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# The Stochastic Inventory Routing Problem on Electric Roads

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**Abstract**

In this work we introduce a green inventory routing problem termed the Stochastic Inventory Routing Problem on Electric Roads (S-IRP-ER), in which a hybrid vehicle navigates a road network with charging opportunities in some road sections, to cover the non-stationary stochastic demand of a single product for a set of retailers in the network. We model the problem using isochrone graphs to represent the real road network. In an isochrone graph nodes are located such that the time to travel any arc is constant all over the network. This allows for tracking the battery level of the vehicle serving retailers, as it charges and discharges continuously while travelling. We formulate a mathematical programming heuristics and prove its effectiveness. We use our model on a realistic instance of the problem, showcasing the different strategies that a vehicle may follow depending on fuel costs in relation to the costs of electricity.

*Keywords:* inventory, routing, electric roads, stochastic demand, mixed-Integer linear programming

## 1. Introduction

In recent years, growing concerns on climate change led to the introduction of new CO2 emission targets in many countries. For instance, in the UK the Government recently established a net-zero emission target for 2050 in an amendment of the Climate Change Act 2008 (UK Government, 2019). For the transportation sector, one of the largest contributors of greenhouse gas (GHG) emissions, this means a complete transformation is needed in coming years. The adoption of electric vehicles (EVs), coupled with increasing levels of renewable energy production, is seen as a potential solution to this challenge. However, limitations associated with EVs — such as battery range — are tighter for heavy good vehicles (HGVs). At the same time, HGVs are the vehicles causing the greatest environmental impact: in the EU, freight transport by HGVs accounts for about a quarter of all CO2 emissions caused by road transport (Schroten et al., 2012). Still, some engineering solutions could see HGVs harnessing electric power without relying on large electric batteries that compromise vehicle payload and weight, and that put stress on natural resources needed to manufacture them. These solutions, named as Electric Roads, eRoads, eHighways or Electric Road Systems (ERS), allow compatible vehicles to use power from the electric grid to travel along, while reducing or even

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eliminating the need to stop for recharging. These systems can be classified into three categories: overhead catenary cables, conductive power from road, and inductive power from road.

One of the engineering companies leading this technology, with an overhead catenary cables solution, is Siemens (Siemens, 2022), with a system termed 'eHighway'. In recent years, overhead catenary cables systems have been trialled for HGVs use in countries such as Germany (Scania, 2020) and Sweden (Vattenfall, 2021). In the UK, the Government has awarded funding through Innovate UK to a consortium to lead a feasibility plan for decarbonising road freight transport in the country by electrifying motorways. The consortium includes the Centre for Sustainable Road Freight (Cambridge University and Heriot-Watt University), Siemens Mobility, Scania, Costain, ARUP, Milne Research, SPL Powerlines, CI Planning, BOX ENERGI and Possible (Lee, 2022). The plan, outlined in (Ainalis et al., 2020) calls for covering about 65% of the roads travelled by HGVs (7500 km of road) with catenary cables by the late 2030s, with a construction investment estimated at £19.3 billion. A recent study shows that it is possible to reduce carbon emissions of road freight transportation by 80-90% by 2050, if long-haul vehicles are electrified (Keyes & et al., 2018).

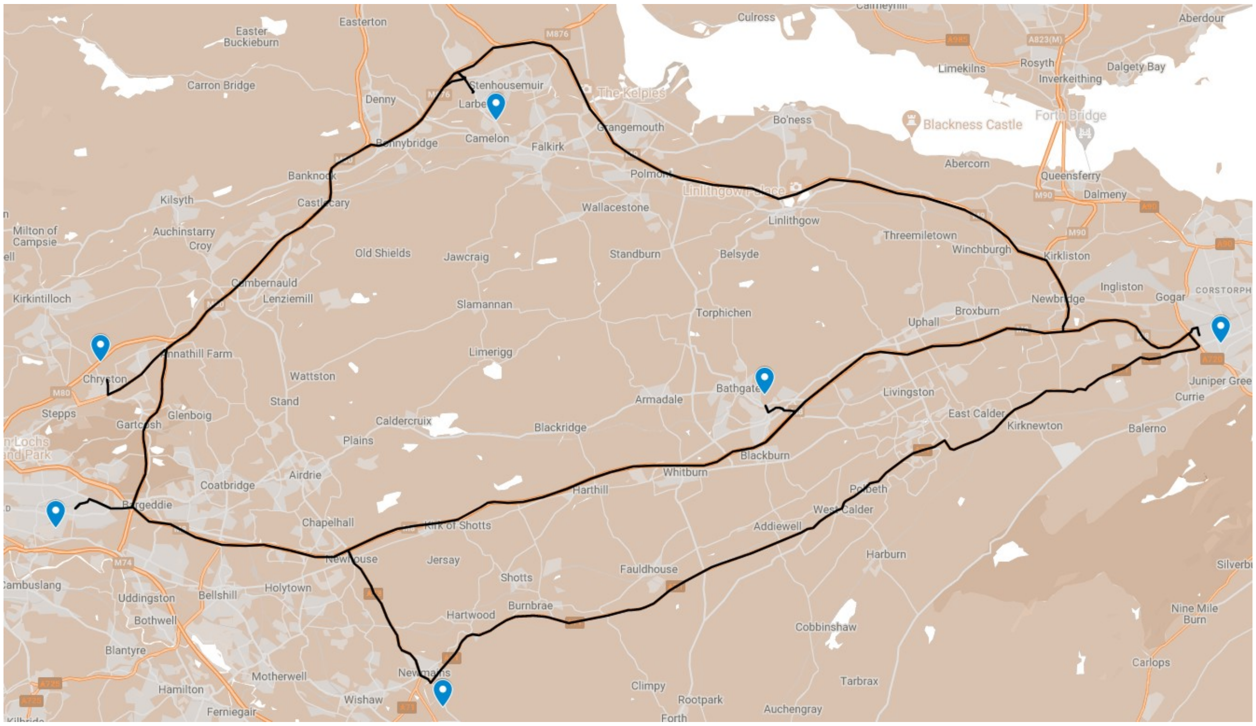
HGV vehicles are equipped with a pantograph system that allows them to connect and disconnect from installed catenary cables during transit without the need for stopping. While connected, the cables power the vehicle and can charge an electric battery that is used to power the vehicle on conventional roads. To ensure an HGV can continue travelling when the ERS network and the battery is not enough, especially during the transition period, an extra propulsion system (such as a diesel engine) can also be leveraged. An example of such vehicle is the hybrid Scania R 450, used in the first trials in Germany (Scania, 2020).

The decarbonisation of road freight transport by deploying ERS on road networks and using hybrid vehicles poses many interesting research questions in transportation, ranging from routing to network design. In this paper, we explore some of the challenges that emerge from this concept from the perspective of routing and inventory control. In the context of hybrid HGVs navigating ERS networks, where some roads are electrified, but others require the use of battery or fuel, we focus on a lot sizing problem where we aim to fulfill the stochastic demand for a single product faced by multiple retailers located on a road network by using an optimal mix of different energy sources to power the delivery truck.

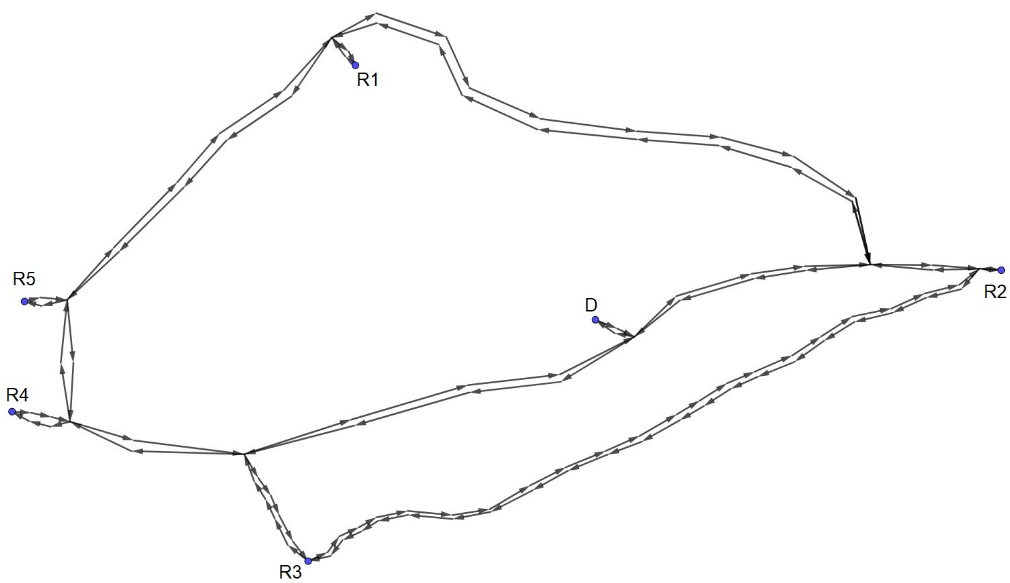
### *1.1. Conceptual framework*

Traditional routing problems such as the vehicle routing problem (VRP) and the ones that include an inventory component such as the inventory routing problem (IRP) rely on the assumption that the shortest path between two retailers in the network is known and set beforehand; this path is typically modelled as a single arc in the network graph, connecting the two retailer nodes. This assumption cannot be made in an ERS network, which embeds electrified road segments offering the opportunity to recharge, as longer paths leading to the same destination may become appealing depending on a vehicle energy needs. Moreover, for a hybrid vehicle that only relies on fuel when battery is not sufficient, the most economical path between any origin and destination pair in a road network will depend on the initial level of battery. An alternative

Figure 1: Example of a map (a) and an isochrone graph (b) representation of a region




(a)




(b)

modeling framework is needed in which the level of granularity of the model allows for tracking energy flows (i.e. battery charge and discharge) at an appropriate level as a vehicle moves through the network.

To address this gap, we adopt a new modelling strategy for inventory routing and introduce a new problem termed the Stochastic Inventory Routing Problem on Electric Roads (S-IRP-ER). In contrast to traditional routing problems, we model our system on *isochrone* graphs, like the one depicted in Figure 1. These graphs are constructed in such a way that the time needed to travel from any node to an adjacent one is constant. In Figure 1b this can be clearly seen by comparing the roads connecting nodes R2 and R3 and the parallel one above: on the road above, vehicles can cover more distance in a time period and arcs are longer than on the road below. Under this modelling strategy only a few nodes in the graph represent key locations for the system modelled, such as retailers, depots, fuel or charging stations (e.g. in Figure 1b, retailers are nodes R1, . . . , R5, and the depot is node D). The remaining nodes are used to capture road segments and are needed to establish the position of an agent (e.g. a vehicle) in the road network. In our application, the isochrone graph is used to delimit the sections in the network that are electrified and represent charging opportunities for the vehicle. Over a finite and discrete time horizon, we then model the stochastic demand faced by retailers and the movement of a hybrid HGV visiting them.

### *1.2. Contributions and outline*

We contribute to the literature as follows:

- we introduce the Stochastic Inventory Routing Problem on Electric Roads (S-IRP-ER), which accounts for battery and fuel requirements of a vehicle transiting an ERS network to fulfil demand of retailers;
- we formulate the problem via a novel framework in inventory routing that leverages isochrone graphs. These graphs are closer representations of a real road network than the abstraction of complete graphs commonly used in the IRP/VRP literature, and allow for tracking decision variables in greater detail;
- we introduce a mixed integer linear programming (MILP) heuristic to tackle this problem, and demonstrate the effectiveness of the proposed method on an extensive test bed, by contrasting its solutions against the optimal solutions obtained via stochastic dynamic programming (SDP); and
- we test our MILP-based heuristic on a realistic case study set in the area of Kent (UK), and perform a sensitivity analysis on fuel cost.

The rest of this document is structured as follows. In Section 2 we survey literature on inventory and routing problems and position our contribution. Section 3 describes the problem setting of the S-IRP-ER. In Section 4 we present an SDP formulation. Section 5 presents our heuristic, based on a MILP formulation. A computational study and results proving the effectiveness of the heuristic are provided in Section 6. In Section 7 we present a case study of the problem and investigate the effect of the costs of fuel on the solutions found by our model. Finally, our conclusions are drawn in Section 8.

## 2. Literature survey

Since its introduction by Dantzig & Ramser (1959), the VRP has been a central framework for routing research (see e.g. Laporte, 2009). In its basic version, the problem consists of devising a least-cost set of routes on a given network visiting a set of retailers and with all routes starting and finishing at a single location. Mathematically, VRP problems are typically defined on *complete graphs*, with the abstraction of each arc between two retailers representing the shortest path of a sparsely connected graph representing the original road graph (Toth & Vigo, 2002). This is the common case in the literature: the cost weights of the arcs represent distance, or other quantities (e.g. travel time, fuel consumption) that, in the context of the problem, are correlated with distance.

Another important stream in the routing literature is on green logistics and refuelling decisions. Regarding the first, the Pollution Routing Problem (PRP), introduced by Bektaş & Laporte (2011), sets a different approach for the common objectives of the VRP by incorporating a comprehensive model, adopted from Barth et al. (2004), to estimate energy consumption and GHG emissions of the vehicles. Since then, this approach has been followed in other green logistic studies that are not direct extensions, or variants, of the PRP (e.g. Goeke & Schneider, 2015; Lu et al., 2020). Regarding refuelling decisions, the Green-VRP (Erdoğan & Miller-Hooks, 2012) is introduced as a problem in which the aim is to minimise environmental impact while scheduling routes of vehicles with a limited refueling infrastructure: in addition to the required retailer visits, the vehicles may make a stop at refueling nodes when needed.

There are studies that aim to incorporate an inventory element to routing problems. The IRP and its variants (Coelho et al., 2014) is perhaps the most well studied problem that combines both inventory and routing elements. In its basic form, the IRP consist of a set of retailers holding inventory of a single product that is depleted on each period by the demand they face. There is a single warehouse, from which each vehicle starts and finishes a route on any single period in order to visit retailers. The network design and routing on each stage is the same as in VRP problems.

The S-IRP-ER is a green logistics problem that shares features with the IRP and the G-VRP literature, but departs from it by utilising a different conceptual framework. As introduced in Section 1.1, rather than designing the network as a *complete graph abstraction*, we utilise *isochrone graphs*. In contrast to the IRP literature, in which in each period a complete route is covered, the S-IRP-ER is presented as a *small bucket* framework (Belvaux & Wolsey, 2001), in which time intervals are small, treating in detail what happens during them as a vehicle travels short road segments. Regarding the G-VRP literature, the S-IRP-ER differs from it not only in our methodological approach, but also in the practical case where it arises: rather than recharging the battery by stopping at particular nodes in the network for a certain period of time, recharging happens instead as the vehicle travels along certain arcs. This framework is more general since it allows modelling of recharging stations as nodes in which a vehicle can stop for one of more periods and charge the battery during the time spent there. It is finally worth mentioning that, to the best of our knowledge, there are no studies that address a stochastic inventory routing problem with recharging opportunities.

The use of isochrone graphs in the S-IRP-ER is not arbitrary. As discussed in other works concerned with shortest paths for EVs (Artmeier et al., 2010; Eisner et al., 2011a,b), the most economical route between two locations depends on the starting battery level of the vehicle; this makes routing decisions dynamic. Therefore, these problems cannot be formulated on a complete graph, since the abstraction of an arc between two retailers representing the amount of charge/discharge of the battery cannot be made. The graph structure of the S-IRP-ER must capture the original road structure, allowing for road segment modeling and turn-by-turn directions. These turn-by-turn decisions are taken over a discrete finite time horizon, which also tracks the battery level and the load inventory of the vehicle. This allows the vehicle to visit a retailer or transit a particular arc multiple times if needed (e.g. to allow for multiple deliveries to the same retailer), and differentiates our setting from capacitated VRP problems, in which decision variables such as inventory load in a vehicle can only be preserved if the vehicle visits each retailer only once.

Another key difference of our work with respect to VRP, IRP, and green logistics literature is on the nature of demand. In these problems, the demand of each retailer node is either known, or stationary. In the S-IRP-ER, retailers face non-stationary stochastic demand, defined over a discrete finite time horizon. To the best of our knowledge, no other studies in these streams of literature address this demand setting.

In summary, our work presents the first model in the inventory routing literature that considers routing on isochrone graphs with recharging opportunities over arcs on a general setting of non-stationary demand at retailer locations.

## 3. Problem definition and formulation

We present the Stochastic Inventory Routing Problem on Electric Roads (S-IRP-ER) as a transportation problem in which some road sections feature ERS technology, allowing a single vehicle to be powered by grid electricity while charging its electric battery alongside. The aim of the vehicle is to deliver a single product to a set of retailers facing demand of that product under uncertainty. In this section we first provide the problem definition, and then introduce an SDP formulation.

### *3.1. Problem definition*

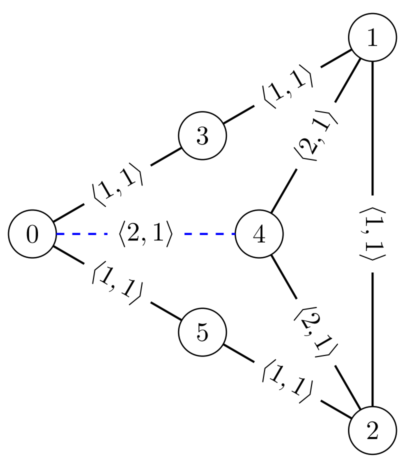
We consider a directed graph $G = \langle \mathcal{N}, \mathcal{A} \rangle$ representing a road network as an isochrone graph, with $\mathcal{N}$ as the set of nodes and $\mathcal{A}$ the set of arcs as segments of road. Isochrone graph means that each segment of the road is travelled in one unit of time. A subset of the nodes $\mathcal{C} \subseteq \mathcal{N}$ represents retailers, and there is a node, denoted as 0, representing a depot. Other nodes in $\mathcal{N} \setminus \mathcal{C} \cup \{0\}$ are used to represent $G$ as an isochrone graphs (with segments that are travelled in one time unit) and also to delimit ERS sections.

We consider a discrete planning horizon of $T$ periods. At each period $t$, each retailer $c \in \mathcal{C}$ faces a random demand $d_t^c$ for a single product. A vehicle movement from its current node position to an adjacent one takes one time period. If the origin node is the depot or a retailer, the vehicle can load or deliver some inventory, respectively. Each retailer $c \in \mathcal{C}$ has an inventory capacity of $k_c$ units, and the vehicle has a hold capacity for inventory of $K$ units. In each period, after a possible delivery to a retailer, the demand of all retailers takes place. If a retailer does not hold enough inventory to cover the demand of that period, the unsatisfied demand is considered lost and a penalty cost $p$ per unit is incurred.

The vehicle has an electric battery with an energy capacity of $B$ (in kWh units). When the vehicle traverses an arc $(i, j)$ it requires $r_{ij}(M) = \alpha_{ij} M + \beta_{ij}$ kWh from the battery, where $M$ is the unladen weight of the vehicle (in kg) and $\alpha_{ij}$ and $\beta_{ij}$ are known parameters (see Appendix A). Parameters $s_{ij}$ account for the energy supplied to the vehicle (in kWh) when it traverses an electrified arc. The level of electric battery of the vehicle is updated after traversing an arc, accounting for the supplied and required electric energy at the time. If the level of battery is not sufficient the vehicle uses fuel (e.g. diesel) to reach the destination node; the quantity of fuel consumed is computed on the basis of the extra energy required to arrive at the new node, after the battery is depleted. Although recharging opportunities present themselves only on ERS arcs, it would be straightforward to extend the model to include charging stations: spending one or more periods at a charging station node would charge the battery according to the power the station can provide and the granularity of the periods. The cost of a kWh of energy provided by grid electricity or battery and fuel is given by parameters $C^e$ and $C^f$ respectively, and they are used to derive the transportation costs of the vehicle.

The goal of the S-IRP-ER is to find a route for the vehicle, and a replenishment plan for retailers, which minimises the costs of travelling, and the costs of unfulfilled demand over the time horizon.

Figure 2: Problem instance on a ERS network. Tuples on arcs are $\langle \alpha_{ij}, \beta_{ij} \rangle$; supplied energy in the dashed blue arc is $s_{04} = 20$, while in the remaining arcs is 0.




**Example 1.** *The following example is presented with the illustrative purposes of depicting the S-IRP-ER. For the sake of simplicity, the values of the parameters are abstract, and demand deterministic.*

*We consider the simple road network presented in Figure 2, in which the depot is located at node 0 and the only retailers of the network are located at nodes 1 and 2, with inventory capacities* $k_1 = k_2 = 5$. *Demand is* $d_t^c = 1$, *for* $c = 1, 2$ *and* $t = 1, \ldots, T$; *and the penalty cost per unit of lost sales is* $p = 25$. *The unladen*

Table 1: Optimal solution for Example 1

| | $t=0$ | $t=1$ | $t=2$ | $t=3$ | $t=4$ |
|---|---|---|---|---|---|
| V. Position | N/A | 0 | 4 | 1 | 2 |
| V. inv. upload | N/A | 3 | 0 | 0 | 0 |
| Delivery | N/A | 0 | 0 | 2 | 1 |
| Battery level | N/A | 0 | 11 | 2 | 0 |
| Weight $M = w + L$ | N/A | 4 | 4 | 2 | 1 |
| Vehicle Load (L) | 0 | 3 | 3 | 1 | 0 |
| Inv. ret. 1 | 2 | 1 | 0 | 1 | 0 |
| Inv. ret. 2 | 3 | 2 | 1 | 0 | 0 |
| Required energy | N/A | 9 | 9 | 3 | 0 |
| Travel costs | N/A | 9 | 9 | 7 | 0 |
| Penalty costs | N/A | 0 | 0 | 0 | 0 |

*weight of the vehicle is* $w = 1$*; the vehicle has a battery capacity* $B = 20$ *and inventory capacity* $K = 4$*. Travel costs are* $C^e = 1$ *and* $C^f = 5$*. We consider a time horizon of* $T = 4$ *periods. The vehicle starts at the depot node in period 1, with no load and a depleted battery. Retailer 1 starts with 3 units of inventory and Retailer 2 with 2 units.*

*The optimal solution for the problem is presented in Table 1. On the first period, the vehicle starts at the depot and loads 3 units of inventory. Then transits to node 3 using a ERS road. In period 2, the vehicle starts with 9 units of battery (on the previous travel the ERS road provided 20 units but 9 were required to power the vehicle) and transits to node 1. In period 3, the vehicle delivers 2 units of inventory to Retailer 1 and transits to node 2 using 2 units of battery and 1 of fuel. In period 4 the vehicle delivers inventory to Retailer 2, and the vehicle does not transit to a new node. There are no penalty costs incurred in any periods since the vehicle delivers to both retailers before demands exceeds inventory and enough to cover demand until the end of the time horizon.*

*The optimal solution when* $(0, 4)$ *is not electrified follows similarly and is presented in Table 2. In this case the path of the vehicle does not transit* $(0, 4)$ *since the arc is not electrified and travelling costs arising from the former path would be higher.*

## 4. A Stochastic Dynamic Programming formulation

The S-IRP-ER can be formulated via stochastic dynamic programming (SDP) over a planning horizon of $T$ periods by defining the *state space* of the problem, the *feasible actions* $A_s$ associated to each state $s$, the *transition probabilities* $p^a_{s,s'}$ and *immediate costs* $c^a_s$ of each state-action pair $\langle s, a \rangle$ and the *objective function* defined over the value function

Table 2: Optimal solution for Example 1 with $s_{04} = 0$

| | $t = 0$ | $t = 1$ | $t = 2$ | $t = 3$ | $t = 4$ |
|---|---|---|---|---|---|
| V. Position | N/A | 0 | 3 | 1 | 2 |
| V. inv. upload | N/A | 3 | 0 | 0 | 0 |
| Delivery | N/A | 0 | 0 | 2 | 1 |
| Battery level | N/A | 0 | 0 | 0 | 0 |
| Weight $M = w + L$ | N/A | 4 | 4 | 2 | 1 |
| Vehicle Load (L) | 0 | 3 | 3 | 1 | 0 |
| Inv. Ret. 1 | 2 | 1 | 0 | 1 | 0 |
| Inv. Ret. 2 | 3 | 2 | 1 | 0 | 0 |
| Required energy | N/A | 5 | 5 | 3 | 0 |
| Travel costs | N/A | 25 | 25 | 18 | 0 |
| Penalty costs | N/A | 0 | 0 | 0 | 0 |

$$V_t(s) = \min_{k \in A_s} \{c_s^k + \sum_{s'} p_{s,s'}^k V_{t+1}(s')\}. \tag{1}$$

Before defining these components, we provide a detailed explanation of the dynamics of the vehicle and the retailers. At the beginning of each period the vehicle can deliver product to a retailer, if the starting node is a retailer node; or load inventory, if the starting node is the depot node. The total weight of the vehicle for the period is updated after these possible actions.[1] In both cases, the quantities are subject to the storage capacity of the retailers $k_c$ and the vehicle inventory capacity $K$. After any possible delivery during the period is accounted for, the demand at each retailer (modeled by random variables $d_t^i$ for retailer $i$ and period $t$) takes place. If a retailer does not hold enough inventory to cover the demand of each period, the unsatisfied demand of that period is considered lost and a penalty cost $p$ per unit is incurred. The vehicle finally transits to an adjacent node at the end of the period, updating the level of battery for the next period, and incurring travelling costs.

We next model the S-IRP-ER by using SDP.

- *States*: states are given by tuples $(t, V_{\text{pos}}, V_{\text{inv}}, V_{\text{bat}}, I_{\text{ret}})$ where $t$ is the current period associated with the state, $V_{\text{pos}}$ is the starting position of the vehicle at time $t$, $V_{\text{inv}}$ is the starting product inventory on hold of the vehicle before any load or delivery of product takes place, $V_{\text{bat}}$ the battery level of the vehicle at $V_{\text{pos}}$ and $I_{\text{ret}}$ is a tuple with the inventory of each retailer before any delivery at time $t$.

[1]Demand uncertainty affects vehicle load and delivery decisions, and therefore the total weight of the vehicle in each period, which is one of the main factors affecting energy requirements as discussed by Bektaş & Laporte (2011).

- *Actions*: an action $a \in A_s$, related to state $s$, is a tuple $(V'_{\text{pos}}, V_{\text{up}}, Q_{\text{ret}})$ where $V'_{\text{pos}}$ is the destination node of the vehicle, $V_{\text{up}}$ is the inventory loaded up to the vehicle and $Q_{\text{ret}}$ is the possible delivery quantity at $V_{\text{pos}}$.

- *Transition probabilities*: from a state $s$ and action $a$, the transition probabilities to a new state $s'$, $p^a_{s,s'}$ are derived directly from the random demand distributions.

- *Immediate costs*: the immediate costs of a state-action pair $(s, a)$ are given by the costs of travel from $V_{\text{pos}}$ to $V'_{\text{pos}}$ with weight derived from $V_{\text{inv}}$ and accounting for the weight that is loaded up or delivered to a retailer, and the expected penalty costs after updating $I_{\text{ret}}$ with the possible delivery $Q_{\text{ret}}$ at node $V_{\text{pos}}$.

With all components defined, the optimal policy for each state can be derived by applying a backward or forward recursion algorithm on Eq. (1), that is the value function. However, this approach only allow us to solve small instances of the problem. In the next section, we explore a MILP heuristic for the S-IRP-ER.

## 5. A MILP-based heuristic for the S-IRP-ER

In this section, we present a mixed-integer linear programming (MILP) based heuristic for the S-IRP-ER. A summary of the symbols used in this model is presented in Appendix B. We define the adjacency matrix of graph $\mathcal{G}$ via binary variables $\delta_{ij}$. We also denote $N$ and $C$ as the cardinalities of $\mathcal{N}$ and $\mathcal{C}$, the number of nodes and retailers.

In the S-IRP-ER, the demand faced by each retailer $i$ at period $t$ is a random variable denoted as $d^i_t$. The lost sales at each retailer in any given period are also random variables, and become a central aspect when formulating stochastic programming models in inventory control. Given a random variable $\omega$ and a scalar $q$, the first order loss function is defined as

$$\mathcal{L}_\omega(q) = E[\max(\omega - q, 0)]. \tag{2}$$

In a stochastic lot sizing setting, this function represents the expected inventory shortfall, given starting inventory at hand $q$, when random demand is $\omega$. Reciprocally, the complementary first order loss function expresses the expected net inventory after demand

$$\hat{\mathcal{L}}_\omega(q) = E[\max(q - \omega, 0)]. \tag{3}$$

In what follows, we adopt a "static uncertainty" strategy (Bookbinder & Tan, 1988) in which replenishment periods and replenishment quantity decisions for all retailers are fixed at the beginning of the planning horizon, before demand random variables for any period are realised. In practical terms, this means that the route the vehicle follows is decided beforehand as well, and since the replenishment quantities $Q^i_t$ are fixed, the load of the vehicle and energy requirements can be calculated at the beginning of the planning horizon.

The objective function minimises the cost associated with carbon emissions emitted during the time horizon plus the cost of lost sales at the retailers

$$\min \sum_{t=2}^{T}\sum_{i=1}^{N}\sum_{j=1}^{N} C^e s_{ij} T_{t-1}^{ij} + \sum_{t=1}^{T} C^e E_t^b + \sum_{t=1}^{T} C^f E_t^f + \sum_{t=1}^{T}\sum_{i=1}^{C} p[I_t^i]^-. \tag{4}$$

The first term of the objective function accounts for the emissions caused by powering the vehicle on ERS roads. The second and third term account for the emissions related to the used battery and the used fuel respectively. The last term accounts for the penalty cost of the expected lost sales.

We next introduce constraints related to the movement of the vehicle, product inventory, and energy use of the vehicle.

Eqs. (5)-(10) deal with the movement of the vehicle over the time horizon

$$\sum_{i=1}^{N} V_t^i = 1 \qquad t = 1, \ldots, T; \tag{5}$$

$$\sum_{j=1}^{N} T_{t-1}^{ij} = V_{t-1}^i \qquad i = 0, \ldots, N-1; t = 2, \ldots, T; \tag{6}$$

$$T_{t-1}^{ij} \geq V_{t-1}^i + V_t^j - 1 \qquad i, j = i = 0, \ldots, N-1; t = 2, \ldots, T; \tag{7}$$

$$T_{t-1}^{ij} \leq V_{t-1}^i \qquad i, j = i = 0, \ldots, N-1; t = 2, \ldots, T; \tag{8}$$

$$T_{t-1}^{ij} \leq V_t^j \qquad i, j = i = 0, \ldots, N-1; t = 2, \ldots, T; \tag{9}$$

$$T_t^{ij} \leq \delta_{ij} \qquad i, j = 0, \ldots, N-1; t = 1, \ldots, T-1. \tag{10}$$

At each period of the planning horizon, the vehicle is situated at one and only one of the nodes $i \in \mathcal{N}$. We model this by considering binary variables $V_t^i$, being equal to one if the vehicle is present at node $i \in \mathcal{N}$ at the beginning of period $t$, and $T_t^{ij}$ as a binary variable set to one if the vehicle transits from node $i$ to node $j$ during period $t$. Eqs. (5) reflects the fact that the vehicle is, at any point in time, at one and only one node of graph $\mathcal{G}$. Eqs. (6) ensures that for each period the vehicle transits to a new node from where it starts. Eqs. (7)-(9) link transit $T_t^{ij}$ and position $V_t^i$ variables. Eqs. (10) ensures that the vehicle only transits valid arcs, observing the adjacency matrix of the graph.

Eqs. (11)-(18) are concerned with product inventory levels

$$L_t \leq K V_t^0 \qquad t = 1, \ldots, T; \tag{11}$$

$$Q_t^i \leq k_i V_t^i \qquad i = 1, \ldots, C; t = 1, \ldots, T; \tag{12}$$

$$l + \sum_{k=1}^{t} L_k - \sum_{i=1}^{N}\sum_{k=1}^{t-1} Q_k^i \leq K \qquad t = 1, \ldots, T; \tag{13}$$

$$l + \sum_{k=1}^{t} L_k - \sum_{i=1}^{N}\sum_{k=1}^{t-1} Q_k^i \geq 0 \qquad t = 1, \ldots, T. \tag{14}$$

Eqs. (11) ensure the vehicle can only load inventory to the vehicle while visiting the depot node, and only up to the vehicle capacity $K$. Eqs. (12) ensure the vehicle can only deliver inventory in one period if it is visiting a retailer node, and up to their capacity. The transition of inventory levels of the vehicle are tracked by Eqs. (13) and (14), ensuring the total cargo does not exceed the capacity of the vehicle.

Eqs. (15), (16) and (17) approximate expected lost sales $[I_t^i]^-$, expected inventory $[I_t^i]^+$ and calculate the expected inventory that would exceed the capacity of each retailer when a delivery is made $e_t^i$

$$[I_t^i]^- = \mathcal{L}_{d_{1t}^i}\left(s_i + \sum_{k=1}^{t} Q_k^i + \sum_{k=1}^{t-1}[I_{t-1}^i]^- - \sum_{k=1}^{t} e_k^i\right) \qquad t = 1,\ldots,T; i = 1,\ldots,C; \tag{15}$$

$$[I_t^i]^+ = \hat{\mathcal{L}}_{d_{1t}^i}\left(s_i + \sum_{k=1}^{t} Q_k^i + \sum_{k=1}^{t-1}[I_{t-1}^i]^- - \sum_{k=1}^{t} e_k^i\right) \qquad t = 1,\ldots,T; i = 1,\ldots,C; \tag{16}$$

$$e_t^i = \max\left([I_t^i]^+ + Q_t^i - s_i, 0\right) \qquad t = 1,\ldots,T; i = 1,\ldots,C. \tag{17}$$

Eqs. (15) use the loss function with the random variables $d_{1t}^i = d_1^i + \ldots + d_t^i$. The idea of the approximation is to calculate the lost sales $[I_t^i]^-$ of a single period comprising the demand of periods $1,\ldots,t$ with a starting inventory $s_i$, and all deliveries $Q_k^i$ up to period $t$, plus the expected lost sales $[I_{t-1}^i]^-$ until period $t-1$ (see Rossi et al., 2019, Lemma 5), and reducing the inventory by the sum of the expected inventory exceeding capacity of retailer $i$ and times $1,\ldots,t$. Similarly, Eqs. (16) approximate expected inventory of each retailer and period $[I_t^i]^+$. Eqs. (17) are used to calculate the expected inventory that would exceed the capacity of each retailer when a delivery is made, and used in the approximations of Eqs. (15) and Eqs. (16). It follows from the fact that, in a lost sales setting, shortages reset at the end of each period and $Q_t^i$ and $s_i$ are constants (see Rossi et al., 2019, Lemma 6). The loss function and the complementary loss function used in Eqs. (15) and (16) are nonlinear. We follow the approach of Rossi et al. (2014) to linearise these functions to be able to use them in our stochastic programming model.

Eqs. (18) update the total weight of the vehicle at the end of each period, after a possible delivery or load up of inventory

$$W_t = w + l + \sum_{k=1}^{t} L_k - \sum_{i=1}^{N}\sum_{k=1}^{t} Q_k^i \qquad t = 1,\ldots,T. \tag{18}$$

Eqs. (19)-(22) track the battery level inventory over the time horizon, which is used to derive the energy used from battery and fuel for each period

$$b_t^u = b_{t-1} + \sum_{i=1}^{N}\sum_{j=1}^{N} T_{t-1}^{ij}\left(s_{ij} - \alpha_{ij}W_{t-1} - \beta_{ij}\right) \qquad t = 2,\ldots,T; \tag{19}$$

$$b_t = \min\{\max\{b_t^u, 0\}, B\} \qquad t = 2,\ldots,T; \tag{20}$$

$$E_{t-1}^b = \max\{b_{t-1} - b_t, 0\} \qquad t = 2,\ldots,T; \tag{21}$$

$$E_{t-1}^f = \max\left(-b_t^u, 0\right)/\lambda \qquad t = 2,\ldots,T. \tag{22}$$

The variables $E_t^b$ represent the energy used by the vehicle (in kWh) from its battery on the travel performed during period $t$. In contrast, variables $E_t^f$ represent the mechanical energy the vehicle needs to finish traversing an arc when the battery has been depleted. In order to determine these variables, we define further auxiliary variables and parameters. First of all, variables $b_t^u$ represent the unconstrained battery level of the vehicle, calculated in Eqs. (19) from the actual level of battery of the previous period, $b_{t-1}$, plus and minus supplied and required energy from the total mass of the vehicle $W_{t-1}$ at the beginning of the period. Expanding this equation, the product of a binary and a continuous variable appear, $T_{t-1}^{ij}W_{t-1}$, which are linearised in our model considering $L_t$ variables, the weight of the vehicle at the end of period $t$, are bounded between zero and the maximum weight of the vehicle $W$. In Eqs. (20), variables $b_t$ are calculated as the $b_t^u$ bounded between 0 and $B$, representing the actual level of battery for each period. Eqs. (21) calculate battery usage in each period, considering that if the battery was charged traversing a ERS arc, the battery consumed during that period should be zero. In Eqs. (22), if $b_t^u < 0$, the interpretation is that during period $t$ the battery was depleted and $-b_t^u$ kWh were still needed to complete travelling the arc on battery. Dividing by the corresponding battery energy losses factor $\lambda$, we obtain $E_t^f$ variables as the required mechanical energy to complete the travel when battery is not sufficient.

Finally, Eqs. (23)-(27) represent the domains of the variables used

$$T_t^{ij}, V_t^i \in \{0,1\} \qquad c = 1,\dots,C; t = 1,\dots,T; \tag{23}$$

$$b_t, b_t^u, E_t^f, E_t^b, L_t \geq 0 \qquad t = 2,\dots,T; \tag{24}$$

$$b_t \leq B \qquad t = 2,\dots,T; \tag{25}$$

$$Q_t^i, S_t^i \geq 0 \qquad i = 1,\dots,C; t = 1,\dots,T; \tag{26}$$

$$0 \leq S_t^i \leq d_t^i \qquad i = 1,\dots,C; t = 1,\dots,T. \tag{27}$$

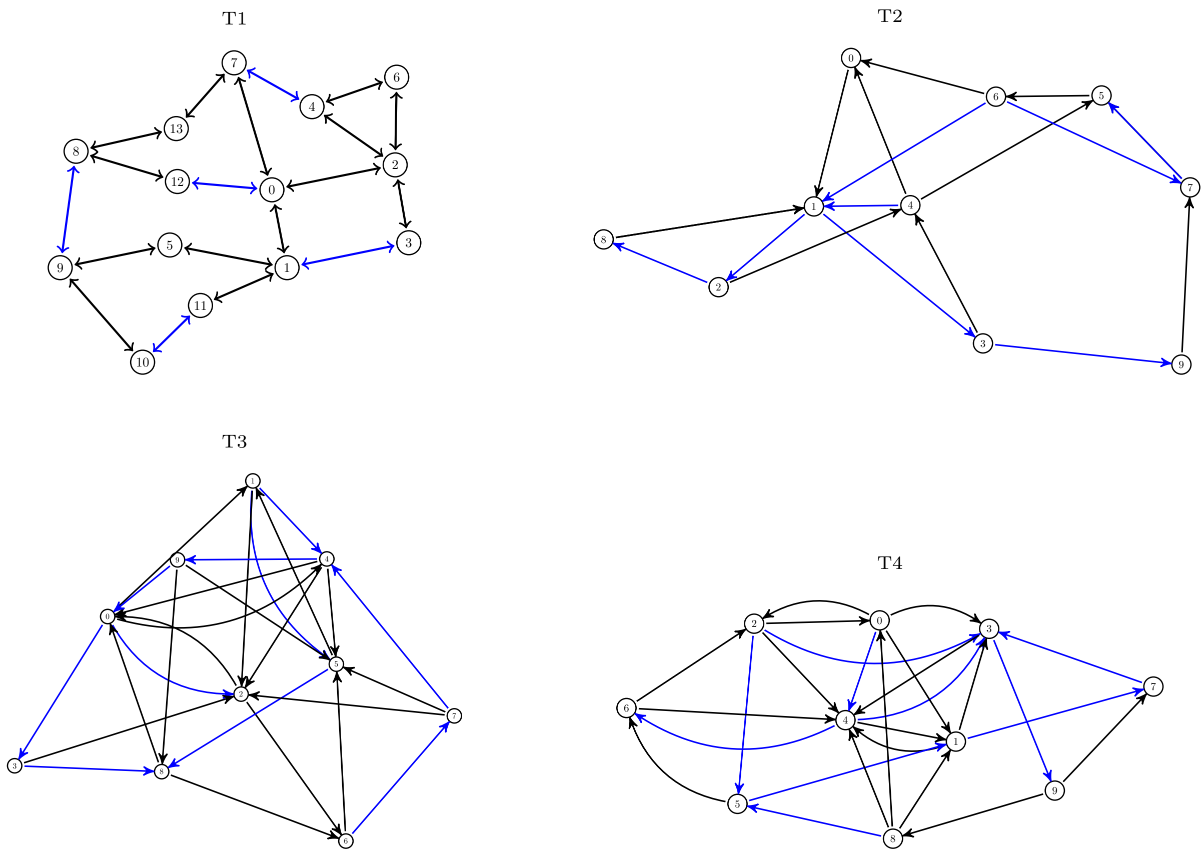
The MILP model consists of the objective function (4) subject to the constraints set in Eqs. (5)-(27).

## 6. Numerical experiments: effectiveness of the MILP-based heuristic

With the aim of investigating the effectiveness of the MILP-based heuristic designed for the S-IRP-ER we present the following computational study. Our goal is to compare the cost of the optimal plan obtained by the heuristic against the optimal cost obtained via SDP.

To ensure that the state space of the SDP implementation is finite, we introduce the following assumptions in relation to inventory and battery state variables. Please note that this is done without loss of generality, and for the sake of comparability, as these assumptions are not required in the MILP model. First, the stochastic demand faced by retailers is expressed using finite discrete random variables. Both retailers and vehicle product inventory are discretised by using the same units. Second, the battery of the vehicle also needs to be discretised, and as a consequence the required energy will be expressed in terms of these battery levels. Equation (A.3) needs to be rounded to the closest battery level unit for each possible weight state of the vehicle, but this constraint cannot be captured in a MILP model. Therefore, we introduce new integer parameters $\varepsilon_{ij}^{W_t}$ see Eq. (28) and rely on Constraint Programming (Rossi et al., 2006) to model and solve these instances.

Figure 3: Network topologies




All our experiments have been carried out on a PC Intel(R) Core(TM) i7-8665U CPU @ 1.90GHz with 32 GB of RAM, using IBM ILOG CPLEX Studio 12.7 with a time limit of 10 minutes and default constraint programming settings otherwise. The design of the experiments is described in Section 6.1, and the results obtained are presented and discussed in Section 6.2.

### *6.1. Design of experiments*

The experiments are designed over a time horizon comprising $T = 9$ periods, on four road networks depicted as the graphs in Figure 3. Road network T1 has been generated for this study, while the other three were used in the computational study of (Rossi et al., 2019). On each road network, we consider six sets of retailers chosen randomly. The demand at the retailers follow normalised Poisson distributions truncated to a maximum value of 8 units. We use 10 segments for the piecewise linearisation of the loss functions associated with the stochastic demand. The inventory capacity of each retailer is 8 units while the capacity of the vehicle is 10 units.

The test bed comprises 432 instances, the full factorial of the values considered for the following parameters: road networks, sets of retailer nodes, initial inventory level of retailers, unit penalty costs — in monetary units (MU) — and demand patterns faced by retailers over the planning horizon, as Poisson

Table 3: Values of parameters considered in the test bed

| | |
|---|---|
| Initial inventory at {R1,R2} | {{0, 0},{5, 5}} |
| Demand distributions | (D1) $\lambda_{R1}$ = {2,2,2,2,2,2,2,2,2}, $\lambda_{R2}$ = {2,2,2,2,2,2,2,2,2} (D2) $\lambda_{R1}$ = {1,1,2,2,3,3,4,4,5}, $\lambda_{R2}$ = {5,4,4,3,3,2,2,1,1} (D3) $\lambda_{R1}$ = {1, 1, 2, 1, 1, 2, 2, 3, 1}, $\lambda_{R2}$ = {1, 1, 2, 1, 1, 2, 2, 3, 1} |
| Unit penalty cost | $p$ = {10, 20, 30} |

distribution described by their mean parameters. The possible values considered are described in Table 3.

For our tests we consider a vehicle with a Gross Vehicle Weight (GVW) of 22 tonnes (t) with a maximum payload of 10t and a battery size of 150 kWh Ainalis et al. (2020). The cost of one kWh of energy from ERS or battery is taken as 1 MU ($C^e = 1$), while one kWh of fuel costs 3 MU ($C^f = 3$). Regarding the battery of the vehicle, we consider 20 possible levels for the state of charge.

As mentioned at the beginning of this Section we must ensure that operations of charge and discharge of battery are closed for the state space considered. To this end we introduce new parameters:

$$\varepsilon_{ij}^{W_t} = \text{round}\left( (\alpha_{ij} W_t + \beta_{ij}) \frac{20}{150} \right), \tag{28}$$

which account for required energy, expressed in battery level units, of each arc and vehicle weight possible. Equation (19) is adapted accordingly

$$b_t^u = b_{t-1} + \sum_{i=1}^{N} \sum_{j=1}^{N} T_{t-1}^{ij} \left( s_{ij} - \varepsilon_{ij}^{W_{t-1}} \right). \tag{29}$$

### *6.2. Results and discussion*

The results of our computational study are summarised in Table 4, which shows the mean, median and standard deviation of the percentage error of the solutions obtained by the MILP-based heuristic when compared with the optimal solution obtained by the SDP formulation, when pivoting over all the parameters considered in our test bed. Overall, the heuristic obtains solutions whose costs are on average 3.72% above the optimal. It is worth noticing that whereas the optimal solution is achieved by reviewing the state of the system on every period, after the demand at the retailers is realised and being able to adapt decisions, our MILP-based heuristic focuses on a static uncertainty strategy (Bookbinder & Tan, 1988), providing a fixed policy set at the beginning of the planning horizon. Our MILP-based heuristic could be implemented within a receding horizon framework: in this setting the MILP-based heuristic is used over periods $t, \ldots, T$ (starting with $t = 1$) but only decisions for the current period $t$ are implemented. After the demand is observed the

Table 4: Pivot table of mean, median and standard deviation of percentage error (MPE, MdPE, SD respectively) of the solutions obtained by the MILP-based heuristic for the computational study

| | MPE | MdPE | SD |
|---|---|---|---|
| Network | | | |
| T1 | 3.47% | 3.42% | 2.14% |
| T2 | 3.48% | 3.19% | 2.05% |
| T3 | 3.18% | 2.53% | 2.57% |
| T4 | 4.73% | 3.88% | 4.12% |
| | | | |
| Initial inv. | | | |
| (0,0) | 3.94% | 3.41% | 2.87% |
| (5,5) | 3.49% | 3.03% | 2.91% |
| | | | |
| Penalty | | | |
| 10 | 1.67% | 1.30% | 1.57% |
| 20 | 3.98% | 3.51% | 2.45% |
| 30 | 5.50% | 5.22% | 3.06% |
| | | | |
| Demand pattern | | | |
| D1 | 3.70% | 3.39% | 2.67% |
| D2 | 3.50% | 3.02% | 2.71% |
| D3 | 3.95% | 3.29% | 3.28% |
| | | | |
| General | 3.72% | 3.25% | 2.90% |

state of the system is updated — inventory level of retailers and vehicle load, position of the vehicle on the graph and level of battery — and the same process is successively repeated over periods $t+1,\dots,T$ until we reach the end of the time horizon. This approach has proven to improve cost performances of lot sizing problem policies such as the one considered in our heuristic (Dural-Selcuk et al., 2020), and while this could represent a future research direction, we do not expect results to provide insights over and above what is already known in the literature.

## 7. Case study

In this section we investigate how the relation of the costs of electricity and fuel, and the penalty costs, affect the plan provided by the MILP-based heuristic. Our new experiments are set up in the region of Kent, in the United Kingdom, creating a realistic instance of the problem, and provide an example of how the delivery quantities and the order of visits to retailers can impact the costs of transportation and reduce the usage of fuel on a ERS network. Figure 4 shows a map of the region, highlighting the major roads used

Table 5: Summary of the solutions, indicating quantities delivered to retailers and the order of visits to the main nodes: Dover (D), Folkestone (F), Ashford (A), Canterbury (C), Maidstone (M), Sittingbourne (M)

| Instance number | Initial inv. | $p$ | $C^f$ | Vehicle load | Total cost | Penalty cost | Transport cost | Deliveries to retailers F | A | C | M | S | Visit order |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | I1 | 0.1 | 3 | 2428.75 | 318.33 | 158.89 | 159.44 | 854.75 | 828.27 | 745.73 | 0.00 | 0.00 | D,F,A,C,D |
| 2 | I1 | 0.1 | 6 | 2417.85 | 369.94 | 228.86 | 141.08 | 849.16 | 828.27 | 740.42 | 0.00 | 0.00 | D,F,A,C,D |
| 3 | I1 | 0.1 | 10 | 0.00 | 381.43 | 381.43 | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | N/A |
| 4 | I1 | 0.5 | 3 | 3363.47 | 636.57 | 369.35 | 267.22 | 857.16 | 840.59 | 751.04 | 378.52 | 536.16 | D,F,A,C,S,M |
| 5 | I1 | 0.5 | 6 | 2432.81 | 1136.8 | 794.36 | 342.44 | 642.92 | 823.39 | 844.17 | 122.32 | 0.00 | D,C,A,F,A,M |
| 6 | I1 | 0.5 | 10 | 2115.73 | 1202.2 | 925.82 | 276.38 | 833.72 | 751.04 | 530.97 | 0.00 | 0.00 | D,F,A,C,D |
| 7 | I2 | 0.1 | 3 | 1786.06 | 391.72 | 233.42 | 158.3 | 556.83 | 537.71 | 691.53 | 0.00 | 0.00 | D,F,A,C,D |
| 8 | I2 | 0.1 | 6 | 0.00 | 395.15 | 395.15 | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | N/A |
| 9 | I2 | 0.1 | 10 | 0.00 | 395.15 | 395.15 | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | N/A |
| 10 | I2 | 0.5 | 3 | 2969.73 | 889.89 | 597.66 | 292.23 | 172.92 | 378.52 | 1027.99 | 529.76 | 860.54 | D,C,S,M,A,F |
| 11 | I2 | 0.5 | 6 | 1817.74 | 1399.5 | 1133.9 | 265.6 | 430.73 | 526.47 | 860.54 | 0.00 | 0.00 | D,C,A,F,D |
| 12 | I2 | 0.5 | 10 | 2160.93 | 1430.4 | 1000.3 | 430.1 | 556.83 | 543.12 | 117.39 | 517.98 | 425.61 | D,F,A,M,S,C |

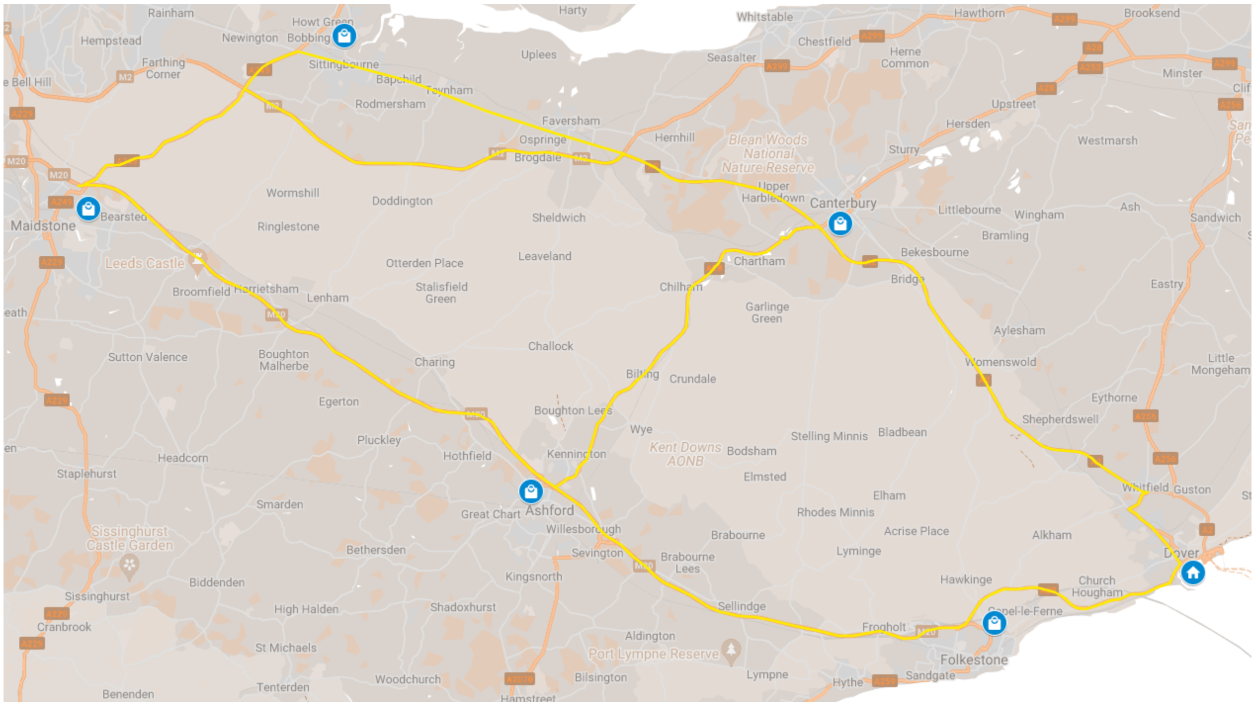
by HGVs and the graph representation of these roads, in splits of around 5 km between adjacent nodes, making a total of 52 nodes. The depot location is situated at the port of Dover, and we consider five retailer locations situated in densely populated areas of Kent. The electrified stretches of road are marked in blue, and they provide 200 kW of power. The HGV considered has a GVW of 17t, with a load capacity of 5t, and it is equipped with a 150 kWh battery. Each retailer has a stocking capacity of 2000 kg of product.

The time horizon for our experiments is 25 periods. All five retailers face demand on each period that follow independent normal distributions $\mathcal{N}(50, 2)$. We consider two configurations of initial inventory: (I1) all retailers start with 500 kg. of product and (I2) Canterbury and Sittingbourne are out of stock, while the remaining retailers start with 800 kg. The kWh cost of electricity is $C^e = 1$, and consider three values for the cost of fuel $C^f = \{3, 6, 10\}$. We also consider two values for the stockout penalty cost $p = \{0.1, 0.5\}$. In all 12 instances the vehicle starts at period 1 at the depot location, with a depleted electric battery and no loaded inventory. We run CPLEX solver with default settings and a time limit of 1 hour.

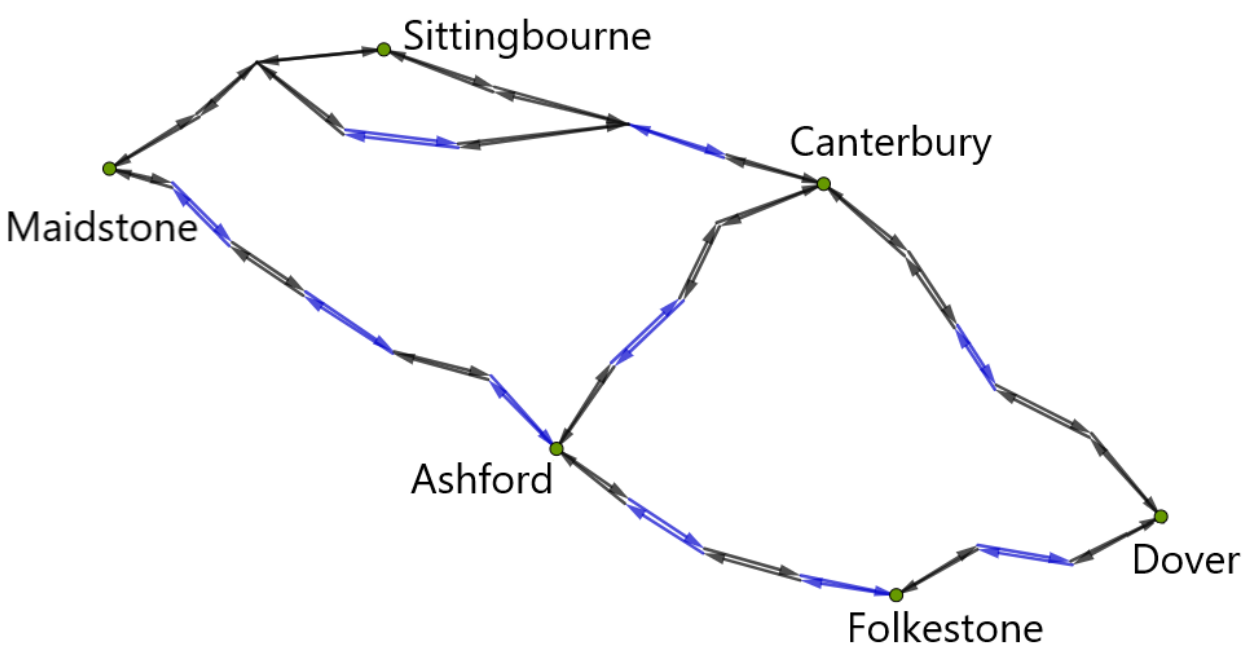
Table 5 summarises the solutions reached by the MILP-based heuristic for the 12 instances, showing the quantities that the vehicle delivers to each retailer and a sketch of the route followed by indicating the order in which the depot and set of retailers are visited. Some trends can be observed in the solutions. First of all, the vehicle load decreases with lower penalty costs and higher fuel costs, to the limiting case of not delivering to any retailers in instances 3, 8 and 9.

Fuel costs can also have an impact on the routes taken. In I2 instances (instances 7-12), Canterbury starts with no inventory and it is usually visited before other retailers (instances 8, 10, 11). However, when fuel costs are high, the vehicle either does not deliver (instances 3, 8 and 9), or starts visiting Folkestone and Ashford (instance 12) a route that allows for more battery charging and less fuel usage. This behaviour reveal the sensitivity of the model to fuel price, when it becomes uneconomical to run, and hence the vehicle does not leave the depot.

Figure 4: Map (a) and isochrone graph (b) representation of main HGV roads in Kent




(a)




(b)

## 8. Conclusions

In this paper we have dealt with a transportation problem — the S-IRP-ER — related to emerging technologies which allows to charge electric vehicles as they travel. We proposed a novel modelling framework based on isochrone graphs to address the issue that the shortest path between any pair of origin and destination in the road network is not always constant. Our framework allows for a small bucket modelling approach to energy and product inventory control. To solve the problem, we provided a MILP-based heuristic for dealing with uncertainty of demand at retailers of the product delivered. The effectiveness of the MILP-based heuristic was demonstrated by carrying out experiments on an extensive test bed comprising different demand distributions patterns over the time horizon, varying set of retailer nodes, different initial inventory at retailer nodes, and a range of values for the penalty cost. The results show that the MILP-based heuristic achieves solutions that, on average, are only 3.72% more costly than the optimal solutions. We further tested our MILP-based heuristic on a realistic case study and showcased how the costs of fuel could impact the routes chosen on an ERS network when delivering to retailers in Kent. Future research may investigate the benefit brought by a receding horizon setting (Dural-Selcuk et al., 2020) and dynamic cut generation strategies (Tunc et al., 2018) to enhance scalability of our MILP-based heuristic. Another research line would be on the study of metaheuristics that could provide good solutions for very large instances in reasonable time. Finally, in its current form, our framework cannot accommodate for changes in travel time across the network, for instance due to traffic conditions. A new approach allowing for dynamic, and possibly stochastic, travel times would represent a valuable contribution to this stream of literature.

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## Appendix A. Energy consumption model

Travelling costs, either in terms of emissions or economic costs of fuel and electricity usage, are directly related to the energy requirements (kWh) necessary to power a vehicle. To estimate these values we use a simple physics model derived from Barth et al. (2004) and in line with the followed approach by Bektaş & Laporte (2011). At any given time, the mechanical power $P$ of a vehicle, as a function of its acceleration $a$ ($m/s^2$) and velocity $v$ ($m/s$) can be estimated as follows:

$$P(a,v) = Mav + Mgv\sin\theta + 0.5C_dA\rho v^3 + MgC_r\cos\theta v, \tag{A.1}$$

where $M$ is the mass of the vehicle (kg.), $\theta$ is the road angle in degrees, $C_d$ is a drag coefficient specific for the vehicle, $A$ its frontal area ($m^2$), $g$ is the gravitational constant ($9.81m/s^2$) and $C_r$ the rolling resistance of the wheels of the vehicle with the road surface. For our purposes, we assume the vehicles transit each arc $(i,j)$ with slope $\theta = \theta_{ij}$ cruising at constant speed $v_{ij}$ and therefore, the acceleration is zero, $a = 0$. We simply denote the mechanical power on node $(i,j)$ as

$$p_{ij} = P(0, v_{ij}). \tag{A.2}$$

The electric power, and ultimately the battery power that is needed to maintain constant speed $v_{ij}$, depend on multiple factors. As in (Goeke & Schneider, 2015), we estimate the battery power required to sustain $v_{ij}$ in arc $(i,j)$ accounting for engine and battery efficiency with a single parameter $\lambda$, obtaining $P^b_{ij} = \lambda p_{ij}$. Finally, the energy required to traverse $(i,j)$ using electric battery, expressed in Joules, can be obtained by multiplying the required power from the battery by the time in seconds spent transiting the arc, $d_{ij}/v_{ij}$, and we express it as a function of the weight of the vehicle $M$:

$$\begin{aligned} r_{ij}(M) = P^b_{ij}(d_{ij}/v_{ij}) &= \lambda(Mg\sin\theta_{ij} + 0.5C_dA\rho v^2_{ij} + MgC_r\cos\theta_{ij})d_{ij} \\ &= \alpha_{ij}M + \beta_{ij}, \end{aligned} \tag{A.3}$$

where $\alpha_{ij} = \lambda d_{ij} g(\sin\theta_{ij} + C_r\cos\theta_{ij})$ and $\beta_{ij} = \lambda d_{ij} 0.5C_dA\rho v^2_{ij}$ are arc constants.

In absence of battery energy, the vehicle uses fuel to continue its trip. An estimation of fuel usage and emissions is given by Barth et al. (2004). Given the engine power output $P$ of the vehicle, the fuel rate is calculated as

$$F \approx (kNV + (P/\epsilon + P_a)/\eta)U, \tag{A.4}$$

where $k$ is the engine friction, $N$ is the engine speed, $V$ the engine displacement, $\epsilon$ the drivetrain efficiency of the vehicle, $P_a$ is the amount of engine power related with engine running losses and vehicle accessories such as air conditioner, $\eta$ is a measure of efficiency for diesel engines and $U$ is a factor that depend on $N$ and other constants.

For the scope of this problem, where travelling speeds are relatively similar on different road segments, we follow a pragmatic approach and approximate fuel directly from the power output $P$, or equally, as a linear transformation of the mechanical energy needed to traverse a specific arc.

## Appendix B. Symbols used in the MILP formulation

In this appendix, for the sake of reader's convenience, we list all symbols used in our MILP formulation.

Parameters

| | |
|---|---|
| $C$ | Number of retailers |
| $N$ | Number of nodes |
| $T$ | Number of time periods |
| $\delta_{ij}$ | Binary parameter indicating if $(i, j) \in \mathcal{A}$ |
| $B$ | Capacity of the battery (kWh) of the vehicle |
| $p_{ij}$ | Mechanical power needed to transit $(i, j)$ at constant speed $v_{ij}$ |
| $s_{ij}$ | Supplied energy (kWh) on arc $(i, j) \in \mathcal{A}$ |
| $r_{ij}(M)$ | Needed battery (kWh) to traverse $(i, j) \in \mathcal{A}$ with total mass $M$ |
| $C^e$ | Emissions caused per kWh of electric energy used |
| $C^f$ | Parameter for estimating fuel emissions from required energy |
| $d_t^c$ | Total demand (kg.) on retailer $c \in \mathcal{C}$ in period $t$ |
| $w$ | Weight of the vehicle without inventory |
| $K$ | Capacity of the vehicle |
| $k_c$ | Capacity of retailer $c$ |
| $s_i$ | Initial inventory level of retailer $i$ |
| $l$ | Initial inventory level of the vehicle |
| $p$ | Penalty costs per unit of lost sales |

Variables

| | |
|---|---|
| $V_t^i$ | Binary variable set to one iff the vehicle is at node $i$ at time $t$ |
| $T_t^{ij}$ | Binary variable set to one iff the vehicle transits from $i$ to $j$ by end of time $t$ |
| $b_t$ | Battery level (in kWh) at time $t$ |
| $E_t^b$ | energy used by the vehicle (in kWh) from electricity at time $t$ |
| $E_t^f$ | energy used by the vehicle (in kWh) from fuel at time $t$ |
| $b_t^u$ | Auxiliary variable: unbounded battery level at time $t$ |
| $b_t^+$ | Auxiliary variable: positive battery level at time $t$ |
| $Q_t^c$ | Quantity of product (kg.) delivered to retailer $c \in \mathcal{C}$ at time $t$ |
| $S_t^c$ | Unsatisfied demand (kg.) on retailer $c \in \mathcal{C}$ at time $t$ |
| $L_t$ | Inventory of the vehicle (kg. at time $t$, after replenishment is done |
| $W_t$ | Weight of the vehicle at the end of period $t$ |