



THE UNIVERSITY *of* EDINBURGH

## Edinburgh Research Explorer

### Lattice Input on the Inclusive $\tau$ Decay $V_{us}$ Puzzle

**Citation for published version:**

A. Boyle, P. Del Debbio, L. Garron, N. J. Hudspith, R. Kerrane, E. Maltman, K & M. Zanotti, J 2014, 'Lattice Input on the Inclusive  $\tau$  Decay  $V_{us}$  Puzzle', *International Journal of Modern Physics: Conference Series*, vol. 2013. <https://doi.org/10.1142/S2010194514604414>

**Digital Object Identifier (DOI):**

[10.1142/S2010194514604414](https://doi.org/10.1142/S2010194514604414)

**Link:**

[Link to publication record in Edinburgh Research Explorer](#)

**Document Version:**

Early version, also known as pre-print

**Published In:**

International Journal of Modern Physics: Conference Series

**General rights**

Copyright for the publications made accessible via the Edinburgh Research Explorer is retained by the author(s) and / or other copyright owners and it is a condition of accessing these publications that users recognise and abide by the legal requirements associated with these rights.

**Take down policy**

The University of Edinburgh has made every reasonable effort to ensure that Edinburgh Research Explorer content complies with UK legislation. If you believe that the public display of this file breaches copyright please contact [openaccess@ed.ac.uk](mailto:openaccess@ed.ac.uk) providing details, and we will remove access to the work immediately and investigate your claim.



International Journal of Modern Physics: Conference Series  
 © World Scientific Publishing Company

## Lattice Input on the Inclusive $\tau$ Decay $V_{us}$ Puzzle

P. A. BOYLE, L. DEL DEBBIO and R.J. HUDSPITH

*Physics and Astronomy, University of Edinburgh  
 Edinburgh EH9 3JZ, UK*

N. GARRON

*School of Mathematics, Trinity College  
 Dublin 2, Ireland*

E. KERRANE

*Instituto de Física Teórica UAM/CSIC, Universidad Autónoma de Madrid  
 Cantoblanco E-28049 Madrid, Spain*

K. MALTMAN\*and J.M. ZANOTTI

*CSSM, University of Adelaide,  
 Adelaide, SA 5005, Australia*

Received Day Month Year

Revised Day Month Year

Recent analyses of flavor-breaking hadronic- $\tau$ -decay-based sum rules produce values of  $|V_{us}| \sim 3\sigma$  low compared to 3-family unitarity expectations. An unresolved systematic issue is the significant variation in  $|V_{us}|$  produced by different prescriptions for treating the slowly converging  $D = 2$  OPE series. We investigate the reliability of these prescriptions using lattice data for various flavor-breaking correlators and show the fixed-scale prescription is clearly preferred. Preliminary updates of the conventional  $\tau$ -based, and related mixed  $\tau$ -electroproduction-data-based, sum rule analyses incorporating B-factory results for low-multiplicity strange  $\tau$  decay mode distributions are then performed. Use of the preferred FOPT  $D = 2$  OPE prescription is shown to significantly reduce the discrepancy between 3-family unitarity expectations and the sum rule results.

The conventional inclusive hadronic  $\tau$  decay determination of  $|V_{us}|$ <sup>1</sup> is obtained by applying the finite energy sum rule (FESR) relation, involving polynomial weight  $w(s)$  and kinematic-singularity-free correlator  $\Pi(s)$  with spectral function  $\rho(s)$ ,

$$\int_0^{s_0} w(s)\rho(s) ds = -\frac{1}{2\pi i} \oint_{|s|=s_0} w(s)\Pi(s) ds, \quad (1)$$

to the flavor-breaking (FB) difference  $\Delta\Pi_\tau \equiv \left[ \Pi_{V+A;ud}^{(0+1)} - \Pi_{V+A;us}^{(0+1)} \right]$ , where

\*Permanent address: Math and Statistics, York University, Toronto, ON Canada M3J 1P3

2 *K. MALTMAN*

$\Pi_{V/A;ij}^{(J)}(s)$  are the spin  $J = 0, 1$  components of the flavor  $ij$ , vector (V) or axial vector (A) current-current 2-point functions. The spectral functions,  $\rho_{V/A;ij}^{(0+1)}$ , hence also  $\Delta\rho_\tau$ , are related to the normalized differential decay distributions,  $dR_{V/A;ij}/ds$ , of flavor  $ij$  V- or A-current-induced  $\tau$  decay widths,  $R_{V/A;ij} \equiv \Gamma[\tau^- \rightarrow \nu_\tau \text{ hadrons}_{V/A;ij}(\gamma)]/\Gamma[\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e(\gamma)]$ , by

$$dR_{V/A;ij}/ds = 12\pi^2 |V_{ij}|^2 S_{EW} \left[ w_\tau(y_\tau) \rho_{V/A;ij}^{(0+1)}(s) - w_L(y_\tau) \rho^{(0)}(s) \right] / m_\tau^2, \quad (2)$$

with  $y_\tau = s/m_\tau^2$ ,  $V_{ij}$  the  $ij$  CKM matrix element,  $w_\tau(y) = (1-y)^2(1+2y)$ ,  $w_L(y) = y(1-y)^2$ , and  $S_{EW}$  a short-distance electroweak correction factor. The  $J = 0$  (longitudinal) contributions in (2) are well known phenomenologically and, due to problems with the corresponding  $D = 2$  OPE series, usually subtracted from  $dR/ds$  <sup>1,2</sup>. The subtracted result,  $dR_{V/A;ij}^{(0+1)}/ds$ , allows the construction of  $J = 0 + 1$  reweighted analogues,  $R_{V+A;ij}^w(s_0) = \int_0^{s_0} ds [w(s)/w_\tau(y_\tau)] dR_{V+A;ij}^{(0+1)}(s)/ds$ , for any  $w(s)$  and  $s_0 < m_\tau^2$ . Defining  $\delta R_{V+A}^w(s_0) = \left[ R_{V+A;ud}^w(s_0)/|V_{ud}|^2 \right] - \left[ R_{V+A;us}^w(s_0)/|V_{us}|^2 \right]$ , one has, for  $s_0$  large enough to allow use of the OPE on the RHS of (1), <sup>1</sup>

$$|V_{us}| = \sqrt{R_{V+A;us}^w(s_0) / \left[ \frac{R_{V+A;ud}^w(s_0)}{|V_{ud}|^2} - \delta R_{V+A}^w(s_0) \right]}. \quad (3)$$

This relation has usually been employed in un-reweighted form, with  $w = w_\tau$ , and the single value  $s_0 = m_\tau^2$  <sup>1</sup>. This has the advantage that  $R_{V+A;ud,us}^{w_\tau}(m_\tau^2)$  is determinable from branching fraction information alone, but the disadvantage of precluding tests of the  $s_0$ - and  $w(s)$ -independence of the analysis, which could otherwise be used to investigate potential systematic uncertainties (in particular, those associated with the treatment of OPE contributions). Such self-consistency tests were carried out in Refs. 2, 3, 4, and non-trivial  $w(s)$ - and  $s_0$ -dependences observed, suggesting shortcomings in the experimental data and/or OPE representation.

The most obvious potential OPE problem lies in the rather slow convergence of the  $D = 2$  OPE series. In terms of the running  $\overline{MS}$  quantities  $m_s(Q^2)$  and  $\bar{a} \equiv \alpha_s(Q^2)/\pi$ , the  $D = 2$  series, which is known to 4-loops, is given by

$$[\Delta\Pi_\tau(Q^2)]_{D=2}^{OPE} = \frac{3}{2\pi^2} \frac{m_s^2(Q^2)}{Q^2} \sum_{k=0} c_k^\tau \bar{a}^k \quad (4)$$

with  $c_k^\tau = 1, 7/3, 19.93, 208.75$  for  $k = 0 \dots 3$  <sup>5</sup>. Since  $\bar{a}(m_\tau^2) \simeq 0.10$ ,  $c_3^\tau \bar{a}^3 > c_2^\tau \bar{a}^2$  at the spacelike point on the contour for all  $s_0 \leq m_\tau^2$ . The problematic convergence complicates the assessment of  $D = 2$  truncation errors, and manifests itself, e.g., in the  $\sim 0.0020$  difference in  $|V_{us}|$  values obtained using two alternate (CIPT or FOPT) versions of the 4-loop-truncated,  $w_\tau$ -weighted series.

An alternate determination employs the FB combination  $\Delta\Pi_{\tau-EM} \equiv 9\Pi_{EM} - 5\Pi_{ud;V}^{(0+1)} + \Pi_{ud;A}^{(0+1)} - \Pi_{us;V+A}^{(0+1)}$  in place of  $\Delta\Pi_\tau$  <sup>6</sup>. Inclusive electroproduction cross-sections fix the electromagnetic (EM) spectral function. By construction, the  $\Delta\Pi_{\tau-EM}$   $D = 2$  series is strongly suppressed, having the form (4), with  $c_k^\tau \rightarrow$

$c_k^{\tau-EM} = 0, -1/3, -4.384, -44.943$  for  $k = 0 \dots 3$ . The  $D = 4$  series is also strongly suppressed. OPE contributions to  $\Delta\Pi_{\tau-EM}$  FESRs, hence also estimated OPE errors, are thus very small<sup>6</sup>, and the resulting  $|V_{us}|$  errors essentially entirely experimental. A check of this predicted suppression is thus of interest.

We investigate the relative merits of the fixed-scale (FOPT-like) and local-scale ( $\mu^2 = Q^2$ , i.e., CIPT-like) treatments of the  $\Delta\Pi_{\tau}$   $D = 2$  series, and the level of  $\Delta\Pi_{\tau-EM}$  suppression, by comparing OPE expectations and lattice data for the two correlator combinations over a range of Euclidean  $Q^2$ . Five RBC/UKQCD domain wall fermion ensembles are employed, three, with  $m_{\pi} = 293, 349, 399$  MeV, having  $1/a = 2.31$  GeV<sup>8</sup>, and two, with  $m_{\pi} = 171, 248$  MeV, having  $1/a = 1.37$  GeV<sup>7</sup>. For technical reasons, conserved-local versions of the flavor  $us$  2-point functions are numerically challenging and hence, for  $\Delta\Pi_{\tau}$ , local-local versions are used. To check that this does not produce residual lattice artifacts which would impact our conclusions, we have also performed the OPE-lattice comparison, using conserved-local data, for the alternate flavor-diagonal FB combination  $\Delta\Pi_{diag} \equiv \Pi_{V;\ell\ell} - \Pi_{V;ss}$ , whose  $D = 2$  series is very similar to that of  $\Delta\Pi_{\tau}$  ( $c_k^{\tau} \rightarrow c_k^{diag} = 1, 8/3, 24.32, 253.69$  for  $k = 0 \dots 3$  in (4)). The results confirm those of the local-local study.

Representative OPE-lattice data comparisons for  $\Delta\Pi_{\tau}$  are shown, for the  $1/a = 2.13$  GeV,  $m_{\pi} = 293$  MeV ensemble, in Fig. 1. The left (right) panel comparison employs the fixed-scale (local-scale) prescription for the  $D = 2$  OPE series. The fixed-scale versions match much better the  $Q^2$  dependence of the lattice results, with the 3-loop-truncated version thereof best matching the overall normalization.

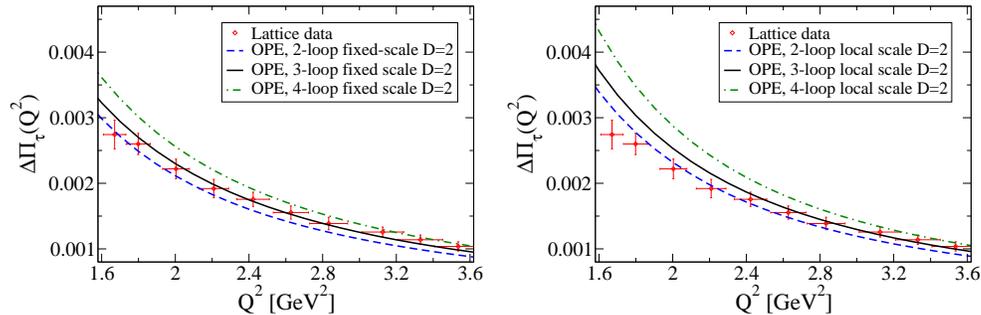
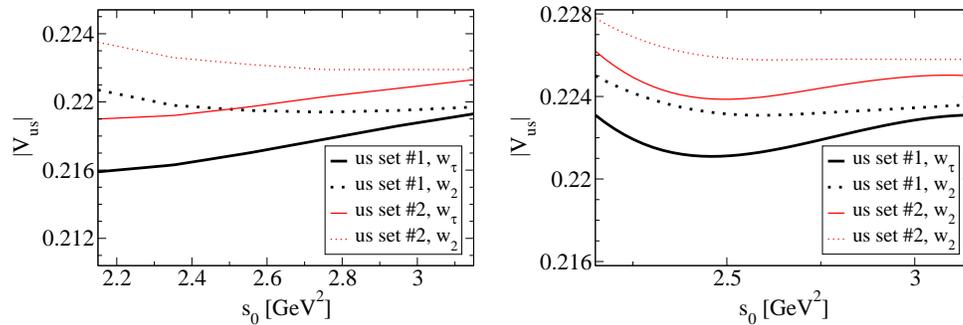


Fig. 1. OPE and lattice  $\Delta\Pi_{\tau}$  data,  $1/a = 2.31$  GeV,  $m_{\pi} = 293$  MeV ensemble,  $O(\bar{a}^{1,2,3})$   $D = 2$  OPE truncation, fixed-scale (left panel) or local-scale (right panel)  $D = 2$  prescription

The comparison of lattice data for  $\Delta\Pi_{\tau}$  and  $\Delta\Pi_{\tau-EM}$  confirms the very strong suppression of  $\Delta\Pi_{\tau-EM}$ <sup>4</sup> (see Ref. 4 for the relevant figure).

4 *K. MALTMAN*

 Fig. 2.  $|V_{us}|$  from preliminary updates of the FB  $\Delta\Pi_\tau$  and  $\Delta\Pi_{\tau-EM}$  FESRs

We turn to preliminary updates of the  $|V_{us}|$  analyses. For the  $D = 2$  OPE series, we employ the 3-loop-truncated FOPT prescription favored by lattice data, and for the  $ud$  spectral integrals, OPAL data<sup>9</sup>, as updated in Ref. 10. For the  $us$  spectral integrals, recent B-factory results are used for the  $K\pi$ <sup>11</sup>,  $K^-\pi^-\pi^+$ <sup>12</sup> and  $K_s\pi^-\pi^0$ <sup>13</sup> exclusive mode distributions, and ALEPH results<sup>14</sup>, updated for current branching fractions (BFs), for all other modes. Contributions from the latter lie higher in the spectrum, and have much larger errors. The B-factory distributions are unit normalized, and also require current BFs for their overall scales. We work with BFs obtained in a  $\pi_{\mu 2}$ ,  $K_{\mu 2}$ -constrained HFAG fit, supplemented by the update to  $B[\tau^- \rightarrow K_s^0\pi^-\pi^0\nu_\tau]$  produced by the recent Belle result<sup>13</sup>. Other non-trivial shifts in the  $us$  BFs also remain possible. To illustrate the changes to  $|V_{us}|$  that could result, we consider also an alternate set of  $us$  BFs with the recent larger, but not yet finalized, BaBar results<sup>15</sup> for  $B[\tau^- \rightarrow K^-n\pi^0\nu_\tau]$ ,  $n \leq 3$ , used in place of those of the HFAG fit. The first set of  $us$  BFs is labelled “ $us$  BF set #1” below, the second, alternate set “ $us$  BF set #2”. Changes to the  $us$  BFs alter the inclusive  $us$  spectral distribution, and hence can affect both the magnitude of  $|V_{us}|$  and the  $s_0$ -dependence of the results. The significantly larger preliminary BaBar  $K^-\pi^0$  BF is particularly relevant for the FB FESRs considered here, which weight more strongly the low- $s$  part of the spectrum. We consider FESRs employing the weights  $w_\tau$  and  $w_2(y) = (1 - y)^2$ .  $w_2$  weights less strongly the higher- $s$ , large-error region of the  $us$  spectral distribution. Differences between results obtained using the two different weights can thus point to issues with the  $us$  spectral distribution.

$|V_{us}|$  results obtained from the  $w_\tau$  and  $w_2$  versions of the  $\Delta\Pi_\tau$  FESR are shown, as a function of  $s_0$ , and also the choice of the input  $us$  BF set, in the left panel of Fig. 2. Similar results for the  $\Delta\Pi_{\tau-EM}$  FESR are shown in the right panel.  $w_2$  results, which are less sensitive to the large-error high- $s$  region, show better  $s_0$ -stability in both cases. For  $w_\tau$ ,  $s_0$ -stability is also better for the  $\Delta\Pi_{\tau-EM}$  case,

where OPE contributions are suppressed. The convergence of  $w_\tau$  results to the more stable  $w_2$  ones as  $s_0 \rightarrow m_\tau^2$ , seen for both the  $\Delta\Pi_\tau$  and  $\Delta\Pi_{\tau-EM}$  FESRs, suggests the possibility of residual OPE problems in the  $w_\tau$  case, where cancellations on the contour play a larger role. Finally we note that results obtained using the FOPT prescription preferred by the lattice data agree better with 3-family unitarity expectations than do those (not shown here) obtained using CIPT, as do those obtained using  $us$  BF set #2 in place of  $us$  BF set #1. More details of these analyses will be presented elsewhere.

We close by stressing the preference for FOPT over CIPT for the  $D = 2$  OPE series. The prescription which underlies CIPT (of summing logarithmic terms to all orders while truncating the series of non-logarithmic terms), though plausible, is motivated by heuristic arguments not generally valid for divergent series<sup>16</sup>, and performs poorly when tested against lattice data for the FB correlators.

### Acknowledgments

Computations were performed using the STFC's DiRAC facilities at Swansea and Edinburgh. PAB, LDD, and RJH were supported by an STFC Consolidated Grant, and by the EU under Grant Agreement PITN-GA-2009-238353 (ITN STRONGnet); EK by the Comunidad Autónoma de Madrid under the program HEPHACOS S2009/ESP-1473 and the EU under Grant Agreement PITN-GA-2009-238353 (ITN STRONGnet); KM by NSERC (Canada); and JMZ by the Australian Research Council grant FT100100005.

### References

1. E. Gamiz *et al.*, *JHEP* **0301**, 060 (2003); *Phys. Rev. Lett.* **94**, 011803 (2005); *PoS KAON (2008)*, 008 (2008); E. Gamiz, arXiv:1303.3971 [hep-ph].
2. K. Maltman, C.E. Wolfe, *Phys. Lett.* **B639**, 283 (2006), *ibid.* *B650*, 27 (2007); K. Maltman *et al.*, *Int. J. Mod. Phys.* **A23**, 3191 (2008), *Nucl. Phys. Proc. Suppl.* **189**, 175 (2009).
3. K. Maltman, *Nucl. Phys. Proc. Suppl.* **218**, 146 (2011).
4. P.A. Boyle *et al.*, *PoS Confinement X (2012)*, 100 (2012).
5. K.G. Chetyrkin and A. Kwiatkowski, *Z. Phys.* **C59**, 525 (1993) and hep-ph/9805232; P.A. Baikov, K.G. Chetyrkin and J.H. Kuhn, *Phys. Rev. Lett.* **95**, 012003 (2005).
6. K. Maltman, *Phys. Lett.* **B672**, 257 (2009).
7. R. Arthur *et al.*, *Phys. Rev.* **D87**, 094514 (2013).
8. Y. Aoki *et al.*, *Phys. Rev.* **D83**, 074508 (2011).
9. K. Ackerstaff *et al.* (OPAL Collab.), *Eur. Phys. J.* **C7**, 571 (1999).
10. D. Boito *et al.*, *Phys. Rev.* **D85**, 093015 (2012).
11. B. Aubert, *et al.* (BaBar Collab.), *Phys. Rev.* **D76**, 051104 (2007) 051104; D. Epifanov, *et al.* (Belle Collab.), *Phys. Lett.* **B654**, 65 (2007).
12. I.M. Nugent (for the BaBar Collaboration), arXiv:1301.7105 [hep-ex].
13. S. Ryu (for the Belle Collaboration), arXiv:1302.4565 [hep-ex].
14. R. Barate *et al.* (ALEPH Collab.), *Eur. Phys. J.* **C11**, 599 (1999).
15. A. Adametz, U. Heidelberg PhD thesis, July 2011 and the BaBar Collab., in progress.
16. M. Beneke and M. Jamin, *JHEP* **0809**, 044 (2008).