Abstract relations

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S.I. : ENABLING MATHEMATICAL CULTURES

Abstract relations: bibliography and the infra-structures of modern mathematics

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Abstract
Beginning at the end of the nineteenth century, systematic scientific abstracting played a crucial role in reconfiguring the sciences on an international scale. For mathematicians, the 1931 launch of the *Zentralblatt für Mathematik* and 1940 launch of *Mathematical Reviews* marked and intensified a fundamental transformation, not just to the geographic scale of professional mathematics but to the very nature of mathematicians' research and theories. It was not an accident that mathematical abstracting in this period coincided with an embrace across mathematical research fields of a distinctive form of symbolic and conceptual abstraction. This essay examines the historical, institutional, embodied, and conceptual bases of mathematical abstracting and abstraction in the mid-twentieth century, placing them in historical context within the first half of the twentieth century and then examining their consequences and legacies for the second half of the twentieth century and beyond. Focused on scale, media, and the relationship between mathematical knowledge and its forms of articulation, my analysis connects the changing social structure of modern mathematical research communities to their changing domains of investigation and resources for representation and collective understanding.

Keywords Bibliography · Infrastructure · Structuralism · Abstracting · Abstraction

1 A searing question

At the start of the first International Congress of Mathematicians, in Zürich in 1897, Ferdinand Rudio (1898) laid out the defining problems and prospects for international mathematics. "I would like to draw your attention to one point, perhaps the most

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important of all," he announced as his opening address surged toward its climax.[1] It was "a searing question whose solution requires energetic initiative" from the world's mathematicians: "the question of mathematical bibliography."

At a time of "enormous production where the works are so dispersed," asserted Rudio, "a rapidly and continually functioning bibliographic repertoire is essential." Rudio dreamed of a bibliographical apparatus that would allow mathematicians to know "everything that has appeared in a given domain not only in the latest years, but also in the latest months or even the latest weeks." Such an apparatus, he claimed, "cannot be established except by an international institution."

Rudio's searing question came to define a central arc across twentieth-century mathematics, with new international institutions exchanging the latest mathematics across geographically expanding and temporally compressing scales. Today's mathematicians are connected by seemingly global electronic infrastructures. For many, the routines of mathematical research involve regularly consulting the stream of new work in their areas of interest on the arXiv preprint repository, a startlingly precise realization of the bibliographic repertoire for which Rudio called over a century ago. Together with the American Mathematical Society's *MathSciNet* and its European counterpart *zbMATH,* as well as other bibliographical resources less specific to mathematics, the arXiv gives mathematicians purchase on a vast dispersed production of modern mathematics across time and space.

Mathematical bibliography indexes a defining infrastructural and institutional transformation in twentieth-century mathematics—one shared in many ways with other sciences of this period (see e.g. Hepler-Smith 2016), though these are beyond the scope of this essay. International publications were a crucial mechanism, both practically and ideologically, for mathematicians to envision and build a discipline that could be called international on more than just paper (see Peiffer et al. 2018). Bibliographic apparatus enabled and responded to changing scales of mathematical research, travel, cooperation, and collaboration. The problems of bibliography drove mathematical organizations and motivated international partnerships. Mathematicians produced more and more mathematics in more and more places, and bibliography helped them to keep up with each other and to interact meaningfully at great distances, creating the conditions for yet more production in turn.

But the global transformation of mathematics was not just a change in volume, and bibliography did not just reflect a growing field. I here propose that modern infrastructures for managing and distributing knowledge about mathematical production altered, in turn, how mathematicians produced that knowledge and what that knowledge meant. By changing how mathematicians accessed and engaged the mathematical literature, modern mathematical bibliography changed the nature of mathematics. Specifically, by intensifying attention to structural relationships among theories and their producers, these *infra*structures contributed to research programs and conceptual frameworks for abstract *structures* in mathematical research.

This is a programmatic essay, articulating a historiographical provocation around the notion of "abstract relations" that reframes existing research and offers new

[1] Rudio's address appears in German and French in the Congress's proceedings. I have translated the remarks into English here.

directions for historical investigation and interpretation. My argument about abstract relations has some implications that can be studied directly, but it is most meaningful as an orientation that can guide how one understands the defining transformations of modern mathematics. A historiographical program, in the sense developed here, is not a concrete plan of research, but rather a conceptual intervention that fruitfully changes how one views both past and future studies. I base this provocation and perspective linking modern mathematical structures to their corresponding infrastructures on my own past and ongoing research, as well as that of other historians, sociologists, and philosophers of modern mathematics. By articulating "abstract relations" as a meaningful historiographical orientation, I aim to move bibliography—a longstanding and indispensable tool for studies *about* mathematics—into the center of understanding modern mathematics itself.

2 Mediated theory

To the extent mathematical knowledge is shared—that is, to the extent it has a social reality as knowledge—it must be shared by means of something. With conversations, gestures, sketches, chalk talks, letters, printed documents, sculpted models, and a great variety of other interactive settings and objects, mathematicians create shared understandings of concepts, methods, and principles that enable further personal and collective knowledge-making. These activities can differ considerably for different individuals in different historical and geographical contexts, but mathematicians' defining project of creating and sharing mathematical knowledge permits some generalizations. Following their means of communication and those means' associated media allows analyses of mathematical understanding that foreground the multifarious forms of comprehension and incomprehension in theory building and communication (e.g. Barany and MacKenzie 2014; De Freitas and Sinclair 2014; Greiffenhagen 2008, 2014; Merz and Knorr Cetina 1997; Rosental 2008).

In pedagogical or research contexts, mathematical communication is dominated by attempts to bridge circumstances of mutual ignorance and incomprehension. I have elsewhere (Barany and MacKenzie 2014) distinguished between two different kinds of mathematical media associated with different genres of communication and understanding. Stabilized media—principally mathematical publications—require a substantial and often effaced labor of "writing up" that renders mathematical abstractions in a form that can reliably travel from place to place. In each site of interpretation, however, such written-up mathematics requires a converse process—what I termed "reading down"—to be meaningful and useful. Such interpretation relies on situated media, which allow dynamic but correspondingly immobile representations and manipulations that facilitate new personal and interpersonal understanding. Situated media include blackboards, scrap paper, and other substrates that are primarily manipulated and interpreted "in the moment" in a particular context, sometimes in interaction with interlocutors. Mathematicians learn to move between stabilized and situated media in their work, naturalizing the considerable creative labor of mediated translation that allows them to create and convey concepts, methods, and ideas.

Rather than serving as a ready repository of settled mathematical knowledge, the mathematical literature here becomes a resource that must be mobilized through deliberate activity. At a remove from workable mathematical understanding, the stabilized forms of mathematical articles and references provide starting points for processes of continual re-mediation. Here, re-mediation indicates both a process of transferring representations from one medium to another and the concomitant process of repairing, remedying, or remediating fragmentary mathematical understandings. Mathematicians adapt and redeploy formulations and approaches according to the idiosyncratic understandings they have developed for their own phenomena of interest and expertise.

Sometimes the process is straightforward, requiring little supplementary labor to grasp and operationalize a circulated result. Other times, as becomes unusually visible in response to a claimed breakthrough for a major unsolved problem, reading down can be a difficult and contested process involving many people and considerable resources. The recent online Polymath projects of blog-based "massively collaborative mathematics" offer a striking record of attempts to perform both reading down and writing up in public, with many (but by no means all) typically-private aspects of this labor documented in online fora (see, e.g., Barany 2010; Polymath 2014; Martin and Pease 2013; Martin 2015; Pease et al. 2018).

In past and present alike, mathematicians have undertaken the work of communicating and re-mediating based on a belief that they were working on *the same thing* as others (see Barany 2018). This belief justifies the effort to circulate texts and reconcile interpretations, and also makes it possible to look past those efforts in characterizing the substance of mathematical activity. Mathematicians come to this belief through common training, personal contacts, experiences of collaboration, and other means of linking and sorting people and ideas. As mathematicians developed new relationships to bibliography and the mathematical literature in the twentieth century, I propose, they reconfigured what it would mean to work on the same thing and how they understood the relationships between the many same things of mathematics. Conversely, following how mathematicians built and engaged mathematical literatures can give historical access to aspects of their activity not readily visible in that activity's published residues alone.

3 Springer's modernity

Many of the most notable features of modern mathematics, including the role of abstraction and the relationship between pure and applied subjects, trace to the German university town of Göttingen and the two mathematical titans who presided there at the turn of the twentieth century, David Hilbert and Felix Klein (Rowe 1989). Their respective mathematical programs, in their own ways, proved both mathematically and institutionally ambitious, driven by a universalism that tied together theories and research efforts through foundational reasoning that cut across conceptual, professional, and national boundaries. The ferment associated with Göttingen underwrote a new structural approach to mathematical theory, especially in the field of Algebra, with profound consequences for the practice and philosophy of modern mathematics (Corry 2004).

Around the end of the Great War, Göttingen's internationally-oriented mathematics community became the hub for a new publishing venture spearheaded by Ferdinand Springer, Jr., grandson of the founder of the Springer publishing house (Munroe 2007). Adding mathematical publications to his catalogue, Springer, Jr. lay claim to what continues to this day as a profitable niche catering to mathematical researchers. This effort gave rise to a profound new development in mathematical bibliography with the 1931 launch of the *Zentralblatt für Mathematik und ihre Grenzgebiete,* based editorially in Göttingen. Siegmund-Schultze (1994) argues that the *Zentralblatt* marked a radical departure from previous bibliographical enterprises, especially the then-dominant, Berlin-based, half-century-old *Jahrbuch über die Fortschritte der Mathematik*. Where the *Jahrbuch's* publishers aimed to produce a comprehensive (albeit German-centered) view of annual developments in the discipline to serve as a durable reference at the cost of long publication delays, the *Zentralblatt* editors released a rolling record of new mathematics as rapidly as they could produce it. Instead of relying like older bibliographical projects had on subsidies from scientific societies, Springer found that such a guide to the current literature justified its cost as an authoritative advertisement for the press's own publications (cf. Fyfe et al. 2017).

The *Zentralblatt* quickly proved that the kind of rapid, wide-ranging, international bibliographic apparatus for which Rudio called in 1897 could become an indispensable and self-sustaining matrix joining a dispersed variety of publications into a unified mathematical literature. While the reviewing journal relied on an international pool of editors and the even broader disciplinary connections they maintained, it did not require the kind of top-down international coordination Rudio presumed necessary. By publishing authors from across Europe and by advertising their publications in a common forum and format that entered them into ongoing research conversations, Springer and the *Zentralblatt*'s editors helped to sediment a common market and a common research community, reinforcing international activity by making it mutually available to participating scholars.

At the same time Springer launched the *Zentralblatt,* the publisher released a monograph that Corry (2004) identifies as a watershed in the rise of mathematical structures, Göttingen-trained Bartel van der Waerden's two-volume (1930–1931) *Moderne Algebra*. The first volume was not reviewed in the *Zentralblatt,* but the second drew an extended notice on pages 8–10 of the first issue of the review journal's second volume. For Corry, van der Waerden's book marked a milestone for "the reflexive character of mathematics," using mathematical methods to make claims about the discipline of mathematics as a whole. New mathematical theories of structures from what Corry calls the "body of mathematics" defined and developed new philosophies of mathematical structuralism for what Corry terms the "image of mathematics." Mathematicians powerfully asserted that the domains and concepts of mathematics themselves obeyed structural principles susceptible to mathematical analysis, and correspondingly that studying these structures constituted the essence of mathematics.

Analyzing the life and worldview of the *Zentralblatt*'s founding managing editor, Otto Neugebauer, Siegmund-Schultze (2016) observes the biographical and institutional convergence in Göttingen of modern international theoretical programs and modern international reviewing infrastructures. This, he notes, came despite the apparent difference between backward-looking bibliographic labor and forward-looking original scholarship. However, seen another way, the modernization associated with mathematical structures and structuralism was very much of a piece with the modernization in mathematical infrastructure associated with Springer, the *Zentralblatt,* and further financial and institutional developments in and around Göttingen. This link between mathematical structures and structuralisms and their supporting infrastructures suggests a corresponding reflexive relationship between the material, personal, and institutional *bases* of mathematics and the discipline's *image* and *body*.

New mathematical infrastructures supported and required new means of practicing mathematics and new conceptions of what mathematics was. At each level—basis, body, and image—the operative object is a kind of *abstract relation*. In structuralist images of mathematics, abstract relations connect principles, fields, and kinds. In the structural body of mathematics, such relations tie together ideal concepts and entities. Post-1930 mathematical infrastructures, meanwhile, joined mathematicians through relations mediated by the production and circulation of bibliographic abstracts in the mold of entries from the *Zentralblatt.* Just as mathematicians applied theoretical developments from the body of mathematics to the image of mathematics and vice versa, I propose, their infrastructural relations emphasized and facilitated distinct kinds of theoretical abstraction.

4 The political economy of abstracting

Publishing's place in the modern mathematical profession meant that the infrastructural stratum of abstract relations encompassed a tremendous range of activities in a globalizing discipline. Early editorial tensions between the *Zentralblatt* and *Jahrbuch* underscored some of the stakes.[2] Rapid dissemination was more meaningful in a distributed mathematical community that relied on epistolary relationships and occasional travel, but was less urgent for a concentrated and relatively inward-looking Berlin mathematical elite that could rely on other sources of professional intelligence and sociality. The *Jahrbuch*'s relationship to the Prussian Academy of Sciences gave its editors official resources and obligations toward national scientific prerogatives, distinct from the commercial and international orientations of Springer and the *Zentralblatt*. The *Zentralblatt*'s comparative emphasis on foreign authors and reviewers became a repeated and intense point of conflict as mathematicians in Germany responded in different ways to the Nazi party's rise in the 1930s.

These latter conflicts culminated in a 1938 political crisis surrounding the *Zentralblatt,* including a mass resignation of editors. This crisis, in turn, precipitated a concerted effort in 1938–1939 to relocate Neugebauer to the United States and to launch, under his direction, an American reviewing journal closely modeled on the *Zentralblatt* to replace the compromised German publication (Siegmund-Schultze

[2] Some features I here associate with the *Zentralblatt* appear in the *Jahrbuch*'s history as well. I thank Harald Kümmerle for detailing to me a striking example: Emmy Noether developed her theory of abstract rings in Göttingen independently of Masazo Sono in Tokyo, but began citing Sono's work shortly after reviewing a 1924 article of his for the *Jahrbuch,* and even helped inform visiting Japanese mathematicians in Germany of their countryman's work.

1994, 2016). Founded in 1940, that new journal, *Mathematical Reviews,* became a powerful icon and instrument of American mathematical internationalism.

Mathematical Reviews differed in several important respects from the *Zentralblatt*. Administered and promoted through the American Mathematical Society, *Mathematical Reviews* quickly developed as a pivotal resource for American mathematicians to assert primacy on a world stage. Indeed, initially they pursued projects like *Mathematical Reviews* self-consciously as a replacement for German institutions that leading Americans had previously seen to hold such a role. Rockefeller Foundation discussions regarding subventions for an American counterpart to the *Zentralblatt* called the matter "one instance of a general situation of … the transferance [sic] to this country of responsibility for the maintenance and protection of certain cultural values which historically have been chiefly located in Europe."[3] Such a transatlantic shift in what multiple American elites began to call "the center of gravity of mathematics" depended heavily on the largess of powerful philanthropies including the Rockefeller Foundation and the Carnegie Corporation of New York, and later on substantial direct and indirect aid from the United States military (see Barany 2016).[4]

For mathematicians across the rapidly-expanded sphere of American bibliographical hegemony after 1940, *Mathematical Reviews* was the crucial gateway to the mathematical literature. The American Mathematical Society used the journal as a venue for partnering with other mathematical societies, sponsors, and related organizations both domestic and foreign. The title page for volume 7 (1946), for instance, listed as co-sponsors the Mathematical Association of America, the Institute of Mathematical Statistics, and mathematical societies and scientific academies from Lima to London and Argentina to Amsterdam. Internationally-oriented entities signaled their relevance and joined in a wide-reaching scholarly community by cooperating with the American Mathematical Society and contributing to the logistics or finances of its review journal.

After World War II, as the number of mathematical publishers and publications soared and even the best resourced libraries struggled to maintain what felt like comprehensive collections of the latest books and journals, *Mathematical Reviews* helped researchers to stay reasonably current in their fields to a far greater extent than was possible by exchanging news and articles by correspondence or other person-to-person means alone. Librarians and researchers developed routines, complete with pre-printed postal apparatus, for converting bibliographical entries into requests for texts from afar. At various points, the American Mathematical Society explored means of providing articles directly to researchers, for instance as microfilms, using *Mathematical Reviews* as a catalogue and index for such services. Systematic current bibliography gave mathematicians the means to engage with research outside of their personal cor-

[3] Warren Weaver officer diary excerpt, 23 Feb 1939, in Rockefeller Foundation Archives (hereafter RF Archives), Record Group 1.1, series 200, box 125, folder 1550, Rockefeller Archive Center, Sleepy Hollow, NY. Siegmund-Schultze (1994, p. 323; 2016, pp. 88–89) discusses this diary entry as well.

[4] For the "center of gravity" phrase, see e.g. R.G.D. Richardson, "Memorandum Regarding a Mathematical Abstracting Journal," Dec 1938, in RF Archives, Record Group 1.1, series 200, box 125, folder 1549; Weaver, "International Mathematical Review Journal," 18 May 1939, RF Archives, Record Group 1.1, series 200, box 126, folder 1551. On early philanthropic support for *Mathematical Reviews,* see American Mathematical Society Records, Ms. 75, box 15, folders 33 and 49, John Hay Library, Brown University, Providence, RI.

respondence networks, local and national communities, and whatever regional and international publications they happened to have on hand.

Conversely, the ambition to assemble and centralize the mathematical literature through rapid abstracting required Neugebauer and his successors to maintain far-reaching networks of their own to collect new works and distribute them for review. Editors cultivated a steady stream of new publications; developed schemes for classifying them by topic, language, and other desiderata; compiled and sorted extensive index files of potential reviewers to match to that sorted literature; and deployed a vast system of procedures and accommodations to shepherd the literature through the process of reviewing and presentation in the review journal. This required disciplinary expertise and labor from highly trained mathematicians as well as considerable bureaucratic expertise and labor from secretarial and other supporting staff.

The same Rockefeller Foundation note about Americans' taking responsibility from Europeans for protecting cultural values captured the extended effects of bibliographic infrastructure for a growing discipline. An American equivalent to the *Zentralblatt* would be "an international coordinating and synthesizing influence," not "a mere mechanical bibliographic aid."[5] The distributed and resource-intensive labor of coordinating and synthesizing the mathematical literature was, at the same time, a means of coordinating and synthesizing the disciplinary communities that produced and consumed that literature.

5 Sociable structuralism

Not all forms of mathematics could travel equally well through the review literature, so not all forms of mathematics could be shared equally well across the coordinating and synthesizing infrastructure that review journals provided. I propose that mathematical abstracts, in these settings, relied upon and reinforced specific forms of mathematical abstraction. This link between abstraction and abstracting underwrote social and intellectual relationships among mathematicians, helping them to sustain particular kinds of research programs. Through reviews, mathematicians simultaneously imagined and enacted a structured literature, a structured field of knowledge, and a structured discipline.

A typical abstract in a reviewing journal began with a typographically distinguished citation indicating the work's author, title, and venue. Implicitly or explicitly, these citations told readers how difficult the work might be to access—linguistically, logistically, technically, or otherwise—and how they might hope to access it. Next came a brief summary of the work's context and contributions, typically no longer than an extended paragraph, followed by the name and sometimes the location of the reviewer.

Reviews often contained definitions and equations, but rarely stated results in detail. There was no room for proofs or thorough explanations of methods and techniques. Rather, the most significant and informative labor of reviewing involved situating the work in question within a broader literature, using shared terminology, eponymy, and abbreviated citations to connect the work in question to others. Such contextualizations

[5] Warren Weaver officer diary excerpt, 23 Feb 1939, op cit.

often evaluated the work's contribution in relation to these other touchpoints, and reviewers also indicated where a work was redundant with other elements of the mathematical literature, or even mistaken.

Editors initially sorted abstracts into broad categories according to editorial expertise and conventional divisions of research fields, corresponding to the table of contents for the review journal. The problem of classification loomed large from the start.[6] As the American mathematical community laid the groundwork for *Mathematical Reviews*, an interested observer in Cambridge, England, worried in 1939 about "the incompetence of their subject classification as compared with Neugebauer's" and later expressed his "hope [that] their subject classification is as near an ultra-filter as possible," invoking modern set-theoretic jargon in an atypically precise early realization of this essay's provocation about the links between the *basis* and *body* of mathematics.[7]

Varying volumes of material could make rigid classifications impractical.[8] Editors considered, for instance, whether mathematical physics should have its own category or be regarded only as a subcategory of the different branches of mathematics, such as the study of differential equations, on which mathematical physicists drew. This distinction affected the review journal's organization and the picture of mathematics it gave, but also affected editors' ability to identify and recruit reviewers for the papers in question.[9] Common classificatory assumptions and keywords connected the index-card apparatus of reviewing to the index-bound apparatus of sorting abstracts for publication, and arranging topics and fields was always simultaneously a matter of arranging people and the professional relations between them. Over the latter half of the twentieth century, successive editors developed such classifications into an elaborate hierarchical classification system reflecting finer distinctions among literatures and research communities, reflecting but also supporting specific kinds of specialization. This system, which came to be called the Mathematics Subject Classification, eventually became a crucial point of cooperation and decennial revision between the publishers of *Mathematical Reviews* and the *Zentralblatt*.

Such explicit and sustained attention to categorizing mathematical publications corresponded considerably in time, place, and personnel with the elaboration of new mathematical theories regarding the structure of mathematics as a whole, most notably the Category Theory at the culmination of Corry's account of mathematical structures. Even those who did not work directly on structural theories of mathematics developed structural views of their subjects through routine engagement with the review literature, which required them to think of their work through its relations to different branches of mathematical theory and practice. Using bibliographical categories to identify relevant scholarship trained such scholarship around those categories, reinforcing the categories' salience through the iterative labor of research and publication.

[6] There are significant literatures on the history and sociology of classification in science and its relation to infrastructures, e.g. Bowker and Star (1999) and Csiszar (2017).

[7] Smithies to Tukey, 24 Jan and 15 Aug 1939, folder A66, Papers of Frank Smithies, St John's College Library, Cambridge, quotations by permission of the Master and Fellows of St John's College.

[8] Boas to Smithies, 23 Jun 1940, folder A10, Papers of Frank Smithies, St John's College Library, Cambridge.

[9] Boas to Smithies, 17 Aug 1940 and 17 Apr 1945, folder A10, Papers of Frank Smithies, St John's College Library, Cambridge.

Place-rooted customs and practices of problem-solving and theorizing continued to matter, but research that reached between such places did so through infrastructures that put abstract relations—in the multiple senses here considered—at the fore. For structuralist mathematicians, classifying and relating became more thoroughgoing means of tying together ideas and methods (cf. McLarty 2008). Category theory and bibliographical category practice thus represented two distinct but related ways of deriving unity and order in a variegated discipline.

6 Unifying infra-structures

While unity and order have been recurring preoccupations for scholars of many stripes in many centuries, they may have appeared especially topical for those at the nexus of bibliography and structuralism in the early-to-mid twentieth century. One of Neugebauer's colleagues worried in 1931 "of a total decay of mathematics into isolated disciplines" (quoted in Siegmund-Schultze 2016, p. 74). Interwar Göttingen was an especially formative setting for several members of the Bourbaki collaboration, who militated so ardently for the unity of mathematics as to drop the "s" from the term in their *Elements de Mathématique* textbooks (Beaulieu 1989; Siegmund-Schultze 2001; Goldstein 2009). A 1926 survey of European mathematicians still listed Göttingen's David Hilbert as an expert in "all fields,"[10] but the designation came at a time when such polymaths were routinely celebrated or mourned as the last of their kind. In 1912, the president of the Cambridge Philosophical Society opined that "The Science of Mathematics is now so wide and is already so much specialised that it may be doubted whether there exists to-day any man fully competent to understand mathematical research in all its many diverse branches," although the recently-deceased Henri Poincaré might have fit the bill (Darwin 1913). Poincaré, indeed, was not just famously wide-ranging in his mathematical expertise but famously obsessed with mathematical and scientific bibliography, even drawing notice for his bibliographic undertakings in Rudio's 1897 remarks (Rudio 1898, p. 41; Csiszar 2010, ch. 5).

Even as they index varieties of fragmentation, bibliographies are unifying infrastructures. Viewed through mathematical abstracts, the fundamental character of new work derived from its relation to the literature—present, past, and future. Abstracts' categorical placement in a section of a review journal embedded them synchronically in the current literature. Citations within abstracts embedded them diachronically in past literature and conversations. Indications of a work's most significant interventions, contributions, or even errors embedded abstracts in future research and potential new publications. From each perspective, relations among bibliographical abstracts corresponded to relations among mathematical ideas, methods, and fields.

Reviewers were not innocent or disinterested actors in the relational work of reviewing (Audin 2012; Paumier 2014, §2.3; Barany 2018, pp. 289–293). Reviews allowed reviewers to promote their own work and that of collaborators and allies to a large potential audience, even if such work appeared initially in venues with com-

[10] "Leaders in the Field of Mathematics Listed Under Countries and Institutions." 26 Jan 1926. International Education Board Archives, box 8, folder 110, Rockefeller Archive Center, Sleepy Hollow, NY.

paratively minor circulation. Citations within reviews, especially citations to one's own or one's collaborators' work, could establish priority and relevance. Strategically worded summaries and identifications let reviewers consolidate competing interpretations or formulations, helping emerging areas of research appear—and consequently become—more unified. These consolidations often amounted to annexations, where reviewers could claim competing research agendas as contributions to their own favored programs and theories, even programs and theories of which the reviewed work's author might be unaware. In this way, for instance, Laurent Schwartz created a literature around his new theory of distributions in the late 1940s and early 1950s that drew together a variety of competing theories in topology, functional analysis, and related fields, using reviews to show readers how to view others' work as a contribution to his own theoretical program by classifying, situating, and interrelating those works and their results accordingly (Paumier 2014; Barany 2018). Schwartz's reviewing was perhaps an extreme form of the inevitable labor at the core of abstracting: embracing the reviewed work in a structured disciplinary context through the eyes of the reviewer. Such annexations could be highly effective, altering how mathematicians across a field of study viewed others' work and even altering mathematicians' conceptions of how their own research fit into a bigger picture.

Bibliography and abstracting could thus powerfully shape how communities of mathematicians established that they were studying *the same thing*, even allowing quite different things to be synthesized through reviews into coherent theoretical unities (see Barany 2018). As mathematicians' research communities became more geographically dispersed and the discipline became more internationally integrated, such bibliographic syntheses took an outsized importance as especially reliable and unified means of consolidating communities and theories alike. Crucially, only certain kinds of mathematical production could be shared and concretized through the review literature. Absent detailed background, complicated expositions, or elaborate constructions—all of which remained very much a part of mathematical research and exposition in other settings—mathematical abstracts created social and theoretical unities on the basis of the relatively constrained contents they could convey. Mathematical communities themselves took on features of mathematical abstracting, mediated by textual exchanges and identifications of central concepts and common reference points. Mathematicians' identities as individuals hinged to a new extent on their manifestations as printed names in networked texts—so much so that the number and significance of mathematical pseudonyms (most famously Nicolas Bourbaki) flourished in the decades after 1930 (Barany 2020).

7 A sense of the literature

Abstracting thus enforced relational views of mathematics and mathematicians where context in the written literature was key. I have argued elsewhere (Barany 2018) that this context-intensive relational approach to mathematical theory emerged in precisely the mid-century transformations in modern mathematics associated with review journals and mathematical structuralism. I claimed that this development should be understood in terms of the multiple meanings of the term *sense* (cf. Wagner 2017). Specifically, mathematicians learned to identify a concept's "having a sense"—an established context or framework of interpretation—with its "making sense"—being foundationally valid and justifiable in principle. Mathematicians could thereby insist that their work fit into a solid and (often-implicitly) unified foundation where everything *made sense* while focusing their reasoning and arguments around identifying and developing specialized, idiosyncratic, community-driven contexts—or *senses*—of interpretation.

These latter activities, focused around *having a sense*, depended on precisely the kinds of relational labor and thought realized and reflected through emerging bibliographic practices. These, in turn, hinged on two further meanings of sense: giving or getting "a sense of" an idea—an approximate or working understanding—and knowing through sensory experience—including the distinctive sensory apparatus of blackboards, abstracts in reviewing journals, and other media discussed in this essay. The limitations and constraints of abstracting media, viewed this way, become resources for abstraction in mathematical theory. Because the sensory apparatus of mathematical abstracts can only give mathematicians *a sense of* (a rough idea of) a concept or method, they focus on citations and relations that indicate *the sense of* (the meaning-giving context of) that concept or method.

Mathematicians had many ways to indicate a term's context, but it is nonetheless notable how regularly they turned to variations of sense (in English), sens (in French), and Sinn (in German) to do so in mathematical abstracts and reviews in this pivotal period. One may conjecture that this linguistic formulation tracks wider developments and distinctions across fields and genres of mathematical text, with sense-specification increasing with structural approaches and condensed formats. Bibliometrically, one would expect mathematicians' changing relationships to bibliographies to manifest in changing patterns of citation, reflecting broader and more textually integrated international research communities. Read against the grain, bibliographic data can show patterns of participation, inclusion, and circulation in a globalizing mathematical discipline, as well as changing definitions of and relationships between fields and specialisms. New and improved full-text repositories of mathematical writing, not least the *MathSciNet* and *zbMATH* databases descended respectively from *Mathematical Reviews* and the *Zentralblatt,* open wide vistas for exploring these and related conjectures through a variety of quantitative and qualitative methods.

Rudio's "searing question" of bibliography remains, more than a century later, a vital one for mathematics, as well as its history, sociology, and philosophy. Bibliography connects mathematicians and mathematical institutions to each other, motivates and supports collective endeavors, and fosters potent images of past, present, and future mathematics. To understand modern mathematics, it is necessary to follow its abstract relations and the infra-structures they enact and make possible.

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