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Achilles, the Tortoise and the Time Machine: A Carrollian Dialogue.

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ABSTRACT: Drawing on near-contemporaneous works by Lewis Carroll and H. G. Wells, this paper uses an imaginary dialogue between Achilles and the Tortoise to explore the supertask possibilities offered by combining Zeno's (and Carroll's) original Eleatic race set-ups with unlimited Wellsian time travel. Besides offering new thoughts on how to address some perennial worries about time travel, e.g. by paying due attention to the distinction between counterfactual and causal dependence, this paper also offers a new form of Eleatic 'paradox' in which Achilles is condemned to run an unending (i.e. infinite time) race at constant finite velocity over a racetrack of finite length. This time round, the Tortoise shows Achilles just how Wellsian time machines can also be infinity machines, which allow unsuspected scope for 'bifurcated' supertasks and unending Eleatic races.

KEY WORDS: Time travel, supertasks, Zeno, David Lewis, Lewis Carroll.

The scene: the infinite mathematical plane where the protagonists of classic paradoxes do time.

Dramatis personae:

Achilles – the world’s fastest runner and notorious paradox-gull.¹

The Tortoise – an inventor, antagonist to Achilles and the world’s slowest runner.

[*Enter Achilles left, jogging. Enter Tortoise right, plodding and wearing on its shell some apparatus with two dials and a lever.*]

Achilles: Good morrow, Tortoise. What brings you here?

Tortoise: If you’ll pardon the zeugma, Achilles, four sturdy legs, the will to power and my newly-minted time machine bring me here.

A: A time machine, indeed? Does more than *tell* the time, I trust? [*Laughs – solo.*]

T: Oh much more. It sends its victim ... I beg your pardon; it lets its heroic wearer voyage at will through history. Like its great progenitor in Wells (1895), it offers unlimited travel in any dimension of time or space as its operator determines.²

A: A truly remarkable device ... if it works. I take it these are its controls?

T: Indeed. These upper and lower dials determine respectively the journey’s total spatial and temporal dimensions. So for example, if you want to jump five seconds back in time and arrive five metres to the right, you simply turn both dials’ pointers clockwise until they each indicate ‘5’. If you want to travel five seconds into the future and five metres to the left, you turn the dials anti-clockwise until they indicate ‘5’. Settings can be varied in any arithmetical or geometrical combination. Once the dials are set, this lever is pressed and the machine creates a wormhole shortcut through space and time that takes the traveller smoothly to the specified time/place. Once the machine is activated, the journey is effectively instantaneous, with departure and arrival apparently simultaneous for the wearer.

A: That seems clear enough, if somewhat staggering in its implications. But I'm puzzled: if I travel (say) five years into the future *and* yet I get there instantaneously, wouldn't I create a durational paradox thereby? How can an instantaneous journey also take five years?

T: Because the two intervals concerned measure the duration of the journey in two different frames of reference. Both frames offer correct measurements. For me as spectator, five full years would elapse between your journey's beginning and end. Whereas the journey would be instantaneous for you.

A: Now *that* sounds suspiciously like the personal/external time distinction introduced by David Lewis,³ in which I never did believe. I recall Lewis says that time travel requires that a journey must have different durations in the traveller's 'personal time' and to an external observer. However, I think there is only *time*: one, objective and indivisible. To me, 'personal time' sounds like some barely-intelligible extra temporal dimension and hence something I can safely dismiss as devoid of possible experiential or scientific content. (So there.)

T: [*Sotto voce*:] 'Think so still, till experience change thy mind.'⁴ [*Aloud*:] Well, I thought Lewis merely meant that time travel requires different ways in which duration can be registered. In forward time travel, external and personal time have the same direction but different durations, whereas external and personal time have different directions (and maybe durations) in backward time travel. While a time-journey always has positive or zero personal duration, a backward time journey ends before it begins viewed in external time.

Talk of 'personal time' need not imply that only persons can time-travel. In many ways it's regrettable that more philosophers haven't followed Murray MacBeath (1982)⁵ and talked of 'particular time', i.e. time *particular* to the

travelling object, rather than personal time in contrast to external time. Be that as it may, the gradual rotting of a damp time-travelling cake would be a record of ‘personal time’ (or particular time) just as much as any human traveller’s physiological or psychological processes. If it helps, you might view the personal/external time distinction as resembling that between durations measured in different relativistic reference frames. That durational differences occur between reference-frames with different velocities is a central prediction of special relativity and supported by experiments on atomic clocks and π -mesons in cosmic ray showers. Add this to general relativistic and quantum models like Kurt Gödel’s (1949) and Igor Novikov’s (1989) depictions of worlds and wormholes (respectively) which contain closed timelike curves apt for backward time travel, and time travel becomes an apt object for physics.⁶ But please don’t take my word for it – why not try my machine?

A: Thank you. Call me unambitious but I’ve always had a yen to visit a spot five metres to my right at a time ten seconds in the past.

T: Be my guest. [*The Tortoise doffs, and Achilles dons, the time machine. As if by magic, a second Achilles, ‘Achilles₍₂₎’ for short, appears exactly five metres to starboard of the original Achilles₍₁₎, and wearing an identical-looking machine.*]

A₍₂₎: Great Olympus, it works.

A₍₁₎: [*Staring at Achilles₍₂₎*] Can that be ...?

} [*Simultaneously.*]

A₍₂₎: [*Staring at Achilles₍₁₎*] So it was me ...

T: Indeed, it can be, and indeed it was. [*Addressing Achilles₍₁₎*] Now, your ten seconds are nearly up so quickly, press the lever, primordial Achilles.

A₍₁₎: I suppose I'd better. [*He does so, and promptly vanishes.*]

A₍₂₎: That was remarkable. For a few moments, there were two of me.

T: Well, two *stages* of you at any rate, (four-dimensionally speaking). My machine lets your spatiotemporal path (or 'worldline') fold back towards itself like a hairpin so that two distinct moments in your personal time can rendezvous at the selfsame moment of external time. As clear a demonstration of Lewisian time travel as one could wish for. (Look - your subscripts have disappeared now that there's only one of you again.)

A: (Why, so they have.) Okay, you've convinced me – I clearly remember experiencing both sides of that conversation, albeit from different points of view and at different personal times. But I now have another worry: what if later-me had tried to stop original-me pressing the lever so original-me couldn't travel back to become later-me? Suppose that my later self had tried to prevent my earlier self time-travelling, in the classic 'Grandfather Paradox' fashion?

T: Well, then either (e.g.) I'd have pressed the lever or some short-circuit would have activated the machine anyway. In which set of cases, your efforts to stop the machine's activation would have proved unavailing. Alternatively, had they been availing, then that other athletic-looking chap in sandals would have proved to have been some other person, (or stage thereof). The operative counterfactuals run something like this: if later-you had tried to prevent your own earlier journey then either some other factor besides your volitions and actions would have triggered the machine or the beings that we might call 'Achilles₍₁₎' and 'Achilles₍₂₎' would have proved to be stages of two different people.

A: How very paradoxical.

T: With all due respect, Achilles, I think you'll find that any *other* alternative would have been paradoxical. (Such as one and the same stage of you simultaneously pressing and not pressing the same lever.)

A: So I could have done otherwise than I did without contradiction following?

T: Oh yes, there are several things that you could have done and several more that you could have tried to do. However, there are some things that you couldn't *successfully* have attempted without contradiction ensuing. Your later self could not have prevented your earlier self time-travelling, given that it was *your* earlier self. Just as if Achilles₍₂₎ had assassinated Achilles₍₁₎, then Achilles₍₁₎ would not have gone on to be Achilles₍₂₎. Conversely, if Achilles₍₁₎ does go on to be Achilles₍₂₎, then Achilles₍₂₎ cannot assassinate Achilles₍₁₎. If time travel counterfactuals often sound odd, it may simply be because what are normally-ordered antecedents and consequents in external time may 'back-track' viewed in personal time, or vice versa. Likewise, we should remember that the relation between antecedent and consequent in a counterfactual is not always a causal one.

A: Sorry Tortoise, you've lost me.

T: Well, in your own case, we might have another true but oddly-entangled counterfactual, such as: 'If you had successfully disabled that person's time machine, then that person would not have been (an earlier stage of) you'. It should not be thought in this case that the antecedent event of machine-disabling would somehow be the *cause* of the consequent failure of identity – as though disabling the machine somehow reached back in time and replaced (paradoxically) what had been a stage of yours at a particular time with a stage of a different person at precisely the same time. There is clearly a relation of dependence between the

events here but it's not (always and invariably) a causal one running from antecedent to consequent. A time traveller can affect an event in the past, i.e. help to make it what it was, but a traveller cannot replace one event at a time with another event at the same time. However, such is not a restriction peculiar to time travellers, since presumably nobody anywhere can effect the replacement of one event at a time with another event at precisely the same time.

In part, it may be that worries about the constraints on backward time travellers simply proceed from an unexamined assumption that casual dependence simply is counterfactual dependence. So it might help dissolve the apparent problem if we simply reflect that success would have required a different past to go with it.

A: So you think I'm just as free an agent when I time-travel as I am normally?

T: Well, I might not go quite *that* far but let us just say that optionality can survive for time travellers. Being unable to succeed is not the same as being unable to try.

A: But as it happens, you made me press the lever at just the right moment. What would have happened if you hadn't? What power would have stayed my hand if I'd tried to change the machine's settings so earlier-me didn't travel back in time?

T: Well, as we're currently standing on the mathematical plane, let me put it like this: please draw for me a *Euclidean* (I would emphasise) triangle with the following lengths of sides: 3 centimetres, 4 centimetres ... and 700 kilometres.

A: But I can't. One side is orders of magnitude bigger than the others put together.

T: Exactly: you'll find something stays your hand in that case too. What *won't* happen is that you successfully draw the pseudo-triangular monster I've just described and then look on aghast as the geometry police appear, erase the infamous thing you've made and carry you off shrieking to the purgatory of

paradoxical geometers. Time travel constraints seem very odd but then we don't usually think of spatial movements or human actions as under logical constraints. And yet they are: for example, whatever determinate (finite) size that you might happen to possess, it seems logically possible that you could have had a greater size. However, logic prevents you, always and everywhere, from being bigger than yourself. Although even saying that logic 'constrains' human actions positively invites potentially misleading metaphors whereby logic has 'iron' laws and acts like some Orwellian dictator of the actual. Note this is not merely the claim that logic will sort all the constraints out or that the constraints may not appear deeply odd – although one might feel that recognising that logic forbids outcome X but continuing to press for an explanation as to just why X does not occur seems to involve a category mistake of some kind.

The past is simply what happened: had you taken different decisions in your personal future, then your history in the external past would have been different. In an important sense, logic does not tie your hands or exact some kind of terrible vengeance but nonetheless there are *apparently* describable sets of input actions that cannot logically be jointly actualised. Put it another way: some inputs which are permissible individually, or in selected combinations, prove to be jointly incompatible or impossible taken as a whole. Such is not a feature unique to time travel cases, even if we can imagine the constraints growing rather baroque.

A: Well, it all sounds reasonable enough, phrased like that ...

[In a flash, another, markedly older-looking, Achilles-stage appears, with another identical-looking machine – 'Achilles_(α)']

*A*_(α): Oh Fates be kind and let me not be too late. You must heed me, Achilles of my distant personal past, for I am a future self of yours. (Pray call me ‘Achilles-alpha’.) Logical constraints be hanged, for I have come from millennia in your personal future, using my free round-trip to the past, to implore you not to accept the Tortoise’s wager at any price. I bear proof of the dire Fate I would avert.

[*Produces scrap of newspaper from his tunic.*] Read this.

A: [*Glances at paper:*] ‘Eleatic News’, eh? And bearing tomorrow’s date.

[*Reads from paper:*] ‘Achilles Vanishes ... Mystery Disappearance of Immortal Paradox-Protagonist ... “It Was Only A Harmless Wager” Says Baffled Chelonian. “Although Achilles May Be Gone, For Me He Will Have Lived Forever” ...’ Oh, the rest of the article is worn away. How vexing. What wager is this?

*A*_(α): You haven’t heard the wager yet ...? Am I then arrived too early?

T: No he hasn’t, yes you are, and off you pop, Achilles_(α). [*Activates machine.*]

*A*_(α): Of course, I remember now: I paid a later-me no heed when I was you, Achilles.

Consistency is not mocked after all. Oh woe is me and all my later stages ...

[*As Achilles_(α) raises his hands aloft mournfully, he and his machine vanish.*]

A: Who was that sorry-looking customer?

T: Oh just some person-stage or other. Nothing for you to fret about.

A: He seemed to think he was me, and from a dreadful (if obscure) future to boot.

T: Your admirers are legion, Achilles, and some are ... eccentric.

A: But he looked just like me, only vastly older, greyer and more haggard.

T: A *bit* like you, perhaps. Yours is a popular look, Achilles, and much copied.

A: You weren't planning a wager with me then, by any chance?

T: Oh, a *wager*. Now there's an idea.

A: Well, were you planning a wager with me or weren't you?

T: Well, a small one perhaps. On the outcome of a thousand-metre race, actually.

A: Experience has taught me to be very wary of races or wagers with you, Tortoise.

T: Your caution does you credit, Achilles, but remember that it is reality itself that poses you these challenges and not your humble servant.

A: Whilst remaining on my guard, I must nonetheless ask: what is this wager?

T: I wager you cannot win a thousand-metre race run against me even if *I* (the world's slowest runner) give *you* (the world's fastest) a five hundred metre head-start *and* the use of my time machine. (I know it's traditional in these cases that I have a head-start over you but today I feel generous.) Furthermore, you may add extra stipulations at will. If you win, you get to keep my time machine forever.

[*Sotto voce:*] And indeed if you lose.

A: A thousand-metre race? With a five hundred metre head-start in my favour? And your time machine to boot? Plus room to add further stipulations as I choose?

T: Such are my terms. Be my guest, and stipulate away.

A: First, my worldline must suffer no discontinuities.

T: Granted. What, in deference to its formulator, we might dub the 'Max Black continuity assumption'.⁷

A: Second, my velocity and acceleration must be finitely bounded. (Even an immortal can find unbounded acceleration and velocity distinctly uncomfortable.)

T: Granted gladly. Indeed, you can set the finite bounds on both your velocity and acceleration at any values as high or as low as you please.

A: That seems very generous, Tortoise.

T: Not at all. [*Sotto voce:*] Generous, forsooth? We'll see.

A: Third, my running must be one task, not some questionable sum of an infinite number of distinct sub-tasks crammed into a finite time by being performed faster and faster. I will have none of what Thomson (1954/5) dubbed 'supertasks'.⁸

T: I can happily confirm that Thomson-style supertasks will not be required of you. I promise you that you won't have to run faster and faster without limit or effect progressively swifter changes in velocity, for examples.

A: Fourth: my journey must remain within finite temporal bounds. (Just to forestall you sending me infinitely far into the past or the future.)

T: Granted without reservation: within finite temporal bounds you shall remain.

A: Fifth (and finally): I must activate the time machine myself.

T: Granted. I'll even throw in a free ten-minute round-trip vacation to the past, which can be inserted into your run at any point you choose. Can I make a stipulation now?

A: It would seem churlish of me to refuse it unheard.

T: It's quite a small one. You perceive this wall?

A: You mean the wall here which I can't quite see over and which encloses an area that seemingly stretches off to infinity on my right? *That* wall?

T: That's the one. I'd appreciate it if you would let our race be run on a course that lies on the other side of that wall. It's just that this is where we store the infinity ...

[*Coughs.*] ... I'm sorry, the *special* machines, like Thomson's (1954/5) Lamp or Black's (1951) marble-moving machines 'Alpha' and 'Beta'. I can set the time machine's controls accordingly if you like. [*He does so.*] Ready for the off?

A: Yes, I think we've covered everything.

T: Good. (You keep thinking that.) Five minutes after the machine's next activation, you'll be translated over the wall directly to the 500-metre marker on the racecourse, where I'll be waiting in position. We'll start racing once you arrive. Meantime, we'll run over a few pre-flight checks. By the way, I've incorporated a minor safety-feature: once activated, the machine can't be removed until it has completed its programmed course. We wouldn't want you getting lost.

A: That seems reasonable. Oh, a sudden thought ... I hope this time you're not going to inveigle me into some premise-circular inferential task or other? It's just that I've been caught that way before.⁹

T: You mean by some such cunning ploy as the one I used in Carroll (1895), don't you? No, nothing like that this time, don't you worry. My stipulations are made.

A: Very well, I accept. I'll activate the machine ... *now*. [*He presses the lever.*]

T: Excellent. Now the machine is activated, one or two small matters, Achilles, before you pop off. First, you didn't check what settings I'd given the dials.

A: But I didn't think ...

T: Yes, I rather thought you wouldn't. There's a significant difference between permission to *activate* the machine (which I freely granted) and permission to *control* it (which I rather fear I withheld). I took the liberty of programming the machine so the only spatiotemporal movements it permits its wearer during the assigned task (barring the aforesaid vacation) are thus: approximately five minutes

hence, you will arrive exactly midway along a one-thousand-metre racecourse at a time we'll call ' t '. You and I will begin running towards the finish line, which is 500 metres away for you and one thousand metres away for me. Alas, one hundred metres from the finish, the machine will take you back in time to a point ten seconds after t (call it ' t_2 ') and to a spot on the starting-line precisely five metres to the right of your original arrival-point. You will then again run towards the finish but on your second approach, the machine will return you to time t_2 when you are fifty metres from finishing, then twenty-five metres on your third approach, and then 12.5 metres on your fourth, and so on. If you ever remain stationary for more than ten personal minutes, a failsafe mechanism will return you to the starting block immediately on top of the first of your stages at time t_2 . So if you don't want to become an unsightly (and infinitely tall in the fullness of personal time) tower of Achilles-stages on the starting-block, you'd better keep moving.

Your remaining distances to the finish-line will decline in a constant geometrical 'Thomson Series'. I dub it thus because the spatial distances to the finish will shrink just like the diminishing durations of the ever-faster operations of Thomson's (1954/5) Lamp. Each operation of the Lamp subsequent to the first one takes half the time of its immediate predecessor. If the Lamp completes its first operation in t seconds, its second is complete in $t/2$, its third in $t/4$, and so on until it has completed unboundedly many operations by the end of $2t$ seconds. Did I not say that the machine has its geometrical as well as arithmetic settings? Well, I chose a 'Thomson Series' geometrical setting for you.

As your total speed and acceleration will have finite bounds, the increasing length of each stage and the infinite number of stages mean your total run will have infinite spatial length and comprehend an infinite amount of your personal time,

(although of course only finite external time). If it's any consolation at all, you will gradually get nearer to the finish-line as each sub-run concludes. Your journey (measured in your personal time) will indeed be continuous but infinite, although finite and *apparently* discontinuous in external time. (But you didn't believe in a personal/external time distinction ... Experience is a great teacher.)

When I promised you that your run would remain within finite temporal bounds, I was of course referring to bounds marked in *external* and not in personal time. Had you thought to stipulate a finite total *personal* time for your run, I'd have been sunk. Likewise, it's a shame that you didn't think to specify finite *spatial* as well as temporal bounds for our race. Because then you would have escaped my clutches altogether, if we were to rule out your successive stages getting progressively smaller without limit.

A: Oh I wish I'd thought of any of that.

T: I'm sure you do. Another escape clause would have been to insist that your objective must get *continually* closer throughout your run, because then your run would have to have been an ever-tightening spiral about a point and your velocity would have been continually changing. All these choices could have been yours, and yet they weren't. Now I call that pre-flight check ... and mate.

A: You really do seem to have thought of everything. Can I have just two more, absolutely last, questions? (The kind that might occur to a reader ...)

T: Certainly, Achilles, ask away.

A: First, I must seemingly make an infinite run while you enjoy mere moments - is not this a supertask when you explicitly promised me freedom from such?

T: At the risk of being pedantic, Achilles, I granted that (my emphasis) ‘*Thomson-style* supertasks will not be required of you’. In two crucial respects, your run will not be a Thomson-style supertask. Firstly, Thomson’s Lamp performs infinitely many distinct (on/off) operations, whereas you will simply run uninterruptedly (barring your aforesaid vacation). If completing a mere legato run were to count as a supertask then my mouth would be completing a supertask as I speak, since it too completes a motion which could (theoretically) be broken-down into infinitely many sub-motions. Granting you freedom from such Pickwickian supertasks would leave you unable to run, or even to breathe.

Secondly, even if you were a staccato runner and thus allowed to interrupt your run with regular halts for planting flags or having a snooze, your run would still not be a supertask *in your frame of reference*, since it would require an infinite amount of your personal time. A *Thomson-style* supertask (like that of the ever-faster Lamp) is a supertask in anybody’s frame of reference, whereas what you are about to commence will only look like a supertask from outside (e.g. to me). Thomson’s Lamp completes infinitely many tasks in finite time whether measured in its own or an observer’s time; you however only run an infinite run in a finite amount of an external observer’s time.

Furthermore, where Thomson’s Lamp performs infinitely many tasks in finite time, you compress infinite personal time into finite external time: your task has a limit in external time but no well-defined end-point in personal time. So, I promised you freedom from traditional supertasks and freedom from traditional supertasks you shall have – instead, you will undertake what Earman and Norton (1996: 232) helpfully dub a ‘bifurcated supertask’, i.e. one which is only ‘super’ when viewed by an external observer.¹⁰ In Aristotelian terms, your journey may

appear to be a *completed* (or actual) infinity viewed in external time but your journey is only *potentially* infinite viewed along your worldline. At any point anywhere along your worldline, there will always be more running ahead of you. If you'd only asked me to rule out *all* kinds of supertasks, including the observer-relative bifurcated ones, then I'd have been sunk.

A: This seems a rather technical excursus, Tortoise.

T: Perhaps, but I offer it for two reasons. Firstly, we seem to have contracted a Faustian bargain, you and I, and it is of the nature of such bargains that the wagger gets the letter of what was sought but misses the spirit. (You thought that in being granted freedom from Thomson-style supertasks, you were thereby granted freedom from supertask *simpliciter*.) Secondly, the notion of a bifurcated supertask offers both an interesting extension of the notion of a supertask and a very useful support to the applicability of the notion of personal and external time. After all, physics may be far friendlier to bifurcated supertasks than to the more familiar variety.

A: Very well. Second final question: If I have infinitely many stages crammed into finite time, whither mass-energy conservation in all this *ex nihilo* Achilles-making?

T: Well, again we need to remember that we dealing with a case of disjoint personal time and external time. One way to express conservation of mass-energy says (in effect) that the amount of mass-energy in an isolated system will remain constant over time. But over *whose* time? Just as we can distinguish between kinds of supertask (i.e. the classical and the bifurcated) as they appear in different frames of reference, so we can distinguish between mass-energy conservation measured in external time (i.e. between timeslices) and mass-energy conservation

measured in personal time (i.e. along a worldline). Now granted it may *look* in external time as if mass-energy was being created by your run but measured along your worldline, there's always only going to be one of you. We are already immortals after all and will each enjoy an infinite worldline howsoever it gets distributed. (In any event, an infinite universe could well have infinite mass-energy on each and every timeslice anyway so infinite multiplication of your mass along certain timeslices will scarcely change things.) Anyway, I make that five minutes. Good luck, Achilles, it's going to be a long run (personally speaking). Over the wall with you. Give my best wishes to Achilles_(α) when you're him ... in a million (personal) years or so.

[The machine and the woebegone Achilles vanish. The Tortoise vaults over the wall and lands on the starting-block. Achilles reappears halfway along the track and starts running. Ten seconds later, a countable infinity of Achilles appear spaced out at five metre intervals to his right. The Tortoise saunters towards the finish.]

A: *[Already tiring:]* How many more stages in this infernal race are there?

T: I'd say about Aleph-null, give or take. Cheer up, Achilles.

[A dismayed Achilles sees his other stages stretching off into infinite distance, right.]

A: Are they all me?

T: They all *will* be you, Achilles, in the fullness of time.

Chorus of Achilles's (Aleph-null of them): Alas, first Achilles, that you let the Tortoise choose the machine's settings, for now are you irrevocably ensnared for an infinite personal time, as are we all.

A: Oh woe is ... Hang on though, that can't have been infinitely many of me all speaking at once. If we've just heard from aleph-null of me simultaneously and

each one was audible individually, why aren't we both completely deafened, nay utterly obliterated, by the infinite force of the sound waves thus generated?

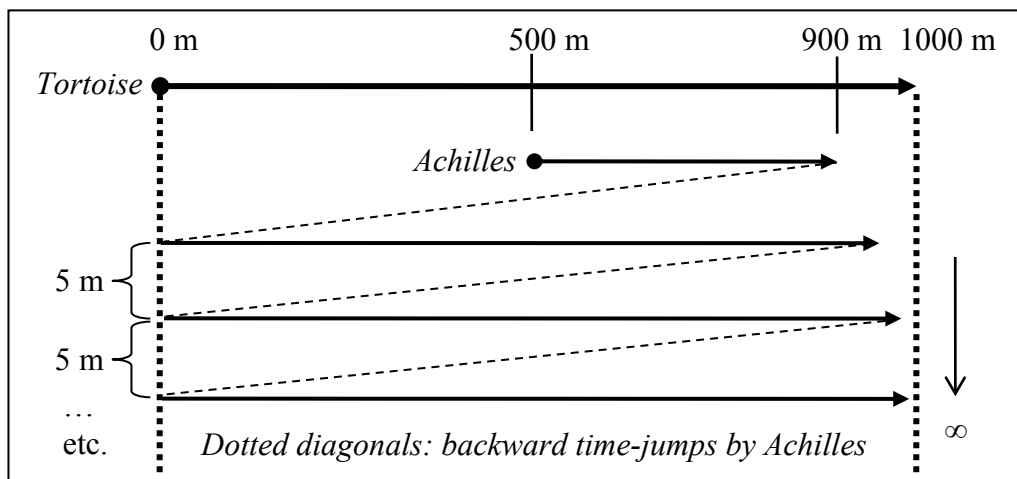
T: Oh that's an easy one, Achilles: your future selves kindly arrange things so each one only makes half as much noise as the Achilles to his immediate left, hence all the sounds made by the infinitely-many future Achilles-stages sum to a finite limit.

A: Oh, I had hoped that our surviving meant that there could only have been finitely many of my future selves. What am I (we?) to do now?

Chorus of Aleph-null Achilles-stages: I say beware all advice from any Achilles to starboard of my position. Everything said by an Achilles to my right is false.

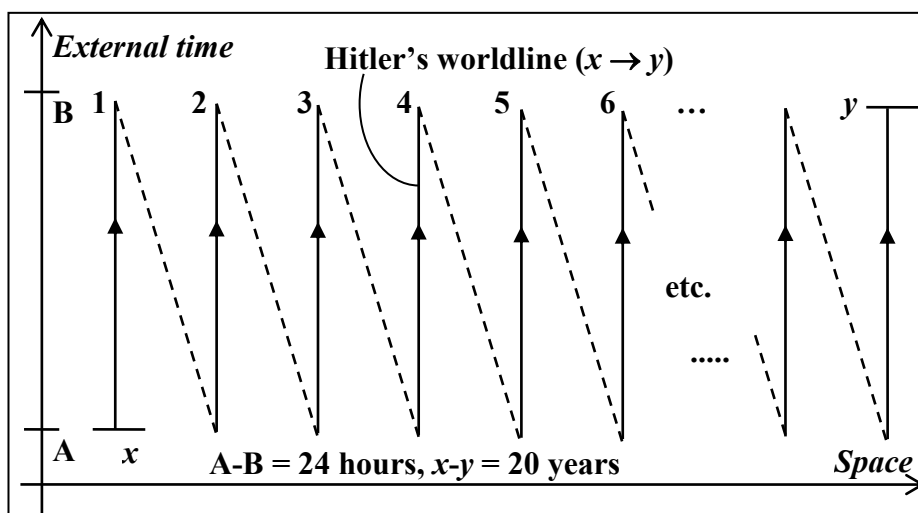
A: Oh, to think I feared that I might wind up enacting a new twist on an old paradox of Zeno's, and now I find that I'm ensnared in a spatial analogue of Yablo's (1993) non-self-referential Paradox as well.¹¹ Consistency between times (and locations in space) condemns me to eternal inconsistency of utterance ...

T: Be thankful, Achilles. Not everyone gets to enact two paradoxes at once. If you find it difficult to visualize your future course, try this. The closer you approach the race's end point in one direction, the further removed you become from its end point in another, orthogonal direction. [*Holds up diagram:*]



A: Tortoise, have you read Lester Del Rey's (1942) 'My Name is Legion'?¹²

T: The remarkable short story in which Adolf Hitler is punished for his crimes by time machine and has his last 20 personal years compressed into one external day? Wherein Hitler, failing to grasp that he's time-travelling, thinks his different stages are all distinct persons and tries forming them into a legion? Wherein one 24-hour external span simultaneously holds c. 7,300 Hitler-stages and ends with an early Hitler executing for disobedience his final, aged and senile, stage? With results not unlike this ...? [*Holds up another diagram:*]



A: Yes, that's the one.

T: No, I haven't.

A: Well, I rather think *someone* has.

T: Be that as it may, Achilles, try doing something useful while you run. I gather the status of Goldbach's Conjecture is still unsettled. So you might want to see if every even integer greater than two can indeed be expressed as the sum of two primes ...

A: I fear I'm going to be what Sorensen (2005: 122) calls a 'pseudo-immortal', i.e. one who is doomed to live a personal eternity in a finite interval of external time,

albeit one for whom each external second in a finite span of time will recur infinitely many times.¹³ Oh well, I suppose it could be worse. Let me see: 4 is the sum of 2 and 2; 6 is the sum of 3 and 3 ...

[Achilles runs resignedly off to become the aleph-null-strong Achilles-chorus. In the far distance, Achilles_(α) is briefly seen conversing with what looks like a version of the Tortoise, itself wearing the time machine. Each Achilles runs towards the finish line. However, with due allowance for external/personal time discrepancies, all disappear at progressively-decreasing distances from the finish only to reappear at the same external moment, t_2 , but five metres to the right of their previous run. The sounds of mutual distrust and exhaustive decomposition of even integers greater than two rumble off into infinite distance but pass after a finite time. The Tortoise reaches the winning-post.]

T: Well, I win - that wasn't too taxing. The 'Eleatic News' should receive my press-release any moment, just in time for tomorrow's edition. Then tomorrow my future self will collect a carefully worn newspaper and travel back to earlier today to press it upon Achilles_(α). Thereafter, tomorrow's Tortoise will travel back in time to give today's-Tortoise (i.e. me) the time machine ready to start the wager with Achilles. (With the results that we've just seen. Ah me, the demands of consistency.) And where is Achilles at this moment, externally speaking? I simply couldn't say, and nor can anyone else – rather like the state of Thomson's Lamp after the supertask, his externally-current state, or any other post-task state of his (personal or external), is left quite undefined by terms of the task itself.¹⁴ Given Achilles' velocity, I can determine precisely where he will have been at every instant at a finite personal time that elapsed after the race began – but where, if anywhere, Achilles may reside 'after' infinite personal time is unspecified, even if 'after' has any application here. Remember too that in Achilles' frame of reference, the run has infinite duration. Likewise, as his personal time blows-up to

arbitrary size, his spatial remove from the line along which he ran his first stage of the run similarly becomes unbounded. If we could somehow compare our elapsed personal times since the race began, I could truly say that Achilles is, in some suitably extended sense, running the race ‘even now’ even if he never happens to occupy the external time at which I now speak.

We Tortoises have long felt that Carroll (1895) and Wells (1895) shared more than mere contemporaneity: Wellsian unlimited access to time and space offers surprising scope for Eleatic outcomes too. Such unlimited time travel would allow arbitrarily great lengthening of one’s personal time *vis-à-vis* external time by in effect letting one trade (external) temporal extension for spatial extension. Given such travel, persons could live infinitely many days in a universe with a finite lifespan. Really determined and/or fated travellers (like our friend Achilles) might re-encounter the same (finitely many) external times infinitely many personal times. Hence another wrinkle on the personal/external time distinction: extended possibilities for supertasks and Eleatic races. Allowing arbitrarily-many operations can make a time machine an infinity machine too. It also suggests that being able to begin a finitely-long race and to run it do not entail being able to complete it, even in a finite interval of (external) time and at constant velocity. Normally an object with constant velocity cannot spend infinite time (or traverse an infinite length) between two lines that keep a fixed finite distance apart. However, as Achilles now knows, unrestricted time travel lets one run an infinite race between two lines a fixed finite distance apart *at constant finite velocity* (in one’s own frame of reference of course). Bless my shell, for every seemingly watertight wager-patch, another Tortoise-powered puncture ...

I must see if I can build some more machines and market them to a few select parties. Sisyphus and Tantalus spring to mind, or Mephistopheles perhaps ...

[*Exit.*]¹⁵

Notes and References:

¹Cf. Lewis Carroll, 'What The Tortoise Said To Achilles' (Mind, Vol. 4, 1895, 278-80). Please note that *this* Achilles is deemed to be a (literally and naturally immortal) continuation (however unworthy) of the character of that name in Carroll (1895). Thus, Achilles herein is not to be assumed to be the all-too-mortal Achilles of (e.g.) the *Iliad*, who tradition says was subsequently killed by an arrow in the heel. Thanks to an anonymous reader of an earlier version of this paper for that point.

²H. G. Wells, *The Time Machine*, (first book publication London: Heinemann, 1895).

³'The Paradoxes of Time Travel', *American Philosophical Quarterly*, Vol. 13, 1976, 145-52.

⁴Christopher Marlowe, *Doctor Faustus*, B-Text, Act II, scene 1, line 123, quoted from *Doctor Faustus: A Norton Critical Edition*, edited by David Scott Kastan, New York: W. W. Norton and Co., 2005, 72.

⁵Murray MacBeath, 'Who Was Dr Who's Father?', *Synthese*, Vol. 51, 1982: 397-430.

⁶Kurt Gödel, 'An Example of a New Type of Cosmological Solutions of Einstein's Field Equations of Gravitation', *Reviews of Modern Physics*, Vol. 21, 1949, 447-50; Igor D. Novikov, 'An Analysis of the Operation of a Time Machine', *Soviet Physics - JETP*, Vol. 68, 1989, 439-443.

⁷Cf. "For any material thing, whether machine or person, that set out to do an infinite number of acts would be committed to performing a motion that was discontinuous and therefore impossible", Max Black, 'Achilles and the Tortoise', *Analysis*, Vol. 11, 1951, 91-101, page 101.

⁸See J. Thomson, 'Tasks and Supertasks', *Analysis*, Vol. 15, 1954/5, 1-13.

⁹Cf. the progress that the Tortoise makes Achilles follow in Carroll (1895, 279-280). By dint of accepting a hypothetical inference as a further premise, Achilles is persuaded to move from granting premise (C) "(C) If A and B are true, Z must be true" to the further premise (D) "(D) If A and B and C are true, Z must be true". The Tortoise further persuades Achilles thus "(E) If A and B and C and D are true, Z must be true. Until I've granted that, of course I needn't grant Z. So it's quite a necessary step, you see?" and so seemingly *ad infinitum*.

¹⁰John Earman and John D. Norton, 'Infinite Pains: The Trouble with Supertasks', *Benacerraf and His Critics*, A. Morton and S. Stich (eds.), Oxford, Blackwell, 1996, 231-61.

¹¹Cf. Stephen Yablo, 'Paradox Without Self-Reference', *Analysis*, 53, 1993, 251-2.

¹²First published in *Astounding Science Fiction*, (June 1942), reprinted in *The Early Del Rey*, Vol. 1, New York, Ballantine Books, 1975, 270-99.

¹³Roy Sorensen, 'The Cheated God: Death and Personal Time', *Analysis*, Vol. 65, 2005, 119-25.

¹⁴ Cf. Paul Benacerraf, 'Tasks, Supertasks and the Modern Eleatics', *The Journal of Philosophy*, Vol. 59, 1962, 765-84.

¹⁵Many thanks to two anonymous referees for the *Journal of Philosophical Research* for very helpful comments on an earlier version of this paper. An initial version of this paper was produced during a

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