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1 **Anomalous wave statistics following sudden depth transitions:**
2 **Application of an alternative Boussinesq-type formulation**

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8 **Abstract** Recent studies of water waves propagating over sloping seabeds have shown that
9 sudden transitions from deeper to shallower depths can produce significant increases in the
10 skewness and kurtosis of the free surface elevation and hence in the probability of rogue
11 wave occurrence. Gramstad et al. (2013, *Phys. Fluids* 25 (12): 122103) have shown that the
12 key physics underlying these increases can be captured by a weakly dispersive and weakly
13 nonlinear Boussinesq-type model. In the present paper, a numerical model based on an al-
14 ternative Boussinesq-type formulation is used to repeat these earlier simulations. Although
15 qualitative agreement is achieved, the present model is found to be unable to reproduce ac-
16 curately the findings of the earlier study. Model parameter tests are then used to demonstrate
17 that the present Boussinesq-type formulation is not well-suited to modelling the propagation
18 of waves over sudden depth transitions. The present study nonetheless provides useful in-
19 sight into the complexity encountered when modelling this type of problem and outlines a
20 number of promising avenues for further research.

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21 **Keywords** Rogue wave · freak wave · Boussinesq-type equations · skewness · kurtosis

22 **1 Introduction**

23 Long considered the stuff of legend, rogue waves are now recognised as a serious hazard
24 to ships and offshore structures. Historical reports of giant, powerful waves appearing first
25 without warning and then suddenly vanishing have since been supported by theory and ex-
26 periment (Dysthe *et al.*, 2008; Kharif *et al.*, 2009). In recent decades, numerous studies have
27 explored both the physical mechanisms which might produce such waves and the statisti-
28 cal parameters that may be used to estimate their occurrence probability. Comprehensive
29 reviews are provided by Dysthe *et al.* (2008), Kharif *et al.* (2009), Slunyaev *et al.* (2011),
30 Onorato *et al.* (2013), and Adcock & Taylor (2014), amongst others.

31 Rogue waves are typically defined as those having heights which are more than twice
32 the local significant wave height (e.g. Holthuijsen, 2007) but their study is complicated by
33 a limited number of real-world measurements (Kharif *et al.*, 2009) and conflicting views as
34 to how much information can be inferred from these (Dysthe *et al.*, 2008). The key ques-
35 tion at present is whether such observations represent ‘classical’ extremes which can be
36 described by conventional models and statistics, or ‘freak’ waves requiring new theories
37 and approaches (Haver & Andersen, 2000; Dysthe *et al.*, 2008; Kharif *et al.*, 2009). Some
38 authors take the view that rogue waves are rare instances of random superposition in seas of
39 weakly nonlinear waves (Christou & Ewans, 2014; Fedele *et al.*, 2016) whilst others hypoth-
40 esise that certain waves, such as the well-known Draupner wave, must have been produced
41 by some other forcing mechanism (Adcock *et al.*, 2011; Cavaleri *et al.*, 2016).

42 Other possible rogue wave generating mechanisms include modulational instability; in-
43 teractions with variable bathymetry, opposing currents, or between crossing seas; wind forc-
44 ing; or some combination of these factors (Dysthe *et al.*, 2008; Kharif *et al.*, 2009; Onorato
45 *et al.*, 2013; Fedele *et al.*, 2016). Attempts to derive a single, unifying theory are compli-
46 cated by the facts that geometric focusing cannot explain the transient nature of rogue waves
47 (Janssen & Herbers, 2009), that modulational instability requires an improbable set of ini-
48 tial conditions (deep-water waves with a narrow spectral bandwidth and narrow directional

49 spreading) (Dysthe et al., 2008), and that rogue waves can be produced even when several
50 of the foregoing factors are absent (Mori & Janssen, 2006; Kharif et al., 2009).

51 The simplest theory assumes that the dynamics of ocean surface waves are purely linear,
52 that the free surface elevation is a stationary, Gaussian process, and that the wave amplitudes
53 are well approximated by the Rayleigh distribution (Ochi, 2005; Holthuijsen, 2007). How-
54 ever, because ocean waves are inherently (weakly) nonlinear (Trulsen, 2018), wave-wave
55 interactions or other mechanisms can result in considerable deviations from the Gaussian
56 model (Fedele et al., 2016). Some authors have suggested that rogue waves may be a re-
57 sult of non-equilibrium dynamics: if waves are somehow forced into an unstable state, their
58 statistics can deviate in such a way as to suggest an increased likelihood of extreme events
59 (Janssen & Herbers, 2009; Viotti & Dias, 2014). The kurtosis of the free surface elevation
60 is a convenient metric by which to quantify such deviations: an increase in free surface kur-
61 tosis signifies an increase in the probability of rogue wave occurrence (Onorato et al., 2004;
62 Mori & Janssen, 2006).

63 Waves propagating into shallower water are known to be transformed by shoaling and
64 nonlinear effects (Dean & Dalrymple, 1991; Dingemans, 1997) but recent studies have
65 shown that sudden transitions between deeper and shallower domains can also produce
66 strongly non-Gaussian wave statistics. Physical experiments by Trulsen et al. (2012), Zhang
67 et al. (2019), and Trulsen et al. (2020) showed significant increases in free surface skew-
68 ness and kurtosis for irregular waves near the crest of an inclined seabed of 1-in-20 slope
69 connecting otherwise flat domains, and these findings have been supported by numerical
70 simulations due to Sergeeva et al. (2011), Gramstad et al. (2013), Viotti & Dias (2014),
71 Ducrozet & Gouin (2017), Zhang et al. (2019), and Zheng et al. (2020). Similar results
72 have also been obtained in experimental and numerical studies of waves propagating over
73 submerged bars (Ma et al., 2014, 2015), shoals (Janssen & Herbers, 2009; Raustøl, 2014;
74 Fallahi, 2016; Trulsen et al., 2020), compound slopes (Kashima et al., 2014), and vertical
75 steps (Zheng et al., 2020).

76 The foregoing local increases in skewness and kurtosis usually coincide with local en-
77 hancements of higher harmonic content related to the sudden decreases in depth and cor-

78 responding increases in nonlinearity (Gramstad et al., 2013; Zhang et al., 2019; Trulsen et
79 al., 2020). In fact, Zheng et al. (2020) have recently shown that second-order terms in wave
80 steepness are responsible for the change in the statistical properties near the depth transition
81 for the cases examined by Trulsen et al. (2012) and Gramstad et al. (2013). These deviations
82 are also expected to depend on the initial steepness, spectral bandwidth, and directionality
83 of the waves (Ducrozet & Gouin, 2017; Støle-Hentschel et al., 2018; Trulsen et al., 2020;
84 Zheng et al., 2020), the gradient of the seabed slope, and the depth beyond the slope: for
85 milder slopes and deeper depths beyond the slopes, there may be no local maxima, or per-
86 haps even local minima, in skewness and kurtosis (Zeng & Trulsen, 2012; Gramstad et al.,
87 2013; Raustøl, 2014; Fallahi, 2016; Trulsen et al., 2020).

88 In this paper, the phenomenon of increased free surface skewness and kurtosis following
89 a sudden depth transition is explored further using an accurate yet computationally efficient
90 Boussinesq-type model, following the work of Gramstad et al. (2013), whose model appears
91 to be the simplest of those describing such anomalous statistical deviations. The aim is to
92 first reproduce the findings of Trulsen et al. (2012) and Gramstad et al. (2013) and then
93 extend the parameter space in our numerical simulations to provide further insight into the
94 underlying physics. The paper is structured as follows: §2 provides a brief description of
95 the numerical model, set-up of the numerical simulations, and grid convergence and sponge
96 layer calibration tests; §3 compares the present findings with those of Trulsen et al. (2012)
97 and Gramstad et al. (2013) and summarises the results of a model parameter study; and §4
98 presents the discussion, conclusions, and recommendations for further work.

99 **2 Model**

100 **2.1 Numerical model**

101 The present simulations are performed using OXBOU, a depth-integrated hybrid numerical
102 model designed to simulate the propagation in one horizontal dimension of ocean surface
103 gravity waves from intermediate to shallow and zero water depth. A brief overview of the
104 model features will suffice here; detailed descriptions of the numerical implementation and

105 verification and validation tests are given by Orszaghova (2011), Orszaghova et al. (2012),
 106 and Fitzgerald et al. (2016).

107 The OXBOU model uses two sets of governing equations and two numerical schemes:
 108 unbroken waves are simulated using weakly dispersive, weakly non-linear Boussinesq-type
 109 equations, which are solved using a fourth-order finite difference method, whilst broken
 110 waves are modelled as bores using the non-dispersive, non-linear shallow water equations,
 111 which are solved using a shock-capturing finite volume scheme (Orszaghova et al., 2012).
 112 The model switches from the Boussinesq-type to shallow water equations when certain
 113 depth or free surface slope criteria are met, but the present simulations involve non-breaking
 114 waves solely and so employ only the Boussinesq-type model. The numerical scheme in-
 115 corporates a moving boundary piston paddle wavemaker, which is facilitated by a mapping
 116 between stretching-compressing physical and fixed computational sub-domains, and is ca-
 117 pable of producing waves with approximately correct second-order bound harmonics (see
 118 Orszaghova et al., 2012). The scheme also includes an absorbing-generating sponge layer
 119 which allows incident waves to propagate freely inshore whilst simultaneously removing
 120 offshore-travelling reflections (see Fitzgerald et al., 2016).

121 OXBOU solves the Boussinesq-type equations of Madsen & Sørensen (1992), which
 122 were selected for their enhanced linear dispersion characteristics and computational ef-
 123 ficiency (Borthwick et al., 2006; Orszaghova et al., 2012). Following Orszaghova et al.
 124 (2012) and Fitzgerald et al. (2016), these equations are presented in a well-balanced, stage-
 125 discharge (η, q) form as

$$\eta_t + q_x = \psi(\eta_o - \eta), \quad (1)$$

$$q_t + \left(\frac{q^2}{d} + \frac{1}{2}g(\eta^2 - 2\eta b) \right)_x = -g\eta b_x - \frac{\tau_b}{\rho} + \frac{1}{3}h^2 q_{xxt} + \frac{1}{3}hh_x q_{xt} \\ + B \left(h^2 q_{xxt} + gh^3 \eta_{xxx} + 2gh^2 h_x \eta_{xx} \right) + \psi(q_o - q), \quad (2)$$

126 where $\eta = b + h + \zeta$ is the free surface elevation above a prescribed horizontal datum (with
 127 b the depth of the datum below the seabed, h the still water depth, and ζ the free surface ele-

128 vation above still water level); q is depth-integrated velocity; ψ is the sponge layer damping
 129 strength; $d = h + \zeta$ is the total depth; g is acceleration due to gravity; τ_b is bed stress; ρ is
 130 the fluid density; the subscripts t and x denote partial derivatives with respect to time and
 131 horizontal distance, respectively; the subscript o refers to solutions imposed by the sponge
 132 layers; and B is a linear dispersion coefficient such that the wave celerity, c , is given by

$$\frac{c^2}{gh} = \frac{1 + Bk^2h^2}{1 + \left(B + \frac{1}{3}\right)k^2h^2}, \quad (3)$$

133 where k is the wave number. Setting $B = 1/15$ embeds the [2,2] Padé approximant of the exact
 134 linear dispersion relation within the momentum equation, whereas setting $B = 0$ recovers the
 135 classical equation derived by Peregrine (1967) (Orszaghova *et al.*, 2012).

136 2.2 Set-up of numerical simulations

137 Following Gramstad *et al.* (2013), the first set of simulations is designed to replicate the
 138 physical experiments described by Trulsen *et al.* (2012), which were performed in the shal-
 139 low water basin at the Maritime Research Institute Netherlands (MARIN). These experi-
 140 ments considered three cases of long-crested irregular waves propagating from a piston-type
 141 wavemaker (at $x = 0$ m) first over a deeper flat domain, then over a 1-in-20 inclined seabed
 142 slope (from $x = 143.41$ m to 149.4 m), and finally over a shallower flat domain leading to
 143 an absorbing beach (at $x = 173.41$ m). In all three experimental cases, the still water depths
 144 before and after the slope were $h = 0.6$ and 0.3 m, respectively, and the nominal input sig-
 145 nificant wave height was $H_s = 0.06$ m. Cases 1, 2 and 3 were distinguished by the nominal
 146 peak periods of their input wave spectra: $T_p = 1.27$, 1.70 , and 2.12 s, respectively. Wave
 147 records were obtained from eight gauges placed along the length of the basin, and the influ-
 148 ence of the depth transition on the probability of rogue wave occurrence was examined by
 149 calculating the skewness and kurtosis of the free surface elevation and exceedance function
 150 of the (Hilbert) wave envelope at each location.

151 In repeating these experiments, the present study follows closely the methodology de-
 152 scribed by Trulsen *et al.* (2012) but uses OXBOU to output results at 1 m spatial intervals,

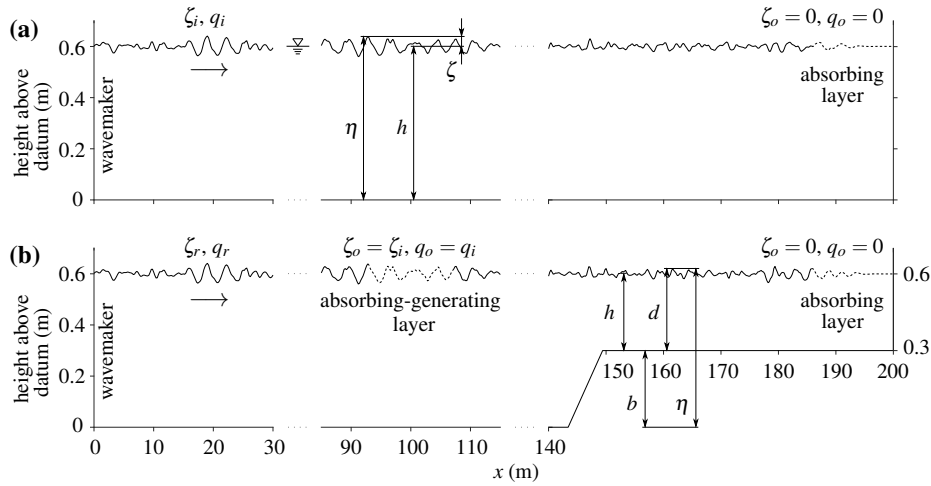


Fig. 1: Schematic diagram showing a simulation performed using coupled (a) incident and (b) run-up domains. Identical irregular waves are produced by the moving boundary wavemakers (left), and absorbing (right) and absorbing-generating sponge layers (centre) are used to eliminate reflections from the ends of the tanks and submerged seabed slope.

153 and moves the seabed slope 0.01 m closer to the wavemaker to facilitate the use of uniform
 154 (fixed) computational grids. The simulations for each case are performed as follows. The
 155 wavemaker is used to generate identical irregular waves in both an incident domain and a
 156 run-up domain. In the incident domain, the numerical wave tank (from $x = 0$ m to 200 m) is
 157 assigned a flat seabed profile ($h = 0.6$ m), whilst in the run-up domain, the tank comprises
 158 deeper ($h = 0.6$ m) and shallower ($h = 0.3$ m) sections connected by a 1-in-20 seabed slope
 159 (from $x = 143.4$ m to 149.4 m). In both domains, the bed is frictionless and the waves prop-
 160 agate into an absorbing sponge layer (from $x = 185.8$ m to 200 m), which gradually reduces
 161 ζ and q to zero to ensure that there are no reflections either from the end of the tank or
 162 the absorbing layer itself. Meanwhile, in the run-up domain, reflections from the slope are
 163 removed by an additional absorbing-generating sponge layer (from $x = 92.9$ m to 107.1 m),
 164 which adjusts the free surface elevation, ζ_r , and depth-integrated velocity, q_r , to match those
 165 in the incident domain, ζ_i and q_i (Fig. 1).

166 Irregular waves are produced as the sum of wave components obtained from a truncated
 167 JONSWAP spectrum with peak frequency $f_p = 1/T_p$ and upper and lower cut-off frequen-
 168 cies $f_{max} = 3f_p$ and $f_{min} = 0.5f_p$. The JONSWAP function is given by

$$S(f) = \alpha \frac{g^2}{(2\pi)^4} \frac{1}{f^5} \exp\{-1.25(f_p/f)^4\} \gamma^{\exp\{-(f-f_p)^2/2(\sigma f_p)^2\}}, \quad (4)$$

169 where f is the component frequency, α is the energy scale parameter, $\gamma = 3.3$ is the peak
 170 shape parameter, and σ is the peak width factor, which is assigned values of $\sigma = 0.07$ for $f \leq$
 171 f_p and $\sigma = 0.09$ for $f > f_p$ (Ochi, 2005; Holthuijsen, 2007). Pseudo-random wave signals
 172 are generated using the random-amplitude/random-phase approach of Tucker et al. (1984),
 173 in which the amplitudes and phases of the linear components are determined, respectively,
 174 from a Rayleigh distribution with scale parameter $\sqrt{S(f)\Delta f}$, where Δf is the frequency
 175 domain sampling interval, and a uniform distribution on $[0, 2\pi]$ (Fitzgerald et al., 2016). The
 176 corresponding linear wavemaker signal is then calculated using the Biésel transfer function,
 177 and a large number of harmonic components is chosen to ensure that the repeat period of the
 178 signal is greater than the duration of the simulation. This linear signal can also be corrected
 179 by applying a second-order transfer function approximated from the wavemaker theory of
 180 Schäffer (1996) but, for ease of computation, only first-order accurate wavemaker signals
 181 are considered initially.

182 2.3 Grid convergence and sponge calibration tests

183 Model solutions converged for a uniform computational grid spacing of 0.02 m and a time
 184 step of ~ 0.0066 s. Figure 2a shows the excellent agreement in free surface time series ob-
 185 tained when computational grids of resolution 0.018 m, 0.02 m, and 0.022 m (which repro-
 186 duce the tank using 11,000, 10,000, and 9,000 grid points, respectively) are used to simulate
 187 an example focused wave group, which is created by bringing 128 harmonic wave com-
 188 ponents from the Case 2 spectrum to a linear focus amplitude of 0.03 m at the toe of the
 189 seabed slope ($x = 143.4$ m). Wave records from a point just beyond the crest of the slope
 190 ($x = 150$ m) show excellent agreement, with root mean square error (RMSE) values ranging
 191 from $\sim 2.47 \times 10^{-5}$ m to $\sim 5.68 \times 10^{-5}$ m, as do the corresponding frequency-domain re-
 192 sults, which are not shown for brevity. Excellent results are also obtained in tests for mass

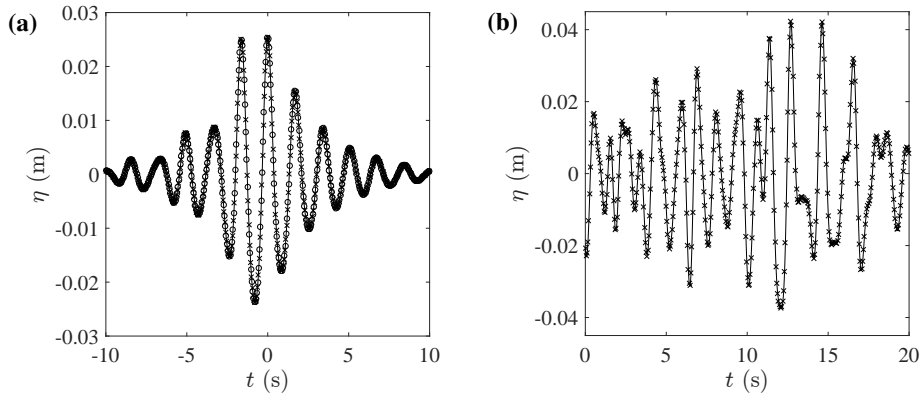


Fig. 2: Free surface elevation time histories at $x = 150$ m showing excellent agreement between (a) records of a crest-focused group simulated on computational grids of resolution 0.018 m (circles), 0.02 m (line), and 0.022 m (crosses), and (b) subsequent repeat periods (crosses, line) of a periodic irregular wave signal.

193 conservation, reversibility, and the accumulation of round-off error, with model errors typi-
 194 cally much less than 1%.

195 The absorbing and absorbing-generating sponge layers are then calibrated to ensure that
 196 they are able to damp effectively waves passing through without altering the incoming wave
 197 field. The absorbing-generating layer, which is used only in the run-up domain and placed
 198 such that its midpoint lies halfway along the one-dimensional tank (Fig. 1), is assigned a
 199 triangular strength profile (such that ψ increases and decreases linearly and symmetrically
 200 about the midpoint of the layer), whilst the identical absorbing layers, which are placed at
 201 the ends of the tanks in both the incident and run-up domains, are given linearly increasing
 202 strength profiles (Fitzgerald et al., 2016).

203 Calibration is undertaken by comparing, for different sponge layer lengths, L_s , and in-
 204 tegrated sponge layer strengths, $\bar{\psi}$, the wave records obtained from points upstream and
 205 downstream of the sponge layers. With the absorbing-generating layer switched off, a crest-
 206 focused wave group is first propagated from left to right through the absorbing layers, which
 207 are temporarily moved 20 m upstream so that measurements can be taken both upstream and
 208 downstream of the layers, and measurements are taken in the run-up domain as the waves are
 209 damped to zero. With the absorbing layers calibrated and moved back to the end of the tank,
 210 the reflected wave group, which is obtained from an additional simulation with no sponge

211 layers, is then propagated from right to left through the absorbing-generating layer, which
 212 is set to damp the waves to the conditions in the incident domain (in this case, still water).

213 Excellent absorption properties are achieved by setting, for all layers, $L_s = 4\lambda_p = 14.2$ m
 214 and $\bar{\psi} = 4\omega_p = 14.8$ rad/s, where λ_p is the the peak wavelength and ω_p is the peak angular
 215 frequency of the Case 2 spectrum. Following Fitzgerald *et al.* (2016), a periodic irregular
 216 wave signal with repeat period $\sim 2.17 \times 10^2$ s is then used to determine the efficacy of the
 217 sponge layer absorption by testing for repeatability in the wave record at a given gauge.
 218 Figure 2b shows the excellent agreement (RMSE $\approx 2.64 \times 10^{-4}$ m) in free surface time
 219 series obtained between subsequent repeat periods in the wave record at $x = 150$ m in the
 220 run-up domain, which confirms that the reflections from the end of the tank and submerged
 221 seabed slope are negligible.

222 3 Results

223 3.1 Comparison with the results of Trulsen *et al.* (2012) and Gramstad *et al.* (2013)

224 The three experimental cases performed at MARIN are simulated by first discretising their
 225 input spectra into 2^{14} harmonic wave components to produce irregular wave signals and
 226 corresponding linear paddle signals with repeat periods $\sim 1.67 \times 10^4$ s, 1.11×10^4 s, and
 227 1.39×10^4 s, respectively (Figs. 3a, 3b). OXBOU is then used to run each simulation for a
 228 duration of $T_d = 1.10 \times 10^4$ s with the linear dispersion coefficient tuned for optimal dis-
 229 persion: $B = 1/15$. With the three simulations complete, the wave records are compiled and
 230 the first 200 s of each is neglected, following Trulsen *et al.* (2012), which leaves, at each
 231 grid point, records of duration $\sim 8.48 \times 10^3$, 6.36×10^3 , and 5.90×10^3 peak wave periods,
 232 respectively. Figure 3c shows, for the Case 2 simulation, the convergence of the normalised
 233 mean, standard deviation, skewness, and kurtosis of the free surface elevation with number
 234 of time samples in the wave record at $x = 150$ m. Each statistic is normalised by the corre-
 235 sponding value obtained for the entire record, and it is clear that the $\sim 1.644 \times 10^6$ samples
 236 are sufficient to provide robust estimates for each experimental case.

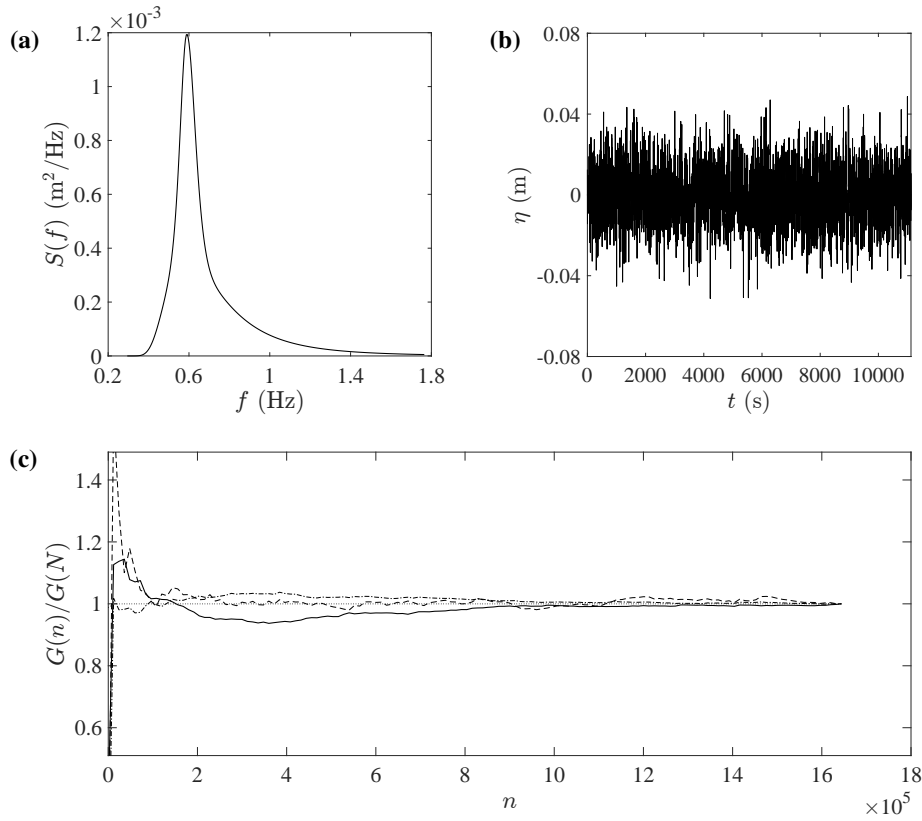


Fig. 3: Example plots from the present Case 2 simulation showing (a) the input JONSWAP spectrum, (b) the nominal input wave signal, and (c) the convergence of the statistical moments G with the number of time samples n in the wave record (which has a total of $N \approx 1.644 \times 10^6$ samples) at $x = 150$ m: mean (dotted line), standard deviation (dashed-dotted line), skewness (dashed line), and kurtosis (solid line).

237 Figure 4 then compares, for each case, the simulated variations in variance, skewness,
 238 and kurtosis along the length of the tank with those obtained from the Boussinesq-type nu-
 239 merical simulations of Gramstad et al. (2013) and the physical experiments of Trulsen et
 240 al. (2012). The results from the present Boussinesq-type simulations are shown with 95%
 241 confidence intervals determined using histograms produced by calculating the same statis-
 242 tics for 1000 bootstrap samples, which are obtained by random sampling with replacement
 243 of 5% of the available data. Although the trends for each statistic are qualitatively similar,
 244 the present profiles do not match those reported by Trulsen et al. (2012) and Gramstad et al.
 245 (2013): the skewness results are consistently lower and initially negative, and the kurtosis

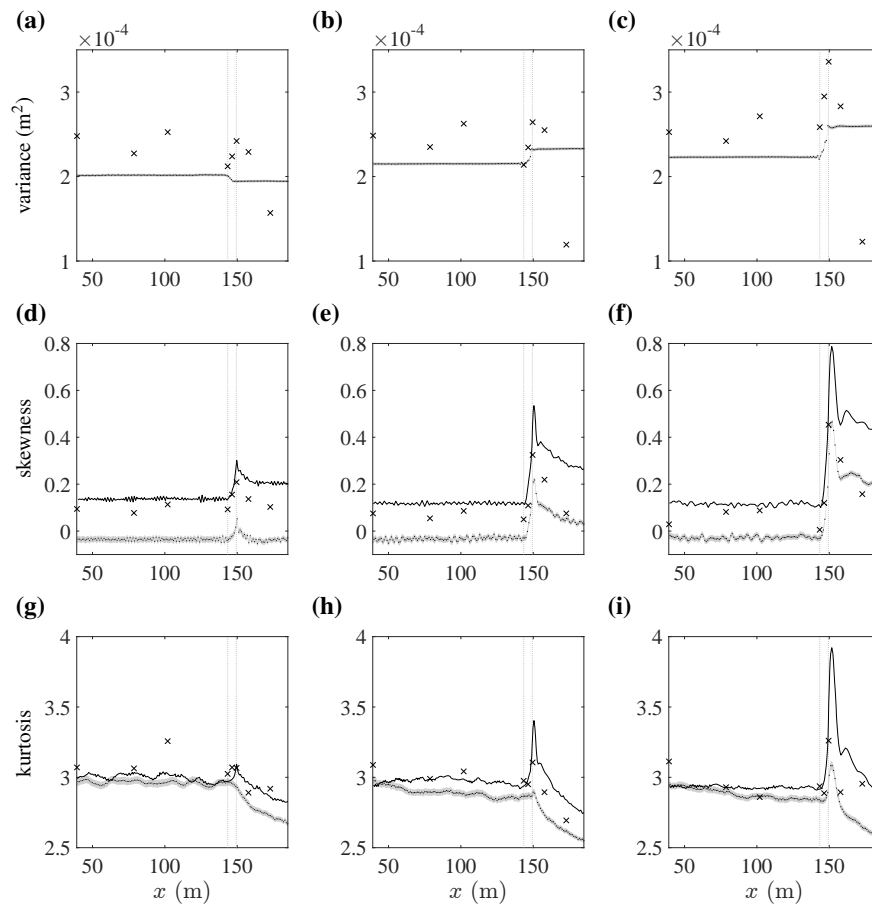


Fig. 4: Profiles of free surface elevation statistics: variance (**a**, **b**, **c**), skewness (**d**, **e**, **f**), and kurtosis (**g**, **h**, **i**) for Cases 1 (left column), 2 (centre column), and 3 (right column). Results are obtained from the physical experiments of Trulsen *et al.* (2012) (crosses), the Boussinesq-type simulations of Gramstad *et al.* (2013) (solid lines), and the present Boussinesq-type simulations (dots with 95% confidence intervals shaded in grey). The vertical dotted lines mark the positions of the toe (left) and crest (right) of the submerged seabed slope.

246 profiles exhibit greater reductions along the tank and much less prominent spikes near the
 247 crest of the submerged seabed slope.

248 3.2 Case 2 parameter study

249 To investigate these discrepancies, a parameter study based on the Case 2 simulation is used
 250 to examine the effects of various model inputs on the kurtosis profiles obtained for irreg-

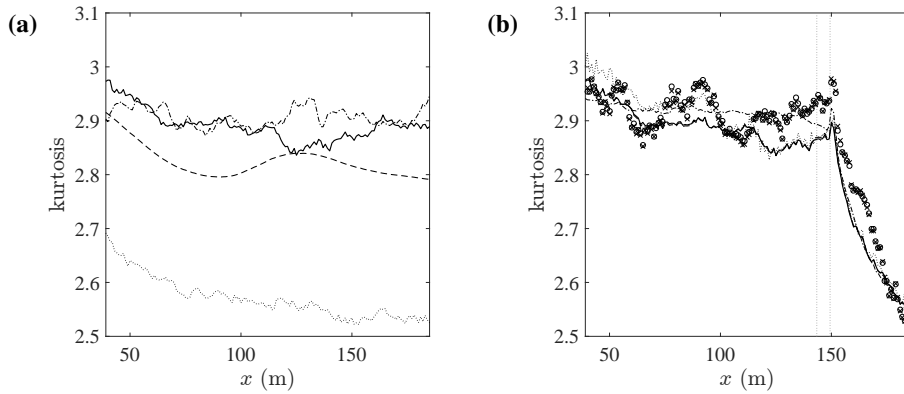


Fig. 5: Kurtosis profiles from the Case 2 parameter study. **(a)** Flat domain: still water depth, $h = 0.6$ m (solid line); narrower input spectrum (dashed line); lower input kurtosis (dashed-dotted line); and $h = 0.3$ m (dotted line). **(b)** Submerged seabed slope: single realisation (solid line); quasi-ensemble average of the single realisation divided into fifths (dashed line); ensemble average of five alternate, independent realisations (dashed-dotted line); reduced sponge layer strengths (dotted line); and shorter simulations using first- (circles) and second-order (crosses) accurate wavemaker signals.

251 ular waves propagating over a flat, horizontal bed (Fig. 5a) as well as over the submerged
 252 seabed slope (Fig. 5b). For a flat domain with still water depth $h = 0.6$ m, the kurtosis pro-
 253 file obtained for $x < 143.4$ m (Fig. 5a: solid line) is practically identical to that obtained
 254 in the Case 2 simulation (Fig. 5b: solid line), which confirms that the upstream kurtosis
 255 profile is unaffected by the reflections from the submerged slope. This flat-bed simulation
 256 also demonstrates a reduction in kurtosis along the length of the tank: the kurtosis decreases
 257 from the input value of ~ 3 and appears to stabilise at a value of ~ 2.9 towards the end of the
 258 domain. Repeating this simulation with a lower input value of kurtosis (which is done by re-
 259 placing the original wavemaker signal with the negatively skewed wave record subsequently
 260 obtained at $x = 160$ m) yields a more uniform profile, which further suggests an equilibrium
 261 kurtosis value of ~ 2.9 for this case. However, this equilibrium value is found to depend, as
 262 in earlier studies (see Janssen, 2003; Zeng & Trulsen, 2012), on both the still water depth
 263 (Fig. 5a: dotted line) and the bandwidth of the input wave spectrum (Fig. 5a: dashed line).

264 For simulations including the submerged seabed slope, the kurtosis profiles appear in-
 265 sensitive to the location of the generating-absorbing sponge layer and the end-of-tank bound-
 266 ary condition. A similar profile is also obtained when the strengths of the absorbing and
 267 absorbing-generating layers are reduced by 90% (Fig. 5b: dotted line), which implies that

268 the observed reduction in kurtosis is not the result of excess numerical damping. Dividing
269 each wave record from the Case 2 simulation into five equal sections and taking the quasi-
270 ensemble average of these fifths yields a similar profile (Fig. 5b: dashed line), as does taking
271 the ensemble average across five alternate, independent realisations (Fig. 5b: dashed-dotted
272 line). This demonstrates that the present results do not depend on the type of measurement
273 taken. Moreover, the kurtosis profiles obtained from shorter-duration (for ease of computa-
274 tion) simulations using first- and second-order accurate wavemaker signals are very similar
275 (Fig. 5b: circles; crosses), which implies that neither are the observed trends due to error
276 waves produced by the first-order accurate wavemaker (see Orszaghova *et al.*, 2014).

277 **4 Discussion and conclusions**

278 The kurtosis profiles obtained in each experimental case agree qualitatively with those of
279 Trulsen *et al.* (2012) and Gramstad *et al.* (2013) but the present numerical model is clearly
280 unable to capture accurately the spikes near the crests of the submerged seabed slopes (Figs.
281 4g, 4h, 4i). A parameter study has confirmed that the present results do not depend on the
282 type of measurement taken, the position or damping strengths of the sponge layers, or the
283 order of accuracy of the wavemaker signal (Fig. 5b). Further discrepancies are also evi-
284 dent: for the depths considered here, second-order bound harmonics are expected to posi-
285 tively skew the probability distribution function for the free surface elevation (Onorato *et al.*,
286 2005) but the present skewness results are initially negative (Figs. 4d, 4e, 4f). Replication
287 of an example irregular wave simulation with the ‘fully nonlinear’ OceanWave3D model
288 (see Engsig-Karup *et al.*, 2009) (comparison not shown for brevity) confirms that OXBOU
289 produces consistently lower values of free surface elevation skewness and kurtosis.

290 The discrepancies between the present results and those of Gramstad *et al.* (2013) most
291 likely stem from differences in the underlying momentum equations. The exact source of
292 these discrepancies, however, is difficult to determine. When examining the propagation of
293 irregular waves over a compound slope, Kashima *et al.* (2014) found that the present equa-
294 tion set returned values of skewness and kurtosis which were considerably lower than those
295 obtained in the corresponding physical experiment. These lower values were explained as

296 being the result of insufficient nonlinearity in the numerical simulations, but Gramstad et
297 al. (2013) were able to use a similar weakly nonlinear model to reproduce the results of
298 Trulsen et al. (2012). Further, in deriving the present equation set, Madsen & Sørensen
299 (1992) adopted a mild slope assumption which retained only the lowest-order spatial deriva-
300 tives of the water depth. This means that the present model is unable to capture the effects
301 of the sudden depth transition as well as that of Gramstad et al. (2013), which retains these
302 high-order terms. It is also worth noting that two of the present three experimental cases con-
303 sider water depths which exceed the depth limit ($k_p h < 1$, where k_p is the peak wavenumber
304 of the input spectrum) recommended to ensure the accuracy of the present equation set (see
305 Madsen & Sørensen, 1992, 1993).

306 Using a boundary element method with fast multipole acceleration to solve Laplace's
307 equation for potential flow with fully nonlinear boundary conditions, Zheng et al. (2020)
308 have recently predicted the local changes in the statistical properties of irregular waves
309 propagating over a range of submerged slopes in close agreement with the experiments
310 by Trulsen et al. (2012). In doing so, Zheng et al. (2020) have demonstrated that these lo-
311 cal changes are driven by second-order terms, which may help to explain why the peaks in
312 skewness and kurtosis cannot be accurately captured by the present Boussinesq-type model.
313 The present equation set includes a linear dispersion coefficient, B , which may be tuned to
314 produce either enhanced dispersion characteristics or approximately correct second-order
315 bound harmonics (Yao, 2007). Herein, B is assigned a value of 1/15 for optimal dispersion.
316 It is reasonable to assume that if the bound waves are inaccurate, significant errors in skew-
317 ness and kurtosis will arise near the sudden depth transition, because the peaks in skewness
318 and kurtosis at this location are likely a consequence of the release of second-order bound
319 waves by the depth transition (Zheng et al., 2020). Although there is no value of B which
320 can make the present equation set equivalent to that of Gramstad et al. (2013), it is possible
321 to match the linear dispersion relations by setting $B = 0.057$. However, this is found to make
322 no appreciable difference to the present results and does not address the need to correct the
323 bound waves. Frequency domain comparisons between OceanWave3D and OXBOU (again

324 not shown for brevity) demonstrate that there is also no value of B which gives satisfactory
325 agreement on sub-harmonic and super-harmonic content.

326 Modelling this sudden depth transition problem is challenging because it requires an
327 accurate yet computationally efficient numerical code which is able to incorporate the ef-
328 fects of both dispersion and nonlinearity on the evolution of the wave field. The work of
329 Gramstad *et al.* (2013) has shown that the key physics underlying this localised increase in
330 the probability of rogue wave occurrence can be captured by a weakly dispersive, weakly
331 nonlinear Boussinesq-type model. There are, however, many different sets of Boussinesq-
332 type equations and the present study demonstrates the importance of making an appropriate
333 selection. Although OXBOU is a very useful tool for modelling nearshore wave propaga-
334 tion, run-up, and overtopping, it is clear that the underlying equation set is not well-suited to
335 modelling the propagation of waves over a sudden depth transition. It is thus recommended
336 that this problem be revisited using a revised version of OXBOU based on an improved set
337 of Boussinesq-type equations. The equations of Schäffer & Madsen (1995), for instance,
338 provide the same enhanced linear dispersion characteristics as those of Madsen & Sørensen
339 (1992) but are not limited to mildly sloping seabeds. It should also be noted, however, that
340 the accuracy of any numerical model will depend on the means by which the spatial and tem-
341 poral derivatives are calculated (Borthwick *et al.*, 2006), and that sudden depth transitions
342 invariably prove challenging for any low-order finite difference scheme. Shock-capturing
343 schemes offer an alternative approach but are generally less accurate and may introduce
344 further complications.

345 In future studies, it would prove valuable to compare statistical results not only between
346 different Boussinesq-type formulations but also between weakly and highly nonlinear mod-
347 els, following Viotti & Dias (2014), Ducrozet & Gouin (2017), and Zheng *et al.* (2020), as
348 well as with physical experiments, following Zhang *et al.* (2019) and Trulsen *et al.* (2020). It
349 would also be interesting to explore whether idealised, multi-layer numerical models, such
350 as SWASH (Zijlema *et al.*, 2011), can provide additional insight. Future work should exam-
351 ine not only the extreme amplitudes but also the shapes and periods of these rogue waves,
352 which are crucial in understanding the strength of the wave impact and the resilience of ships

353 and offshore structures (Kharif et al., 2009; Adcock & Taylor, 2014). The effects of direc-
354 tionality must also be considered because large waves evolve differently in unidirectional
355 and directionally spread seas (Adcock & Taylor, 2014), and studies have shown that even
356 a small amount of counter-propagating wave energy can result in a significant reduction in
357 free surface kurtosis (Ducrozet & Gouin, 2017; Støle-Hentschel et al., 2018). Finally, real-
358 world observations should be included wherever possible in studies of rogue wave formation
359 and occurrence probability (Slunyaev et al., 2011) because it is the ocean that provides the
360 most representative conditions with which to test and revise new theories.

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370 **References**

- 371 Adcock, TAA, Taylor, PH (2014) The physics of anomalous ('rogue') ocean waves. Rep.
372 Prog. Phys. 77: 105901, <https://doi.org/10.1088/0034-4885/77/10/105901>.
- 373 Adcock, TAA, Taylor, PH, Yan, S, Ma, QW, Janssen, PAEM (2011) Did the
374 Draupner wave occur in a crossing sea? Proc. R. Soc. A 467: 3004–3021,
375 <https://doi.org/10.1098/rspa.2011.0049>.
- 376 Borthwick, AGL, Ford, M, Weston, BP, Taylor, PH, Stansby, PK (2006) Solitary wave trans-
377 formation, breaking, and run-up at a beach. Proc ICE - Marit. Engng 159(3): 97–105,
378 <https://doi.org/10.1680/maen.2006.159.3.97>.
- 379 Cavaleri, L, Barbariol, F, Benetazzo, A, Bertotti, L, Bidlot, J-R, Janssen, P, Wedi, N (2016)
380 The Draupner wave: A fresh look and the emerging view. J. Geophys. Res. Oceans 121:
381 6061–6075, <https://doi.org/10.1002/2016JC011649>.

- 382 Christou, M, Ewans, K (2014) Field measurements of rogue water waves. *J. Phys. Oceanogr.*
383 44(9): 2317–2335, <https://doi.org/10.1175/JPO-D-13-0199.1>.
- 384 Dean, RG, Dalrymple, RA (1991) *Water wave mechanics for engineers and scientists*. World
385 Scientific.
- 386 Dingemans, MW (1997) *Water wave propagation over uneven bottoms*. World Scientific.
- 387 Ducrozet, G, Gouin, M (2017) Influence of varying bathymetry in rogue wave occurrence
388 within unidirectional and directional sea-states. *J. Ocean Engng Mar. Energy* 3: 309–324,
389 <https://doi.org/10.1007/s40722-017-0086-6>.
- 390 Dysthe, K, Krogstad, HE, Müller, P (2008) Oceanic rogue waves. *Annu. Rev. Fluid Mech.*
391 40: 287–310, <https://doi.org/10.1146/annurev.fluid.40.111406.102203>.
- 392 Engsig-Karup, AP, Bingham, HB, Lindberg, O (2009) An efficient flexible-order
393 model for 3D nonlinear water waves. *J. Comp. Phys.* 228(6): 2100–2118,
394 <https://doi.org/10.1016/j.jcp.2008.11.028>.
- 395 Fallahi, S (2016) *Freak waves over nonuniform depth with different slopes*. Master's thesis,
396 University of Oslo, Norway.
- 397 Fedele, F, Brennan, J, Ponce De León, S, Dudley, J, Dias, F (2016) Real world
398 ocean rogue waves explained without the modulational instability. *Sci. Rep.* 6: 27715,
399 <https://doi.org/10.1038/srep27715>.
- 400 Fitzgerald, CJ, Taylor, PH, Orszaghova, J, Borthwick, AGL, Whittaker, C, Raby, AC (2016)
401 Irregular wave runup statistics on plane beaches: Application of a Boussinesq-type model
402 incorporating a generating-absorbing sponge layer and second-order wave generation.
403 *Coast. Engng* 114: 309–324, <http://dx.doi.org/10.1016/j.coastaleng.2016.04.019>.
- 404 Gramstad, O, Zeng, H, Trulsen, K, Pedersen, GK (2013) Freak waves in weakly nonlinear
405 unidirectional wave trains over a sloping bottom in shallow water. *Phys. Fluids* 25(12):
406 122103, <https://doi.org/10.1063/1.4847035>.
- 407 Haver, S, Andersen, OJ (2000) Freak waves: Rare realizations of a typical population or typ-
408 ical realizations of a rare population? In: *Proceedings of the 10th International Offshore*
409 *and Polar Engineering Conference*, Seattle, WA, USA.
- 410 Holthuijsen, LH (2007) *Waves in oceanic and coastal waters*. Cambridge University Press.

- 411 Janssen, PAEM (2003) Nonlinear four-wave interactions and freak
412 waves. *J. Phys. Oceanogr.* 33(4): 863–884, [https://doi.org/10.1175/1520-0485\(2003\)33<863:NFIAFW>2.0.CO;2](https://doi.org/10.1175/1520-0485(2003)33<863:NFIAFW>2.0.CO;2).
- 414 Janssen, TT, Herbers, THC (2009) Nonlinear wave statistics in a focal zone. *J. Phys. Oceanogr.* 39(8): 1948–1964, <https://doi.org/10.1175/2009JPO4124.1>.
- 416 Kashima, H, Hirayama, K, Mori, N (2014) Estimation of freak wave occurrence from deep to shallow water regions. *Coast. Engng Proc.* 1(34): 36, <https://doi.org/10.9753/icce.v34.waves.36>.
- 419 Kharif, C, Pelinovsky, E, Slunyaev, A (2009) *Rogue waves in the ocean*. Springer Science & Business Media.
- 421 Ma, Y, Dong, G, Ma, X (2014) Experimental study of statistics of random waves propagating over a bar. *Coast. Engng Proc.* 1(34): 30, <https://doi.org/10.9753/icce.v34.waves.30>.
- 423 Ma, Y, Ma, X, Dong, G (2015) Variations of statistics for random waves propagating over a bar. *J. Mar. Sci. Tech.* 23(6): 864–869, <https://doi.org/10.6119/JMST-015-0610-3>.
- 425 Madsen, PA, Sørensen, OR (1992) A new form of the Boussinesq equations with improved linear dispersion characteristics. Part 2. A slowly-varying bathymetry. *Coast. Engng* 18(3–4): 183–204, [https://doi.org/10.1016/0378-3839\(92\)90019-Q](https://doi.org/10.1016/0378-3839(92)90019-Q).
- 428 Madsen, PA, Sørensen, OR (1993) Bound waves and triad interactions in shallow water. *Ocean Engng* 20(4): 359–388, [https://doi.org/10.1016/0029-8018\(93\)90002-Y](https://doi.org/10.1016/0029-8018(93)90002-Y).
- 430 Mori, N, Janssen, PAEM (2006) On kurtosis and occurrence probability of freak waves. *J. Phys. Oceanogr.* 36(7): 1471–1483, <https://doi.org/10.1175/JPO2922.1>.
- 432 Ochi, MK (2005) *Ocean waves: The stochastic approach*. Cambridge University Press.
- 433 Onorato, M, Osborne, AR, Serio, M (2005) On deviations from Gaussian statistics for surface gravity waves. arXiv preprint nlin/0503071, <https://arxiv.org/pdf/nlin/0503071.pdf>.
- 435 Onorato, M, Osborne, AR, Serio, M, Cavaleri, L, Brandani, C, Stansberg, CT (2004) Observation of strongly non-Gaussian statistics for random sea surface gravity waves in wave flume experiments. *Phys. Rev. E* 70: 067302, <https://doi.org/10.1103/PhysRevE.70.067302>.
- 438

- 439 Onorato, M, Residori, S, Bortolozzo, U, Montina, A, Arecchi, FT (2013) Rogue waves and
440 their generating mechanisms in different physical contexts. *Phys. Rep.* 528(2): 47–89,
441 <https://doi.org/10.1016/j.physrep.2013.03.001>.
- 442 Orszaghova, J (2011) Solitary waves and wave groups at the shore. DPhil thesis, University
443 of Oxford, UK.
- 444 Orszaghova, J, Borthwick, AGL, Taylor, PH (2012) From the paddle to the beach – A
445 Boussinesq shallow water numerical wave tank based on Madsen and Sørensen’s equa-
446 tions. *J. Comp. Phys.* 231(2): 328–344, <https://doi.org/10.1016/j.jcp.2011.08.028>.
- 447 Orszaghova, J, Taylor, PH, Borthwick, AGL, Raby, AC (2014) Importance of second-order
448 wave generation for focused wave group run-up and overtopping. *Coast. Engng* 94(2):
449 63–79, <http://doi.org/10.1016/j.coastaleng.2014.08.007>.
- 450 Peregrine, DH (1967) Long waves on a beach. *J. Fluid Mech.* 27(4): 815–827,
451 <https://doi.org/10.1017/S0022112067002605>.
- 452 Raustøl, A (2014) Freake bølger over variabelt dyp. Master’s thesis (in Norwegian), Univer-
453 sity of Oslo, Norway.
- 454 Schäffer, HA (1996) Second-order wavemaker theory for irregular waves. *Ocean Engng*
455 23(1): 47–88, [https://doi.org/10.1016/0029-8018\(95\)00013-B](https://doi.org/10.1016/0029-8018(95)00013-B).
- 456 Schäffer, HA, Madsen, PA (1995) Further enhancements of Boussinesq-type equations.
457 *Coast. Engng* 26(1–2): 1–14, [https://doi.org/10.1016/0378-3839\(95\)00017-2](https://doi.org/10.1016/0378-3839(95)00017-2).
- 458 Sergeeva, A, Pelinovsky, E, Talipova, T (2011) Nonlinear random wave field in shallow
459 water: Variable Korteweg-de Vries framework. *Nat. Hazards Earth Syst. Sci.* 11: 323–
460 330, <https://doi.org/10.5194/nhess-11-323-2011>.
- 461 Slunyaev, A, Didenkulova, I, Pelinovsky, E (2011) Rogue waters. *Contemp. Phys.* 52(6):
462 571–590, <https://doi.org/10.1080/00107514.2011.613256>.
- 463 Støle-Hentschel, S, Trulsen, K, Rye, LB, Raustøl, A (2018) Extreme wave statistics
464 of counter-propagating, irregular, long-crested sea states. *Phys. Fluids* 30: 067102,
465 <https://doi.org/10.1063/1.5034212>.
- 466 Trulsen, K (2018) Rogue waves in the ocean, the role of modulational instability, and abrupt
467 changes of environmental conditions that can provoke non equilibrium wave dynamics.

- 468 In: *The ocean in motion*. Springer Oceanography.
- 469 Trulsen, K, Raustøl, A, Jorde, S, Rye, LB (2020) Extreme wave statis-
470 tics of long-crested irregular waves over a shoal. *J. Fluid Mech.* 882: R2,
471 <http://dx.doi.org/10.1017/jfm.2019.861>.
- 472 Trulsen, K, Zeng, H, Gramstad, O (2012) Laboratory evidence of freak
473 waves provoked by non-uniform bathymetry. *Phys. Fluids* 24(9): 097101,
474 <http://dx.doi.org/10.1063/1.4748346>.
- 475 Tucker, MJ, Challenor, PG, Carter, DJT (1984) Numerical simulation of a random sea: A
476 common error and its effect upon wave group statistics. *Appl. Ocean Res.* 6(2): 118–122,
477 [https://doi.org/10.1016/0141-1187\(84\)90050-6](https://doi.org/10.1016/0141-1187(84)90050-6).
- 478 Viotti, C, Dias, F (2014) Extreme waves induced by strong depth transitions: Fully nonlinear
479 results. *Phys. Fluids* 26(5): 051705, <https://doi.org/10.1063/1.4880659>.
- 480 Yao, Y (2007) Boussinesq-type modelling of gently shoaling extreme ocean waves. DPhil
481 thesis, University of Oxford, UK.
- 482 Zeng, H, Trulsen, K (2012) Evolution of skewness and kurtosis of weakly nonlinear uni-
483 directional waves over a sloping bottom. *Nat. Hazards Earth Syst. Sci.* 12(3): 631–638,
484 <https://doi.org/10.5194/nhess-12-631-2012>.
- 485 Zhang, J, Benoit, M, Kimmoun, O, Chabchoub, A, Hsu, H-C (2019) Statistics of extreme
486 waves in coastal waters: Large scale experiments and advanced numerical simulations.
487 *Fluids* 4(2): 99, <https://doi.org/10.3390/fluids4020099>.
- 488 Zheng, Y, Lin, Z, Li, Y, Adcock, TAA, Li, Y, van den Bremer, TS (2020) Fully nonlinear
489 simulations of extreme waves provoked by strong depth transitions: The effect of slope.
490 *Phys. Rev. Fluids* 5: 064804, <https://doi.org/10.1103/PhysRevFluids.5.064804>.
- 491 Zijlema, M, Stelling, G, Smit, P (2011) SWASH: An operational public domain code for
492 simulating wave fields and rapidly varied flows in coastal waters. *Coast. Engng* 58(10):
493 992–1012, <https://doi.org/10.1016/j.coastaleng.2011.05.015>.