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Negative Existentials and Non-denoting Terms

Paul Schweizer

Institute for Language, Cognition and Computation
School of Informatics, University of Edinburgh
e-mail: paul@inf.ed.ac.uk

Abstract. Logical and semantical issues surrounding non-denoting terms have been investigated since ancient times, in both the Western and Indian philosophical traditions. And in a more applied formal setting, such issues have also gained importance in constructive mathematics, as well as computer science and software engineering. The paper first presents a strategic exploration of logical treatments of reference failure in Western thought, and then goes on to provide a comparative examination of the issue in the Indian tradition, particularly with respect to the dispute between the Yogācāra-Sautrāntika school of Buddhism and the Nyāya school of Hinduism. The paper concludes by advancing a formalization of the Buddhist *apoha* semantical theory in terms of a dual-domain Free logic.

1 The Analysis of Non-Existence in Western Logic

It is a distinctive feature of human language and thought that we can introduce terms purporting to designate some object or entity in the world, but where no such object or entity exists. And we can then go on to use such terms to make grammatically well formed assertions which appear to be *meaningful*, and indeed many of these meaningful assertions about non-existent objects seem to be either *true* or *false*. This phenomenon poses some deep challenges for philosophy and logical theory which have been recognized and investigated since ancient times, in both the Western and Indian traditions. And in a more applied formal setting, such issues have also gained importance in constructive mathematics, as well as computer science and software engineering. In the context of ancient Greek philosophy, a well known version of the problem is articulated in Plato's riddle of non-being, often referred to as the predicament of 'Plato's beard'. Let us suppose that Plato was always a clean shaven individual and never sported facial hair. In such a case, we would seem to be asserting a true proposition with the negative existential statement 'Plato's beard did not exist'. But if Plato's beard did not exist, then exactly what are we talking about when we say that he didn't have one? And how can we make any coherent assertion involving the term 'Plato's beard' when, by hypothesis, it fails to denote? Even the cogency of the seemingly innocent 'Plato did not have a beard' seems threatened.

1.1 Sense, Reference and Definite Descriptions

Frege's [1] distinction between sense and reference provides a powerful and far reaching response to scenarios such as Plato's beard. In accord with Kant's maxim that existence is not a predicate applying to individual objects, Frege analyzed assertions of existence in terms of the extensions of concepts. Hence to say that aardvarks exist is, in effect, to say that the 'cognitive content' or sense (*Sinn* in Frege's terminology and intension in Carnap's) expressed by the term 'aardvark' is true of at least one object in the universe of discourse. So there are individuals in the range of the existential quantifier that satisfy or 'fall under' the concept 'aardvark'. And conversely, to say that unicorns do not exist is to say that no individuals in the range of the existential quantifier fall under this concept, and hence that its extension is empty. This makes existence a second order claim about concepts rather than objects. 'Aardvark' and 'unicorn' are general terms to which singular terms can attach to form atomic statements. Frege applies the distinction between sense and reference to singular terms as well, such as 'Pegasus' or 'Sherlock Holmes'. To say that Pegasus does not exist is again to say that no individual in the domain of discourse falls under the 'Pegasus' concept. In other words, 'Pegasus' has a sense but no reference.

This dual level analysis provides an elegant explanation of why terms with empty extensions can still contribute to *meaningful* discourse. At the level of intension or sense, there is still semantic content associated with terms such as 'Pegasus' and 'unicorn'. According to Frege's principle of compositionality, the semantic value of a complex whole is a function of the semantic values of its respective parts and their mode of combination. Propositions (or 'complete thoughts') are the intensions of declarative sentences, and the sense of a non-denoting term such as Pegasus can still contribute to the intensional level of sentences in which it occurs, to yield a meaningful proposition. And indeed, this supplies a very elegant explanation of the semantic content conveyed by literature and other forms of fictional discourse.

A proposition is the intension of a declarative sentence, while for Frege its extension or reference is a truth value. In accord with the above principle of compositionality, failure of reference for singular terms must turn the method of designating the reference of a sentence involving such terms into a partial function on the range {True, False}. Since Pegasus has a sense but no reference, the sense can contribute to a proposition, while the lack of reference entails that functional combination at this level fails, and sentences involving Pegasus will lack a truth value. If 'Pegasus' has no referent then neither does the sentence 'Pegasus is winged', so that its truth value is undefined or '*u*'. Bivalence must be sacrificed if genuinely non-denoting terms are allowed, and the principle of strict compositionality requires lack of reference to recursively propagate in the manner of an infectious disease. If the referent of 'Pegasus is winged' is *u*, then the value of, e.g. 'Pegasus is winged or snow is white' must also be u, because if there is an input missing to the disjunctive truth function then there can be no output. This yields a version of Kleene's system of weak 3 valued logic.

In Russell's [2] response to Frege the level of intension is not invoked, and instead Russell focuses purely on referential considerations. His 'logically proper name' is a pure indexical referring to immediate aspects of raw sensation, while standard and logically improper names are analyzed along the lines of definite descriptions. On Russell's account, expressions involving the definite article, such as 'the ϕ', are treated according to the standard existence and uniqueness constraints, $\exists x(\phi x \wedge \forall y(\phi y \leftrightarrow y = x))$. This analysis yields a formula rather than a singular term, and to make a further assertion *about* 'the ϕ', requires an appropriate augmentation of the base formula. Hence 'The ϕ is Ψ' is formalized as $\exists x(\phi x \wedge \forall y(\phi y \leftrightarrow y = x) \wedge \Psi x)$. If there is no object in the domain of discourse satisfying both the existence and uniqueness constraints, then 'the ϕ' is a vacuous description and the corresponding formula above will be false, as will any further formula attempting to assert something about 'the ϕ'. There is no present King of France, and if we let Kx symbolize the property in question, then $\exists x(Kx \wedge \forall y(Ky \leftrightarrow y = x))$ is rendered false by the falsity of the first conjunct. Consequently 'The present King of France is just' and 'The present King of France is not just' both turn out false (on both narrow and wide readings of negation), and now uniform falsity, rather than lack of truth value, propagates through the account.

But, contra both Frege and Russell, there is an intuitive sense in which we might want to make *true* assertions using non-denoting terms, such as those involving basic logical properties like self identity: 'The present King of France is identical to the present King of France', or statements using fictional names that affirm details of the literary context, like 'Sherlock Holmes was a brilliant detective'. It is also convenient to retain the logical form of a genuine *singular term* for both proper names and definite descriptions. But this won't work in classical logic for expressions that don't refer. If **t** is a singular term standing, say, for 'Plato's beard', then the negative existential mentioned above, *viz.*, $\neg\exists x(x = \mathbf{t})$ is a *contradiction* in classical first-order logic with identity, since it's a basic requirement of the model theory that **t** be assigned some object in the domain. This highlights a crucial asymmetry in the classical approach, where general terms are allowed to have empty extensions while singular terms are not.

1.2 Free Logic

As Lambert [3] perspicuously observes, the branch of non-classical logic known as Free logic is largely motivated in response to this asymmetry. The traditional logic of general terms supposed that the inference from $\forall y(\phi y \rightarrow \Psi y)$ to $\exists y(\phi y \wedge \Psi y)$ was valid, because the terms ϕ and Ψ were thought to have *existential import*. But this imposes an unwanted restriction on the range of applicability of formal reasoning, and on the modern and broader approach no such import is presupposed. The general terms ϕ and Ψ are allowed to be true of nothing, and hence the inference is invalidated. For example, since there are no unicorns, the actual world is a model of the sentence 'Every unicorn is an aardvark', formalized as $\forall y(Uy \rightarrow Ay)$, while it is false that $\exists y(Uy \wedge Ay)$, so the actual world serves as a counterexample to the inference. On the modern approach, an *additional premise* of the form $\exists y(Uy)$ is required to restore existential import and yield the valid (but unsound) piece of reasoning: $\forall y(Uy \rightarrow Ay)$, $\exists y(Uy)$ $\therefore$ $\exists y(Uy \wedge Ay)$.

However, classical first-order logic with identity retains a somewhat curious exception to the need for an additional premise. If the (potentially complex) 1-place predicate expression ϕy is replaced with the complex 1-place predicate $y = \mathbf{t}$, then the original inference pattern $\forall y(y = \mathbf{t} \rightarrow \Psi y)$ $\therefore$ $\exists y(y = \mathbf{t} \wedge \Psi y)$ goes through on its own. The expression ' = **t**' has existential import in the traditional sense, while in general the expressions ϕy and Ψy do not. This traditional residue derives from the asymmetrical fact that singular terms are required to denote while general terms can be empty. $\exists y(y = \mathbf{t})$ is a truth of classical logic for every singular term **t** in the language, and hence does not need to be introduced as an extra premise. This can itself be viewed as an undue restriction on the range of applicability of formal reasoning, since it is not possible to carry out intuitively plausible inferences concerning objects that do not or might not exist in the actual world. And in the same manner as above, the natural strategy is to devise a logic free of existence assumptions with respect to its terms, both *singular* and general (Lambert [4]).

In Free logic, the quantifiers are interpreted in the normal way, as ranging over some domain of discourse $\boldsymbol{D}$, normally construed as the set of 'existent objects'. But the singular terms may denote objects outside of $\boldsymbol{D}$, or fail to denote altogether. This de-coupling of singular reference from the range of the quantifiers undermines two fundamental inference patterns of classical logic, namely Universal Instantiation (UI) and Existential Generalization (EG). According to UI, $\forall y\phi y$ $\therefore$ $\phi\mathbf{t}$ is a valid inference. But it fails in Free logic because the quantifier $\forall$y only ranges over objects $e \in \boldsymbol{D}$, whereas '**t**' may not refer to any such *e*. So from the fact that every $e \in \boldsymbol{D}$ has property ϕ, it does not follow that **t** does. And according to EG, $\phi\mathbf{t} \therefore \exists y\phi y$ is a valid inference. But similarly this fails in Free logic because, e.g., **t** may denote a nonexistent object not in the range of $\exists y$, thus allowing for the possibility of true premise and false conclusion.

Analogous to the foregoing transition from traditional to modern logic in the case of general terms, now that singular terms are also free of existence presuppositions, an *additional premise* is required to restore validity. Existential import with respect to singular terms is expressed via an existence predicate for individuals (in violation of Kantian notions), normally using Russell's '*E!*' notation. With the use of identity, the existence predicate can be defined as $E!(\mathbf{t}) :=_{\text{def}} \exists y(y = \mathbf{t})$. In the case of both UI and EG, $E!(\mathbf{t})$ is the suppressed premise required to yield an inference pattern valid in the context of Free logic. Hence UI_{Free} has the form $\forall y\phi y$, $E!(\mathbf{t})$ $\therefore$ $\phi\mathbf{t}$, and EG_{Free} has the form $\phi\mathbf{t}$, $E!(\mathbf{t})$ $\therefore$ $\exists y\phi$y. It is now possible to directly articulate the fact that Pegasus does not exist with the formula $\neg E!(\mathbf{t})$, letting **t** denote the mythical flying horse. And while it's true that neither Plato's beard nor Pegasus exist, it's nonetheless *false* that $\exists x \neg E!(x)$.

1.3 Definite Descriptions Revisited

As noted earlier, Russell's 1905 theory of definite descriptions analysed expression such as 'the ϕ' in terms of a formula rather than a singular term. However, it is often convenient to be able to render such expressions as genuine terms, and have a uniform treatment of simple terms such as individual constants or proper names, along with complex singular terms such as definite descriptions and function terms. In *Principia Mathematica*, Russell [5] introduced his variable-binding, term-forming 'iota' operator to do just that. If it's provable that the existence and uniqueness conditions are satisfied, then a Russellian iota operator '$\boldsymbol{i}$' yields a complex singular term as follows: if $\vdash \exists x(\phi x \wedge \forall y(\phi y \leftrightarrow y = x))$ then $\boldsymbol{i}x\phi x$, read as 'the x such that ϕx', or simply 'the ϕ' is defined (contextually) as *that* unique x. The definite description '$\boldsymbol{i}x\phi x$' can then be used as a legitimate complex singular term for making assertions such as $\exists y(y = \boldsymbol{i}x\phi x)$, $\Psi(\boldsymbol{i}x\phi x)$, and the seemingly innocuous $\phi(\boldsymbol{i}x\phi x)$. In the special case of definite descriptions, a 1-place predicate ϕx is used to define a 0-place function, i.e. a singular term. In the general case, an n-ary relation $R^n(x_1,..., x_{n-1},y)$ can be used to define an $(n - 1)$-ary *total* function $f^{\,n-1}$, if R^n satisfies the corresponding existence and uniqueness constraints $\forall x_1 \ldots \forall x_{n-1}\exists y\ \forall z[\mathrm{R}^n(x_1,...x_{n-1},y) \wedge (R^n(x_1,...x_{n-1},z) \rightarrow z = y)]$ in which case $f^{\,n-1}(x_1,..., x_{n-1}) = y$ and the set of $(n - 1)$-ary total functions can be viewed as a proper subset of the set of n-place relations.

However, not all functions that we might wish to consider are total, and this can be due to a failure of either constraint. Furthermore, such failures might not be known to us at the time the function term is introduced. For example, 0-place definite descriptions are often vacuous, as in 'the greatest prime number', although prior to Euclid's proof the semantic status of this description was not definitively known. The function $f(x) = x^{-1}$ on the reals is partial, since it is not defined in the case of $x = 0$, and the description 'the x such that $x^2 = 2$' fails the uniqueness constraint. Nonetheless it is often expedient to perform logical and mathematical manipulations involving partial functions, and thus in the general case Russell's constraints seem unduly restrictive. For example, on Russell's account, it is a logical truth that $\exists y(y = \boldsymbol{i}x\phi x)$. However, it might be useful to be able to introduce the term $\boldsymbol{i}x\phi x$ *without* first proving that the existence condition is satisfied, *a la* Free logic, and then employ the term to articulate the discovery that $\neg E!(\boldsymbol{i}x\phi x)$, if it's later found that no such object exists.

In the context of providing a foundation for mathematics, Frege sought to avoid the truth value gaps mentioned above that result from descriptions that fail to denote, and his solution was to assign a 'dummy value' from the realm of existents. This is akin to the current strategy in computer science of assigning an 'error object' in such cases (see Gumb [6]). The (generic) Free logic approach is to dispense with existence assumptions for such terms and use the existence predicate to preserve valid patterns of inference. This is also the intuitive strategy adopted by Troelstra and van Dalen [7] with their E-logics in the context of constructive mathematics. Within Free logic there are various choices regarding descriptions that fail to denote. Making all atomic formulas containing empty descriptions *false* yields a 'negative' free description theory equivalent to Russell. In contrast, making all identities between empty descriptions *true* yields a 'positive' description theory analogous to Frege's solution above, although instead of taking the 'dummy value' from the realm of existents, it is now more natural to use a nonextistent object, *as per* the semantics outlined below. So called 'neutral' Free description theories constitute yet a third option, where bivalence is sacrificed and statements involving empty terms lack a truth value, as on Frege's strictly compositional approach.

1.4 Inner and Outer Domains

From the point of view of classical semantics, there are two distinct ways in which singular terms can fail to denote. First, a term can be genuinely empty, in the sense that it maps to nothing at all, in which case the semantical interpretation function on the set of terms is itself partial. Second, it can map to something, but this 'something' is not in the realm of actual or proper existents, and hence is outside the range of the (classical) quantifiers. In this case the semantical interpretation function can be total, but with a range that exceeds the scope of the quantifiers. This is in broad accord with Meinong's [8] famous and influential distinction between existentent and subsistent objects. Subsistence is a wide ontological category that includes both concrete and abstract objects, where concrete objects both exist and subsist, while abstract entities merely subsist. Meinong's idea serves as an inspiration behind a standard version of Free logic in which the semantic structures have both an 'inner' and 'outer' domain, and where the inner domain $\boldsymbol{D_i}$ specifies the universe of existent objects over which the quantifiers range. There are technical choices to be made concerning the relation between $\boldsymbol{D_i}$ and the outer domain $\boldsymbol{D_o}$, and it's possible to make them disjoint, or to adopt the Meinongian picture and let $\boldsymbol{D_i} \subseteq \boldsymbol{D_o}$. In the current exposition the latter option will be selected, and we will allow $\boldsymbol{D_i}$ (although not $\boldsymbol{D_o}$) to be empty, thereby evading yet another philosophically dubious presupposition of classical logic, namely that at least one object must exist, which presupposition is embodied in the logical truth $\exists y(y = y)$. A straightforward semantics for this type of dual-domain Free logic can be specified as a direct extension of the classical approach, where the objects not belonging to the inner domain cannot be accessed by the quantifiers, but where such objects *can* be accessed by the interpretation function, both to serve as the referents of singular terms, and to appear in the extensions of predicate expressions.

A Free logic interpretation for the respective first-order language with identity $\mathcal{L}$, is a triple $< \boldsymbol{D_i}, \boldsymbol{D_o}, \boldsymbol{f} >$, where $\boldsymbol{D_i}$ is a (possibly empty) set of existent objects, $\boldsymbol{D_o}$ is a (non-empty) set of subsistent objects, and $\boldsymbol{D_i} \subseteq \boldsymbol{D_o}$. $\boldsymbol{f}$ is an interpretation function such that for every individual constant $\boldsymbol{c}$ of $\mathcal{L}$, $\boldsymbol{f}(\boldsymbol{c}) \in \boldsymbol{D_o}$, and for every n-place predicate $\boldsymbol{P}^n$ of $\mathcal{L}$, $\boldsymbol{f}(\boldsymbol{P}^n) \subseteq \boldsymbol{D_o}^n$. Given an interpretation $< \boldsymbol{D_i}, \boldsymbol{D_o}, \boldsymbol{f} >$, the valuation function V assigns truth values to formulas Θ of $\mathcal{L}$ in the following manner (truth functional combinations are evaluated as normal):

(i) if Θ is of the form $\boldsymbol{P}^n\, \boldsymbol{c}_1,\ldots, \boldsymbol{c}_n$, then $V(\Theta)$ = True iff $< \boldsymbol{f}(\boldsymbol{c}_1),\ldots, \boldsymbol{f}(\boldsymbol{c}_n) > \in \boldsymbol{f}(\boldsymbol{P}^n)$. $V(\Theta)$ = False otherwise;

(ii) if Θ is of the form $\boldsymbol{c}_1 = \boldsymbol{c}_2$, then $V(\Theta)$ = True iff $\boldsymbol{f}(\boldsymbol{c}_1) = \boldsymbol{f}(\boldsymbol{c}_2)$. $V(\Theta)$ = False otherwise;

(iii) if Θ is of the form $E!(\boldsymbol{c})$, then $V(\Theta)$ = True iff $\boldsymbol{f}(\boldsymbol{c}_1) \in \boldsymbol{D_i}$. $V(\Theta)$ = False otherwise;

(iv) if Θ is of the form $\forall \boldsymbol{v}\phi \boldsymbol{v}$, then $V(\Theta)$ = True iff for every $e \in \boldsymbol{D_i}$, $V_e^{\boldsymbol{a}}(\Phi\mathbf{v}/\boldsymbol{a}) = 1$, where $\boldsymbol{a}$ is a *new* individual constant, $\Phi\mathbf{v}/\boldsymbol{a}$ is the result of substituting $\boldsymbol{a}$ for every free occurrence of $\mathbf{v}$ in Φ, and $V_e^{\boldsymbol{a}}$ is the valuation function on the interpretation $< \boldsymbol{D_i}, \boldsymbol{D_o}, \boldsymbol{f}^* >$ which is exactly like $< \boldsymbol{D_i}, \boldsymbol{D_o}, \boldsymbol{f} >$ except that $\boldsymbol{f}^*(\boldsymbol{a}) = e$. $V(\Theta)$ = False otherwise.

In this 'positive' Free logic, predications involving nonexistent objects can be evaluated as *true* on the basis of set membership, in the typical Tarskian fashion. For example, suppose the merely subsistent Pegasus is an element of $\boldsymbol{D_o}$ but not $\boldsymbol{D_i}$, the 1-place predicate Wx stands for the property of 'being winged', $\boldsymbol{f}(\boldsymbol{c}_1)$ = Pegasus, and $\boldsymbol{f}(W)$ = {Pegasus, ... }. Then 'Pegasus is winged' is formalized as $W\boldsymbol{c}_1$ and is evaluated as True, while $E!(\boldsymbol{c}_1)$ comes out False. Leblanc and Thomason [9] provide a 7-schema axiomatization of Free logic with identity which is sound and complete with respect to this semantics. It incorporates the $\mathrm{UI}_{\mathrm{Free}}$ rule previously discussed, as well as the axiom $\forall xE!x$.

1.5 Actual versus Possible

Another well developed framework for dealing with objects that can be referred to but do not actually exist is supplied by modal logic, and the discussion of Western logic will finish with a brief examination of possible world semantics. Although in the actual world Plato did not possess a beard, it's nonetheless *possible* that he could have grown one, say like Aristotle's, and so there's a plausible sense in which Plato's beard 'exists' in alternative possible worlds. Similarly, there have never been flying horses in this world, but if biological evolution had taken a somewhat different course then there *might* have been. Indeed, the possibility of a winged horse seems no more outlandish than the palaeontological fact of flying dinosaurs, and thus Pegasus is a possible though non-actual creature.

There are a number of options and technical choices that must be made when providing a semantics for quantified modal logic, and Kripke's [10] groundbreaking work adopts some key choices that embody principles of Free rather than classical logic. The most distinctive of these concerns the extensions of predicates. Each world w in a modal structure has a domain $\boldsymbol{D}_w$ of objects that exist at that index. Let $\boldsymbol{UD}$ be the union of all domains $\boldsymbol{D}_w$ for worlds in the structure. Then the binary interpretation function $\mathcal{J}(w, \boldsymbol{P}^n)$ can assign an object e to the extension of the predicate $\boldsymbol{P}^n$ at some world w, even though $e \notin \boldsymbol{D}_w$ and hence e does not exist at that world. The only restriction is that $\mathcal{J}(w, \boldsymbol{P}^n) \subseteq \boldsymbol{UD}^n$. Conversely, a predication can turn out to be *false* in a world w, when evaluated with respect to an object $e \notin \boldsymbol{D}_w$, but where *e does exist* at another world w' which has access to w. In addition, Kripke upholds the principle that the quantifiers have existential import and are thereby restricted at each world to the set $\boldsymbol{D}_w$. This combination of features is in harmony with the positive dual-domain semantics for Free logic described above, where $\boldsymbol{UD}$ corresponds to the outer domain D_o, while $\boldsymbol{D}_w$ constitutes the inner domain $\boldsymbol{D}_i$ of locally existent objects over which the quantifiers range. One of the prime advantages of this combination of choices is that it allows both the Barcan formula, $\forall x\Box\Psi x \rightarrow \Box\forall x\Psi x$, and its equally implausible converse to be refuted, thereby yielding the maximum degree of articulation with respect to scope interactions between the quantifiers and the modal operators. Neither the Barcan formula nor its converse are derivable in Free logic, whereas both are valid in straightforward modal extensions of classical logic (see Schweizer [11] for further discussion).

2 The Analysis of Non-Existence in Classical Indian Philosophy

In classical Indian philosophy, the riddle of non-being was a historical focal point of controversy, particularly between rival Buddhist and Hindu schools. The remainder of the paper will explore the polemical exchange between the Yogācāra-Sautrāntika school of Buddhism and the orthodox Nyāya *darśana* of Hinduism. The exposition relies primarily on Matilal [12,13], Siderits [14,15] and Tillemans [16] as sources.

2.1 The *Apoha* Semantics of Dharmakīrti

In the 7th century A.D. the Yogācāra-Sautrāntika philosopher Dharmakīrti provided an extended development of the nominalistic theory of his predecessor Dinnāga. *Apoha* nominalism emerged within an ontological framework of radical particularism, in which each existent is held to be absolutely unique and distinct from every other, and thus it is not strictly true to say that two objects have a property in common. Not only are there no universals or abstract entities crowding the metaphysical heavens, but there are not even genuine similarities or resemblances between distinct objects to underwrite our everyday use of property terms. On this beautifully self-consistent analysis, the conventional use of property terms is explained in purely negative fashion. Every object differs absolutely from every other, but objects differ from each other in different ways, and these assorted modes of differing sustain, through two applications of negation, our use of ordinary language predicates.

The *apoha* analysis is based on the idea that the conventionally correct use of a term is acquired through various learning episodes, where encounters with particular objects give rise to a mental paradigm which guides the language user's verbal behavior. This paradigm serves as an internal representation or conceptual 'image', whose primary function is to exclude incompatible representations and thereby specify some portion of the term's *anti-extension*. For example, use of the term 'cow' is based on a particular mental paradigm, which does not encode the abstract features which all cows (are mistakenly supposed to) share, but rather which guides our ability to exclude other objects, and hence judge that a given table or chair is a non-cow. So the particular paradigm or conceptual construct is first used to exclude non-cows, and the extension of the general term 'cow' is obtained through a second application of negation, as the set of all things which are not non-cows. In this manner, the extension of the general term is obtained without commitment to any genuine properties or positive similarities shared by members of the set of cows.

Of course, this immediately leads to the question, 'On what basis does the paradigm exclude some objects and not others, if not by tacit appeal to relevant similarities?' Dharmakīrti's consistent, though semantically somewhat unsatisfying answer, is that exclusion is a *causal* property of the representation as an actual cognitive structure, so that incompatibility between representations is not a logical or semantical trait, but rather is more akin to a repulsive mechanical force. Dharmakīrti's view is not an 'idea' theory of meaning, and the mental paradigm is not a type of pictorial image accessible to consciousness. According to Siderits [14], 'for the Buddhists the psychological machinery that explains our use of words to refer... is purely causal in nature and semantically invisible' (p. 99). Thus unlike the Fregean model, *apoha* semantics is predominantly concerned with reference, and we discover nothing about sense or conceptual *content* when we discover that, say, two terms 'A' and 'B' are co-extensive. Instead this is always a quasi-empirical finding.

As one of the basic metaphysical tenants of Buddhism, the Yogācāra-Sautrāntika school embraced the principle that existence is purely momentary. On this view, the world is not materially preserved from one moment to the next, but rather consists of a series of discrete 'instants' (*kṣaṇas*) of existence, followed by complete annihilation before the next instant occurs. In combination with this view of reality as a kind of Heraclitean flux, the Yogācāra-Sautrāntikas also supported the widespread Indian distinction between brute sensation (*nirvikalpika*) and determinate perception (*savikalpika*). According to this distinction, the raw data supplied by sensory contact with the world must be ordered with respect to a verbal/conceptual scheme, before various objects can be perceived *as* members of their respective categories. This imposition of a conceptual framework on the chaotic field of raw sensation is required to provide the propositional content of ordinary perceptual experience, while the basal level of indeterminate sensation is strictly ineffable. Thus the ordinary objects which we experience in propositionally structured perception do not exist independently of our conceptual activities. Only the instantaneous and ultimately unique particulars are real, and, for reasons quite analogous to Russell's arguments concerning logically proper names, are not referred to with ordinary singular terms, while the enduring and composite objects which we perceive and talk about in everyday speech are diagnosed as conceptual constructs.

2.2 Negative Existentials and Non-denoting Terms

When the foregoing analysis of the objects of perception and reference is combined with *apoha* nominalism, the result is an elegant treatment of negative existentials, which the Yogācāra-Sautrāntikas defended against rivals, especially those of the Nyāya school. The Naiyāyikas held that some absences, *viz*. those which can be associated with existing counter-positive instances, are real and can be directly perceived. Thus when I say I can see that, for example, there is no gorilla in the doorway, this absence itself is said to be directly perceived, because there is a clearly defined counter-positive phenomenon, namely, the way the doorway would look if there were any particular gorilla standing in it. In contrast, the Yogācāra-Sautrāntikas maintain that absences are never perceived but only inferred. And the inferential mechanisms involved stem directly from the two-step negation of *apoha* semantics.

On the Yogācāra-Sautrāntika view, my non-perception of the gorilla is nothing other than my perception of the actual doorway in question. When I judge that I see a doorway, this is an instance of determinate perception, and as such it necessarily involves the mental paradigm governing my use of the term 'doorway'. This paradigm is a conceptual construction which enables me to apply the term under the correct assertability conditions. Thus the perceptual data with which I am now presented must be such that it is not excluded by the doorway paradigm, *i.e.* it must be such that it is not a non-doorway. But since a gorilla is included in the non-doorway class, I can rightly judge that there is no gorilla present in my immediate visual field, simply on the basis of my determinate perception of this doorway. Because of the exclusionary machinery of *apoha* semantics, the perception of the doorway *simplicitor* is a sufficient condition for inferring the non-presence of a gorilla (or any other non-doorway construction).

The *apoha* semantic analysis applies to singular as well as general terms (see, e.g., Tillemans [16]), and can be uniformly extended to cases where the subject of the assertion has no counter-positive instance, either because the subject is purely fictional (but possible), or because it is impossible. The feature which distinguishes conceptual constructions which are 'actual' is that the assertability conditions for terms denoting actual objects are constrained by direct causal interactions with ultimately existing particulars, while in the case of fictional objects the assertability conditions are governed purely by linguistic conventions. Thus to make the statement 'Kripke exists' is to hold that the conceptual construction designated by the term 'Kripke' (which we would normally take to be an actual individual in our naive, pretheoretic belief that enduring and composite entities such as human individuals are real) is causally tied to the non-linguistic world of ultimate particulars, in such a way that stimuli from this world, combined with salient linguistic conventions, yield the result that the statement is at present correctly assertable, while at some undetermined future point it will not be. In a related vein, to assert that 'Pegasus does not exist' involves holding that the 'Pegasus construction' is not directly tied to the world of ultimate particulars, and the rules governing its use are constrained purely by discourse conventions. In this case, even though Pegasus does not exist, the discourse conventions warrant the assertion that 'Pegasus is winged', since the Pegasus concept is psychologically generated in response to the story in which Pegasus is presented as a flying horse. This allows statements such as 'Pegasus is winged' to have the same type of subject as factually grounded assertions, since in both cases the subject is a conceptual construction. Thus, in a manner analogous with Meinong's view that 'being so' is independent of 'being', the statement 'Pegasus is winged' is construed as both 'true' and *about* a genuine 'object'.

In the vernacular of the dispute between the Buddhist and Nyāya schools, a 'horned hare' is a stock example of a fictitious object, and according to the Buddhists the predication 'The hare's horn is sharp' is a normal sentence that we may employ in our discourse for various purposes. In contrast, advocates of the realist Nyāya school such as Vācaspati argue that the subject term of a sentence must refer to something actual, and if not, then the sentence is in need of philosophical paraphrasing in a manner strikingly akin to Russell's 1905 view. In order to cogently assert that 'the hare's horn does not exist', this must be analysed as the claim that 'each thing that is a horn does not belong to a hare' (Matilal [12], p. 81). As in Russell, this analysis (implicitly) relies on quantified variables rather than singular terms to express the lack of reference. So as in Russell, predications involving fictitious objects turn out uniformly false: both 'The hare's horn is sharp' and 'The hare's horn is not sharp' are evaluated as falsehoods. At this level there is nothing paradoxical about the analysis, and Russell's theory provides an explicit formalization of the basic idea. However, the Naiyāyikas did acknowledge a subtle but 'superficial' self-contradiction when expressed as the general principle that 'nothing can be truly affirmed or denied of a fictitious entity', since this is itself presumably intended as a true statement about fictitious entities.

The Yogācāra-Sautrāntika analysis of statements involving possible but non-actual objects is then carried over to statements about impossible objects, where the stock example is 'Devadatta, the son of a barren woman'. The fact that the specification of such an object is not self-consistent does not prevent the formation of an attendant conceptual construct (since the construction itself does not possess the incompatible traits), and thus we can make comprehensible assertions about Devadatta, wherein these assertions will have a constructed subject in many ways comparable to Kripke or Pegasus. Attempted *application* of the 'Devadatta' concept will result in the discovery of a null extension, since everything must be excluded. However, I would argue that the Yogācāra-Sautrāntika semantical theory does not seem to possess the resources needed to distinguish merely possible but non-actual entities like a hare's horn or Meinong's Golden Mountain, from impossible objects such as the son of a barren woman. The impossibility and hence non-existence of the latter is due to the mutual incompatibility of the *meanings* involved. Appeal to the level of sense or intension reveals that the description can perforce be satisfied by no object, and hence the purported individual is impossible. But the purely exclusionary mechanisms of the *apoha* account are not sufficient to distinguish cases of contingent non-existence from the analytically unsatisfiable, since the two are extensionally identical. To capture the definitional impossibility of 'the son of a barren woman' would require the introduction of something like Carnap's 'meaning postulates' to specify the salient natural language *content* carried by these terms.

2.3 *Apoha* Semantics and Free Logic

Siderits contrasts the Yogācāra-Sautrāntika view with Meinong's account, and argues that the Buddhist view has all of the virtues of Meinongianism with none of its vices. Assertions about nonexistent objects are given subjects and truth conditions in accord with common sense (as in Meinong), but not at the price of an 'ontological slum', bloated with subsistent but nonexistent objects. This latter claim is far from clear however, since the objects of predication do exist *qua* 'conceptual constructions'. Thus according to Dharmakīrti, 'Pegasus' does not refer to some attenuated individual residing in the nether world of abstract entities, but rather designates a private *mental* object of some kind. Furthermore, there is now not just *one* salient object of reference for the entire linguistic community, but instead there is one for every linguistic agent, just as there is an idiosyncratic Kripke concept, Everest concept, Zeus concept, etc. Thus the Buddhist view seems to constitute a type of psychologically instantiated Meinongianism, where the objects of reference are multiplied rather than decreased. This approach is perhaps more realistic than a logically idealized account with a single semantical structure for an entire linguistic community, although the sense in which it is genuinely 'nominalist' is in need of clarification. From an externalist point of view it is nominalist, since terms do not refer to external, mind-independent entities. But from an internalist perspective, linguistics expressions are interpreted as referring to conceptual constructs, which are the psychological analogues of ordinary, everyday objects.

Hence, I would propose a dual-domain Free logic as an appropriate way to *formally model* the individual nominalistic ideolects. In contrast to the division between existent and non-existent oblects underlying the Free logic domains, on the *apoha* view all cognitive representations exist as mental structures and hence are ontologically commensurate as such. So the demarcation between actual and 'non-actual' must be delineated as above – the inner domain $\boldsymbol{D_i}$ is comprised of the conceptual constructions such as Kripke and Everest which are causally tied to the non-linguistic world of ultimate particulars, while the outer domain $\boldsymbol{D_o}$ is comprised of conceptual constructs such as Pegasus and Zeus which are not directly tied to the world of particulars, and where the rules governing their use are constrained purely by linguistic conventions. On the *apoha* account, 'impossible objects' such as 'Devadatta' and

Meinong's 'the round square' are countenanced as well, and should be mapped to constructs inhabiting the outer domain. As noted above, the constructs themselves do not possess the incompatible attributes in question and hence are not themselves impossible. So it is both fitting and necessary to block the deducibility of the aforementioned and seemingly innocuous notion that $\phi(\boldsymbol{i}x\phi x)$, which in fact is not benign and will lead to paradox in the case of inconsistent descriptions. This is achieved by adopting Lambert's Law $\forall y(y = \boldsymbol{i}x\phi x \leftrightarrow \forall x(\phi x \leftrightarrow x = y))$ as a basic principle of Free description theory. In terms of the semantics for definite descriptions, the following clause is added to the foregoing specification of a Free logic interpretation:

(v) if $\forall x(\phi x \leftrightarrow x = \boldsymbol{c})$ is true, where $\boldsymbol{c}$ is an individual constant, then $\boldsymbol{f}(\boldsymbol{i}x\phi x) = \boldsymbol{f}(\boldsymbol{c})$, and if $\neg\exists x(\forall y\phi y \leftrightarrow y = x)$, then $\boldsymbol{f}(\boldsymbol{i}x\phi x) = e$, where $e \in \boldsymbol{D}_o \setminus \boldsymbol{D}_i$, and where the interpretation function $\boldsymbol{f}$ on the set of individual constants must be surjective with respect to $\boldsymbol{D}_o$.

The nominalism of the Buddhist view indicates that the predicate extensions given by the interpretation function $\boldsymbol{f}$ and the attendant set membership conditions used in the formal definition of truth for atomic formulas should *not* be seen as reflecting some literal correspondence theory of truth. Instead, they simply encode the internalized discourse conventions of the speaker's linguistic community. In this manner it is possible to provide the basics of a formal semantics for natural language that models the *apoha* view, and hence provides a structure that can reflect the conventional truth conditions for sentences involving non-denoting terms.

References

1. Frege, G.: On Sense and Meaning. In: P. Geach and M. Black (eds.) Translations from the Philosophical Writings of Gottlob Frege. Basil Blackwell, Oxford (1960)
2. Russell, B.: On Denoting. *Mind* 14, 479–93 (1905)
3. Lambert, K.: Free Logic: Selected Essays. CUP, Cambridge (2003)
4. Lambert, K.: Existential Import Revisited. *Notre Dame Journal of Formal Logic* 4, 288–292 (1963)
5. Russell, B. and Whitehead, A.: *Principia Mathematica*. CUP, Cambridge (1910)
6. Gumb, R.: Free Logic in Program Specification and Verification. In: E. Morscher and A. Hieke (eds.) New Essays in Free Logic (pp. 157-189). Kluwer, London (2001)
7. Troelstra, A.S. and van Dalen, D.: Constructivism in Mathematics. Elsevier, Dordrecht (1999)
8. Meinong, A.: On the Theory of Objects. In: R. Chisholm (ed.) Realism and the Background of Phenomenology (pp. 76–117). Free Press, Glencoe (1960.
9. Leblanc, H and Thomason, R.: Completeness Theorems for Some Presupposition-free Logics. *Fundamenta Mathematicae* 62, 125–64 (1968)
10. Kripke, S.: Semantic Considerations on Modal Logic. *Acta Philosophica Fennica* 16: 83–94 (1963)
11. Schweizer, P.: Free Logic and Quantification in Syntactic Modal Contexts. In: E. Morscher and A. Hieke (eds.) New Essays in Free Logic (pp. 69-83). Kluwer, London (2001)
12. Matilal, B.K.: Logic, Language and Reality. Motilal Banarsidass, Delhi (1985)
13. Matilal, B.K.: The Character of Logic in India. SUNY, Albany (1998)
14. Siderits, M.: Indian Philosophy of Language. Kluwer, Dordrecht (1991)

15. Siderits, M.: Śrughna by Dusk. In: M. Siderits, T. Tillemans and A. Chakrabarti (eds.) Aphoha: Buddhist Nominalism and Human Cognition (pp. 283-304). Columbia University Press, New York (2011)
16. Tillemans, T.: Dharmakīrti. In: E. Zalta (ed.) *The Stanford Encyclopedia of Philosophy*. <http://plato.stanford.edu/archives/spr2014/entries/dharmakiirti/> (2014)