A probabilistic account of the concept of cross-transfer and inferential interactions for trace materials

Citation for published version:
Taroni, F, Juchli, P & Aitken, C 2020, 'A probabilistic account of the concept of cross-transfer and inferential interactions for trace materials', *Law, Probability & Risk*, vol. 19, no. 3-4, pp. 221-233.
https://doi.org/10.1093/lpr/mgaa015

Digital Object Identifier (DOI):
10.1093/lpr/mgaa015

Link:
Link to publication record in Edinburgh Research Explorer

Document Version:
Peer reviewed version

Published In:
Law, Probability & Risk

Law, Probability and Risk (2020) **00**, 1–13 doi:10.1093/lpr/mgaa015

A probabilistic account of the concept of cross-transfer and inferential interactions for trace materials

FRANCO TARONI†
School of Criminal Justice, University of Lausanne, le Batochime, Lausanne, Switzerland

PATRICK JUCHLI[1]
PwC Switzerland - Data & Analytics, Zürich, Switzerland and

AND

COLIN AITKEN
School of Mathematics and Maxwell Institute, University of Edinburgh, King's Buildings, Edinburgh, UK

[Received on 6 September 2019; revised on 6 May 2020; accepted on 21 May 2020]

The analysis of inferential interactions plays an important role in the description of the line of reasoning for a forensic evaluator in a case involving the cross-transfer of evidence. It is possible the two items of evidence may mean more to an evaluator when considered jointly than they do if considered separately. An approach to the evaluation of evidence, with particular attention to the factors that need to be considered, is described for a case involving the cross-transfer of evidence. A formula is given which may be used to define the possible interactions between the evidence transferred in each direction and hence ease the interpretation of such evidence. Numerical examples are given from a classical fibre evidence scenario.

Keywords: cross-transfer; inferential interaction; evaluation of forensic results; probability; Bayesian networks

1. Cross-transfer of trace material: scenario and statement of the problem

Imagine the general situation in which a direct contact between two persons or objects, or an object and a person, may have occurred. In such cases, a so-called cross- or two-way transfer of trace material may occur. As an example, consider a case described in [1], but simplified here in order to ease the discussion of the general idea. A stolen vehicle is used in a robbery on the day of its theft. An hour later it is abandoned. The vehicle is found by the police a few hours later. On the polyester seats, which were recently cleaned with a car vacuum cleaner, extraneous textile fibres are collected. The car owner lives alone and has never lent his vehicle to anyone. The owner wears nothing but cotton. The day following the robbery a person of interest, PoI, is apprehended, their red

† Corresponding author. Email: franco.taroni@unil.ch
[1] The contribution to this article by Patrick Juchli is independent from and unrelated to his function at PwC Switzerland.

woollen pullover and their denim jeans are confiscated. On the driver's seat, one group of relevant foreign fibres is collected. It consists of a large number, n_1 say, of red woollen fibres. These findings, denoted E_1, are a combination of the form $\{y_1, x_1\}$, where y_1 refers to the recovered fibres on the car seat and x_1 refers to known (control) material from the red woollen pullover of the PoI. The evaluation will assume that the group of fibres found on the driver's seat is the potential result of a transfer from the offender's clothing; the recovered group of fibres appears to be relevant in the context of the case at hand. Further, an assumption is made that, if they were the offender, the PoI was wearing the seized pullover of interest at the time of the offence. The association between the pullover and the PoI is not questioned. A discussion on this particular topic is presented in [2].

On the pullover of the PoI, the scientist finds a so-called foreign fibre group, that is a number, n_2 say, of fibres which can be distinguished from fibres from a known source (either associated directly with the PoI or associated with an object associated with the PoI). The group consists of n_2 extraneous black fibres. These fibres correspond, in some sense, to the fibres of which the driver's seat of the stolen car is composed. This finding, denoted E_2, is a combination of the form $\{y_2, x_2\}$ where y_2 refers to the n_2 recovered fibres on the clothing of the PoI and x_2 refers to known (i.e. control) material from the driver's seat.

Let the competing propositions for this case refer to an activity, such as 'The PoI sat on the driver's seat of the stolen car (H_p)', and 'The PoI never sat on the driver's seat of the stolen car (H_d)'. Because of the reciprocal nature of transfer, the two sets of recovered traces (fibres in this case) should be considered as dependent, with respect to the main propositions H. Given H_p, for example, and trace material found that characterizes a potential transfer in one direction, then the forensic scientist might expect to find trace material characterizing an event of transfer in the other direction. Stated otherwise, the presence of material transferred in one direction provides information about the expectation of the potential presence of material transferred in the other direction [3]. On the other hand, if H_d holds (i.e. the PoI has nothing to do with the case), then knowledge about material found on the car seat should not affect one's expectations to find material on the PoI's pullover. A formal analysis will clarify the relevance of this aspect.

2. The probative value

2.1 *General form of the Bayes' factor*

The Bayes' factor, often called 'likelihood ratio' in forensic literature, is a measure that can be used to quantify the value of scientific results. Formulating the Bayes' factor for the scenario of interest here will require that the findings E_2 be conditioned on findings E_1. This stems from the view stated in Section 1 according to which recovered material that characterizes a potential transfer in one direction might affect a forensic scientist's expectation to find trace material characterizing transfer in the other direction. Therefore, the value expressed for the second item of trace material should take into account the results observed (i.e. obtained measurements) for the first item of trace material. The choice of which evidence is E_1 and which is E_2 depends on the structure of the investigation. The crime scene is investigated and traces (E_1) are collected. A PoI is then identified and traces (E_2) are collected. The ordering of the subscripts reflects the temporal aspect of the investigation.

The joint value of findings E_1 and E_2 can be formulated in terms of the following Bayes' factor (V, short for 'value'):

$$V_{12} = \frac{Pr(E_2|E_1, H_p)}{Pr(E_2|E_1, H_d)} \times \frac{Pr(E_1|H_p)}{Pr(E_1|H_d)}. \tag{1}$$

For shortness of notation, V_{12} can also be written as the product $V_{2|1} \times V_1$, where $V_{2|1}$ is the Bayes' factor for evidence y_2 given knowledge of evidence y_1 and V_1 is the Bayes' factor for evidence y_1[2].

2.2 *Component Bayes' factor for transfer to the scene* (V_1)

In literature on the topic on unidirectional transfer (e.g. [1, 4]), the second ratio in Equation (1) is given as follows:

$$\frac{Pr(E_1|H_p)}{Pr(E_1|H_d)} = \frac{b_0 t_{n_1} + b_{1,n_1}\gamma_1 t_0}{b_0\gamma_1 t'_{n_1} + b_{1,n_1}\gamma_1 t'_0}. \tag{2}$$

Here, γ_1 is the assigned population proportion for the characteristics seen in y_1, among extraneous groups of fibres of similar size found on seats of stolen cars. The term t_{n_1} refers to the probability of transfer (including persistence and recovery) of a group of n_1 fibres. Similarly, the term t_0 refers to the probability of no transfer (under proposition H_p). Under the alternative proposition H_d, probabilities t'_{n_1} and t'_0 refer, respectively, to the event of transfer and no transfer from an alternative source involved in the case (i.e. different from the PoI). Probabilities b_0 and b_{1,n_1} refer to so-called background presence, that is, material coming from alternative sources, which are not crime-related, and which can be distinguished from fibres deriving from the owner. A background group of fibres may be of size n_1 or 0, with the latter representing the absence of background material on the inspected receptor surface.

Note that in this probabilistic development of the equations, it is simply considered that a group of fibres has either been transferred in its entirety during the commission of the crime or was already there. There is, of course, a possibility that just a subgroup of fibres was transferred and the remainder were there beforehand. This possibility is not considered here.[3]

It is useful to note that Equation (2) refers to a situation that can also be described as 'potential transfer *to* the scene', or 'material potentially left by the offender'. This emphasis on the direction of transfer is important because, given H_d, the potential of transfer from the true offender (different from the PoI) is considered. This is different for situations of the kind 'transfer *away* from the scene', where material is found on PoI. In the latter case, given H_d (i.e. the suspect has nothing to do with the case), transfer is *not* taken into account, only presence by chance alone. Glass found on a PoI is a typical example for this (e.g. [5]). Thus, for situations in which material is found on a PoI, and the alternative proposition implies that the person is not involved in the case (e.g. they were not a bystander in a case involving breaking glass), the denominator of the Bayes' factor takes a simpler form than that shown in Equation (2). It will only involve terms for background presence and the

[2] A full exposition of Equation (1) would include a symbol I to denote background information. This is omitted here for brevity of notation.

[3] Such a situation can typically be the case with scenarios involving particles such as paint flakes and tools. Imagine—for the purpose of illustration—the presence of two stains of white paint on a crowbar and that these stains are indistinguishable with respect to their optical and chemical characteristics. In consequence, it is questionable as to whether it can be stated that either both stains were there before the criminal action or both were transferred during the criminal action.

rarity of the corresponding analytical features. The discussion of the denominator of $V_{2|1}$ in Section 2.3 will further clarify this aspect.

Equation (2) is a general formula and it can be useful to note that, under particular circumstances, it reduces to $1/\gamma_1$. A first condition for this simplification is the absence of background material on the receptor surface (i.e. $b_{1,n_1} = 0$ and $b_0 = 1$). A second condition is that the probability for the event of transfer given H_p is the same as that for the event of transfer (persistence and recovery) given H_d. The probability of transfer t from the PoI's garment is considered to be the same as t', transfer from the true offender's garment. The first condition may be difficult to accept. However, it is reasonable to consider $1/\gamma$ as a limiting result because, often, probabilities for no transfer (t_0, t'_0) may be low so that the terms involving b_{1,n_1} in both the numerator and denominator may become negligible.

Traditionally, simplifying assumptions were sought in order to present Bayes' factor in a practically affordable and easily memorable way. The discussion here will make simplifying assumptions for ease of exposition. However, formalisms are now available, in particular Bayesian networks (e.g. [6–8]), which allow one to deal with the full arithmetic specification.

2.3 *Conditional component Bayes' factor for transfer away from the scene ($V_{2|1}$)*

The first ratio on the right-hand side of Equation (1) accounts for a group of n_2 fibres (y_2) present on the PoI's clothing. In the numerator it is assumed that this group of fibres is, potentially, the result of transfer while the PoI sat on the car's seat.

In the denominator, the presence of y_2 is considered as being part of the background presence. If the PoI did not sit on the car's seat, the fibres on their pullover are due to chance alone. Thus, the denominator can be written as

$$b^*_{1,n_2}\gamma_2.$$

This expression shows that an alternative event of transfer, comparable to transfer from the true offender under H_d in case of material found on the scene (see development for V_1 in Section 2.2), is not an issue in the context of material found on a PoI. Note further that the '*' in the above notation is used to distinguish the probability assignment for background on the PoI's clothing from that used for background presence on the car seat. The factor γ_2 stands for the rarity of the analytical features of y_2 among extraneous groups of fibres on clothing of persons comparable to the PoI.

The numerator $Pr(E_2|E_1, H_p)$ needs a more detailed analysis. More formally, it can be developed as:

$$Pr(E_2|E_1, T_2, H_p)Pr(T_2|E_1, H_p) + Pr(E_2|E_1, \overline{T}_2, H_p)Pr(\overline{T}_2|E_1, H_p), \tag{3}$$

where T_2 is the event of transfer from the car seat to the PoI's pullover. Given T_2, the observation E_1 of the fibres on the car seat (corresponding to the suspect's pullover) does not influence the conditional probability of E_2. Thus, T_2 screens off E_2 from E_1 and one can write $Pr(E_2|E_1, T_2, H_p) = Pr(E_2|T_2, H_p)$ and $Pr(E_2|E_1, \overline{T}_2, H_p) = Pr(E_2|\overline{T}_2, H_p)$. Equation (3) thus becomes

$$\underbrace{Pr(E_2|T_2, H_p)}_{b^*_0} Pr(T_2|E_1, H_p) + \underbrace{Pr(E_2|\overline{T}_2, H_p)}_{b^*_{1,n_2}\gamma_2} Pr(\overline{T}_2|E_1, H_p) \tag{4}$$

with b^*_0 and $b^*_{1,n_2}\gamma_2$ following common notation for, respectively, the probabilities of zero background and background of one group of comparable size and with compatible analytical features. The probability of the event of transfer T_2, conditional on H_p and E_1, that is $Pr(T_2|E_1,H_p)$, can be extended by considering the event of transfer T_1, that is the event of transfer of a group of foreign fibres to the car seat, conditional on the result E_1. Assuming T_1 to screen off T_2 from E_1, the term $Pr(T_2|E_1,H_p)$ thus becomes:

$$\overbrace{Pr(T_2|T_1,H_p)}^{u_{n_2|T_1}}\, Pr(T_1|E_1,H_p) + \overbrace{Pr(T_2|\overline{T}_1,H_p)}^{u_{n_2|\overline{T}_1}}\, Pr(\overline{T}_1|E_1,H_p), \tag{5}$$

with $u_{n_2|\cdot}$ representing a conditional transfer probability. This assignment is conditional on the state of the variable T_1. This conditional transfer probability is highly case dependent as it is strongly influenced by the kind of textile materials involved, in particular their properties (e.g. sheddability). The conditional transfer probability thus requires a case-tailored assessment.

The probability $Pr(T_1|E_1,H_p)$ is obtained using Bayes' theorem. Using common notation and assignments, the following transformation can be applied:

$$Pr(T_1|E_1,H_p) = \frac{Pr(E_1|T_1,H_p)Pr(T_1|H_p)}{Pr(E_1|T_1,H_p)Pr(T_1|H_p) + Pr(E_1|\overline{T}_1,H_p)Pr(\overline{T}_1|H_p)} = \frac{b_0 t_n}{b_0 t_n + b_1\gamma_1 t_0}. \tag{6}$$

Denote the ratio (6) as a. The value of the evidence $V_{2|1}$ is then

$$\frac{b_0^*[u_{n_2|T_1}a + u_{n_2|\overline{T}_1}(1-a)] + b_{1,n_2}^*\gamma_2\{1 - [(u_{n_2|T_1}a) + u_{n_2|\overline{T}_1}(1-a)]\}}{b_{1,n_2}^*\gamma_2}. \tag{7}$$

For the sake of simplicity, consider again the previously supposed extreme situation with $b_0 = 1$ and $b_1 = 0$, concerning the background material on the driver's seat. In such a situation, $Pr(T_1|E_1,H_p) = 1$ and, thus, $Pr(\overline{T}_1|E_1,H_p) = 1 - Pr(T_1|E_1,H_p) = 0$ and $a = 1$. This expresses the view that if there was no background material on the driver's seat, but fibres corresponding to those of the PoI's pullover are found, then it is the event of transfer that led to this finding. Consequently, Equation (5) becomes $Pr(T_2|E_1,H_p) = u_{n_2|T_1}$ and hence $Pr(\overline{T}_2|E_1,H_p) = (1 - u_{n_2|T_1})$. Thus, in summary, the numerator as specified by Equation (4) becomes $b_0^* u_{n_2|T_1} + b_{1,n_2}^*\gamma_2(1 - u_{n_2|T_1})$.

Note that in such a situation, the probability $u_{n_2|\overline{T}_1}$ does not appear in the final expression. However, if the assumption of no background (i.e. $b_0 = 1$ and $b_1 = 0$) for the car seat is relaxed, then an assigned value for $u_{n_2|\overline{T}_1}$ is needed. This latter term refers to the event of transfer from the car seat to the criminal given that no transfer occurred in the opposite direction.

Combining the various simplifications made above, one can rewrite Equation (1) as follows:

$$V = V_{2|1} \times V_1 = \frac{b_0^* u_{n_2|T_1} + b_{1,n_2}^*\gamma_2(1 - u_{n_2|T_1})}{b_{1,n_2}^*\gamma_2} \times \frac{1}{\gamma_1}\,. \tag{8}$$

3. The graphical model

The conditional probabilities expressed in the previous equations allow the forensic scientist to model the cross-transfer scenario as in Fig.1 which illustrates a possible Bayesian network for this scenario.

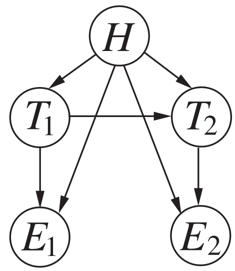

FIG. 1. Bayesian network for a cross-transfer situation. Each node of the network has two states. Node H: with states H_p, 'The PoI sat on the car seat', and H_d, 'the PoI never sat on the car seat'. Node T_1, with states T_1, 'There was a transfer from the offender to the car seat', and $\overline{T}_1$, 'There was not a transfer to the car seat'. Node E_1, with states E_1, 'The fibres found on the car seat are found to correspond to those of the PoI's', and $\overline{E}_1$, 'The fibres found on the car seat are not found to correspond to those of the PoI's pullover'. Nodes T_2 and E_2 denote, respectively, transfer to the PoI's pullover and the finding of corresponding fibres on the PoI's pullover.

Figure 1 consists of a combination of two networks of the kind $H \rightarrow T$, $H \rightarrow E$, $T \rightarrow E$ but retaining only a single node for the main proposition H. The assumed relevance relationship between the two sets of findings is expressed in terms of a connection between the two network fragments. The main consideration of dependency is given by the arrow between the nodes T_1 and T_2, representing events of transfer. This dependency expresses the view that the occurrence of an event of transfer in one direction can affect one's assessment of the occurrence of an event of transfer in the opposite direction. The extent of this influence depends, however, on the way in which the node tables are specified. A discussion on this aspect and notably on the existence of an arrow connecting the transfer nodes T_1 and T_2 is presented in Section 4.

Equation (5) clarifies that, potentially, the probability for the event of transfer T_2 from the seat to the PoI (under H_p) can vary according to the truth or otherwise of T_1. That is, $u_{n_2|T_1}$ can be different from $u_{n_2|\overline{T}_1}$. If, however, one judges these probabilities to be the same, then the link between T_1 and T_2 would not be needed because one would suppose that knowledge of the state of the variable T_1 would not affect one's assessment of the probability of the event T_2. Note that given H_d (i.e. the PoI did not sit on the car seat), a zero probability is assigned to the event T_2 because the event is supposed not to have taken place.

The conditional transfer probability $u_{n_2|\overline{T}_1}$ may appear difficult to conceptualize, but note that with particular assumptions (i.e. $b_0 = 1$ and $b_1 = 0$) that reduce Equation (6) to 1, only a single conditional transfer probability, $u_{n_2|T_1}$, needs to be assigned. It is often useful to examine the behaviour of the Bayes' factor, Equation (8), for different values of its components.

Figure 2 describes values for the Bayes' factor V_{12} through a two-way sensitivity analysis; here, parameters b_0^* and $u_{n_2|T_1}$ vary and it can be noticed that the presence of the recovered trace material E_2 characterizing a possible cross-transfer leads to an increase—under the specified conditions on b_0 and b_1 presented in Sections 2.2 and 2.3—in the probative value, $\log_{10}(V_1) = 2$ offered by the first item of evidence E_1.

It can be seen that the more the probability (b_0^*) of an absence of background material with characteristics similar to the control fibres from the car seat on the material associated with the PoI increases (from 0.01 to 0.99), the more the joint Bayes' factor increases too. The potential presence of extraneous fibres as contamination on clothes increases the uncertainty related to the action under investigation and so the Bayes' factor is reduced. The impact of the conditional transfer probability

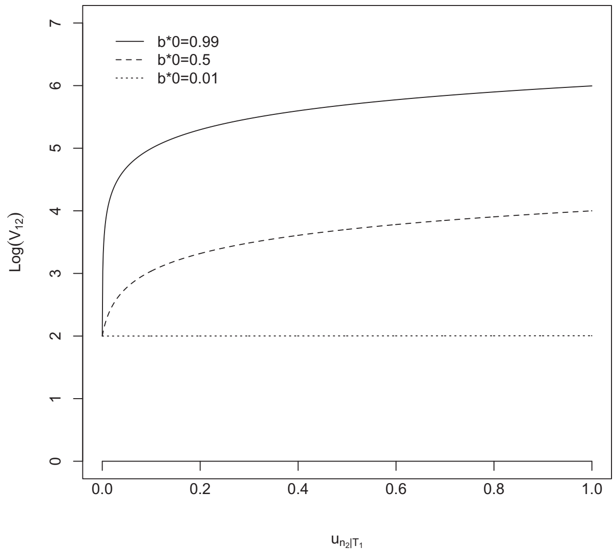

FIG. 2. Two-way sensitivity analysis. Values for the logarithm (to base 10) of the joint Bayes' factor V_{12} (for E_1 and E_2) depending on values of $b_0^* = 0.01, 0.50, 0.99$ and $0 \leq u_{n_2|T_1} \leq 1$. Proportions $\gamma_1 = \gamma_2 = 0.01$.

($u_{n_2|T_1}$) of fibres to material associated with the PoI, given the transfer of a group of fibres from PoI to the car seat, has a relatively minor impact on the joint Bayes' factor mainly because such a probability is conditional on a previous occurrence of a transfer T_1.

4. Interaction measure: an example

Consider, for sake of illustration, the reduced form of the Bayes' factor due to the acceptance of the assumptions (a) that values b_0 and b_1 are as expressed as in Equation (8) ($b_0 = 1$ and $b_1 = 0$), (b) that, if the suspect was involved in the theft of the car, then he was wearing the seized pullover and so the association between the pullover and the suspect is not questioned, and (c) that the forensic scientist is able to observe y_1 and y_2 features every time they are faced with such physical or chemical characteristics, and also that no error in characterisation is made (the scientist is free of error).

Consider also the following assignments for the parameters of interest: $b_0^* = 0.9$, $b_{1,n_2}^* = 0.1$, $u_{n_2|T_1} = 0.8$, $\gamma_1 = \gamma_2 = 0.01$. Using Equation (8), the joint Bayes' factor for evidence E_1 and E_2, say V_{12}, equals $\approx$72, 000. This means that the joint consideration of the two items of fibres evidence, E_1 and E_2 very strongly supports the proposition H_p that the suspect sat on the car seat; the proposition H_p is supported by a factor greater than 72,000.

The relationship between E_1 and E_2 can be studied to detect the presence of a special form of inferential interaction between the two items of evidence. In order to examine if and to what degree a potential effect applies in a given case, the logarithm of the Bayes' factor, the 'weight of evidence', ([9]) is used as a metric. The weight of evidence in favour of H_p is written $W_{E_1|H_p}$ for the logarithm of the Bayes' factor $Pr(E_1|H_p)/Pr(E_1|H_d)$, $W_{E_2|H_p}$ for the logarithm of the Bayes' factor $Pr(E_2|H_p)/Pr(E_2|H_d)$, and $W_{E_2|E_1,H_p}$ for the logarithm of the Bayes' factor $Pr(E_2|E_1,H_p)/Pr(E_2|E_1,H_d)$.

The inferential interactions based on this concept of weight of evidence may be represented by a metric known as the *interaction measure*, denoted here with R [10].

Recall Equation (1) and consider the multiplication law of probability. The weight of evidence that the events E_1 and E_2 provide in favour of hypothesis H_p can be written as

$$W_{E_1,E_2|H_p} = W_{E_1|E_2,H_p} + W_{E_2|H_p} = W_{E_2|E_1,H_p} + W_{E_1|H_p}. \tag{9}$$

From this, it follows that

$$W_{E_2|H_p} - W_{E_2|E_1,H_p} = W_{E_1|H_p} - W_{E_1|E_2,H_p}. \tag{10}$$

By considering Equation (9), it can also be seen that $W_{E_1|E_2,H_p} = W_{E_1,E_2|H_p} - W_{E_2|H_p}$. By substituting $W_{E_1|E_2,H_p}$ by the difference $W_{E_1,E_2|H_p} - W_{E_2|H_p}$, one obtains

$$W_{E_1|H_p} + W_{E_2|H_p} - W_{E_1,E_2|H_p} = W_{E_2|H_p} - W_{E_2|E_1,H_p}. \tag{11}$$

Equation (11) covers the properties required to define the types of inferential interactions in terms of the weight of evidence, in a single identity statement. It thus provides an explanation for these interactions. The derivations of the inferential measures available from Equation (11) are given in [10, 11].

Consider as a specific case the cross-transfer scenario described above. In this scenario it is of interest to quantify the weight of evidence in favour of H_p provided by evidence E_2 alone and the weight of evidence in favour of H_p provided by evidence E_2 knowing that evidence E_1 occurred. Events E_1 and E_2 are said to be 'inferentially synergistic' given the hypothesis H_p if the inequality $W_{E_2|E_1,H_p} > W_{E_2|H_p}$ holds, meaning that $W_{E_1,E_2|H_p} > W_{E_1|H_p} + W_{E_2|H_p}$; the joint weight of evidence E_1 and E_2 is greater than the sum of the individual weights for evidence E_1 and E_2.

The interaction measure R can be calculated for the scenario of interest. Equation (12) quantifies the difference between the weight of evidence in favour of H_p provided by evidence E_2 alone and the weight of evidence in favour of H_p provided by evidence E_2 knowing that evidence E_1 occurred relative to the weight of evidence E_2. This is what Schum [12] called the 'redundance measure' denoted $R_{E_2|E_1}$. Different values of $R_{E_2|E_1}$ indicate different types of inferential interaction between events E_1 and E_2 given the main propositions H.

The measure compares the weight of evidence for E_2 given the observation of evidence E_1, $W_{E_2|E_1,H_p}$, against the weight of evidence for E_2 for a situation in which nothing is known about evidence E_1, $W_{E_2|H_p}$. The measure is expressed in the following terms:

$$R_{E_2|E_1} = \frac{W_{E_2|H_p} - W_{E_2|E_1,H_p}}{W_{E_2|H_p}} = 1 - \frac{W_{E_2|E_1,H_p}}{W_{E_2|H_p}}. \tag{12}$$

where $W_{E_2|H_p} \neq 0$.

4.1 *Definitions of symbols used*

Before some numerical examples are given it will be helpful to provide a summary list of the symbols used and their definitions. This is done in Table 1.

TABLE 1 *Symbols used in the determination of the weight of evidence and measure of interaction*

Symbol	Definition
E_1	Recovered extraneous fibres on car seat, control fibres on red pullover belonging to PoI, putative transfer from PoI to car seat.
E_2	Recovered extraneous fibres on material associated with PoI, control fibres on car seat, putative transfer from car seat to PoI.
γ_1	Assigned population proportion for the characteristics seen in the control fibres on the red woollen pullover of the PoI.
γ_2	Assigned population proportion for the characteristics seen in the control fibres on the car seat.
n_1	Number of extraneous fibres recovered on the car seat.
n_2	Number of extraneous fibres recovered on material associated with PoI.
b_0	Probability of an absence of background material with characteristics similar to the control fibres from the red woollen pullover of the PoI on the car seat.
b_0^*	Probability of an absence of background material with characteristics similar to the control fibres from the car seat on the material associated with the PoI.
b_{1,n_1}	Probability of the presence of one group of n_1 fibres of background material with characteristics simlar to the control fibres from the red woollen pullover of of the PoI, but not from the PoI, on the car seat, $= 1 - b_0$.
b^*_{1,n_2}	Probability of the presence of one group of n_2 fibres of background material with characteristics similar to the control fibres from the car seat, but not from the car seat, on the material associated with the PoI, $= 1 - b_0^*$.
T_1	Event of transfer of one group of foreign fibres from the red woollen pullover of the PoI to the car seat.
$\overline{T}_1$	Event of no transfer of one group of foreign fibres from the red woollen pullover of the PoI to the car seat.
$u_{n_2\|T_1}$	Conditional transfer probability of n_2 fibres to material associated with the PoI, given transfer of group of fibres from PoI to the car seat.
$u_{n_2\|\overline{T}_1}$	Conditional transfer probability of n_2 fibres to material associated with the PoI, given no transfer of group of fibres from PoI to the car seat.
t_0	Probability of no transfer of fibres from the red woollen pullover of the PoI to the car seat.
t'_0	Probability of no transfer of fibres from alternative source than PoI to car seat.
t_{n_1}	Probability of transfer, persistence and recovery of group of n_1 fibres from PoI to the car seat.
t'_{n_1}	Probability of transfer, persistence and recovery of group of

(*Continued*)

Table 1 *(continued)*

Symbol	Definition
	n_1 fibres from alternative source than PoI to car seat.
t_0^*	Probability of no transfer of fibres from the car seat to material associated with the PoI.
$t_{n_2}^*$	Probability of transfer, persistence and recovery of group of n_2 fibres from the car seat to PoI, $= 1 - t_0^*$.
x_1	Control material from the red woollen pullover of the PoI.
x_2	Control material from the car seat.
y_1	n_1 recovered fibres on the car seat.
y_2	n_2 recovered fibres on the red woollen pullover of the PoI.

4.2 *Examples of the calculation and interpretation of the values of* R

The measure of interaction R is defined in Equation (12) as

$$R_{E_2|E_1} = \frac{W_{E_2|H_p} - W_{E_2|E_1,H_p}}{W_{E_2|H_p}} = 1 - \frac{W_{E_2|E_1,H_p}}{W_{E_2|H_p}} = 1 - \frac{\log(V_{2|1})}{\log(V_2)}. \tag{13}$$

where $W_{E_2|H_p} \neq 0$ and

$$V_{2|1} = \frac{b_0^* u_{n_2|T_1} + b_{1,n_2}^* \gamma_2 (1 - u_{n_2|T_1})}{b_{1,n_2}^* \gamma_2}$$

and

$$V_2 = \frac{b_0^* t_{n_2}^* + b_{1,n_2}^* \gamma_2 t_0^*}{b_{1,n_2}^* \gamma_2},$$

with symbols as defined in Table 1 with V_2 given as an analogous result to Equation (2) with $t_0'^* = 1$ and $t_{n_2}'^* = 0$. The base of the logarithms is not important as it is the relative values that matter. In the calculations below, the base 10 was used.

Note that if $u_{n_2|T_1}$, the conditional probability of transfer of fibres from the car seat to material associated with the PoI, given transfer of fibres from the PoI to the car seat, equals $t_{n_2}^*$, the unconditional probability of transfer of fibres from the car seat to material associated with the PoI, then $R = 0$ and there is conditional independence; the arrow between nodes T in the Bayesian network of Fig. 1 is superfluous.

Let $\gamma_2 = 0.01$ throughout. Table 2 lists differing circumstances of transfer with the associated assignments of probabilities.

The corresponding values of the interaction measure R are given in Table 3.

A value of $R < 0$ occurs when $W_{E_2|E_1,H_p}$ and $W_{E_2|H_p}$ are of the same sign and $|W_{E_2|E_1,H_p}| > |W_{E_2|H_p}|$. The support given by E_2 to H_p, conditional on E_1, is greater than the support of E_2 on its own. The relationship between E_1 and E_2 is said to be 'synergistic'. A value of $1 > R > 0$ occurs when $W_{E_2|E_1,H_p}$ and $W_{E_2|H_p}$ are of the same sign and $|W_{E_2|E_1,H_p}| < |W_{E_2|H_p}|$. Both E_2 and E_1 support H_p or both support H_d. However, the support given by E_2 to H_p (or H_d), conditional on E_1,

TABLE 2 *Description of differing circumstances of transfer and corresponding probability assignments*

Case	Description	Probability assignments
1	Low probability of background fibres on PoI High conditional probability of transfer from car seat to PoI given transfer of fibres from PoI to car seat Low unconditional probability of transfer from car seat to PoI	$b_0^* = 0.9 = 1 - b_{1,n_2}^*$ $u_{n_2\|T_1} = 0.9.$ $t_{n_2}^* = 0.1 = 1 - t_0^*.$
2	Low probability of background fibres on PoI High conditional probability of transfer from car seat to PoI given transfer of fibres from PoI to car seat Moderately high unconditional probability of transfer from car seat to PoI	$b_0^* = 0.9 = 1 - b_{1,n_2}^*$ $u_{n_2\|T_1} = 0.9.$ $t_{n_2}^* = 0.8 = 1 - t_0^*.$
3	Low probability of background fibres on PoI High conditional probability of transfer from car seat to PoI given transfer of fibres from PoI to car seat Moderate unconditional probability of transfer from car seat to PoI	$b_0^* = 0.9 = 1 - b_{1,n_2}^*$ $u_{n_2\|T_1} = 0.9.$ $t_{n_2}^* = 0.7 = 1 - t_0^*.$
4	High probability of background fibres on PoI High conditional probability of transfer from car seat to PoI given transfer of fibres from PoI to car seat Moderate unconditional probability of transfer from car seat to PoI	$b_0^* = 0.1 = 1 - b_{1,n_2}^*$ $u_{n_2\|T_1} = 0.9.$ $t_{n_2}^* = 0.7 = 1 - t_0^*.$
5	High probability of background fibres on PoI Low conditional probability of transfer from car seat to PoI given transfer of fibres from PoI to car seat High unconditional probability of transfer from car seat to PoI	$b_0^* = 0.1 = 1 - b_{1,n_2}^*$ $u_{n_2\|T_1} = 0.1.$ $t_{n_2}^* = 0.9 = 1 - t_0^*.$
6	High probability of background fibres on PoI Moderate conditional probability of transfer from car seat to PoI given transfer of fibres from PoI to car seat High unconditional probability of transfer from car seat to PoI	$b_0^* = 0.1 = 1 - b_{1,n_2}^*$ $u_{n_2\|T_1} = 0.7.$ $t_{n_2}^* = 0.9 = 1 - t_0^*.$

is less than the support of E_2 on its own. The relationship between E_1 and E_2 is said to be one of 'partial redundance'. A value of $R > 1$ occurs when $W_{E_2|E_1,H_p}$ and $W_{E_2|H_p}$ are of different signs, one supports H_p and the other supports H_d. The relationship between E_1 and E_2 is said to be one of 'directional change'.

Cases 1 to 3 in Table 2 are cases where there is a very high assigned probability of no background material. Thus the probability of the presence of background material is very low. The transfer of fibres from the car seat to the PoI conditional on transfer of material from the PoI to the car seat has a high assigned probability of 0.9 ($u_{n_2|T_1}$). The unconditional probability $t_{n_2}^*$ of material from the car seat to the PoI has varying values of 0.1 in case 1, 0.8 in case 2 and 0.7 in case 3, giving measures of interaction of −0.485, −0.017 and –0.039, respectively. The higher the conditional probability is relative to the unconditional probability the greater the synergy. Case 4 has a low assigned probability for no background material associated with the PoI. Case 4 may be compared with Case 3. Both cases have the same conditional and unconditional transfer probabilities of material from the car seat to the PoI. Case 4 has a much lower assigned probability (0.1) of no background material than case 3 (0.9) and case 4 has a more synergistic measure of interaction than case 3 (–0.107 compared

TABLE 3 *Measures of interaction R for differing circumstances of transfer as defined in Table 2*

Case	1	2	3	4	5	6
R	–0.485	–0.017	–0.039	–0.107	0.698	0.097
Comment	Very synergistic	Slightly synergistic	Very slightly synergistic	Synergistic	High partial redundance	Slight partial redundance

with –0.039). Cases 5 and 6 have low assigned probabilities for no background material associated with the PoI. In case 5 the conditional probability of transfer from the car seat to the PoI is very low (0.1) compared with the unconditional probability (0.9) which has led to a high level of partial redundance. In case 6 the conditional probability of transfer from the car set to the PoI is slightly smaller (0.7) compared with the unconditional probability (0.9) which has led to a slight level of partial redundance.

5. Conclusion

The analysis of inferential interactions plays an important role in the description of the line of reasoning for a forensic evaluator. The two items of evidence may mean more to an evaluator when considered jointly (as previously presented), than they do if considered separately.

The probabilities defined in Table 1, with sample values given in Table 2 are subjective. They are said to be 'assigned' by the investigator or forensic scientist based on their experience of other cases of transfer. The importance of the transfer material may be assessed by the value of the interaction measure *R* that is determined by the assigned probabilities. High values of the conditional transfer probability relative to the unconditional transfer, lead to high negative values of *R*, or synergistic values which suggests the transfer evidence is important. Values of the conditional transfer probability which are not much higher than the unconditional transfer, lead to low negative values of *R*, or synergistic values which suggests the transfer evidence is not very important.

Acknowledgements

In memory of David Schum to whom we owe a debt of gratitude for all that he has done for the subject of probabilistic reasoning.

The authors thank the Swiss National Science Foundation for its support through grant n. IZSEZ0-19114, the Editor in Chief of *Law, Probability and Risk* and an anonymous referee for their fruitful comments and suggestions to increase the clarity of this paper.

REFERENCES

[1] CHAMPOD C and TARONI F. A probabilistic approach to the evaluation of fibre evidence. In J ROBERTSON, C ROUX, and K G WIGGINS, editors, *Forensic examination of fibres*, pages 395–425. CRC Press, Boca Raton, 2018.

[2] TARONI F, BIEDERMANN A, BOZZA S, COMTE J, and GARBOLINO P. Uncertainty about the true source - A note on the likelihood ratio at the activity level. *Forensic Science International*, **220**:173–179, 2012.

[3] COOK R, EVETT I W, JACKSO G, JONES P J, and LAMBERT J A. Case pre-assessment and review in a two-way transfer case. *Science & Justice*, **39**:103–111, 1999.

[4] Evett I W. A quantitative theory for interpreting transfer evidence in criminal cases. *Applied Statistics*, **33**:25–32, 1984.
[5] Curran J M, Hicks T N, and Buckleton J S. *Forensic interpretation of glass evidence*. CRC Press, Boca Raton, FL, 2000.
[6] Cowell R G, Dawid A P, L Lauritzen S, and Spiegelhalter D J. *Probabilistic networks and expert systems*. Springer, New York, 1999.
[7] Kjrulf U B and Madsen A L. *Bayesian networks and influence diagrams. A guide to construction and analysis*. Springer, New York, 2008.
[8] Taroni F, Biedermann A, Bozza S, Garbolino P, and Aitken C G G. *Bayesian networks for probabilistic inference and decision analysis in forensic science*. John Wiley & Sons, Chichester, 2014.
[9] Good I J. Weight of evidence: a brief survey (with discussion). In J.M Bernardo, M.H DeGroot, D.V Lindley, and A.F.M Smith, editors, *Bayesian statistics 2*, pages 249–270. North Holland, Amsterdam, 1985.
[10] Juchli P. *Combining evidence*. PhD thesis, The University of Lausanne, School of Criminal *Justice, Lausanne*, 2016.
[11] Juchli P, Taroni F, and Aitken C G G. Measuring evidential phenomena in masses of evidence and complex reasoning patterns. Technical report, School of Criminal Justice, The University of Lausanne, Lausanne, 2020.
[12] Schum D A. *Evidential foundations of probabilistic reasoning*. John Wiley & Sons, Inc., New York, 1994.