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Whither Job Destruction?  
Unemployment, Job Flows and Hours in a New Keynesian Model.

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Abstract
Labour markets play a key role in business cycle analysis. Although a focal point of research on unemployment over the past decade, endogenous job destruction has recently fallen into disfavour, since its introduction leads to a positively sloped Beveridge curve. We show that introducing variation in hours per worker - a second margin for labour input adjustment - in combination with endogenous job destruction generates a negatively sloped Beveridge curve, a data consistent correlation structure for job flows and captures many aspects of the cyclical behaviour of hours per worker. This improved performance is robust to wage rigidity (which raises the variability of unemployment and labour market tightness) and a wide range of empirically plausible labour supply elasticities - but not completely inelastic labour supply implicit in much of the literature on labour market search.

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1 Introduction.

Unemployment is a fundamental concern for individuals and policy makers. Fluctuations in unemployment provide a key motivation for business cycle analysis. Despite this many models of cyclical phenomena do not even consider unemployment or, importantly, frictions associated with finding employment once unemployed - see, inter alia, Gali (2003), Woodford (2003). The equilibrium labour market search framework with matching frictions, expounded in Pissarides (2000), is a natural framework for thinking about the properties of unemployment and other labour market variables. Yet research using this approach often avoids an explicit treatment of variation in hours per worker, which is an important component of overall variation in labour input at business cycle frequencies, Cho and Cooley (1994). Shimer (2005) highlights a series of anomalies arising in the canonical equilibrium unemployment model with job destruction associated with capturing the variability of unemployment and vacancies and the correlation between them.\footnote{Shimer (2005) proposes wage rigidity and exogenous job destruction to circumvent these problems.} Using a New Keynesian set up to impose the discipline of general equilibrium on our analysis, we show that introducing a second margin - hours per worker - for labour input adjustment in a model with labour market search, matching frictions and endogenous job destruction, alters the use of the extensive margin in response to shocks and makes it possible to capture key properties of the relationships between unemployment and vacancies, hours and employment and job creation and job destruction, without distorting the behaviour of other macroeconomic aggregates at business cycle frequencies. Our results appear robust both to the introduction of varying degrees of wage rigidity and across a wide range of plausible values of labour supply elasticity.

For much of the last decade, an important strand of the literature has considered how to endogenise gross job flows in an equilibrium labour market search framework, Mortensen and Pissarides (1994, 1999), and examined the consequences of this reformulation of the basic model for labour market and macroeconomic issues, Mortensen and Pissarides (1999), Hall (1999), Den Haan et al. (2000). These developments were a response to extensive empirical evidence on the
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The (cyclical) properties of US gross job flows summarised in Davis et al. (1996). This highlighted the greater volatility of job destruction compared to job creation at business cycle frequencies and the countercyclical behaviour of job destruction. Implicit in these developments is the view that variation of inflows to unemployment - which depends on job destruction - is critical to the understanding of unemployment dynamics.

More recent research regards endogenous job destruction as, at best, an unhelpful distraction in the quest to understand unemployment. Using US data from 2001 Shimer (2005) finds that cyclical variation in unemployment is driven by outflows, not inflows: he finds that variation in job destruction is offset by variation in quits. Using an equilibrium labour market search model Shimer argues that shocks to job destruction produce a positive correlation between unemployment and vacancies - an upward-sloping Beveridge curve. Krause and Lubik (2006) show that this phenomenon also arises in a New Keynesian model (without variation in hours per worker).

The implications of Shimer’s study for the role of job destruction have attracted less attention than his finding that unemployment, vacancies and labour market tightness are insufficiently variable in the canonical model. Hall (2005) and Shimer (2005) attribute this absence of variability to the fact that vacancies are sensitive to wages, which, under period-by-period Nash bargaining are too flexible. They propose a revised canonical model of labour market search with matching frictions, wage rigidity and exogenous job destruction. Their proposal has aroused some controversy in the equilibrium labour market search literature, see, inter alia, Mortensen and Nagypal (2005). It has been more readily accepted within the business cycle literature. Using a New Keynesian framework Krause and Lubik (2006) show that under Nash-bargained wages, marginal cost is no longer directly related to the current real wage, so that real wage rigidity does not generate increased inflation persistence. Gertler and Trigari (2006) investigate the consequences of staggered Nash-bargaining of wages in a real business cycle environment.

However, Davis, Faberman and Haltiwanger (2006) question the generality of Shimer and Hall’s empirical evidence. Using US data spanning 1948-2005 they show that job destruction (and hence inflows to unemployment) are important in determining unemployment during severe recessions. This raises doubts as to the wisdom of deliberately suppressing endogenous variation.
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of job destruction when trying to understand the cyclical properties of labour market variables. The question we address is whether it is possible to construct plausible labour market behaviour at business cycle frequencies whilst retaining endogenous job destruction.

Fluctuation in hours per worker accounts for a substantial proportion of the variation in labour input at business cycle frequencies, Cho and Cooley (1994). Table (2) displays the cyclical behaviour of hours employment and total hours for US data over the period 1972-1994 - we use this period because gross job flows data are publicly available only for this period. The volatility of hours per worker relative to output is 0.57, and that of employment is 0.68 while the relative volatility of total hours worked is 1.06. The instantaneous correlation is 0.43, indicating that hours per worker and output move together at business cycle frequencies. Despite its role in labour input variation hours per worker is frequently omitted from models with labour market search, presumably on grounds of parsimony.

We consider a model with labour market search, endogenous job destruction and variable hours. To show that we are doing more than simply exploiting extra degrees of freedom provided by these features we demonstrate that the model can account for a broader range of phenomena than the unemployment-vacancy relationship considered by Shimer (2005), Hall (2005), Krause and Lubik (2006). We examine the observed variability of unemployment and vacancies, hours, wages and gross job flows as well as the correlation of hours with employment, vacancies with employment and of the correlation structure of gross job flows with each other and unemployment. Save for hours, each element of this diverse set of properties has been considered in at least one paper above. In one sense this provides a check on the extent to which previous results are robust to the proposed innovations in model structure, but we go beyond these studies by investigating the joint behaviour of all these variables simultaneously. To impose modelling discipline we embed the labour market in a New Keynesian dynamic general equilibrium framework. This differs from the equilibrium labour market search model used by Mortensen and Pissarides (1994), Shimer (2005) and Hall (2005) in allowing a role for consumption smoothing, capital accumulation, monetary shocks and nominal rigidities. This structure permits us to examine whether any improvement in labour market performance distorts the behaviour of other macroeconomic aggregates and also
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facilitates comparison with the benchmark alternative of a frictionless Walrasian labour market.\textsuperscript{2}

We show that introducing an intensive margin for the adjustment of labour input, enables the model to capture the interaction of hours and employment at business cycle frequencies. This is because the presence of the intensive margin alters the pattern of variation on the extensive margin(s). With the intensive margin suppressed, a rise in unemployment occurs through a rise in job creation and an even bigger rise in job destruction. This reflects the relatively low cost of altering the rate of job destruction (relative to that of job creation). It gives rise to a (counterfactual) positive correlation of job creation and job destruction. Since job creation tends also to be high when the number of vacancies open is high, the Beveridge curve also has a positive slope in the absence of hours variation. Once the intensive margin is introduced, existing firms can use the intensive rather than the extensive margin to adjust to shocks. This alters both rents available to existing matches and incentives to destroy existing matches and create new ones, and it attenuates the use of the extensive margin. So when hours per worker can vary job destruction is less volatile, and is negatively correlated with both unemployment and job creation. As a result, despite the presence of endogenous job destruction, hours variation produces a negatively sloped Beveridge curve.

One potential problem with this approach is that variation on the intensive margin may may substitute for variation on the extensive margin and attenuate fluctuations in unemployment - worsening the unemployment - vacancy - tightness volatility puzzle highlighted by Shimer (2005). This does happen. The variability of unemployment declines, from 60\% of that in the data to 33\% of that in the data as the intensive margin for labour input is opened up in a baseline New Keynesian model with endogenous job destruction and flexible wages. Yet in Shimer’s analysis - which uses a labour market, rather than business cycle model, only 10\% of the variability of US unemployment is captured. So the unemployment volatility problem is less quantitatively

\textsuperscript{2} This paper uses a New Keynesian framework with labour market frictions. In terms of its focus on the cyclical behaviour of labour market variables its closest antecedent is that by Krause and Lubik (2005) discussed above. Trigari (2005) and Walsh (2005) also develop New Keynesian models with labour market search and endogenous job destruction. Trigari (2005) introduces hours variation. However, like Walsh, her interest is primarily in the propagation mechanism for inflation and output. These authors demonstrate that this combination of real and nominal rigidities goes some way to addressing the output and inflation persistence problems observed in early New Keynesian models. These studies build on the results of Den Haan et al. (2000) who develop a real DGE a model with endogenous job destruction and demonstrate the role of consumption smoothing and capital accumulation in the output propagation mechanism.
significant in our New Keynesian setup than in the canonical equilibrium labour market search model. In any case, labour market tightness is unaffected because the decline in unemployment variability associated with the introduction of variation in hours per worker is offset by a fall in the correlation of unemployment with vacancies and a rise in the variability of vacancies.

In the light of the interest in the literature in wage rigidity as a solution to the unemployment volatility puzzle, Shimer (2005), Hall (2005), Krause and Lubik (2006), we examine whether our results are robust to and/or improved upon by wage rigidity. We confirm that with or without wage rigidity, the introduction of variation in hours per worker renders both the correlations between unemployment and vacancies and between job creation and job destruction negative. However, wage rigidity improves the ability of the model to capture the magnitudes of these statistics, as well as the variability of unemployment, vacancies, tightness and wages. Next, since a wide range of estimates of the elasticity of labour supply are available in the microeconometric literature, see Blundell and McCurdy (1999), Domeij and Floden (2005), we demonstrate that the effects of introducing variation in hours results are robust across a wide range of labour supply elasticities - but that a positively sloped Beveridge curve and job flows correlation arise when labour supply is very inelastic, as in Shimer (2005) and Krause and Lubik (2006).

The model is outlined in Section 2. Calibration and solution method are discussed in Section 3. Section 4 presents and discusses the results and assess the contribution of various features in accounting for US business cycles facts. Section 5 concludes.

2 Model

To study the interaction of cyclical variation in hours per worker and employment decisions (through variation in job creation and job destruction), and examine the robustness of results to wage rigidity, we embed the treatment of labour market search frictions in a New Keynesian framework, (we call this model NKS). Besides providing modelling discipline this allows us to contrast model performance under labour market frictions model with that under frictionless labour markets as in the canonical New Keynesian model, (hereafter NK).

There are 4 types of agent in the NKS economy: intermediate good producers, final goods
producers, households and a government. Production of the intermediate good requires capital and labour input. Labour input can be varied on both extensive and intensive margins. The strength of variation on the intensive margin can be 'controlled' by altering the elasticity of labour supply (preferences over leisure). Frictions in the formation of new matches are captured by an aggregate matching function - where the probability of a firm filling a vacancy, and the probability of an unemployed worker finding a job depend on the relative numbers of these two types. The number of vacancies is determined by a free entry condition - which drives the expected value of opening a new vacancy to zero. The number of vacancies is sensitive to wages. In the NKS model we assume wages are determined by Nash Bargaining each period. In later sections we introduce the possibility that wages display 'rigidity'. The flow into unemployment arises through destruction of existing matches. This rate is determined endogenously.

Otherwise the NKS economy is as a standard New Keynesian economy. Households derive utility from leisure, final goods consumption and holding (real) money balances. They may supply hours of work to intermediate good producers. They can save by accumulating capital which they rent to intermediate good producers and they purchase a basket of final goods from final good producers. Final good producers are monopolistically competitive. They each costlessly produce a differentiated final good by using only the homogeneous intermediate good and set the price of their product intermittently according to a Calvo price adjustment rule. Intermediate good producers are price takers in product and factor markets. The government issues money, collects seigniorage revenues and rebates these to households. It undertakes no other function.

With this basic structure in mind we discuss in turn the specification of goods and labour markets, the decision problem of households, our assumptions about the actions of the government and finally the equilibrium characterisation of the economy.

2.1 Goods and Labour Markets

2.1.1 The Intermediate sector

Production Production of intermediate goods takes place in matched firm-worker pairs - or, for notational ease, matches. Each match consists of one worker and one firm, who together
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engage in production until the employment relationship is terminated. By assumption, both firms and workers are restricted to a single employment relationship at any given time. Matches are subject to aggregate productivity and idiosyncratic cost shocks, \( Z_t \) and \( X_t \) respectively, each with unit mean. Following Den Haan et al. (2000), Trigari (2006), assume that idiosyncratic cost disturbances are serially uncorrelated. Date \( t \) production occurs after realisation of the date \( t \) shocks. A match facing a high realisation of \( X \) at date \( t \), greater than some threshold value \( \bar{X}_t \), may decide to terminate the matching relationship - see below. At date \( t \) an ongoing match - facing idiosyncratic shock \( X_t < \bar{X}_t \) can combine capital, \( \hat{K} (X_t) \) and hours of labour input, \( H (X_t) \), to produce

\[
Y^I (X_t) = AZ_t \hat{K} (X_t)^\alpha H (X_t)^{1-\alpha} + F - X_t
\]

units of intermediate good.\(^3\) The parameter \( \alpha \) represents the elasticity of match output with respect to capital input and \( A \) and \( F \) are positive constants. Matches are price takers and sell their homogeneous intermediate output at (nominal) price \( P_t^I \). The formal separation of the job-destruction and price-setting decision problems is maintained for tractability, but is consistent with the view that prices are not set at the level of an individual match.

Suppose that the match specific capital stock, \( \hat{K} (X_t) \) is chosen to maximise current profits of the match:

\[
\max_{\hat{K}} \left\{ \frac{AZ_t \hat{K} (X_t)^\alpha H (X_t)^{1-\alpha} + F - X_t}{\mu_t} - \frac{R^K \hat{K} (X_t)}{P_t} - \frac{W (X_t) H (X_t)}{P_t} \right\}.
\]

where \( \mu_t = \frac{P_P}{P_I} \) is the markup of the index of final goods prices over the price of the intermediate good (the reciprocal of marginal cost), \( R^K_t \) is the (nominal) rental rate on capital (user cost of capital), \( W (X_t) \) is the match specific (nominal) wage. The first order condition for this problem is

\[
\hat{K} (X_t) = \left[ \frac{\alpha AZ_t}{\mu_t R^K_t / P_t} \right]^{\frac{1}{1-\alpha}} H (X_t) \tag{1}
\]

The optimal choice of capital-hours ratio \( \frac{\hat{K} (X_t)}{H (X_t)} \) depends only on aggregate conditions, and is decreasing in the markup and the real return on capital and increasing in aggregate productivity.

\(^3\) An additive idiosyncratic shock avoids wide variation of hours across matches, Cooley and Quadrini (1999).
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Using (1) current profits are

$$\Pi^t(X_t) \equiv (1-\alpha) \left[ \frac{AZ_t}{\mu_t} \right]^{1-\alpha} \left[ \frac{\alpha}{R^K/P_t} \right]^{\alpha} H(X_t) + \frac{F - X_t}{\mu_t} - \frac{W(X_t) H(X_t)}{P_t} \right]$$  \hspace{1cm} (2)

**Value Functions**  Next we describe the value functions for firms’ and workers’ decision problems.

Let $h^t = \{h_0, ..., h_t\}$ denote the history of events up to date $t$, where $h_t$ is the event realisation at date $t$. The date 0 probability of observing $h^t$ is given by $d_t$. The initial state $h^0$ is given so that $d_0(h^0) = 1$. Henceforth, in order to simplify the notation, define the operator $E_t[.] \equiv \sum_{h_{t+1}} d(h_{t+1}|h^t)$ as the mathematical expectation over all possible states of nature conditional on $h^t$.

In equation (3) $V^U_t$, the date $t$ value of unemployment, expressed in final goods, comprises the consumption value of utility from search, the discounted present value of ongoing unemployment next period, $V^U_{t+1}$, and the difference between the value of employment, $V^W(X)$, and that of unemployment in the event that the worker matches this period (with probability $\kappa^U_t$) and the match survives to produce next period (with probability $(1 - \rho^x) F(X_{t+1})$):

$$V^U_t = \frac{(1-\phi)^{1-\phi}}{1-\phi} \frac{1}{\Lambda_t} + \beta E_t \left[ \frac{\Lambda_{t+1}}{\Lambda_t} \left[ V^U_{t+1} + \kappa^U_t (1 - \rho^x) \int_{X_{t+1}}^{X_t} [V^W(X) - V^U_{t+1}] dF(X) \right] \right]. \hspace{1cm} (3)$$

Matching and production occur simultaneously, so that a match which is formed in period $t$ cannot produce until period $t+1$, after aggregate and idiosyncratic shocks have been realised. As a result a new match survives with probability $(1 - \rho^x) F(X_{t+1})$.

Let $V^W(X_t)$ denote the date $t$ value, expressed in terms of consumption goods, to a worker of employment in an ongoing match with idiosyncratic cost shock $X_t$.

$$V^W(X_t) = \frac{W(X_t) H(X_t)}{P_t} + \kappa^H \frac{1 - H(X_t))^{1-\phi}}{1-\phi} \frac{1}{\Lambda_t}$$

$$+ \beta E_t \left[ \frac{\Lambda_{t+1}}{\Lambda_t} \left[ V^U_{t+1} + (1 - \rho^x) \int_{X_{t+1}}^{X_t} [V^W(X) - V^U_{t+1}] dF(X) \right] \right]. \hspace{1cm} (4)$$

The worker supplies $H(X_t)$ hours of labour to the firm for real hourly wage $\frac{W(X_t)}{P_t}$. Both wage and hours are outcomes of a bargaining process - see below. Hours worked generates income, but hours spent in the workplace reduce utility. These concerns are captured in the first two terms in (4). The remainder of the date $t$ value to an employed worker from the ongoing match is the
The free entry condition on vacancies drives the value of a vacancy to zero, \( V_t^{V} = 0 \), \( \forall t \), so the Bellman equations for \( V^J(X_t) \), and \( V_t^{V} \) become

\[
V^J(X_t) = \Pi^J(X_t) + (1 - \rho^s) \beta E_t \left[ \frac{\Lambda_{t+1}}{\Lambda_t} \int_{X_{t+1}}^{X_t} V^J(X) dF(X) \right] \tag{5}
\]

\[
\kappa = \kappa_t^V (1 - \rho^s) \beta E_t \left[ \frac{\Lambda_{t+1}}{\Lambda_t} \int_{X_{t+1}}^{X_t} V^J(X) dF(X) \right]. \tag{6}
\]

Moreover, using (5), we can re-write (6) as a Bellman equation for \( \kappa_t^V \):

\[
\frac{\kappa}{\kappa_t^V} = \beta (1 - \rho^s) E_t \left[ \frac{\Lambda_{t+1}}{\Lambda_t} \int_{X_{t+1}}^{X_t} \left( \Pi^J(X) + \frac{\kappa}{\kappa_t^V} \right) dF(X) \right]. \tag{7}
\]
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Bargaining: Hours and Wages As in the canonical equilibrium unemployment model, Mortensen and Pissarides (1999), we assume that for each match engaged in production, the firm and worker adopt Nash bargaining over hours worked and the hourly wage. We discuss how to model wage rigidity in Section (4.2). The division of the match surplus, 

\[ S(X_t) = V^W(X_t) - V^U_t + V^J(X_t) - V^V_t = V^W(X_t) - V^U_t + V^J(X_t), \] 

is determined on a period by period basis as:

\[
\max_{W(X_t), H(X_t)} [V^W(X_t) - V^U_t]^\eta [V^J(X_t) - V^V_{t+1}]^{1-\eta}.
\]

The first order conditions for hours and wages respectively are

\[
\eta V^J(X_t) \left[ \frac{W(X_t)}{P_t} - \kappa_H \frac{(1-H(X_t))^{-\varphi}}{\Lambda_t} \right] = - \left\{ (1-\eta) \frac{V^W(X_t) - V^U_t}{(1-H(X_t))^{-\varphi}} \right\} - \left[ (1-\alpha) \frac{AZ_t}{\mu_t} \right]^{\frac{1}{1-\varphi}} \frac{\kappa}{\Lambda_t} \frac{\alpha}{R_t^{1-\varphi} / P_t} \frac{1-\psi}{1-\varphi} - \frac{W(X_t)}{P_t},
\]

\[
\eta V^J(X_t) = (1-\eta) \left( V^W(X_t) - V^U_t \right).
\]

Optimal hours worked are thus

\[
\kappa_H \frac{(1-H(X_t))^{-\varphi}}{\Lambda_t} = \kappa_H \frac{(1-H_t)^{-\varphi}}{\Lambda_t} = (1-\alpha) \left[ \frac{AZ_t}{\mu_t} \right]^{\frac{1}{1-\varphi}} \frac{\alpha}{R_t^{1-\varphi} / P_t} \frac{1-\varphi}{1-\varphi} \quad \forall X_t \leq \bar{X}_t.
\]

Equation (11) says that, under Nash bargaining, the marginal rate of substitution between consumption and hours worked is equal to the marginal product of labour. Hours per worker in ongoing matches are decreasing in the rental rate on capital, and the markup, but increasing in aggregate productivity. Variation in hours per worker is decreasing in \( \varphi \), so choice of \( \psi \) can be used to shut down the intensive margin in our experiments. Hours per worker are independent of the match specific shock: \( H(X_t) = H_t \). From (1) capital is also independent of the match specific shock: \( \dot{K}(X_t) = \bar{K}_t \). Note also that this outcome is efficient.

Using equations (3) and (4) we write

\[
V^W(X_t) - V^U_t = \frac{W(X_t) H_t}{P_t} + \kappa_H \frac{(1-H_t)^{-\varphi}}{1-\varphi} \frac{1}{\Lambda_t} - \frac{(1-c)^{1-\varphi}}{1-\varphi} \frac{1}{\Lambda_t} + (1-\kappa^U_t) (1-\varphi) E_t \left[ \frac{\Lambda_{t+1}}{\Lambda_t} \int_{X_{t+1}} [V^W(X) - V^V_{t+1}] dF(X) \right].
\]

Using (10) and (6) it follows that

\[
V^W(X_t) - V^U_t = \frac{W(X_t) H_t}{P_t} + \kappa_H \frac{(1-H_t)^{-\varphi}}{1-\varphi} \frac{1}{\Lambda_t} - \frac{(1-c)^{1-\varphi}}{1-\varphi} \frac{1}{\Lambda_t} + \frac{\eta}{1-\eta} \left( 1-\kappa^U_t \right) \frac{\kappa}{\kappa^W} \frac{1-\mu_t}{1-\mu_t}.
\]
Lastly, combining (5) and (6)

\[ V^J(X_t) = (1 - \alpha) \left[ \frac{AZ_t}{\mu_t} \right] \frac{1}{\pi} \left[ \frac{\alpha}{R_t^P / P_t} \right] \frac{1}{\pi} H_t + \frac{\mathcal{F} - X_t}{\mu_t} - \frac{W(X_t) H_t}{P_t} + \frac{\kappa}{\kappa_t^1} \]

So the optimal wage for a match with idiosyncratic cost realisation \(X_t\) becomes

\[ \frac{W(X_t) H_t}{P_t} = \eta \left[ (1 - \alpha) \left[ \frac{AZ_t}{\mu_t} \right] \frac{1}{\pi} \left[ \frac{\alpha}{R_t^P / P_t} \right] \frac{1}{\pi} H_t + \frac{\mathcal{F} - X_t}{\mu_t} + \frac{\kappa}{\kappa_t^1} \right] + (1 - \eta) \left[ \frac{(1 - e)^{1 - \varphi}}{1 - \varphi} \frac{1}{\lambda_t} - \kappa_H \frac{(1 - H_t)^{1 - \varphi}}{1 - \varphi} \frac{1}{\lambda_t} \right]. \]

Define aggregate labour income as \(\frac{W_t H_t}{P_t} = H_t \int X_t W(X_t) dF(X)\). Then

\[ \frac{W_t H_t}{P_t} = \left\{ \eta \left[ (1 - \alpha) \left[ \frac{AZ_t}{\mu_t} \right] \frac{1}{\pi} \left[ \frac{\alpha}{R_t^P / P_t} \right] \frac{1}{\pi} H_t + \frac{\mathcal{F} - X_t}{\mu_t} + \frac{\kappa}{\kappa_t^1} \right] + (1 - \eta) \left[ \frac{(1 - e)^{1 - \varphi}}{1 - \varphi} \frac{1}{\lambda_t} - \kappa_H \frac{(1 - H_t)^{1 - \varphi}}{1 - \varphi} \frac{1}{\lambda_t} \right] \right\} F(X_t). \]

(12)

The first term within the first square brackets on the right hand side of equation (12) represents the workers’ share of the market value of production, the second term reflects the market value of idiosyncratic costs (relative to steady state), and the third term reflects the impact of labour market tightness. The remaining term reflects the worker’s reservation wage.

**Separation** For values of the idiosyncratic cost shock above a certain threshold level, \(\bar{X}_t\), separation occurs. The condition \(S(\bar{X}_t) = 0\), pins down this threshold value of the match specific shock. Combining (8) and (10), \(V^J(X_t) = (1 - \eta) S(X_t)\). So \(\bar{X}_t\) is determined by the condition

\[ V^J(\bar{X}_t) = 0 : \]

\[ (1 - \alpha) \left[ \frac{AZ_t}{\mu_t} \right] \frac{1}{\pi} \left[ \frac{\alpha}{R_t^P / P_t} \right] \frac{1}{\pi} H_t + \frac{\mathcal{F} - \bar{X}_t}{\mu_t} - \frac{W(\bar{X}_t) H_t}{P_t} + \frac{\kappa}{\kappa_t^1} = 0. \]

This equation indicates that a job is destroyed when costs are sufficiently high that the value of production net of idiosyncratic cost shock and wage equals the (expected) cost of posting a vacancy. Substituting for the match specific wage, the threshold value \(\bar{X}_t\) is determined by

\[ (1 - \eta) \left[ (1 - \alpha) \left[ \frac{AZ_t}{\mu_t} \right] \frac{1}{\pi} \left[ \frac{\alpha}{R_t^P / P_t} \right] \frac{1}{\pi} H_t + \frac{\mathcal{F} - \bar{X}_t}{\mu_t} - \frac{W(\bar{X}_t) H_t}{P_t} + \frac{\kappa}{\kappa_t^1} \right] - \eta \kappa_{\kappa_t^1} + \frac{\kappa}{\kappa_t^1} = 0. \]

(13)

### 2.1.2 Labour Market Flows

The match specific production, bargaining and separation decisions described above depend on the probability that unemployed workers find jobs and the probability that vacancies are filled.
Here we discuss these probabilities and the associated labour market flows.

Define the number of matches at the beginning of period $t$ as $N_t \in [0, 1]$. We allow some job destruction in the form of quits which are taken as exogenous and independent of the match-specific productivity. We capture this by allowing a fraction, $\rho^x$, of matches to separate prior to the realisation of period $t$ (productivity) shocks. Subsequently, idiosyncratic productivity disturbances are realised, and a match may choose to break up if the value of the match surplus is negative. Endogenous separation thus occurs with probability $\rho^n (\bar{X}_t) = 1 - \int \bar{X} dF (X)$, where $dF (\cdot)$ is the probability density function over $X$. The overall separation rate in period $t$ is

$$\rho_t = \rho^x + (1 - \rho^x) \left(1 - F (\bar{X}_t)\right).$$

(14)

Next consider the matching frictions. We model this rigidity using an aggregate matching function. Matching occurs at the same time as production. We assume, following Pissarides (2000), that there is a continuum of potential firms, with infinite mass, and a continuum of workers of unit mass. Unmatched firms choose whether or not to post a vacancy and incur a cost $\kappa$ per period. Free entry of unmatched firms determines the size of the vacancy pool. Define the mass of firms posting vacancies in period $t$ as $V_t$. Let the mass of searchers, unmatched workers, be $U_t$. All unmatched workers may enter the matching market in period $t$ - even if their match dissolved at the start of period $t$, so

$$U_t = 1 - (1 - \rho_t) N_t.$$

(15)

New matches in date $t$ begin production in date $t + 1$, while unmatched workers remain in the worker matching pool. The flow of successful matches created in period $t$ is given by the constant returns matching function

$$\mathcal{M}_t = \mathcal{M} (e \cdot U_t)^\gamma V_t^{1 - \gamma}.$$

(16)

where $\gamma, e \in (0, 1)$ and $\mathcal{M} > 0$. The parameter $e$ represents the efficiency with which unemployed workers engage in search. The number of employed workers at the start of period $t + 1$ is

$$N_{t+1} = (1 - \rho_t) N_t + \mathcal{M}_t.$$

(17)
Denote the probability that a vacancy is filled in date $t$ as
\[ \kappa_t^V = \frac{M_t}{V_t}, \] (18)
and the probability that an unemployed worker enters employment in period $t$ as
\[ \kappa_t^U = \frac{M_t}{U_t}. \] (19)
Gross job destruction is the employment relationships that separate less exogenous separations that rematch within period
\[ JD_t = \rho_t N_t - \kappa_t^V \rho^x N_t = \rho_t - \kappa_t^V \rho^x. \] (20)
Gross job creation is the flow of new matches (as a fraction of existing employment) less matches due to firms filling vacancies that resulted from exogenous separations
\[ JC_t = \frac{M_t - \kappa_t^V \rho^x N_t}{N_t} = \frac{M_t}{N_t} - \kappa_t^V \rho^x. \] (21)

### 2.1.3 Final Goods Sector

Assume that there is a continuum of final goods producers, with unit mass. Final good firm $z$ acquires the wholesale good at price $P_t^I$ and costlessly transforms it into the divisible final good $z$ which is then sold directly to households at price $p_t(z)$. Define $P_t = \left( \int_0^1 p_t(z)^{1-\varepsilon} \, dz \right)^\frac{1}{1-\varepsilon}$ as the utility based price index associated with both the investment and consumption composites. The market for final goods is characterised by monopolistic competition - $\varepsilon$ represents the elasticity of substitution across varieties of final good. Aggregate demand for the final good $z$ in period $t$ is
\[ y_t(z) = c_t(z) + i_t(z), \]
where $c_t(z)$ represents consumption demand for final good $z$ output and $i_t(z)$ represents gross investment demand for final good $z$ output. The optimal choices of consumption and investment expenditures on final good $z$ are then
\[ c_t(z) = \left( \frac{p_t(z)}{P_t} \right)^{-\varepsilon} C_t, \quad i_t(z) = \left( \frac{p_t(z)}{P_t} \right)^{-\varepsilon} I_t, \]
where aggregate consumption, $C_t = \left( \int_0^1 c_t(z)^{\frac{\varepsilon}{1-\varepsilon}} \, dz \right)^\frac{1}{\varepsilon}$, aggregate investment, $I_t = \left( \int_0^1 i_t(z)^{\frac{\varepsilon}{1-\varepsilon}} \, dz \right)^\frac{1}{\varepsilon}$ and aggregate final good output $Y_t = \left( \int_0^1 y_t(z)^{\frac{\varepsilon}{1-\varepsilon}} \, dz \right)^\frac{1}{\varepsilon}$ are composite indices of final goods.
Suppose that final goods prices exhibit nominal rigidities which follow a Calvo style adjustment scheme. Assume that with probability \((1 - \omega)\) a final good producer can set the price of its output in period \(t\). Let this probability be independent of when the firm last adjusted price. Then the average price for final goods producers who do not adjust their price is simply \(P_{t-1}\). Suppose that the average price set by firms who do adjust price is \(p_t^*\).

Since pure forward looking price adjustment schemes seem not to account adequately for observed inflation dynamics, and to facilitate comparability with other studies, we employ a hybrid scheme (following Gali and Gertler (1999)). Assume that a fraction \((1 - \tau)\) of the final goods producers are forward looking and set prices optimally (to maximise expected discounted profits given the probability of future adjustment).\(^4\) Define the price set by forward looking producer \(z\) at date \(t\) as \(p_t^*(z)\). Since all forward looking firms setting price at date \(t\) face the same expected future demand and cost conditions they choose the same price, so \(p_t^*(z) = p_t^*\), where

\[
p_t^* = \frac{\varepsilon}{1 - \varepsilon} E_t \sum_{s=0}^{\infty} \omega^s \beta^s \frac{\lambda_{t+s}}{\lambda_t} \left( \frac{p_t^*}{p_{t+s}^*} \right)^{1 - \varepsilon} Y_{t+s} P_{t+s}^I.
\]

(22)

The remaining fraction \(\tau\) of firms setting price in period \(t\) are assumed to set prices equal to the average of the last period reset prices, corrected for inflation, \(\pi_{t-1}\):

\[
p_t^b = \bar{p}_{t-1} \pi_{t-1}.
\]

(23)

The average price set in period \(t\) is \(p_t = \left[ (1 - \tau) (p_t^*)^{1 - \varepsilon} + \tau (p_{t-1}^b)^{1 - \varepsilon} \right]^{\frac{1}{1 - \varepsilon}}\), and the aggregate retail price index evolves according to

\[
P_t^{1 - \varepsilon} = (1 - \omega) (\bar{p}_t)^{1 - \varepsilon} + \omega P_{t-1}^{1 - \varepsilon}.
\]

(24)

2.2 Households

Assume that the economy contains a continuum of households of unit mass. Households own all retail and wholesale firms. They can save by accumulating capital - which they rent to wholesale firms - by holding (nominal) contingent claims, or non-interest bearing money balances. To avoid the distributional issues that arise because some workers are unmatched, we assume that complete asset markets allow workers to insure themselves against (cross-section) variation in the marginal

\(^4\) This structure is used by Trigari (2003), Walsh (2005) uses Christiano et al. (2005) approach.
utility of consumption. Under this simplifying assumption, household behaviour can be analysed in terms of a representative consumer. Assume that the representative consumer derives utility from consumption, leisure, and services provided by holding real money balances; that the instantaneous utility function exhibits habit persistence in consumption. Let the household chooses consumption and money balances to maximise expected utility over her lifetime

\[
E^τ \left[ \sum_{t=0}^{τ} \beta^t \left( \frac{(C_t^h - \kappa C_{t-1}^h)}{1-\phi} + \frac{\kappa M_t^h}{1-\xi} \left( \frac{M_t^h}{P_t} \right)^{1-\xi} + U_t \left( 1 - e \right)^{1-\varphi} - (1 - U_t) \frac{\kappa H_t}{1-\varphi} (1 - H_t)^{1-\varphi} \right) \right].
\]

Here \( \beta \), the discount factor, \( \kappa_C \), the strength of the habit, \( \kappa_M \), the relative weight on money balances in the utility function, \( \kappa_H \), the relative weight on leisure in the utility function, \( \phi \), the inverse of the elasticity of intertemporal substitution, \( \varphi \), the elasticity of substitution of hours per worker and \( \xi \) are all positive constants. Variables superscripted \( h \) are elements of the household decision problem. When employed, hours of work are determined through bargaining (with wholesale firms) rather than being unilaterally determined by the individual household.

The representative consumer maximises expected lifetime utility subject to the following sequence of constraints

\[
P_t C_t^h + P_t I_t^h + E_t \left( v_{t,t+1} D_{t+1}^h \right) + M_t^h = R^K_t K_{t+1}^h + I_t^h + D_t^h + M_{t-1}^h + P_t T_t^h,
\]

\[
K_{t+1}^h = (1 - \delta) K_t^h + I_t^h + \frac{\chi (I_t^h - \delta K_t^h)^2}{2 K_t^h}, \quad t \geq 0.(25)
\]

Here \( D_{t+1}^h \) represents nominal payoff in period \( t+1 \) of the portfolio held at the end of period \( t \), \( v_{t,t+1} \), is the stochastic discount factor for one period ahead nominal payoffs relevant to the representative household. \( M_t^h \) represents holdings of nominal money balances at the end of period \( t \), \( P_t T_t^h \) represents a lump-sum nominal transfer from households due to rebated seigniorage revenues. \( C_t^h \) and \( I_t^h \) are household consumption expenditures and gross fixed capital formation expenditures respectively, \( K_{t+1}^h \) represents capital stock carried held at the end of period \( t \). The positive constants \( \delta \) and \( \chi \) capture (geometric) capital depreciation and the magnitude of (quadratic) adjustment costs (on net investment). \( R^K_t \) is the rental return on capital. \( T_t^h \) is the household’s nominal income (labour income, plus firms’ profits net of expenditures on vacancies).
The solution to the representative consumer’s problem is characterised by first-order conditions for bond holdings, $D^h_t$, consumption $C^h_t$, money balances $M^h_t$, investment $I^h_t$ and capital stock $K^h_{t+1}$. Define the gross return on a riskless asset paying off one unit of currency in date $t+1$ as

$$R^n_t = \frac{1}{E_t [v_{t,t+1}]}$$

where $E_t [v_{t,t+1}]$ is the price of that asset, and the date $t$ shadow value of capital at date $t+1$ as $Q_t$. The resulting first order conditions can be written as:

$$1 = \beta R^n_t E_t \left[ \frac{P_t}{P_{t+1}} \frac{\Lambda_{t+1}}{\Lambda_t} \right].$$

$$\Lambda_t = (C^h_t - \kappa C^h_{t-1})^{-\phi} + \beta E_t \left[ (C^h_{t+1} - \kappa C^h_t)^{-\phi} \right].$$

$$\kappa M \left( \frac{M^h_t}{P_t} \right)^{-\xi} - \Lambda_t = \beta E_t \left[ \Lambda_{t+1} \frac{P_{t+1}}{P_t} \right]$$

$$\Lambda_t = Q_t \left[ 1 + \frac{I^h_t - \delta K^h_t}{P_t} \right]$$

$$Q_t = \beta E_t \left[ \Lambda_{t+1} \frac{R^K_{t+1}}{P_{t+1}} + Q_{t+1} \left[ 1 - \delta - \frac{\chi}{2} \left( \frac{I^h_{t+1}}{K^h_{t+1}} \right)^2 \right] \right]$$

2.3 Monetary and Fiscal Policy and Exogenous Driving Processes

We set government spending to zero and assume that the government maintains a balanced budget by rebating seigniorage revenues to households in the form of lump-sum transfers. The government budget constraint is thus $P_t T_t = M_t - M_{t-1}$, where $M_t$ is the aggregate money stock. Monetary policy is specified by

$$M_t = M_{t-1} e^{\upsilon_t}$$

where $\upsilon_t$ evolve according to the AR(1) process

$$\upsilon_t = \rho_{\upsilon} \upsilon_{t-1} + \epsilon_{\upsilon,t}.$$
Whither Job Destruction?
Unemployment, Job Flows and Hours in a New Keynesian Model.

The logarithm of aggregate productivity also follows an AR(1) process:

$$\ln Z_t = \rho \ln Z_{t-1} + \varepsilon_{Z,t}$$  \hspace{1cm} (33)

where $\varepsilon_{\nu,t}$ and $\varepsilon_{Z,t}$ are independent mean zero processes.

2.4 Equilibrium

Equilibrium in the rental market for capital requires that

$$K_t = (1 - \rho x) N_t \int_0^{X_t} K_t dF(X) = (1 - \rho x) F(X_t^\rho) N_t K_t.$$  \hspace{1cm} (34)

Using (1) gives

$$K_t = (1 - \rho x) F(X_t^\rho) N_t \left[ \frac{\alpha AZ_t}{\mu R^K_t / P_t} \right]^{\frac{\alpha}{1-\alpha}} H_t.$$  \hspace{1cm} (35)

denotes aggregate intermediate output net of vacancy posting costs.\(^8\) Nominal intermediate good producers’ profit can be written as the sum of output net of vacancy posting costs, less aggregate wage payments and capital rental payments: $P_t \Pi^I_t = P_t Y_t - (1 - \rho x) N_t W_t H_t - R^K_t (1 - \rho x) N_t \int_0^{X_t} K_t dF(X)$. Using these insights and cancelling terms gives

$$\mathcal{I}_t = P_t Y_t - R^K_t K_t$$

In equilibrium, the household budget constraint reduces to the aggregate (final) goods market equilibrium condition

$$Y_t = C_t + I_t$$  \hspace{1cm} (36)

\(^8\) Note $Y_t^I = \int_0^\lambda y_0(z) dz$. Using the demand function for final good $y_0(z) = \left( \frac{w(z)}{y(z)} \right)^{-\epsilon} Y_t$, we have $Y_t^I = \int_0^\lambda \left( \frac{w(z)}{y(z)} \right)^{-\epsilon} Y_t dz = \left( \frac{\tilde{p}(z)}{P_t} \right)^\epsilon Y_t$, where $\tilde{p}_t = \int_0^\lambda p_t(z)^{-\epsilon} dz$, is an auxiliary price index.
Thus the system of equations governing equilibrium in the economy consists of the numbered equations (2), (7) and (11) - (36).

3 Calibration & Model Solution Method

We log-linearise the model about its (zero-inflation, zero growth) steady state and use impulse response analysis and dynamic simulations to tease out the dynamic structure of the economy. Model solution requires choice of several parameters governing steady state values of labour and goods market variables; nominal rigidity, and household preferences. We also specify the processes governing idiosyncratic productivity and money supply growth. These parameters are chosen to match properties of the US economy over the sample period. The parameter values are summarised in Table 1, Appendix A contains discussion of the rationale for and origins of these choices.

4 Results

4.1 Qualitative Response to Monetary and Productivity Shocks

To enhance our understanding of the mechanisms at work in the NKS economy and the behaviour of the general equilibrium system, we study the impulse responses to monetary and productivity shocks under the baseline calibration. These are illustrated in Figure (1).

Panels (a) - (e) show the NKS response to a 1% monetary growth innovation. First consider the behaviour of expenditures. With nominal rigidity in price-setting, output, consumption and investment increase in response to a monetary innovation - panel (a). Habit persistence penalises rapid changes in consumption and leads to a hump-shaped response in consumption, whereas the output and investment responses are front-loaded. This reflects the absence of microeconometric evidence supporting a habits based approach to investment. Long-run neutrality of money guarantees that steady state capital stock is unaffected by the monetary shock, and, despite ongoing depreciation, the sharp initial rise in investment is followed, after 6 quarters, by a period of disinvestment. With
nominal rigidities in price setting, a money growth expansion raises marginal cost (reduces the markup) and inflation. Although this markup effect is frontloaded, the backward-looking element in price-setting leads to a hump-shaped inflation response - panels (d) and (e).

Next consider labour market variables. In the absence of changes to exogenous productivity, the increase in production is associated with a rise in both employment and hours (as well as capital accumulation) panel (d) and (e). Increased employment is achieved through action on both extensive margins with a sharp rise in job creation and a sharp decline in job destruction - panel (c). The rise in job creation is accompanied by an increase in vacancy posting and triggered by high rents which accrue to existing matches and arise because matching frictions prevent instantaneous adjustment to innovations. The decline in unemployment and rise in vacancies raises labour market tightness and makes it relatively difficult to fill vacancies; so both vacancies and job creation rebound after their initial increases - panels (b) and (c). This subsequent lack of new job creation, combines with a near steady-state job-destruction rate to eliminate the initial rise in employment (after about 6 quarters) and leads to above steady-state unemployment for the remainder of the transient response. The rise in hours per worker and the increase in labour market tightness lead (through equation (12)), to a front-loaded rise in the real wage - panel (d).

Panels (f) - (j) of Figure (1) show the NKS response to a 1% productivity shock. Output, investment (after a brief decline) and consumption display a persistent hump-shaped response, with investment displaying the largest percentage deviation - panel (f). With nominal rigidities in price setting, the productivity shock generates a rise in the markup (decline in marginal cost) - panel (i) and disinflation - panel (j) - which is hump-shaped due to backward looking price setting behaviour. Since money is neutral in the long-run, cumulative inflation is zero.

There is a front-loaded decline in the real wage - panel (i). This is associated with an immediate decline in hours worked and initial decline in employment (and fall in labour market tightness) - panel (i). These features are consistent with Gali’s (1999) account of the impact of technology shocks under nominal rigidities. Since hours worked returns to steady state while capital and productivity are higher than in steady state, real wage rises above steady state for a period. The initial decline in employment below steady state is more than reversed after 4 quarters as new
matches continue to be formed to take advantage of temporarily high productivity and capital stock. A period of above steady state employment ensues, as existing matches continue to enjoy high levels of capital stock and aggregate productivity levels. In the immediate aftermath of the productivity shock, inauspicious employment prospects lead to a decline in vacancies - consistent with a decline in job creation. However, the initial rise in unemployment reduces labour market tightness and makes it relatively easy for firms to fill vacancies; so vacancies and job creation rebound after their initial declines to several percentage points above steady state panels (g) and (h). This ongoing job creation, combined with a rate of job-destruction that subsequently remains near the steady state level eliminates the initial unemployment (after about 4 quarters) allowing below steady state unemployment for the remainder of the transient response.

In summary, adjustment to shocks involves variation of hours and unemployment. The latter arises through variation in both job creation and job destruction. The balance of these reflect the relative costs of adjustment at each margin, we explore this issue in more detail below.

4.2 Business Cycle Moments

How do labour market frictions affect relative volatilities of labour market variables and other macroeconomic aggregates? What is the effect of (1) suppressing variation in hours worked and (2) imposing real wage rigidity on the slope of the Beveridge curve, the contemporaneous correlation of hours with employment, and job creation with job destruction? Evidence on these issues is collected in Table (2).

Column (2) of Table (2) records US Data, Data, corresponding to the sample period (1972:2-1993:4).9 Every other column of Table (2) corresponds to a particular model variant.10 These variants are chosen to illustrate the limit behaviour of the baseline model approximating to particular features that are common in the literature. Column (3), NKS, reports results for the baseline calibration. Column (4), NK, is a standard New Keynesian model with a frictionless Walrasian...

9 The length of this sample period is dictated by the availability of the job flows series.
10 All statistics (for model simulations and data) are computed from Hodrick Prescott detrended data, expressed as percentage deviations from steady state (or trend in the case of the data). The business cycle statistics are computed by averaging across 200 simulations. In conducting the simulations of NKS, the standard deviation of productivity shocks in the baseline model outlined in Section (2) is set to 1.1% in order to match the variability of (HP-detrended) quarterly GDP in US data, which is 1.61% over the sample period (1972:2 - 1993:4). This value of $\sigma_Z$ is used in all model variants, in order to be able to compare the relative strength of the amplitude and propagation mechanisms in each variant.
labour market.\textsuperscript{11} This model omits parameters $\rho$, $N$, $\kappa^V$, $\rho^x$, $\sigma_X$, $\gamma$ and $\eta$, relating to search, bargaining and separation, but is otherwise calibrated identically to NKS. In particular, total hours are identical (in steady state) for NK and NKS. Column (5), FixH, presents results for the case where we suppress variation of hours by adopting preferences that yield inelastic labour supply (while allowing flexible Nash-bargained wages). Column (6) RigidW presents results for the case where wage rigidity is imposed (while allowing variation in hours per worker). Column (7), FixWH, presents results for the case where variation in hours is suppressed and wage rigidity imposed. This last model corresponds most closely to the set up favoured by Shimer (2005), Hall (2005), Krause and Lubik (2006). For a given column in Table (2), the entry in the row labelled output indicates the variability of output in column X ($\neq \text{Data}$) relative to the variability of output in the US Data. The other entries in column X (except the final 3) correspond to the variability relative to that of output generated by model X. The adoption of identical parameterisation of the driving processes facilitates comparison across different models, as well as allowing comparison within models across variables. The Final three entries in each column are simple correlation statistics for labour market variables.

Table (2) here.

4.2.1 Comparison of Data, Frictional and Walrasian Models

Consider NKS, NK and Data columns of Table (2). NKS outperforms NK along several dimensions in matching the moments in Data. First, given $\sigma_Z$, it generates greater output variability than NK while capturing the relative variability of consumption and investment. Second NKS does a better job of capturing the variability of inflation than NK - where the reliance on variation in hours, and the direct link between hours, wages, and marginal costs, requires more extreme real wage variation, a more immediate rise in marginal costs in response to shocks and a greater front loading of inflation. Third, by decomposing labour input variation into changes along extensive and intensive margins, NKS reduces the variability of hours compared with NK. For NKS, employment variability is close to that in US Data. Yet hours are more variable in NKS than in
US Data, as are real wages - a consequence of the flexible wage assumption in NKS. Fourth, NK cannot begin to account for unemployment, gross job flows and the like. Unemployment, vacancies and labour market tightness exhibit less variability in NKS than US Data. Yet while Shimer (2005) finds that unemployment and vacancies generate only 10% of that observed in the data in a labour market equilibrium model with flexible wages, the fraction of the variability that can be generated in the fully articulated NKS macroeconomic model is around 30%. This suggests that the unemployment - vacancy - labour market tightness puzzle identified by Shimer, while important, is substantially less dramatic in a fully articulated macroeconomic model. These numbers are somewhat higher than Krause and Lubik (2006), which suggests that habit persistence in consumption may augment employment variability - as Krause and Lubik omit this feature.12 Fifth, the variability of job flows is of similar magnitude to that in the data, although job creation is overvolatile and job destruction is insufficiently volatile. Sixth, for the baseline parameterisation of NKS the correlations of unemployment with vacancies, of job creation with job destruction and of hours with employment have the same signs as in US Data, although none closely match the magnitudes observed in the data: the correlations of unemployment with vacancies and of hours with employment are too small while that of job creation with job destruction is too large.13

Overall, this evidence supports the prevalent view that inclusion of search, matching frictions and job destruction into a New Keynesian framework improves performance over a basic New Keynesian model with a frictionless Walrasian labour market. It also highlights the value of embedding labour market frictions in a more general macroeconomic framework. Firstly, with variation of hours on the intensive margin the counterfactual implications of endogenous job destruction are reversed; secondly the severity of the unemployment variability puzzle is reduced.

4.2.2 Comparison with Fixed Hours Worked Variant

The impulse responses for NKS show that firms use variation of hours to facilitate short-run adjustment to shocks. Mortensen and Pissarides (1999), Shimer (2005), Den Haan et al. (1999), Walsh (2005), Krause and Lubik (2006) implicitly suppress variation of hours per worker, by

12Capital accumulation reduces the variability of (un)employment, vacancies & labour market tightness, Holt (2006).
13Labour market tightness, $\theta = \frac{\sigma_v}{\sigma_u}$, so log deviation of tightness, unemployment, vacancies and the slope of the Beveridge curve are related by the identity: $\sigma_\theta = \sqrt{\sigma_u^2 + \sigma_v^2 - 2 \cdot \sigma_u \cdot \sigma_v \cdot \rho(\hat{u}, \hat{v})}$.
excluding an intensive margin for labour input. Since the models in those papers differ in many ways from the one we study, we examine how suppressing variation in hours per worker alters the business cycle moments from those obtained with the baseline NKS set up. To do this we create a variant of NKS in which hours variation is suppressed, call this FixH. FixH approximates complete suppression (omission) of hours variation by setting $\psi = 100$ - which corresponds to a labour supply elasticity of 0.02. The variability of hours in FixH is zero to two decimal places.

Several results emerge from this experiment. First, suppressing variation in hours results in increased variation on the extensive margin. With the variability of both employment and unemployment rising compared to NKS, the inability to adjust costs by altering hours worked clearly leads to greater use of the extensive margin. Note that employment displays higher variability than in US Data, although unemployment is less variable than in US Data. The variability of vacancies declines, relative to that in either NKS or US Data. This appears surprising, but also arises in Krause and Lubik (2006), who impose the suppressed hours feature that we are examining in this Section. Also labour market tightness remains at roughly the same value as in NKS, which suggests that the reduction in the variability of vacancies offsets the increase in the variability of unemployment.

Secondly, a positively sloped Beveridge curve, and positive correlation of job creation with job destruction result from suppressing variation in hours per worker (with endogenous job destruction). This is interesting. The Beveridge curve effect is consistent with Shimer’s view of the deleterious effect of endogenous job destruction, but our experiment shows that the result appears to be specific to an environment in which an intensive margin is omitted. Of course, hours per worker exhibit substantial variation in US Data at business cycle frequencies so the unrealistic positively sloped Beveridge curve arises only for an unreasonable simplification.

The positive correlation of job creation with job destruction once hours are suppressed hints at the mechanism underlying the upward sloping Beveridge curve. We examine this mechanism in greater detail in Section (4.3). The (increased) variation in employment can arise either through changes in the variability of job creation or that of job destruction, or both. Suppression of hours variation raises the absolute variability of gross job flows, particularly for job destruction.
However, as we discuss below, it is the correlation structure for job creation rather than job destruction that is distorted most by the suppression of hours per worker.

Turning to other macroeconomic aggregates. Suppressing hours variation forces labour input adjustment to occur on the extensive margin, yet this has little effect on the variability of output, as the increased variation in employment offsets the decline in hours variation. Equally, while labour market variables are affected in important ways, the presence or absence of the intensive margin has little effect on the relative variability of consumption or investment. Both labour market tightness and elasticity of labour supply affect the bargained wage in equation (12). The real wage is less volatile in FixH than in NKS, but remains over three times more volatile than US Data. There is little impact on inflation.14

4.2.3 Comparison with Wage Rigidity Variant

In both NKS and FixH the standard deviation of wages is too large, while the variability of unemployment and vacancies is too small. Shimer, (2006), Hall (2005) and Krause and Lubik (2006), show that wage rigidity can address these problems, and find that if job destruction is endogenous then the Beveridge curve is upward sloping. These studies implicitly suppress variation in hours per worker. Here we study the effect of introducing wage rigidity into the NKS model which allows hours per worker and job destruction to vary endogenously.

To study the effect of wage rigidity, we assume that the wage is a weighted average of the pure Nash-bargained wage (as derived above), and a wage norm. The wage norm chosen is \( \bar{W}_t = W_{t-1} \), which is independent of idiosyncratic shocks and exhibits dynamics of its own. This follows Krause and Lubik (2006). For a worker that is part of a match with idiosyncratic cost shock \( X_t \), idiosyncratic earnings are then

\[
\frac{W(X_t) H_t}{P_t} = \alpha^w \left[ \eta \left( 1 - \alpha \right) \left[ \frac{\Delta Z_t}{\mu_t} \right]^{\frac{1}{1-\alpha}} - \frac{\alpha}{R_t^e/P_t} \right]^{\frac{1}{\alpha}} H_t + \frac{\xi}{\mu_t} + \kappa \frac{K_t}{\mu_t} + \frac{1 - \eta}{1 - \varphi} \left[ \frac{1 - \varphi}{1 - \varphi} \Lambda_t - \kappa H_t \right] + (1 - \alpha^w) \frac{W_{t-1} H_t}{P_t},
\]

where \( \alpha^w \) parameterises the degree of wage rigidity present. If \( \alpha^w = 0 \) then only aggregate factors (the wage norm) affects the present wage, whereas if \( \alpha^w = 1 \), then the wage norm plays no part.
and the wage is purely determined by Nash bargaining as outlined in Section (2). In place of (12)
the aggregate earnings are written
\[ W_t H_t = \begin{cases} \alpha w \left[ (1 - \alpha) \left( \frac{A Z_t}{\mu t} \right)^{\frac{\alpha}{\alpha + \rho}} \left( \frac{\alpha}{R^F t^F / P_t} \right)^{\frac{1}{\rho}} H_t \right] + (1 - \alpha w) \frac{W_{t-1} H_t}{P_t} \end{cases} F \left( X_t \right) \] (37)

Of the other equations that form part of the system governing the behaviour of the economy, the
only one that is modified by the presence of the (backward looking) wage norm is the separation
condition, (13), which becomes
\[ \left\{ \begin{array}{l} (1 - \alpha w) \left[ (1 - \alpha) \left( \frac{A Z_t}{\mu t} \right)^{\frac{\alpha}{\alpha + \rho}} \left( \frac{\alpha}{R^F t^F / P_t} \right)^{\frac{1}{\rho}} H_t \right] + \frac{1}{\mu_t} \left[ F - \frac{e_t X_t F(X_t)}{F(X_t)} \right] + (1 - \alpha w) \frac{W_{t-1} H_t}{P_t} \end{array} \right\} = 0. \] (38)

Column (6) of Table (2) displays the results of this alternative to Nash wage bargaining under
the title RigidW, where we let \( \alpha w = 0.005 \). First we examine whether variation in hours per
worker attenuates the variation in unemployment sufficiently that wage rigidity ceases to 'solve' the
unemployment - vacancy - tightness puzzle. In fact Table (2) shows that even when job destruc-
tion is endogenous and hours are variable, wage rigidity raises the variability of unemployment,
vacancies and labour market tightness compared with either NKS or FixH. Unemployment and
labour market tightness are around two thirds of the figure in US Data, while vacancies rise to
over half of that in Data. This rise in the variability of vacancies under wage rigidity appears
consistent with the mechanism identified and exploited by Shimer (2005), whereby with flexible
wages, wage variation absorbs most of the effect of shocks and reduces the incentive for firms to
post vacancies.15

Next we ask whether the introduction of wage rigidity overturns the result that variation in
hours per worker generates a downward sloping Beveridge curve despite endogenous job destruc-
tion? Table (2) shows that not only is the downward sloping Beveridge curve preserved under
wage rigidity, but also that the negative correlation of unemployment with vacancies becomes
stronger (more consistent with US Data) than in NKS. The correlation of job creation with job
destruction in RigidW also remains negative and becomes a little weaker than NKS - and more
15Krause and Lubik (2006) report that the volatility of vacancies declines once wage rigidity is imposed.
consistent with US Data. As we discuss below, the behaviour of job flows underlies the slope of the Beveridge curve. Although wage rigidity appears to affect the variability of job flows to the same extent as FixH, it distorts the correlation structure much less than FixH because variation in hours per worker is used to soak up some of the initial response to shocks.\textsuperscript{16}

Now consider whether wage rigidity results in realistic variation in hours per worker despite the availability of the intensive margin or instead overrestricts hours variation. First note that wage variability is not eliminated in RigidW, but is closer to the figure in Data than either of NKS or FixH. Lower variation in wages tends to limit the variability of hours. The variation in hours in RigidW is also closer to Data than the overvolatile case, NKS, or the undervolatile FixH. Thus, besides its desirable effect on the volatility of unemployment and vacancies, imposing wage rigidity in an NKS model, with labour supply elasticity of 0.5, generates realistic hours variation.

Although the variability of employment is the same under wage rigidity with variation in hours per worker - RigidW - as with flexible wages and no variation in hours per worker - FixH - the performance of other labour market variables is markedly different. In RigidW firms can use hours per worker to adjust to shocks, as in NKS, but in addition wage rigidity leads the correlation of employment and hours takes the correct sign. This suggests that hours per worker and employment act as complements in adjustment to shocks in a manner consistent with US Data.

Finally, turning to other macroeconomic aggregates. We find that inflation exhibits the same volatility in RigidW as in NKS and FixH. This extends - to a more general macroeconomic environment - Krause and Lubik’s result on the irrelevance of real wage rigidity for inflation dynamics. Output is about 7% higher in RigidW than in NKS and FixH. This reflects the greater variability of labour input (on the extensive margin). Finally, the relative volatilities of consumption and investment are unaffected by the treatment of wages.

\textsuperscript{16}The increased labour input variation on the extensive margin has implications for the variability of gross job flows. As with FixH, wage rigidity tends to raise the importance of job destruction relative to job creation as a means of affecting employment. This suggests that variation in job destruction (the elimination/preservation of existing matches) is the cheaper of the two means of using the the extensive margin to adjust labour input.
4.2.4 Comparison with Wage Rigidity - Fixed Hours Worked Variant

If wage rigidity is imposed - $\alpha^w = 0.005$ - and hours variation is suppressed - $\psi = 100$ (labour supply elasticity is 0.02) - then we obtain the model variant in column (7) $\text{FixWH}$. This combination comes closest to the set up employed by Shimer (2005), Hall (2005), Krause and Lubik (2005) when discussing the impact of wage rigidity. The relative volatilities are closest to those obtained in $\text{FixH}$. For the correlation of unemployment with vacancies and for job creation with job destruction, the impact of suppressing hours variation while imposing real wage rigidity lies between the desirable effects of $\text{RigidW}$ and the undesirable consequences of $\text{FixH}$.

4.2.5 Section Summary

Our results suggest that the baseline $\text{NKS}$ model does a reasonable job of matching the relative volatility of a number of macroeconomic variables, but generates insufficient variability in unemployment and vacancies. Low unemployment variability can be addressed to some extent by decreasing the elasticity of labour supply to suppress variation in hours - $\text{FixH}$ - but this results in a decline in the variability of vacancies, and an upward sloping Beveridge curve. A preferable approach is to impose real wage rigidity while allowing labour input to vary on both intensive and extensive margins - $\text{RigidW}$. This comes closest to solving the unemployment-vacancies-tightness puzzle, generates a downward sloping Beveridge curve with a plausible magnitude (despite endogenous job destruction), captures the variation of hours at business cycle frequencies, and the interaction of hours with employment but does not distort other macroeconomic aggregates.

4.3 Labour Market Cross-Correlations

In this section we study the cross correlation structure of job creation and job destruction with each other, with unemployment and with inflation. We compare the behaviour in US $\text{Data}$, $\text{NKS}$, $\text{FixH}$, and $\text{RigidW}$ so as to shed light on the mechanism that determines the slope of the Beveridge curve. This section also acts to unify two strands of the literature. When research is directed towards the cyclical behaviour of job flows (and particularly job destruction) researchers have studied the dynamic cross correlations that we consider here, Den Haan et al. (1999), Trigari (2005), Walsh (2005), do not consider the cyclical properties unemployment - vacancies - tight-
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ness. Yet when the main research questions concern the properties of unemployment, vacancies and tightness it is common to ignore the dynamic correlation structure of job flows or suppress endogenous job destruction altogether, Hall (2005), Shimer (2005), Krause and Lubik (2006). This is unfortunate because these issues are related. The cross-correlations are illustrated in Figure (2).\(^\text{17}\)

Figure (2) here

First we consider the properties of the baseline model \textbf{NKS} in relation to US \textbf{Data}. Panel (a) of Figure (2) shows the correlation of leads and lags of job creation with contemporaneous job destruction. For instance, in US \textbf{Data}, leads of job creation are positively correlated with current job destruction but negatively correlated with contemporaneous and lagged job destruction. This suggests that job creation is above average in advance of a spike in job destruction. From panel (c) since job destruction is positively correlated with unemployment in US \textbf{Data} (high during and after a recession), job creation is high in advance of a recession - panel (b). \textbf{NKS} seems to capture the broad features of the data. It captures various phenomena such as (i) positive correlation between leads of job creation and unemployment while lags are negatively correlated with unemployment; (ii) the positive correlation of current and lagged job destruction with unemployment; (iii) the relatively weak relationship between inflation and gross job flows and (iv) the sign, if not the magnitude of the dynamic cross correlations of inflation and unemployment. The cross correlations for \textbf{NKS} frequently display the same sign as that in US \textbf{Data}. However, in panel (a) much of the 'action' in the correlation structure for \textbf{NKS} arises around the contemporaneous cross-correlation - as the overstrong contemporaneous correlation of job creation and job destruction is offset by overly weak correlations between lags of job correlation and job destruction. Such discrepancies between \textbf{NKS} responses and US \textbf{Data} reflect the lack of persistence of the job creation and job destruction responses revealed in the impulse responses in Figure (1). Evidence of this rebound effect is also present in the relationship between job flows and other variables where much more of the movement in the correlograms occurs around the contemporaneous correlation for \textbf{NKS} than for US \textbf{Data}.

\(^{17}\)\textbf{FixWH} is omitted for clarity as its dynamic cross-correlation structure is so similar to that of \textbf{FixH}.
Next, we consider how differences in the margins for adjustment and in wage rigidity across model variants affect the correlation structure of gross job flows and how this is related to unemployment and vacancy dynamics. From a simple accounting viewpoint, a rise in unemployment can be achieved either through a rise in job destruction and a fall in job creation, or by one of these in isolation with no change in the other, by a fall in job creation combined with a smaller fall in job destruction or by a rise in job creation combined with a larger rise in job destruction. The first of these would tend to give rise to a negative contemporaneous correlation of job creation and job destruction, while the last (two) of these would tend to produce a positive contemporaneous correlation of job creation with job destruction. Figure (2) and Table (2) reveal that the first of these is what occurs in US Data, and in NKS and RigidW, whereas the last of these options captures what occurs in FixH without variation in hours.

Why does this happen? Suppressing the variation of hours increases labour input variation on the extensive margin, raising the variability of job destruction Table (2). Figure (2) panel (c) shows that this does not alter the correlation structure of job destruction with unemployment or inflation (relative to that in NKS). But the volatile response of job destruction obtained when suppressing hours variation can only be sustained if there is an increase in job creation prior to, contemporaneously with and following a rise in unemployment - in contrast to US Data. So in FixH job creation rises before, during and after a recession, and job creation and job destruction appear (weakly) positively correlated. While changes in job creation are not equal to changes in vacancies, the two are related, and the positive correlation between job creation and unemployment is sufficient to overturn the normally negative correlation of unemployment with vacancies found in US Data, and in NKS and RigidW. This unfortunate feature of the relationship between job creation and unemployment for FixH is also replicated in the correlation of job creation with inflation, and is exacerbated by the fact that the contemporaneous association between unemployment and inflation is itself too strong in the model.

Turning to the case of wage rigidity. Table (2) indicates that real wage rigidity (RigidW) also generates a rise in the variability of employment and job destruction. However, Figure (2) shows that this does not distort the behaviour of job flows (especially job creation) in the manner
observed for FixH - so that the correlation structure of job creation with job destruction is closer to that in the data. The opportunity to vary hours per worker in response to shocks in the face of comparatively smooth wage variation produces a smoother dynamic correlation structure that is as close or closer to US Data than obtained for either NKS or FixH. A minor exception to this is the relationship between job destruction and unemployment. Here the instantaneous correlation is similar to that in US Data, NKS and FixH while lagged job destruction is less strongly positively correlated with unemployment than is the case for NKS and FixH.

Section Summary Detailed examination of the dynamic cross correlations lends some support to the NKS model with flexible wages, but provides stronger support for the version with wage rigidity, RigidW, which matches key features of US Data more closely. It also suggests that models which lack an intensive margin (FixH, FixWH and many papers in the literature) are misspecified owing to the consequences of an overreliance on the extensive margin for adjustment of labour input.

4.4 Robustness Issues

Next we examine whether the results we have obtained are robust to variation in key parameters. The results discussed so far favour the RigidW variant over the baseline NKS model, and both of these to FixH - without variation in hours per worker. Since our results are obtained for particular combinations of wage rigidity and labour supply elasticity, and since a range of parameterisations could be supported by different empirical studies, we consider two issues. First, whether the desirable properties arising under rigid wage (RigidW) are sensitive to the parameterisation of wage rigidity. Second, whether the desirable results arising with mildly elastic labour supply are robust to other values of labour supply elasticity.

4.4.1 How much wage rigidity?

The degree of wage rigidity is captured by the parameter $\alpha^w$, which was assumed to take a value on the unit interval - with $\alpha^w$ close to 0 representing relatively rigid wages. In the absence of any compelling evidence on its value, and to highlight the contrast with NKS, it was set at $\alpha^w = 0.005$, so that the backward looking component of wages contributes 99.5% to wage
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dynamics. We examine the sensitivity of the variables in Table (2) to wage rigidity, by allowing \( \alpha_w \) to take values on the interval \([0.005, 1]\). Here 0.005 corresponds to almost complete wage rigidity (RigidW above) and \( \alpha_w = 1 \) corresponds to flexible wages in the baseline NKS model.

Figure (3) here.

Figure (3) illustrates the results. Panel (d) displays instantaneous correlation for unemployment with vacancies (the Beveridge curve), job creation with job destruction, and employment with hours. For most values of \( \alpha_w \), these statistics take on plausible values (of the expected sign and magnitude) close to those in US Data. Only with little wage rigidity (\( \alpha_w > 0.9 \))- in the neighbourhood of the NKS calibration - do these statistics take on less realistic values, with the hours-employment correlation tending to zero and the correlation of unemployment with vacancies becoming smaller than that of job creation with job destruction. Panel (c) captures the relative variability of employment, hours and wages. Of these, unsurprisingly, the wage is the most sensitive to the degree of wage rigidity. A less rigid wage (higher \( \alpha_w \)) is a more variable wage. However, no value of \( \alpha_w \) allows wage variability to fall to the values in the data.\(^{18}\) Employment variability is less sensitive to \( \alpha_w \) but is also somewhat too high relative to US Data. Panel (b) captures the relative volatilities of unemployment, vacancies, job creation and job destruction. Unemployment and vacancies are closest to the values attained in US Data when \( \alpha_w = 0.005 \), as in RigidW, while job destruction is then too high and job creation is too low. At intermediate values of wage rigidity (\( 0.005 < \alpha_w < 0.9 \)) job destruction remains too high and job creation too low, while unemployment and vacancies decline to the levels observed in NKS. Panel (a) confirms that the volatility of inflation relative to output is not sensitive to the degree of wage rigidity as captured by \( \alpha_w \).

So from Figure (3) it appears that of the desirable properties arising in RigidW, all except the volatility of unemployment and vacancies are robust to the introduction of substantial amounts of wage flexibility. This evidence is broadly supportive of the RigidW calibration.\(^{18}\) Perhaps an alternative, more backward-looking wage norm would help in this regard.
4.4.2 Labour Supply Elasticity

Both RigidW and the baseline NKS model are calibrated to a labour supply elasticity of 0.5 (corresponding to $\psi = 4$).\textsuperscript{19} This figure can be justified from microeconometric evidence Domeij and Floden (2005). Alternatives in the range $[0, 1]$ have been used elsewhere Blundell and McCurdy (1999). It is lower than the unit-elastic specification adopted in Andolfatto’s (1995) real business cycle model that incorporates labour market search with hours variation (with exogenous job destruction), and also in standard New Keynesian models without labour market frictions Woodford (2003). However, this choice is not uncontroversial. A labour supply elasticity of 0 is implicit in studies in the labour market search literature that omit the intensive margin for adjusting labour input. In Figure (4) we examine the sensitivity of the model with rigid wages - RigidW - to variation in labour supply elasticity.

The previous arguments in this section point to the importance of wage rigidity and endogenous job destruction as key assumptions driving empirically plausible results in Tables (2), which shows that inelastic labour supply generates the absence of hours variation akin to that found in models without an extensive margin, and that it leads key correlations to take the magnitudes and even signs inconsistent with US Data. A key question then is how rapidly these undesired properties set in as labour supply becomes less elastic, or equivalently how robust are the desirable properties of RigidW to labour supply elasticity?

We consider values of $\epsilon_H \in [0.02, 1]$ in order to be able to incorporate the model variants RigidW and FixWH. The results are contained in Figure (4). Panel (d) shows that the slope of the Beveridge curve, the correlation of job creation with job destruction and the correlation of hours with employment are close to the values observed in US Data for a wide variety of labour supply elasticities (at least for $\epsilon_H \in [0.1, 1]$). Only for very inelastic labour supply do these values become implausible, and at such values hours exhibit much less variation than is present in US Data.\textsuperscript{20} Panels (a) and (c) show that employment, wages and inflation are insensitive to the elasticity of labour supply. Panel (b) shows that job destruction is too volatile.

\textsuperscript{19}Labour supply elasticity, $\epsilon_H$, is $\frac{1}{\psi} \left[ \frac{1}{H} \right] = \frac{2}{\psi}$. In the limit as $\psi \to \infty$, $\epsilon_H \to 0$, and variation in hours is eliminated.

\textsuperscript{20}In the neighbourhood of unit elastic labour supply parameterisations the slope of the Beveridge curve and the correlation of job creation and destruction become too small.
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In summary, the desirable properties of RigidW appear robust to a wide range of plausible labour supply elasticities, but model performance deteriorates if labour supply is sufficiently inelastic to eliminate variation in hours - as in FixWH.

5 Conclusions

In this paper we show that one simple modification - the introduction of an intensive margin for labour input (i.e., variable hours) - allows a New Keynesian model with labour market search to capture the properties of job flows and of unemployment and vacancies - both volatilities and correlations. Modelling both aspects of labour market behaviour simultaneously has until now been infeasible. At the same time this modification allows us to capture salient features of the cyclical properties of hours.

To introduce hours, we adopt the dynamic general equilibrium discipline imposed by a New Keynesian model. This explicit macroeconomic setting seems to be an appropriate environment for the evaluation of labour market phenomena at business cycle frequencies. Indeed the New Keynesian framework appears to ameliorate the unemployment and vacancy volatility problem uncovered by Shimer (2005), since the standard deviations of unemployment and vacancies are around one third of that in US data under the baseline NKS model not the one tenth found by Shimer in the context of a labour market equilibrium model. Nonetheless, Shimer’s result on wage rigidity holds: wage rigidity raises the volatility of unemployment and vacancies. In the RigidW set up (with wage rigidity) it is possible to account for half of the variability of unemployment and vacancies present in the data. The introduction of variable hours has an additional payoff here. If the intensive margin is suppressed, FixWH, then wage rigidity tends to generate volatile unemployment but suppresses the volatility of vacancies as in Krause and Lubik (2006), whereas with the intensive margin, the variability of vacancies is almost as large as that of unemployment - true to the data.

The desirable properties obtained under real wage rigidity, with an intensive margin and endogenous job destruction are robust across a wide range of labour supply elasticities. This is useful, since there seems to be controversy over the appropriate value of this. However, the results
are not robust to extremely inelastic labour supply which (unrealistically) suppresses hours - a short cut implicit in much of the literature on labour market search. In addition, except for the variability of unemployment and of vacancies, the desirable properties are robust to a substantial amount of wage flexibility.
References


6 Appendix A: Calibration & Model Solution Method

6.1 Labour Market Flows

We specify the following labour market parameters $\rho$, $\rho^s$, $\sigma_X$, $N$, $\kappa^V$, $\epsilon$ and $\gamma$. We use this information to compute $F(\bar{X})$, $\bar{X}$, $\mathcal{F}$, $U/N$, $V/N$, $JC$, $JD$, $\mathfrak{M}$.

Following Den Haan et al. (2000) we assume that 10% of employment relationships separate each quarter: $\rho = 0.1$.

In steady state the probability of separation, $\rho$, can be written as

$$
\rho = \rho^s + (1 - \rho^s) (1 - F(\bar{X}))
$$

Following Den Haan et al. (2000), let the probability of exogenous separation, $\rho^s = 0.068$. So the probability of endogenous separation is

$$
1 - F(\bar{X}) = (\rho - \rho^s) / (1 - \rho^s) = 0.034.
$$

To compute $\bar{X}$ and $\mathcal{F}$, we assume that $X$ is lognormally distributed ($\mu_X, \sigma_X$). We normalise $\mu_X = 1$. There is little empirical evidence to guide the choice of $\sigma_X$. Den Haan et al. (2000) and Walsh (2005) set $\sigma_X = 0.1$ and $0.12$ respectively; but $X$ enters multiplicatively in their models, and in NKS this would generate too much variability in labour flows data. We set $\sigma_X = 0.05$ to match the variability of labour market flows data. Then, in steady state, the threshold value of the idiosyncratic cost shock is $\bar{X} = F^{-1}(0.034) = 1.09$. The elasticity $\epsilon_X = 1.57$. I set $\mathcal{F}$ to equal the average cost shock for those matches that produce in steady state, $\mathcal{F} = \frac{E[X|X < \bar{X}]}{F(\bar{X})} = 0.997$, so that the cost shock does not impact on steady state wholesale production.

We also set $N = 0.8$, lower than Den Haan et al. (2000) (0.94) close to Trigari (2005) (0.75) and higher than Andolfatto (1995) (0.54). From equation (15) the ratio of searchers to employment $U/N$ is $\frac{U}{N} = \frac{1}{\mathfrak{M}} - (1 - \rho)$. Therefore the ratio of workers searching for jobs to employed workers, $U/N$, is 0.35. In steady state, the employment evolution equation (17) can be written as

$$
\rho = \frac{\mathfrak{M}}{\epsilon U} \left( \frac{V}{N} \right)^{1-\gamma}.
$$

We set the parameter $\gamma$ in the matching function at 0.6, in the light of Petrongolo and Pissarides.
survey of empirical results and $e = 0.5H$. Then $\mathcal{M} = 1.001$.

### 6.2 Preferences

The preferences of the representative household are characterised by the parameters $\beta$, $\psi$, $H$, $\xi$, $\kappa_C$, $C$, $\phi$, $\Lambda$ and $\kappa_H$. The final two are computed from the steady state of the system.

We set the discount factor $\beta = 0.989$, in line with the literature. Let us assume that a worker has a unit time endowment and in steady state spends a third of her time working: $H = 1/3$. For the preferences specified, the elasticity of hours per worker, $\epsilon_H$, is $\frac{1}{\psi} \left[ \frac{1-H}{H} \right]$. Estimates of this elasticity vary with gender and other variables, but is typically less than unity. We take the value $\psi = 4$ as a baseline, which gives $\epsilon_H = 0.5$. The elasticity, $\xi$, of demand for real money balances with respect to the marginal utility of consumption is set at 1. In line with Christiano et al. (2005), we set the curvature parameter for the instantaneous utility function, $\phi = 1$, and $\kappa_C = 0.5$. We also normalise the steady state value of the aggregate consumption index to $C = 1$, then from the steady state version of (27), $\Lambda = \frac{1-\beta \kappa_C}{(1-\kappa_C)C^\psi} = 1.011$. Computation of $\kappa_H$ will be discussed below.

### 6.3 Capital Accumulation

The rate of depreciation, $\delta$, is set at the standard value of 0.025. In the baseline parameterisation adjustment costs are set to $\chi = 12$. In steady state, from the capital accumulation equation (25), we that $I/K = \delta$. From the steady state of equation (29) $Q = \Lambda$. In steady state, the capital euler equation (30) determines the real rental rate on capital as $R^K/P = \beta^{-1} - (1 - \delta) \approx 0.036$.

### 6.4 Price Rigidity & Price Setting

Calibration of nominal rigidities in price setting by retailers involves specification of $\omega$, $\tau$ and $\varepsilon$.

The extent of nominal rigidity in the goods market is determined by $\omega$, which captures the fraction of final goods firms in any period that do not adjust their price and $\tau$ which refers to the fraction of final goods firms which set prices in a backward-looking manner. Empirical evidence from studies using aggregate data suggests that prices last for 9-12 months on average corresponding to $\omega \in [2/3, 3/4]$. We take $\omega = 3/4$ as a baseline value. Following Gali and Gertler’s
estimates we set $\tau = 0.5$.

In steady state the markup is $\mu = \frac{\varepsilon}{(\varepsilon - 1)}$. We assume that, $\varepsilon$, the elasticity of demand equals 6, so $\mu = 1.2$.

### 6.5 Production, Bargaining and Equilibrium

Next we set $\rho_Z, \rho_U, \alpha$ and $\eta$. Then using (7), (12), (13), (25), (34), (35) and (36) we compute $\kappa, \kappa_H, A, K, W/PY$ and $C/Y$.

Following Cooley and Quadrini (1999), the money supply growth process is assumed to follow an AR(1) process with the autoregressive parameter $\rho_\upsilon = 0.49$, with mean zero normally distributed innovations with standard deviation $\sigma_\upsilon = 0.006$, while aggregate productivity also follows an AR(1) process, with $\rho_Z = 0.95$. We normalise the steady state value of $Z$ to unity. The standard deviation of productivity is chosen in order to match the standard deviation of US quarterly HP-detrended GDP data. Let us set the elasticity of output with respect to capital, $\alpha = 1/3$, and worker bargaining power (and share of the match surplus) $\eta = 0.6$.

\[
K = (1 - \rho^x) F(\bar{X}) N \left[ \frac{\alpha A}{\mu R^K / P} \right]^{1 - \alpha} H. \tag{39}
\]

Combining (25), (35) and (36) with the definition of the auxiliary price index, $\tilde{P}$, and the capital market equilibrium condition we have the steady state condition

\[
Y = C + \delta K = \left( \frac{\tilde{P}}{P} \right)^\varepsilon A \left[ (1 - \rho^x) F(\bar{X}) NH \right]^{1 - \alpha} K^\alpha - \kappa \mu V \tag{40}
\]

This can be combined with (39) to give

\[
A^{1 - \alpha} = \frac{C + \kappa \mu V \left( \frac{\tilde{P}}{P} \right)^\varepsilon}{\left( \frac{\alpha A}{\mu R^K / P} \right)^{1 - \alpha} \left[ \left( \frac{\tilde{P}}{P} \right)^\varepsilon \frac{\mu R^K}{\alpha P} - \delta \right]} (1 - \rho^x) F(\bar{X}) NH. \tag{41}
\]

The steady state versions of (12) and (7) combined with (2) are

\[
\frac{\kappa}{\kappa^\alpha} = \beta (1 - \rho^x) \left[ F(\bar{X}) \frac{\kappa}{\kappa^\alpha} + (1 - \alpha) \left( \frac{A}{\mu} \right)^{1 - \alpha} \left( \frac{\alpha}{R^K / P} \right)^{1 - \alpha} H \right] - \frac{WH}{P} \tag{42}
\]

\[
\frac{WH}{P} = \eta \left[ (1 - \alpha) \left( \frac{A}{\mu} \right)^{1 - \alpha} \left( \frac{\alpha}{R^K / P} \right)^{1 - \alpha} H + \frac{V}{\kappa} \right] - \left[ \frac{1}{\Lambda} \left[ (1 - \eta) \left( 1 - \psi \right) - \kappa \left( 1 - H \right)^{1 - \psi} \right] \right]. \tag{43}
\]
Combining (42) and (43) to eliminate $\frac{WH}{P}$

$$
\kappa \left[ \frac{1}{\psi} - \frac{\beta (1-\rho^2) F(X)}{\kappa^p} + \eta \beta (1-\rho^2) F(\bar{X}) \right] = (1-\eta) \beta (1-\rho^2) F(\bar{X}) \left[ \frac{1}{\alpha} \left( \frac{A}{\mu} \right) \right]^{\frac{1}{\alpha}} - \kappa_H \frac{(1-H)^{1-\psi}}{1-\psi} H
$$

Combining (41) and the steady state version of (13) to eliminate $\tilde{k}$ gives

$$
\frac{1-\eta}{\Lambda} \left[ (1-\epsilon)^{1-\psi} \frac{(1-H)^{1-\psi}}{1-\psi} - \kappa_H \right] = \left[ \frac{\eta \kappa V}{1-\psi} - \frac{\kappa - (1-\eta) \frac{\bar{X} - X}{\mu}}{1-\psi} C + \kappa \mu V \right] \left[ \frac{\left( \frac{\bar{X}}{\mu} \right)^{1-\psi} - \epsilon}{(1-\rho^2) F(X) N} \right]
$$

Next, substituting (41) and (45) in (44), to eliminate $A$ and $\kappa_H$ and gives the following expression for $\kappa$ in terms of known parameters:

$$
\kappa = -\beta (1-\rho^2) F(\bar{X}) (1-\eta) \left( \frac{\bar{X} - X}{\mu} \right) \kappa^V = 0.025
$$

Combining this value for $\kappa$ with (45) determines $\kappa_H = 1.400$. Then using (43) $W/P = 2.881$.

Finally, substituting for $\kappa$ in (41) $A = 1.745$, from (39) $K = 8.692$ and from (40) $Y = 1.217$. It follows that $C/Y = 0.821$.

### 6.6 Model Solution Method

The log-linearised approximation to the system of equations, (2), (7) and (11) - (36), is stacked in the form

$$
\mathcal{A} \mathcal{E}_t [\mathcal{Y}_{t+1}] = \mathcal{B} \cdot \mathcal{Y}_t + \mathcal{C} \cdot \mathcal{Z}_t
$$

Where $\mathcal{Z}_t$ is a vector of exogenous state variables ($\bar{z}_t$ and $\bar{v}_t$) and $\mathcal{Y}_t$ is a vector of endogenous jump ($\bar{y}_t$, $\bar{c}_t$, $\bar{r}_t$, $\bar{H}_t$, $\bar{h}_t$, $\bar{u}_t$, $\bar{v}_t$, $\bar{m}_t$, $\bar{q}_t$, $\bar{\pi}_t$, $\bar{T}_t$, $\bar{\pi}_t$, $\bar{\theta}_t$, $\bar{\lambda}_t$) and state ($\bar{\theta}_t$, $\bar{\theta}_t$, $\bar{\pi}_{t-1}$, $\bar{\pi}_{t-1}$, $\bar{\pi}_{t-1}$, $\bar{\pi}_{t-1}$) variables, and $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ are conformable matrices of coefficients. The system is solved with MATLAB, 7.0.1, using McCallum’s (1998) undetermined coefficients approach based on Klein’s (1997) generalised Schur decomposition method.

\[\text{21} \text{The full system, } \mathcal{Y}_t, \text{ includes a definition of the inflation rate in terms of the price index } (\bar{\pi}_t = \bar{p}_t - \bar{p}_{t-1}) \]

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<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
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<td>$\rho$</td>
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<td>$\gamma$</td>
<td>0.6</td>
<td>$\omega$</td>
<td>0.75</td>
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<tr>
<td>$\rho^x$</td>
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<td>$\beta$</td>
<td>0.989</td>
<td>$\tau$</td>
<td>0.5</td>
</tr>
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<td>$\sigma_X$</td>
<td>0.05</td>
<td>$\phi$</td>
<td>1</td>
<td>$\epsilon$</td>
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<td>$\kappa_C$</td>
<td>0.5</td>
<td>$\alpha$</td>
<td>0.3</td>
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<td>$\kappa^V$</td>
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<td>$C$</td>
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<td>$\chi$</td>
<td>12</td>
<td>$\rho_w$</td>
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<td>$\delta$</td>
<td>0.025</td>
<td>$\sigma_v$</td>
<td>0.006</td>
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Table 1: Calibration: Assigned Parameters
Whither Job Destruction?
Unemployment, Job Flows and Hours in a New Keynesian Model.

<table>
<thead>
<tr>
<th>Standard Deviation w.r.t. GDP or Correlation</th>
<th>Data</th>
<th>NKS</th>
<th>NK</th>
<th>FixH</th>
<th>RigidW</th>
<th>FixWH</th>
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</thead>
<tbody>
<tr>
<td>Output</td>
<td>1.00</td>
<td>1.00</td>
<td>0.93</td>
<td>1.00</td>
<td>1.07</td>
<td>1.02</td>
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<td>2.81</td>
<td>2.79</td>
<td>2.87</td>
<td>2.86</td>
<td>2.82</td>
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<tr>
<td>Consumption</td>
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<td>0.64</td>
<td>0.65</td>
<td>0.65</td>
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<tr>
<td>Inflation</td>
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<td>0.31</td>
<td>0.48</td>
<td>0.28</td>
<td>0.29</td>
<td>0.27</td>
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<td>Wage</td>
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<td>2.19</td>
<td>5.18</td>
<td>1.53</td>
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<td>1.19</td>
</tr>
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<td>0.65</td>
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<td>0.46</td>
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<td>0.88</td>
<td>-</td>
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<td>1.13</td>
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<tr>
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<td>-</td>
<td>4.22</td>
<td>5.79</td>
<td>4.20</td>
</tr>
<tr>
<td>Vacancies</td>
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<td>6.37</td>
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<td>6.30</td>
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<td>6.00</td>
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<td>Unemployment - Vacancies</td>
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<td>-</td>
<td>0.68</td>
<td>-0.77</td>
<td>-0.40</td>
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<tr>
<td>Job Creation - Job Destruction</td>
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<td>-0.63</td>
<td>-</td>
<td>0.05</td>
<td>-0.50</td>
<td>-0.15</td>
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<tr>
<td>Hours - Employment</td>
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<td>0.05</td>
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<td>0.04</td>
<td>0.73</td>
<td>-0.36</td>
</tr>
</tbody>
</table>

Table 2: Business Cycle Statistics
All statistics computed from HP detrended series, smoothing parameter 1600.
NKS NKS: New Keynesian model with labour market search and variable hours per worker.
NK: New Keynesian model with frictionless hours in Walrasian labour market.
FixH: NKS with fixed hours per worker.
RigidW: NKS with wage rigidity.
FixWH: NKS with wage rigidity and fixed hours per worker.
Whither Job Destruction?
Unemployment, Job Flows and Hours in a New Keynesian Model.

Figure 1: NKS Impulse Responses for Money Growth and Productivity
Whither Job Destruction?
Unemployment, Job Flows and Hours in a New Keynesian Model.
Figure 4: Role of Labour Supply Elasticity under Wage Rigidity and Endogenous Job Destruction.