Frontloading, supposition, and contraction

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Frontloading, Supposition, and Contraction∗

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Abstract

In *Constructing the World*, Chalmers observes that our knowledge exceeds the core evidence provided by our senses and introspection. Thus, on the basis of core evidence, one also can know (S) that water covers the majority of the Earth. This knowledge, Chalmers suggests, requires a great deal of apriori knowledge. Chalmers argues that even if one suspends belief in one’s core evidence, one can nevertheless reason from a description of this evidence to an ordinary claim such as S. Chalmers concludes that the ordinary claim must be apriori entailed by a description of the core evidence. However, I propose that careful thinking about belief suspension reveals that empirical information can contaminate the reasoning from the core evidence to the ordinary claim S, even if belief in the core evidence is suspended. One result is that empiricists and externalists may freely appeal to thought experiments without having to concede that there are substantive apriori truths.

Neo-rationalist philosophers have recently attempted to rehabilitate the notion of apriori knowledge, knowledge that is warranted independently of experience. They have assigned the apriori a rather wide theoretical role. It has been held that apriori knowledge can (a) ground linguistic understanding and conceptual competence; (b) explain modal knowledge and counterfactual reasoning; and (c) distinguish reductive explanation from mere necessitation.1 For instance, it has been thought that one is competent with the concept *water* just in case one knows apriori that a substance in a given scenario s is water if and only if it has certain surface properties s: it is wet, it fills the lakes and rivers, it is drinkable, and so on. In this way, one’s competence with the concept *water* is purportedly explained via the concept’s apriori links to more basic recognition concepts: *being wet*, *filling lakes and rivers*, *being drinkable*, and so on. One’s modal knowledge, for instance, that *water cannot be an animal*, is purportedly explained by one’s ability to know apriori that whatever is water is not an animal. Finally, the reducibility of water to *H₂O* is explained by the purported fact that the truths about *H₂O* not only necessarily, but also apriori, entail the truths about waters.

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1See (Jackson 1998b; 1998a: 82) and (Chalmers and Jackson 2001). Dissenting views include (Block and Stalnaker 1999), (Schroeter 2006; 2013; 2014), (Soames 2005), and (Williamson 2008).
This wide theoretical role for the apriori suggests that the class of apriori truths exceeds the narrow range of the least controversial cases such as logical truths (such as every engineer is an engineer), definitional truths (every bachelor is unmarried), and mathematical truths (7 + 5 = 12). Indeed, it has been held that it is apriori that water fills lakes, rivers, and streams; that gold is a yellow metal; and that Hesperus rises in the evening. Many of these claims were put into jeopardy by Quinean (1953b) arguments from revisability. It is not plausibly an apriori truth, for instance, that water fills lakes and rivers, since there is possible evidence that would tell against this hypothesis. Externalist arguments—developed by Putnam (1975a) and Kripke (1980)—put further pressure on these purportedly apriori claims. These arguments seemingly establish that agents often lack discriminatory knowledge about the extensions of their expressions and concepts. Agents who are competent with the relevant concepts may nonetheless be ignorant that water fills lakes and rivers, that gold is a metal, and that Hesperus rises in the evening. This ignorance plausibly suggests that these truths are knowable only aposteriori.

In a number of works and most prominently in his *Constructing the World*, David Chalmers argues that the range of apriori truths greatly exceeds the least controversial cases. Chalmers’s strategy is the most developed implementation of a broad neo-rationalist strategy for resisting Quinean and externalist objections. The general strategy attempts to show that our ability to make a judgment about a world (whether considered as possible or as actual) based on a condensed description somehow reveals that one has apriori knowledge linking the content of the judgment to the content of the condensed description. Since counterfactual (and counteractual) reasoning is widely used in philosophy, the broad strategy suggests there is a great deal of substantive apriori knowledge beyond the core cases of logical, definitional, and mathematical truths.

In his implementation of the argument, Chalmers (2012: 130) argues that ordinary knowledge about the distribution of water, tables, chairs, mountains, and so forth is grounded in what he calls core evidence which concerns ‘subjects’ introspective evidence about their own phenomenal states […] and perceptual evidence about the distribution of primary and secondary qualities in the environment.² Chalmers argues from the fact that ordinary truths are knowable on the basis of core evidence to the claim that a conditional whose antecedent (D) is a complete description of the core evidence in a privileged vocabulary and whose consequent (S) is some ordinary claim is knowable apriori. Thus, it is purportedly knowable apriori that if (D) the majority of the Earth is covered by a substance that fills lakes and rivers, that is wet, that speakers

²Chalmers (2012: 131) is open to the possibility that some knowledge is not actually grounded in core evidence, but rather on recognition capacities that exploit phenomenal introspection and perceptual experience. He therefore modifies his claim to suggest that even if knowledge is not actually grounded in core evidence, it could be. The modification will not affect the discussion.
refer to as ‘water’, that produces precisely these sensations, that satisfies the tests for being \( H_2O \), and so on, then \((S)\) water covers the majority of the Earth. The ‘and so on’ is to be filled out by a comprehensive description of all core evidence available in the scenario. In particular, one is supposed to be able to reason from the supposition of the antecedent in such a conditional to high credence in its conclusion. The reasoning is supposed to require no empirical information, since all of this information has been ‘frontloaded’ into the antecedent. To support this conclusion, Chalmers argues that the reasoning is rational, even if one suspends beliefs in all of one’s core empirical evidence. Chalmers concludes that the truth or falsity of most ordinary claims is apriori inferable from a comprehensive description of the core evidence in terms of a privileged vocabulary.

I discuss the neo-rationalist argument that a conditional whose antecedent is one’s core evidence and whose consequent is some ordinary claim outside of one’s core evidence is knowable a priori. I first address a more general form of this argument. I then show that Chalmers’s more sophisticated version of this argument crucially appeals to the notion of belief suspension. The conditional is supposedly apriori because agents who suspend their empirical beliefs can rationally deduce \( S \) from \( D \). Chalmers’s belief suspension argument makes it difficult to see how the empiricist and externalist challenges to his project—in (Block and Stalnaker 1999) and, especially, (Schroeter 2006; 2013; 2014)—could possibly work. So long as it is conceded that the reasoning from \( D \) to \( S \) is rational even in a context in which belief in \( D \) has been withdrawn, how can one deny that the reasoning is a priori?

In this paper, I propose that careful thinking about belief suspension reveals just how empirical information can contaminate the reasoning from \( D \) to \( S \), even in contexts in which belief in \( D \) is suspended. That is, on closer inspection, the models proposed by Block and Stalnaker and by Schroeter look far better than it might initially seem from Chalmers’s discussion. In particular, I argue that belief suspension is best understood as a species of belief contraction in which one transitions from a high information state to a low information state. Contraction is well studied by both proponents of the AGM framework developed in Alchourrón, Gärdenfors, and Makinson (1985) and by proponents of rival frameworks. I show that understanding belief suspension as contraction allows one to more explicitly unpack the formal assumptions behind the neo-rationalist argument. Once these assumptions are laid bare, it becomes possible to see how the reasoning from a supposition of the complete description \( D \) to the ordinary truth \( S \) can require empirical information. In particular, the ‘frontloading’ argument is best understood in the context of debates over the ‘recovery’ postulate, which is motivated by the principle that belief contraction should minimise information loss. So even if full belief in \( D \) is suspended, many downstream effects of this belief can be retained in
the suspended belief state. Since the inference from $D$ to $S$ can smuggle in empirical information, I argue that the neo-rationalists have not proved that it is apriori. Moreover, I suggest that this is the best way to understand empiricist and externalist lines of resistance to this sort of neo-rationalism. One result of this paper is that empiricists and externalists may freely appeal to standard thought experiments that involve revisions of current beliefs without having to concede that these thought experiments make use of apriori principles.

1 The Frontloading Argument

Chalmers begins with the observation that ordinary agents know a variety of claims on the basis of a sparser evidence set. Thus, one might know ($S$) that water covers the majority of the Earth on the basis of one’s introspective knowledge of internal states and one’s perceptual knowledge of the distribution of primary and secondary qualities in one’s environment. Let $D$ stand for a complete description of this evidence in a restricted vocabulary (which does not use—but may mention—words such as ‘water’). Chalmers would frame this by saying that one can rationally have a high credence in $S$ in light of having acquired total evidence specified by $D$.\(^3\) Chalmers follows probablists in modelling a rational credal state as a probability function $cr$ which assigns values between 0 and 1 to a total set of claims expressed in a language, where $cr(\phi) = 0$ indicates that $\phi$ is certainly false and $cr(\phi) = 1$ indicates that $\phi$ is certainly true.\(^4\) In addition to absolute credences, a conditional credence function $cr(\cdot|\psi)$ corresponds to one’s credence on the supposition that $\psi$.

Given that agents rationally have high absolute credence in $S$ on the basis of evidence $D$, Chalmers argues they be able to rationally have a high conditional credence $cr(S|D)$ whose justification is independent of $D$. Chalmers (2012: 162) states the key premise in this argument as follows (changing abbreviations to $D$ and $S$ and subtly altering the notation for conformity):

**Frontloading**: If having a high credence $cr(S)$ is justified by $D$, then having a high conditional credence $cr(S|D)$ is justified independently of $D$.

Since one’s credence in a material conditional is at least as high as one’s corresponding conditional credence (where defined) $-$ $cr(S|D) \leq cr(D \rightarrow S)$,\(^5\) one may rationally have a high credence in the target material conditional independently of one’s total experience and, therefore, apriori.

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\(^3\)See (Moss 2013) for discussion of how conditional credences can constitute knowledge.

\(^4\)Chalmers normally takes the objects of credence to be sentences rather than propositions. Nothing I have to say will turn on this issue.

\(^5\)(Milne 2003) attributes this point to Popper.
Chalmers spends a great deal of time arguing for a thesis he calls *conditional scrutability*, that one may rationally have a high credence in $S$ given $D$ (presumably without having knowledge of $D$). But Chalmers (2012: 158) rightly concedes that this fails to establish that the knowledge is empirically uncontaminated:

> The obvious worry is that armchair knowledge does not entail apriori knowledge. Conditional Scrutability allows that the justification of the conditional belief (and rational conditional credences) may depend on the subject’s existing empirical evidence and beliefs.

Indeed, this is the challenge issued by many of Chalmers’s critics, who concede that one may know the conditional without knowing the truth of the antecedent but deny that this knowledge is apriori. Chalmers offers two arguments intended to show that one may rationally have a high credence in $S$ given $D$ independently of any empirical knowledge. One argument, which I address briefly, appeals to the Bayesian principle of conditionalisation in order to show that one may rationally have a high credence in $S$ given $D$ *temporally prior* to any experience. The other, more important, argument appeals to suppositional reasoning.

### 1.1 Reverse Conditionalisation

I begin with the argument from conditionalisation. Chalmers (2012: 213) formulates the principle of conditionalisation as follows:

**Conditionalisation:** If a subject is fully rational, and if the subject acquires total evidence specified by $D$ between $t_1$ and $t_2$ […], then $cr_2(S) = cr_1(S|D)$.

Chalmers attempts to extrapolate a backwards looking principle that governs how an agent could rationally arrive at her credence in $p$ given that she has acquired $q$ as evidence. Chalmers’s first attempt at formulating such a principle simply postulates that if one rationally arrives at a credence $cr(S) = \Phi$ at $t_2$ after updating with $D$, then one’s credence in $S$ given $D$ must have been $\Phi$ at some time $t_1$ prior to acquiring $D$.

**Reverse Conditionalisation:** If $cr(S) = \Phi$ at $t_2$, and one acquires total evidence $D$ between $t_1$ and $t_2$, then $cr(S|D) = \Phi$ at $t_1$. (Chalmers 2012: 163)

![Figure 1: Reverse Conditionalisation](image)

This reverse conditionalisation principle would allow Chalmers to infer that high conditional credence in $S$ given $D$ is justified independently of $D$ since it must have been rational to have such a credence *temporally* prior to acquiring evidence $D$. 
The problem is that conditionalisation is ‘forward looking’ in the sense in the sense that it assesses the rationality of updating on certain information given that the agent has a certain credal distribution. As Chalmers notes, reverse conditionalisation is a much stronger constraint on rationality. Conditionalisation cannot be applied if one simply fails to have a conditional credence at the prior state. This leaves it open that one acquires a conditional credence in $S$ given $D$ only after acquiring evidence $D$. This would undermine, at very least, Chalmers’s temporal priority claim.

Chalmers (2012: 163) considers the possibility that one’s prior conditional credence is undefined due to a failure to be able to entertain certain propositions.

Some versions of Bayesianism require that all conditional and unconditional credences are defined, and these versions will be committed to reverse conditionalization. But there is logical space for exceptions. For example, an opponent might suggest that an exception will arise if the subject is unable to even entertain $[S]$ at $t_1$ because they lack crucial concepts.

But there are many reasons for thinking that one might not always have definite conditional credences prior to acquiring any empirical evidence whatsoever. For instance, the claim one is conditionalising on, $D$, is a maximal statement of the evidence possessed by a given agent. But, what credence should one have in $D$ prior to the acquisition of any empirical evidence? Seemingly, there are uncountably many potential total evidence sets one could have. The probability of any one of them is 0. So, one’s conditional credence in $S$ given $D$ requires one to conditionalise on a claim one has 0 credence in. Many traditional versions of Bayesianism treat such conditional credences as undefined. Following (Hájek 2003), one could hold that some of them are defined, and this seems to be Chalmers’s preferred option. But there is no reason to think that all such conditional credences are defined nor that those that are defined are always defined independently of experience. At very least, there is no reason issuing from the Bayesian principle of conditionalisation for supposing that they are defined prior to any experience or justified independently of experience. Many Bayesians admit that one’s assessment of the background space of possibilities caries empirical presuppositions.\(^6\)

Recognising that there is no straightforward argument from reverse conditionalisation, Chalmers suggests a synchronic frontloading principle, which requires only that one have a high posterior conditional credence in $S$ given $D$.

\(^6\)See, for instance, Lance (1995) and Titelbaum (2008: 558, footnote 3; 2013: §5.3). Related considerations arise in the context of the imprecise credence literature such as (Schoenfield 2012). But nothing I have committed to mandates the move to this framework. Chalmers (2014: 682, footnote 2) suggests that Neta (2014) may be thinking along these lines as well.
**Synchronic Frontloading**: If \( cr(S) = \Phi \) at \( t_2 \), and one acquires total evidence \( D \) between \( t_1 \) and \( t_2 \), and if \( cr(S|D) = \Phi \) is defined at \( t_2 \), then \( cr(S|D) = \Phi \) at \( t_2 \), with justification independent of \( D \). (Chalmers 2012: 164)

The synchronic principle would do the work of the reverse conditionalisation principle without requiring defined antecedent credences. However, the claim that the justification for high posterior conditional credence in \( S \) given \( D \) is *independent* of \( D \) cannot readily be extracted from the Bayesian framework. On the rival picture, one begins one’s epistemic life as a rational agent only when one has certain empirical information in hand. Upon acquiring total evidence \( D \), one also rationally has a high credence in \( S \). Of course, one’s conditional credence in \( S \) given \( D \) will also be high. Yet Bayesianism is silent about where that high conditional credence comes from. Chalmers would need an independent argument to show that the high credence comes from background apriori rational constraints, as Chalmers thinks, rather than empirical information. The belief suspension argument could provide such a bridge.

### 1.2 Belief Suspension

Chalmers’s belief suspension argument again takes as a premise that a rational agent may assign high credence to \( S \) on the basis of updating with total empirical evidence \( D \). This agent should be able to *suspend* belief in \( D \) and after having done so, reintroduce \( D \) as a supposition, and reason to a high credence in \( S \). Chalmers concludes that the high credence in \( S \) given \( D \) does not depend on the total evidence specified by \( D \).

Given that \( D \) justifies \( S \), then one could in principle \((i)\) suspend judgment concerning \( D \), \((ii)\) suppose (for the purposes of conditional reasoning) that \( D \), \((iii)\) conclude (under this supposition) that \( S \), with justification provided by \( D \)’s support for \( S \), and \((iv)\) discharge the supposition, yielding a justified conditional belief in \( S \) given \( D \). This conditional belief is justified even though one has suspended judgment concerning \( D \), so that \( D \) played no non-suppositional role in its support. So the conditional belief in \( S \) given \( D \) is justified independently of \( D \). (Chalmers 2012: 162)

Given that the reasoning from \( D \) to \( S \) proceeds after one suspends one’s empirical evidence \( D \), Chalmers argues that the high conditional credence in \( S \) given \( D \) is justified apriori.

If the reasoning survives suspension of one’s current empirical beliefs, this suggest that it is not essentially justified by those beliefs. (*ibid*: 160)
Chalmers concludes that one may rationally have high credence in $S$ given $D$ in the absence of any experience. The crucial premises in Chalmers’s argument are the following.

**Limited Recovery**: Even upon suspending belief in one’s total empirical evidence $D$, one can still reason one’s way from the supposition that $D$ to $S$.

**Aprioricity**: If one can reason one’s way from $D$ to $S$ having suspended belief in all core empirical evidence, then one can have an apriori high credence in $S$ given $D$.

Both steps have been challenged. Chalmers (2012: 162) takes **Limited Recovery** to be his most contentious premise. Neta (2014) challenges Chalmers’s **Limited Recovery** thesis, denying that one can indeed reason from $D$ to $S$ given that one suspended one’s belief in $D$. Without going into the details, Neta’s major objection seems to presuppose that if the reasoning from $D$ to $S$ requires empirical information, then one should not be able to reason from $D$ to $S$ in the contracted belief state.

On the other hand, Schroeter (2006; 2013; 2014) develops interesting informal models which lead to doubt about **Aprioricity**. Schroeter (2006: 565ff) explicitly grants that one can conditionally reason from a complete description of the world in terms of the basic evidence such as $D$ to an ordinary claim such as $S$. But she provides a model—the ‘improv model’—suggesting that the reasoning subtly depends on the reasoner’s actual experience. On this model, agents in the actual world have a wealth of water-beliefs. In evaluating any given conditional whose antecedent is a complete-evidence description and whose consequent is some water claim such as *water covers the majority of the world*, the agents will appeal to any of these beliefs in the assessment of the relevant conditionals or conditional credences. Thus, the conditional credences are empirically contaminated. Moreover, Schroeter suggests that no particular belief or set of beliefs is ‘held fixed’ as relevant for assessing these counterfactuals. She therefore suggests that agents who have had different background experiences may come to different conclusions about whether $D$ entails $S$. This is not, of course, a surefire mark of aposterioricity. Some background experiences may constitute defeaters for the reasoning. But if Schroeter could make the case that the legitimacy of the reasoning from $D$ to $S$ is sensitive to the subject’s background experience, then that would be some evidence that the reasoning is empirically contaminated.

## 2 Belief Suspension as Contraction

These challenges to Chalmers’s framework can be clarified by observing that moving from a credal state $cr$ in which $D$ is treated as believed to one in which belief in $D$ has been suspended results in a contraction of
one's credal state. Following Gärdenfors (1988), let $cr_p^-$ correspond to the contracted credal state in which endorsement of $p$ has been withdrawn and $cr_p^+$ correspond to the expanded credal state which updates $cr$ by adding the information $p$. (For Bayesians, if $cr(.|p)$ is defined, then $cr_p^+ (. ) = cr(.|p)$.) In suspending belief in $D$, one temporarily retreats from a credal state $cr$ in which $D$ is taken as certain to a weaker credal state $cr_D^-$ in which less information is taken as certain. Of course, in moving to this weaker credal state, many beliefs other than just $D$ will have to be withdrawn as well. When one comes to believe $D$, one also comes to believe various things that entail $D$. Withdrawing one’s belief in $D$ will mandate withdrawing these ambient beliefs. **Limited Recovery** is a claim about how many of these ambient beliefs need to be withdrawn. It says that contraction retains enough information that when $D$ is reintroduced as a supposition, one can still reason one’s way to $S$. That is, limited recovery says that the result of removing one’s total empirical information, $D$, and then re-introducing it returns the original state: $(cr_D^-)_D^+ = cr$.²

**Figure 2: Belief Suspension**

Current Belief State: $cr$

\[ \xrightarrow{\text{Suspended Belief State: } cr_D^-} \text{ Suppositional Belief State: } (cr_D^-)_D^+ \]

I will argue that a good case can be made for **Limited Recovery**. At any rate, I will argue that there are many plausible models of contraction on which **Limited Recovery** holds.

However, the motivations for these models threaten **Aprioricity**. I will argue that updating with $D$ can severely distort one’s prior credences. One should not expect contracting by $D$ once again to result in an empirically uncontaminated credal state. In particular, I will argue that the very motivations for **Limited Recovery** also suggest that updating on and then withdrawing belief in different total evidence statements may result in different credal states. To a certain extent, then, the argument of this paper substantiates what is predicted by Schroeter’s improv model.

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²The relationship between one’s full belief and one’s credences may be complicated. See (Leitgeb 2014). Thus, the exact constraints on ceasing to believe that $p$ are likely to be more complicated than merely lowering one’s credence in $p$ below 1. It has also been questioned whether entering a state of suspending belief in $p$ is equivalent to taking oneself into a state where $p$ is no longer believed. See (Friedman 2013).

²One clarification: Chalmers means for one to suspend belief the totality of one’s core evidence. In the belief contraction framework, this would require contracting on the *disjunction* of the statements of one’s core evidence. Reintroducing them, would require reintroducing the *conjunction*. So the $D$ contracted on will be weaker than the $D$ that gets expanded. I will gloss over this complication and nothing I say turns on it.
2.1 Constraints on Contraction

Levi (1991: 117) describes two sorts of reason an agent may wish to contract her belief state. The contraction may be *coerced* because she is confronted with apparent evidence that \( A \) whose negation \( \neg A \) she has previously held as certain (or she finds that her corpus is otherwise inconsistent). On standard forms of Bayesianism, the agent cannot merely conditionalise on \( A \) without lapsing into absurdity.\(^9\) She must revise her belief set. On the standard model of (Levi 2006b: 125), one revises one’s belief set to include \( A \), by first contracting it to remove \( \neg A \) and then updating on \( A \).

Other contractions are *uncoerced*. One may wish to contract to give a hypothesis that is inconsistent with one’s current certainty set ‘a fair hearing’ (Levi 2006b: 169). Thus, one may wish to entertain what follows from the supposition that \( A \), even though one’s current certainties include \( \neg A \). This type of case will be at issue when we consider Schroeter’s claim that agents who have different total evidence \( D^* \) may rationally come to different verdicts from those who have total evidence \( D \) on whether \( S \) is true given \( D \). Another kind of uncoerced contraction occurs when one’s belief may already entail \( A \), but one may wish to see what follows from the supposition that \( A \) in one’s contracted belief set. This latter kind of revision is at issue in Chalmers’s belief suspension argument.

The ‘basic’ AGM postulates for belief contraction articulated in (Alchourrón et al. 1985: 513) provide a useful starting point for discussion.\(^{10}\) The framework initially modeled a belief state, \( K \), as a set of sentences or propositions closed under a consequence relation, \( \vdash \). The basic postulates are the following (following the presentation in (Gärdenfors 1988: §3.4), but re-ordered)

\[(K^{-1})\] For any sentence \( A \) and any belief set \( K \), \( K_A^- \) is a belief set.

\[(K^{-2})\] \( K_A^- \subseteq K \).

\[(K^{-3})\] If \( A \notin K \), then \( K_A^- = K \)

\[(K^{-4})\] If \( \nvdash A \), then \( A \notin K_A^- \)

\[(K^{-5})\] If \( \vdash A \leftrightarrow B \), then \( K_A^- = K_B^- \)

\[(K^{-6})\] If \( A \in K \), then \( (K_A^-)^+_A = K \)

\(^9\) (Hájek 2003) holds that it is possible to have credences conditional on a claim to which one assigns 0 credence. On this view, one can conditionalise on a claim \( A \) assigned 0 credence without lapsing into absurdity. Yet, coerced contraction will still be necessary for cases in which one considers a claim that is inconsistent with one’s evidence.

\(^{10}\) See also (Fermé and Hansson 2011: 297)
The first five basic postulates \((K^{-1})-\(K^{-5}\) are relatively uncontroversial. \((K^{-1})\) says that the result of contracting \(K\) is also a belief set. \((K^{-2})\) says that this belief set is included in the original belief set. \((K^{-3})\) says that if one contracts \(K\) by trying to remove a claim that is not endorsed, the contraction is vacuous in that it returns \(K\) unaltered. \((K^{-4})\) says that contraction of \(K\) by \(A\) yields a belief set that does not include \(A\), so long as \(A\) is not a logical truth. \((K^{-5})\) says that the result of contracting a belief is the same as the result of contracting a logically equivalent belief. An operation satisfying \((K^{-1})-\(K^{-5}\) is a withdrawal (Makinson 1987: 6). I discuss the sixth postulate in the next section.

These postulates describe constraints on a contraction function, but do not uniquely identify it.\(^{11}\) This is even clearer in the probabilistic framework as developed by (Gärdenfors 1988: §5.7), where \((cr^{-5})\) corresponds to the recovery postulate.

\[
\begin{align*}
\text{(cr}^{-1}) & \text{ For all credence functions } cr \text { and all sentences } A, cr^{-A} \text { is a credence function.} \\
\text{(cr}^{-2}) & cr^{-A}(A) < 1 \text { if } \not\models A. \\
\text{(cr}^{-3}) & \text{If } \vdash A \leftrightarrow B, \text { then } cr^{-A} = cr^{-B}. \\
\text{(cr}^{-4}) & \text{If } cr(A) < 1, \text { then } cr^{-A} = cr. \\
\text{(cr}^{-5}) & \text{If } cr(A) = 1, \text { then } (cr^{-A})^{-A} = cr
\end{align*}
\]

The credence function in this case is less tightly characterised because in contracting belief that \(A\), the constraints require only that \(cr^{-A}(A) < 1\), they do not impose a constraint on the ‘magnitude’ of this belief (Gärdenfors 1988: 118ff). Moreover, they leave open the absolute and conditional probabilities of background beliefs.\(^ {12}\)

### 2.2 Recovery and Limited Recovery

The final contraction postulate \((K^{-6})\) is highly controversial. It says that the result of removing a belief and then reintroducing it always returns the original belief set. It is an attempt to answer the question of how much information one should remove. As Gärdenfors (1988: 58) says, this is the main goal for any account of contraction:

\([^ {11}\text{The characterisation can be tightened by imposing an ‘entrencement ordering’ on belief sets letting these constrain legitimate contractions as in (Gärdenfors 1984). (Grove 1988) models the AGM framework in a possible worlds framework and is able to characterise the contraction function in terms of a system of spheres.}\]

\([^ {12}\text{Given the principle of information economy, one can more tightly constrain belief change by imposing a measure on the distance between probability functions.}\]
The main problem concerning contractions of belief sets is that, when retracting a belief $A$ from a belief set $K$, there may be other beliefs in $K$ that entail $A$ (or other beliefs that jointly entail $A$ without separately doing so). So, if we want to [...] keep the contracted state of belief closed under logical consequences, it is necessary to give up other beliefs as well.

Postulate ($K^{-6}$), known as the ‘recovery postulate’, is an attempt to minimise the information lost. Gärdenfors (1988: 58) describes the postulate as issuing from a principle of informational economy: the retraction should be, in some sense, minimal.

We want the criterion of informational economy to apply also to contractions, so that no beliefs should be given up unnecessarily.

Even opponents of recovery, such as Levi (1991: 150) agree that minimal information should be lost. They simply disagree on the characterisation of information at issue.\footnote{Similarly, proponents of finite belief bases rather than belief sets can appeal to information loss (Hansson 2008: §2.3).} The premise in Chalmers’s argument that I called Limited Recovery is a special case of the recovery postulate where the information being retracted is the totality of one’s empirical information. Limited Recovery says that if $cr(S)$ is high and $D$ is one’s total empirical evidence, then $(cr^D_D)^{-1}(S)$ must also be high. Although there are many objections to the full recovery postulate, I will argue that they do not extend to this more restricted principle.

The recovery postulate is highly controversial because it is subject to a variety of purported counterexamples. These mostly involve cases in which contracting on some claim also requires contracting on a stronger claim. Suppose that agent $A$ learns that a six-sided, fair die was cast. $A$ learns ($p$) that the die landed on ‘5’. $A$ then contracts her belief ($q$) that the die did not land on ‘2’. Contracting on $q$ requires contracting on $p$, since $cr_A(q|p) = 1$. But now consider what happens when the information $q$ that the die did not land on ‘2’ is re-introduced. According to recovery postulate, $q$, the claim that the die landed on ‘5’, should also be re-introduced. This strikes many as implausible. In the abstract, one cannot reason one’s way from the claim that the die did not land on ‘2’ to the claim that the die landed on ‘5’.

The important point, however, is that Chalmers is restoring the totality of an agent’s empirical information. In the case of the die, observe that the contraction of just $p$ is equivalent to the contraction which withdraws belief both in $p$ and in $q$, or $cr_p^{-} = cr_{p\lor q}^{-}$. Restoring the totality of one’s empirical information, including both $p$ and $q$ returns the original belief set. Thus, it seems that the standard objections to recovery do not undermine Limited Recovery.
Though the recovery postulate is controversial, it is not universally rejected.\footnote{14} As I have said, it is motivated by the idea that belief contraction should be minimal, it should distort as little as possible. Many would share this motivation even if they object to the full recovery postulate. They would do so, by challenging the notion of ‘information’. In the AGM framework, a belief set is taken to have less information value than a set that contains it.\footnote{15} It seems, however, that anyone motivated by such a minimal revision principle should embrace something like Limited Recovery, since it does not raise the sorts of problematic potential counterexamples introduced by the full recovery postulate. This suggests that any plausible version of the principle of informational economy will validate Limited Recovery.\footnote{16}

2.3 Recovery and Empirical Contamination

If this is right, there is no reason to believe that withdrawing endorsement of one’s total evidence \(D\), namely \(c_D\), is some unadulterated credal state that is unaffected by one’s prior credence in \(D\). It is simply one of the minimal alterations of one’s current belief state that retracts certainty from \(D\). Notice that this is exactly what is predicted by Schroeter’s improv model. An agent has many experiences of colourless, odourless, drinkable portions of fluid. She hears her community members using a word ‘water’ in the presence of the fluid. She immerses herself in their practice and becomes able to apply the word ‘water’ on the basis of sight or touch, though occasionally having to revise her view. She undertakes scientific investigation into the chemical properties of the bits of fluid that she recognises as instances of water. On the basis of the totality of evidence, she conjectures that water is a common chemical kind comprising the various portions of colourless, odourless fluid around. She learns that these portions cover the majority of the Earth, so she concludes that water covers the majority of the Earth. Given all of this information, she is able to form beliefs conditional on possible total evidence sets that either extend or revise her own total evidence set. She can even do so upon suspending her beliefs in her current evidence set, because this suspension preserves information sufficient to reason from a description of a possible total evidence state \(D\) to an ordinary claim such as \(S\) ‘water covers the majority of the Earth’.

One might imagine a subject with the (logical closure of) the following belief set.

\footnote{14}See discussion in (Makinson 1987), (Levi 1991: §4.5), and (Hansson 2008: §2.3),\footnote{15}See discussion in (Makinson 1987).\footnote{16}Some views of belief contraction such as (Rott and Pagnucco 1999) do not support any version of the principle of informational economy. It would be interesting to investigate whether these support Limited Recovery. Another important question is what happens to Limited Recovery if contraction is taken as an operation on belief bases rather than belief sets or credal distributions. A belief base is a subset of beliefs taken as the bases for other beliefs. These might, for instance, be the explicitly represented beliefs. So construed, removing \(D\) may leave enough information intact to infer \(S\) from the reintroduction of \(D\) as a supposition. But if one takes belief bases to be the evidential base of a theory, then it is possible that removing \(D\) would remove everything needed to infer \(S\) when \(D\) is reintroduced as a supposition along the lines Neta suggests. See (Makinson 1987) and (Makinson 2011: §3.2.), which discusses this framework in a probabilistic setting.
In suspending her belief about core evidence, she must contract this set so that the remaining claims do not entail any core evidence claims. This will require revising some beliefs that are not core evidence. The principle of informational economy says that she should not do this unnecessarily. The retracted belief set may be as follows.

<table>
<thead>
<tr>
<th>Core Evidence</th>
<th>Beliefs</th>
</tr>
</thead>
<tbody>
<tr>
<td>There is a wet sensation in my arm.</td>
<td>Water is causing a wet sensation in my arm.</td>
</tr>
<tr>
<td>Lakes, rivers, and streams are filled with colourless, odourless liquid.</td>
<td>Water fills lakes rivers and streams.</td>
</tr>
<tr>
<td>Speakers use the sound ‘water’ in the presence colourless, odourless liquid.</td>
<td>Water is a colourless, odourless liquid, if it exists.</td>
</tr>
<tr>
<td>Speakers withhold the term ‘water’ from things that don’t obey scientific tests for being H₂O.</td>
<td>Water is whatever is same-substance related to the samples called ‘water’ around me.</td>
</tr>
<tr>
<td>The wet substance satisfying the scientific tests for being H₂O covers the majority of the Earth.</td>
<td>Water covers the majority of the Earth.</td>
</tr>
</tbody>
</table>

Given LIMITED RECOVERY, enough information should be left in the belief set (or credal state) to reinstate belief in (S) water covers the majority of the Earth when belief in D is reinstated.

This understanding of the debate offers Schroeter a reply in her recent exchange with Chalmers. When Schroeter explicitly considers the frontloading argument, she suggests that the conditional credence is justified, in part, by empirical information about the actual world state used to represent that water covers the majority of the Earth, such as the fact that the state is partially caused by the subject’s interaction with a
colourless, odourless substance of a common chemical kind. Schroeter argues:

Ideal epistemic methods for updating beliefs [...] suggest that empirical information about the relevant token representational states must be both included in $[D]$ and known to be actually true for a conditional credence in $[cr(S|D)]$. (Schroeter 2014: 8, footnote 8)

Chalmers challenges Schroeter, arguing that this additional information can simply be frontloaded into the antecedent of the conditional:

If a subject can reason from $[D]$ to $[S]$ with background justification from $[K]$, then in principle she could suspend judgment about $[K]$ and come to be justified in believing if $[D]$, then $[S]$.  

(Chalmers 2014: 684)

But the present model suggests a different way to understand Schroeter’s talk about one’s conditional credence depending on empirical knowledge of the actual world. The reasoning from $D$ to $S$ is empirically dependent, but not in the sense that it requires some specific auxiliary empirical premise $K$ that can be frontloaded into the antecedent of the relevant conditional without reducing the conditional to a logical truth. Rather, the inference is empirically dependent because it is only rational in contractions of some credal states and not others. When one contracts one’s total evidence from a high information state, one yields a credal state that supports the inference of $S$ from $D$. This may be done, for instance, by leaving in place a suitably entrenched conditional belief whose antecedent is a consequence of $D$ (say, that there is a common substance covering the majority of the Earth with the typical surface properties of water) and whose consequent is $S$, that water covers the majority of the Earth. Or, the linkage may be more subtle involving diverse conditional credences. But the empirical information cannot be incorporated into the antecedent $D$ of the conditional without the risk of reducing the entire conditional to a logical truth.

This last point is important. Contracting credal state $cr$ to remove $D$ leaves in place many of the “downstream effects” induced by originally updating with $D$. It is not a pure, uncontaminated credal state. Re-introducing $D$ as a supposition may—let’s suppose—allow one to reason one’s way to all of the original conclusions. This introduces a possible response for Chalmers: what if we try to find a way to return the totality of the agents’s credal states to what they were before updating with any evidence? This would mean contracting not only on $D$, but on any downstream effects of $D$, returning the agent to the edenic initial credal state.

If—as I believe—the inference from $D$ to $S$ is empirical, then this would mean that the agent in her initial credal state is unable to reason from the supposition of $D$ to $S$. It would follow one cannot infer $S$
on the basis of $D$ in this prior state. It would follow that one cannot infer $S$ from $D$ on the basis of a total contraction of this kind (unless $D$ was so strong as to logically entail $S$). So in a sense, my strategy is two-fold. I grant Chalmers that there is a perfectly good sense in which agents can contract on their core evidence $D$ and yet reason their way from the supposition that $D$ to $S$. They can do so because contracting on $D$ leaves in place enough empirical information for the reasoning. In this sense, I grant Limited Recovery but deny Apriority. On the other hand, there may be a sense in which one can contract on $D$ and everything else. If one can do this this, then I deny that Limited Recovery holds: one cannot reason one’s way from $D$ to $S$ in this absolutely contracted credal state. But it does not seem plausible to me that this absolute contraction regularly occurs in ordinary cases of belief suspension and suppositional reasoning.

2.4 Empirical Variation

It would be plausible to hold that the suppositional reasoning is empirically dependent, if the rationality of the reasoning could be shown to vary with one’s background empirical information. This would involve showing that the legitimacy of the suppositional reasoning is sensitive to the real world information of the rational agent. The focus thus far has been on whether agents who believe $S$ on the basis of total evidence described by $D$ can suspend belief in $D$ and yet still reason from the mere supposition that $D$ to $S$. Schroeter’s case would be significantly strengthened if updating and then suspending different information $D^*$ yields a different verdict. This would imply that one’s ability it rationally deduce $S$ from $D$ in a suspended belief state is dependent on one’s background empirical information. This type of empirical dependence is not a surefire indicator of aposterioricity. It could be that there is some default warrant for having high conditional credences but which can be undermined depending on one’s experiences. Nonetheless, empirical variability is a mark of aposterioricity, creating a strong presumption in favour of the inference being aposteriori.

Indeed, Chalmers himself seems to hold that the relevant conditional beliefs can be known by agents who have different total empirical information. That is, if one has total information $D^*$ conflicting with $D$, then Chalmers (2012: 139) thinks that the supposition that $D$ ‘simply overrides’ one’s knowledge that $D^*$ insofar as the two conflict. This would suggest that the inference from $D$ to $S$ should be valid regardless of one’s incoming experiences.\footnote{See (Gärdenfors 1986: 81) for a related discussion about information dependence in the AGM framework.}

Schroeter’s informal model is supposed to make plausible that one’s conditional credences are information-dependent in this way, however. Actual agents have had a variety of experiences which taught them the point of having a concept water. These experiences are what the agents take as evidence for being water when
it is the same chemical kind as the colourless orderless liquid around here. Schroeter suggests that these experiences constitute empirical evidence that anything of that chemical kind is indeed water. But Schroeter (2014: 9-10) imagines agents with different experiences. In particular, these agents find that the members of their community use the word ‘water’ to pick out a manifest kind, applying to all and only those things that share the surface properties of what they call ‘water’. Given their background information about what is relevant to being water, Schroeter’s model predicts that the suppositional reasoning might yield a different result. Schroeter (2006: 575) would suggest that the mere fact that these agents yield different verdicts on the relevant conditionals is not decisive evidence that they mean something different: ‘No particular pattern of assumptions about water leaps out at us as rationally unrevisable on pain of changing the subject’. (Indeed, their experiences may be systematically misleading about the way the members of the community as a whole use the concept water.)

The model of belief suspension as belief contraction makes it possible to see how these results could diverge. Consider two agents. Agent 1 is in credal state $cr_1$ with total evidence $D$. She then suspends credence in $D$, temporarily entering contracted credal state $cr_1 \overline{D}$. Finally, she assumes $D$ as a supposition entering credal state $(cr_1 \overline{D})^+_D$. Agent 2 is in credal state $cr_2$ with total evidence $D^*$. She then suspends credence in $D^*$, temporarily entering contracted credal state $cr_2 \overline{D^*}$. Finally, she assumes $D$ as a supposition entering credal state $(cr_2 \overline{D^*})^+_D$. The question at hand is whether the two agents both have the same credence in $S$, or $(cr_1 \overline{D})^+_D(S) = (cr_2 \overline{D^*})^+_D(S)$.

Figure 3: Empirical Variation

Current Belief State: $cr_1$

\[ \Downarrow \]

Suspended Belief State: $cr_1 \overline{D}$ \quad \rightarrow \quad Suppositional Belief State: $(cr_1 \overline{D})^+_D$

Current Belief State: $cr_2$

\[ \Downarrow \]

Suspended Belief State: $cr_2 \overline{D^*}$ \quad \rightarrow \quad Suppositional Belief State: $(cr_2 \overline{D^*})^+_D (\neq (cr_1 \overline{D})^+_D)$

If belief contraction is governed by the principle of informational economy, then $cr_1 \overline{D}$ is the state that minimally changes $cr_1$ by removing $D$ and $cr_2 \overline{D^*}$ is the state that minimally changes $cr_2$ by removing $D^*$. There is no reason to expect these to be the same. So there is no reason to expect the suppositional reasoning...
to be insensitive to one’s background empirical information.

In practice, we may imagine that \( cr_1 \) is the credal state above while \( cr_2 \) is as follows:

\[
\begin{align*}
\text{Core Evidence} & \quad \text{Beliefs} \\
\text{Speakers use the sound ‘water’ in} & \quad \text{Water is a colourless,} \\
\text{the presence colourless, odourless} & \quad \text{odourless liquid, if it} \\
\text{liquid.} & \quad \text{exists.} \\
\text{Speakers don’t care about the ma-} & \quad \text{A substance’s mate-} \\
\text{terial composition of what they} & \quad \text{rial constitution is} \\
\text{call ‘water’}. & \quad \text{irrelevant to its be-} \\
\text{There is a substance covering the} & \quad \text{ing water.} \\
\text{majority of the Earth that satisfies} & \\
\text{the test for being \( H_2O \) but doesn’t} & \\
\text{look, taste, or smell like the sub-} & \\
\text{stance called ‘water’ around here.} & \\
\end{align*}
\]

Contracting on this belief set would yield something like the following.

\[
\begin{align*}
\text{Core Evidence} & \quad \text{Beliefs} \\
\text{Speakers use the sound ‘water’ in} & \quad \text{Water is a colourless,} \\
\text{the presence colourless, odourless} & \quad \text{odourless liquid, if it} \\
\text{liquid.} & \quad \text{exists.} \\
\text{Speakers don’t care about the ma-} & \quad \text{A substance’s mate-} \\
\text{terial composition of what they} & \quad \text{rial constitution is} \\
\text{call ‘water’}. & \quad \text{irrelevant to its be-} \\
\text{There is a substance covering the} & \quad \text{ing water.} \\
\text{majority of the Earth that satisfies} & \\
\text{the test for being \( H_2O \) but doesn’t} & \\
\text{look, taste, or smell like the sub-} & \\
\text{stance called ‘water’ around} & \\
\text{here.} & \\
\end{align*}
\]

If Agent 2 re-introduces \( D \) to this suspended belief state, the result will be very different from if Agent 1, who started out believing \( D \), reintroduces \( D \) on a number of fronts. Agent 2, for instance, may not be able to reason her way to the conclusion that \( water = H_2O \), that water covers the majority of the Earth, or a number of other conclusions. For this reason, I suggest that Chalmers’s frontloading argument does not foreclose the possibility one’s ability to reason from \( D \) to \( S \) is empirically dependent, in the sense of being sensitive to background empirical information.
3 Conclusion

Chalmers has argued that an agent who suspends belief in her core evidence $D$ can nonetheless reason her way from the supposition that $D$ to an ordinary claim $S$. He concludes that one may have high apriori conditional credence in $S$ given $D$. In this paper, I have effectively conceded **Limited Recovery**, the claim that one can reason from $D$ to $S$ in the suspended belief state. However, I have denied that it follows that one’s credence in $S$ given $D$ is apriori. In particular, I have argued that standard theories of belief contraction reveal how reasoning may be empirically contaminated even when one suspends one’s belief about the actual world. That is, on standard theories of belief contraction, contracting on a claim such as $D$ need not return us to the initial credal state held prior to endorsing $D$. Rather, it returns the believer to the minimal departure of her current state that withholds full belief in $D$. This paper therefore reinforces the conclusions of empiricist and externalist responses to neo-rationalist projects such as (Block and Stalnaker 1999) and (Schroeter 2006; 2013; 2006) unintelligible. These responses can be incorporated into plausible, formal models of belief change that satisfied **Limited Recovery**.

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References


