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A Decentralized Market for a Perishable Good

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A Decentralized Market for a Perishable Good*

Ahmed W. Anwar and József Sákovics

Abstract

We characterize the steady state of a market with random matching and bargaining, where the sellers' goods can perish overnight. Generically, the quantity traded is suboptimal, prices are dispersed and there is a dead-weight loss caused by excess supply or demand. In the limit as the cost of staying in the market tends to zero, only the amount of trade tends to the efficient level, the other two non-competitive characteristics remain. We discuss the implications of these findings on the foundations of competitive equilibrium and on the robustness of the results in the literature on durable-good markets.

KEYWORDS: perishability, decentralized market, queuing externality

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1 Introduction

The literature on dynamic decentralized markets for a homogeneous good assumes that the good traded does not lose its value over time: it is non-perishable. Under this assumption, it has been shown that – in general – when information is complete and frictions are small, the outcome of strategic interaction in a steady-state market is approximately efficient (thus providing a non-cooperative foundation for competitive equilibrium)[1]. In practice, however, it is often the case that a delay of the transaction would imply a lower value placed on the item, at least in expected terms. There are two salient ways this can happen. The first one – depreciation – corresponds to the situation where there is a deterministic, gradually decreasing relation between an item's age and its (expected) value. While this scenario is a good representation of some goods – say, a closed-end bond – the assumption of predictability is rather strong, and the analysis of the model is too complex, as items of different age coexisting in the market essentially make it into one of differentiated products. Instead, we concentrate on the other possibility, where decay is sudden and it occurs in a random manner. We call this phenomenon perishability. Obvious cases are when the object itself may "malfunction" or "rot". Think of a washing machine or fresh produce. Our model is sufficiently general to encompass both type of goods, despite the noticeable difference that a washing machine is a durable good (modulo perishability) as it provides both the seller and the buyer with a flow of utility, while milk is a consumption good which provides utility on a single date.

As further applications, note that the decrease in the good's value may also come as a result of a change in preferences – as in the case of fashion goods – or a change in market structure – as in the case of industries with a high R&D component.[2] Finally, an extreme form of perishability is the one that characterizes individualized services: if the time-slot of a dentist is not used for treating a patient, it is irretrievably lost.

In short, the analysis of the trade of perishable goods is empirically relevant, and therefore called for. We re-examine the question of decentralized dynamic price formation when the good traded is perishable to some degree. Namely, each single item may become worthless in each period with some

[1] See Gale (1987, 2000).

[2] These latter examples are special, as the shocks affecting identical items are (perfectly) correlated. This phenomenon is usually called obsolescence in the literature.

(given) probability.[3]

Our main finding is that a market for a perishable good is inherently inefficient. Moreover, this inefficiency is not generated by the "wrong" trades taking place.[4] Rather, it stems from a queuing externality: the dead-weight loss created by too many buyers entering the market. This externality is so large that as the waiting costs disappear, not only does the buyer queue become infinite but it grows at such a rate that the aggregate cost of waiting does not vanish.

We begin by illustrating the issues with an example in Section 2. We continue by a brief review of the literature in Section 3. Section 4 introduces our model and the efficiency benchmark. In Section 5 we analyze the steady state of our market. We explore the implications of our results for the foundations of competitive equilibrium in Section 6. Section 7 concludes.

2 A didactic example

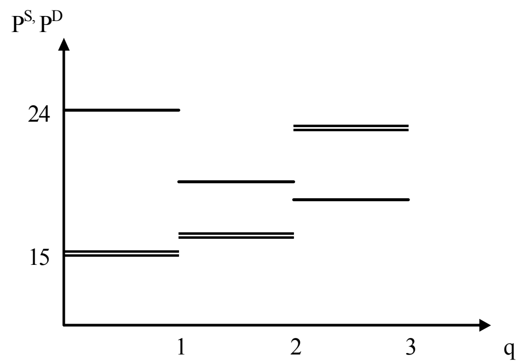
The purpose of this section is twofold: i) to provide an illustration of the main inefficiency result, ii) to shed some light on the process by which the steady-state is reached in equilibrium.

Consider a market for fresh milk. There are 3 newly generated units of demand for milk every day with marginal valuations $\{24, 20, 19\}$ and 3 newly generated units of potential supply with marginal valuations $\{15, 16, 23\}$. This is illustrated in Figure 1. The 3 potential buyers and 3 potential sellers must decide whether to enter a decentralized market that may include buyers from previous days who have yet to purchase milk. The milk turns sour overnight, so there are no milk units available from previous days. Market participation costs the buyers $c_b = 1$ and the sellers $c_s = 0.1$ per day. Each day, the buyers and sellers *in* the market are randomly matched into pairs, and if the number of buyers and sellers is unequal, then the remainder do

[3] While we set up our model with random shocks that are independent, our analysis also captures the correlated case when the new good, which makes the old one obsolete, is replacing the old one immediately and where its supply and demand are also the same as those of the old one.

[4] This is what is driving the results of Gale and Sabourian (2005), who show that in a market game of one-time entry (without a steady-state), heterogeneity of valuations is sufficient to cause inefficiency, as high valuation buyers may trade with high valuation sellers, and low valuation sellers with low valuation buyers, increasing the amount of trade but decreasing the aggregate gains from trade.

not trade. Matched traders first observe each other's valuations of the good and then engage in bargaining where a trader gets to make a take-it-or-leave-it offer to his trading partner and both traders are equally likely to be the proposer. If the offer is accepted, trade takes place and both agents leave the market. If the offer is rejected, then the buyer joins the matching pool the following day and the seller leaves the market.



Supply and demand

Starting at date 1, the efficient outcome is clearly where the two higher valuation buyers and the two lower valuation sellers enter and both offers are accepted. Since there will be no market participants who failed to trade, the same will be true on day 2, 3 and so on. Hence the Pareto efficient flow of surplus per day will be $24 - 15 + 20 - 16 - 2 \times 1 - 2 \times .1 = 10.8$.

Now let us look at this market in equilibrium. Let $V_t^B(b)$ be the value of participation in the market for a buyer who is in the market on day t and has valuation b (this could be a new entrant or a buyer who is yet to find a match). Let $V_t^S(s)$ be the value of participation in the market for a seller with valuation s who enters the market (with fresh milk) on day t. On day one, a seller who makes a proposal asks for $b - V_2^B(b)$ (offer the buyer a surplus equal to his continuation value), provided that this does not involve a loss for the seller – whose continuation value is zero, as the good is perishable. A buyer who makes a proposal offers the seller her valuation, s (that is, no additional surplus), provided that this gives the buyer at least $V_2^B(b)$ as this is what the buyer would get if he failed to trade. Hence in a perfect Bayesian equilibrium[5]

$$V_1^S(s) = \frac{1}{2}\pi_1^s E_b[\max\{b - s - V_2^B(b), 0\}] - 0.1 \quad (1)$$

$$V_1^B(b) = \frac{1}{2}\pi_1^b E_s[\max\{b - s, V_2^B(b)\}] + (1 - \frac{1}{2}\pi_1^b)V_2^B(b) - 1 \quad (2)$$

where π_1^s and π_1^b are the probabilities that the sellers and buyers, respectively, are matched on day one. Consider the following candidate for an equilibrium: 2 buyers $\{24, 20\}$ and 2 sellers $\{15, 16\}$ enter each period, a buyer with valuation 19 enters only in period 1 and all matches end in trade. Then we have a process that is stationary from the point of view of the buyers since – whether or not the valuation-19 buyer has already traded and been replaced by a valuation-20 or -24 buyer – $\pi_t^b \equiv \frac{2}{3}$ and the sellers are the same each period. Solving (2) gives $V^B(24) = 5.5$, $V^B(20) = 1.5$ and $V^B(19) = 0.5$. So it is indeed optimal for the buyer with valuation 19 to enter the market on the first day if there is no future entry by a valuation-19 buyer. The sellers' value on day t will be given by

$$V_t^S(s) = \frac{1}{2}[(1 - \frac{1}{3^t})((\frac{1}{2}(24 - s - 5.5) + \frac{1}{2}(20 - s - 1.5))) + \frac{1}{3^t}(19 - s - 0.5)] - 0.1$$

because there is a 1 in 3^t chance that she is matched with the valuation-19 buyer who entered in the first period.[6] $V_t^S(s)$ is positive for $s = 15$ and $s = 16$. Since $b - s > V_2^B(b)$ for all participating buyers and sellers it is true that all matches end in trade. We also need to check that the other potential traders are happy with their non-participation. The valuation-23 seller does not wish to enter. She would not trade with a buyer with valuation 20 or 19, as this is less than her marginal valuation, while a buyer with valuation 24 would not accept a price above 23.[7] Finally, consider the case where a second valuation-19 buyer enters on day 2 but there is no further excess entry. Then $V_2^B(24) = 4.5$, $V_2^B(20) = 0.5$ and $V_2^B(19) = -0.5$. Since further entry only

[5] E_b (E_s) denotes the expectation operator over the buyer (seller) types who are in the market in any given period in equilibrium. For simplicity, we assume that market participants cannot observe who are actually in the market. In the general model this will not be an issue (in equilibrium) as there will be a continuum of traders.

[6] In each period the probability that the valuation-19 buyer does not trade is 1/3. In the current period there is a probability of 1/3 that he is matched with a given seller.

[7] He would have at least a 1 in 2 chance of meeting the seller with a valuation of 16 or less in the next period and so is guaranteed a continuation payoff of at least $\frac{1}{2} \times \frac{1}{2} \times 8 - 1 = 1$.

makes matters worse, the valuation-19 buyer would definitely make a loss from entering in period 2.[8] Hence, we have shown that the efficient outcome is not an equilibrium, as a deviation by the valuation-19 buyer in the first period is profitable. Instead we have an equilibrium where a single buyer with valuation 19 enters in period 1, resulting in a permanent buyer queue. The equilibrium does not give rise to a steady state because $V_t^S(s)$ is changing over time. The reason for this variation is that the probability that the buyer with $b = 19$ is still in the market is diminishing over time. Eventually, this buyer must trade and is replaced by either a $b = 20$ or a $b = 24$ buyer in the queue. Thus, the system approaches a steady state with $\lim_{t\to\infty} V_t^S(15) = 1.65$ and $\lim_{t\to\infty} V_t^S(16) = 1.15$.

The flow of surplus per day in this steady state is $5.5+1.5+1.65+1.15 = 9.8$. Hence, ignoring the efficiency loss caused by the fact that there is a point at which the buyer with valuation 19 trades (displacing a trade with a buyer of higher valuation), there is still a loss of 1 per day, relative to the efficient steady state. The only departure from the efficient steady state is the entry of a buyer with valuation 19 in period 1. However, this entry creates a large externality which significantly reduces the per period flow of surplus as *each* buyer now expects to incur the cost $1 + \frac{1}{3} + \left(\frac{1}{3}\right)^2 + ... = \frac{3}{2}$, because with probability $\frac{1}{3}$ they do not trade each day. In the efficient case the cost is only 1, as they trade on the same day.

The striking result is that the inefficiency persists even if we reduce the costs towards zero, as in equilibrium this simply makes the queue longer. In the above example a buyer with valuation 19 will enter if $\frac{c_b}{\pi_t^b} > \frac{1}{2}(\frac{1}{2}(19 - 16) + \frac{1}{2}(19 - 15)) = \frac{7}{4}$. Every period where a buyer with valuation 19 enters, the queue increases by 1 and π_t^b falls. A buyer with valuation 19 does not enter on day t if $\pi_t^b = \frac{2}{t+2} < \frac{4c_b}{7}$. Hence, in equilibrium, the last period in which a buyer with valuation 19 enters is given by $\max[t \in I \mid t < \frac{14}{4c_b} - 2]$, where I is the set of integers. If we take $c_b = 0.1$, we obtain that a buyer with valuation 19 enters on each of the first 33 days. As above, the system eventually approaches a steady state, now with $V^B(24) = 5$, $V^B(20) = 1$, $V^S(15) = 1.9$, $V^S(16) = 1.4$. Note that despite the lower cost of staying in the market the buyers are worse off as they have a lower probability of matching. This, in turn, improves the sellers' bargaining position, leading to a higher

[8] Delaying his entry in the hope that the incumbent 19 valuation buyer will have already traded makes no sense, as he will have been replaced by a buyer with an even higher valuation.

steady-state payoff for them. The aggregate welfare effect is still negative, though. The loss per day relative to the efficient steady state will be the waiting cost of the 33 unserved buyers: 3.3. In the limit as the participation costs go to zero the queue becomes infinitely long and the aggregate loss per day approaches $\lim_{c_b \to 0} \left(\frac{14}{4c_b} - 2 \right) c_b = 3.5$.

3 A brief literature review

The literature on the non-cooperative foundations for competitive equilibrium originates from Diamond (1971) and Rothschild (1973). Previously, attempts at explaining how a market could find and settle at an equilibrium price were in the tradition of tatônnement processes, where some "Walrasian auctioneer" would collect data on demands and supplies at proposed prices and then adjust these prices according to excess demand or supply. The main feature of this approach is that agents are supposed to react to announced prices as if there was going to be trade at those prices, even though this is only the case after infinitely many rounds without trade. In a finite market, however, rational, strategic agents would clearly have an incentive not to report truthfully. Therefore, it is unclear whether the equilibrium of a finite market with strategic agents would converge to the competitive equilibrium as the number of agents tends to infinity. As Rothschild (1973, p. 1285-6) puts it, "a satisfactory model of adjustment to equilibrium will have at least three parts: a discussion of the rules which market participants follow when the market is out of equilibrium; a description of how a market system in which individuals follow these rules operates; and, of course, a convergence theorem."

Diamond's (1971) seminal contribution both showed the way forward and signalled the accompanying difficulties. He presented a model of a market where interaction was decentralized, with randomly matched traders where switching trading partners was costly. This approach eliminated the need for an artificial auctioneer, but at the cost that it did not converge to the competitive equilibrium as frictions disappeared! The local monopoly power of price-setting sellers did not vanish as the cost of partner switching became negligible. Rubinstein and Wolinsky (1985) have generalized the intuition that the distorting effect of local bargaining power does not vanish to the case where instead of price setting by the seller there is a non-cooperative bargaining game played between a matched buyer and seller. Gale (1987) then pointed out that the non-Walrasian outcome of Rubinstein and Wolinsky (1985) should not be taken at face value. In the steady state of a dynamic market game it should not be the stocks of supply and demand that clear. Rather it should be the flows which clear the market, which indeed do in the Rubinstein-Wolinsky model and its generalizations (see Gale, 2000, for an in-depth discussion). In the third millennium the main thrust of this literature has been on the one hand to incorporate many-to-one matching and thus auctions (which also makes possible the easy incorporation of asymmetric information) into the mechanism (see De Fraja and Sákovics, 2001, and Satterthwaite and Shneyerov, 2004), while on the other hand to examine the implications of bounded rationality see (Sabourian, 2004, Gale and Sabourian, 2005). In this paper, we take an alternative route instead. We stick with one-to-one matching and complete information bilateral bargaining but relax the assumption that the good traded is non-perishable.

4 The general model

Take a market that evolves over (discrete) time, say, days. Buyers are seeking to purchase one unit of an indivisible, homogeneous good, and sellers have one unit for sale each. Any item that has not been sold during the day perishes overnight with probability $d \in [0, 1]$. Every day a new cohort of (a continuum of) potential buyers and sellers appear who (simultaneously) decide whether to enter the market – considering their outside option normalized to zero. The aggregate inverse demand function of each new cohort of potential buyers is $P^d(.)$, assumed continuous and strictly decreasing, while the aggregate inverse supply function of each new cohort of potential sellers is $P^s(.)$, assumed continuous and strictly increasing. We will make a distinction between goods that are consumed at one point in time, *consumption goods*, and goods that are consumed everyday until they perish, *durable goods.* The crucial difference between the two cases is that with a consumption good a seller who has not traded will have the choice of consuming the good today or participating in the market tomorrow (whereas with a durable good, if she fails to trade today she can consume the good for one more day before trading tomorrow and so she faces no trade-off). Consequently, the outside option (relative to continuing in the market) of the seller of a consumption good would equal her valuation for it, while the sellers of durable goods can be thought of as having an outside option of zero. For ease of exposition, we normalize all outside options to zero, meaning that in the former case the value of participation in the market is the value in excess of the sellers' valuation.

Let us digress here to look at how this model covers the examples given in the introduction. Milk is a consumption good and a washing machine is a durable good. In both these examples, the supply function is a seller valuation curve. In the case of milk, the seller valuation is simply derived from its one off consumption value whereas with a washing machine it is the present discounted value from using the washing machine daily. The model also covers examples where the supply function is a seller cost curve as long as the cost (either production or transaction) is incurred once trade is agreed. This situation is technically equivalent to the durable good case because no extra cost is incurred by waiting. We can think of services as the limiting case of this where $d = 1$ since if a seller fails to trade during a particular time slot, this time slot is lost for ever.

The traders who decide to enter the market join those who have entered before but have yet to trade. Then, all the incumbent traders participate in an anonymous, one-to-one random matching process, where each trader on the same side of the market has equal probability of being matched (π_s and π_b for sellers and buyers, respectively)[9] and is matched to each trader on the other side of the market with equal probability. For simplicity, we also assume that the matching technology is efficient, that is, the traders on the short side of the market always find a partner. Matched traders first observe each other's valuations of the good and then engage in bilateral bargaining. Bargaining takes the following simple form: with probability $\lambda \in (0, 1)$ the buyer – $(1 - \lambda)$ the seller – makes a take-it-or-leave-it offer to his trading partner. If the offer is accepted, trade takes place and both agents leave the market. If the offer is rejected, then together with the unmatched traders they join the matching pool the following period.[10] Finally, market participation is costly: buyers and sellers incur a cost of c_b and c_s, respectively, per period while they are in the market.

Denote the difference between the (inverse) demand and supply functions by $G(.)$, that is, $G(x) \equiv P^d(x) - P^s(x)$. Then the (constrained) efficient flow of trade is given by $x^e = G^{-1}(c_b + c_s)$, that is, the quantity at which

[9] We concentrate the analysis on the steady state, so the matching probabilities do not vary over time.

[10] In the seller's case, provided that her good has not perished and that she decides not to consume the (consumption) good.

the trade surplus of the marginal participants is equal to the total cost of participating. As the costs approach zero, $x^e \rightarrow x^c$, which is the *static* market clearing equilibrium corresponding to the per-period flow of demand and supply: $P^d(x^c) = P^s(x^c) = p^c$. The competitive benchmark for our *dynamic* model can then be straightforwardly defined as the outcome where the trades in every period are given by x^e (x^c in the limit) AND where there are no traders waiting (overnight) at any time.

5 The steady state and its characteristics

We wish to examine the limiting steady-state behavior of this market, especially in terms of its efficiency. More precisely, we are looking for a perfect Bayesian equilibrium, where the composition of the market and the amount traded is constant over time and the participation costs are very small. We will compare this outcome to the efficient one.[11]

We begin by setting up the steady state equations. The traders' value functions will depend on whether they are trading a consumption good, C or a durable good, D. Denote the buyers' and sellers' value functions upon entry in a stationary equilibrium by $V_i(b)$ and $V_i(s)$, respectively, where $i \in \{C, D\}$. We then have

$$\begin{aligned} V_i(s) &= \pi_s(1-\lambda)E_b[\max\{b - s - V_i(b), C_i(s)\}] + \\ &\quad (1 - \pi_s(1-\lambda))C_i(s) - c_s \end{aligned} \tag{3}$$

and

$$\begin{aligned} V_i(b) &= \pi_b \lambda E_s[\max\{b - s - C_i(s), V_i(b)\}] + \\ &\quad (1 - \pi_b \lambda)V_i(b) - c_b \end{aligned} \tag{4}$$

where $C_i(s)$ is the seller value at the point where she has failed to trade. In the durable good case, this is simply $C_D(s) = (1-d)V_D(s)$. In the consumption good case (with types denoting valuations) there is an additional cost of waiting to trade in the next period, ds; if the item is not consumed today it may perish and will not be consumed at all. If this cost is sufficiently high then the seller will consume the good today and get s rather than

[11]We are going to ignore the trivial equilibrium, where there is zero measure of entry on either side of the market (and hence there is no trade either).

participate in the market tomorrow and get $(1-d)(V_C(s)+s)$. Hence $C_C(s) = \max[(1-d)(V_C(s)+s), s] - s = \max[(1-d)V_C(s) - ds, 0]$.

The value functions are constructed as follows. With probability $\pi_s(1-\lambda)$, a seller gets matched and is allowed to name the price. She will either make an offer, which leaves the buyer indifferent about accepting $(b - V_i(b))$ (which gives her $(b - V_i(b) - s)$), or take $C_i(s)$. If she is either unmatched or matched but has to take the buyer's offer or leave it, then her payoff is $C_i(s)$. A buyer will offer $s + C_i(s)$ (which gives him $b - s - C_i(s)$) and will get $V_i(b)$ if he makes an unacceptable offer, remains unmatched or has to take or leave a seller offer.

Rearranging (4), we obtain

$$b - V_i(b) = \frac{c_b}{\pi_b \lambda} + E_s[\min\{C_i(s) + s, b - V_i(b)\}]. \tag{5}$$

If the minimum operator in (5) selected the second argument for all b and s then this would imply $\frac{c_b}{\pi_b \lambda} = 0$ which is not true. Hence $b - V_i(b)$ must be constant (otherwise, the derivative with respect to b would differ on the two sides of the equation). Denoting the marginal traders by b^* and s^*, by definition we have that $V_i(b^*) = 0, V_i(s^*) = 0$. Therefore, $b - V_i(b) \equiv b^* - V_i(b^*) \equiv b^*$. This implies that we can drop the expectations operator from (3). Note that $V_i(b) = b - b^*$ is strictly increasing in b and so only buyers with a valuation, $b \geqslant b^*$ will choose to participate.

For all participating sellers, $b - V_i(b) - s = b^* - s \geqslant C_i(s)$ (if for any s, $b^* - s < C_i(s)$ then – by (3) – either $V_D(s) = C_D(s) - c_s$ which implies $V_D(s) = -\frac{c_s}{d}$ or $V_C(s) = C_C(s) - c_s$ which implies $V_C(s) = -c_s$ or $V_C(s) = -\frac{c_s + ds}{d}$). Consequently, the first maximum operator in (3) and minimum operator in (5) always select the first arguments. Rearranging the "operatorless" (5) and (3), we have

$$V_i(b) = b - E_s[C_i(s) + s] - \frac{c_b}{\pi_b \lambda} \tag{6}$$

$$V_i(s) = \pi_s(1-\lambda)(b^* - s) + (1 - \pi_s(1-\lambda))C_i(s) - c_s \tag{7}$$

The following result (proved in the Appendix) establishes some monotonicity properties which will be useful in the subsequent analysis.

Lemma 1 *i) $V_i(s)$ is strictly decreasing in s.*

ii) If $d > 0$, then the price offered by any buyer b, $C_i(s) + s$, is independent of b, non-decreasing in s and strictly increasing in s in a non-degenerate interval, $[\widehat{s}, s^*]$. If $d = 0$ then $C_i(s) + s \equiv s^*$.

A solution to (6) and (7) with $V(b^*) = V(s^*) = 0$ and either $\pi_s = 1$, $\pi_b \leqslant 1$ or $\pi_b = 1$, $\pi_s \leqslant 1$ is a steady state equilibrium[12], (since $V_i(b)$ is strictly increasing in b and $V_i(s)$ is strictly decreasing in s, buyers with $b \geqslant b^*$ and sellers with $s \leqslant s^*$ will enter each period). We focus on equilibria where the trading frictions are small:

Proposition 1 *When the participation costs are sufficiently small, there is a unique steady-state equilibrium (with trade). In this equilibrium there is a buyer queue and the steady-state quantity traded is* $z = G^{-1}(\frac{c_s}{1-\lambda})$.

When frictions are larger, steady-state equilibria with seller queues can also exist. We characterize these in the appendix.

Proof. For the marginal seller, $V_i(s^*) = C_i(s^*) = 0$. Plugging this back into (7) evaluated at $s = s^*$ yields

$$b^* - s^* = \frac{c_s}{\pi_s(1-\lambda)} \tag{8}$$

Consider the case where there is no seller queue, $\pi_s = 1$. In a steady-state with no seller queue, the measure of entering buyers and sellers must be the same and from (8) this measure must be $z = G^{-1}(\frac{c_s}{1-\lambda})$ (since $b^* = P^d(z)$, $s^* = P^s(z)$ and $\pi_s = 1$). This measure of trade must be positive to avoid a degenerate solution, hence the sellers' participation cost must not be too large: $c_s < (1-\lambda)G(0)$. Rearranging (6) we have

$$\pi_b^* = \frac{c_b}{(P^d(z) - E_s[C_i(s) + s])\lambda}.$$

Since – by Lemma 1ii – $s^* > C_i(s) + s$ for all participating sellers and since $b^* \geq s^*$ it follows that $b^* > E_s[C_i(s)+s]$. Hence $P^d(z) > E_s[C_i(s)+s]$. From the proof of Lemma 1 it follows that $P^d(z) - E_s[C_i(s)+s]$ is independent of c_b (as long as $\pi_s^* = 1$). Hence, if $c_b < \lambda(P^d(z) - E_s[C_i(s)+s])$ then $\pi_b^* < 1$ and we have an equilibrium with a buyer queue. This is the unique solution to the steady-state equations with $\pi_s = 1$. To show that this is the unique steady-state equilibrium (with trade) we must rule out an equilibrium with a seller queue, $\pi_b = 1$, $\pi_s < 1$. Assume that for a given value of c_b, such an equilibrium existed. From (6), (with $\pi_b = 1$) we have

$$c_b = \lambda\left(b^* - E_s[C_i(s) + s]\right). \tag{9}$$

[12] Efficient matching rules out the case where $\pi_b < 1$ and $\pi_s < 1$.

All we have left to do in order to complete the proof is to establish a positive lower bound for the right-hand side of this equation. Rewrite (9) as

$$c_b = \lambda \left(b^* - s^* + s^* - E_s[C_i(s) + s]\right). \tag{10}$$

From (8) we know that $b^* - s^* \geq 0$, while from Lemma 1*ii* we have that $s^* > E_s[C_i(s) + s]$ for all s^* consistent with trade. Hence, unless $b^* = s^*$, the proof is complete. Assume $b^* = s^*$, which will only happen if $c_s = 0$ (see (8)). This implies that s^* equals p^c, which is significantly above the lowest seller type and therefore bounded away from zero. Thus, since by Lemma 1, $C_i(s) + s$ is strictly increasing on a non-degenerate interval, $[\widehat{s}, s^*]$, $s^* - E_s[C_i(s) + s]$ is bounded away from zero. ■

The steady-state characteristics can also be explained intuitively. In a steady-state equilibrium the marginal traders must have participation values of zero. Since all the buyers have the same cost c_b, the same bargaining power λ and the same probability of trading π_b and they face the same distribution of sellers, it follows that the difference between a buyer's valuation b and steady-state participation value $V(b)$ must be the same for all buyers. Combining this with the fact that the marginal buyer, b^*, has a participation value of zero we have $b - V(b) = b^*$. This implies that a seller will ask the same price, b^*, from every buyer. To see that the buyer does not offer the same price to every seller, consider the extreme situation, when the good lasts only a single day ($d = 1$). In this case, the buyer only has to offer the seller her cost, s. Hence a buyer will make a smaller surplus when trading with the marginal seller than when trading with a seller with a lower value. This continues to hold when the perishing rate is less than 1 but greater than zero. In agreement with the literature on non-perishable goods (see, for example, Mortensen and Wright, 2002), when $d = 0$, this price dispersion disappears.[13]

When the marginal buyer is matched with the marginal seller, his expected surplus is $\lambda(b^* - s^*) - c_b = \lambda \frac{c_s}{1-\lambda} - c_b$ which will be positive when c_b is sufficiently small. Since he makes a greater surplus from other sellers his expected surplus will be positive when c_b is sufficiently small, even when $c_s = 0$. Consequently, there must be a buyer queue to ensure $V(b^*) = 0$.

In the $d = 0$ case it is well known that, whether we have a sellers' or a buyers' market is determined by the Hosios condition.[14] We reproduce this

[13] More precisely, the price will be the independent outcome of the same lottery in every match.

[14] This condition was first derived in Hosios (1990).

result here.

Corollary 1 *If $d = 0$, in steady state the Hosios condition holds:*

$$\frac{\pi_b}{\pi_s} = \frac{(1-\lambda)c_b}{\lambda c_s}.$$

Proof. When $d = 0$, $V_i(.) \equiv C_i(.)$. Hence (6) and (7) become

$$V_i(b) = b - E_s[V_i(s) + s] - \frac{c_b}{\pi_b \lambda} \tag{11}$$

and

$$V_i(s) = b^* - s - \frac{c_s}{\pi_s(1-\lambda)}. \tag{12}$$

Substituting the latter into the former and evaluating it at $b = b^*$, we get

$$V_i(b^*) = \frac{c_s}{\pi_s(1-\lambda)} - \frac{c_b}{\pi_b \lambda} = 0.$$

■

In words, we have a balanced market ($\pi_b = \pi_s = 1$) if and only if the bargaining powers are proportional to the costs of staying in the market. The side of the market which has a higher relative bargaining power than that will have to suffer queues, to keep the market in its steady state. It is important to note the discontinuity: if $d = 0$, it is the Hosios condition that determines which side of the market has a queue, if $d > 0$ then it is the buyers' waiting cost on its own. That is, if we fix c_b at a sufficiently small value then – by Proposition 1 – independently of the value of c_s we will have a buyer queue, when $d > 0$, but we would get a seller queue if $c_s < \frac{(1-\lambda)c_b}{\lambda}$ when $d = 0$. The intuition for this lack of continuity comes from the fact that the perishability of her good and the participation costs of the seller both weaken the seller's bargaining power. When $d > 0$, and the buyers are strong (low c_b), the seller would need a subsidy to be able to compensate for this. Simply taking c_s to zero is not sufficient to put the sellers into the driver's seat.

The above argument not only demonstrates that the Hosios condition does not apply when the good is perishable but also shows that it is not robust to the order in which we take limits.

Corollary 1 also implies that in a balanced market with $d = 0$, we also have $z = x^e$. That is, not only is there no loss due to waiting (by definition), but given the fixed costs, there is also the efficient amount of trade. To see this, note that by Proposition 1, in a balanced market it must be the case that $G(z) = \frac{c_s}{1-\lambda}$. Using the Hosios condition (evaluated at $\pi_s = \pi_b = 1$), we also have $G(x^e) = c_b + c_s = \frac{\lambda c_s}{(1-\lambda)} + c_s = \frac{c_s}{1-\lambda}$.

However, the efficient outcome is not possible when $d > 0$.

Corollary 2 *If $d > 0$, then it is not possible to have both a balanced market ($\pi_s = \pi_b = 1$) and the efficient measure of trade (x^e).*

Proof. In a balanced market the measure of trade must be $z = G^{-1}(\frac{c_s}{1-\lambda})$ and efficiency requires that the measure of trade is $x^e = G^{-1}(c_s + c_b)$. Hence $\frac{c_s}{1-\lambda} = c_s + c_b$. Re-arranging gives $\frac{c_b}{\lambda} = \frac{c_s}{1-\lambda}$. Hence $b^* - s^* = \frac{c_s}{(1-\lambda)} = \frac{c_b}{\lambda}$ or $s^* = b^* - \frac{c_b}{\lambda}$. Substituting this into (6) gives $V(b^*) = s^* - E_s[C_i(s) + s]$, which by Lemma 1 must be greater than zero. Hence, when we have the efficient volume of trade, there must be a buyer queue in the steady-state equilibrium. ■

The intuition for this follows from our earlier discussion. If we have a steady state where the volume of trade is x^e and there is no seller queue, then the marginal buyer and marginal seller generate a net surplus of zero. We know that the marginal seller will make zero on average when matched with the marginal (or any other) buyer, so the marginal buyer must also make zero on average, when matched with the marginal seller. However, if this were the case then the "marginal" buyer would make positive surpluses when matched with sellers with lower values and we would have further entry, leading to a buyer queue. Note that price dispersion is crucial here, as without price dispersion the last part of the argument would not apply. In that case, if the marginal buyer makes zero on average when matched with the marginal seller, then he will make zero on average when matched with any seller (as the expected price would be the same) and no queue would be necessary.

We now look at the steady-state equilibrium in the limit as the participation costs go to zero.

Let $\overline{p}$ denote the average transaction price in the market and let $\overline{p}^*$ denote its limit as the participation costs tend to zero (along any path).

Proposition 2 *In the limiting equilibrium as the costs of participation tend to zero (along any path), the amount of trade taking place coincides with the competitive one (x^c). Unless $d = 0$, the limiting queue of the buyers is infinite and the aggregate waiting cost (per period) is given by $W = (p^c - \overline{p}^*)\, x^c > 0$.*

Proof. When $d > 0$, from Proposition 1 we must have a buyer queue in the limit equilibrium. It is straightforward from the definition of z that in a sellers' market the limiting amount of trade is $G^{-1}(0) = x^c$, $b^* \to p^c$ and $s^* \to p^c$. The welfare loss caused by the permanent excess demand is the aggregate participation cost of all buyers in each period.[15] Let $B = z/\pi_b$ be the measure of buyers in the steady state equilibrium. Substituting this into (6) and rearranging gives

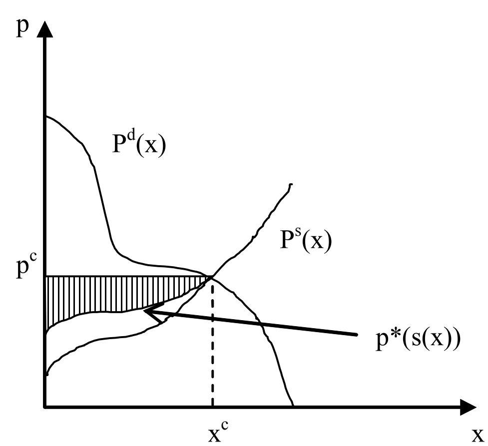
$$Bc_b = (b^* - E_s[C_i(s) + s])\,\lambda z. \tag{13}$$

We also have

$$\begin{aligned}(p^c - \overline{p}^*)\,x^c &= (p^c - \lambda E_s[C_i(s) + s] - (1-\lambda)p^c)x^c \\ &= (p^c - E_s[C_i(s) + s])\,\lambda x^c \\ &= \lim_{(c_b,c_s)\to(0,0)} Bc_b.\end{aligned}$$

By Lemma 1 $p^c > E_s[C_i(s) + s]$. Hence, $(p^c - \overline{p}^*)\,x^c > 0$. ■

In Figure 2 one can visualize the functioning of the limit equilibrium. Note that the per period loss, W, is the (shaded) area between the expected price curve, $\lim_{(c_b,c_s)\to(0,0)} p(s(x))$, and the competitive price.



The dead-weight loss in the limit as the costs disappear

[15] Alternatively, we could define the welfare loss as the participation cost of traders who do not actually trade in that period. As the limiting amount of trade is finite, the limiting value of the aggregate cost would be unaltered.

The intuition for this is simple. The marginal buyer makes an expected surplus of $b^* - \overline{p}$. This must be balanced with an expected loss (which is the same for all buyers) of $c_b + \sum_{n=1}^{\infty}(1-\pi_b)^n c_b = \frac{c_b}{\pi_b}$. In the limit, we have

$$p^c - \overline{p}^* = \lim_{(c_s,c_b)\to(0,0)} \frac{c_b}{\pi_b}.$$

Now, note that the aggregate loss "newly generated" in every period is $x^c \lim_{(c_s,c_b)\to(0,0)} \frac{c_b}{\pi_b}$. Since we are in steady state, this aggregate loss must equal the aggregate delay cost suffered in every period, W. Putting these together, we have that

$$W = (p^c - \overline{p}^*)\, x^c,$$

which corresponds to the shaded area in Figure 2.[16] This reinforces the point (made after Corollary 2) that price dispersion is crucial to the inefficiency of the market. The figure illustrates that any measures that reduce the price dispersion (reducing λ or d) will also reduce the loss.

6 Implications for the strategic foundations of competitive equilibrium

The above result may sound like a serious blow to the non-cooperative foundations of competitive equilibrium. After all, why should perishability prevent the market from reaching efficiency as per period frictions disappear? In this section, we examine both the intuition for and the robustness of our inefficiency result, further clarifying our understanding of competitive equilibrium.

One way to understand the inefficiency caused by perishability is to realize that technically, perishability is equivalent to the discounting of the sellers' future utility. In other words, our results would not change if we reinterpreted $1 - d$ as the sellers' discount factor (holding the buyers' discount factor at one) and assumed that the good was non-perishable. Since discounting is a friction, it is not surprising that it should lead to inefficiency.[17]

[16] An alternative way of thinking of the marginal buyer's problem at the limiting equilibrium is that she is given a lottery ticket at cost c_b and with probability π_b she earns a prize of $p^c - \overline{p}^*$.

[17] Note that our model – even with the asymmetric discounting interpretation – is distinct from Bose (1996), whose analysis is centered on heterogeneous time preferences on the same side of the market.

One has to be careful with how far to take this argument though. For example, we should not say that in light of the above we should only be interested in the limit as the period length – and therefore both discounting and additive costs of delay – tends to zero. The reasons for this are twofold.

First, there are markets where looking at the limit as perishability disappears would make no sense. Think of markets for (individualized) services.[18] These neatly fit our model, if we allow the sellers to return to the market after having served a customer. Note that the assumption of repeating sellers makes no difference to the analysis as long as the market interaction is anonymous. Now, if a dentist, say, cannot treat a patient between 9:30 and 10:00, she cannot postpone delivery of the treatment to 10:00-10:30, since in equilibrium she expects to treat another patient by then. Hence the opportunity to provide the 9:30-10:00 service only exists between 9:30 and 10:00. In our model this is the case where the *good* perishes with probability 1. Setting $d = 1$, the aggregate loss in the limit is $(p^c - E_s[s\,|s^* = P^s(x^c)])\,\lambda x^c$. The bottom line is that in a decentralized model of services, there will be an important dead-weight loss even in the limit as the waiting costs tend to zero. Consequently, it should not be thought of – even approximately – as a competitive market.

The second reason against hastily taking limits is that it is not necessary. Consider a model where the death rate is changing over time – but the buyers cannot tell the age of the good.[19] Although such a process is not stationary, we now show that the steady-state equilibrium of the stationary model where the death rate is constant is also an equilibrium of this model, whenever the market has the buyers queueing.

Proposition 3 *For waiting costs close enough to zero, the model where the perishing rate is variable but non-decreasing over time – and the buyers cannot tell the age of the good – has a steady-state (Perfect Bayesian) equilibrium, which leads to the same outcome as the unique steady-state (subgame-perfect) equilibrium (with trade) in the model where the death rate is constant at $d_1 > 0$.*

Proof. Let $V_i(.)$ be the steady-state equilibrium value function of a trader in the case where the death rate is always d_1. From Proposition 1 we have

[18] See Ponsatí and Sákovics (2005) for a related model with vertically differentiated service providers.

[19] Note that in our main model knowing the age of the good provides no additional information (by the stationarity of the perishing process).

a buyer queue for low enough costs of waiting. Hence, in the steady-state equilibrium the sellers always trade in period one and thus the buyers are always matched with a new entrant:

$$V_i(s) = (1-\lambda)E_b[b - s - V_i(b)] + \lambda C_i(s) - c_s$$

$$V_i(b) = \pi_b \lambda E_s[b - s - C_i(s)] + (1 - \pi_b\lambda)V_i(b) - c_b.$$

Now, consider whether the same outcome is supported by an equilibrium of the non-stationary model. Let $V_i^n(s)$ be the value function for the owner of a good in the n^{th} period of its life and $C_i^n(s)$ the value at the point where they have failed to trade in period n. By hypothesis, the sellers are always matched in the first period. Consider the value functions when the buyers always offer the sellers $C_i^1(s)$:

$$V_i^1(s) = (1-\lambda)E_b[\max\{b - s - V_i(b), C_i^1(s)\}] + \lambda C_i^1(s) - c_s$$

$$V_i^2(s) = (1-\lambda)E_b[\max\{b - s - V_i(b), C_i^2(s)\}] + \lambda \max\{C_i^1(s), C_i^2(s)\} - c_s$$

$$\vdots$$

$$V_i^n(s) = (1-\lambda)E_b[\max\{b - s - V_i(b), C_i^n(s)\}] + \lambda \max\{C_i^1(s), C_i^n(s)\} - c_s$$

$$\vdots$$

$$V(b) = \pi_b \lambda E_s[\max\{b - s - C_i^1(s), V_i(b)\}] + (1 - \pi_b\lambda)V_i(b) - c_b.$$

Now, since $1 - d_n \geq 1 - d_{n+1}$, we have that $C_i^1(s) \geq C_i^n(s)$, for all n. As a result, all the max operations are resolved in favor of the (stationary) left-hand argument – corresponding to trade occurring in every match, thereby validating the hypothesis that all the sellers trade in their first period in the market, implying that $V_i^1(s) \equiv V_i^n(s) \equiv V_i(s)$ for all n.

■

Intuitively, since in the hypothetical equilibrium the buyers are queuing, all the sellers are matched and thus, as long as they always trade, the buyers are always faced with sellers of "new" goods, and thus have no new incentive to deviate. The only thing that can go wrong is that a seller who in period one prefers to trade rather than to wait, in a later period prefers to wait (because the death rate has dropped), but this is ruled out by the realistic assumption of a non-decreasing death rate. As a result, a seller indeed never stays in the market for more than one period, and thus her entry decision will also only depend on her first-period perishing rate.

By Proposition 3, if we take the limit as $d_1 \to 0$ in the non-stationary model, then the welfare loss converges to zero even though $d_n > 0$, $n = 2, 3, \ldots$. That is, we do not need all the perishability to disappear in order to regain efficiency. The (almost) *certain* option to costlessly switch bargaining partners *at least once* is necessary and sufficient to obtain the competitive outcome.

One way to think of this in terms of our dentist example, is that each dentist needs to take a break once in every two periods, but she is indifferent between taking this break now or in the next period. If she fails to sell in the first period, she uses this as her break but then definitely wants to provide the service in the next period. This market is equivalent to $d_n = 0$ and $d_{n+1} = 1$, for n odd. In the efficient equilibrium half the dentists would work in odd periods and half in even periods.

It is eye-opening to note that Proposition 3 does not hold with $d_1 = 0$. That is, the equilibrium of the standard non-perishable good model is in general not an equilibrium of the non-stationary model. To see this, note that if the good is not perishable the long side of the market is decided by the Hosios condition. Consequently, no matter how low the costs are, we may have the sellers queuing. In that case, in the non-stationary model there *is* a new incentive to deviate in period 2, as it is no longer a credible belief that all the sellers are "new".

7 Conclusion

We have made a number of achievements in this paper. First, we have provided the analysis of the steady state of a decentralized market for perishable goods (including services). Second, we have shown that such a market is inherently inefficient. Third, we have shown that even if – using the discounting interpretation – the existence of inefficiency is not too surprising, the manifestation of it is. Rather than an inefficient quantity being traded, the welfare loss appears in the form of queueing. We have also shown that the standard model's prediction about which side of the market is the short side (c.f. Hosios condition) is not robust to perturbations in the perishability dimension.[20] Finally, we have also provided a tight result on what exactly is necessary for the decentralized market to provide a non-cooperative foundation of competitive equilibrium: that each agent should be guaranteed a second opportunity to trade.

8 Appendix

8.1 Proof of Lemma 1

First consider the durable good case. Solving for $V_D(s)$ using (7) we have

$$V_D(s) = \frac{\pi_s(1-\lambda)(b^* - s) - c_s}{d + \pi_s(1-\lambda)(1-d)}$$

which is strictly decreasing in s. Now consider $C_D(s) + s$

$$C_D(s) + s = (1-d)V_D(s) + s = \frac{(1-d)(\pi_s(1-\lambda)b^* - c_s)}{d + \pi_s(1-\lambda)(1-d)} + ds$$

which is strictly increasing in s if $d > 0$. In this case, $\widehat{s} = P^S(0)$. When $d = 0$, $C_D(s) + s \equiv b^* - c_s / \ \pi_s(1-\lambda) = s^*$.

Now consider the consumption good case. If $(1-d)V_C(s) - ds > 0$ then solving for $V_C(s)$ using (7) we have

$$V_C(s) = \frac{\pi_s(1-\lambda)b^* - c_s}{d + \pi_s(1-\lambda)(1-d)} - s \tag{14}$$

If $(1-d)V_C(s) - ds < 0$ then solving for $V_C(s)$ using (7) we have

$$V_C(s) = \pi_s(1-\lambda)(b^* - s) - c_s$$

In both scenarios $V_C(s)$ is strictly decreasing in s.

Now consider $C_C(s)+s$ in the two scenarios when $d > 0$: if $(1-d)V_C(s) - ds > 0$

$$C_C(s) + s = (1-d)V_C(s) - ds + s = \frac{(1-d)(\pi_s(1-\lambda)b^* - c_s)}{d + \pi_s(1-\lambda)(1-d)}$$

[20] This non-robustness carries over to the discounting interpretation. Hence, when buyers and sellers discount at different rates, Gale's (1987) arguments in favor of looking at the market equilibrium at the limit of no discounting do not fully apply.

and if $(1-d)V_C(s) - ds < 0$

$$C_C(s) + s = s.$$

Finally consider the pivotal seller, $\widehat{s}$ for whom $(1-d)V_C(\widehat{s}) - d\widehat{s} = 0$. Plugging this into (14) and solving for $\widehat{s}$ gives

$$\widehat{s} = \frac{(1-d)(\pi_s(1-\lambda)b^* - c_s)}{d + \pi_s(1-\lambda)(1-d)} = \frac{(1-d)\pi_s(1-\lambda)s^*}{d + \pi_s(1-\lambda)(1-d)} < s^*. \tag{15}$$

Hence for $s > \widehat{s}$, $(1-d)V_C(s) - ds < 0$ and $C_C(s) + s = s$ which is strictly increasing in s. For $s < \widehat{s}$, $C_C(s) + s = \widehat{s}$ is constant in s. When $d = 0$, we are always in the latter case and $C_C(s) + s \equiv s^*$.

8.2 Steady-states with seller queues

Here we look at what happens when c_b is not sufficiently small to ensure a buyer queue. Let x_s and x_b be the steady-state measure of entering buyers and sellers. Then $b^* = P^d(x_b)$ and $s^* = P^s(x_s)$. Substituting this into (6), the condition for $V_i(b^*) = 0$ is

$$P^d(x_b) = E_s[C_i(s) + s] - \frac{c_b}{\pi_b \lambda} \tag{16}$$

and using (8) the condition for $V(s^*) = 0$ is

$$P^d(x_b) - P^s(x_s) = \frac{c_s}{\pi_s(1-\lambda)} \tag{17}$$

In the balanced case where there is no buyer or seller queue, $\pi_b = \pi_s = 1$ and $x_s = x_b = z$. If

$$c_b > \lambda(P^d(z) - E_s[C_i(s) + s])$$

then $P^d(z) < 0$ and a buyer with value $P^d(z)$ will not enter. From this point, it is not possible to get a steady state with a buyer queue as this will only decrease $P^d(z)$. Hence in any steady-state equilibrium $x_b < z$. This also implies that $P^d(x_b) - P^s(x_s)$ will have to rise and to ensure the condition $V_i(s^*) = 0$ there will be a seller queue, $\pi_s < 1$. However, there is an additional complication with a seller queue that arises from the fact that some sellers who fail to trade leave the market and so the stock of sellers decreases. In the durable good case the number leaving is simply $L_D = (S - x_b)d$. In the consumption good case we will also lose the sellers who decide to leave the market and consume the good. From (15) the measure of sellers we lose is $x_L = x_s - (P^s)^{-1}(\widehat{s})$ and $L_C = x_L + (S - x_b - x_L)d$. Hence we have an additional condition for a steady-state equilibrium,

$$x_s - x_b = L_i \tag{18}$$

A steady-state equilibrium, (x_s, x_b, S) is a solution to (16), (17) and (18) with $\pi_b = 1$ and $\pi_s = x_s/S$.

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