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# Education for sustainable development (ESD) in mathematics education: what could it look like?

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Over the past two decades, there has been a loud call around us to integrate Education for Sustainable Development (ESD) across all subjects in the primary and secondary curricula. However, researchers have argued that sustainability and mathematics education remain largely unconnected in actual classrooms. The question as to what sustainable mathematics education could look like in the 21st century remains unanswered. There is a pressing need to rethink, re-envision mathematics teaching and learning for 21st-century learning priorities.

In this note, we first present a theoretical synthesis of the specialized literature on the learning and teaching of mathematics. We then attempt to contribute to this particular area by challenging the traditional boundaries of the philosophy of mathematics, followed by some thoughts on ESD in mathematics teacher education. In doing so, we seek to call attention to the need for integrating ESD into mathematics education. We hope, through our call in this note, that more researchers will be stimulated to revisit the integration of ESD into mathematics education, and, consequently, to conceptualize how, and where, efforts should be focused in order to integrate aspects of ESD with mathematics teaching and learning in schools.

**Keywords:** Education for sustainable development (ESD); mathematics teacher education; teachers' wider professional responsibilities; sustainable mathematics education; sustainability

## 1. Introduction

In recent years, Education for Sustainable Development (ESD) has been the term used internationally and by the United Nations. The roots of ESD can be traced back to the agenda of the UN Decade of Education for Sustainable Development 2005–2014. In contradistinction to the traditional approach of teacher-led teaching, ESD aims to empower “learners to take informed decisions and responsible actions for environmental integrity, economic viability and a just society, for present and future generations, while respecting cultural diversity” [1]. Nowadays, we see a huge growth in various adaptations of ESD across many subject areas and

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education systems [2]; however, there has been limited progress in ESD so far in actual classrooms [3-4]. It has also been argued that the quantity and quality of ESD provision in teacher education have been “patchy” [5]. This statement may be shocking, but, unfortunately, other research has verified it: Laurie et al. [2] conducted interviews with education leaders and practitioners in eighteen countries about their views on the outcomes and implementation of ESD and concluded that there is a sense of urgency about the necessity to enhance teachers’ professional development for ESD.

While recent studies have offered a wealth of empirical evidence about the benefits of integrating ESD across all subjects in the primary and secondary curricula [1], in a subject area such as mathematics, “it seems difficult to conceive of how this could be done” [6, p. 136]. Renert [7, p. 20] also argued that “sustainability and mathematics education remain largely unconnected in the research literature”. Our review of ESD-related studies also shows that the number of available studies in ESD for teachers of mathematics in particular, and for mathematics education in general, is small and disproportionate to the number of studies that document problems of practice for which solutions are needed. Issues concerning how ESD can be integrated with mathematics teaching and learning in schools remain poorly understood. There is a pressing need to rethink, re-envision mathematics teaching and learning for 21st-century learning priorities.

In recent years, education systems around the world have taken action to integrate ESD as part of their responsibility [8]. For instance, the international ESD agenda has informed curriculum developments in Australia and Scotland. Sustainability as a cross-curricular priority was introduced to Australia’s curriculum in 2010. In Scotland, “Learning for Sustainability (LfS) [is] cross-cutting themes in Scotland’s CfE [Curriculum for Excellence] which provides an overarching philosophical, pedagogical and practical framework for embedding ESD in the school curriculum” [5, p. 5]. LfS as a core part of teachers’ professional standards has been embedded at all levels of Scottish Education since 2012 as a response to the UN’s call for ESD.

ESD emphasizes students’ engagement in discussion, analysis, and application of learning and knowledge through interdisciplinary activities [2]. However, recent research shows that there has been limited progress in ESD in actual classrooms [5 & 7]. It is not surprising that the complexities of ESD and interdisciplinary approaches continue to pose challenges for mathematics teachers to work with others across traditional disciplinary boundaries, to bring

about critical dialogue across issues, and thus to show links between the subject and ESD [7 & 9]. This note, therefore, takes a step forward by renewing attention on the place of ESD within mathematics education.

## **2. Paradigm shift in mathematics teaching and learning**

Researchers interested in the development of students' mathematical understanding have long distinguished between knowledge memorized by rote and knowledge acquired meaningfully. In the early 1900s, Thorndike [10] applied the theory of behaviorism to learning. He believed that drill and practice would help to build "bonds," or connections, within the brain and thus create accuracy in mathematics. The bonds that were developed through repetition could help students to achieve computational fluency and solve word problems. Baroody [11, p. 4] considered that "the most efficient way to accomplish bond formation is through direct instruction and drill". Dissatisfaction with the drill theory led Brownell [12] to propose meaning theory, whereby instruction should focus on promoting the meaningful memorization of skills. Piaget's constructivist theory [13] and Vygotsky's social-learning theory [14] also adopt this perspective.

Over the past few decades, constructivist perspectives on learning and teaching have been a popular topic among mathematics educators, psychologists and researchers [15-18] and, as a result, have contributed to shaping mathematics reform efforts in many countries around the world [19-20]. A key feature of constructivism in education is its completely different approach to traditional pedagogies. Its great strength, as commonly recognized, is to provide students with more opportunities to construct knowledge meaningfully [21]. Nevertheless, it is argued that, when adopting constructivist teaching, it is difficult for teachers to be sure that all students develop necessary, or useful, skills; hence, it is considered by some researchers to be ineffective when compared to direct instruction [22-23]. Some have stated that a constructivist approach overemphasizes the acquisition of conceptual knowledge and problem-solving at the expense of skills mastery [22 & 24-25]. Ewing [26] and Pearson [27] have asserted that practices largely comprised of teachers' direct instruction are the most effective approach to high achievement in mathematics.

According to these findings, we suggest that it is necessary to differentiate between "achievement goal orientation" and "performance goal orientation". In the literature, the former is identified as being "task involved" [28], "learning oriented" [29], and "mastery

focused” [30], while the latter is characterized as being “ego involved” [31], “performance oriented” [32], and “ability focused” [33]. Linnenbrink [34] noted that “achievement goal orientation” focuses on developing competence, while “performance goal orientation” aims to demonstrate it. Whereas early work, such as [35], conceptualized achievement and performance goal orientations as bipolar continuums, Button, Mathieu, and Zajac [36, p. 28] suggested that they are separate dimensions—they are “neither mutually exclusive, nor contradictory and [their relative merits depend] on the strength of situational conditions”. Indeed, in line with Button et al. [36], we believe that constructivist student-centered and traditional teacher-centered approaches pursue different goal orientations—the former focusing on “achievement goal orientation,” the latter on “performance goal orientation.”

ESD is an interdisciplinary approach to learning and teaching that develops students’ knowledge in collaborative and novel ways with the aim of empowering them to see ESD as a way of being and thinking about the world and their actions. However, in recent years, “teaching to the test” in mathematics education has become an increasingly common phenomenon in many education systems across the world, and the practice has made mathematics teaching strongly academically oriented [37]. It is evident that there is a difference in goal orientation between ESD and “teaching to the test” in mathematics education. This difference invites us to re-think what the goals of, and the vision for, mathematics education should be in the 21st century.

Admittedly, we live in an information age. The transition from the industrial age to the information age has required the development of 21st-century learning skills, such as critical thinking, creative thinking, collaborating, and communicating. However, we also live in troubling times. There has been a loud call around us to attend to the problems of ecological sustainability. During the UN Decade (2005–2014), it was seen that a number of formal education systems reported on their nations’ progress and began to take ESD into account as part of their responsibility. The 2030 Agenda for Sustainable Development was introduced in 2016 in order to stimulate action over the next fifteen years (2016–2030).

How might mathematics education contribute to our understanding of, and our responses to, sustainable development challenges? In acknowledging that “social, technical, economic or ecological problems [...] call for new mathematics to improve our perception, control and regulation of the problematic situation” [38, p. 20], we argue that ESD in mathematics

education can lead the sustainable development movement by providing appropriate content that allows students to acquire the skills needed to understand the concepts related to sustainable development and the aspects of the world around them. Furthermore, in ESD, the conception of mathematics is reconfigured to expand the traditional philosophy of mathematics to accommodate social, economic, and environmental dimensions. As a result, students will be better prepared for the global issues that they will face during their lifetimes. Nevertheless, what would and should ESD in mathematics education look like in the 21st century? This question remains unanswered, as “there has been little sustained mathematics education research” [39, p. 156]. In the following sections, we attempt to contribute to this particular area by challenging the philosophy of mathematics, followed by some thoughts on ESD in mathematics teacher education.

### **3. ESD in mathematics education: reconfiguring and rethinking the philosophy of mathematics for the 21st century**

In mathematics education, in line with Renert [7], we note that ESD—which encourages or allows learners to engage in interdisciplinary activities and discussions about sustainability—is rather uncommon in mathematics. One reason for this deficit, in our view, is that the integration of ESD into mathematics education would require a redefinition of the scope of mathematics. Despite increasing awareness of the importance of sustainability, there is no clear rationale for ESD in mathematics education.

As Ernest stated [40, p. 79], “one of the traditional problems of the philosophy of mathematics is the question of how wholly abstract mathematics can have any effect on the world”. Skovsmose’s performative interpretation of mathematics [41] may provide a partial answer. In contrast to the historical claims of pure mathematics to certainty and the nature of mathematical objects, Skovsmose [41] offers a different perspective on the philosophy of mathematics. He argues that mathematics is performative in that its adoption and application by human beings can lead to altered political priorities and can transform social realities. Mathematical readings of a critical situation (what Skovsmose calls a crisis) are not expected to be neutral or value-free. Instead, they establish perspectives that may help people acquire relevant knowledge, practice critical thinking, manage uncertainty, and act responsibly regarding the situation.

Mathematics, influenced by Platonism, was popularly viewed as consisting of abstract mathematical objects [7]. Plato’s assumptions about mathematics as abstract objects ruled out

causal properties linking them to their environment. Such mathematical “absolutism” has been under question. A number of mathematicians and philosophers have argued that mathematics learning and teaching acquires an alignment with its cultural practices through communicative and dialogic interactions [42-43]. Barwell [39] suggested that critical mathematics education can provide opportunities for students to gain insight into the role of mathematics in their lives and, in particular, to draw their attention to the climate change occurring within Earth’s ecosystem. Renert’s example of fractal geometry [7] also shows how we can connect mathematics education to the environment.

From our experience as educators and instructors when training prospective and in-service mathematics teachers, we found that there is general agreement among mathematics teachers and the majority of the general public that mathematics learning plays an essential role in helping the younger generations function effectively in a technological society, and, by so doing, contributes to their country’s economic growth. There is little doubt about this view. Yet, several mathematics researchers have raised concerns that there seems to be a trend for reducing mathematics to a tool for finding an answer. For instance, Gellert [38, p. 20] argued that “thinking of mathematics only as a powerful tool for solving economic problems is a truncated conception of mathematics-in-society”. In line with Gellert [38], we also believe that the call for sustainability in mathematics teaching and learning should include a critical questioning of the relationship between mathematics, technology and society. Gellert [38, p. 20] argued as follows:

If the call for a sustainable mathematics education includes a critical questioning of the relationship between mathematics, technology and society, and if it does not reduce mathematics to a remedy and an answer, then this mathematics education has the potential to break with many myths about mathematics and to reconcile the mathematics educator’s task with the desire to act in an ecologically sustainable way.

Skovsmose [40] contended that mathematics is not simply applied to social problems and issues. Its involvement through mathematical modeling and measurement is performative. This performative interpretation of mathematics highlights the importance of repositioning and deepening the conception of mathematics as an integral part of human social life [40-41]. Therefore, in order to integrate ESD into mathematics education, we will need to reimagine mathematics as a general human concern, to see it as providing an opportunity to learn critical thinking, problem-solving, and contextual understanding, as providing a lingua franca that

supports relationships between disparate communities and facilitates learning within other disciplines [44].

Arguably, if we are to take up sustainable development challenges, we cannot afford to continue with no or limited progress in ESD in mathematics education. Over the past few decades, the existing philosophy of mathematics has largely concerned itself with the philosophy of pure mathematics [41]. A philosophy of applied mathematics has yet to be incorporated into the standard philosophy of mathematics [40]. Yaro, Amoah, and Wagner [45] created authentic mathematics tasks to explore how peace, sustainability, and local issues can inform science and mathematics teaching. They highlighted the challenge of adjusting the teachers' conception of mathematics from an abstract, independent subject to a subject that brings about critical dialogue across disciplines and across issues.

Moving toward a mathematics education incorporating ESD requires a “mind shift,” and success lies largely in a fundamental reconfiguring and expanding of the traditional boundaries of the philosophy of mathematics. The social, economic, and environmental responsibilities of mathematics and the philosophical implications of mathematical applications also need to be recognized in mathematics teaching and learning. Some thoughts on ESD in mathematics teacher education are discussed below.

#### **4. Some thoughts on ESD in mathematics teacher education**

The widespread interest in sustainability has led to different terms and concepts being used to express the ideas of sustainability in students' learning. Researchers in a wide range of subject areas (including mathematics) understand the ideas of sustainability in the curriculum in a variety of different ways [6]. For instance, the terms “sustainable development” and “sustainability”, often appear interchangeably in the literature, and are sometimes even said to be synonymous [43]. In line with the definition given by UNESCO, we agree that ESD is a vision for the future: a key concept for thinking and acting about how we can balance environmental, societal, and economic issues as we move toward to an improved quality of life in the future. In this note, we define ESD perspectives in mathematics education as a way of thinking about the world that arises from transforming the way in which we understand and use mathematics.



As with any new initiative in education, the key to success lies in the willingness and capacity of teachers to engage with the initiative. Although, as we have mentioned earlier, there have been instances of good practice during the past few decades, researchers have largely neglected the issue of the actual activities and learning processes of ESD in mathematics teaching and learning. It is still unclear what is involved for teachers, as they are required to align their practice with ESD in the classrooms. As noted by Yaro et al. [45, p. 227], the teachers in their study were not always comfortable in supporting and implementing the mathematical tasks for peace and sustainability; they would discount or downplay those tasks, viewing them “as outside the realm of formal school sanctioned activities”. There is no doubt that teachers need support to develop their knowledge of current educational priorities such as ESD. Therefore, by reviewing and synthesizing the studies relating to sustainability as well as teacher education, we attempt to contribute to this particular area by sharing our thoughts on ESD in mathematics teacher education. In particular, we argue that in order for ESD to be routinely incorporated into mathematics education, more attention needs to be paid to the following three points.

First, as mentioned earlier, the quantity and quality of ESD provision in teacher education are “patchy” [5]. Consequently, the need for integrating ESD perspectives into teacher education programs should be widely recognized by mathematics educators in the field of teacher education. As mathematics educators, we have often been asked: “What’s maths got to do with ESD?” ESD in mathematics education calls for a new key concept for thinking and acting about how we can reorient mathematics toward environmentally and socially conscious thinking to engage the younger generations in ethical action for a better future. This new concept must push back the boundaries of the traditional philosophy of mathematics and accommodate the power of mathematical applications in guiding human lives and the social world [40-41]. The programs in initial teacher education and the continuous teacher professional development should respond to the call for ESD by providing opportunities for mathematics teachers to enact their understanding of ESD. It would not be possible to expect mathematics teachers to take action to address the ESD perspectives in their teaching and to evaluate critically the impact of their ESD practice on students’ learning of mathematics unless mathematics teacher education pays more attention to this area.

Second, one reason why ESD is rather uncommon in mathematics is that lessons that encourage or allow learners to engage in interdisciplinary activities and discussion about what they are learning are challenging to teach, even for experienced teachers [7& 43]. We are all aware that

the world is changing, and so is education. This change requires interdisciplinary learning to underpin and permeate education [1]. Mathematics education is no exception. In line with Skovsmose [41 & 46], from the perspective of critical mathematics education, we argue that mathematics teaching and learning should span a range from mathematical behavior, defined by “references to pure mathematics”, to a mathematics education that draws on “references to reality”. Skovsmose [41] drew on the application of number theory in cryptography to challenge Hardy’s view [47] of mathematics as a neutral subject that has value independent of possible applications. Furthermore, mathematics is performative: its critical nature can make available unexpected possibilities; can contribute to the understanding and interpretation of, and the reaction to, a critical situation; can serve particular political, business, or environmental interests; and can bring about an awareness of the devastating risks inherent in, or the remedial actions available to address, the situation under investigation.

Staata [9, p.7] investigated students’ interdisciplinary writings in a college algebra class (e.g., by using algebraic terminology of increasing and decreasing functions to explore a young immigrant’s story) and stressed that “when a student demonstrates integrated, interdisciplinary understanding, their writing [...] expresses an authentic and affecting sense of productive disposition towards mathematics”. Staata [9] also noted that mathematics teachers may feel uncomfortable teaching content that incorporates a field of study outside of mathematics. In addition, it has been reported that mathematics teachers have difficulties in exploring the connections between different areas of subject knowledge and disciplines and ways of approaching interdisciplinary work [45]. Ongoing research on ESD in mathematics teacher education needs to be conducted in order to provide support for teachers to develop their understanding of how to develop realistic and coherent contexts for learning across disciplines and across issues.

Third, equity and social justice are core components of ESD [48]. Within teacher education, the development of a professional knowledge that extends beyond the classroom is crucial if teachers are to engage with the complexities of the communities in which they teach. As noted by Salter et al. [49, p. 94], teacher education should promote “the status of wider professional experiences and the future of such experiences within teacher education programs”. In line with Salter et al. [49], we also argue that in addition to teaching the subject, teachers also have responsibilities for providing their students with opportunities to engage in learning about, and

participating in, wider social, ecological, and economic matters. It is important to engage students in real world issues with the aim of enhancing their learning experiences and outcomes.

The overarching goal of ESD is to integrate the values of sustainable development into all aspects of education with a view to creating more sustainable and fairer societies [48]. Consequently, when responding to the call for ESD in mathematics education, the fulfillment of teachers' wider professional responsibilities should be recognized and emphasized as one important element in mathematics teacher education, along with the development of teachers' subject and pedagogical knowledge. Without an awareness of teachers' wider professional responsibilities, teachers may encounter difficulties in valuing and promoting the inclusion of individuals with different ages, disabilities, ethnicities, genders, races, religions, and sexual identities in the classroom, the school, the community, and the wider society.

## **5. Conclusion**

ESD in mathematics education is complex. It continues to pose challenges for many teachers, especially mathematics teachers, as mathematics teaching has often been criticized for being teacher-centered and content-oriented [6]. Over the past two decades, research has barely scratched the surface of the fundamental questions facing mathematics teaching and learning in the area of ESD. How can mathematics education help learners of all ages to respond to sustainable development challenges, to lead healthy and fulfilled lives, to nurture sustainable livelihoods, and to achieve human fulfillment for all? Our note explores these questions but does not go far enough to answer them. Yet, in this note, we have attempted to contribute to this particular area by challenging the existing philosophy of mathematics and by calling attention to the need for ESD in mathematics education. Some thoughts on ESD in mathematics teacher education are also presented with the aim of opening up possibilities for meaningful discussions and actions toward ESD in mathematics education.

We hope, through our call in this note, that more researchers will be stimulated to revisit the integration of ESD into mathematics education, and, consequently, to conceptualize how, and where, efforts should be focused in order to integrate aspects of ESD with students' mathematics learning in schools. More research into the integration of ESD into mathematics education will no doubt contribute further to an understanding of this highly complex and demanding area of education. Doing so would also require deep reflection on the part of

researchers and teachers as to how mathematics teaching and learning may incorporate the ESD perspective in actual mathematics classrooms.

### **Disclosure statement**

The authors certify that they have no affiliations with/or involvement in any organization or entity with any financial interest, or non-financial interest in the subject matter or materials discussed in this manuscript.

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