on the derivations of  $\Gamma \vdash V : A$  and  $\Gamma \mid B \vdash M : A$ . To state a semantics preservation result, we also define a translation on configurations; the translations on parallel composition and name restrictions are homomorphic. An actor configuration  $\langle a, M, \vec{V} \rangle$  is translated as a buffer  $a(\llbracket \vec{V} \rrbracket)$ , (writing  $\llbracket \vec{V} \rrbracket = \llbracket V_0 \rrbracket, \dots, \llbracket V_n \rrbracket$  for each  $V_i \in \vec{V}$ ), composed in parallel with the translation of  $M$ , using  $a$  as the mailbox channel. We can now see that the translation preserves typeability of configurations.

► **Theorem 18** ( $\llbracket - \rrbracket$  preserves typeability (configurations)).

If  $\Gamma; \Delta \vdash C$  in  $\lambda_{act}$ , then  $\llbracket \Gamma \rrbracket; \llbracket \Delta \rrbracket \vdash \llbracket C \rrbracket$  in  $\lambda_{ch}$ .

We describe semantics preservation in terms of a simulation theorem: should a configuration  $\mathcal{C}_1$  reduce to a configuration  $\mathcal{C}_2$  in  $\lambda_{act}$ , then there exists some configuration  $\mathcal{D}$  in  $\lambda_{ch}$  such that  $\llbracket \mathcal{C}_1 \rrbracket$  reduces in zero or more steps to  $\mathcal{D}$ , with  $\mathcal{D} \equiv \llbracket \mathcal{C}_2 \rrbracket$ . To establish the result, we begin by showing that  $\lambda_{act}$  term reduction can be simulated in  $\lambda_{ch}$ .

► **Lemma 19** (Simulation of  $\lambda_{act}$  term reduction in  $\lambda_{ch}$ ).

If  $\Gamma \vdash M_1 : A$  and  $M_1 \rightarrow_M M_2$  in  $\lambda_{act}$ , then given some  $\alpha$ ,  $\llbracket M_1 \rrbracket \alpha \rightarrow_M^* \llbracket M_2 \rrbracket \alpha$  in  $\lambda_{ch}$ .

Finally, we can see that the translation preserves structural congruences, and that  $\lambda_{ch}$  configurations can simulate reductions in  $\lambda_{act}$ .

► **Lemma 20.** If  $\Gamma; \Delta \vdash C$  and  $C \equiv \mathcal{D}$ , then  $\llbracket C \rrbracket \equiv \llbracket \mathcal{D} \rrbracket$ .

► **Theorem 21** (Simulation of  $\lambda_{act}$  configurations in  $\lambda_{ch}$ ).

If  $\Gamma; \Delta \vdash \mathcal{C}_1$  and  $\mathcal{C}_1 \rightarrow \mathcal{C}_2$ , then there exists some  $\mathcal{D}$  such that  $\llbracket \mathcal{C}_1 \rrbracket \rightarrow^* \mathcal{D}$ , with  $\mathcal{D} \equiv \llbracket \mathcal{C}_2 \rrbracket$ .

## 6 From $\lambda_{ch}$ to $\lambda_{act}$

The translation from  $\lambda_{act}$  into  $\lambda_{ch}$  emulates an actor mailbox using a channel to implement operations which normally rely on the context of the actor. Though global, the translation is straightforward due to the limited form of communication supported by mailboxes. Translating from  $\lambda_{ch}$  into  $\lambda_{act}$  is more challenging, as would be expected from Figure 2. Each channel in a system may have a different type; each process may have access to multiple channels; and (crucially) channels may be freely passed between processes.

### 6.1 Extensions to the core language

We require several more language constructs: sums, products, recursive functions, and iso-recursive types. Recursive functions are used to implement an event loop, and recursive types to maintain a term-level buffer. Products are used to record both a list of values in the buffer and a list of pending requests. Sum types allow the disambiguation of the two types of messages sent to an actor: one to queue a value (emulating **give**) and one to dequeue a value (emulating **take**). Sums are also used to encode monomorphic variant types; we write  $\langle \ell_1 : A_1, \dots, \ell_n : A_n \rangle$  for variant types and  $\langle \ell_i = V \rangle$  for variant values.

Figure 12 shows the extensions to the core term language and their reduction rules; we omit the symmetric rules for **inr**. With products, sums, and recursive types, we can encode lists. The typing rules are shown for  $\lambda_{ch}$  but can be easily adapted for  $\lambda_{act}$ , and it is straightforward to verify that the extended languages still enjoy progress and preservation.



## 11:16 Mixing Metaphors

Syntax

Types  $A, B, C ::= \dots \mid A \times B \mid A + B \mid \text{List}(A) \mid \mu X.A \mid X$   
 Values  $V, W ::= \dots \mid \text{rec } f(x).M \mid (V, W) \mid \text{inl } V \mid \text{inr } W \mid \text{roll } V$   
 Terms  $L, M, N ::= \dots \mid \text{let } (x, y) = V \text{ in } M \mid \text{case } V \{ \text{inl } x \mapsto M; \text{inr } y \mapsto N \} \mid \text{unroll } V$

Additional value typing rules

$$\frac{}{\Gamma \vdash V : A} \quad \frac{\text{REC} \quad \Gamma, x : A, f : A \rightarrow B \vdash M : B}{\Gamma \vdash \text{rec } f(x).M : A \rightarrow B} \quad \frac{\text{PAIR} \quad \Gamma \vdash V : A \quad \Gamma \vdash W : B}{\Gamma \vdash (V, W) : A \times B} \quad \frac{\text{INL} \quad \Gamma \vdash V : A}{\Gamma \vdash \text{inl } V : A + B} \quad \frac{\text{ROLL} \quad \Gamma \vdash V : A\{\mu X.A/X\}}{\Gamma \vdash \text{roll } V : \mu X.A}$$

Additional term typing rules

$$\frac{}{\Gamma \vdash M : A} \quad \frac{\text{LET} \quad \Gamma \vdash V : A \times A' \quad \Gamma, x : A, y : A' \vdash M : B}{\Gamma \vdash \text{let } (x, y) = V \text{ in } M : B} \quad \frac{\text{CASE} \quad \Gamma \vdash V : A + A' \quad \Gamma, x : A \vdash M : B \quad \Gamma, y : A' \vdash N : B}{\Gamma \vdash \text{case } V \{ \text{inl } x \mapsto M; \text{inr } y \mapsto N \} : B} \quad \frac{\text{UNROLL} \quad \Gamma \vdash V : \mu X.A}{\Gamma \vdash \text{unroll } V : A\{\mu X.A/X\}}$$

Additional term reduction rules

$$\frac{}{M \rightarrow_M M'} \quad \begin{aligned} &(\text{rec } f(x).M).V \rightarrow_M M\{(\text{rec } f(x).M)/f, V/x\} \\ &\text{let } (x, y) = (V, W) \text{ in } M \rightarrow_M M\{V/x, W/y\} \\ &\text{case } (\text{inl } V) \{ \text{inl } x \mapsto M; \text{inr } y \mapsto N \} \rightarrow_M M\{V/x\} \\ &\text{unroll } (\text{roll } V) \rightarrow_M \text{return } V \end{aligned}$$

Encoding of lists

$$\begin{aligned} \text{List}(A) &\triangleq \mu X.1 + (A \times X) & [\ ] &\triangleq \text{roll } (\text{inl } ()) & V :: W &\triangleq \text{roll } (\text{inr } (V, W)) \\ \text{case } V \{ [\ ] \mapsto M; x :: y \mapsto N \} &\triangleq \text{let } z \leftarrow \text{unroll } V \text{ in case } z \{ \text{inl } () \mapsto M; \text{inr } (x, y) \mapsto N \} \end{aligned}$$

■ **Figure 12** Extensions to core languages to allow translation from  $\lambda_{\text{ch}}$  into  $\lambda_{\text{act}}$ .

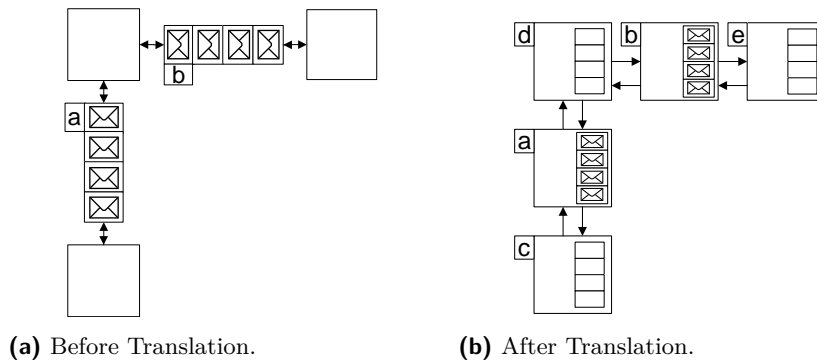
## 6.2 Translation strategy ( $\lambda_{\text{ch}}$ into $\lambda_{\text{act}}$ )

To translate typed actors into typed channels (shown in Figure 13), we emulate each channel using an actor process, which is crucial in retaining the mobility of channel endpoints. Channel types describe the typing of a *communication medium* between communicating processes, where processes are unaware of the identity of other communicating parties, and the types of messages that another party may receive. Unfortunately, the same does not hold for mailboxes. Consequently, we require that before translating into actors, *every channel has the same type*. Although this may seem restrictive, it is both possible and safe to transform a  $\lambda_{\text{ch}}$  program with multiple channel types into a  $\lambda_{\text{ch}}$  program with a single channel type.

As an example, suppose we have a program which contains channels carrying values of types `Int`, `String`, and `ChanRef(String)`. It is possible to construct a recursive variant type  $\mu X.\langle \ell_1 : \text{Int}, \ell_2 : \text{String}, \ell_3 : \text{ChanRef}(X) \rangle$  which can be assigned to all channels in the system. Then, supposing we wanted to send a 5 along a channel which previously had type `ChanRef(Int)`, we would instead send a value `roll ⟨ℓ1 = 5⟩` (where `roll V` is the introduction rule for an iso-recursive type). Appendix A [15] provides more details.

## 6.3 Translation

We write  $\lambda_{\text{ch}}$  judgements of the form  $\{B\} \Gamma \vdash M : A$  for a term where all channels have type  $B$ , and similarly for value and configuration typing judgements. Under such a judgement, we can write `Chan` instead of `ChanRef(B)`.



■ **Figure 13** Translation strategy:  $\lambda_{\text{ch}}$  into  $\lambda_{\text{act}}$ .

**Meta level definitions.** The majority of the translation lies within the translation of `newCh`, which makes use of the meta-level definitions `body` and `drain`. The `body` function emulates a channel. Firstly, the actor receives a message `recvVal`, which is either of the form `inl V` to store a message  $V$ , or `inr W` to request that a value is dequeued and sent to the actor with ID  $W$ . We assume a standard implementation of list concatenation ( $\#$ ). If the message is `inl V`, then  $V$  is appended to the tail of the list of values stored in the channel, and the new state is passed as an argument to `drain`. If the message is `inr W`, then the process ID  $W$  is appended to the end of the list of processes waiting for a value. The `drain` function satisfies all requests that can be satisfied, returning an updated channel state. Note that `drain` does not need to be recursive, since one of the lists will either be empty or a singleton.

**Translation on types.** Figure 14 shows the translation from  $\lambda_{\text{ch}}$  into  $\lambda_{\text{act}}$ . The translation function on types  $\langle - \rangle$  is defined with respect to the type of all channels  $C$  and is used to annotate function arrows and to assign a parameter to `ActorRef` types. The (omitted) translations on sums, products, and lists are homomorphic. The translation of `Chan` is `ActorRef( $\langle C \rangle$  + ActorRef( $\langle C \rangle$ ))`, meaning an actor which can receive a request to either store a value of type  $\langle C \rangle$ , or to dequeue a value and send it to a process ID of type `ActorRef( $\langle C \rangle$ )`.

**Translation on communication and concurrency primitives.** We omit the translation on values and functional terms, which are homomorphisms. Processes in  $\lambda_{\text{ch}}$  are anonymous, whereas all actors in  $\lambda_{\text{act}}$  are addressable; to emulate `fork`, we therefore discard the reference returned by `spawn`. The translation of `give` wraps the translated value to be sent in the left injection of a sum type, and sends to the translated channel name  $\langle W \rangle$ . To emulate `take`, the process ID (retrieved using `self`) is wrapped in the right injection and sent to the actor emulating the channel, and the actor waits for the response message. Finally, the translation of `newCh` spawns a new actor to execute `body`.

**Translation on configurations.** The translation function  $\langle - \rangle$  is homomorphic on parallel composition and name restriction. Unlike  $\lambda_{\text{ch}}$ , a term cannot exist outwith an enclosing actor context in  $\lambda_{\text{act}}$ , so the translation of a process evaluating term  $M$  is an actor evaluating  $\langle M \rangle$  with some fresh name  $a$  and an empty mailbox, enclosed in a name restriction. A buffer is translated to an actor with an empty mailbox, evaluating `body` with a state containing the (term-level) list of values previously stored in the buffer.

Although the translation from  $\lambda_{\text{ch}}$  into  $\lambda_{\text{act}}$ , is much more verbose than the translation from  $\lambda_{\text{act}}$  to  $\lambda_{\text{ch}}$ , it is (once all channels have the same type) a *local transformation* [12].

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Translation on types (wrt. a channel type  $C$ )

$$\llbracket \text{Chan} \rrbracket = \text{ActorRef}(\llbracket C \rrbracket + \text{ActorRef}(\llbracket C \rrbracket)) \quad \llbracket A \rightarrow B \rrbracket = \llbracket A \rrbracket \rightarrow^{\llbracket C \rrbracket} \llbracket B \rrbracket$$

Translation on communication and concurrency primitives

$$\begin{aligned} \llbracket \text{fork } M \rrbracket &= \text{let } x \leftarrow \text{spawn } \llbracket M \rrbracket \text{ in return } () & \llbracket \text{take } V \rrbracket &= \text{let } \text{selfPid} \leftarrow \text{self in} \\ \llbracket \text{give } V W \rrbracket &= \text{send } (\text{inr } \llbracket V \rrbracket) \llbracket W \rrbracket & & \text{send } (\text{inr } \text{selfPid}) \llbracket V \rrbracket; \\ \llbracket \text{newCh} \rrbracket &= \text{spawn } (\text{body } (\llbracket \_ \rrbracket, \llbracket \_ \rrbracket)) & & \text{receive} \end{aligned}$$

Translation on configurations

$$\begin{aligned} \llbracket \mathcal{C}_1 \parallel \mathcal{C}_2 \rrbracket &= \llbracket \mathcal{C}_1 \rrbracket \parallel \llbracket \mathcal{C}_2 \rrbracket & \llbracket (\nu a)C \rrbracket &= (\nu a)\llbracket C \rrbracket & \llbracket M \rrbracket &= (\nu a)(\langle a, \llbracket M \rrbracket, \epsilon \rangle) \\ & & & & & a \text{ is a fresh name} \\ \llbracket a(\vec{V}) \rrbracket &= \langle a, \text{body } (\llbracket \vec{V} \rrbracket, \llbracket \_ \rrbracket), \epsilon \rangle & \text{where } \llbracket \vec{V} \rrbracket &= \llbracket V_0 \rrbracket :: \dots :: \llbracket V_n \rrbracket :: \llbracket \_ \rrbracket \end{aligned}$$

Meta level definitions

$$\begin{aligned} \text{body} &\triangleq \text{rec } g(\text{state}) . & \text{drain} &\triangleq \lambda x. \\ \text{let } \text{recvVal} \leftarrow \text{receive in} & & \text{let } (\text{vals}, \text{pids}) = x \text{ in} & \\ \text{let } (\text{vals}, \text{pids}) = \text{state in} & & \text{case vals } \{ & \\ \text{case } \text{recvVal} \{ & & \llbracket \_ \rrbracket \mapsto \text{return } (\text{vals}, \text{pids}) & \\ \text{inl } v \mapsto \text{let } \text{vals}' \leftarrow \text{vals} \text{ ++ } [v] \text{ in} & & v :: \text{vs} \mapsto & \\ \text{let } \text{state}' \leftarrow \text{drain } (\text{vals}', \text{pids}) \text{ in} & & \text{case pids } \{ & \\ g(\text{state}') & & \llbracket \_ \rrbracket \mapsto \text{return } (\text{vals}, \text{pids}) & \\ \text{inr } \text{pid} \mapsto \text{let } \text{pids}' \leftarrow \text{pids} \text{ ++ } [\text{pid}] \text{ in} & & \text{pid} :: \text{pids} \mapsto \text{send } v \text{ pid;} & \\ \text{let } \text{state}' \leftarrow \text{drain } (\text{vals}, \text{pids}') \text{ in} & & \text{return } (\text{vs}, \text{pids}) & \\ g(\text{state}') \} & & \} \} & \end{aligned}$$

■ **Figure 14** Translation from  $\lambda_{\text{ch}}$  into  $\lambda_{\text{act}}$ .

## 6.4 Properties of the translation

Since all channels in the source language of the translation have the same type, we can assume that each entry in the codomain of  $\Delta$  is the same type  $B$ .

► **Definition 22** (Translation of typing environments wrt. a channel type  $B$ ).

1. If  $\Gamma = \alpha_1 : A_1, \dots, \alpha_n : A_n$ , define  $\llbracket \Gamma \rrbracket = \alpha_1 : \llbracket A_1 \rrbracket, \dots, \alpha_n : \llbracket A_n \rrbracket$ .
2. Given a  $\Delta = a_1 : B, \dots, a_n : B$ , define  $\llbracket \Delta \rrbracket = a_1 : (\llbracket B \rrbracket + \text{ActorRef}(\llbracket B \rrbracket)), \dots, a_n : (\llbracket B \rrbracket + \text{ActorRef}(\llbracket B \rrbracket))$ .

The translation on terms preserves typing.

► **Lemma 23** ( $\llbracket - \rrbracket$  preserves typing (terms and values)).

1. If  $\{B\} \Gamma \vdash V : A$ , then  $\llbracket \Gamma \rrbracket \vdash \llbracket V \rrbracket : \llbracket A \rrbracket$ .
2. If  $\{B\} \Gamma \vdash M : A$ , then  $\llbracket \Gamma \rrbracket \mid \llbracket B \rrbracket \vdash \llbracket M \rrbracket : \llbracket A \rrbracket$ .

The translation on configurations also preserves typeability. We write  $\Gamma \asymp \Delta$  if for each  $a : A \in \Delta$ , we have that  $a : \text{ChanRef}(A) \in \Gamma$ ; for closed configurations this is ensured by CHAN. This is necessary since the typing rules for  $\lambda_{\text{act}}$  require that the local actor name is present in the term environment to ensure preservation in the presence of **self**, but there is no such restriction in  $\lambda_{\text{ch}}$ .

► **Theorem 24** ( $\llbracket - \rrbracket$  preserves typeability (configurations)).

If  $\{A\} \Gamma; \Delta \vdash C$  with  $\Gamma \asymp \Delta$ , then  $\llbracket \Gamma \rrbracket; \llbracket \Delta \rrbracket \vdash \llbracket C \rrbracket$ .

It is clear that reduction on translated  $\lambda_{\text{ch}}$  terms can simulate reduction in  $\lambda_{\text{act}}$ .

► **Lemma 25.** *If  $\{B\} \Gamma \vdash M_1 : A$  and  $M_1 \longrightarrow_M M_2$ , then  $\langle M_1 \rangle \longrightarrow_M \langle M_2 \rangle$ .*

Finally, we show that  $\lambda_{\text{act}}$  can simulate  $\lambda_{\text{ch}}$ .

► **Lemma 26.** *If  $\Gamma; \Delta \vdash C$  and  $C \equiv D$ , then  $\langle C \rangle \equiv \langle D \rangle$ .*

► **Theorem 27** (Simulation ( $\lambda_{\text{act}}$  configurations in  $\lambda_{\text{ch}}$ )).

*If  $\{A\} \Gamma; \Delta \vdash C_1$ , and  $C_1 \longrightarrow C_2$ , then there exists some  $D$  such that  $\langle C_1 \rangle \longrightarrow^* D$  with  $D \equiv \langle C_2 \rangle$ .*

**Remark.** The translation from  $\lambda_{\text{ch}}$  into  $\lambda_{\text{act}}$  is more involved than the translation from  $\lambda_{\text{act}}$  into  $\lambda_{\text{ch}}$  due to the asymmetry shown in Figure 2. Mailbox types are less precise; generally taking the form of a large variant type.

Typical implementations of this translation use synchronisation mechanisms such as futures or shared memory (see Section 7.1); the implementation shown in the Hopac documentation uses ML references [1]. Given the ubiquity of these abstractions, we were surprised to discover that the additional expressive power of synchronisation is not *necessary*. Our original attempt at a synchronisation-free translation was type-directed. We were surprised to discover that the translation can be described so succinctly after factoring out the coalescing step, which precisely captures the type pollution problem.

## 7 Extensions

In this section, we discuss common extensions to channel- and actor-based languages. Firstly, we discuss synchronisation, which is ubiquitous in practical implementations of actor-inspired languages. Adding synchronisation simplifies the translation from channels to actors, and relaxes the restriction that all channels must have the same type. Secondly, we consider an extension with Erlang-style selective receive, and show how to encode it in  $\lambda_{\text{act}}$ . Thirdly, we discuss how to nondeterministically choose a message from a collection of possible sources, and finally, we discuss what the translations tell us about the nature of behavioural typing disciplines for actors. Establishing exactly how the latter two extensions fit into our framework is the subject of ongoing and future work.

### 7.1 Synchronisation

Although communicating with an actor via asynchronous message passing suffices for many purposes, implementing “call-response” style interactions can become cumbersome. Practical implementations such as Erlang and Akka implement some way of synchronising on a result: Erlang achieves this by generating a unique reference to send along with a request, *selectively receiving* from the mailbox to await a response tagged with the same unique reference. Another method of synchronisation embraced by the Active Object community [33, 10, 32] and Akka is to generate a *future variable* which is populated with the result of the call.

Figure 15 details an extension of  $\lambda_{\text{act}}$  with a synchronisation primitive, `wait`, which encodes a deliberately restrictive form of synchronisation capable of emulating futures. The key idea behind `wait` is it allows some actor  $a$  to block until an actor  $b$  evaluates to a value; this value is then returned directly to  $a$ , bypassing the mailbox. A variation of the `wait` primitive is implemented as part of the Links [9] concurrency model. This is but one of multiple ways of allowing synchronisation; first-class futures, shared reference cells, or selective receive can achieve a similar result. We discuss `wait` as it avoids the need for new configurations.

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Additional types, terms, configuration reduction rule, and equivalence

$$\begin{aligned} \text{Types} &::= \text{ActorRef}(A, B) \mid \dots & \text{Terms} &::= \text{wait } V \mid \dots \\ \langle a, E[\text{wait } b], \vec{V} \rangle \parallel \langle b, \text{return } V', \vec{W} \rangle &\longrightarrow \langle a, E[\text{return } V'], \vec{V} \rangle \parallel \langle b, \text{return } V', \vec{W} \rangle \\ &(\nu a)(\langle a, \text{return } V, \vec{V} \rangle) \parallel \mathcal{C} \equiv \mathcal{C} \end{aligned}$$

Modified typing rules for terms

$$\boxed{\Gamma \mid A, B \vdash M : A}$$

$$\begin{array}{c} \text{SYNC-SPAWN} \\ \frac{\Gamma \mid A, B \vdash M : B}{\Gamma \mid C, C' \vdash \text{spawn } M : \text{ActorRef}(A, B)} \end{array} \quad \begin{array}{c} \text{SYNC-WAIT} \\ \frac{\Gamma \vdash V : \text{ActorRef}(A, B)}{\Gamma \mid C, C' \vdash \text{wait } V : B} \end{array} \quad \begin{array}{c} \text{SYNC-SELF} \\ \frac{}{\Gamma \mid A, B \vdash \text{self} : \text{ActorRef}(A, B)} \end{array}$$

Modified typing rules for configurations

$$\boxed{\Gamma; \Delta \vdash \mathcal{C}}$$

$$\begin{array}{c} \text{SYNC-ACTOR} \\ \frac{\Gamma, a : \text{ActorRef}(A, B) \vdash M : B \quad (\Gamma, a : \text{ActorRef}(A, B) \vdash V_i : A)_i}{\Gamma, a : \text{ActorRef}(A, B); a : (A, B) \vdash \langle a, M, \vec{V} \rangle} \end{array} \quad \begin{array}{c} \text{SYNC-NU} \\ \frac{\Gamma, a : \text{ActorRef}(A, B); \Delta, a : (A, B) \vdash \mathcal{C}}{\Gamma; \Delta \vdash (\nu a)\mathcal{C}} \end{array}$$

Modified translation

$$\begin{aligned} \llbracket \text{ChanRef}(A) \rrbracket &= \text{ActorRef}(\llbracket A \rrbracket + \text{ActorRef}(\llbracket A \rrbracket, \llbracket A \rrbracket), \mathbf{1}) \\ \llbracket A \rightarrow B \rrbracket &= \llbracket A \rrbracket \rightarrow^{C, \mathbf{1}} \llbracket B \rrbracket \\ \llbracket \text{take } V \rrbracket &= \text{let } \text{requestorPid} \leftarrow \text{spawn} ( \\ &\quad \text{let } \text{selfPid} \leftarrow \text{self in} \\ &\quad \text{send } (\text{inr } \text{selfPid}) \llbracket V \rrbracket; \\ &\quad \text{receive}) \text{ in} \\ &\quad \text{wait } \text{requestorPid} \end{aligned}$$

■ **Figure 15** Extensions to add synchronisation to  $\lambda_{\text{act}}$ .

We replace the unary type constructor for process IDs with a binary type constructor  $\text{ActorRef}(A, B)$ , where  $A$  is the type of messages that the process can receive from its mailbox, and  $B$  is the type of value to which the process will eventually evaluate. We assume that the remainder of the primitives are modified to take the additional effect type into account. We can now adapt the previous translation from  $\lambda_{\text{ch}}$  to  $\lambda_{\text{act}}$ , making use of `wait` to avoid the need for the coalescing transformation. Channel references are translated into actor references which can either receive a value of type  $A$ , or the PID of a process which can receive a value of type  $A$  and will eventually evaluate to a value of type  $A$ . Note that the unbound annotation  $C, \mathbf{1}$  on function arrows reflects that the mailboxes can be of *any* type, since the mailboxes are unused in the actors emulating threads.

The key idea behind the modified translation is to spawn a fresh actor which makes the request to the channel and blocks waiting for the response. Once the spawned actor has received the result, the result can be retrieved synchronously using `wait` *without* reading from the mailbox. The previous soundness theorems adapt to the new setting.

► **Theorem 28.** *If  $\Gamma; \Delta \vdash \mathcal{C}$  with  $\Gamma \asymp \Delta$ , then  $\llbracket \Gamma \rrbracket; \llbracket \Delta \rrbracket \vdash \llbracket \mathcal{C} \rrbracket$ .*

► **Theorem 29.** *If  $\Gamma; \Delta \vdash \mathcal{C}_1$  and  $\mathcal{C}_1 \longrightarrow \mathcal{C}_2$ , then there exists some  $\mathcal{D}$  such that  $\llbracket \mathcal{C} \rrbracket \longrightarrow^* \mathcal{D}$  with  $\mathcal{D} \equiv \llbracket \mathcal{C}_2 \rrbracket$ .*

The translation in the other direction requires named threads and a `join` construct in  $\lambda_{\text{ch}}$ .

Additional syntax

Receive Patterns  $c ::= \langle \ell = x \rangle \text{ when } M \mapsto N$   
 Computations  $M ::= \text{receive } \{\vec{c}\} \mid \dots$

Additional term typing rule

$$\frac{\text{SEL-RECV} \quad \vec{c} = \{\langle \ell_i = x_i \rangle \text{ when } M_i \mapsto N_i\}_i \quad i \in J \quad \Gamma, x_i : A_i \vdash_{\text{P}} M_i : \text{Bool} \quad \Gamma, x_i : A_i \mid \langle \ell_j : A_j \rangle_{j \in J} \vdash N_i : C}{\Gamma \mid \langle \ell_j : A_j \rangle_{j \in J} \vdash \text{receive } \{\vec{c}\} : C}$$

Additional configuration reduction rule

$$\frac{\exists k, l. \forall i. i < k \Rightarrow \neg(\text{matchesAny}(\vec{c}, V_i)) \wedge \text{matches}(c_l, V_k) \wedge \forall j. j < l \Rightarrow \neg(\text{matches}(c_j, V_k))}{\langle a, E[\text{receive } \{\vec{c}\}], \vec{W} \cdot V_k \cdot \vec{W}' \rangle \longrightarrow \langle a, E[N_l\{V_k'/x_l\}], \vec{W} \cdot \vec{W}' \rangle}$$

where

$$\vec{c} = \{\langle \ell_i = x_i \rangle \text{ when } M_i \mapsto N_i\}_i \quad \vec{W} = V_1 \cdot \dots \cdot V_{k-1} \quad \vec{W}' = V_{k+1} \cdot \dots \cdot V_n \quad V_k = \langle \ell_k = V_k' \rangle$$

$$\text{matches}(\langle \ell = x \rangle \text{ when } M \mapsto N, \langle \ell' = V \rangle) \triangleq (\ell = \ell') \wedge (M\{V/x\} \longrightarrow_{\text{M}}^* \text{return true})$$

$$\text{matchesAny}(\vec{c}, V) \triangleq \exists c \in \vec{c}. \text{matches}(c, V)$$

■ **Figure 16** Additional syntax, typing rules, and reduction rules for  $\lambda_{\text{act}}$  with selective receive.

## 7.2 Selective receive

The `receive` construct in  $\lambda_{\text{act}}$  can only read the first message in the queue, which is cumbersome as it often only makes sense for an actor to handle a subset of messages at a given time.

In practice, Erlang provides a *selective receive* construct, matching messages in the mailbox against multiple pattern clauses. Assume we have a mailbox containing values  $V_1, \dots, V_n$  and evaluate `receive`  $\{c_1, \dots, c_m\}$ . The construct first tries to match value  $V_1$  against clause  $c_1$ —if it matches, then the body of  $c_1$  is evaluated, whereas if it fails,  $V_1$  is tested against  $c_2$  and so on. Should  $V_1$  not match any pattern, then the process is repeated until  $V_n$  has been tested against  $c_m$ . At this point, the process blocks until a matching message arrives.

More concretely, consider an actor with mailbox type  $C = \langle \text{PriorityMessage} : \text{Message}, \text{StandardMessage} : \text{Message}, \text{Timeout} : 1 \rangle$  which can receive both high- and low-priority messages. Let *getPriority* be a function which extracts a priority from a message.

Now consider the following actor:

```
receive {
  ⟨PriorityMessage = msg⟩ when (getPriority msg) > 5 ↦ handleMessage msg
  ⟨Timeout = msg⟩ when true ↦ ()
};
receive {
  ⟨PriorityMessage = msg⟩ when true ↦ handleMessage msg
  ⟨StandardMessage = msg⟩ when true ↦ handleMessage msg
  ⟨Timeout = msg⟩ when true ↦ ()
}
```

This actor begins by handling a message only if it has a priority greater than 5. After the timeout message is received, however, it will handle any message—including lower-priority messages that were received beforehand.

## 11:22 Mixing Metaphors

Translation on types

$$\llbracket \text{ActorRef}(\langle \ell_i : A_i \rangle_i) \rrbracket = \text{ActorRef}(\langle \ell_i : \llbracket A_i \rrbracket \rangle_i) \quad \llbracket A \times B \rrbracket = \llbracket A \rrbracket \times \llbracket B \rrbracket \quad \llbracket A + B \rrbracket = \llbracket A \rrbracket + \llbracket B \rrbracket$$

$$\llbracket \mu X. A \rrbracket = \mu X. \llbracket A \rrbracket \quad \llbracket A \rightarrow^C B \rrbracket = \llbracket A \rrbracket \rightarrow^{\llbracket C \rrbracket} \text{List}(\llbracket C \rrbracket) \rightarrow^{\llbracket C \rrbracket} (\llbracket B \rrbracket \times \text{List}(\llbracket C \rrbracket))$$

where  $C = \langle \ell_i : A_i \rangle_i$ , and  $\llbracket C \rrbracket = \langle \ell_i : \llbracket A_i \rrbracket \rangle_i$

Translation on values

$$\llbracket \lambda x. M \rrbracket = \lambda x. \lambda mb. (\llbracket M \rrbracket mb) \quad \llbracket \text{rec } f(x). M \rrbracket = \text{rec } f(x). \lambda mb. (\llbracket M \rrbracket mb)$$

Translation on computation terms (wrt. a mailbox type  $\langle \ell_i : A_i \rangle_i$ )

$$\begin{aligned} \llbracket V W \rrbracket mb &= \text{let } f \Leftarrow (\llbracket V \rrbracket \llbracket W \rrbracket) \text{ in } f mb \\ \llbracket \text{return } V \rrbracket mb &= \text{return } (\llbracket V \rrbracket, mb) \\ \llbracket \text{let } x \Leftarrow M \text{ in } N \rrbracket mb &= \text{let } resPair \Leftarrow \llbracket M \rrbracket mb \text{ in let } (x, mb') = resPair \text{ in } \llbracket N \rrbracket mb' \\ \llbracket \text{self} \rrbracket mb &= \text{let } selfPid \Leftarrow \text{self} \text{ in return } (selfPid, mb) \\ \llbracket \text{send } V W \rrbracket mb &= \text{let } x \Leftarrow \text{send } (\llbracket V \rrbracket) (\llbracket W \rrbracket) \text{ in return } (x, mb) \\ \llbracket \text{spawn } M \rrbracket mb &= \text{let } spawnRes \Leftarrow \text{spawn}(\llbracket M \rrbracket [ \ ]) \text{ in return } (spawnRes, mb) \\ \llbracket \text{receive } \{\vec{c}\} \rrbracket mb &= \text{find}(\vec{c}, mb) \end{aligned}$$

Translation on configurations

$$\begin{aligned} \llbracket (\nu a) \mathcal{C} \rrbracket &= \{(\nu a) \mathcal{D} \mid \mathcal{D} \in \llbracket \mathcal{C} \rrbracket\} \\ \llbracket \mathcal{C}_1 \parallel \mathcal{C}_2 \rrbracket &= \{\mathcal{D}_1 \parallel \mathcal{D}_2 \mid \mathcal{D}_1 \in \llbracket \mathcal{C}_1 \rrbracket \wedge \mathcal{D}_2 \in \llbracket \mathcal{C}_2 \rrbracket\} \\ \llbracket \langle a, M, \vec{V} \rangle \rrbracket &= \{\langle a, \llbracket M \rrbracket [ \ ], \llbracket \vec{V} \rrbracket \rangle\} \cup \\ &\quad \{\langle a, (\llbracket M \rrbracket \vec{W}_i^1, \vec{W}_i^2) \mid i \in 1..n \rangle\} \end{aligned} \quad \text{where} \quad \begin{aligned} \vec{W}_i^1 &= \llbracket V_1 \rrbracket :: \dots :: \llbracket V_i \rrbracket :: [ \ ] \\ \vec{W}_i^2 &= \llbracket V_{i+1} \rrbracket \cdot \dots \cdot \llbracket V_n \rrbracket \end{aligned}$$

■ **Figure 17** Translation from  $\lambda_{\text{act}}$  with selective receive into  $\lambda_{\text{act}}$ .

Figure 16 shows the additional syntax, typing rule, and configuration reduction rule required to encode selective receive; the type `Bool` and logical operators are encoded using sums in the standard way. We write  $\Gamma \vdash_{\text{P}} M : A$  to mean that under context  $\Gamma$ , a term  $M$  which does not perform any communication or concurrency actions has type  $A$ . Intuitively, this means that no subterm of  $M$  is a communication or concurrency construct.

The `receive`  $\{\vec{c}\}$  construct models an ordered sequence of receive pattern clauses  $c$  of the form  $(\langle \ell = x \rangle \text{ when } M) \mapsto N$ , which can be read as “If a message with body  $x$  has label  $\ell$  and satisfies predicate  $M$ , then evaluate  $N$ ”. The typing rule for `receive`  $\{\vec{c}\}$  ensures that for each pattern  $\langle \ell_i = x_i \rangle \text{ when } M_i \mapsto N_i$  in  $\vec{c}$ , we have that there exists some  $\ell_i : A_i$  contained in the mailbox variant type; and when  $\Gamma$  is extended with  $x_i : A_i$ , that the guard  $M_i$  has type `Bool` and the body  $N_i$  has the same type  $C$  for each branch.

The reduction rule for selective receive is inspired by that of Fredlund [16]. Assume that the mailbox is of the form  $V_1 \cdot \dots \cdot V_k \cdot \dots \cdot V_n$ , with  $\vec{W} = V_1 \cdot \dots \cdot V_{k-1}$  and  $\vec{W}' = V_{k+1} \cdot \dots \cdot V_n$ . The `matches`( $c, V$ ) predicate holds if the label matches, and the branch guard evaluates to true. The `matchesAny`( $\vec{c}, V$ ) predicate holds if  $V$  matches any pattern in  $\vec{c}$ . The key idea is that  $V_k$  is the first value to satisfy a pattern. The construct evaluates to the body of the matched pattern, with the message payload  $V_k'$  substituted for the pattern variable  $x_k$ ; the final mailbox is  $\vec{W} \cdot \vec{W}'$  (that is, the original mailbox without  $V_k$ ).

Reduction in the presence of selective receive preserves typing.

► **Theorem 30** (Preservation ( $\lambda_{\text{act}}$  configurations with selective receive)). *If  $\Gamma; \Delta \mid \langle \ell_i : A_i \rangle_i \vdash \mathcal{C}_1$  and  $\mathcal{C}_1 \longrightarrow \mathcal{C}_2$ , then  $\Gamma; \Delta \mid \langle \ell_i : A_i \rangle_i \vdash \mathcal{C}_2$ .*

**Translation to  $\lambda_{\text{act}}$ .** Given the additional constructs used to translate  $\lambda_{\text{ch}}$  into  $\lambda_{\text{act}}$ , it is possible to translate  $\lambda_{\text{act}}$  with selective receive into plain  $\lambda_{\text{act}}$ . Key to the translation is

```

find( $\vec{c}$ ,  $mb$ )  $\triangleq$ 
(rec findLoop( $ms$ )).
  let ( $mb_1, mb_2$ ) =  $ms$  in
  case  $mb_2$  {
    [ ]  $\mapsto$  loop( $\vec{c}$ ,  $mb_1$ )
     $x :: mb'_2 \mapsto$ 
      let  $mb' \leftarrow mb_1 ++ mb'_2$  in
      case  $x$  {branches( $\vec{c}$ ,  $mb'$ ,
         $\lambda y.(\text{let } mb'_1 \leftarrow mb_1 ++ [y] \text{ in}$ 
           $\text{findLoop}(mb'_1, mb'_2))$ )} ([ ],  $mb$ )
  }

label( $\langle \ell = x \rangle \text{ when } M \mapsto N$ ) =  $\ell$ 
labels( $\vec{c}$ ) = noDups([label( $c$ ) |  $c \leftarrow \vec{c}$ ])
matching( $\ell$ ,  $\vec{c}$ ) = [ $c$  | ( $c \leftarrow \vec{c}$ )  $\wedge$  label( $c$ ) =  $\ell$ ]
unhandled( $\vec{c}$ ) = [ $\ell$  | ( $\langle \ell : A \rangle \leftarrow \langle \ell_i : A_i \rangle_i$ )  $\wedge$   $\ell \notin \text{labels}(\vec{c})$ ]

branches( $\vec{c}$ ,  $mb$ ,  $default$ ) = patBranches( $\vec{c}$ ,  $mb$ ,  $default$ )  $\cdot$  defaultBranches( $\vec{c}$ ,  $mb$ ,  $default$ )
patBranches( $\vec{c}$ ,  $mb$ ,  $default$ ) =
  [ $\langle \ell = x \rangle \mapsto$  ifPats( $mb, \ell, x, \vec{c}, default$ ) | ( $\ell \leftarrow \text{labels}(\vec{c})$ )  $\wedge$   $\vec{c}_\ell = \text{matching}(\ell, \vec{c}) \wedge x$  fresh]
defaultBranches( $\vec{c}$ ,  $mb$ ,  $default$ ) = [ $\langle \ell = x \rangle \mapsto default \langle \ell = x \rangle$  | ( $\ell \leftarrow \text{unhandled}(\vec{c})$ )  $\wedge x$  fresh]

```

■ **Figure 18** Meta level definitions for translation from  $\lambda_{act}$  with selective receive to  $\lambda_{act}$  (wrt. a mailbox type  $\langle \ell_i : A_i \rangle_i$ ).

reasoning about values in the mailbox at the term level; we maintain a term-level ‘save queue’ of values that have been received but not yet matched, and can loop through the list to find the first matching value. Our translation is similar in spirit to the “stashing” mechanism described by Haller [19] to emulate selective receive in Akka, where messages can be moved to an auxiliary queue for processing at a later time.

Figure 17 shows the translation formally. Except for function types, the translation on types is homomorphic. Similar to the translation from  $\lambda_{act}$  into  $\lambda_{ch}$ , we add an additional parameter for the save queue.

The translation on terms  $[M] mb$  takes a variable  $mb$  representing the save queue as its parameter, returning a pair of the resulting term and the updated save queue. The majority of cases are standard, except for **receive**  $\{\vec{c}\}$ , which relies on the meta-level definition **find**( $\vec{c}$ ,  $mb$ ):  $\vec{c}$  is a sequence of clauses, and  $mb$  is the save queue. The constituent **findLoop** function takes a pair of lists ( $mb_1, mb_2$ ), where  $mb_1$  is the list of processed values found not to match, and  $mb_2$  is the list of values still to be processed. The loop inspects the list until one either matches, or the end of the list is reached. Should no values in the term-level representation of the mailbox match, then the **loop** function repeatedly receives from the mailbox, testing each new message against the patterns.

Note that the **case** construct in the core  $\lambda_{act}$  calculus is more restrictive than selective receive: given a variant  $\langle \ell_i : A_i \rangle_i$ , **case** requires a single branch for each label. Selective receive allows multiple branches for each label, each containing a possibly-different predicate, and does not require pattern matching to be exhaustive.

We therefore need to perform pattern matching elaboration; this is achieved by the **branches** meta level definition. We make use of list comprehension notation: for example,



$[c \mid (c \leftarrow \vec{c}) \wedge \text{label}(c) = \ell]$  returns the (ordered) list of clauses in a sequence  $\vec{c}$  such that the label of the receive clause matches a label  $\ell$ . We assume a meta level function **noDups** which removes duplicates from a list. Case branches are computed using the **branches** meta level definition: **patBranches** creates a branch for each label present in the selective receive, creating (via **ifPats**) a sequence of if-then-else statements to check each predicate in turn; **defaultBranches** creates a branch for each label that is present in the mailbox type but not in any selective receive clauses.

**Properties of the translation.** The translation preserves typing of terms and values.

► **Lemma 31** (Translation preserves typing (values and terms)).

1. If  $\Gamma \vdash V : A$ , then  $[\Gamma] \vdash [V] : [A]$ .
2. If  $\Gamma \mid \langle \ell_i : A_i \rangle_i \vdash M : B$ , then  $[\Gamma], mb : \text{List}(\langle \ell_i : [A_i] \rangle_i) \mid \langle \ell_i : [A_i] \rangle_i \vdash [M] mb : ([B] \times \text{List}(\langle \ell_i : [A_i] \rangle_i))$ .

Alas, a direct one-to-one translation on configurations is not possible, since a message in a mailbox in the source language could be either in the mailbox or the save queue in the target language. Consequently, we translate a configuration into a set of possible configurations, depending on how many messages have been processed. We can show that all configurations in the resulting set are type-correct, and can simulate the original reduction.

► **Theorem 32** (Translation preserves typing). If  $\Gamma; \Delta \vdash \mathcal{C}$ , then  $\forall \mathcal{D} \in [\mathcal{C}]$ , it is the case that  $[\Gamma]; [\Delta] \vdash \mathcal{D}$ .

► **Theorem 33** (Simulation ( $\lambda_{\text{act}}$  with selective receive in  $\lambda_{\text{act}}$ )). If  $\Gamma; \Delta \vdash \mathcal{C}$  and  $\mathcal{C} \longrightarrow \mathcal{C}'$ , then  $\forall \mathcal{D} \in [\mathcal{C}]$ , there exists a  $\mathcal{D}'$  such that  $\mathcal{D} \longrightarrow^+ \mathcal{D}'$  and  $\mathcal{D}' \in [\mathcal{C}']$ .

**Remark.** Originally we expected to need to add an analogous selective receive construct to  $\lambda_{\text{ch}}$  in order to be able to translate  $\lambda_{\text{act}}$  with selective receive into  $\lambda_{\text{ch}}$ . We were surprised (in part due to the complex reduction rule and the native runtime support in Erlang) when we discovered that selective receive can be emulated in plain  $\lambda_{\text{act}}$ . Moreover, we were pleasantly surprised that types pose no difficulties in the translation.

### 7.3 Choice

The calculus  $\lambda_{\text{ch}}$  supports only blocking receive on a *single* channel. A more powerful mechanism is *selective communication*, where a value is taken nondeterministically from *two* channels. An important use case is receiving a value when either channel could be empty.

Here we have considered only the most basic form of selective choice over two channels. More generally, it may be extended to arbitrary regular data types [42]. As Concurrent ML [45] embraces rendezvous-based synchronous communication, it provides *generalised selective communication* where a process can synchronise on a mixture of input or output communication events. Similarly, the join patterns of the join calculus [14] provide a general abstraction for selective communication over multiple channels.

As we are working in the asynchronous setting where a **give** operation can reduce immediately, we consider only input-guarded choice. Input-guarded choice can be added straightforwardly to  $\lambda_{\text{ch}}$ , as shown in Figure 19. Emulating such a construct satisfactorily in  $\lambda_{\text{act}}$  is nontrivial, because messages must be multiplexed through a local queue. One approach could be to use the work of Chaudhuri [8] which shows how to implement generalised choice using synchronous message passing, but implementing this in  $\lambda_{\text{ch}}$  may be difficult due to the asynchrony of **give**. We leave a more thorough investigation to future work.

$$\frac{\Gamma \vdash V : \text{ChanRef}(A) \quad \Gamma \vdash W : \text{ChanRef}(B)}{\Gamma \vdash \text{choose } V W : A + B}$$

$$E[\text{choose } a b] \parallel a(W_1 \cdot \vec{V}_1) \parallel b(\vec{V}_2) \longrightarrow E[\text{return } (\text{inl } W_1)] \parallel a(\vec{V}_1) \parallel b(\vec{V}_2)$$

$$E[\text{choose } a b] \parallel a(\vec{V}_1) \parallel b(W_2 \cdot \vec{V}_2) \longrightarrow E[\text{return } (\text{inr } W_2)] \parallel a(\vec{V}_1) \parallel b(\vec{V}_2)$$

■ **Figure 19** Additional typing and evaluation rules for  $\lambda_{\text{ch}}$  with choice.

## 7.4 Behavioural types

Behavioural types allow the type of an object (e.g. a channel) to evolve as a program executes. A widely studied behavioural typing discipline is that of *session types* [26, 27], which are channel types sufficiently expressive to describe *communication protocols* between participants. For example, the session type for a channel which sends two integers and receives their sum could be defined as `!Int.!Int.?Int.end`. Session types are suited to channels, whereas current work on session-typed actors concentrates on runtime monitoring [39].

A natural question to ask is whether one can combine the benefits of actors and of session types—indeed, this was one of our original motivations for wanting to better understand the relationship between actors and channels in the first place! A session-typed channel may support both sending and receiving (at different points in the protocol it encodes), but communication with another process’ mailbox is one-way. We have studied several variants of  $\lambda_{\text{act}}$  with *polarised* session types [43, 36] which capture such one-way communication, but they seem too weak to simulate session-typed channels. In future, we would like to find an extension of  $\lambda_{\text{act}}$  with behavioural types that admits a similar simulation result to the ones in this paper.

## 8 Related work

Our formulation of concurrent  $\lambda$ -calculi is inspired by  $\lambda(\text{fut})$  [40], a concurrent  $\lambda$ -calculus with threads, futures, reference cells, and an atomic exchange construct. In the presence of lists, futures are sufficient to encode asynchronous channels. In  $\lambda_{\text{ch}}$ , we concentrate on asynchronous channels to better understand the correspondence with actors. Channel-based concurrent  $\lambda$ -calculi form the basis of functional languages with session types [17, 35].

Concurrent ML [45] extends Standard ML with a rich set of combinators for synchronous channels, which again can emulate asynchronous channels. A core notion in Concurrent ML is nondeterministically synchronising on multiple synchronous events, such as sending or receiving messages; relating such a construct to an actor calculus is nontrivial, and remains an open problem. Hopac [28] is a channel-based concurrency library for F#, based on Concurrent ML. The Hopac documentation relates synchronous channels and actors [1], implementing actor-style primitives using channels, and channel-style primitives using actors. The implementation of channels using actors uses mutable references to emulate the `take` function, whereas our translation achieves this using message passing. Additionally, our translation is formalised and we prove that the translations are type- and semantics-preserving.

Links [9] provides actor-style concurrency, and the paper describes a translation into  $\lambda(\text{fut})$ . Our translation is semantics-preserving and can be done without synchronisation.

The actor model was designed by Hewitt [23] and examined in the context of distributed systems by Agha [2]. Agha et al. [3] describe a functional actor calculus based on the  $\lambda$ -calculus augmented by three core constructs: `send` sends a message; `letactor` creates a new

actor; and `become` changes an actor’s behaviour. The operational semantics is defined in terms of a global actor mapping, a global multiset of messages, a set of *receptionists* (actors which are externally visible to other configurations), and a set of external actor names. Instead of `become`, we use an explicit `receive` construct, which more closely resembles Erlang (referred to by the authors as “essentially an actor language”). Our concurrent semantics, more in the spirit of process calculi, encodes visibility via name restrictions and structural congruences. The authors consider a behavioural theory in terms of operational and testing equivalences—something we have not investigated.

Scala has native support for actor-style concurrency, implemented efficiently without explicit virtual machine support [20]. The actor model inspires *active objects* [33]: objects supporting asynchronous method calls which return responses using futures. De Boer et al. [10] describe a language for active objects with cooperatively scheduled threads within each object. Core ABS [32] is a specification language based on active objects. Using futures for synchronisation sidesteps the type pollution problem inherent in call-response patterns with actors, although our translations work in the absence of synchronisation. By working in the functional setting, we obtain more compact calculi.

## 9 Conclusion

Inspired by languages such as Go which take channels as core constructs for communication, and languages such as Erlang which are based on the actor model of concurrency, we have presented translations back and forth between a concurrent  $\lambda$ -calculus  $\lambda_{\text{ch}}$  with channel-based communication constructs and a concurrent  $\lambda$ -calculus  $\lambda_{\text{act}}$  with actor-based communication constructs. We have proved that  $\lambda_{\text{act}}$  can simulate  $\lambda_{\text{ch}}$  and vice-versa.

The translation from  $\lambda_{\text{act}}$  to  $\lambda_{\text{ch}}$  is straightforward, whereas the translation from  $\lambda_{\text{ch}}$  to  $\lambda_{\text{act}}$  requires considerably more effort. Returning to Figure 2, this is unsurprising!

We have also shown how to extend  $\lambda_{\text{act}}$  with synchronisation, greatly simplifying the translation from  $\lambda_{\text{ch}}$  into  $\lambda_{\text{act}}$ , and have shown how Erlang-style selective receive can be emulated in  $\lambda_{\text{act}}$ . Additionally, we have discussed input-guarded choice in  $\lambda_{\text{ch}}$ , and how behavioural types may fit in with  $\lambda_{\text{act}}$ .

In future, we firstly plan to strengthen our operational correspondence results by considering operational completeness. Secondly, we plan to investigate how to emulate  $\lambda_{\text{ch}}$  with input-guarded choice in  $\lambda_{\text{act}}$ . Finally, we intend to use the lessons learnt from studying  $\lambda_{\text{ch}}$  and  $\lambda_{\text{act}}$  to inform the design of an actor-inspired language with behavioural types.

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