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Robust Transceiver Design for Full Duplex Multi-user MIMO Systems

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***Abstract*—We consider a weighted sum-rate maximization problem for a multi-user multiple-input multiple-output (MIMO) cellular system where a full-duplex (FD) base-station (BS) serves multiple half-duplex (HD) uplink (UL) and downlink (DL) users simultaneously while taking the imperfect channel knowledge into consideration. By exploiting the relationship between weighted sum-rate and weighted minimum-mean-squared-error problems, joint design of transceiver matrices can be obtained through an iterative convergent algorithm. Simulation results confirmed the importance of accurate channel estimation in FD systems.**

***Keywords*—Full-duplex, imperfect CSI, MIMO, multi-user.**

I. Introduction

Amongst the emerging technologies for next-generation wireless networks, full-duplex (FD) communication is considered as a promising technique to potentially double the speed of wireless systems, since it enables available spectral resources to be fully utilized in time and frequency [1].

FD multi-user systems, where a FD capable base-station (BS) communicates with half-duplex (HD) uplink (UL) and downlink (DL) users at the same time slot over the same frequency band, have been investigated in [2]-[6]. The authors in [2]-[6] assume that perfect channel-state-information (CSI) is available at the transmitters, which is practically impossible due to the inaccurate channel estimation. Therefore, robust transceiver designs that take into account imperfect channel knowledge are of interest, which have not been reported (to the best of our knowledge) so far for FD cellular systems.

In this work, we propose a robust precoder scheme for the FD multiple-input multiple-output (MIMO) multi-user system to maximize the weighted sum-rate of the network subject to power constraints at the BS and UL users under norm-bounded channel estimation errors. Similar to [7], we adopt an iterative approach to solve this non-convex optimization problem which is proven to converge, wherein a convex sub-problem is solved at each step. Numerical results are presented to show the importance of channel estimation in FD systems.

Notation: Matrices and vectors are denoted as bold capital and lowercase letters, respectively. $(\cdot)^T$ is the transpose, and $(\cdot)^H$ is the conjugate transpose. $\mathbf{I}_N$ and $\mathbf{0}_{N\times M}$ are the $N\times N$ identity and $N\times M$ zero matrix, respectively; $\text{tr}(\cdot)$ is the trace; $|.|$ is the determinant; $vec(\cdot)$ stacks the elements of a matrix to one long column vector. $\otimes$ denotes the Kronecker product. $\|\mathbf{X}\|_F$ and $\|\mathbf{x}\|_2$ denote the Frobenius norm of matrix $\mathbf{X}$ and the Euclidean norm of vector $\mathbf{x}$, respectively.

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II. System Model

We consider a multi-user MIMO system, in which a BS operating in FD mode serves K UL and J DL HD users simultaneously. The BS is equipped with M_0 and N_0 transmit and receive antennas, respectively. The number of antennas at the k-th UL and the j-th DL user are denoted by M_k and N_j, respectively. $\mathbf{H}_k^{UL}\in\mathbb{C}^{N_0\times M_k}$ and $\mathbf{H}_j^{DL}\in\mathbb{C}^{N_j\times M_0}$ represent the k-th UL and the j-th DL channel, respectively. $\mathbf{H}_0\in\mathbb{C}^{N_0\times M_0}$ is the self-interference channel between the transmitter and receiver antennas of BS. $\mathbf{H}_{jk}^{DU}\in\mathbb{C}^{N_j\times M_k}$ denotes the co-channel interference (CCI) channel from the k-th UL user to the j-th DL user.

The source symbols $\mathbf{s}_k^{UL}\in\mathbb{C}^{d_k^{UL}}$ and $\mathbf{s}_j^{DL}\in\mathbb{C}^{d_j^{DL}}$ for the k-th UL and the j-th DL user, respectively are assumed to be independent and identically distributed (i.i.d.) with unit power. Denoting the precoders for the data streams of the k-th UL and j-th DL user as $\mathbf{V}_k^{UL}=\left[\mathbf{v}_{k,1}^{UL},\ldots,\mathbf{v}_{k,d_k^{UL}}^{UL}\right]\in\mathbb{C}^{M_k\times d_k^{UL}}$, and $\mathbf{V}_j^{DL}=\left[\mathbf{v}_{j,1}^{DL},\ldots,\mathbf{v}_{j,d_j^{DL}}^{DL}\right]\in\mathbb{C}^{M_0\times d_j^{DL}}$, respectively, the signal received by the BS and the j-th DL user can be written, respectively, as

$$\mathbf{y}_0=\sum_{k=1}^{K}\mathbf{H}_k^{UL}\mathbf{V}_k^{UL}\mathbf{s}_k^{UL}+\mathbf{H}_0\sum_{j=1}^{J}\mathbf{V}_j^{DL}\mathbf{s}_j^{DL}+\mathbf{n}_0, \tag{1}$$

$$\mathbf{y}_j^{DL}=\mathbf{H}_j^{DL}\sum_{j=1}^{J}\mathbf{V}_j^{DL}\mathbf{s}_j^{DL}+\sum_{k=1}^{K}\mathbf{H}_{jk}^{DU}\mathbf{V}_k^{UL}\mathbf{s}_k^{UL}+\mathbf{n}_j^{DL}, \tag{2}$$

where $\mathbf{n}_0\sim\mathcal{CN}\left(\mathbf{0},\sigma_{n_0}^2\mathbf{I}_{N_0}\right)$ and $\mathbf{n}_j^{DL}\sim\mathcal{CN}\left(\mathbf{0},\sigma_{n_j}^2\mathbf{I}_{N_j}\right)$ denote the additive white Gaussian noise vector at the the BS and the j-th DL user, respectively.

The received signals are processed by linear decoders, denoted as $\mathbf{U}_k^{UL}=\left[\mathbf{u}_{k,1}^{UL},\ldots,\mathbf{u}_{k,d_k^{UL}}^{UL}\right]\in\mathbb{C}^{N_0\times d_k^{UL}}$, and $\mathbf{U}_j^{DL}=\left[\mathbf{u}_{j,1}^{DL},\ldots,\mathbf{u}_{j,d_j^{DL}}^{DL}\right]\in\mathbb{C}^{N_j\times d_j^{DL}}$ by the BS and the j-th DL user, respectively. Therefore, the estimate of data streams of the k-th UL and the j-th DL user are given as $\hat{\mathbf{s}}_k^{UL}=\left(\mathbf{U}_k^{UL}\right)^H\mathbf{y}_0$ and $\hat{\mathbf{s}}_j^{DL}=\left(\mathbf{U}_j^{DL}\right)^H\mathbf{y}_j^{DL}$, respectively. Using these estimates, the signal-to-interference-plus-noise ratio (SINR) values of the m-th stream of the k-th user in the

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channel $X,\ X \in \{UL, DL\}$ can be written as

$$\gamma_{k,m}^X = \frac{\left|\left(\mathbf{u}_{k,m}^X\right)^H \mathbf{H}_k^X \mathbf{v}_{k,m}^X\right|^2}{\left(\mathbf{u}_{k,m}^X\right)^H \mathbf{\Sigma}_k^X \mathbf{u}_{k,m}^X + \sum_{n\neq m}^{d_k^X} \left|\left(\mathbf{u}_{k,m}^X\right)^H \mathbf{H}_k^X \mathbf{v}_{k,n}^X\right|^2},$$

where $\mathbf{\Sigma}_k^{UL}$ denotes the covariance matrix of the interference-plus-noise terms at the k-th UL user given as[1]

$$\mathbf{\Sigma}_k^{UL} = \sum_{j\neq k}^{K} \mathbf{H}_j^{UL} \mathbf{V}_j^{UL} \left(\mathbf{V}_j^{UL}\right)^H \left(\mathbf{H}_j^{UL}\right)^H + \sum_{j=1}^{J} \mathbf{H}_0 \mathbf{V}_j^{DL} \left(\mathbf{V}_j^{DL}\right)^H \mathbf{H}_0^H + \sigma_{n_0}^2 \mathbf{I}_{N_0}. \quad (3)$$

The WSR optimization problem can be formulated as:

$$\max_{\substack{\mathbf{V}^{UL},\mathbf{U}^{UL}\\ \mathbf{V}^{DL},\mathbf{U}^{DL}}} \sum_{k=1}^{K} w_k^{UL} \sum_{m=1}^{d_k^{UL}} \log\left(1+\gamma_{k,m}^{UL}\right) + \sum_{j=1}^{J} w_j^{DL} \sum_{m=1}^{d_j^{DL}} \log\left(1+\gamma_{j,m}^{DL}\right) \quad (4)$$

$$\text{s.t.} \quad \sum_{m=1}^{d_k^{UL}} \left(\mathbf{v}_{k,m}^{UL}\right)^H \mathbf{v}_{k,m}^{UL} \leq P_k,\ k \in \mathcal{S}^{UL}, \quad (5)$$

$$\sum_{j=1}^{J} \sum_{m=1}^{d_j^{DL}} \left(\mathbf{v}_{j,m}^{DL}\right)^H \mathbf{v}_{j,m}^{DL} \leq P_0, \quad (6)$$

where w_k^{UL} and w_j^{DL} are the weights of the k-th UL and the j-th DL user, respectively, and $\mathbf{V}^X = \left\{\mathbf{v}_{k,m}^X : \forall(k,m)\right\}$ and $\mathbf{U}^X = \left\{\mathbf{U}_{k,m}^X : \forall(k,m)\right\},\ X \in \{UL, DL\}$. The constraints P_k and P_0 are the transmit power constraints at the k-th UL user and at the BS, respectively. We use $\mathcal{S}^{UL}$ and $\mathcal{S}^{DL}$ to represent the set of K UL and J DL channels, respectively.

III. Joint Beamforming Design

We will first simplify the notations similar to [3] by combining UL and DL channels. Denoting

$$\mathbf{H}_{ij} = \begin{cases} \mathbf{H}_j^{UL}, i \in \mathcal{S}^{UL},\ j \in \mathcal{S}^{UL}, \\ \mathbf{H}_0, \quad i \in \mathcal{S}^{UL},\ j \in \mathcal{S}^{DL}, \\ \mathbf{H}_{ij}^{DU}, i \in \mathcal{S}^{DL},\ j \in \mathcal{S}^{UL}, \\ \mathbf{H}_i^{DL}, i \in \mathcal{S}^{DL},\ j \in \mathcal{S}^{DL}, \end{cases} \mathbf{n}_i = \begin{cases} \mathbf{n}_0, \quad i \in \mathcal{S}^{UL}, \\ \mathbf{n}_i^{DL}, \quad i \in \mathcal{S}^{DL}, \end{cases}$$

$$\tilde{N}_i\left(\tilde{M}_i\right) = \begin{cases} N_0\left(M_i\right), & i \in \mathcal{S}^{UL}, \\ N_i\left(M_0\right), & i \in \mathcal{S}^{DL}, \end{cases}$$

and referring to $\mathbf{V}_i^X$, $\mathbf{U}_i^X$, $\gamma_{i,m}^X$, $\mathbf{\Sigma}_i^X$, $d_i^X,\ X \in \{UL, DL\}$ as $\mathbf{V}_i$, $\mathbf{U}_i$, $\gamma_{i,m}$, $\mathbf{\Sigma}_i$, d_i, the SINR of the m-th stream in the i-th link, $i \in \mathcal{S} \triangleq \mathcal{S}^{UL} \bigcup \mathcal{S}^{DL}$ can be written as

$$\gamma_{i,m} = \frac{\left|\mathbf{u}_{i,m}^H \mathbf{H}_{ii} \mathbf{v}_{i,m}\right|^2}{\mathbf{u}_{i,m}^H \underbrace{\left(\mathbf{\Sigma}_i + \sum_{n\neq m}^{d_i} \mathbf{H}_{ii} \mathbf{v}_{i,n} \mathbf{v}_{i,n}^H \mathbf{H}_{ii}^H\right)}_{\mathbf{\Sigma}_{i,m}} \mathbf{u}_{i,m}}, \quad (7)$$

where $\mathbf{\Sigma}_i = \sum_{j\in\mathcal{S}, j\neq i} \mathbf{H}_{ij} \mathbf{V}_j \mathbf{V}_j^H \mathbf{H}_{ij}^H + \sigma_{n_i}^2 \mathbf{I}_{\tilde{N}_i}$. Using the simplified notations, the optimization problem (4)-(6) can be rewritten as

$$\max_{\substack{\mathbf{v}_{i,m},\mathbf{u}_{i,m}\\ i\in\mathcal{S}}} \quad \sum_{i\in\mathcal{S}} w_i \sum_{m=1}^{d_i} \log_2\left(1+\gamma_{i,m}\right) \quad (8)$$

$$\text{s.t.} \quad \sum_{m=1}^{d_i} \mathbf{v}_{i,m}^H \mathbf{v}_{i,m} \leq P_i,\ i \in \mathcal{S}^{UL}, \quad (9)$$

$$\sum_{i\in\mathcal{S}^{DL}} \sum_{m=1}^{d_i} \mathbf{v}_{i,m}^H \mathbf{v}_{i,m} \leq P_0. \quad (10)$$

The following result adopted from [8, Theorem 1] and used in [5], [6] allows us to express (4)-(6) in terms of only linear precoders.

Proposition 1: For any $\mathbf{V}_i$ that is a solution of (11)-(13), there is a solution of (4)-(6) that share the same objective and constraint values, and thus (4)-(6) and (11)-(13) are equivalent. In particular, $\mathbf{v}_{i,m}$ can be obtained by taking the m-th column of $\tilde{\mathbf{V}}_i = \mathbf{V}_i\mathbf{D}$, where $\mathbf{D}\mathbf{\Lambda}\mathbf{D}^H$ is the eigen-decomposition of $\mathbf{V}_i^H \mathbf{H}_{ii}^H \mathbf{\Sigma}_i^{-1} \mathbf{H}_{ii} \mathbf{V}_i$, and $\mathbf{u}_{i,m}$ can be obtained from $\mathbf{u}_{i,m} = \mathbf{\Sigma}_{i,m}^{-1} \mathbf{H}_{ii} \mathbf{v}_{i,m}$, where $\mathbf{\Sigma}_{i,m}$ is defined in (7).

$$\max_{\mathbf{V}} \quad \sum_{i\in\mathcal{S}} w_i \log\left|\mathbf{I}_{d_i} + \mathbf{V}_i^H \mathbf{H}_{ii}^H \mathbf{\Sigma}_i^{-1} \mathbf{H}_{ii} \mathbf{V}_i\right| \quad (11)$$

$$\text{s.t.} \quad \text{tr}\left\{\mathbf{V}_i^H \mathbf{V}_i\right\} \leq P_i,\ i \in \mathcal{S}^{UL}, \quad (12)$$

$$\sum_{i\in\mathcal{S}^{DL}} \text{tr}\left\{\mathbf{V}_i^H \mathbf{V}_i\right\} \leq P_0, \quad (13)$$

where $\mathbf{V} = \{\mathbf{V}_i : i \in \mathcal{S}\}$. Note that Proposition 1 states that decoupled capacity composed of linear transmit and receive beamforming vectors in (8) is fully equivalent to the mutual information in (11), which only involves the linear precoders as optimization variables. Based on Proposition 1, we can solve the sum-rate maximization problem (8)-(10) by solving the problem (11)-(13) and then construct linear precoders and receive beamforming vectors in (8) from the resulting solution.

A. Imperfect CSI Model

The CSI for all channels is assumed to be imperfectly known at the BS, and based on the imperfect CSI knowledge, the BS computes the optimum transceiver matrices in a centralized manner, and then distributes them to the users via control links. The imperfect CSI is modeled using a deterministic norm-bounded error model [7] is expressed as

$$\mathbf{H}_{ij} = \left\{\tilde{\mathbf{H}}_{ij} + \mathbf{\Delta}_{ij} : \|\mathbf{\Delta}_{ij}\|_F \leq \tau_{ij}\right\}, \quad (14)$$

[1]The covariance matrix of the aggregate interference-plus-noise terms of the j-th DL user, $\mathbf{\Sigma}_j^{DL}$ can be written similarly, i.e., by changing $\mathbf{H}_j^{UL}$, $\mathbf{V}_j^{UL}$ and $\mathbf{H}_0$ with $\mathbf{H}_j^{DL}$, $\mathbf{V}_i^{DL},\ i \neq j$ and $\mathbf{H}_{jk}^{DU},\ k = 1, \ldots, K$, respectively.

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where $\tilde{\mathbf{H}}_{ij}$ and $\boldsymbol{\Delta_{ij}}$ denote the estimated CSI and the channel error matrix with uncertainty bound τ_{ij}, respectively.

With the imperfect CSI, the objective function of the optimization problem (11)-(13) is replaced with

$$\max_{\mathbf{V}} \min_{\|\boldsymbol{\Delta_{ij}}\|_F \le \tau_{ij}} \sum_{i\in\mathcal{S}} w_i \log\left|\mathbf{I}_{d_i} + \mathbf{V}_i^H\mathbf{H}_{ii}^H\boldsymbol{\Sigma}_i^{-1}\mathbf{H}_{ii}\mathbf{V}_i\right|. \quad (15)$$

By using the well-known relationship between the weighted sum-rate and weighted minimum mean-squared-error (MSE) problems [9], we transform the robust weighted sum-rate problem in (15) into an equivalent robust weighted MSE problem, which is expressed as

$$\max_{\mathbf{V}} \min_{\|\boldsymbol{\Delta_{ij}}\|_F \le \tau_{ij}} \max_{\mathbf{W},\mathbf{U}} \sum_{i\in\mathcal{S}} w_i(-\text{tr}\{\mathbf{W}_i\mathbf{E}_i\} + \log|\mathbf{W}_i| + d_i) \quad (16)$$

where $\mathbf{U}(\mathbf{W}) = \{\mathbf{U}_i(\mathbf{W}_i) : i\in\mathcal{S}\}$, $\mathbf{W}_i \in \mathbb{C}^{d_i\times d_i}$ is a weight matrix, and $\mathbf{E}_i$ is the MSE matrix of the i-th link defined as

$$\mathbf{E}_i = \left(\mathbf{U}_i^H\mathbf{H}_{ii}\mathbf{V}_i - \mathbf{I}_{d_i}\right)\left(\mathbf{U}_i^H\mathbf{H}_{ii}\mathbf{V}_i - \mathbf{I}_{d_i}\right)^H + \mathbf{U}_i^H\boldsymbol{\Sigma}_i\mathbf{U}_i.$$

Since the formulation in (16) is intractable, we look at the lower bound of the inner min-max problem by interchanging the min-max terms, and express the problem as

$$\max_{\mathbf{V},\mathbf{W},\mathbf{U}} \min_{\|\boldsymbol{\Delta_{ij}}\|_F \le \tau_{ij}} \sum_{i\in\mathcal{S}} w_i\left(-\text{tr}\{\mathbf{W}_i\mathbf{E}_i\} + \log|\mathbf{W}_i| + d_i\right). \quad (17)$$

To simplify the problem further, we write tr $\{\mathbf{W}_i\mathbf{E}_i\}$ as

$$\begin{aligned}
&\text{tr}\{\mathbf{W}_i\mathbf{E}_i\} \\
&\overset{(a)}{=} \sum_{j\in\mathcal{S}} \text{tr}\left\{\mathbf{W}_i\left(\mathbf{U}_i^H\mathbf{H}_{ij}\mathbf{V}_j - \delta_{ij}\mathbf{I}_{d_i}\right)\left(\mathbf{U}_i^H\mathbf{H}_{ij}\mathbf{V}_j - \delta_{ij}\mathbf{I}_{d_i}\right)^H\right\} \\
&\quad + \sigma_{n_i}^2 \text{tr}\left\{\mathbf{W}_i\mathbf{U}_i^H\mathbf{U}_i\right\} \\
&\overset{(b)}{=} \sum_{j\in\mathcal{S}} \left\|\mathbf{B}_i^H\Big(\mathbf{U}_i^H\left(\tilde{\mathbf{H}}_{ij} + \boldsymbol{\Delta_{ij}}\right)\mathbf{V}_j - \delta_{ij}\mathbf{I}_{d_i}\Big)\right\|_F^2 + \sigma_{n_i}^2\|\mathbf{U}_i\mathbf{B}_i\|_F^2 \\
&\overset{(c)}{=} \sum_{j\in\mathcal{S}} \Bigg\| \underbrace{vec\left(\mathbf{B}_i^H\left(\mathbf{U}_i^H\tilde{\mathbf{H}}_{ij}\mathbf{V}_j - \delta_{ij}\mathbf{I}_{d_i}\right)\right)}_{\mathbf{d}_{ij}} \\
&\quad + \underbrace{\left(\mathbf{V}_j^T \otimes \left(\mathbf{B}_i^H\mathbf{U}_i^H\right)\right)}_{\mathbf{D}_{ij}} vec\left(\boldsymbol{\Delta_{ij}}\right)\Bigg\|_2^2 + \sigma_{n_i}^2\|\mathbf{U}_i\mathbf{B}_i\|_F^2, \qquad (18)
\end{aligned}$$

where (a) is obtained by plugging $\boldsymbol{\Sigma}_i$ in (7) and using δ_{ij} as the Kronecker delta function; (b) is obtained by using the equality tr $\left\{\mathbf{A}^H\mathbf{A}\right\} = \|\mathbf{A}\|_F^2$ and writing $\mathbf{W}_i = \mathbf{B}_i\mathbf{B}_i^H$; and (c) is obtained by using the identities $\|\mathbf{A}\|_F^2 = \|vec(\mathbf{A})\|_2^2$ and $vec(\mathbf{ABC}) = \left(\mathbf{C}^T\otimes\mathbf{A}\right)vec(\mathbf{B})$. Using (18), the inner minimization problem in (17) can be rewritten as

$$\begin{aligned}
\min_{\|\boldsymbol{\Delta_{ij}}\|_F \le \tau_{ij}} \sum_{i\in\mathcal{S}} w_i \Bigg(&-\sum_{j\in\mathcal{S}} \left\|\mathbf{d}_{ij} + \mathbf{D}_{ij}vec\left(\boldsymbol{\Delta_{ij}}\right)\right\|_2^2 \\
&- \sigma_{n_i}^2\|\mathbf{U}_i\mathbf{B}_i\|_F^2 + 2\log|\mathbf{B}_i| + d_i\Bigg). \quad (19)
\end{aligned}$$

Using epigraph form and introducing slack variable λ_{ij}, the problem (19) can be written as

$$\begin{aligned}
&\max_{\boldsymbol{\lambda}} \sum_{i\in\mathcal{S}} w_i\left(-\sum_{j\in\mathcal{S}}\lambda_{ij} - \sigma_{n_i}^2\|\mathbf{U}_i\mathbf{B}_i\|_F^2 + 2\log|\mathbf{B}_i| + d_i\right) \quad (20) \\
&\text{s.t.} \quad -\left\|\mathbf{d}_{ij} + \mathbf{D}_{ij}vec\left(\boldsymbol{\Delta_{ij}}\right)\right\|_2^2 \le \lambda_{ij},\ \|\boldsymbol{\Delta_{ij}}\|_F \le \tau_{ij},\ \forall(i,j),
\end{aligned}$$

where $\boldsymbol{\lambda} = \{\lambda_{ij} : \forall(i,j)\}$. Using Schur complement lemma, the constraint in (20) is expressed in linear matrix inequalities (LMI) form:

$$\begin{bmatrix}\lambda_{ij} & \mathbf{d}_{ij}^H \\ \mathbf{d}_{ij} & \mathbf{I}_{d_id_j}\end{bmatrix} + \begin{bmatrix}0 & vec\left(\boldsymbol{\Delta_{ij}}\right)^H\mathbf{D}_{ij}^H \\ \mathbf{D}_{ij}vec\left(\boldsymbol{\Delta_{ij}}\right) & \mathbf{0}_{d_id_j\times d_id_j}\end{bmatrix} \succeq 0. \quad (21)$$

To further simplify (21), we use the following lemma:

Lemma 1 ([10]): Given matrices $\mathbf{P}$, $\mathbf{Q}$, $\mathbf{A}$ with $\mathbf{A} = \mathbf{A}^H$, the semi-infinite LMI of the form of

$$\mathbf{A} \succeq \mathbf{P}^H\mathbf{X}\mathbf{Q} + \mathbf{Q}^H\mathbf{X}^H\mathbf{P}, \qquad \forall\mathbf{X} : \|\mathbf{X}\|_F \le \rho,$$

holds if and only if $\exists\epsilon \ge 0$ such that

$$\begin{bmatrix}\mathbf{A} - \epsilon\mathbf{Q}^H\mathbf{Q} & -\rho\mathbf{P}^H \\ -\rho\mathbf{P} & \epsilon\mathbf{I}\end{bmatrix} \succeq 0. \quad (22)$$

By choosing

$$\begin{aligned}
\mathbf{A} &= \begin{bmatrix}\lambda_{ij} & \mathbf{d}_{ij}^H \\ \mathbf{d}_{ij} & \mathbf{I}_{d_id_j}\end{bmatrix},\ \mathbf{P} = \left[\mathbf{0}_{\tilde{N}_i\tilde{M}_j\times 1},\ \mathbf{D}_{ij}^H\right], \quad (23) \\
\mathbf{X} &= vec\left(\boldsymbol{\Delta_{ij}}\right),\ \mathbf{Q} = \left[-1,\ \mathbf{0}_{1\times d_id_j}\right], \quad (24)
\end{aligned}$$

we can apply Lemma 1 to (21), and the resulting overall optimization problem is formulated as

$$\begin{aligned}
\max_{\mathbf{V},\mathbf{B},\mathbf{U},\boldsymbol{\lambda},\boldsymbol{\epsilon}} \sum_{i\in\mathcal{S}} w_i \Bigg(&-\sum_{j\in\mathcal{S}}\lambda_{ij} - \sigma_{n_i}^2\|\mathbf{U}_i\mathbf{B}_i\|_F^2 \\
&+ \log\left|\mathbf{B}_i\mathbf{B}_i^H\right| + d_i\Bigg) \quad (25) \\
\text{s.t.} \quad &\begin{bmatrix}\lambda_{ij} - \epsilon_{ij} & \mathbf{d}_{ij}^H & \mathbf{0}_{1\times\tilde{N}_i\tilde{M}_j} \\ \mathbf{d}_{ij} & \mathbf{I}_{d_id_j} & -\tau_{ij}\mathbf{D}_{ij} \\ \mathbf{0}_{\tilde{N}_i\tilde{M}_j\times 1} & -\tau_{ij}\mathbf{D}_{ij}^H & \epsilon_{ij}\mathbf{I}_{\tilde{N}_i\tilde{M}_j}\end{bmatrix} \succeq 0,\ \forall(i,j), \quad (26) \\
&\|vec(\mathbf{V}_i)\|_2^2 \le P_i,\ i\in\mathcal{S}^{UL}, \quad (27) \\
&\sum_{i\in\mathcal{S}^{DL}} \|vec(\mathbf{V}_i)\|_2^2 \le P_0,\ \epsilon_{ij} \ge 0,\ \forall(i,j), \quad (28)
\end{aligned}$$

where $\boldsymbol{\epsilon} = \{\epsilon_{ij} : \forall(i,j)\}$, and $\mathbf{B} = \{\mathbf{B}_i : i\in\mathcal{S}\}$. Although the problem in (25)-(28) is non-convex, it becomes a convex function of each optimization variable when the other two are fixed. Therefore we can apply the coordinate ascend method to update the transceiver matrices iteratively. In particular, when $\mathbf{V}$ and $\mathbf{U}$ are fixed, $\mathbf{B}$ can be solved using MAX-DET algorithm [11], when $\mathbf{B}$ and $\mathbf{U}$ ($\mathbf{B}$ and $\mathbf{V}$) are fixed, $\mathbf{V}$ ($\mathbf{U}$) can be computed by solving the resulting Semidefinite programming (SDP) problem. Since the alternating iterative updates lead to a monotonic increase of the objective function in (25), and the fact that it is bounded above guarantees the convergence of the proposed algorithm to a stationary point.

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TABLE I. COMPLEXITY PARAMETERS

	Number of variables (n)	**Dimension of blocks** (a_l)
$\mathbf{V}$	$\sum_{i\in\mathcal{S}} 2\tilde{M}_i d_i + 2\lvert\mathcal{S}\rvert^2$	$a_l = d_i d_j + \tilde{N}_i \tilde{M}_j + 1,\ (i,j)\in\mathcal{S}$, $a_l = M_i d_i^{UL} + 1,\ i\in\mathcal{S}^{UL}$, $a_l = M_0 \sum_{j\in\mathcal{S}^{DL}} d_j^{DL} + 1$.
$\mathbf{U}_i$	$2\tilde{N}_i d_i + 2\lvert\mathcal{S}\rvert$	$a_l = d_i d_j + \tilde{N}_i \tilde{M}_j + 1,\ (i,j)\in\mathcal{S}$.
$\mathbf{B}_i$	$2d_i^2 + 2\lvert\mathcal{S}\rvert$	$a_l = d_i d_j + \tilde{N}_i \tilde{M}_j + 1,\ (i,j)\in\mathcal{S}$.

B. Computational Complexity

Since the proposed algorithm solves a SDP problem in each step (SDP is a special case of MAX-DET [11]), we focus on the complexity analysis of a standard SDP problem:

$$\min_{\mathbf{x}\in\mathcal{R}^n} \mathbf{c}^T\mathbf{x} \quad \text{s.t.} \quad \mathbf{A}_0 + \sum_{i=1}^{n} x_i \mathbf{A}_i \succeq \mathbf{0}, \ \ \|\mathbf{x}\|_2 \le R, \tag{29}$$

where $\mathbf{A}_i$ denotes the symmetric block-diagonal matrices with L diagonal blocks of size $a_l \times a_l,\ l = 1,\ldots,L$. The number of elementary arithmetic operations necessary for solving this problem is upper-bounded by [12]

$$\mathcal{O}(1)\left(1+\sum_{l=1}^{L} a_l\right)^{1/2} n\left(n^2 + n\sum_{l=1}^{L} a_l^2 + \sum_{l=1}^{L} a_l^3\right). \tag{30}$$

For example, in computing $\mathbf{V}_i$, the number of diagonal blocks L is equal to $|\mathcal{S}|^2 + \left|\mathcal{S}^{UL}\right| + 1$. For the $|\mathcal{S}|^2$ LMI constraints in (26), the dimension of blocks are $a_l = d_i d_j + \tilde{N}_i\tilde{M}_j + 1,\ (i,j)\in\mathcal{S}$. For the UL power constraint, the dimension of the blocks are $a_l = M_i d_i^{UL} + 1,\ i\in\mathcal{S}^{UL}$. For the BS power constraint, the dimension of the block is $a_l = M_0\sum_{j\in\mathcal{S}^{DL}} d_j^{DL} + 1$. The unknown variables to be determined are of size $n = \sum_{i\in\mathcal{S}} 2\tilde{M}_i d_i + 2|\mathcal{S}|^2$. The analysis of the other subproblems can be carried out in a similar manner. Then, the complexity parameters for solving the problem (25)-(28) using SDP method are given in Table I.

IV. SIMULATION RESULTS

In this section, we compare the proposed FD setup with the equivalent HD system under the 3GPP LTE specifications for small cell deployments, which is considered to be especially suitable for deployment of FD technology due to low transmit powers, short transmission distances and low mobility [2]. A single hexagonal cell having a BS in the center with randomly distributed UL and DL users is simulated. The parameters for the system model and the path-loss model for each link are adopted from [13, Table II]. The elements of the nominal small-scale fading channels, except the self-interference channel, are randomly generated according to zero-mean, unit-variance, i.i.d. Gaussian distributions. For the nominal self-interference channel, we adopt the model in [1], in which the self-interference channel is distributed as $\tilde{\mathbf{H}}_0 \sim \mathcal{CN}\left(\sqrt{\frac{\sigma_{SI}^2 K_R}{1+K_R}}\hat{\mathbf{H}}_0, \frac{\sigma_{SI}^2}{1+K_R}\mathbf{I}_{N_0}\otimes\mathbf{I}_{M_0}\right)$, where K_R is the Rician factor, $\hat{\mathbf{H}}_0$ is a deterministic matrix, and σ_{SI}^2 is introduced to parametrize the capability of a certain self-interference cancellation design. The uncertainty sizes are related to the quality of channels, i.e., $\tau_{ij} = s\|\hat{\mathbf{H}}_{ij}\|_F,\ s\in[0,1)$. We apply the following values as our system parameters: $M_k = N_j = 2,\ M_0 = N_0 = 2,\ K = J = 2, K_R = 1$ and $\hat{\mathbf{H}}_0$ to be the matrix of all ones for all experiments. The resulting system performance is averaged over 100 channel realizations.

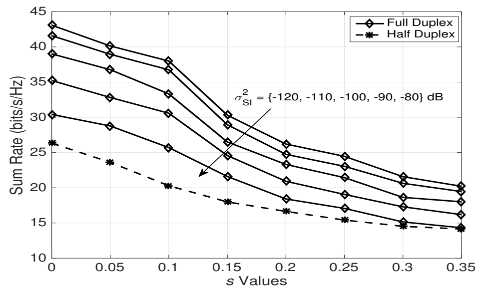

Fig. 1. Comparison between FD and HD systems with respect to s.

It can be seen from Fig. 1 that as the size of the uncertainty region increases, the FD system suffers more, and the gap between FD and HD systems decreases. This degradation in performance of the FD system is explained as follows. Since there are more interference channels (self-interference and CCI) in FD systems, as the uncertainty level of the channels increases, the system performance of the FD system degrades more. This indicates that the channel estimation is a critical factor for successful deployment of FD systems.

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