



THE UNIVERSITY *of* EDINBURGH

Edinburgh Research Explorer

## Finite-top-mass effects in NNLO Higgs production

**Citation for published version:**

Marzani, S, Ball, RD, Del Duca, V, Forte, S & Vicini, A 2009, 'Finite-top-mass effects in NNLO Higgs production', *Nuclear Physics B - Proceedings Supplements*, vol. 186, pp. 98-101.  
<https://doi.org/10.1016/j.nuclphysbps.2008.12.019>

**Digital Object Identifier (DOI):**

[10.1016/j.nuclphysbps.2008.12.019](https://doi.org/10.1016/j.nuclphysbps.2008.12.019)

**Link:**

[Link to publication record in Edinburgh Research Explorer](#)

**Published In:**

Nuclear Physics B - Proceedings Supplements

**General rights**

Copyright for the publications made accessible via the Edinburgh Research Explorer is retained by the author(s) and / or other copyright owners and it is a condition of accessing these publications that users recognise and abide by the legal requirements associated with these rights.

**Take down policy**

The University of Edinburgh has made every reasonable effort to ensure that Edinburgh Research Explorer content complies with UK legislation. If you believe that the public display of this file breaches copyright please contact [openaccess@ed.ac.uk](mailto:openaccess@ed.ac.uk) providing details, and we will remove access to the work immediately and investigate your claim.



## Finite-top-mass effects in NNLO Higgs production

Simone Marzani, <sup>a</sup> Richard D. Ball, <sup>a</sup> Vittorio Del Duca, <sup>b</sup> Stefano Forte, <sup>c</sup> Alessandro Vicini <sup>c</sup>

<sup>a</sup>School of Physics and Astronomy, The University of Edinburgh,  
Edinburgh EH9 3JZ, Scotland, UK

<sup>b</sup>INFN, Laboratori Nazionali di Frascati  
Via E. Fermi 40, I-00044 Frascati, Italy

<sup>c</sup>Dipartimento di Fisica, Università di Milano and  
INFN, Sezione di Milano, Via Celoria 16, I-20133 Milan, Italy

We construct an accurate approximation to the exact NNLO cross section for Higgs production in gluon-gluon fusion by matching the dominant finite top mass corrections recently computed by us to the known result in the infinite mass limit. The ensuing corrections to the partonic cross section are very large when the center of mass energy of the partonic collision is much larger than the Higgs mass, but lead to a moderate correction at the percent level to the total Higgs production cross section at the LHC. Our computation thus reduces the uncertainty related to these corrections at the LHC from the percent to the per mille level.

The search for the Higgs boson is one of major tasks of the forthcoming experiments at the Large Hadron Collider (LHC) at CERN. The theoretical and experimental effort which has been put into Higgs studies for LHC phenomenology is remarkable. In particular, the determination of higher-order corrections in perturbative QCD has been widely investigated. The dominant Higgs production mechanism in the Standard Model is gluon-gluon fusion through a top loop. The hadronic cross section can be obtained by convolution of the partonic cross section with parton distributions  $f_i(x, \mu^2)$

$$\sigma(\alpha_s; \tau_h, y_t, m_H^2) = \sigma_0(y_t) \sum_{i,j} \int_{\tau_h}^1 \frac{dx_1}{x_1} \int_{\tau_h}^1 \frac{dx_2}{x_2} C_{i,j} \left( \alpha_s; \frac{\tau_h}{x_1 x_2}, y_t \right) f_i(x_1, m_H^2) f_j(x_2, m_H^2) \quad (1)$$

where  $\sigma_0(y_t)$  is the partonic Born cross section [1] and the dimensionless variables  $\tau_h$  and  $y_t$  parametrise the hadronic center-of-mass energy and the dependence on the top mass:

$$\tau_h = \frac{m_H^2}{s}, \quad y_t = \frac{m_t^2}{m_H^2}. \quad (2)$$

The dimensionless coefficient function

$C(\alpha_s; \tau, y_t)$  contains the QCD corrections. The NLO contribution to it was computed in [2] and recently confirmed in [3]. The dominant NLO correction comes from the radiation of soft gluons, which cannot resolve the quark loop in the ggH coupling. Therefore, at least at the inclusive level, the approximation to the exact NLO result obtained [4,5] by taking the limit  $m_t \rightarrow \infty$  turns out to be very accurate. This approximation considerably simplifies the calculation because the ggH coupling becomes pointlike and the corresponding Feynman diagrams have one less loop.

Recently, the NNLO contribution to  $C(\alpha_s; \tau, y_t)$  has been computed in the  $m_t \rightarrow \infty$  limit [6]. The NNLO result appears to be perturbatively quite stable and it should provide a good approximation to the yet unknown exact result; it has been widely used for precision phenomenology at the LHC [7]. However, the infinite  $m_t$  approximation fails in the limit of large partonic center-of-mass energy or equivalently  $\tau \rightarrow 0$ . This is due to the fact that the high energy behaviour of the partonic cross section is completely different according to whether the ggH coupling is pointlike or goes through a quark loop, because in the latter case the quark loop

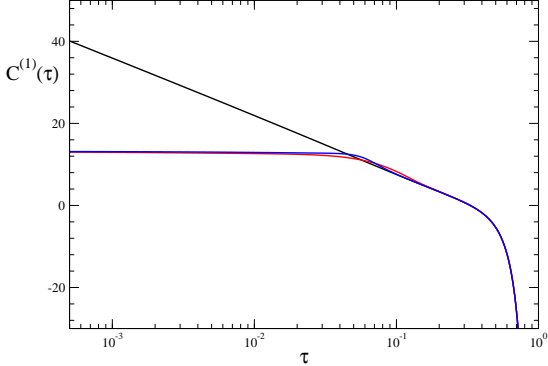


Figure 1. The NLO coefficient function for  $m_H = 130$  GeV. The red curve corresponds to the exact case, the black one to  $m_t \rightarrow \infty$  and the blue one to the approximation Eq. (6) with  $\tau_0 = 0.057$  and  $\omega = 1/100$ .

effectively provides a form factor which softens the interaction. Indeed, the coefficient function behaves respectively as

$$C \underset{\tau \rightarrow 0}{\sim} \begin{cases} \sum_{k=1}^{\infty} \alpha_s^k \ln^{2k-1} \left( \frac{1}{\tau} \right) & \text{if } m_t \rightarrow \infty \\ \sum_{k=1}^{\infty} \alpha_s^k \ln^{k-1} \left( \frac{1}{\tau} \right) & \text{for finite } m_t \end{cases} \quad (3)$$

Equation (3) shows that the difference at high energy between the exact and approximate behaviour is larger at higher orders, so one might expect the relative accuracy of the infinite  $m_t$  approximation to become accordingly worse. In Ref. [10] we have recently computed the leading high energy logarithms Eq. (3) at finite  $m_t$ , using the techniques of high-energy (or  $k_T$ ) factorization [8] (the corresponding coefficients in the  $m_t \rightarrow \infty$  limit had been previously computed in Ref. [9]). We can use this result to construct an improvement of the NNLO result [6], by replacing its spurious double logarithmic growth with the correct high energy behaviour, Eq. (3).

This construction requires a suitable matching procedure, and it gives us an approximation to the exact NNLO result. We shall first perform this improvement on the NLO contribution,

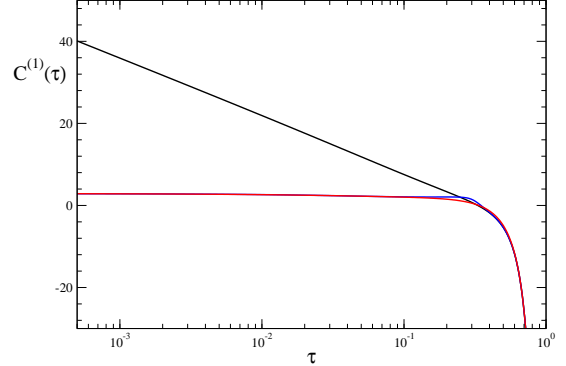


Figure 2. Same as Fig. 1, but with  $m_H = 280$  GeV (here  $\tau_0 = 0.315$  and  $\omega = 1/20$ ).

where the exact result is known: it turns out to give an approximation to the exact NLO partonic cross section which is everywhere accurate to better than 1% for any value of the Higgs mass, thus leading to an approximation to the total cross section which is accurate to the level of 0.05%. We shall then construct a similar improvement of the NNLO term. This improvement changes the total cross section computed up to NNLO by an amount which varies between 0.2% for light Higgs ( $m_H \sim 130$  GeV) to 1% for heavy Higgs ( $m_H \sim 280$  GeV). This is thus the size of the error which is made if the cross section is computed at NNLO using the approximate  $m_t \rightarrow \infty$  result. By varying the matching prescription, we estimate that the ambiguity on this result is at the level of the per mille.

The perturbative expansion of the coefficient function in the gluon-gluon channel is

$$C(\alpha_s; \tau, y_t) = \delta(1 - \tau) + \frac{\alpha_s}{\pi} C^{(1)}(\tau, y_t) + \left( \frac{\alpha_s}{\pi} \right)^2 C^{(2)}(\tau, y_t) + \mathcal{O}(\alpha_s^3). \quad (4)$$

The leading high energy behaviour of the NLO and NNLO contributions is given by

$$\begin{aligned} C^{(1)}(\tau, y_t) &= \mathcal{C}_1^{(1)}(y_t) C_A + \mathcal{O}(\tau) \\ C^{(2)}(\tau, y_t) &= -\mathcal{C}_2^{(2)}(y_t) C_A^2 \ln \tau + \mathcal{C}_1^{(2)}(y_t) + \mathcal{O}(\tau). \end{aligned} \quad (5)$$

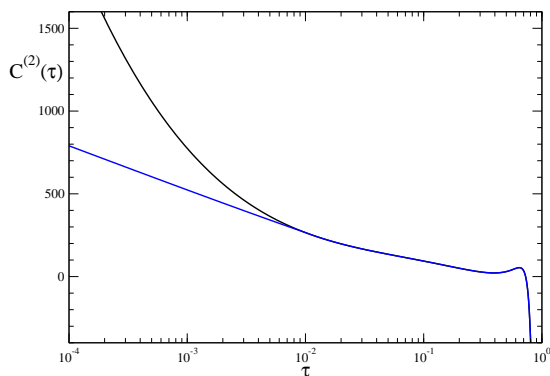


Figure 3. The NNLO coefficient function for  $m_H = 130$  GeV. The black curve corresponds to  $m_t \rightarrow \infty$  and the blue one to the approximation Eq. (6) with  $\tau_0 = 0.011$  and  $\omega = 1/100$ . The exact case is not known at NNLO.

The leading NNLO coefficient  $C_2^{(2)}(y_t)$  was computed in Ref. [10], while the subleading NNLO coefficient  $C_1^{(2)}(y_t)$  is unknown.

The approximate pointlike determination of the coefficient function can be improved by subtracting its spurious small  $\tau$  growth and replacing it with the exact behaviour:

$$C^{\text{app.}}(\tau, y_t) = C(\tau, \infty) + T(\tau, \tau_0) \times \left[ C(\tau, y_t) - \lim_{\tau \rightarrow 0} C_0(\tau, \infty) \right], \quad (6)$$

where  $C_0(\tau, \infty)$  is the sum of contributions to the infinite  $m_t$  result  $C(\tau, \infty)$  which do not vanish when  $\tau \rightarrow 0$ , as given at NLO and NNLO by the terms listed in Eq. (5). Also,  $T(\tau, \tau_0)$  is a matching function, which is introduced in order to tune the point  $\tau_0$  where the small  $\tau$  behaviour sets in. We choose

$$T(\tau, \tau_0) = \frac{1}{2} \left[ 1 + \tanh \left( \frac{\tau_0 - \tau}{\omega} \right) \right], \quad (7)$$

which in the limit of vanishing width  $\omega$  becomes the step function:  $\lim_{\omega \rightarrow 0} T(\tau, \tau_0) = \Theta(\tau - \tau_0)$ .

At NLO, because the exact asymptotic behaviour Eq. (5) is the constant  $C_1^{(1)}(y_t)$ , the

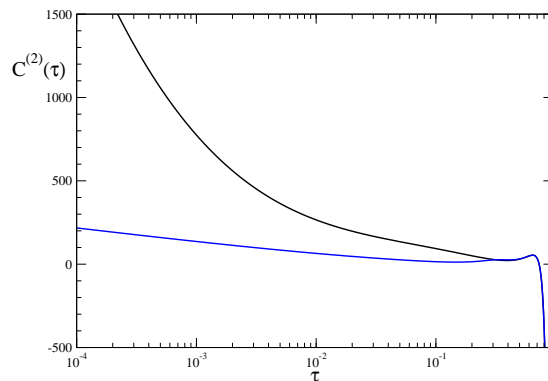


Figure 4. Same as Fig. 3, but with  $m_H = 280$  GeV (here  $\tau_0 = 0.317$  and  $\omega = 1/20$ ).

matching point  $\tau_0$  is naturally determined as the value of  $\tau$  where the pointlike approximation equals this constant. It is clear from Fig. 1 that this choice leads to an excellent approximation to the exact result: in fact, it is accurate to better than 1% for all  $\tau$ .

At NNLO the exact asymptotic behaviour Eq. (5) is a linear rise in  $\ln \tau$ . Hence, reasoning as at NLO, we are led to choose  $\tau_0$  as the point where the log derivative of the  $m_t \rightarrow \infty$  curve matches the asymptotic value  $C_2^{(2)}(y_t)$ :

$$\frac{d}{d \ln \tau} C^{(2)}(\tau, \infty) \Big|_{\tau=\tau_0} = -9C^{(2)}(y_t). \quad (8)$$

This does not fix completely the approximate result at NNLO however, because the subleading constant  $C_1^{(2)}(y_t)$  is unknown. We fix it by requiring that the approximate curve Eq. (6) be continuous at  $\tau = \tau_0$  even when the matching function  $T(\tau, \tau_0) = \Theta(\tau - \tau_0)$ . The NNLO approximation determined thus is compared to the exact result in Figs. 3, 4 for heavy and light Higgs, respectively. A more conservative matching might consist instead of taking for  $\tau_0$  the value found at NLO, and then determining again the subleading constant by continuity. The result found in this way is actually very close to the previous one, and in fact indistinguishable from it in the case of Fig. 4.

	$K^{\text{NLO}}$	$K^{\text{NNLO}}$
$m_H = 130 \text{ GeV}$		
pointlike	1.800	2.140
exact	1.797	n.a.
appr.	1.796	2.136
$m_H = 280 \text{ GeV}$		
pointlike	1.976	2.420
exact	1.958	n.a.
appr.	1.959	2.394

Table 1  
The NLO and NNLO  $K$  factors Eq. (9), computed with center-of-mass energy  $s = 14 \text{ TeV}$ .

Let us now turn to the inclusive cross section. We define a  $K$  factor

$$K(\tau_h; y_t) \equiv \frac{\sigma_{gg}(\tau_h, y_t, m_H^2)}{\sigma_{gg}^0(\tau_h; y_t, m_H^2)}, \quad (9)$$

where  $\sigma_{gg}^0$  is the LO cross-section Eq. (1), computed with LO parton distributions and LO coupling constant. The value of the NLO and NNLO  $K$  factors, determined using the MRST2002 [11] gluon distribution in Eq. (1) are given in Table 1, at the LHC center-of-mass energy  $s = 14 \text{ TeV}$ . In the table the pointlike, exact and approximate cases are shown.

At NLO the discrepancy between the infinite top mass approximation and the exact result is tiny, less than 1% even for a fairly heavy Higgs. If the improved (approximate) NLO result is used, this discrepancy is reduced by a factor three. At NNLO the inclusion of the correct small  $\tau$  dependence of the partonic coefficient function changes the  $K$  factor by an amount which varies between 0.3% for  $m_H = 130 \text{ GeV}$  and 1% for  $m_H = 280 \text{ GeV}$ . If we modify the matching prescription by using the NLO value of  $\tau_0$  also at NNLO the approximate NNLO results of table 1 change by 0.1 %. We can take this as the error which is made by use of the infinite  $m_t$  NNLO formula (there is also a dependence on  $m_t$  in the contribution to  $C^{(2)}$  which is proportional to  $\delta(1-\tau)$ , but at NNLO this contribution is relatively small, unlike at NLO). Dominant uncertainties on the total Higgs cross-section are typically at the percent level [7]. Varying  $\omega$  in the matching function Eq. (7) between 1/20 and 1/100 the NNLO results change by about 0.1 %. We conclude that use of

the improved approximate NNLO reduces the error due to finite mass terms at NNLO to the per mille level. This may be relevant in view of recent progress on the computation of electroweak corrections to this process [12].

Finally, we observe that less inclusive quantities which depend on the  $\tau$  shape of the partonic cross section can be rather more sensitive to finite-mass effects, in particular if they probe the small  $\tau$  tail of a coefficient function. An interesting case in point, which deserves further investigation, is the Higgs rapidity distribution [13].

## REFERENCES

1. J. R. Ellis, M. K. Gaillard and D. V. Nanopoulos, Nucl. Phys. B **106** (1976) 292; M. A. Shifman, A. I. Vainshtein, M. B. Voloshin and V. I. Zakharov, Sov. J. Nucl. Phys. **30** (1979) 711.
2. M. Spira, A. Djouadi, D. Graudenz and P. M. Zerwas, Nucl. Phys. B **453** (1995) 17.
3. R. Bonciani, G. Degrossi and A. Vicini, JHEP **0711** (2007) 095.
4. A. Djouadi, M. Spira and P. M. Zerwas, Phys. Lett. B **264** (1991) 440.
5. S. Dawson, Nucl. Phys. B **359** (1991) 283.
6. C. Anastasiou, K. Melnikov, Nucl. Phys. B **646** (2002) 220; R. V. Harlander, W. B. Kilgore, Phys. Rev. Lett. **88** (2002) 201801; V. Ravindran, J. Smith, W. L. van Neerven, Nucl. Phys. B **665** (2003) 325.
7. R. Harlander, Acta Phys. Pol. B **38** (2007) 693.
8. S. Catani, M. Ciafaloni and F. Hautmann, Nucl. Phys. B **366** (1991) 135.
9. F. Hautmann, Phys. Lett. B **535** (2002) 159.
10. S. Marzani, R. D. Ball, V. Del Duca, S. Forte and A. Vicini, Nucl. Phys. B **800** (2008) 127 [arXiv:0801.2544 [hep-ph]].
11. A. D. Martin, R. G. Roberts, W. J. Stirling, R. S. Thorne, Eur. Phys. J. C **28** (2003) 455.
12. U. Aglietti, R. Bonciani, G. Degrossi and A. Vicini, Phys. Lett. B **595** (2004) 432; S. Actis, G. Passarino, C. Sturm and S. Uccirati, 0809.3667 [hep-ph].
13. C. Anastasiou, K. Melnikov and F. Petriello, Phys. Rev. Lett. **93** (2004) 262002.