



THE UNIVERSITY *of* EDINBURGH

## Edinburgh Research Explorer

### Limited commitment models of the labour market

**Citation for published version:**

Thomas, J & Worrall, T 2007 'Limited commitment models of the labour market' Keele economics research paper , Keele University .

**Link:**

[Link to publication record in Edinburgh Research Explorer](#)

**Document Version:**

Peer reviewed version

**Publisher Rights Statement:**

© Thomas, J., & Worrall, T. (2007). Limited commitment models of the labour market. (Keele economics research paper ). Keele University . 10.1111/j.1467-9485.2007.00440.x

**General rights**

Copyright for the publications made accessible via the Edinburgh Research Explorer is retained by the author(s) and / or other copyright owners and it is a condition of accessing these publications that users recognise and abide by the legal requirements associated with these rights.

**Take down policy**

The University of Edinburgh has made every reasonable effort to ensure that Edinburgh Research Explorer content complies with UK legislation. If you believe that the public display of this file breaches copyright please contact [openaccess@ed.ac.uk](mailto:openaccess@ed.ac.uk) providing details, and we will remove access to the work immediately and investigate your claim.



# Limited Commitment Models of the Labour Market\*

Jonathan P. Thomas (University of Edinburgh)<sup>†</sup>

Tim Worrall (Keele University)<sup>‡</sup>

Version: July 2007

## Abstract

We present an overview of models of long-term self-enforcing labour contracts in which risk-sharing is the dominant motive for contractual solutions. A base model is developed which is sufficiently general to encompass the two-agent problem central to most of the literature, including variable hours. We consider two-sided limited commitment and look at its implications for aggregate labour market variables. We consider the implications for empirical testing and the available empirical evidence. We also consider the one-sided limited commitment problem for which there exists a considerable amount of empirical support.

KEYWORDS: Labour contracts, self-enforcing contracts, business cycle, unemployment.

JEL CODES: E32, J41.

---

\*We gratefully acknowledge the financial support of the Economic and Social Research Council (Research Grant:RES-000-23-0865). We also thank Jim Malcomson, Leena Rudanko and Dongyun Shin for helpful comments on an earlier draft.

<sup>†</sup>Jonathan P. Thomas: Management School and Economics, William Robertson Building, University of Edinburgh, 50 George Square, Edinburgh EH8 9JY. E-Mail: [jonathan.thomas@ed.ac.uk](mailto:jonathan.thomas@ed.ac.uk). Web: <http://homepages.ed.ac.uk/jthomas1>.

<sup>‡</sup>Tim Worrall: School of Economic and Management Studies, Keele University, Keele ST5 5BG. E-Mail: [t.s.worrall@econ.keele.ac.uk](mailto:t.s.worrall@econ.keele.ac.uk). Web: <http://www.keele.ac.uk/depts/ec/cer/timworrall.htm>.

# 1 Introduction

In this paper we consider long-term risk-sharing<sup>1</sup> labour contracts under limited commitment. Firms and workers are allowed to sign, or implicitly agree to, contingent contracts but also to renege on these contracts when it is to their advantage. That is to say, there are no courts to enforce contracts and low mobility or “lock-in” costs. We first develop a general framework for analysing contracts in this class of repeated interactions. The logic of these contracts follows that of repeated games, in that a party called upon to sacrifice current utility to maintain the insurance is prepared to do so in anticipation of receiving reciprocal benefits in the future. However in general first-best risk sharing cannot be achieved, and it is what happens in the second best contracts which is of particular interest. What then follows is a selective overview of the existing literature that considers both the implications for empirical testing and summarises the available empirical evidence.

The study of long-term labour contracts with limited commitment is important because other standard models of the labour market cannot easily account for observed patterns in the data. The data typically show that real wages are only weakly correlated with productivity or even mildly countercyclical. Hours on the other hand are found to be quite strongly positively correlated with productivity. To match this observed pattern in the data using standard real business cycle models requires a very high intertemporal elasticity of substitution for labour supply that is not supported by estimates from micro data. Recently Shimer (2005) has suggested that standard search models under-predict the volatilities of vacancies and unemployment because of the flexibility of wage responses to productivity under Nash bargaining unless implausibly large shocks for productivity are assumed. We therefore consider some of the available empirical evidence on whether these puzzles might be resolved within the limited commitment labour contracting model.

We start by developing a basic two agent (worker/firm) model in which either agent can quit the relationship at any time either at a positive or zero cost. The agents agree initially to a contingent sequence of wages (and potentially a termina-

---

<sup>1</sup>Thus we do not consider the other much analysed motive for contracting, namely to protect relationship specific investments from opportunistic behaviour. For a discussion of this, see MacLeod (2007, Current Issue).

tion rule) which satisfies certain incentive or participation constraints. The outside environment is summarised by the evolution of the respective outside options for the two agents. The basic characterisation of second-best contracts can then be applied to specific models, and we do this to summarise the existing theoretical work in the area. In the development of the model we do not use the dynamic programming framework that is usually employed for this environment, but instead show that the model can be solved by using local variational arguments, thus avoiding the need to establish a number of technical properties of value functions.

Although the basic characterisations of the second-best contracts have been known for some time, in the second part of the paper we consider how the outside options of the agents can be made endogenous in search equilibrium models or competitive models with perfect labour mobility. There has been a recent upsurge of interest in applications of this type of model to macroeconomics, and of testing of the model particularly in the one-sided limited commitment case where workers are mobile but firms can commit. We summarise the main findings of the literature and the empirical evidence which is generally very supportive of the one-sided model.

## 2 A general model of limited commitment

This section considers a general model of limited commitment. We first derive the implications of optimum contracting in a simple model with fixed hours and consider the test of this model by Macis (2006). Section 2.2 considers the modification of the model when hours are variable and reports on the results of Beaudry & DiNardo (1995) on the implied negative correlation between hours and wages. Section 2.3 examines the implications of quitting or renegeing costs.

### 2.1 A baseline model

The model is as follows.<sup>2</sup> There is an infinite horizon,  $t = 1, 2, 3 \dots \infty$ . Workers are risk averse with per period twice differentiable utility function  $u(c)$ ,  $u' > 0$ ,  $u'' < 0$ ,

---

<sup>2</sup>A description of a general limited commitment model of risk-sharing can be found in Ljungqvist & Sargent (2004, Chapter 20).

where  $c \geq 0$  is the income/consumption of the single good received within the period; crucially, it is assumed that they cannot make capital market transactions, so the only possibility for consumption smoothing across states of nature or over time arises if the firm provides insurance. There is no disutility of work, but hours are fixed so that workers are either employed or unemployed (although we relax the assumption of fixed hours below). The firm is assumed to be risk neutral. We consider a single match between one worker and one firm,<sup>3</sup> and for the moment we do not need to fill in the details of the outside environment. There is perfect information within the match. We suppose that output at time  $t$  within this match is  $z(s_t) \geq 0$ , where  $s_t$  is the current state of nature.<sup>4</sup> The state of nature  $s_t$  follows a time-homogeneous Markov process, with finite state space  $S$ , and initial distribution  $p$  over  $S$ , and from state  $s$  state  $r \in S$  is reachable next period with transition probability:  $\pi_{sr} \geq 0$ . Let  $h_t := (s_1, s_2, \dots, s_t)$  be the history at  $t$ . Workers and firms discount the future with common discount factor  $\beta \in (0, 1)$ .

At the start of date 1, after the initial state  $s_1$  is observed,<sup>5</sup> the firm offers the worker a contract  $(w_t(h_t))_{t=1}^T = ((w_1(s_1), w_2(s_1, s_2), w_3(s_1, s_2, s_3), \dots))$ , where  $w_t(h_t) \geq 0$  is the wage at  $t$  after history  $h_t$ , and  $T > 1$  is the (random) date at which the contract is terminated.<sup>6</sup> The within period timing is as follows. At the start of each period, both agents observe the current state of nature,  $s_t$ . At this point either party can quit and take their outside option. Otherwise, they trade at the agreed terms, in which case the value of output  $z(s_t)$  is realised, and the firm then makes a wage payment according to the contract. (Thus we do not allow for example for the firm to renege on its wage payment after the worker has contributed

---

<sup>3</sup>That is we shall treat contracts between each firm and worker separately. The case where contracts with different workers cannot be treated separately is studied in Martins, Snell & Thomas (2005) and Snell & Thomas (2006) and discussed briefly in Section 2.3.

<sup>4</sup>We do not identify the state of nature directly with productivity,  $z$ , as it may be that other firms face different productivity shocks, and so the outside options will not be determined by the match productivity. In fact, even if there is a common productivity shock, because optimal contracts do not necessarily express current wages as a function of only the current state, the outside option may not be a function of the current state only (although in most models the payoff from a new contract, and hence the outside option, will only depend on the current state).

<sup>5</sup>If matches also start at later dates, the characterisation developed below, which depends only on the state prevailing at the time the contract starts, is the same.

<sup>6</sup>So that at  $t = T$ , after observing the current state  $s_t$ , the partnership dissolves and both agents get their outside options.  $T$  is a random variable (a stopping time) so that the length of the contract will in general depend on the history of shocks. At this level of generality, termination must be allowed for as there may be no continuation values that satisfy participation constraints.

to output.) The value (discounted utility) of the outside option for the worker and firm respectively is denoted by  $\chi_w(s)$  and  $\chi_f(s)$  in state  $s$ .<sup>7</sup> We assume that  $\chi_w(s) - C_w > u(0) / (1 - \beta)$ .

Let  $V_t(h_t)$  denote the continuation utility from  $t$  onwards from the contract (assuming it does not terminate at  $t$ ):

$$V_t(h_t) := u(w_t(h_t)) + E \left[ \sum_{t'=t+1}^{T-1} \beta^{t'-t} u(w_{t'}(h_{t'})) + \beta^{T-t} \chi_w(s_T) \mid h_t \right], \quad (1)$$

where  $E$  denotes expectation. Likewise the firm's continuation profit is

$$\Pi_t(h_t) := z(s_t) - w_t(h_t) + E \left[ \sum_{t'=t+1}^{T-1} \beta^{t'-t} (z(s_{t'}) - w_{t'}(h_{t'})) \mid h_t \right]. \quad (2)$$

The contract is said to be *self-enforcing* if the following hold for all dates  $t$ ,  $T - 1 \geq t \geq 1$ , and for all positive probability  $h_t$  (with initial state  $s_1$ ):

$$V_t(h_t) \geq \chi_w(s_t) - C_w, \quad (3)$$

$$\Pi_t(h_t) \geq \chi_f(s_t) - C_f, \quad (4)$$

where  $C_f$  and  $C_w$  are respective directly incurred quitting/mobility costs for the firm and worker.<sup>8</sup> Inequality (3) is the worker's *participation constraint* that says that at any point in the future the contract must offer at least what a worker can get by quitting, net of quitting costs, while (4) is the corresponding constraint for the firm.<sup>9</sup>

---

<sup>7</sup>In much of the existing literature it is assumed that competition among firms drives profits to zero from new matches so  $\chi_f = 0$ . Even with competition, if other inputs such as capital were included in the firm's profits, then the participation constraint for the firm would require that it covers capital costs. This would make the firm's outside option state dependent if say, the interest rate varied with the state. See Calmès (2007) for a model including a fixed capital component where the firm's outside option is state dependent.

<sup>8</sup>Either party can initiate termination, but both suffer the costs. We assume that these are also incurred if the contract is terminated by agreement (i.e. at  $t = T$ ), so they are costs which cannot be avoided on match break-up. It would be equivalent to factor these directly into outside options. See Section 2.3 for discussion of alternative assumptions.

<sup>9</sup>It is also possible to introduce hiring costs for the firm. The contract dynamics do not depend on whether there are hiring costs (unlike quitting costs which may potentially affect the contract dynamics, see Section 2.3 below), but apply as soon as a relationship is established. Thus if the firm incurs hiring costs to establish a relationship, to judge the profitability of the relationship it would have to subtract them from whatever surplus it makes once the relationship is established in the manner to be described.

We are interested in *constrained efficient contracts*, that is to say contracts which are self enforcing and are not Pareto dominated by any other self-enforcing contracts. Efficient contracts are thus solutions to the following problem:

$$\max_{(w_t(h_t))_{t=1}^T} \Pi_1(h_1) \quad \text{Problem A}$$

subject to (3), (4), and

$$V_1(h_1) \geq \bar{V}_1. \quad (5)$$

The term  $\bar{V}_1$  measures how much utility the worker gets from the relationship, and as this is varied across feasible values (i.e. values for which self-enforcing contract exist), all efficient contracts are traced out.<sup>10</sup>

**Lemma 1** *In an efficient contract in which the firm's (worker's) participation constraint is slack at  $t + 1$ , wages cannot fall (rise) between  $t$  and  $t + 1$ .*

**Proof.** Suppose we are at  $h_t$ , and suppose that the firm's participation constraint at  $t + 1$  in some state  $s$  is not binding. By assumption the contract is not terminated at  $t + 1$  (otherwise the constraint would trivially bind). Consider, starting from the optimal contract, reshuffling wages between  $t$ , and  $t + 1$  in state  $s$ , to *backload* them (assume  $w_t > 0$ ). Increase the wage at  $t + 1$  after state  $s$  by a small amount  $\Delta$ , and cut the wage at  $t$  by  $x$  so as to leave the worker indifferent; do not change the contract otherwise:

$$\pi_{s_t s} \beta u'(w_{t+1}(h_t, s)) \Delta - u'(w_t(h_t)) x \simeq 0.$$

This backloading satisfies all worker participation constraints since the worker's utility rises at  $t + 1$ , and so even if her constraint were binding, it will not be violated; at  $t$  her constraint holds as her utility is unchanged, and likewise it is unchanged earlier since utility is held constant over the two periods. The change in profits (viewed from  $h_t$ ) is

$$-\pi_{s_t s} \beta \Delta + x \simeq -\pi_{s_t s} \beta \Delta + \frac{\pi_{s_t s} \beta u'(w_{t+1}(h_t, s)) \Delta}{u'(w_t(h_t))},$$

---

<sup>10</sup>The issue of existence of solutions to this problem for feasible  $\bar{V}_1$  is standard in this environment.

which is positive for  $\Delta$  small enough if

$$\frac{u'(w_{t+1}(h_t, s))}{u'(w_t(h_t))} > 1. \quad (6)$$

If (6) holds (so that wages are falling), then the backloading would raise profits at  $t$ , so the firm's participation constraint would hold at  $t$ , and at  $t + 1$  by assumption the firm's participation constraint is slack, so a small change to the wage will not violate it. Thus all constraints are satisfied by this change, and profits have increased, contrary to the optimality of the original contract. So (6) cannot hold: marginal utility growth cannot be positive, or equivalently, *wages cannot fall*. By a symmetric argument if the worker's participation constraint is slack at  $t + 1$ , then wages cannot rise between  $t$  and  $t + 1$ .<sup>11</sup> ■

Next, we need to characterise more precisely what happens to the wage when one of the participation constraints binds. First, let  $(w_t(\bar{V}_1; s))_{t=1}^T$  be an optimal contract in Problem A starting from state  $s_1 = s$ . This must deliver precisely  $\bar{V}_1$  to the worker, otherwise we can cut the period 1 wage without violating the worker's constraint, thus increasing profits.<sup>12</sup> We define  $\underline{w}_s := w_1(\chi_w(s) - C_w; s)$ , i.e. the period 1 wage specified by an optimal contract starting in state  $s$  which delivers exactly the worker's net outside option,  $\bar{V}_1 = \chi_w(s) - C_w$ . It must be unique by a simple convexity argument (see below). A key observation is the following: it must be optimal *at any date*  $t$  in state  $s$  to set  $w_t = \underline{w}_s$  whenever  $V_t(h_t) = \chi_w(s) - C_w$ . This follows from the fact that the future distribution over states depends only on  $s$ , and that the continuation contract must itself be optimal (otherwise replacing the continuation contract by a lower cost one which delivered the same continuation utility would reduce the initial costs but satisfy all participation constraints). *Thus,  $\underline{w}_s$  is the wage in state  $s$  at any  $t$  if the participation constraint is binding.* Similarly define  $\bar{w}_s$  to be the period 1 wage specified by an optimal contract starting in state  $s$  which delivers profits of exactly  $\chi_f(s) - C_f$ .

---

<sup>11</sup>Suppose that the strict opposite of (6) holds,  $u'(w_{t+1}(h_t, s))/u'(w_t(h_t)) < 1$ . The reverse argument can be used: frontloading would be profitable but it might violate the worker's  $t + 1$  participation constraint, since wages fall at this point. In other words, if the worker's participation constraint is slack at  $t + 1$ , wages cannot rise, but we cannot rule out a rise when this constraint is binding.

<sup>12</sup>Provided  $w_1 > 0$ ; otherwise, since it is assumed that the outside option dominates zero consumption for ever, it is easily shown that there must be a point in the future at which  $w_t > 0$  and the worker's constraint is not binding, so wages can be cut at this point instead.



It can then be established that if an optimal contract offers a higher utility, then it must offer a higher first period wage:

**Lemma 2** *If  $V' > V$ , then  $w_1(V'; s) > w_1(V; s)$ .*

**Proof.** Assume otherwise, so that  $w_1(V'; s) \leq w_1(V; s)$ . Suppose at some point in the future on some path  $h_t$  that  $w_t(V'; s) > w_t(V; s)$  for the first time, and the discounted utility from  $t$  is higher in the  $V'$  contract. This  $h_t$  must exist as the  $V'$  contract offers higher utility. This implies that wage growth between  $t - 1$  and  $t$  is greater in the  $V'$  case, which from Lemma 1 can only be true if one or both of the following occur: (i) the worker's participation constraint binds at  $t$  for the  $V'$  contract; (ii) the firm's constraint binds in the  $V$  contract. In case (i) in the  $V'$  contract, wages are weakly lower than in the  $V$  contract until minimum continuation utility is obtained (so the  $V$  contract cannot offer less from this point); thus discounted utility cannot be greater in the  $V'$  contract, contrary to assumption. In case (ii) in the  $V'$  contract, wages are weakly lower than in the  $V$  contract until maximum continuation utility is obtained in the  $V$  contract, again contrary to assumption. ■

**Proposition 3** *An optimal contract evolves according to the following updating rule. In state  $s \in S$  either (a) the contract (always) terminates, or (b) there is associated a minimum and a maximum wage,  $\underline{w}_s$  and  $\bar{w}_s$  respectively ( $\underline{w}_s \leq \bar{w}_s$ ), such that in an optimal contract if at date  $t + 1$  state  $s_{t+1}$  occurs then  $w_{t+1}$  is updated from  $w_t$  by*

$$w_{t+1} = \begin{cases} \bar{w}_{s_{t+1}} & \text{if } w_t > \bar{w}_{s_{t+1}}, \\ w_t & \text{if } w_t \in [\underline{w}_{s_{t+1}}, \bar{w}_{s_{t+1}}], \\ \underline{w}_{s_{t+1}} & \text{if } w_t < \underline{w}_{s_{t+1}}. \end{cases}$$

**Proof.** If there exist self-enforcing continuation utilities from  $s$  (i.e. if a self-enforcing contract exists) then by definition an efficient contract should continue as each player gets at least their outside option, and cannot be worse off. Otherwise termination must occur. Thus we assume w.l.o.g. that termination does not occur at  $s$  for the remainder of the proof. We start by showing that  $\underline{w}_s$  is unique. Suppose otherwise: then there are two distinct contracts that deliver  $\chi_w(s) - C_w$  to a worker,

both of which satisfy participation constraints and yield the same costs. Take a strict convex combination of these two contracts (i.e., a convex combination of wages at each  $h_t$ ). From (1) and the concavity of  $u(\cdot)$  it is clear this increases a worker's utility, and satisfies the participation constraint at each point. Costs are linear in wages, and hence are unchanged. Thus a small reduction in the initial wage (in state  $s$ ) will still satisfy participation, and will lead to lower costs, a contradiction. So  $\underline{w}_s$  is unique. Likewise  $\bar{w}_s$  is unique. Moreover, by Lemma 2,  $\underline{w}_s \leq \bar{w}_s$  since a contract that delivers the firm  $-C_f$  (i.e., corresponding to  $\bar{w}_s$ ) must deliver the worker at least  $\chi_w(s) - C_w$  (i.e. corresponding to  $\underline{w}_s$ ), otherwise the worker's participation constraint would be violated. Next, suppose that  $\underline{w}_{s_{t+1}} < \bar{w}_{s_{t+1}}$  and  $w_t \in (\underline{w}_{s_{t+1}}, \bar{w}_{s_{t+1}})$ . If the worker's participation constraint at  $t + 1$  in state  $s_{t+1}$  binds,  $w_{t+1} = \underline{w}_{s_{t+1}}$ , i.e., wages fall (as  $w_t > \underline{w}_{s_{t+1}}$ ), but then the firm's constraint is slack ( $w_{t+1} \neq \bar{w}_{s_{t+1}}$ ), so this contradicts Lemma 1 which asserts that wages do not fall. Thus the worker's constraint does not hold, and we know from Lemma 1 that wages cannot rise. Likewise as  $w_t < \bar{w}_{s_{t+1}}$  the worker's constraint cannot bind, and wages cannot fall. Thus for  $w_t \in (\underline{w}_{s_{t+1}}, \bar{w}_{s_{t+1}})$ , wages remain constant. Conversely, if  $w_t \leq \underline{w}_{s_{t+1}}$ , then if the worker's constraint does not hold ( $V_{t+1} > \chi_w(s_{t+1}) - C_w$ ), by Lemma 1 wages cannot rise, so  $w_{t+1} \leq \underline{w}_{s_{t+1}}$ . However,  $V_{t+1} > \chi_w(s_{t+1}) - C_w$  would imply by Lemma 2 (comparing with the contract that delivers  $\chi_w(s_{t+1}) - C_w$ ) that  $w_{t+1} > \underline{w}_{s_{t+1}}$ , a contradiction. So the constraint binds and  $w_{t+1} = \underline{w}_{s_{t+1}}$ . A symmetrical argument establishes that  $w_{t+1} = \bar{w}_{s_{t+1}}$  if  $w_t > \bar{w}_{s_{t+1}}$ . ■

Thus wages evolve in a simple fashion: they remain constant unless this takes the wage outside the interval of efficient wages  $[\underline{w}_s, \bar{w}_s]$  for the current state, in which case the wage changes by the minimum amount needed to bring it into this interval. The only thing remaining to be determined is the initial wage,  $w_1(s_1)$ . This will be determined by  $\bar{V}_1$  in Problem A, and this can in turn be thought of as depending on the bargaining strengths of the two parties or the initial outside options of the two parties. By varying the initial wage all possible splits of the joint surplus will be traced out.<sup>13</sup>

---

<sup>13</sup>For an intuitive derivation of a version of this proposition, including a discussion of when termination occurs, see Malcomson (1999).

The state-dependent wage intervals  $[\underline{w}_s, \bar{w}_s]$  will in general depend on all the parameters of the model including the worker's preferences and the stochastic process for productivity. However, the outside options and the quitting and mobility costs  $C_w$  and  $C_f$ , will also play a crucial role in the determination of these interval endpoints. We will provide the more specific assumptions in various models as we encounter them. In the first paper to analyse a problem of this type, Thomas & Worrall (1988), it is assumed that  $C_w$  and  $C_f$  are zero, and if a worker reneges, thereafter he or she can find work only at the spot market wage, where because of competition among firms, the wage equals current productivity  $z(s_t)$  (which is assumed to be a common shock across all firms). Similarly, if a firm reneges it is assumed it can hire at the spot market rate. This may be motivated as follows. Suppose there are, in addition to infinitely lived workers and firms, at each date  $m$  workers and  $n$  firms,  $n > m$ , who live for only one period. Since there is no enforcement mechanism and no mobility costs, the one-period-lived agents trade at the spot market wage. The infinitely lived agents are competitive and thus treat these spot market wages as given. This is then in line with reputation models of repeated games, and corresponds to the most severe credible punishment. It requires that when an agent reneges he is observed by everyone else, and once he has reneged he has proved himself unreliable and no one will sign a contract with him again. Likewise for a firm which has reneged in the past. The implication is that a worker who reneges will receive a consumption stream equal to productivity at each date, and so  $\chi_w(s)$  equals the discounted expected utility generated by this stream.

The most direct testing of the implications of this two-sided model is by Macis (2006), using Italian panel data on a sample of 1500 firms which includes detailed information on all workers with these firms.<sup>14</sup> He conducts a number of tests. One test, which extends the approach of Beaudry & DiNardo (1991) discussed in Section 3.2 below, is to allow both the best and worst realisations of outside opportunities (proxied by unemployment rates) since the start of the employment relationship. Controlling for current outside opportunities, and other observable characteristics

---

<sup>14</sup>It should be noted that it is difficult to distinguish the limited commitment hypothesis from that of efficient incomplete contracts to overcome hold-up when there are exogenous switching costs (MacLeod & Malcomson (1993); see also Malcomson (1997)). The latter can also however rationalise rigid nominal contracts, something that the risk-sharing approach cannot.

of both workers and firms, current wages respond to both the lowest and the highest unemployment rates recorded since the start of the employment relationship. The fact that both the best and worst labor market conditions since hiring have a significant impact on current wages suggests that both the worker’s and the firm’s outside option constraints matter. It should however be noted that Grant (2003) also used the highest unemployment rate in a similar analysis of U.S. data, and found less evidence for its significance, while Devereux & Hart (2007, Current Issue) did find it to be either insignificant or largely incorrectly signed in U.K. data.<sup>15</sup>

A second implication of the model is that “cohort effects”—differences between wages for different entry cohorts within a firm—will tend not to persist. The wage intervals will be cohort independent, so that a large change in outside opportunities should eliminate any differences if all cohorts need to “renegotiate” (have binding self-enforcing constraints). Consistent with this, Macis finds that the correlation between the unemployment rate prevailing at the time of hiring and current wages declines with tenure. A further test is based on the following observation: if a worker’s wage rose between  $t - 1$  and  $t$ , then according to the model he is constrained at  $t$ . This implies an *asymmetric* response to changes in outside options. Suppose first that unemployment rises in the next period between  $t$  and  $t + 1$  so that the worker’s outside option worsens. This should relax the constraint, and certainly a small change should not imply that the firm’s constraint binds, so the wage will be unchanged; a larger rise in unemployment will however cause the firm’s constraint to bind and the wage to fall. On the other hand, if unemployment falls, the improvement in the worker’s outside option will further tighten the constraint, pushing up the wage, even if this change is small. This is what Macis finds in the data. However the prediction should also work in the opposite direction when wages fall between  $t - 1$  and  $t$ , so that the firm’s constraint can be assumed to be binding, but in this case small increases in unemployment at  $t + 1$  (which should further tighten the firm’s constraint) do not appear to reduce wages.

---

<sup>15</sup>Grant (2003) finds maximum unemployment to matter in a basic individual fixed effects specification, but not if year and tenure dummies are included (whereas the effect of minimum unemployment is largely robust to these additions; see Section 3.2).

## 2.2 Introducing variable hours

The baseline model presented above is important in understanding the behaviour of wages as the insurance motive partially disassociates wages from productivity. It is commonly observed in many countries that labour market fluctuations are characterised by large procyclical variations in hours, but far smaller variations in wages. It has been suggested that the insurance provided in wage contracts can help explain this (Rosen (1985), Azariadis (1975)). Abowd & Card (1987) and Boldrin & Horvath (1995) have tested the implicit contract model of full insurance against the spot market alternative and have found some weak support for the contracting hypothesis over the alternative.

In order to address the behaviour of both wages and hours in the limited commitment model this subsection shows how the baseline model presented above can be extended to allow for joint determination of wages and hours within the contract.<sup>16</sup> In this case a contract will specify not only a profile for wages  $(w_t(h_t))_{t=1}^T$  but also a profile for hours worked  $(H_t(h_t))_{t=1}^T$ . It is assumed that the worker has per-period twice differentiable strictly concave utility function  $u(c, H)$  where work is disliked, so  $u_H < 0$ . It will further be assumed that leisure is a normal good so that the Engel curve for hours worked is downward sloping. In the previous model we were implicitly assuming that hours were fixed, say at 1 unit and  $u(c) = u(c, 1)$ . As before it is assumed that workers cannot engage in capital market transactions so that consumption is equal to earnings,  $c(h_t) = w(h_t)H(h_t)$ . The continuation utilities are defined analogously to equations (1) and (2) but with the per-period payoffs of the worker and the firm are replaced by  $u(c_t(h_t), H(h_t))$  and  $z(s_t)H(h_t) - c_t(h_t)$  respectively. The self-enforcing constraints are then still given by equations (3) and (4) and constrained efficient contracts can be found by solving

$$\max_{(c_t(h_t), H_t(h_t))_{t=1}^T} \Pi_1(h_1) \quad \text{Problem A'}$$

subject to (3), (4), and (5). Again if matches start at a later date the characterisation is exactly the same as it depends only on the state in which the match is initiated.

---

<sup>16</sup>See also Malcomson (1999).

The first thing to note about the solution to Problem A' is that hours will be chosen efficiently so that for every history<sup>17</sup>

$$-\frac{u_H(c_t(h_t), H_t(h_t))}{u_c(c_t(h_t), H_t(h_t))} = z(s_t). \quad (7)$$

To see this consider a pure intratemporal reallocation of consumption and hours that leaves profits unchanged. That is consider a change in consumption of  $\Delta c$  and a change in hours  $\delta H$  such that  $\Delta c = z\Delta H$ . The net effect on utility is approximately  $u_c(c, H)\Delta c + u_H(c, H)\Delta H = (u_c(c, H)z + u_H(c, H))\Delta H$ . Thus if  $-u_H/u_c < z$  a small decrease in hours,  $\Delta H < 0$  would raise utility and if  $-u_H/u_c > z$  a small increase in hours would raise utility. Hence at the optimum (7) must hold. The reason why this condition holds is that the self-enforcing constraints are concerned only with the intertemporal allocation and thus do not interfere with the efficient intratemporal allocation of hours.<sup>18</sup>

It is further possible to find the updating rule analogous to Proposition 3. To do this we define the marginal utility of consumption

$$\lambda_t(h_t) = u_c(c_t(h_t), H_t(h_t)). \quad (8)$$

Associated with each state  $s_{t+1}$  is an interval  $[\underline{\lambda}_{s_{t+1}}, \bar{\lambda}_{s_{t+1}}]$  and the updating rule for  $\lambda$  is given by

$$\lambda_{t+1} = \begin{cases} \bar{\lambda}_{s_{t+1}} & \text{if } \lambda_t > \bar{\lambda}_{s_{t+1}}, \\ \lambda_t & \text{if } \lambda_t \in [\underline{\lambda}_{s_{t+1}}, \bar{\lambda}_{s_{t+1}}], \\ \underline{\lambda}_{s_{t+1}} & \text{if } \lambda_t < \underline{\lambda}_{s_{t+1}}. \end{cases} \quad (9)$$

Here  $\bar{\lambda}_{s_{t+1}}$  is the value of  $\lambda$  which delivers the exactly the worker's outside option and  $\underline{\lambda}_{s_{t+1}}$  is the value that delivers the firm's outside option. The initial value of  $\lambda$  will be determined by the bargaining strength or initial outside options of the parties as reflected by  $\bar{V}_1$  in equation (5).<sup>19</sup> It is easy to see that if hours are fixed then  $\bar{\lambda}_{s_{t+1}} = u'(w_{s_{t+1}})$  and  $\underline{\lambda}_{s_{t+1}} = u'(\bar{w}_{s_{t+1}})$ .

<sup>17</sup>This was first pointed out in Beaudry & DiNardo (1995).

<sup>18</sup>If there were also a moral hazard or adverse selection problem then (7) would not hold and in general there would be an interaction between the intratemporal and intertemporal allocation problems.

<sup>19</sup>Here  $\lambda$  is the inverse of the multiplier on inequality (5) in Problem A'. Thus a lower value of  $\lambda$  corresponds to a greater bargaining strength for the worker. See Sigouin (2004) for a derivation of the updating rule in the case of separable preferences.

To consider the contractual solution for the path of wages and hours, first consider the following two equations

$$u_c(c, H) = \lambda \tag{10}$$

$$-u_H(c, H) = \lambda z. \tag{11}$$

The solutions to the two equations (10) and (11) are the Frisch-type demand functions  $c(\lambda, z)$  and  $H(\lambda, z)$ .<sup>20</sup> It is easy to check that provided leisure is a normal good, the hours function  $H(\lambda, z)$  is increasing in  $\lambda$  and  $z$ . The intuition is that a decrease in  $\lambda$  holding  $z$  fixed, and hence holding the marginal rate of substitution constant, is a pure positive income effect and therefore, because leisure is normal, leads to a decrease in hours worked. Equally, an increase in productivity holding the marginal utility of consumption,  $\lambda$ , fixed leads to a substitution effect and therefore an increase in hours worked. It can also be checked that the function  $c(\lambda, z)$  is decreasing in  $\lambda$  provided consumption is normal.<sup>21</sup> In the limited commitment contractual solution consumption and hours satisfy equations (10) and (11) where at each history consumption equals earnings,  $c_t(h_t) = w_t(h_t)H_t(h_t)$ , and  $\lambda_t(h_t)$  satisfies equation (8) and follows the updating rule given by equation (9). It follows from the equation of earnings and consumption, that provided consumption is normal, the contractual wage rate is decreasing in  $\lambda$  and  $z$ .<sup>22</sup>

The implications of the model have been considered and tested by Beaudry & DiNardo (1995). Consider first the case of complete insurance so that  $\lambda$  is fixed and determined by the initial bargaining position at the time the contract is begun. This may vary from worker to worker. Thus workers who enter the contract with a better bargaining position will in any given state (and hence productivity  $z$ ) have higher wage rates and lower hours. Looking at a cross section of workers therefore it is to be expected that hours are negatively related to wage rates. This is to be

---

<sup>20</sup>These are Frisch-type as Frisch demand functions are derived by keeping the marginal utility of wealth constant and where the marginal rate of substitution equals the real wage (see below).

<sup>21</sup>The effect of an increase in  $z$  on consumption is ambiguous and depends on whether the marginal utility of consumption increases or decreases with hours worked (i.e. on the sign of  $u_{cH}$ ): if utility is separable, consumption is independent of  $z$  for a fixed  $\lambda$ .

<sup>22</sup>This is easy to see if utility is separable in consumption and hours worked: with  $z$  fixed an increase in  $\lambda$  increases the marginal disutility of labour and hence the hours worked. Equally an increase in  $\lambda$  increases the marginal utility of consumption so that consumption or earnings is decreased. Since hours are increased it follows that the wage rate falls. A separable formulation for preferences is used by Sigouin (2004) in his search model (see Section 3.1 below).

contrasted with the standard intertemporal model of labour supply. In that model equations (10) and (11) apply with  $z = w$  and with  $\lambda$  determined by an Euler equation of the form  $\lambda_t = (1 + r_t)\beta E[\lambda_{t+1}]$  where  $r_t$  is the interest rate on borrowing and lending. Since the standard intertemporal model of labour supply allows the worker to self-insure through borrowing and lending, earnings need not equal consumption and therefore it follows directly from equation (11) with  $z$  replaced by the wage rate  $w$  that the Frisch labour supply function  $H(\lambda, w)$  is increasing in  $w$  holding  $\lambda$  fixed provided only that the marginal disutility of work is increasing. This is the intertemporal substitution effect that as wages rise more hours of work are supplied so that wages and hours should be positively associated holding the marginal utility of wealth fixed. Of course  $\lambda$  will not in general be constant over time and therefore the long-run elasticity of wages on hours will depend on the evolution of  $\lambda$ .

Beaudry & DiNardo (1995) also consider the implications of the case where the participation constraints are binding in some states. Depending on the history of states any individual worker may have any  $\lambda_t(h_t) \in [\underline{\lambda}_{s_t}, \bar{\lambda}_{s_t}]$  for a given state and productivity  $z(s_t)$ . This has three important effects. First although different workers initially employed at different dates may have different  $\lambda$ s, as soon as both workers are constrained in a particular state (or the firm is constrained for both workers), their  $\lambda$ s will be equalised and therefore they will have the same wages and hours in subsequent periods. Thus the cross-sectional variation in wages and hours across employees should be lower with increasing tenure. Second, for any worker who is constrained following an increase in productivity, there will be a decrease in  $\lambda$  and two offsetting effects: the hours worked will increase because of the increase in productivity but the decrease in  $\lambda$  will offset this and tend to reduce hours worked. Similarly the wage rate will rise because of the decrease in  $\lambda$  but fall because of the increase in productivity. Thus the model will predict an ambiguous or weak effect of changes in productivity on hours and wage rates. Thirdly, for workers with different starting points the change in  $\lambda$  experienced by different workers will be different. Therefore the consequent growth rates in wages and hours will vary across workers of different tenure.

In testing the relationship between hours and wages Beaudry & DiNardo (1995) use an instrumental variable approach. They use the implications of the limited com-



mitment solution and exploit both variations due to time of entry into a job and cross-sectional variation in on the job wage growth associated with different cohorts (identified by time of entry into a job). Thus they use time of entry dummy variables and year of entry cross-year dummies to instrument for wage growth. Using the Panel Study of Income Dynamics (PSID) 1976–1989 for male heads of household Beaudry & DiNardo (1995) estimate the relationship between hours and wages according to the equation

$$\Delta \ln H_{j,\tau+t} = \alpha_1 \Delta \ln w_{j,\tau+t} + \alpha_2 \Delta \ln z_{j,k,\tau+t} + \alpha_3 \Delta X_{j,\tau+t} + \epsilon_{j,\tau+t}.$$

Hours,  $H_{j,\tau+t}$  measure annual hours at date  $\tau + t$  of worker  $j$  hired in year  $\tau$ ,  $X_{j,\tau+t}$  measures marital and union status and  $\epsilon_{j,\tau+t}$  is the error term. The wage rates,  $w_{j,\tau+t}$  are measured in two alternative ways, either as an annual average or as the reported “point in time” estimate from the survey information. The productivity term  $z_{j,k,\tau+t}$  is decomposed into industry specific terms ( $k$  denotes the industry), and a quadratic experience and tenure profile for each worker. The equation is estimated in log differences to account for worker specific productivity differences at the time of hiring.

They find a statistically significant negative relationship between hours and wages. The test of the validity of their instrumental variable approach shows that typically the instruments for productivity are not only affecting hours through their effect on wages. However when Beaudry & DiNardo restrict data either to non-union contracts or by excluding workers that have recently switched jobs they find that for these subsets the overidentification restrictions are rejected less frequently while the coefficient  $\alpha_1$  remains significantly negative. This offers strong evidence in support of what the limited commitment contracting model would predict. It is however, important to recall that as mentioned above this model is not testing against an alternative. Thus unless assumptions are made about the *long-run* intertemporal elasticity of substitution this cannot be taken as evidence against the spot market model. When the estimates for  $\alpha_1$  are combined with the results of Beaudry & DiNardo (1991) (discussed below in Section 3.2) this suggests that a 1% reduction in unemployment would lead to a 3–4% increase in the wage rate and therefore a reduction in hours worked of between one-half and one per cent absent changes in

productivity. This combination would seem to give quite plausible estimates for the change in hours.

### 2.3 More on quitting costs

Much of the literature assumes that quitting costs are zero (e.g., Thomas & Worrall (1988), Beaudry & DiNardo (1991)) although the search models described below in Section 3.1 implicitly assume there is a cost to quitting as new matches cannot be made immediately. The basic theory considered in Section 2.1 allows for termination costs  $(C_f, C_w)$  which are assumed to be incurred by *both* parties whenever *either* party terminates the relationship and goes for its outside option; as noted this means that the termination costs can be incorporated into the outside options. If, however, there is a direct cost *to renegeing on an agreement* over and above necessary economic costs, for example because of psychic, legal or reputation costs, then this partially affects the previous characterization.<sup>23</sup> In particular, the termination rule is less straightforward. Suppose the direct costs to renegeing are  $p_i, i = w, f$  (these could be made state dependent) and ignore the costs  $C_f, C_w$  considered earlier (or factor them into the outside options  $\chi_i(s_t)$ ). The self-enforcing constraints become

$$\begin{aligned} V_t(h_t) &\geq \chi_w(s_t) - p_w, \\ \Pi_t(h_t) &\geq \chi_f(s_t) - p_f. \end{aligned}$$

Suppose that after observing the current state at  $t$ , we calculate the Pareto frontier conditional on the relationship continuing. That is, consider self-enforcing agreements from state  $s_t$  which do not terminate immediately and calculate the frontier from their payoffs. If it is the case that  $(\chi_w(s_t), \chi_f(s_t))$  lies inside this frontier then termination is inefficient and cannot occur in an efficient agreement, since by definition there is a non-terminating self-enforcing contract which could be followed instead of termination which would be better for both parties. If however  $(\chi_w(s_t), \chi_f(s_t))$  lies above the frontier, then the overall frontier for this state is composed of the undominated points from set composed of the restricted frontier plus

---

<sup>23</sup>A similar argument should apply if there were some enforced compensation on contract breach, as it is only the cost to the reneger that matters. If however there are enforced costs on break-up, such as redundancy payments (i.e., that are incurred even if it is agreed to terminate the relationship), then these should be factored into the outside options.

$(\chi_w(s_t), \chi_f(s_t))$ . As the optimal point on the frontier is generally history dependent, whether termination is optimal may now depend on the previous history, and so we lose the simple termination rule of Proposition 3.<sup>24</sup>

If quitting costs are substantial, the contract will approximate one under full commitment. Suppose however, that the firm employs many workers and that rather than dealing with each employee bilaterally, as we have assumed so far, an employer must treat every employee in the same way. Moreover suppose that this restriction applies even to subsequent hires, so that they must be paid the same as incumbents from the point they join. There is thus a single contract for all workers, and new hires receive a continuation of this contract when they join. Provided the firm needs to hire new workers each period, it must ensure the continuation of the contract matches new hires' outside options. This means, despite incumbents being committed, that there is a participation constraint just as before, where  $\chi_w(s)$  is now the alternative option for new hires. The analysis of this case and empirical implications have been discussed in Martins et al. (2005). Although the worker participation constraint has the same form as the one considered earlier, because it applies to different cohorts at the same time, the wage dynamics and macroeconomic implications are different. Essentially the firm, by attempting to insure incumbents, offers a contract that may not as flexible as would be needed to clear the market for current job seekers. See Snell & Thomas (2006) for an analysis of this case.

### 3 Endogenising the workers' outside option

As explained above in Thomas & Worrall (1988) the worker's outside option  $\chi_w(s)$  is determined by the expected discounted utility a worker would get from being employed henceforth at the spot market wage. Thus the outside option will depend only on the exogenous productivity process. In order to justify this assumption it is necessary to assume that all firms can perfectly observe the worker's past history

---

<sup>24</sup>This discussion assumes that side-payments are not possible. However, if side-payments were feasible it may be that after observing  $s_t$  the contract specifies termination plus a payment from one agent to another, and the penalties  $p_i$  may support this to an extent (for example, the firm will be prepared to transfer up to  $p_f$ ). In this case instead of a single point  $(\chi_w, \chi_f)$  being added to set of payoffs, a curve through  $(\chi_w, \chi_f)$  determined by the trade-off of the side payments between the two agents will be added.

and observe when a worker reneges on a contract, and punish the worker by not offering her anything other than a spot contract. An alternative assumption is that firms treat all new workers in the same way, irrespective of whether or not they have reneged on a previous contract. According to this view, when a worker quits a firm, she can look for a new job offering as much insurance as in the contract from which she just quit. If however, workers and firms could move costlessly to other contracts then no non-spot contracts could be sustained.<sup>25</sup> Therefore it will be necessary to assume either that there are other frictions such as search costs in the labour market, or that firms can commit to contracts. We deal with each of these in turn.

### 3.1 Search frictions

In this section we discuss two papers (Sigouin (2004), Rudanko (2006)) which embed the above model into a matching framework to analyse the association of certain variables with aggregate productivity. Both argue that the two-sided limited commitment model performs better than full commitment models and other versions such as spot contracts, one-sided limited commitment or continuous bargaining. Sigouin (2004) allows hours, but not employment, to vary, while Rudanko (2006) allows employment and vacancies to vary. However in both of these matching models there is also the possibility of an unemployment spell before a new contract is found, so the outside option  $\chi_w(s)$  is less than the utility from a new contract.

Sigouin (2004) develops the model with variable hours by allowing the outside option  $\chi_w(s)$  to be determined by contracts offered by other firms, rather than on a spot market as in the Thomas-Worrall model. He assumes however, that if a worker quits from one firm he or she faces a probability of not being matched with a new firm (even though if matching does occur, it happens without a delay) and being unemployed.<sup>26</sup> This is sufficient to drive a potential wedge between what a worker

---

<sup>25</sup>This assertion assumes that the surplus split in state  $s$  from a new contract is always the same. Otherwise quitters could be punished effectively by starting a new contract so that the other agent gets all the surplus from the relationship. For example, in the Thomas-Worrall model this would imply that punishments are as severe as consignment to trading on the spot market, so the same set of contracts are self-enforcing.

<sup>26</sup>There is no cost to posting a vacancy, but only a fixed fraction of the unemployed are able to make a match, or rather, to ‘see’ wage offers (i.e. they are not directly matched, but are able to enter

can get by remaining in the contract and what is available by quitting, and allows for some insurance to be sustained. Then  $\chi_w(s)$  is determined by what a worker would get by quitting and waiting for a job; because of competition between firms a new job yields the worker the maximum surplus from a self-enforcing contract; however the worker may be unlucky and suffer unemployment, so this is also factored into  $\chi_w(s)$ .

Each worker has a total time endowment which is normalised to one, and can supply up to this amount to a single firm at any date. The productivity per hour worked is  $z(s_t)$  at time  $t$ , which is common to all firms. However there is also a match specific shock, which can reduce productivity to zero (where it remains). If this happens, the match is dissolved. A worker has separable preferences at  $t$  given by

$$E_t \sum_{j=t}^{\infty} \beta^{j-t} [\ln(c_j) + B(1-\eta)^{-1}(1-H_j)^{1-\eta}],$$

where  $c_j$  is consumption and  $H_j$  is hours supplied at time  $j$ . With separable preferences the updating rule of (9) in Section 2.2 implies that each state  $s \in S$  is associated minimum and maximum *earnings*,  $\underline{W}_s$  and  $\overline{W}_s$  ( $\underline{W}_s \leq \overline{W}_s$ ), such that earnings are kept constant if possible and otherwise move by the smallest amount to  $\underline{W}_s$  or  $\overline{W}_s$ . In addition earnings and hours satisfy equation (7) that the marginal rate of substitution equals the marginal product. With the separable specification of preference given above, it follows that

$$(w_{t+1}H_{t+1})B(1-H_{t+1})^{-\eta} = z(s_t).$$

Notice that under a full commitment contract with these preferences a risk-neutral firm will stabilise total earnings while hours will vary procyclically with productivity (according to the intertemporal elasticity of labour supply described above). This leads to the (counterfactual, given the very weak empirical correlation) conclusion that the wage rate is perfectly negatively correlated with hours supplied. On the 

---

into a contract, whereas the unlucky ones cannot). This implies that the unemployment rate does not vary. Essentially he posits a matching function where the matching or “seeing” probability does not depend on the number of vacancies but only on the number seeking work. Moreover, although each entrepreneur can only match with a single worker, there are more entrepreneurs than workers so that competition between entrepreneurs for the fraction of unemployed workers who can see offers drives profits down to zero.

other hand under a spot labour contract, where the wage is always equal to productivity  $z(s_t)$ , these preferences have the property that income and substitution effects of a wage change cancel out (assuming that all income is labour income and there are no taxes, and maintaining the assumption of no borrowing/saving). In this case hours do not vary at all with the wage or productivity (this contradicts the positive correlation between hours and productivity typically found in the data).

As described in Section 2.2 the situation will, however, be somewhat different when there are enforcement constraints, and the result is a mixing of the above two extremes. For relatively small changes in productivity (and assuming that earnings are not already up against the constraint that tightens) such that  $w_t H_t \in [\underline{W}_{s_{t+1}}, \overline{W}_{s_{t+1}}]$ , so neither constraint is binding with strictly positive shadow value, the rule says that earnings stay constant, so there is no income effect, and hours change with productivity according to the intertemporal elasticity of supply. On the other hand, if the change is large enough that a constraint binds, then earnings change and there will be an income effect which reduces to an extent the change in hours. For example, a large increase in productivity may imply only a small increase in hours if earnings rise substantially, so the wage will also rise.<sup>27</sup> In this case there is a positive correlation. The overall effect may then be that the correlation is very weak, in accordance with the evidence.

Rudanko (2006) also embeds the basic model in a model of search. She addresses issues recently raised by Shimer (2005) who argues that the Mortensen-Pissarides model cannot account for the magnitude of unemployment and vacancy fluctuations without assuming unrealistically high volatility in productivity. Hall (2005) argues that some form of wage rigidity may be sufficient to solve this puzzle. The Sigouin model holds unemployment and vacancies fixed, so cannot address these issues. Rudanko looks at different versions of a contracting model in a directed search model of the labour market, following Moen (1997), rather than the random matching model typically used in this literature. The model has similarities with the Sigouin model in that match specific productivity is composed (as the product of) a common (economy wide) component and match component that is unity initially, but transits

---

<sup>27</sup>This depends on how  $[\underline{W}_{s_{t+1}}, \overline{W}_{s_{t+1}}]$  varies with  $z_{t+1}$  but Sigouin shows through numerical simulations that the intuition will be correct in many situations.

to an absorbing state of 0 with a fixed probability each period. As in Sigouin, when this occurs, the match dissolves and the worker looks for a new job. Likewise there are a large number of risk-neutral entrepreneurs operating under constant returns to scale. (Unlike Sigouin, however, hours are fixed, although in the US the extensive margin is more important in accounting for total hours variation than the intensive one.) The model is one of competitive search: At the start of each period, after observing the current aggregate productivity level, firms can choose to post an offer of a wage contract, but have to pay a cost  $k$  for keeping a vacancy open. Worker search can be directed to a particular wage contract  $\sigma$ . There is a matching function defined as follows: if there is a measure  $N_u$  of unemployed agents searching for  $\sigma$  and measure  $N_v$  of vacancies offering  $\sigma$ , the measure of matches taking place this period is given by a Cobb-Douglas matching function

$$m(N_u, N_v) = KN_u^\alpha N_v^{1-\alpha}$$

where  $0 < K < 1$  and  $0 < \alpha < 1$ . Defining  $\theta = N_v/N_u$  to be the vacancy unemployment ratio (“labour market tightness”), the probability that a worker finds a contract  $\sigma$  this period is  $m(\theta) := m(N_u, N_v)/N_u$ , and the corresponding probability for an entrepreneur is  $q(\theta) := m(N_u, N_v)/N_v$ . Thus the payoff to a worker from searching for  $\sigma$  is

$$\mu(\theta(z)) V_\sigma(z) + (1 - \mu(\theta(z))) V_u(z)$$

where  $V_\sigma(z)$  is the discounted worker utility from finding a job with contract  $\sigma$ , while  $V_u(z)$  is the corresponding utility from being unemployed, where both are functions of the prevailing aggregate state  $z$ .  $V_u(z)$  is the discounted utility from consuming the unemployment benefit today and searching again tomorrow. Likewise  $V_\sigma(z)$  is just the expression given in the original model for contract utility with a stochastic termination added, at which point the worker gets  $V_u(z')$  if  $z'$  is the current state as she is unemployed for a period and then has to seek a new job. The firm’s profit per job will depend on the probability that a job is filled,  $q(\theta)$ , and equals  $q(\theta(z)) F_\sigma(z) - k$  where  $F_\sigma(z)$  is the discounted profit from  $\sigma$ , but this is only achieved if a match occurs, but the vacancy cost  $k$  must be incurred in any case. Because of competition among entrepreneurs, this profit is driven to zero in equilibrium. The self-enforcing constraints specify that a worker cannot gain by leaving the contract, which requires that continuation utility must not be below  $V_u(z')$  (the worker is unemployed for

at least a period), and again that the continuation profits of the entrepreneur are non-negative. In addition, for equilibrium to obtain it must be the case that there is no other contract that could be offered which would offer greater profits, where the corresponding  $\theta$  will equate the returns to workers from searching in either market.<sup>28</sup> As in Sigouin, the model endogenises the worker's outside option so that it depends on what she would get by starting a new contract, but again the risk of unemployment (here it will last at least one period) is a sufficient deterrent to allow non-spot contracts to be sustained.

The model is calibrated to U.S. data, and the volatilities of real wages and of the vacancy-unemployment ratio are analysed. Not surprisingly, if there is commitment in the wage contract then wages vary too little with productivity (only new matches are responsible for any variability). The model only comes close to matching the respective empirical correlations of the wage and the vacancy-unemployment ratio with productivity in the two-sided limited commitment model if the replacement ratio is around 80%, which is considerably higher than usually assumed (although Rudanko argues that this is not necessarily an unreasonable number). Intuitively, to get the wage to vary sufficiently, the worker's outside option constraint must bind sufficiently frequently; this requires workers to be relatively indifferent between working and not working.<sup>29</sup>

### 3.2 One-sided limited commitment

We next consider the influential paper by Beaudry & DiNardo (1991) (hereafter BD91). They develop a model of labour contracting where a risk-neutral firm offers insurance to risk-averse employees, but there is no worker commitment and unlike the search models considered above a worker who quits can immediately start work elsewhere (perfect mobility). In terms of our model above, they assume that  $C_f = \infty$  (firm commitment) and  $C_w = 0$  with  $\chi_w(s_t)$  given by the utility from

---

<sup>28</sup>Rudanko shows that only a single contract is ever offered to new matches in equilibrium. Moreover, it is equivalent to a model with undirected search in which a weighted Nash product of surpluses (relative to  $(V_u(z), 0)$ ) is maximised, with weights proportional to the exponents in the matching function, i.e.,  $\alpha$  and  $1 - \alpha$ . So the competitive search framework appears not to be crucial to the results.

<sup>29</sup>The model actually does better as risk-aversion tends to zero; this may be taken as support for the type of hold-up model analysed by MacLeod & Malcomson (1993).



starting a new job (perfect labour mobility). We derive their basic characterisation, which is a generalisation of Holmstrom (1983) who considered a two-period model. We then describe the other ingredients of their model which lead to empirically testable predictions, and finally we discuss the empirical evidence. Their work is particularly important for two reasons. First, they provide strong evidence in favour of the perfect mobility model. Secondly, the paper addresses how wages respond to unemployment levels over the business cycle. There is a voluminous literature that examines how real wages respond to contemporaneous movements in unemployment which generally has not found a very strong relationship, but the results in BD91 suggest that this literature may have been looking at the wrong business cycle variable. If one looks at the lowest unemployment rate since a worker started a job, this appears to show a much stronger effect.

Given that  $C_f = \infty$ , we can treat the value of  $\bar{w}_s$  derived in Section 2.1 as being infinite. (Alternatively, we can just ignore the firm's constraint in all the above arguments, so Lemma 1 directly asserts that wages cannot fall, etc.). Thus the intervals for efficient wages become  $[\underline{w}_s, \infty)$ . The ratchet nature of wages follows from Proposition 3:  $w_{t+1} = \underline{w}_{s_{t+1}}$  if  $w_t < \underline{w}_{s_{t+1}}$ , and otherwise  $w_{t+1} = w_t$ . To pin down the values for the  $\underline{w}_s$ , we need to specify the process for  $\chi_w(s_t)$  and how the contractual surplus is split between worker and firm.

BD91 assume that there are a large number of identical firms and workers, with new workers entering each period to replenish the labour force, replacing workers who die.<sup>30</sup> It is assumed further that because firms operate under constant returns to scale, competition for workers drives profits for a new worker to zero, so any surplus goes to the worker. A worker who quits a firm can immediately seek employment with another firm. Moreover the only source of uncertainty is the common shock to productivity each period. What this implies is that  $\chi_w(s_t)$ , the utility of the worker's outside option, equals the utility from an optimal contract which generates zero profits.<sup>31</sup> Given the updating rule, it is then possible to calculate the initial

---

<sup>30</sup>BD91 also have firm death, but we shall abstract from this in the exposition that follows.

<sup>31</sup>BD91 express the worker's participation constraint equivalently as the fact that the contract must never offer strictly positive profits, looking forward from any point—if it did then the worker would be bid away.

wage of a contract starting in state  $s$  for which discounted expected wages and discounted expected productivity are equal. This must therefore be  $\underline{w}_s$ .

What is perhaps surprising at first glance is how it is possible to offer any insurance at all when the worker can quit and *restart* the contract at a different firm, without any penalty.<sup>32</sup> Normally in repeated game models of cooperation players are induced to take short-term sub-optimal actions (such as paying out on insurance) by the promise of long-term rewards relative to renegeing on this, which yields termination. But here a worker who quits is able to immediately start a new contract with a different firm so that whenever productivity is such that the contract demands a sacrifice by the worker, the worker can quit. The resolution of this apparent paradox is that contracts demand up-front payments by workers in that initially the worker receives a wage below productivity, to be compensated later by the likelihood of wages above productivity.<sup>33,34</sup>

In order to get testable restrictions, it is necessary to link the productivity level in the theoretical model to an observable variable. Notice that the optimal wage contract depends *only* on the productivity process—a very convenient feature. Moreover the labour market must always clear, since at the point of hiring there are no restrictions on wages. However when productivity is high, the wage and expected utility for a new entrant is high. BD91 posit an alternative sector in which a worker could be employed which is subject to a (fixed) decreasing returns technology. Thus a new entrant to the labour market faces a choice between a period in the alternative sector and then getting a contract, versus getting a contract in the original sector right away (by construction of the equilibrium, once a worker has a contract, the option of moving to the alternative sector will offer the same as a new contract, and so is always weakly dominated due to the participation constraint). In equilibrium workers will be indifferent (there are always some workers employed in the alternative sector) so a high wage in the original sector must go along with

---

<sup>32</sup>In fact this intuition is correct in the two-sided case where the firm could also terminate the relationship costlessly. In the Sigouin and Rudanko models discussed above, there is the possibility of unemployment if a worker quits, and this is sufficient to support non-trivial contracts.

<sup>33</sup>This issue has been explored by Krueger & Uhlig (2006) in a general risk-sharing context where both parties to the contract are risk averse.

<sup>34</sup>The feature that workers initially receive wages below productivity with a rising wage profile is of course reminiscent of the agency models where rising wages provide incentives for effort, see e.g. Lazear (1981).

a low level of employment in the alternative sector (due to decreasing returns, this raises the wage), or equivalently, high employment in the original sector. BD91 associate high employment in the original sector with low unemployment.<sup>35</sup> Putting this together, high wages go together with high unemployment levels.

BD91 conclude, then, that with no worker commitment (perfect mobility), where the worker is free to quit at any point, the wage follows a ratchet like process, rising whenever the labour market is tighter than hitherto (since the worker joined the firm), but staying constant otherwise; hence the current wage is determined by the tightest labour market during a worker's tenure. Tightness of the labour market is measured by how low the unemployment rate is. In testing, this perfect mobility model does better than two alternatives: a spot market model in which current unemployment determines wages, and a full commitment model in which unemployment at the time of hiring is the determining factor. In the spot market model, wages are determined solely by the value of a worker's current marginal product, in the full-commitment contracting model, wages are constant but the level is determined by the worker's outside opportunity at the point at which he/she joins the firm. Beaudry & DiNardo (1991) test these three models against each other on U.S. data (PSID/Current Population Survey (CPS)). Perhaps surprisingly, the latter model appears to perform much better than the other two, which we describe in more detail:

**Commitment:** a binding contract is signed when the worker joins a firm. Because the worker is risk-averse, the risk-neutral firm acts as an insurance company, completely stabilising wages. (This results from our above general model by imposing  $C_w = \infty$ , so that  $\underline{w}_s = -\infty$ .) In equilibrium workers will be offered a fixed wage contract (where the wage will equal the expected discounted value of a worker's productivity so firms make zero profits). The wage will be fixed at a level corresponding to conditions *at the point the worker joins the firm*—it equals the best estimate of a worker's lifetime productivity, and under the assumed productivity process this will depend only on his productivity at this point, which is, as explained above, proxied by the unemployment rate,  $U_t$ , at that point.

---

<sup>35</sup>It is tempting to interpret the alternative sector as leisure or some sort of household production, although the decreasing returns to total labour input makes this interpretation difficult.

**Spot market contract:** no long-term contract is possible, so this implies that  $w_t = z(s_t)$ . (If a firm offered say a fixed wage contract, then whenever the wage was less than  $z(s_t)$  the worker could just walk away, and go to another firm, while if the wage was greater than  $z(s_t)$  the firm could sack the worker.) Thus wages fluctuate with  $z(s_t)$  which is proxied by  $U_t$ .

The general model can be expressed as follows: the natural log of the real wage for worker  $j$  at time  $\tau + t$  for a worker who started the job at time  $\tau$  satisfies:

$$\ln w_{j,\tau+t} = \alpha_1 X_{j,\tau+t} + \alpha_2 C(\tau, t) + \varepsilon_{j,\tau+t}$$

where  $X_{j,\tau+t}$  is a vector of individual variables<sup>36</sup>,  $\alpha_1$  is the vector of coefficients on these variables,  $\varepsilon_{j,\tau+t}$  is an error term, and  $\alpha_2$  is the coefficient on the business cycle (i.e., unemployment) variable, with the 3 possibilities for the business cycle variable  $C(\tau, t)$  being:

$$C(\tau, t) = \begin{cases} U_{\tau+t} & \text{spot market model} \\ U_{\tau} & \text{fully binding contract} \\ \min\{U_{\tau+k}, k = 0, 1, \dots, t\} & \text{non-binding on worker} \end{cases}$$

where the unemployment rate is denoted by  $U$ , with  $U_{\tau}$  the rate prevailing at the start of the job and  $U_{\tau+t}$  the rate at time  $\tau + t$  where  $t$  denotes tenure with the employer.

The results are striking. In some specifications<sup>37</sup> in which all three variables are included, the coefficient on the minimum unemployment rate is the only correctly signed (i.e., negative) significant one (PSID, no fixed effects), and in all specifications it is much larger than the other coefficients, implying that a 1% drop in the minimum unemployment rate (e.g., from 4% to 3%) leads to an increase in current wages of between 3% and 8%.

The implications for our understanding of real wage cyclicality are considerable. Typically studies have looked at how wages respond to contemporaneous unemployment movements. For example, using the PSID for men over the period of 1968-69 to 86-87, Solon, Barsky & Parker (1994) found that a one percentage point reduction

---

<sup>36</sup>For individual characteristics, BD91 used experience, experience squared, how much schooling, job tenure, and dummies for industry, region, race, union status, marriage, and metropolitan area (SMSA).

<sup>37</sup>See Table 2 of their paper.

of the unemployment rate leads to a rise in the real wage rate of workers who stayed in their jobs by 1.2 percent (movers appear to be subject to greater procyclical wage movements). Similar estimates are found in Shin (1994) and Devereux (2001). BD91's results suggest that the response of wages to the minimum unemployment rate is substantially larger. On the other hand, as argued in Grant (2003), because the minimum unemployment rate does not actually vary as much as contemporaneous unemployment (consider a worker whose minimum value occurred early in a job spell), minimum unemployment may not explain very much of the variability of aggregate wages over the business cycle.

Several recent empirical studies have largely confirmed the robustness of BD91's main empirical findings over different periods and using different datasets, that the minimum rate of unemployment since hiring is a statistically important determinant of the current wage of an individual (McDonald & Worswick 1999, Grant 2003, Shin & Shin 2007, Devereux & Hart 2007). Both Grant (2003), and Devereux & Hart (2007, Current Issue), however, find more of a role for the current unemployment rate than did BD91. Grant (2003) extends BD91's analysis (using six cohorts from the National Longitudinal Surveys) to cover the time period 1966 to 1998. He finds that the significance and importance of  $\min u$  is broadly robust with respect to the addition of fixed time dummies (to rule out any effects coming through macroeconomic variables, and thus the coefficient on  $\min u$  is estimated only through variation across individuals in each year), of tenure dummies (to capture nonlinear tenure effects), a tenure-unemployment interaction term (to capture tenure effects that vary over the business cycle), and using sub-samples selected on the basis of age, and sex. As mentioned, however, current unemployment levels also have some explanatory power.

A somewhat different methodology was adopted by Shin & Shin (2007), using the PSID for the period 1974-91, which includes one business cycle more than BD91. They run the BD91 regressions over the whole period and get very similar results—but as Grant does, they also find more significant results for contempora-

neous unemployment.<sup>38,39</sup> They also estimate a complementary econometric model, only using the current unemployment rate as a business cycle regressor, but look for asymmetric effects of tight labour markets. Thus they split a job history into periods of tightening and loosening labour markets, and subdivide the former category into two sub-categories, when unemployment is falling but above its minimum for the current job, and when it is below the minimum. Tenure is measured with considerable error in the PSID; thus a mismeasurement in tenure may lead to an incorrect value for  $\min u$ , used in BD91's estimation, whereas here it will lead to the respective periods when unemployment is falling but below or respectively above the historical minimum, to be measured incorrectly. It is argued that the former is more likely to be problematical. The results are that most of the wage adjustment occurs in periods when the unemployment rate falls below the historical minimum level observed since the start of the current job in accordance with the perfect mobility model (according to the model, wages should be constant in other periods): For the sample of male household heads, the estimated coefficient on the unemployment rate is -0.026 (i.e., a one percentage point reduction in the unemployment rate is associated with a 2.64 percent rise in real wages). The coefficient on unemployment when it is falling but not below the historical minimum is much smaller at -.0076, but not significant. For periods of contraction, the coefficient is smaller and insignificant. So again there is a strong confirmation of the perfect mobility model. They also confirm the findings of other studies that the wages of job stayers are procyclical, but less so than those of movers.

---

<sup>38</sup>In comparing their estimates with those of Grant, it is interesting to note that the PSID sample has a higher average age than the National Longitudinal Survey of Youth used by Grant except for the NLSY Older Men. Estimates in Table 2 of Grant (2003) show that the effect of the minimum unemployment rate on current wages dominates those of the other unemployment variables more in the NLSY "Older Men" cohort than in "Young Men" or "Women," and the "Older Men" results are closest to the PSID estimates. This suggests that the BD91 model may work better for older workers.

<sup>39</sup>Shin & Shin (2007) include a trend, which might matter as the period studied has a generally rising unemployment rate, so that the a job's minimum unemployment rate is negatively correlated with time elapsed since the date at which the minimum is attained; as wages are rising omitting this trend might overstate the effect of the minimum unemployment rate. However it makes little difference, as one would anticipate from Grant's analysis with time dummies. Likewise, to rule out nonlinear effects of tenure they find that the addition a squared tenure term does not matter to the worker fixed effect model (although it does to the no-fixed effects specification); again this confirms Grant's findings.

## 4 Closing comments

We presented an overview of models of self-enforcing labour contracts in which risk-sharing is the dominant motive for contractual solutions. A basic two-agent (firm-worker) model was developed which is sufficiently general to encompass the problem considered in most of the literature. We have shown how the solution can be characterised using local variational arguments and therefore avoided the need to establish more complex technical properties of optimum value functions. We have considered how the outside option of the worker is made endogenous in competitive or search markets and considered some of the implications for aggregate hours and wages and productivity and what empirical support exists for the model. The broad conclusion is that the self-enforcing contractual model does help explain some of the observed empirical regularities better than a spot market or full commitment alternatives. There is fairly strong support from a variety of sources and data of the one-sided limited commitment model where workers outside options are determined competitively.

There remain some issues for further study. A weakness of the empirical tests has been to discriminate against alternative assumptions of capital market imperfections, such as credit constraints for employees, or alternative contracting explanations based on hold-up rather than risk aversion. The general success of the empirical method however suggests that it will be useful to explore whether the model can help explain observed patterns in wages at the firm level where it is typically found that larger firms pay higher wages and fast growing firms pay lower wages. An approach along these lines combining contracts with firm credit constraints can be found in Michelacci & Quadrini (2005).

## References

- Abowd, J. M. & Card, D. (1987), ‘Intertemporal labor supply and long-term employment contracts’, *American Economic Review* **77**(1), 50–68.
- Azariadis, C. (1975), ‘Implicit contracts and underemployment equilibria’, *Journal of Political Economy* **83**(6), 1183–1202.
- Beaudry, P. & DiNardo, J. (1991), ‘The effect of implicit contracts on the movement of wages over the business-cycle - evidence from micro data’, *Journal of Political Economy* **99**(4), 665–688.
- Beaudry, P. & DiNardo, J. (1995), ‘Is the behavior of hours worked consistent with implicit contract theory?’, *Quarterly Journal of Economics* **110**(3), 743–768.
- Boldrin, M. & Horvath, M. (1995), ‘Labor contracts and business cycles’, *Journal of Political Economy* **103**(5), 972–1004.
- Calmès, C. (2007), ‘Self-enforcing labour contracts and macroeconomic dynamics’, *International Advances in Economic Research* **13**(2), 200–213.
- Devereux, P. J. (2001), ‘The cyclicalty of real wages within employer-employee matches’, *Industrial Labor Relations Review* **54**(4), 835–850.
- Devereux, P. J. & Hart, R. A. (2007), ‘The spot market matters: Evidence on implicit contracts from Britain’, *Scottish Journal of Political Economy* **54**(5), xxx–xxx. Current Issue.
- Grant, D. (2003), ‘The effect of implicit contracts on the movement of wages over the business cycle: Evidence from national longitudinal surveys’, *Industrial and Labor Relations Review* **56**(3), 393–408.
- Hall, R. E. (2005), ‘Employment fluctuations with equilibrium wage stickiness’, *American Economic Review* **95**(1), 50–65.
- Holmstrom, B. (1983), ‘Equilibrium long-term labor contracts’, *Quarterly Journal of Economics* **98**(3), 23–54.
- Krueger, D. & Uhlig, H. (2006), ‘Competitive risk sharing contracts with one-sided commitment’, *Journal of Monetary Economics* **53**(7), 1661–1691.
- Lazear, E. (1981), ‘Agency, earnings profiles, productivity, and hours restrictions’, *American Economic Review* **71**(4), 606–620.
- Ljungqvist, L. & Sargent, T. J. (2004), *Recursive Macroeconomic Theory*, second edn, MIT Press.
- Macis, M. (2006), Wage dynamics and insurance. Mimeo, University of Chicago.
- MacLeod, W. B. (2007), ‘The three solitudes in contract: Law, data and theory’, *Scottish Journal of Political Economy* **54**(5), xxx–xxx. Current Issue.



- MacLeod, W. B. & Malcomson, J. M. (1993), ‘Investments, hold-up, and the form of market contracts’, *American Economic Review* **83**(4), 811–847.
- Malcomson, J. M. (1997), ‘Contracts, hold-up, and labor markets’, *Journal of Economic Literature* **35**(4), 1916–1957.
- Malcomson, J. M. (1999), Individual employment contracts, in O. Ashenfelter & D. Card, eds, ‘Handbook of Labor Economics’, Vol. 3, Elsevier, chapter 35, pp. 2291–2372.
- Martins, P., Snell, A. & Thomas, J. P. (2005), ‘Wage dynamics, cohort effects, and limited commitment models’, *Journal of the European Economic Association* **3**(2-3), 350–359.
- McDonald, J. T. & Worswick, C. (1999), ‘Wages, implicit contracts, and the business cycle: Evidence from Canadian micro data’, *Journal of Political Economy* **107**(4), 884–892.
- Michelacci, C. & Quadrini, V. (2005), Financial markets and wages, NBER Working Papers 11050, National Bureau of Economic Research, Inc.
- Moen, E. R. (1997), ‘Competitive search equilibrium’, *Journal of Political Economy* **105**(2), 385–411.
- Rosen, S. (1985), ‘Implicit contracts: A survey’, *Journal of Economic Literature* **23**(3), 1144–1175.
- Rudanko, L. (2006), Labor market dynamics under long-term wage contracting and incomplete markets. Mimeo, University of Chicago.
- Shimer, R. (2005), ‘The cyclical behavior of equilibrium unemployment and vacancies’, *American Economic Review* **95**(1), 25–49.
- Shin, D. (1994), ‘Cyclicalities of real wages among young men’, *Economics Letters* **46**(2), 137–142.
- Shin, D. & Shin, K. (2007), ‘Why are the wages of job stayers procyclical?’, *Macroeconomic Dynamics* **11**. forthcoming.
- Sigouin, C. (2004), ‘Self-enforcing employment contracts and business cycle fluctuations’, *Journal of Monetary Economics* **51**(2), 339–373.
- Snell, A. & Thomas, J. P. (2006), Labour contracts, equal treatment and wage-unemployment dynamics, CESifo Working Paper No. 1835, CESifo Munich.
- Solon, G., Barsky, R. & Parker, J. A. (1994), ‘Measuring the cyclicalities of real wages: how important is composition bias?’, *Quarterly Journal of Economics* **109**(1), 1–26.
- Thomas, J. P. & Worrall, T. (1988), ‘Self-enforcing wage contracts’, *Review of Economic Studies* **55**(4), 541–554.