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Fractal Scaling in Crude Oil Price Evolution via Time Series Analysis of Historical Data

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Abstract

This paper presents historical price data for two different crude oil types and examines the stationarity and inherent structure in oil price variation, applying many degrees of time resolution. Time Series Analysis results are then used to identify patterns and analyze the variation timescales. A specific goal of this study is to investigate and demonstrate the presence of fractal scaling. In particular, we postulate and prove that the mean size of the absolute values of price changes obeys a fractal scaling law (a power law) and can be expressed as a function of the analysis time interval (here, the latter is an independently varying parameter, ranging from a day up to a calendar year). The fractal structure of crude oil price variation is confirmed, the drift exponent is computed and the power scaling window of validity is depicted for both types, illustrating the interplay of both short- and long-term effects on the intrinsic structure of crude oil prices before and after 2008.

KEYWORDS: non-Gaussian processes, Time Series Analysis (TSA), crude oil, Brent, West Texas Intermediate (WTI), power law, fractal scaling, price forecasting

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1. INTRODUCTION AND LITERATURE REVIEW

The recent, lasting surge in crude oil and fuel prices has dramatic consequences on national economies, multinational industries as well as international petroleum and chemical process operations, posing a fundamental question in econometrics: do these extreme and unprecedented price variations still follow the established price statistics and fall within the current variation pattern, or is the international oil market and economy entering a new, drastically different historical period?

The inherently stochastic nature of economic phenomena that evolve continuously at multiple frequencies (e.g. stock exchange rates, energy prices) poses great difficulties and formidable challenges to their systematic study, but also renders traditional (descriptive statistical) methods inadequate, inasmuch as quantitative understanding of intrinsic structure in these phenomena is of interest. The availability, consistency and reliability of high-frequency price data streams are perhaps taken for granted in modern times of globalized digital automation, but they can also be elusive prerequisites when compiling scarce historical data. Novel statistical methods have been developed or adapted so as to probe oil price variation effects, e.g. via vector autoregressions (Burbridge & Harrison, 1984). Oil price variability macroeconomics are intensely studied (Lee et al., 1995), because extreme price variations frequently have clear repercussions on inducing energy shocks, which affect financial markets dramatically (Huang et al., 1996) and also have a documented impact on stock market activity (Sadorsky, 1999). Oil price volatility data improves industrial production forecasts (Ferderer, 1996), but its key characteristic is a persistently time-varying behavior (Laopodis,1996). A recent econometric analysis indicates that economies respond to oil supply crises in ways that are very different from other shocks (Backus & Crucini, 1998). Furthermore, oil price variations have nontrivial secondary effects: gasoline prices are found to respond asymmetrically to oil price changes (Borenstein et al., 1997).

Benoit Mandelbrot (1963), in a seminal and widely cited paper, discovered that the seemingly random signals of cotton prices, if appropriately examined by means of advanced statistical techniques, possess inherent structure: the latter can be quantitatively characterized via conventional statistical metrics (e.g. moments) for distributions, and special indices (e.g. drift exponent) for long-term variation. Essentially, Mandelbrot's fractal theory is impressively powerful and accurate for price series analysis, because it enables the study of statistical distribution classes which have well-defined ranges of mean variation, but infinite variance variation. These correspond very well to phenomena described by stochastic price signals, which are essentially bounded for a given period of study (*finite mean variation*), but are known to evolve across several orders of frequency (*infinite variance*). Our fundamental hypothesis (which can be confirmed by descriptive statistics) is that multi-resolution (monthly, weekly, daily, intra-day) oil price signals satisfy the foregoing criteria for several different crude oil types, over long time periods.

Time Series Analysis (TSA) is a methodology focusing on data acquired while monitoring industrial processes or tracking business metrics (NIST, 2008); in this study, we examine the evolution of crude oil prices over several decades. The definitive characteristic of TSA is that it is useful in systematically revealing and quantitatively analyzing intrinsic structure patterns of seemingly uncorrelated data points, which are recorded at variable frequency over extended time periods. Thus, TSA is effective in examining the *autocorrelation*, *stationarity*, *seasonality* and *moment* trends of extremely large data sets, employing a variety of metrics and algorithmic methods which are established for probing multiscale sets (e.g. autocorrelation function, single/multiple exponential smoothing, intrinsic time). Employing TSA methods to identify or predict macroeconomic trends has already been a common and computationally manageable practice for numerous decades. The advent of online, real-time, high-performance computing in finance and the constant generation of vast amounts of data have expanded the scope of TSA towards more demanding and CPU-intensive applications (Müller et al., 1990).

Forecasting oil prices is essential today for corporate production planning, process operations and supply chain optimization in chemical process industries. Crude oil is not just the cornerstone feedstock for many petrochemical products, but also important in process heat management and transportation of most goods. Consequently, the fidelity of price forecasting models impacts continuously not just the financial position, turnover, exposure and profitability of a company, but also corporate decision making in the context of undertaking business ventures, accessing new markets and launching new products and manufacturing processes. Both short-term and long-term price forecasting are arduous but pay dividends if successful, as they clearly benefit production and strategic planning, respectively.

Autoregressive models (NIST, 2008) require frequent reparameterization (to overcome drifts due to the absence of stationarity and presence of seasonality), and they cannot a priori forecast extreme variations observed at low frequencies. Well-known exemplary models that are widely used include the AR, ARMA and ARMAX (Auto-Regressive / Moving Average / with eXogenous input) classes. *Power scaling laws*, when available, can be extremely valuable in either avoiding or calibrating such arbitrary parameterizations, since they can yield credible upper and lower bounds for extended forecasting horizons within their validity domain.

The importance of identifying a non-Gaussian crude oil price variation lies in conclusively disproving the plausible stationarity of this stochastic process, as well as in quantifying variation bounds and intrinsic times affecting their validity. Constraining the envelope of forecasted variation by means of bounds computed from a power scaling law can be extremely beneficial to a valid parameterization: providing an additional safeguard against determining suboptimal parameter sets is very useful when parameterizing nonlinear models via nonconvex optimization.

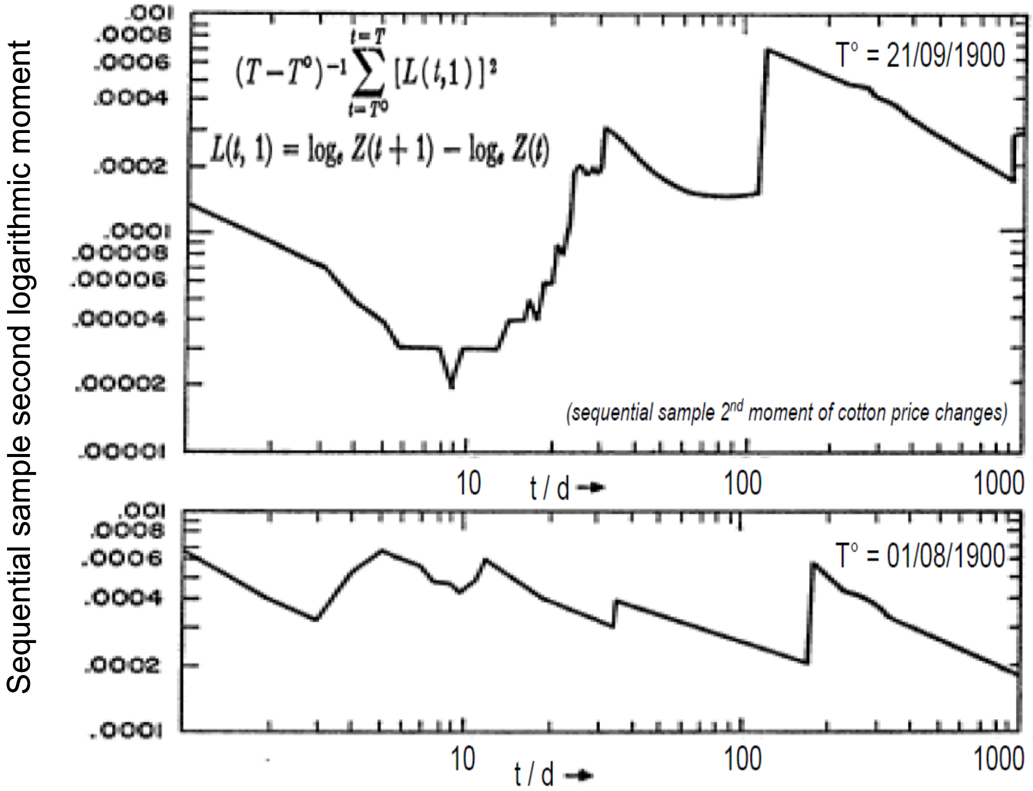

Figure 1: Fractal structure and self-similarity clear in cotton prices for two different reference dates, T° (Mandelbrot, 1963).

Statistical autocorrelation studies of intra-day financial data (Müller et al., 1995) reveal self-similarity: absolute price change values behave as the fractional noise that was defined by Mandelbrot and van Ness (Mandelbrot & Taylor, 1967). The mean absolute price change yields a linear correlation to the observation time, in logarithmic coordinates, indicating that price changes obey a scaling law: although the frequencies involved in economic phenomena span several orders of magnitude, they can be thus conclusively identified and comprehensively studied.

Figure 2 illustrates the implementation of the above TSA method to the U.S. Dollar (USD)-Deutsche Mark (DEM) currency exchange rate, by means of processing a high-frequency data stream: the mean logarithmic absolute price change is linearly correlated to the logarithmic observation window duration, effectively confirming a power law whose characteristics have been quantified. Stochastic processes which do obey scaling laws possess a fingerprint exponent. High-frequency, intra-day price data is necessary in order to probe multiple timescales, and in this case accounts for 6 (out of a total of 14) data points; remarkably, power law deviations are obvious at both frequency spectrum ends.

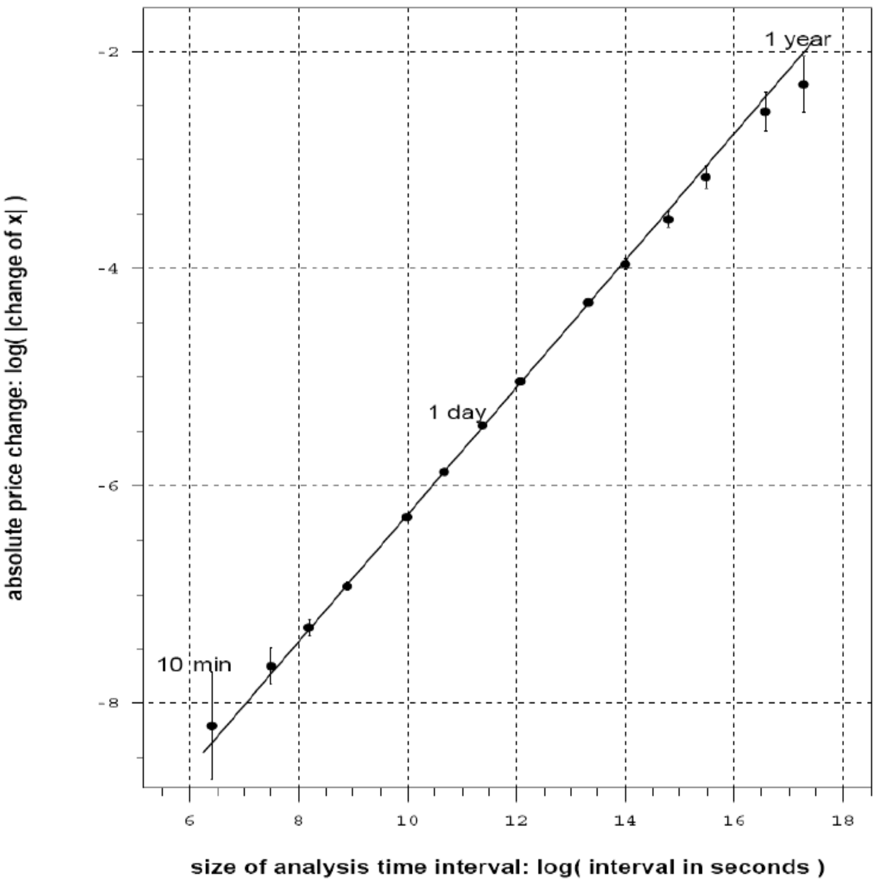

Figure 2**:** Fractal structure identified and power scaling law plotted for the USD-DEM currency exchange rate (Müller et al., 1990).

An excellent literature survey by Müller et al. (1995) illustrates how TSA principles and methodologies have been employed successfully over the past five decades, revealing scaling patterns in a wide variety of non-Gaussian financial time series, including numerous *stock market indices* (S&P 500, Hang Seng), *stocks*, *commodities* (cotton, wool), and numerous *currency exchange rates* (USD-DEM, USD-JPY, DEM-JPY, GBP-USD), over many different time periods. For all identified non-Gaussian patterns, the scaling window (over which the power law is confirmed) can vary greatly, extending from just a few minutes up to months (in case of stocks) or even years (in case of currency exchange rates). *High-frequency, consistent historical price data streams* can be costly to acquire; nevertheless, they should be used as much as possible wherever available, since this greatly enhances the accuracy of TSA methods and algorithms, and it is evidently essential in providing reliable insight for intra-day trading endeavors.

2. TIME SERIES ANALYSIS (TSA): METHODOLOGY AND DATA

The objective of this study is to analyze quantitatively and address conclusively a number of questions about the long-term variation of oil prices and its structure. First of all, we aim to investigate *if crude oil price evolution is a fundamentally non-Gaussian stochastic process*, and how exactly it relates to Mandelbrot's groundbreaking observations confirming fractal scaling in cotton and wool prices. Oil price change distributions and their common descriptive statistical measures (mean, variance, kurtosis, skewness) evolve as a function of the historical period of observation and the data resolution, and depend on the crude oil type studied. These metrics and trends must be thoroughly investigated for the longest period and the finest resolution possible (the highest frequency of data stream available).

Scaling laws describe the absolute size of change as a function of the time interval during which this is measured, and provide a direct relation between the time interval of observation (Δt) and the average volatility measured as a certain power (1/D) of the absolute returns (price changes) observed in this time interval. The scaling law presented by Müller et al. (1990) relates the mean of the (logarithmic) absolute price change to the time-interval size (Δt) over which the price change is observed, assuming the availability of an acceptably large sample:

$$\overline{|x|} = \left(\frac{\Delta t}{T}\right)^{D} \tag{1}$$

Here, $\overline{|x|}$ denotes the *sample mean of (logarithmic) absolute price changes*, while T is the *intrinsic time* of the process (an empirical time constant). For any power scaling law, the *drift exponent* (D) constitutes a fingerprint property which can be used in estimating price changes in a given time interval, and in probing via known bounds the long-term evolution of a stochastic process. Reliable parameter estimation of T and D is achieved by rigorous optimization.

The question of paramount importance in this study is *whether we can conclusively identify fractal scaling in oil price variation*, compute drift exponents, quantify the time period (if any) within which crude oil price variation obeys fractal scaling indeed, and discern long-term trends of both crude oil types. Once these questions and problems are addressed, one can quantitatively understand how crude oil trends relate to those that have been reported in the aforementioned publications on commodities, stocks and currency exchange rates. Another goal is to analyze the isolated effect of the recently experienced extreme price variations which were recorded during the past year (2008) on the stochastic process, its intrinsic structure and the foregoing quantitative metrics (Figure 3). This analysis can reveal qualitative and quantitative drifts in any existing patterns.

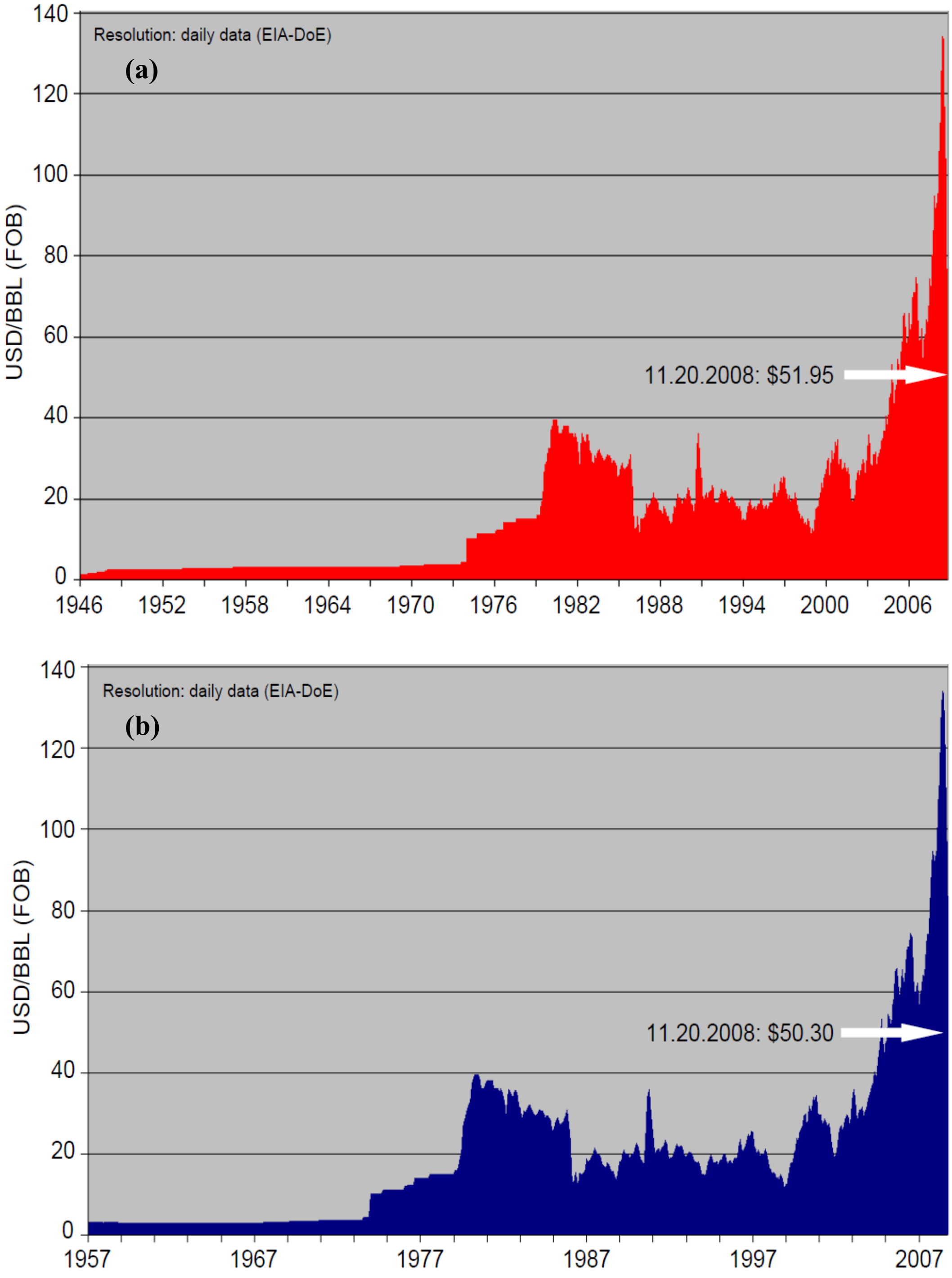

***Figure 3*: Historical price evolution for both crude oil prices considered: (a) WTI-NYMEX (1946-2008) and (b) Brent-UK (1957-2008).**

For the purpose of this study, daily price data for two benchmark crude oil types (WTI and Brent) of the period 1986-2008 have been compiled and analyzed via standard TSA techniques (autocorrelation function for a variable window). The *GFD (Global Financial Data)* data sets for WTI and Brent used are based on Energy Information Administration (EIA – U.S. DoE) publications (Figures 3, 4).

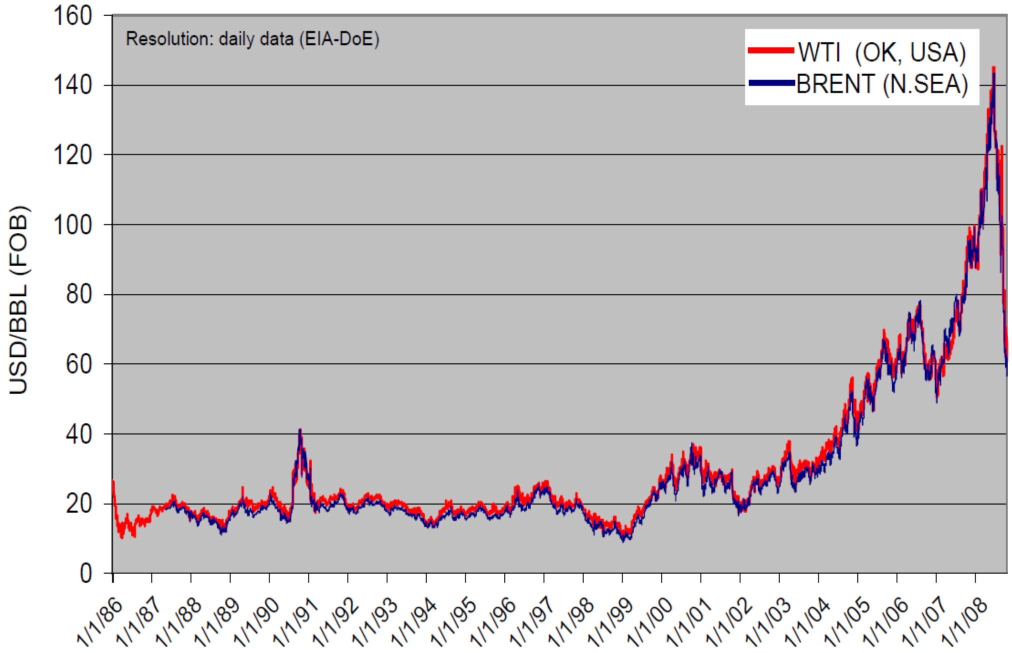

***Figure 4*:** Comparison of the price evolution time series for benchmark crude oil prices over the entire period considered (1986-2008).

3. RESULTS AND DISCUSSION

Time series analysis of the two crude oil price data streams at a daily resolution over the whole period of availability (1986-2008) provides crucial insight and proves the non-Gaussian nature of the stochastic process of oil price variation. Figures 5 and 6 illustrate price change distributions for both crude oil types as a function of the observation window (Δt). Lines show the corresponding Gaussian distributions for the data stream mean and standard deviation (Mandelbrot, 1963).

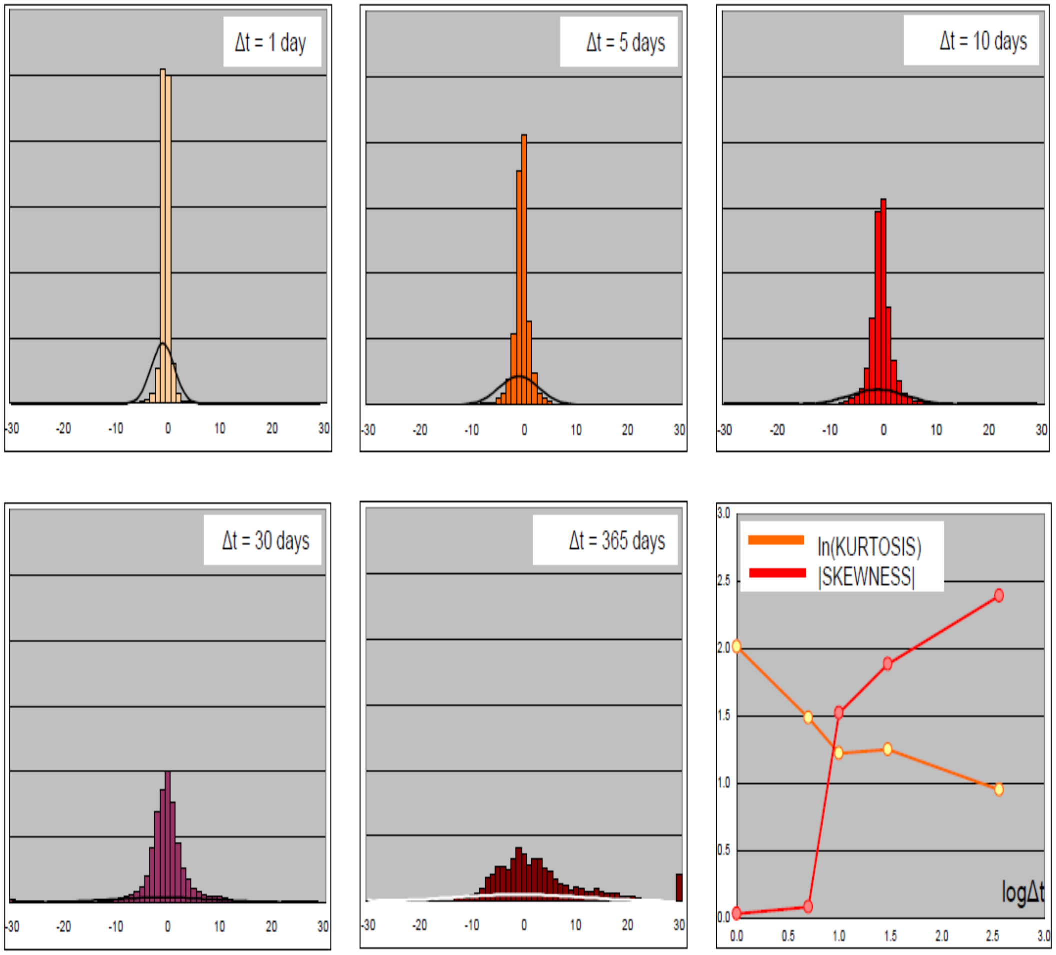

Figure 5: WTI crude oil price change distribution and statistics as a function of the observation time interval, Δt (1986-2008).

The gradual loss of symmetry as the observation window (Δt) increases is evident for both oil types: positive distribution tails dominate over negative ones. Figures 5 and 6 clearly illustrate almost symmetric (albeit clearly non-Gaussian) price change distributions, obtained for short observation times (Δt < 5 days); however, symmetry quickly disappears when observation times exceed a week. Kurtosis and skewness plots of price change distributions as a function of the observation period display identical trends, also indicating that there is a rapid loss of symmetry due to the higher frequency of positive vs. negative changes. For both oil types, price evolution is confirmed to be a non-stationary process and its variance is strongly correlated with the duration of the observation window (for extended observation periods, extreme positive changes are clear in fat tails).

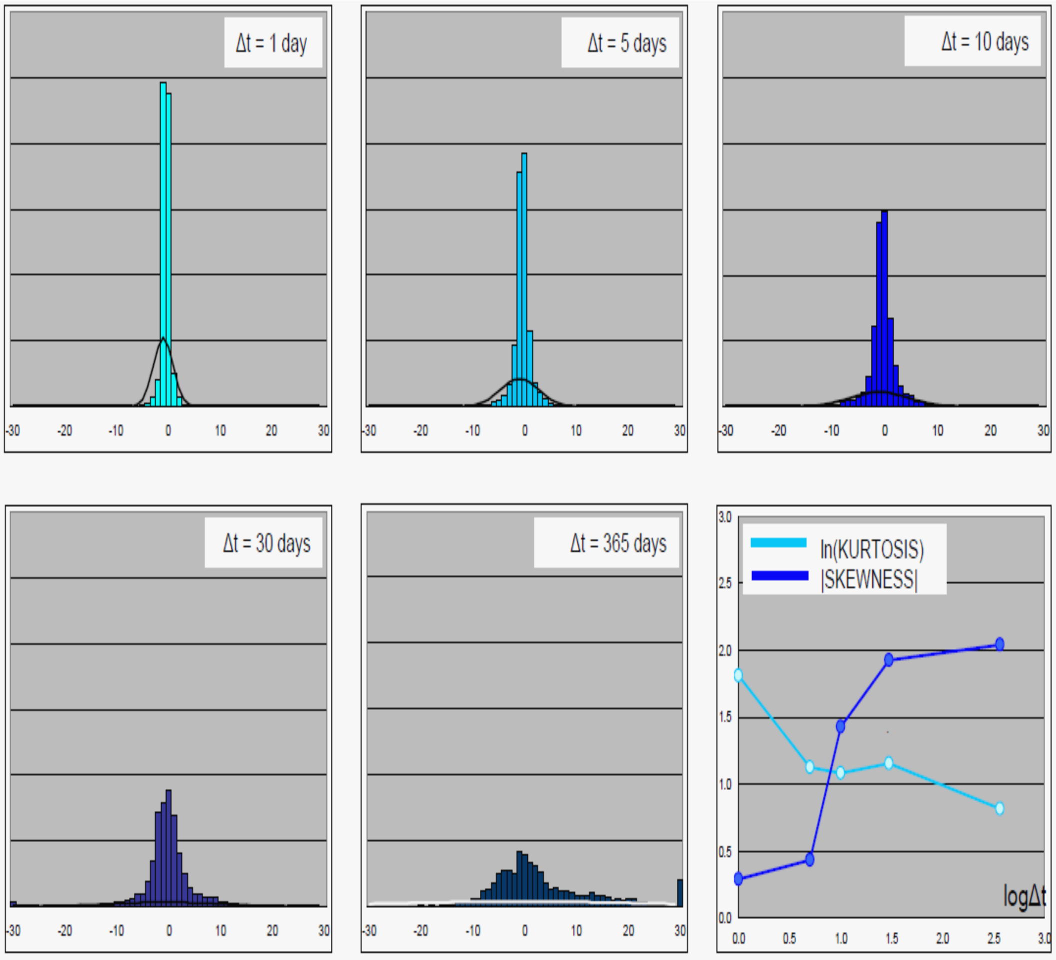

Figure 6: Brent crude oil price change distribution and statistics as a function of the observation time interval, Δt (1987-2008).

Despite the similarity in price evolution trends between crude oil types, the comparison of Figures 5 and 6 also reveals certain differences which are equally interesting and confirm that each commodity has characteristic properties. Kurtosis values for WTI (Figure 5) over a given observation window are slightly but consistently higher than those computed for Brent (Figure 6); the latter has a kurtosis which remains almost constant for intermediate observation periods. Conversely, skewness values for Brent in small observation windows are higher than those computed for WTI, indicating that asymmetry varies appreciably. Clearly, WTI has the highest skewness between the two types for long periods, so its accentuated long-term asymmetric effects surpass those experienced by Brent.

The determination of the drift exponent for a power scaling law (which is suitable for modeling oil price changes as a function of the observation time) is a key part of the present study, confirming our initial hypothesis that crude oil price variation exhibits fractal scaling properties and can be described by a power law. The mean of logarithmic absolute price changes as a function of the observation time (Δt) can be plotted in order to evaluate whether a straight line does fit the processed data (clear indication of a power scaling law) and straightforwardly determine the drift exponent from the line slope in a doubly logarithmic plot. Exactly as it has been documented in the open literature for a number of stock and currency exchange rates, the present analysis confirms the fractal structure and scaling of crude oil price variation for both crude oil types we have studied. Figure 7 illustrates the power law exponent calculation for two different periods: the price surge of 2008 spiked the exponent by more than 17%, from 0.52 to 0.61. Nevertheless, the drift exponent is common for both oil types in a given period, but the power law validity window for WTI is found to be significantly wider. Brent displays negative power law deviations for large windows (Δt > 30 days) in both periods, necessitating the study of these long timescales at a finer resolution.

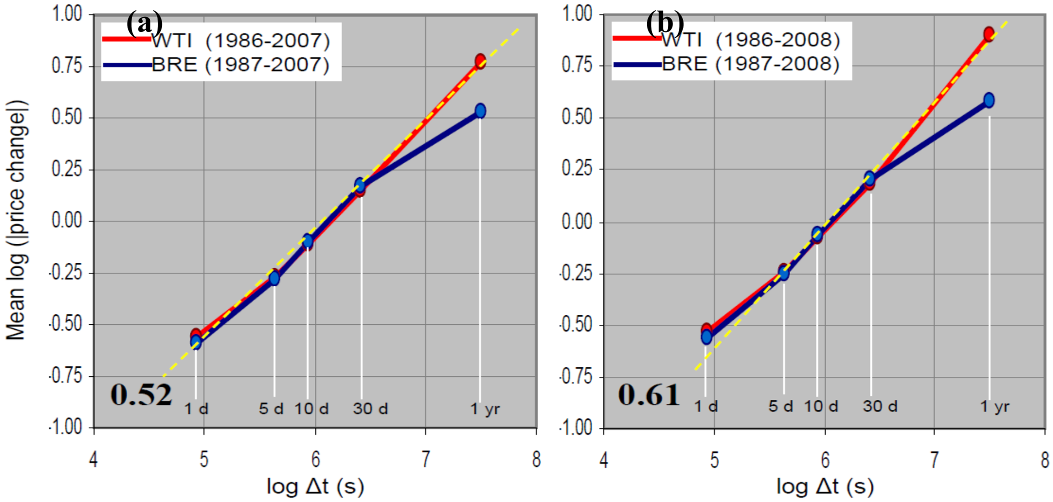

Figure 7: Fractal structure in oil price variation and power law exponent determination for both crude oil types over two time periods.

4. CONCLUSIONS

Identifying the existence of a power scaling law for crude oil price variation is of particular academic interest, because it illustrates a fundamental structural similarity between crude oil types on one hand and various markets on the other. Moreover, a confirmed power scaling is of definite interest to price forecasting, as it surpasses complex, multiparametric (linear or nonlinear) forecasting models, whose parameters must be repeatedly computed via robust optimization methods. A power scaling law is characterized by a drift exponent and an intrinsic time, providing useful bounds on price variation that depend on the prediction horizon.

Crude oil price evolution has been studied via descriptive statistics and TSA of price data streams for two crude oil types at variable observation duration, over a wide time period for which daily historical data is available (1986-2008). Price change distributions have been analyzed for multiple windows (Δt) and power scaling has been conclusively identified for both crude types (namely WTI and Brent) with daily price data from the Global Financial Data (GFD) database. Consistently non-Gaussian price change distributions (fat tails) have been identified for both crude oil types: non-Gaussian behavior is evident even for the smallest observation time window considered (Δt = 1 day), but the initial apparent symmetry of price change distributions vanishes at longer periods (Δt > 5 days). Positive price changes are consistently more frequent than negative ones beyond this threshold duration (1 week), while WTI crude displays consistently higher prices, higher kurtosis and higher skewness than those computed for Brent crude. Power law scaling holds in a wide observation window which differs significantly between crude oil types; also, the drift exponent has increased significantly (from 0.52 to 0.61) due to the extreme price variations observed in a single year (2008). The observed behavior agrees well with published econometric trends and models for stocks, indices and currency exchange rates, indicating there is merit in further study of high-frequency, intra-day oil price data streams for more crude oil types.

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