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Reducing Estimation Risk Using a Bayesian Posterior Distribution Approach: Application to Stress Testing Mortgage Loan Default

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Abstract: We propose a new stress testing method to model coefficient uncertainty in addition to macroeconomic stress. Based on U.S. mortgage loan data, we model the probability of default at account level using discrete time hazard analysis. We employ both the frequentist and Bayesian methods in parameter estimation and default rate (DR) stress testing. By applying the Bayesian parameter posterior distribution, which includes all ranges of possible parameter estimates, obtained in the Bayesian approach to simulating the DR distribution, we reduce the estimation risk coming from employing point estimates in stress testing. Since estimation risk, a commonly neglected source of risk, is addressed in our method, we obtain more prudential forecasts of credit losses. We find that the simulated DR distribution obtained using the Bayesian approach with the parameter posterior distribution has a standard deviation 10.7 times as large as that using the frequentist approach with parameter mean estimates. Moreover, the 99% values at risk (VaR) using the Bayesian posterior distribution approach is around 6.5 times the VaR at the same probability level using the point estimate approach.

Keywords: OR in banking, Stress testing, Estimation risk, Bayesian posterior distribution approach, Probability of default

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1. Introduction

Stress testing is an important operational research tool to assess bank risk levels and to provide a basis to assist decision making by financial institutions and regulators (Breeden, 2016; Ju, Jeon, & Sohn, 2015; Schechtman & Gaglianone, 2012). Stress tests are designed to measure how sensitive risk exposures are to external or internal shocks to a financial system, an individual financial institution, a portfolio or an account (Misina, Tessier, & Dey, 2006). Based on stress testing results, regulators can assess if the financial system is stable enough to tolerate extreme but plausible economic conditions. Banks can decide how much capital they should keep to protect depositors in case such conditions occur. In practice, the financial sector assessment programmes (FSAPs) of the IMF and World Bank, as well as the financial authorities of various countries, regularly apply stress testing on financial institutions to assess the stability of the financial system (Schuermann, 2014; Sorge & Virolainen, 2006).

The focus of stress testing is largely on credit risk which is the most significant risk in banking systems (Sorge & Virolainen, 2006). The credit risk stress testing methods in the literature can be summarized as a three-step procedure (Borio, Drehmann, & Tsatsaronis, 2014; Kapinos, Martin, & Mitnik, 2018). First, empirical models exploring the relationship between an indicator of bank credit risk and macroeconomic variables are built and model coefficients estimated. Many papers (Kanas & Molyneux, 2018; Schechtman & Gaglianone, 2012; Vazquez, Tabak, & Souto, 2012) perform macro stress tests with aggregate data at the system level or with data of groups of financial institutions. Some (Bangia, Diebold, Kronimus, Schagen, & Schuermann, 2002; Bellotti & Crook, 2013, 2014; Breeden, 2016; Ju et al., 2015; Pesaran, Schuermann, Treutler, & Weiner, 2006) implement micro stress testing methods with granular data at an individual account, portfolio or institution level. Studies find that increases in output growth tend to reduce credit risk (Bikker & Hu, 2002; Laeven & Majnoni, 2003; Sorge & Virolainen, 2006), whereas rises in interest rate or unemployment tend to increase credit risk (Bellotti & Crook, 2013; Bikker & Hu, 2002; Pesaran et al., 2006). In the second step, stress scenarios for the macroeconomic variables are constructed on a historical (Sorge & Virolainen, 2006; Bellotti & Crook, 2013, 2014) or hypothetical (Jokivuolle & Viren, 2013; Tsukahara, Kimura, Sobreiro, & Zambrano, 2016) basis using distribution simulation methods (Bellotti & Crook, 2013, 2014; Kanas & Molyneux, 2018) or point prediction methods (Breeden, 2016; Busch, Koziol, & Mitrovic, 2018). The third step is to apply the stress scenarios of the macroeconomic variables to the empirical model to measure the extent of impact they have on the credit risk indicator of interest, such as the probability of default. Previous research has found that credit losses under stress scenarios with shocks from

macroeconomic variables are higher than those under baseline scenarios (Bellotti & Crook, 2013, 2014; Bikker & Hu, 2002; Jokivuolle & Viren, 2013; Laeven & Majnoni, 2003; Sorge & Virolainen, 2006). For example, Bellotti and Crook (2013) found that in the worst 1% economic scenarios, the default rate is 1.73 times the median using account level credit card data. Rösch and Scheule (2004) found the 99% Value at Risk to be between 1.50 and 8.32 times the mean for real estate loans based on aggregate data.

Estimation risk and model risk have been recognised in the financial risk management literature. For instance, Escanciano and Olmo (2010) take into account estimation risk when backtesting market risk models so that how appropriate and conservative those models are can be better assessed. Gourieroux and Zakoïan (2013) argue that using an estimate based on a sample to approximate a true parameter is asymptotically biased and would cause VaR underestimation. Therefore they propose to substitute an adjusted estimate to the true parameter which would result in larger estimates of VaR. Some papers address estimation risk and model risk in credit risk stress testing. Philippon, Pessarossi, and Camara (2017) provide the first assessment method for the stress tests in the European Union which can be seen as an effort to detect model risks in existing stress testing models. In general, they find no evidence of biases in scenario building and loss forecasting. Jacobs, Karagozoglu, and Sensenbrenner (2015) use a Bayesian approach to address estimation risk in stress testing in the sense that they use informative priors to include expert knowledge. However, the importance of estimation risk has not been fully addressed in the credit risk stress testing literature. Although parameter estimation risk is gaining increasing attention, none of the papers introduces this type of uncertainty as a source of stress scenario input into the stress testing procedure as they do the risk of macroeconomic shocks. The majority of the literature on credit risk stress testing uses the frequentist estimation approach in the first step of the three-step procedure to obtain model parameter estimates. Such fixed scalars are used in the simulation of the loss distribution. However, there are estimation errors inherent in coefficient estimates. In stress testing practice in the literature, this estimation risk is ignored since only the mean estimates are substituted into the DR simulations in stress testing. For papers that do consider estimation risk in their modelling (Escanciano & Olmo, 2010; Gourieroux & Zakoïan, 2013), this is still the case. Some papers use a Bayesian method for stress testing (Jacobs et al., 2015; Louzis, 2017; Petropoulos et al., 2018). However, their stress test approaches still employ Bayesian coefficient point estimates without addressing all of the range of possible coefficient estimates.

In practice, although banks seemed well capitalised based on regulatory capital requirements,

during the financial crisis many banks still had insufficient capital which led to their failure or near-failure (Schuermann, 2014). In response to the financial crisis, the U.S. government authorised $475 billion to purchase equity and toxic assets from banks to improve the solvency and liquidity of these banks in case of a possible total collapse of the U.S. financial system. For instance, around $200 billion was spent in purchasing preferred stock and equity warrant from hundreds of banks through the Capital Purchase Program. Purchasing illiquid mortgage-backed securities and assisting residential mortgage loan foreclosures cost more than $65 billion. Around $70 billion was spent in stock purchase of the American International Group, and $40 billion in that of Citigroup and Bank of America. In response to the financial crisis, the UK government announced a bank rescue package of £500 billion to restore confidence in and stabilise the British banking system. £50 billion was made available to recapitalise the banks through common and preferred stocks purchase. The government invested in the banks short of capital, making them partly nationalised. For instance, the Royal Bank of Scotland raised £20 billion capital, and Lloyds banking group £17 billion through the Bank Recapitalisation Fund.

Inadequate bank capital was attributed to many factors, such as insufficient minimum capital ratio requirements, the definition of capital being too wide, excessive leverage, procyclical amplification of financial shocks, insufficient liquidity requirements, etc. (BCBS, 2011). We consider another reason could be the neglect of uncertainty in the regulatory and bank internal risk models in use to assess the potential loss. For instance, the Basel accords require regulatory capital to cover credit risk, operational risk and market risk, etc., but did not include estimation risk which is the uncertainty of the coefficient estimates in the risk models. This type of risk may be present when modelling all types of risks that are required of the banks. We consider that incorporating the coefficient estimation risk in stress testing may increase perceived risk and provide more prudential predictions of losses and required capital, therefore helping financial institutions make better and safer capital planning decisions.

The contribution this paper makes to the operational research literature is to propose that by using a Bayesian approach and a Bayesian coefficient posterior distribution in stress testing, we take into account parameter uncertainty and reduce risk underestimation. That is, a more prudential amount of capital that a bank would need in order to maintain a given risk to protect depositors is estimated than with conventional methods. In Bayesian econometric theory, both parameters and explanatory variables are random variables instead of scalars and variables respectively, as in the frequentist approach. When doing Bayesian stress testing and simulating the estimated default rate distribution, we not only take random draws from the historical

scenarios of the macroeconomic variables but also simulate from the coefficient posterior to take into account other possible coefficient estimates in the coefficient posterior distribution. Therefore by employing Bayesian simulation of coefficients in stress testing, we model the uncertainty of coefficients thus reducing credit risk underestimation that arises from neglecting estimation risk. Moreover, since the number of draws taken from different regions of the posterior is proportionate to the posterior probability of these regions, when we include the less likely coefficient estimates from the posterior distribution, we also take their corresponding low probability into account. Therefore this stress testing method also has the benefit that it avoids risk overestimation.

This paper estimates and stress tests the probability of default at the micro/account level using a dataset of U.S. mortgage loans. The distribution approach is used to form the simulated default rate (DR) distribution and to obtain Value at Risk (VaR) at different percentiles. A discrete time hazard model is employed to analyze the relationship between default behaviour and macroeconomic as well as account level covariates, and to make forecasts for the probability of default. We use both the frequentist and Bayesian methods in estimation and stress testing. A Bayesian approach is used in order to simulate the posterior distribution of the model coefficients. We employ non-informative priors so that the differences between the simulated DR distributions using the frequentist and Bayesian approaches in the stress testing stage do not come from subjectivity introduced in the estimation stage. The coefficient posterior draws obtained in the Bayesian approach are subsequently applied to simulate the Bayesian estimated DR distribution. The Bayesian simulated DR distribution is then compared with the simulated DR distribution obtained using a frequentist approach with coefficient point estimates. In detail our method involves: 1) modelling the probability of default of mortgage loans and estimating the relationship between the probability of default and macro and micro predictors using both frequentist and Bayesian methods; 2) stress testing the impact of rare but plausible macro events as well as coefficient uncertainty on default rate by using historical scenarios of macroeconomic variables, the Bayesian coefficient posterior distribution, and simulated error terms, to simulate the Bayesian estimated DR distribution; 3) comparing the Bayesian DR distribution using a posterior distribution with the frequentist DR distribution using coefficient point estimates and computing the VaRs accordingly.

We find that the estimation and forecast results are similar using a frequentist method and a Bayesian method with non-informative priors. But in stress testing, the 95% and 99% VaRs of the simulated DR distribution obtained using a Bayesian approach with a coefficient posterior

distribution are 3.7 and 6.5 times as large as the VaRs at the same probability levels respectively using the frequentist approach with coefficient mean estimates. The expected monetary values of loss estimated based on the 99% VaRs show that neglecting coefficient uncertainty in stress testing may considerably underestimate credit losses and the capital needed by a bank.

The structure of this paper is as follows. Section 2 outlines the discrete time hazard model employed to estimate the probability of default and defines the estimation risk and a Bayesian approach that can be used to address it. Section 3 describes the stress testing models and procedures using the frequentist and Bayesian approaches. Section 4 describes the data and variables used in this research. Section 5 presents the estimation, prediction, performance, and stress testing results. Section 6 discusses the implications of this paper and ideas for future research. Section 7 provides a summary of the main findings and concludes.

2. Methodology

2.1 Discrete time hazard model, default rate and expected loss

Discrete time hazard model

To estimate probability of default, we use a discrete time hazard model, which is a logistic regression model using panel data (Belloti & Crook, 2013):

$$\log\mathrm{it}(p_{i,t}) = \alpha + \boldsymbol{z}_{t-3}'\boldsymbol{\beta}_1 + \boldsymbol{w}_i'\boldsymbol{\beta}_2 + \boldsymbol{u}_{i,t-3}'\boldsymbol{\beta}_3 + \boldsymbol{g}(\boldsymbol{t})'\boldsymbol{\beta}_4 \quad (1)$$

$p_{i,t}$ denotes the probability of default for account i at the duration time t; $\boldsymbol{z}_{t-3}$ denotes a vector of macroeconomic variables lagged three months; $\boldsymbol{w}_i$ denotes a vector of application variables for account i; $\boldsymbol{u}_{i,t-3}$ denotes a vector of behavioural variables lagged three months for account i; $\boldsymbol{g}(\boldsymbol{t})$ denotes functions of loan duration time.

We arrange the data such that the observations after the first default for any account are set to missing values. This ensures that the econometric model is parameterised using data up until the first default, which is what is required for the single event hazard distribution (See Singer &

Willett, 1993). Functions of duration are included as explanatory variables. In this way, our model is a discrete time survival model with the event of interest being loan default (Bellotti & Crook 2013). Using an appropriate estimation method, and with the accounts' defaults and covariates information in the logistic model, we can predict the probability of default for each account at duration time t , and how much impact each explanatory variable has on the logit. Maximum likelihood estimation is used in the frequentist approach. The random walk chain Metropolis-Hastings algorithm is used in the Bayesian approach. Non-informative priors are employed in the Bayesian method. For details, see Koop, Poirier, and Tobias (2007).

Survival probability

The survival probability to a duration time period can be computed from the probability of default at each time period. The predicted survival probability at duration time $t = q$ for account i is the product of the probability of account i not defaulting in each of the time periods until q :

$$\hat{S}_{i,q} = \prod_{t=1}^{q} (1 - p_{i,t}) \tag{2}$$

The cumulative probability of default is the complement of the survival function during each time period. It provides the probability of default at any time within the duration of q time periods:

$$\hat{H}_{i,q} = 1 - \hat{S}_{i,q} \tag{3}$$

The predicted and observed default rate at the aggregate level

Suppose t_c denotes calendar time. t_{a_i} denotes an account's opening time. $d_{i,t_c - t_{a_i}}$ denotes default of account i at calendar time t_c. $p_{i,t_c - t_{a_i}}$ denotes the probability of default for account i at calendar time t_c. n denotes the number of accounts. Then the default rate at calendar time t_c is computed as the ratio of the number of defaults at calendar time t_c divided by the total number of accounts at risk at that time:

$$R_{t_c} = \frac{1}{n} \sum_{i=1}^{n} d_{i,t_c - t_{a_i}} \tag{4}$$

The predicted probability of default at calendar time t_c at the aggregate level is:

$$P_{t_c} = \frac{1}{n}\sum_{i=1}^{n} p_{i,t_c - t_{a_i}} \tag{5}$$

The expected loss (EL) of an account is calculated as the product of the probability of default (PD), exposure at default (EAD) and loss given default (LGD):

$$EL = PD \times EAD \times LGD \tag{6}$$

2.2 Coefficient uncertainty and Bayesian stress testing

Frequentist methodology

In the frequentist approach, data is repeatable and random while parameters are fixed. The frequentist approach assumes the parameters from a population form a vector of scalars, $\boldsymbol{\theta}$. The estimator $\hat{\boldsymbol{\theta}}$ is a function of the data, which is a repeated sample from a population. As the sample size increases, an unbiased estimator converges to the true parameter $\hat{\boldsymbol{\theta}} \to \boldsymbol{\theta}$. Therefore the coefficient estimate, which is the expectation of the unbiased estimator, tends to the true parameter $E(\hat{\boldsymbol{\theta}}) = \boldsymbol{\theta}$. In reality, the sample size is finite; and a difference between the estimate and the true parameter exists. Estimation risk arises but is ignored in stress testing because coefficient estimation standard errors are ignored.

Bayesian methodology

In the Bayesian approach, the data, which is the observed sample, is fixed, while parameters are random. The Bayesian approach treats the parameters as random variables that have their own distributions since in the Bayesian approach anything uncertain can be expressed using probability (Koop et al., 2007). Suppose $\boldsymbol{y}$ and $\boldsymbol{\theta}$ are the data and a vector of parameters respectively. Based on Bayes' rule, the posterior distribution $f(\boldsymbol{\theta} \mid \boldsymbol{y})$ is proportional to the product of the prior distribution $f(\boldsymbol{\theta})$ and the likelihood distribution $f(\boldsymbol{y} \mid \boldsymbol{\theta})$:

$$f(\boldsymbol{\theta} \mid \boldsymbol{y}) = \frac{f(\boldsymbol{y} \mid \boldsymbol{\theta}) f(\boldsymbol{\theta})}{f(\boldsymbol{y})} \propto f(\boldsymbol{y} \mid \boldsymbol{\theta}) f(\boldsymbol{\theta})$$

To obtain coefficient estimates in the Bayesian approach, instead of random sampling from data as in the frequentist approach, the Bayesian method involves random sampling from the parameter posterior distribution. Each region within the posterior distribution has a probability. Therefore unlike the frequentist approach which assumes there is a true parameter with certainty, in the Bayesian approach no coefficient estimate is the right or wrong one. The more likely parameter regions have higher probabilities while the unlikely ones have lower probabilities.

Bayesian stress testing

In our Bayesian stress testing application that addresses the estimation risk of coefficient uncertainty, we use the Bayesian parameter variables $\boldsymbol{\theta}$ as opposed to the frequentist point estimates $E(\hat{\boldsymbol{\theta}})$. We randomly sample from the posterior distribution of the parameters, and apply these random draws to stress testing to avoid the estimation risk that arises from omitting the differences between the draws, as well as the different probabilities of different regions in the posterior distribution. In other words, with our Bayesian approach in the stress testing application, the simulated value of a dependent variable $\boldsymbol{y}_j$ for each observation j using the kth draw is:

$y_{j,k} = \boldsymbol{x}_j' \boldsymbol{\theta}_k + \varepsilon_{j,k}$, in which $k = 1, 2, ..., K$. (7)

In contrast, with the frequentist stress testing method using coefficient mean estimates it is:

$$y_j = \boldsymbol{x}_j' E(\hat{\boldsymbol{\theta}}) + \varepsilon_j \quad (8)$$

To compare with frequentist stress testing using coefficient mean estimates, we also carry out Bayesian stress testing using Bayesian coefficient posterior mean estimates:

$$y_j = \boldsymbol{x}_j' E(\boldsymbol{\theta} \mid \boldsymbol{y}) + \varepsilon_j \quad (9)$$

3. Stress testing

3.1 Stress testing model

Our stress testing method is a combination of posterior simulation of coefficients, simple random sampling of historical macroeconomic scenarios, and Monte Carlo simulation of the error terms. Consider the latent variable interpretation of the logistic regression over the time period that stress testing is applied to:

$$d^*_{j,i,t_s} = I(y^*_{j,i,t_s} = \boldsymbol{x}'_{j,i,t_s}\boldsymbol{\beta} + \varepsilon_{j,i,t_s} > 0) \tag{10}$$

in which

i denotes the ith account. $i = 1, 2, ..., n$.

j denotes the jth macroeconomic scenario. $j = 1, 2, ..., l$.

$\boldsymbol{x}_{j,i,t_s}$ denotes a vector of covariates including macroeconomic covariates that take their historical values, account application variables $\boldsymbol{w}_i$, account behavioural variables $\boldsymbol{u}_{i,t_s-t_{a_i}-3}$ and duration functions $\boldsymbol{g}(t_s - t_{a_i})$, in which t_s is the calender time stress testing is applied to, and t_{a_i} is the calendar time of the opening of account i.

y^*_{j,i,t_s} denotes the simulated value of the latent variable in the logistic regression for account i in scenario j at calendar time t_s.

d^*_{j,i,t_s} denotes the simulated default behavior for account i in scenario j at calendar time t_s.

d^*_{j,i,t_s} takes the value 1 when an event occurs and 0 when it does not occur:

$$d^*_{j,i,t_s} = \begin{cases} 1 & y^*_{j,i,t_s} > 0 \\ 0 & else \end{cases}.$$

Application variables $\boldsymbol{w}_i$, account behavioural variables $\boldsymbol{u}_{i,t_s-t_{a_i}-3}$ and duration functions $\boldsymbol{g}(t_s - t_{a_i})$ can all be considered account specific variables. Suppose we represent the vector

$\boldsymbol{x}_{j,i,t_s}$ as follows. $\boldsymbol{x}_{j,i,t_s} = \begin{pmatrix} \boldsymbol{m}_j \\ \boldsymbol{a}_{i,t_s} \end{pmatrix}$, in which $\boldsymbol{m}_j$ denotes the values of macroeconomic variables for scenario j. $\boldsymbol{m}_j$ represents the observed values of the macroeconomic variables in a random month in history before t_s. $\boldsymbol{a}_{i,t_s}$ denotes the values of the account-specific variables for account i at the time period stress testing is applied to. $\boldsymbol{a}_{i,t_s}$ includes account application variables $\boldsymbol{w}_i$, account behavioural variables $\boldsymbol{u}_{i,t_s-t_{a_i}-3}$ and duration functions $\boldsymbol{g}(t_s - t_{a_i})$ at calendar time t_s. $\boldsymbol{\beta}^{(m)}$ denotes a $v_1 \times 1$ column vector of parameter mean estimates for the constant and the macroeconomic variables using frequentist estimation. In this paper, $v_1 = 10$. $\boldsymbol{\beta}^{(a)}$ denotes a $v_2 \times 1$ column vector of coefficient mean estimates for account specific variables using frequentist estimation. In this paper, $v_2 = 7$. $\boldsymbol{b}^{(m)}$ is a $v_1 \times 1$ column vector of Bayesian posterior mean estimates for the constant and the coefficients for the macroeconomic variables. $\boldsymbol{b}^{(a)}$ is a $v_2 \times 1$ column vector of Bayesian posterior mean estimates for the coefficients for the account-specific variables. k denotes the kth draw from the K number of random draws from the posterior distribution. $\boldsymbol{b}_k^{(m)}$ denotes a $v_1 \times 1$ column vector of the kth draw from the Bayesian posterior distribution for the constant and the coefficients for the macroeconomic variables. Then Eq. (10) is further written as follows.

(1) Stress testing using the frequentist and Bayesian coefficient mean estimates

Frequentist:

$$d^*_{j,i,t_s} = I(y^*_{j,i,t_s} = \boldsymbol{m}'_j \boldsymbol{\beta}^{(m)} + \boldsymbol{a}'_{i,t_s} \boldsymbol{\beta}^{(a)} + \varepsilon_{j,i,t_s} > 0) \tag{11}$$

Bayesian:

$$d^*_{j,i,t_s} = I(y^*_{j,i,t_s} = \boldsymbol{m}'_j \boldsymbol{b}^{(m)} + \boldsymbol{a}'_{i,t_s} \boldsymbol{b}^{(a)} + \varepsilon_{j,i,t_s} > 0) \tag{12}$$

(2) Bayesian stress testing using the Bayesian coefficient posterior distribution:

$$d^*_{j,k,i,t_s} = I(y^*_{j,k,i,t_s} = \boldsymbol{m}'_j \boldsymbol{b}_k^{(m)} + \boldsymbol{a}'_{i,t_s} \boldsymbol{b}^{(a)} + \varepsilon_{j,k,i,t_s} > 0), \quad k = 1,2,...,K \tag{13}$$

The simulated default rates using the mean estimates and posterior distribution approaches:

1) The simulated default rate at t_s in scenario j using the frequentist and Bayesian coefficient mean estimates is:

$$\hat{R}_{j,t_s} = \frac{1}{n}\sum_{i=1}^{n} d^*_{j,i,t_s} \tag{14}$$

There are l scenarios in total.

2) In the Bayesian posterior distribution approach, the simulated default rate at t_s in scenario j,k, which is in the jth macroeconomic scenario and using the kth Bayesian coefficient draw, is:

$$\hat{R}_{j,k,t_s} = \frac{1}{n}\sum_{i=1}^{n} d^*_{j,k,i,t_s} \tag{15}$$

There are $l * K$ scenarios in total.

3.2 Stress testing procedure

The stress testing procedure we propose is as follows:

1. Estimate the discrete hazard model Eq. (1) based on the training sample using the frequentist and Bayesian approaches and obtain the frequentist and Bayesian coefficient mean estimates as well as the Bayesian posterior draws.
2. Choose a time period in the test sample for stress testing to apply to, which in our case is the end of the test sample time period October 2017. Simulate the default/non-default events of all the accounts alive during the stress testing period in each scenario in both frequentist and Bayesian frameworks by sampling from historical macroeconomic scenarios, posterior distribution (in the Bayesian posterior distribution approach) and error terms. The simulated default event of account i in scenario j for the point estimate approach is based on Eq. (11) – Eq. (12). The

simulated default event of account i in scenario j,k for the Bayesian posterior distribution approach is based on Eq. (13). Next, calculate the simulated default rates across the accounts in each scenario. The simulated default rate in scenario j using the coefficient mean estimate approach is calculated based on Eq. (14). The simulated default rate in scenario j,k using the coefficient distribution approach is calculated based on Eq. (15).

3. Build the frequentist and Bayesian distributions of simulated default rates and compute the VaRs at different probability levels.

The panel data of the macroeconomic variables is arranged as a matrix, $\boldsymbol{M}_{v_1\times T}$, of v_1 macroeconomic variables and T time periods[1]. In order to keep the dependence structure between the macroeconomic variables, we draw the historical values of all the macroeconomic variables simultaneously as opposed to sampling historical values of each variable individually. To give more details, we draw l simple random samples[2] with replacement of the columns[3] of $\boldsymbol{M}_{v_1\times T}$. Each draw represents a macroeconomic scenario $\boldsymbol{m}_j$. All the l scenarios form a matrix of macro scenarios $\boldsymbol{M}_{v_1\times l}$. The values of the v_2 number of account level variables for the n accounts alive at stress testing time t_s is $\boldsymbol{A}_{v_2\times n}$.

In the Bayesian posterior distribution approach, we take K number of draws for the constant and macroeconomic coefficients from the posterior distribution, thus forming a matrix of v_1 number of parameters and each having K draws: $\boldsymbol{B}^{(m)}_{v_1\times K}$. For coefficients for the v_2 number of account level variables, we use their posterior mean estimates: $\boldsymbol{b}^{(a)}_{v_2\times 1}$.

Since there are three components, which are the *macroeconomic component* $\boldsymbol{m}_j'\boldsymbol{\beta}^{(m)}$ (or $\boldsymbol{m}_j'\boldsymbol{b}^{(m)}$, $\boldsymbol{m}_j'\boldsymbol{b}_k^{(m)}$), the *account level component* $\boldsymbol{a}_{i,t_s}'\boldsymbol{\beta}^{(a)}$ (or $\boldsymbol{a}_{i,t_s}'\boldsymbol{b}^{(a)}$), and the *error term component* ε_{j,i,t_s} (or ε_{j,k,i,t_s}) in the latent variable y^*_{j,i,t_s} (or y^*_{j,k,i,t_s}) for each account in each

[1] T equals the number of months in our macroeconomic data before t_s. The first row of $\boldsymbol{M}_{v_1\times T}$ is a vector of $\mathbf{1}$.

[2] l can be larger or smaller or equal to T, and in our case is larger.

[3] Each column represents the values of the macroeconomic variables in a same month.

scenario in Eqs. (11) – (13), to build the simulated default behaviours d^*_{j,i,t_s} and d^*_{j,k,i,t_s} in Eqs. (11) – (13), we first build the three components in y^*_{j,i,t_s} and y^*_{j,k,i,t_s} for all accounts in all scenarios. In the rest of this section, we use the Bayesian posterior distribution approach as an example to illustrate our stress testing procedure. The details for the frequentist point estimate approach are given in Appendix A.

Stress testing procedure for the Bayesian posterior distribution approach:

The *macroeconomic component* is: $\mathbf{M}_{l\times K} = (\boldsymbol{M}_{v_1\times l})' \boldsymbol{B}^{(m)}_{v_1\times K}$ with each scalar in $\mathbf{M}_{l\times K}$ being $\mathrm{m}_{j,k}$

Convert the *macroeconomic component matrix* $\mathbf{M}_{l\times K}$ into a *macroeconomic component vector* $\mathbf{M}_{lK\times 1}$ with each scalar being $\mathrm{m}_{j,k}$: $\mathbf{M}_{lK\times 1} = \begin{pmatrix} \mathrm{m}_{1,1} \\ \vdots \\ \mathrm{m}_{j,k} \\ \vdots \\ \mathrm{m}_{l,K} \end{pmatrix}_{(lK)\times 1}$. In Bayesian stress testing, the coefficient draws also contribute to scenario building. Therefore instead of l scenarios, we have $l\times K$ scenarios in total using the Bayesian posterior distribution method.

The *account level component* is: $\mathbf{A}_{n\times 1} = (\boldsymbol{A}_{v_2\times n})' \boldsymbol{b}^{(a)}_{v_2\times 1}$ with each scalar in $\mathbf{A}_{n\times 1}$ being a_i

The *error term component* is: take $l\times K\times n$ draws from a standard logistic distribution: $\boldsymbol{\varepsilon}_{(lKn)\times 1}$

Repeat ***each scalar*** of the macroeconomic component vector $\mathbf{M}_{lK\times 1}$ by n times, and we obtain:

$$\mathbf{M}_{lKn\times 1}=\begin{pmatrix}\mathrm{m}_{1,1,1}\\ \vdots\\ \mathrm{m}_{1,1,n}\\ \vdots\\ \mathrm{m}_{j,k,1}\\ \vdots\\ \mathrm{m}_{j,k,n}\\ \vdots\\ \mathrm{m}_{l,K,1}\\ \vdots\\ \mathrm{m}_{l,K,n}\end{pmatrix}_{(lKn)\times 1}$$, in which $\mathrm{m}_{j,k,1}=\cdots=\mathrm{m}_{j,k,n}=\mathrm{m}_{j,k}$.

Repeat the account level component $\mathbf{A}_{n\times 1}$ ***as a whole*** lK times, and we obtain:

$$\mathbf{A}_{lKn\times 1}=\begin{pmatrix}\mathrm{a}_{1,1,1}\\ \vdots\\ \mathrm{a}_{n,1,1}\\ \vdots\\ \mathrm{a}_{1,j,k}\\ \vdots\\ \mathrm{a}_{n,j,k}\\ \vdots\\ \mathrm{a}_{1,l,K}\\ \vdots\\ \mathrm{a}_{n,l,K}\end{pmatrix}_{(lKn)\times 1}$$, in which $\begin{pmatrix}\mathrm{a}_{1,1,1}\\ \vdots\\ \mathrm{a}_{n,1,1}\end{pmatrix}_{n\times 1}=\ldots=\begin{pmatrix}\mathrm{a}_{1,j,k}\\ \vdots\\ \mathrm{a}_{n,j,k}\end{pmatrix}_{n\times 1}=\ldots=\begin{pmatrix}\mathrm{a}_{1,l,K}\\ \vdots\\ \mathrm{a}_{n,l,K}\end{pmatrix}_{n\times 1}=\mathbf{A}_{n\times 1}$.

Then add $\mathbf{M}_{lKn\times 1}$, $\mathbf{A}_{lKn\times 1}$, and $\boldsymbol{\varepsilon}_{(lKn)\times 1}$ together. The intuition is that all the n accounts in the stress testing time period t_s face lK number of potential parallel scenarios, and each one of the lK scenarios should have all the n accounts. Specifically, consider n accounts, l macroeconomic scenarios and K draws from the posterior distribution. The full results of the right-hand side of the latent logistic function for all the accounts in all scenarios are:

$$
\begin{matrix} (e) \end{matrix} \qquad \begin{matrix} (f) \end{matrix} \qquad \begin{matrix} (g) \end{matrix} \qquad \begin{matrix} (h) \end{matrix}
$$

$$
\begin{pmatrix} y^*_{1,1,1,t_s} \\ \vdots \\ y^*_{1,1,n,t_s} \\ \vdots \\ y^*_{j,k,1,t_s} \\ \vdots \\ y^*_{j,k,n,t_s} \\ \vdots \\ y^*_{l,K,1,t_s} \\ \vdots \\ y^*_{l,K,n,t_s} \end{pmatrix}_{(lKn)\times 1} = \begin{pmatrix} \mathrm{m}_{1,1,1} \\ \vdots \\ \mathrm{m}_{1,1,n} \\ \vdots \\ \mathrm{m}_{j,k,1} \\ \vdots \\ \mathrm{m}_{j,k,n} \\ \vdots \\ \mathrm{m}_{l,K,1} \\ \vdots \\ \mathrm{m}_{l,K,n} \end{pmatrix}_{(lKn)\times 1} + \begin{pmatrix} \mathrm{a}_{1,1,1} \\ \vdots \\ \mathrm{a}_{n,1,1} \\ \vdots \\ \mathrm{a}_{1,j,k} \\ \vdots \\ \mathrm{a}_{n,j,k} \\ \vdots \\ \mathrm{a}_{1,l,K} \\ \vdots \\ \mathrm{a}_{n,l,K} \end{pmatrix}_{(lKn)\times 1} + \begin{pmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_n \\ \vdots \\ \varepsilon_{(jk-1)n+1} \\ \vdots \\ \varepsilon_{jkn} \\ \vdots \\ \varepsilon_{(lK-1)n+1} \\ \vdots \\ \varepsilon_{lKn} \end{pmatrix}_{(lKn)\times 1} \tag{16}
$$

Each scalar in vector (e) in Eq. (16) represents the simulated value of the latent variable for an account in a scenario. For instance, y^*_{j,k,n,t_s} represents the simulated latent variable for account n in macroeconomic scenario j using the k th draw from the Bayesian posterior at stress testing time t_s. Divide vector (e) equally into lK sections, one for each scenario, with each section having n scalars. Rearrange vector (e) into a new matrix with the number of rows being the number of accounts n, and the number of columns being the number of scenarios lK. That is, put the first n scalars from vector (e) into the first column of the new matrix, the next n scalars into the second column, the jk th n scalars into the jk th column, so on until the last n scalars from vector (e) are put into the last column of the new matrix. In this way, each column of the new matrix has the simulated values of the latent variable for all the accounts in the same scenario, and all columns represent all the lK scenarios:

$$
\begin{pmatrix} y^*_{1,1,1,t_s} & \cdots & y^*_{j,k,1,t_s} & \cdots & y^*_{l,K,1,t_s} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ y^*_{1,1,n,t_s} & \cdots & y^*_{j,k,n,t_s} & \cdots & y^*_{l,K,n,t_s} \end{pmatrix} \tag{17}
$$

Based on Eq. (13), compare the simulated scalars in Eq. (17) with 0 and decide whether each of the n accounts is predicted to default or not to default in each scenario:

$$\begin{pmatrix} d^*_{1,1,1,t_s} & \cdots & d^*_{j,k,1,t_s} & \cdots & d^*_{l,K,1,t_s} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ d^*_{1,1,n,t_s} & \cdots & d^*_{j,k,n,t_s} & \cdots & d^*_{l,K,n,t_s} \end{pmatrix} \tag{18}$$

Subsequently, based on Eq. (15), the simulated default rate in each of the lK scenarios can be computed by averaging over the simulated default behaviours of all the n accounts in each column in Eq. (18), and obtain the simulated default rates in all lK scenarios: $\left(\hat{R}_{1,1,t_s} \quad \cdots \quad \hat{R}_{j,k,t_s} \quad \cdots \quad \hat{R}_{l,K,t_s}\right)$.

Once we have the simulated default rates $\hat{R}_{j,t_s}$ and $\hat{R}_{j,k,t_s}$ in all the l and lK scenarios using the frequentist and Bayesian approaches, we use these simulated default rates to form the empirical simulated default rate distributions and obtain the VaRs.

3. Data and Variables

4.1 Data

The data we use to illustrate our methods are from the Freddie Mac single-family loan level dataset[4]. The loans are fully amortizing long term fixed rate mortgages. We use the mortgage accounts that originated during the 12 months in 2014 as a training sample. We use accounts originated during the 12 months in 2015 as a test sample. For the training sample, we take December 2016 as the observation date. For the test sample, we take October 2017 as the observation date. For each year we use a random sample of 50000 loans. We consider an account is in default if it has in its payment history record no less than 60 days delinquency. Table 1 shows the number of accounts and defaults in each sample cross-sectionally.

Table 1

Training and test samples

	Train (2014)	Test (2015)
Number of accounts	50000	50000
Defaults of accounts	415	295

[4] Dataset url: http://www.freddiemac.com/research/datasets/sf_loanlevel_dataset.page

4.2 Variables

For our discrete time hazard model, the event of interest is default with the event indicator being 1 (default) and the non-event being 0 (non-default).

Table 2 gives a full list of the explanatory variables for this research. We include macroeconomic variables and application as well as behavioural variables of the accounts. To avoid trends, the macroeconomic variables are first differenced. To enable prediction and to avoid endogeneity, macroeconomic variables and behavioural variables are lagged 3 months.

Table 2

Full list of explanatory variables

Group	Variable name	Definition
Macroeconomic	d_l_tbill_3m	Three months treasury bill interest rate
	d_l_unemployment_rate	Unemployment rate
	d_l_CPI	Consumer price index
	d_l_consumer_confidence	Consumer confidence
	d_l_retail_sales	Log of retail sales
	d_l_personal_earnings	Log of personal earnings
	d_l_IPI	Industrial production index
	d_l_dowjones_index	Dow Jones stock price index
	d_l_CS_houseprice_index	House price index
Application	original_debt_to_income_ratio	The sum of monthly debt/sum of monthly income calculated at loan origination
	original_loan_to_value	Original loan amount / appraised loan value or purchase price
Behavioural	l_current_actual_upb	Log of the current unpaid balance of the mortgage
	l_current_interest_rate	Current interest rate
	l_remaining_months	The remaining months from the loan term in the mortgage note
Duration	loan_age	The duration of the loan since its origination
	loan_age_sq	The squared term of loan age

Source: Freddie Mac database for account specific variables, Datastream for macroeconomic variables

5. Results

5.1 Estimation results for the discrete time hazard model

We estimate models on the training sample using a frequentist approach and a Bayesian approach. Table 3 illustrates the estimation results.

Table 3

Estimation Results using the frequentist approach and the Bayesian approach with non-informative priors

Variables (Macro and behavioural variables are lagged 3 months)	Training sample: accounts originated during 2014 with December 2016 as the observation date	
	Frequentist Estimate (std.error)	Bayesian Posterior mean (std.dev)
Intercept	-14.7275682 ***	-14.758733
	(1.2980562)	(1.2752167)
d_l_tbill_3m	0.2126658	0.195955
	(0.9666853)	(0.9538694)
d_l_unemployment_rate	-0.2944642	-0.319338
	(0.4911965)	(0.5026099)
d_l_CPI	-0.0446704	-0.047746
	(0.1540015)	(0.1548964)
d_l_consumer_confidence	0.0079945	0.008596
	(0.0144115)	(0.0145325)
d_l_retail_sales	23.0846659 *	23.398211
	(13.8015895)	(14.0626727)
d_l_personal_earnings	24.9354680	24.469273
	(38.1177126)	(37.9106425)
d_l_IPI	0.1759931	0.182078
	(0.1345957)	(0.1334646)
d_l_dowjones_index	1.7978390	1.888173
	(2.0682810)	(2.0835078)
d_l_CS_houseprice_index	0.6831190 ***	0.679918
	(0.2085248)	(0.2080482)
original_debt_to_income_ratio	0.0346552 ***	0.035027
	(0.0061243)	(0.0060389)
original_loan_to_value	0.0044147	0.004576
	(0.0034320)	(0.0034240)
l_current_actual_upb	-0.1502279 **	-0.151780
	(0.0876762)	(0.0859478)
l_current_interest_rate	1.5111287 ***	1.505791
	(0.1570621)	(0.1572061)
l_remaining_months	-0.0031977 ***	-0.003141
	(0.0011235)	(0.0011293)
loan_age	0.0837527 **	0.085288
	(0.0346547)	(0.0347888)
loan_age_sq	-0.0017150 **	-0.001753
	(0.0009199)	(0.0009265)
	Log likelihood = -3579.424 Prob > chi2 = 0.0000	Number of draws in MCMC = 100000 Burn-in = 200000

The estimation results of the frequentist and Bayesian noninformative methods are very similar since both are based on information contained in the data entirely. In the frequentist approach,

the effects of accounts' individual interest rate and debt to income ratio are significantly positive which is consistent with the expectation that the higher the interest rate and the amount of debt compared to borrowers' income the more likely a borrower is to default. Among the macroeconomic covariates, the house price index and the retail sales have a significantly positive impact on default probability also as expected. Loan duration and its squared term have positive and negative signs respectively showing that the default probability is nonlinear over time. The ratios of posterior means to posterior standard deviations show the variables that have an important impact on default rates in the Bayesian approach are similar to those in the frequentist approach. The coefficient signs of these variables in the Bayesian framework are the same as those in the frequentist framework. Based on the Bayesian coefficients convergence diagnostics, the Markov chain converges successfully, and the Bayesian estimation is reliable.

5.2 Prediction results using frequentist and Bayesian methods

The predicted and observed default rates are calculated based on Eqs. (4) and (5). Fig. 1. and Fig. 2. show that the default rates in the training and test samples are well predicted using both the frequentist and Bayesian methods. The default rate predictions follow the trend and fluctuation of the observed default rate along the time periods.

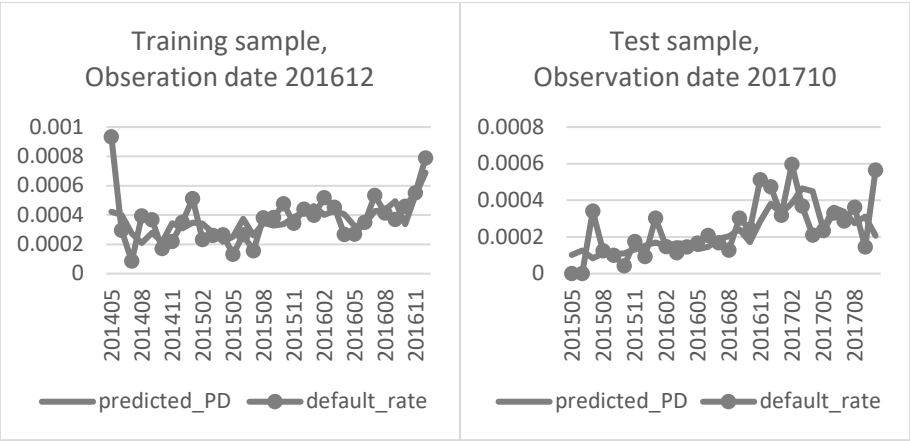

Fig. 1. Predicted and observed DR in the training and test samples using a frequentist approach

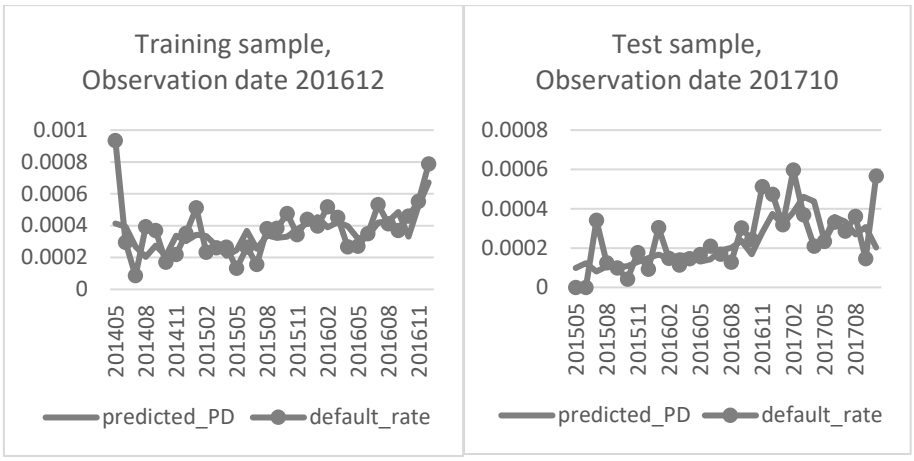

Fig. 2. Predicted and observed DR in the training and test samples using a Bayesian approach

Table 4 presents the mean absolute difference between the predicted and observed default rates in the training and test samples using the frequentist and Bayesian approaches. The default rate predictions are close to the observed default rates on average with the mean absolute difference in the two samples using the two approaches being approximately 0.000095.

Table 4

Mean absolute difference between the estimated and observed default rates

Measure	Sample	Frequentist	Bayesian
Mean absolute difference	Train	0.0000984	0.0000993
	Test	0.0000928	0.0000923

5.3 Performance results using the frequentist and Bayesian methods

Table 5 shows the performance results on the training and test samples using the frequentist and Bayesian methods in the duration of the first 12 months since each account's opening based on Eq. (3).

Table 5

Performance results in the duration of the first 12 months

		Performance measures				
Approach	Sample	H	GINI	AUC	AUCH	K-S
Frequentist	Train	0.1122979	0.3965329	0.6982665	0.7068052	0.3230177
	Test	0.1047113	0.3407828	0.6703914	0.6856272	0.2827534
Bayesian	Train	0.1119204	0.3965452	0.6982726	0.7067148	0.3222444
	Test	0.1042898	0.3405327	0.6702664	0.6852344	0.2839313

The AUC and AUCH measures based on the training sample are around 70% whereas the AUC and AUCH measures based on the test sample are around 68%. The GINI coefficients based on the training sample are around 40% whereas those based on the test sample are about 34%. The H measure results based on the training sample is about 11% whereas that on the test sample is a little above 10%. The performance results show good predictive accuracy of the models.

5.4 Stress testing results using the frequentist and Bayesian methods

We have carried out stress testing in 3 ways using frequentist coefficient mean estimates, Bayesian coefficient mean estimates, and the Bayesian coefficient posterior distribution. We ensure that there are equal numbers of scenarios using all three methods. In the frequentist and Bayesian stress tests using coefficient mean estimates, we take 22500 random draws with replacement from past economic scenarios between Jan 1999 to Sept 2017. In the Bayesian stress test, using the posterior distribution and taking both macroeconomic risk and estimation risk into consideration, for computational efficiency, we take each economic scenario between Jan 1999 to Sept 2017 once (225 observations altogether). The values of the macroeconomic variables in each time period are drawn simultaneously.

In the Bayesian stress test using the posterior distribution approach, we take 100 random draws from the posterior distribution. That is, each of the 225 vectors of macroeconomic values is combined with 100 draws from the posterior distribution. Each draw forming the Bayesian posterior distribution includes all the coefficients. We take each draw of the coefficients simultaneously as opposed to sampling from the marginal posterior distribution of each coefficient individually. For the coefficients for the macroeconomic variables and the constant, values in the posterior random draws are used. For the coefficients for the account specific variables, Bayesian coefficient mean estimates are used.

Stress testing is performed on the test sample. For computational efficiency, we take a random sample of 50% of the accounts in the test sample. We then use all the accounts in this sample that live to the time period upon which stress testing is performed. The time period that stress testing is performed upon is October 2017 which is the observation date of the test sample.

To avoid sampling bias, we apply bootstrapping for stress testing computations. For each one of three approaches (i.e. the frequentist mean estimate approach, the Bayesian mean estimate approach, and the Bayesian posterior distribution approach), the stress testing procedure is

repeated 100 times, each with random simulations for the samplings of macroeconomic scenarios, the Bayesian coefficient posterior distribution (when using the posterior distribution approach) and the error terms. We then collect all the estimated default rates obtained in the 100 computations to build the empirical simulated default rate distribution for each of the three stress testing methods.

5.4.1 Stress testing results

Table 6 presents a comparison of different VaRs, means and standard deviations of the simulated default rate distributions using the frequentist and Bayesian approaches as well as the observed default rate in October 2017.

Table 6

Statistics of the simulated default rate distributions using frequentist mean estimates, Bayesian mean estimates, and random samples from the Bayesian posterior distribution and the observed default rate in the test sample in October 2017

Approaches / Statistics of DR distributions	Frequentist mean estimates	Bayesian mean estimates	Bayesian posterior distribution
Mean of simulated DR distribution	0.000319	0.000314	0.000784
St.d of simulated DR distribution	0.000270	0.000267	0.002887
95% VaR of simulated DR distribution	0.000782	0.000782	0.002884
99% VaR of simulated DR distribution	0.001320	0.001271	0.008554
Observed DR in October 2017 in the test sample		0.000565	

Using the frequentist mean estimates approach, the mean of the simulated DR distribution is lower than the observed default rate. The standard deviation of the distribution is approximately 0.00027. The 95% and 99% VaRs of the DR distribution are about 0.0008 and 0.0013 respectively. Both are larger than the observed default rate. In other words, the stress testing succeeds in yielding VaRs above the observed default rate if we use the frequentist approach without considering estimation risk. The statistics for the simulated DR distribution using Bayesian mean estimates give the same conclusions since firstly both estimation methods rely fully on information contained in the data and secondly, both stress testing methods only consider macroeconomic scenarios without considering estimation risk.

For the simulated DR distribution using the Bayesian approach with random draws from the Bayesian posterior, the distribution mean is about 0.00078, much larger than the simulated DR

distribution mean using the frequentist or Bayesian coefficient mean estimates. The 95% and 99% VaRs are about 0.0029 and 0.0086 respectively. The standard deviation of the distribution is approximately 0.0029, much higher than that of the simulated DR distribution using the frequentist or Bayesian coefficient mean estimates. In this approach, the mean, 95% and 99% VaRs of the simulated DR distribution all successfully exceed the observed default rate when we use random draws from the Bayesian posterior distribution with both macroeconomic scenarios and the estimation risk taken into account. Notice, the 99% VaR obtained using this stress testing method is very close to the observed default rates during the 07/08 financial crisis. For instance, the observed default rate for accounts originated in 2007 with December 2009 as the observation date is 0.008485 at the observation time, based on a random sample of 50000 accounts from the same database.

In summary, the observed default rate in the stress testing period is within the 95% and 99% VaRs both using coefficient mean estimates methods and the Bayesian posterior distribution method. The stress testing results show that statistics such as the VaRs and the standard deviation of the simulated DR distribution increase as estimation risk is introduced.

We propose the following way to measure the relative sizes of macroeconomic stress and estimation risk. The distribution mean of the frequentist simulated DR distribution using coefficient mean estimates represents the expected default rate under normal macroeconomic circumstances with neither macroeconomic stress nor estimation risk considered. We use this value as a benchmark to measure macroeconomic stress and estimation risk. The 99% VaR of the frequentist simulated DR distribution represents the simulated default rate in stressed macroeconomic conditions but without considering coefficient uncertainty. The 99% VaR of the Bayesian simulated DR distribution using the coefficient posterior distribution approach is the simulated default rate both in stressed macroeconomic circumstances and with coefficient uncertainty addressed. Therefore the unexpected loss that comes from macroeconomic stress can be quantified in the traditional way by comparing the distribution mean (0.0003) and the 99% VaR (0.0013) of the frequentist DR distribution. Furthermore, a combination of the stress from macroeconomic stress and the coefficient uncertainty can be quantified by comparing the mean (0.0003) of the frequentist simulated DR distribution, which is the expected default rate in tranquil economic circumstances and without estimation risk, and the 99% VaR (0.0086) of the Bayesian simulated DR distribution that uses the coefficient posterior distribution approach, which both addresses macroeconomic stress and estimation risk. In other words, the size of macroeconomic stress is approximately 0.001 (=0.001320-0.000319) measured in simulated

default rate. The size of a combination of macroeconomic stress and the coefficient uncertainty is around 0.0082 (=0.008554-0.000319) measured in simulated default rate. Therefore, estimation risk contributes much higher than macroeconomic stress to the simulated default rate.

5.4.2 Stress testing results comparison

Fig. 3. compares the simulated DR distributions between using the frequentist and Bayesian point estimate approaches. The simulated DR distributions using frequentist and Bayesian coefficient mean estimates are almost identical since the coefficient estimates are very similar between the frequentist and non-informative Bayesian approaches.

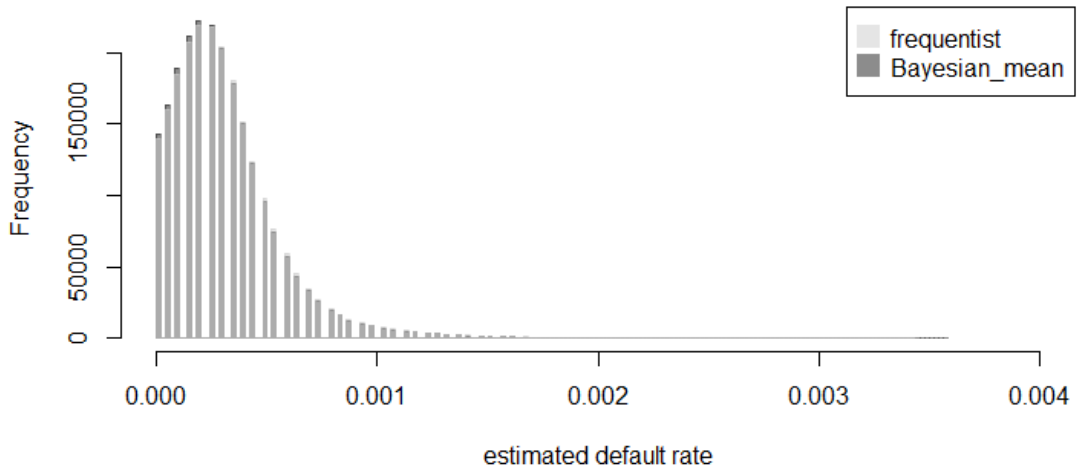

Fig. 3. Histograms of the simulated default rate distributions using frequentist and Bayesian coefficient mean estimates

Fig. 4. compares the simulated DR distributions between using the frequentist point estimate approach and the Bayesian posterior distribution approach. Fig. 5. Shows the tails of the two distributions. It can be seen from the two figures that the simulated DR distribution has a fatter and longer tail when using the Bayesian posterior distribution approach compared to when using the mean estimate approach. At midrange default rates, the simulated DR distribution has higher frequencies using the mean estimate approach than when using the Bayesian distribution approach. At other default rates, the reverse is true.

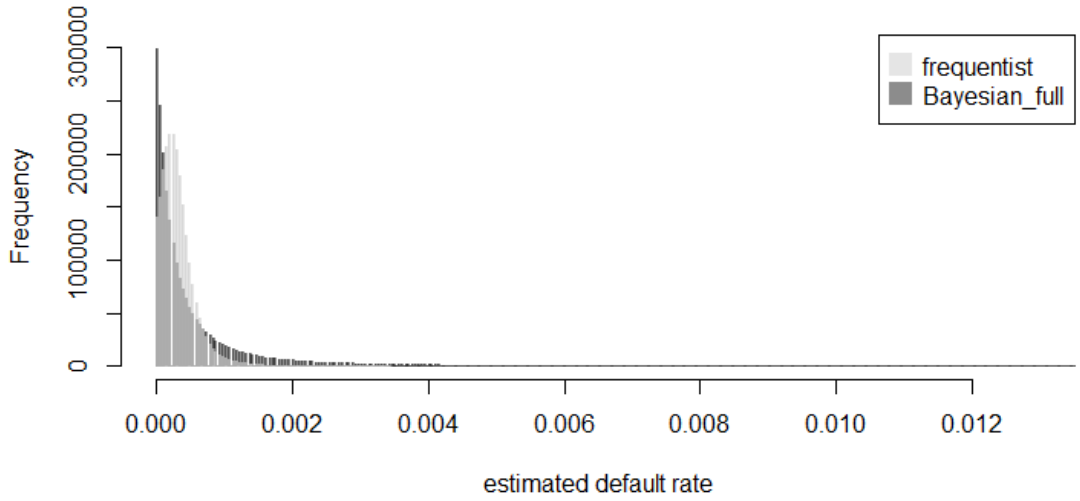

Fig. 4. Histograms of simulated default rate distributions using frequentist coefficient mean estimates and the Bayesian coefficient posterior distribution

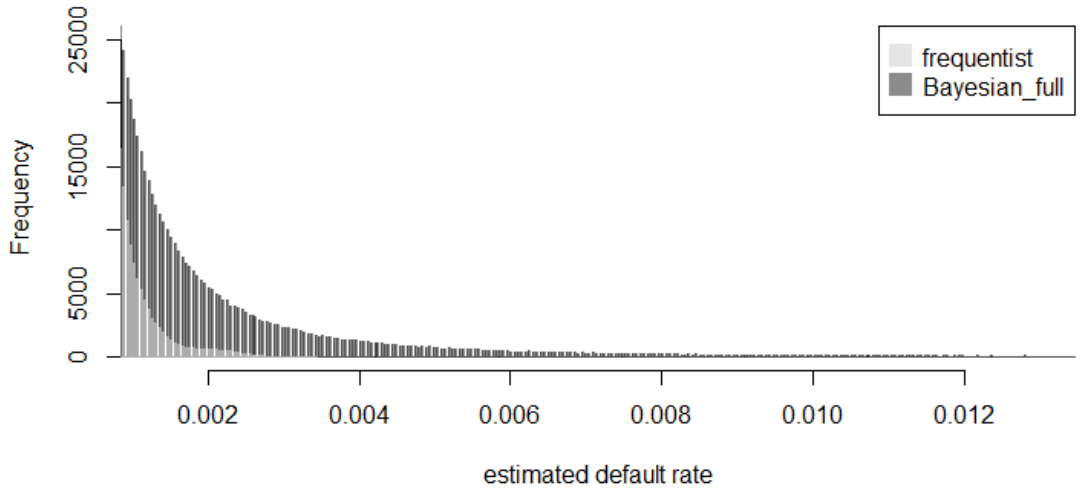

Fig. 5. Tails of the simulated default rate distributions using frequentist coefficient mean estimates and the Bayesian coefficient posterior distribution

Since for the Bayesian posterior distribution approach there are two sources of variation, that is from both the macroeconomic variables and coefficient variables instead of just from the macroeconomic variables alone, the measurements of variation such as the standard deviation and variance are larger.

In the literature, when macroeconomic shocks such as decreases in GDP and increases in interest rates are introduced, the simulated default rates increase (Sorge & Virolainen 2006; Jokivuolle & Viren 2013). Similarly, when introducing estimation risk into stress testing, we expect the simulated default rate of a scenario to increase further.

In the Bayesian posterior distribution approach, higher estimation risk is taken into account which results in higher simulated default rates in scenarios that use draws from areas of the posterior distribution that are far away from the coefficient mean estimates, such as the tails, causing the simulated DR distribution to have higher VaRs compared to using the coefficient mean estimates approach.

Table 7 shows the monetary values of credit loss based on the 99% VaRs for an average account in October 2017. For PD, we use the 99% VaRs of the simulated DR distributions. We assume the EAD of an account is the average current unpaid balance among accounts originated in 2015, which is the population data from which the test sample is taken, and alive in October 2017. We assume the fraction of EAD that is not recovered is 100%.

Table 7

Estimated Monetary values for the 99% VaRs at the observation date based on stress tests using coefficient mean estimates approaches and the Bayesian posterior distribution approach for an average account

EAD (Average current unpaid balance in 201710)	$ 204115.5
The monetary value for 99% VaR (frequentist coefficient mean estimates approach)	$ 269.4
The monetary value for 99% VaR (Bayesian coefficient mean estimates approach)	$ 259.4
The monetary value for 99% VaR (Bayesian posterior distribution approach)	$ 1746.0

The estimated monetary value of the 99% VaR for an account on average is about $ 264 when ignoring estimation risk. The loss is around $ 1746 when estimation risk is included in stress testing. Considering there were 1.065 million accounts that were both originated in 2015 and alive in October 2017 in the Freddie Mac dataset population, it is clear that when the stress testing exercises only address macroeconomic shocks and ignore estimation uncertainty, they underestimate credit loss considerably.

6. Implications and future research

Stress testing is an area of considerable interest for academics, industry practitioners, and

regulators, especially after the 2008 financial crisis. One gap in the stress testing literature is that papers only employ model coefficient point estimates thus neglecting the estimation error surrounding the point estimates. Therefore, the objective of this paper has been to include coefficient estimation risk in stress testing modelling. To reduce coefficient estimation risk in stress testing exercises, we contribute a new stress testing method which employs the Bayesian coefficient posterior distribution instead of point estimate values as the source of coefficients. Since only the mean estimates are used in the conventional stress testing methods, the estimation errors of the coefficient estimates are not addressed. In contrast, in our method we include full ranges of possible values of the coefficients through the use of the coefficient posterior distribution, hence incorporating estimation errors. As an additional source of risk, i.e. estimation risk, in addition to macroeconomic stress, is incorporated into stress testing, we can obtain more conservative estimates of the predicted loss compared to when only shocks to macroeconomic covariates are addressed.

This research gives policy implications for practitioners and regulators. This work gives one additional and possible reason that banks did not have sufficient capital during the financial crisis: the stress testing models in use did not address estimation uncertainty. Moreover, this work not only points out the problem but also provides a stress testing model that includes estimation risk and provides more conservative estimates of credit loss and required capital. It strongly suggests that it is essential to address estimation risk in stress testing since neglecting it could considerably underestimate credit loss. Our work also provides new insight into the application of the Bayesian approach for stress testing. Since in the Bayesian approach model coefficients are treated as random variables instead of fixed values, with the use of a Bayesian approach, we accommodate uncertainty in coefficient estimates in the stress testing model apart from uncertainty in covariate values. This work shows that it is important to take more types of uncertainty into account in stress tests as higher losses are predicted and more capital is required using this method. Since more capital can absorb more loss, banks and depositors can, therefore, be safer against stress.

In this research we focus on the effect of coefficient estimation uncertainty on stress testing. That is, we concentrate on the relationship between the model coefficients and the loss distribution. To include more types of uncertainty, other than estimation risk, into the stress testing method could be an interesting direction for future research. For instance, we may consider the influence of additional useful information, model selection, and variable combination on stress testing.

To give more details, one potential direction of future research is to use informative priors in stress testing to study the impact of additional useful information on stress tests. In our research we single out the influence of estimation risk while controlling for other influences on the simulated distributions. One potential development is to use informative priors on top of including estimation risk to study their combined influence on stress testing results. Another potential topic of future research could be the effect of model variable selection uncertainty on stress testing. In this paper, we use logistic regression as our stress testing model. The possibility of using our stress testing methodology, which includes estimation uncertainty risk, but applied to other models, such as machine learning classifiers, could also be explored.

7. Conclusions

Credit risk stress testing is a topic that attracts a growing research interest in the operational research literature. Our paper contributes to the literature in that we introduce estimation risk into stress testing to reduce credit risk underestimation. We demonstrate how a Bayesian approach and the Bayesian coefficient posterior distribution can be employed in stress testing to account for the potential credit risk underestimation induced by ignoring parameter uncertainty and estimation risk. In the stress testing application, we model both macroeconomic stress and coefficient uncertainty. We apply the Bayesian coefficient posterior distribution instead of coefficient point estimates to the stress test model to include various possible coefficient values and their corresponding probabilities.

In this paper, we use discrete time hazard analysis to model credit default risk over time based on U.S. mortgage loan data. We employ maximum likelihood estimation and the Metropolis-Hasting algorithm respectively for the frequentist and Bayesian approaches. In the Bayesian PD modelling and coefficient estimation, we use Bayesian non-informative priors to ensure the coefficient point estimate results are essentially the same between the frequentist and Bayesian methods, so that the differences in the stress testing results between using posterior distribution and point estimates are mainly due to the accommodation of estimation risk.

In the stress testing step, our Bayesian framework not only takes random draws from the historical scenarios of the macroeconomic variables but also considers estimation risk by simulating from the Bayesian coefficient posterior distribution. By employing Bayesian simulation of coefficients in stress testing, we model the uncertainty of coefficients thus

providing more conservative estimates of credit risk by addressing the estimation variation. Furthermore, since the number of draws from different areas of the posterior is proportionate to the posterior probability of these areas, when we include the less likely coefficient estimates from the posterior distribution, we also take into consideration their corresponding low probability thus avoiding unnecessarily putting high weight on unlikely estimates.

Our main finding is that with the Bayesian stress testing approach using the posterior distribution, we obtain a broader simulated default rate distribution with higher VaRs and larger variance compared to stress testing approaches using coefficient mean estimates. The simulated DR distribution obtained using the Bayesian posterior distribution approach has a standard deviation 10.7 times as large as that using the parameter mean estimates approach. Moreover, the 95% and 99% VaRs of the estimated DR distribution using the Bayesian posterior distribution approach are around 3.7 and 6.5 times the 95% and 99% VaRs using the point estimate approach. The credit loss computed when estimation risk is included is much higher, around 6.5 times as much as the credit loss when estimation risk is ignored.

The results show that if the financial institutions use the traditional stress testing methods without addressing coefficient uncertainty, they could substantially underestimate default rates, and credit loss levels. Therefore it is essential for financial institutions and regulators to include estimation risk in their stress testing applications so that they do not underestimate credit risk and so that the amount of capital they keep accordingly does not fall short of the credit loss.

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Appendix A. Stress testing procedure for the Mean estimate approach:

We use the frequentist mean estimate approach to illustrate in this section. The stress testing process is very similar using the Bayesian mean estimate approach.

The *macroeconomic component* is: $\mathbf{M}_{l\times 1} = (\boldsymbol{M}_{v_1\times l})'\boldsymbol{\beta}^{(m)}_{v_1\times 1}$ with each scalar in $\mathbf{M}_{l\times 1}$ being m_j.

The *account level component* is: $\mathbf{A}_{n\times 1} = (\boldsymbol{A}_{v_2\times n})'\boldsymbol{\beta}^{(a)}_{v_2\times 1}$ with each scalar in $\mathbf{A}_{n\times 1}$ being a_i.

The *error term component* is: $n\times l$ draws from a standard logistic distribution: $\boldsymbol{\varepsilon}_{(ln)\times 1}$.

Repeat ***each scalar*** in the macroeconomic component $\mathbf{M}_{l\times 1}$ n times, and we obtain:

$$\mathbf{M}_{ln\times 1} = \begin{pmatrix} \mathrm{m}_{1,1} \\ \vdots \\ \mathrm{m}_{1,n} \\ \vdots \\ \mathrm{m}_{j,1} \\ \vdots \\ \mathrm{m}_{j,n} \\ \vdots \\ \mathrm{m}_{l,1} \\ \vdots \\ \mathrm{m}_{l,n} \end{pmatrix}_{(ln)\times 1}$$

, in which $\mathrm{m}_{j,1} = \cdots = \mathrm{m}_{j,n} = \mathrm{m}_j$.

Repeat the account level component $\mathbf{A}_{n\times 1}$ ***as a whole*** l times, and we obtain:

$$\mathbf{A}_{ln\times 1}=\begin{pmatrix} \mathrm{a}_{1,1} \\ \vdots \\ \mathrm{a}_{n,1} \\ \vdots \\ \mathrm{a}_{1,j} \\ \vdots \\ \mathrm{a}_{n,j} \\ \vdots \\ \mathrm{a}_{1,l} \\ \vdots \\ \mathrm{a}_{n,l} \end{pmatrix}_{(ln)\times 1}$$

, in which $\begin{pmatrix} \mathrm{a}_{1,1} \\ \vdots \\ \mathrm{a}_{n,1} \end{pmatrix}_{n\times 1}=\ldots=\begin{pmatrix} \mathrm{a}_{1,j} \\ \vdots \\ \mathrm{a}_{n,j} \end{pmatrix}_{n\times 1}=\ldots=\begin{pmatrix} \mathrm{a}_{1,l} \\ \vdots \\ \mathrm{a}_{n,l} \end{pmatrix}_{n\times 1}=\mathbf{A}_{n\times 1}$.

Then add $\mathbf{M}_{ln\times 1}$, $\mathbf{A}_{ln\times 1}$, $\boldsymbol{\varepsilon}_{(ln)\times 1}$ together. The intuition is that each one of the n accounts in the stress testing time period t_s faces l number of potential parallel scenarios, and each one of the l scenarios should have all the n accounts. Specifically, consider n accounts and l macro scenarios. The full results of the right-hand side of the latent logistic function for all the accounts in all the scenarios are:

$$\begin{matrix} (a) & & (b) & & (c) & & (d) \end{matrix}$$

$$\begin{pmatrix} y^*_{1,1,t_s} \\ \vdots \\ y^*_{1,n,t_s} \\ \vdots \\ y^*_{j,1,t_s} \\ \vdots \\ y^*_{j,n,t_s} \\ \vdots \\ y^*_{l,1,t_s} \\ \vdots \\ y^*_{l,n,t_s} \end{pmatrix}_{(ln)\times 1} = \begin{pmatrix} \mathrm{m}_{1,1} \\ \vdots \\ \mathrm{m}_{1,n} \\ \vdots \\ \mathrm{m}_{j,1} \\ \vdots \\ \mathrm{m}_{j,n} \\ \vdots \\ \mathrm{m}_{l,1} \\ \vdots \\ \mathrm{m}_{l,n} \end{pmatrix}_{(ln)\times 1} + \begin{pmatrix} \mathrm{a}_{1,1} \\ \vdots \\ \mathrm{a}_{n,1} \\ \vdots \\ \mathrm{a}_{1,j} \\ \vdots \\ \mathrm{a}_{n,j} \\ \vdots \\ \mathrm{a}_{1,l} \\ \vdots \\ \mathrm{a}_{n,l} \end{pmatrix}_{(ln)\times 1} + \begin{pmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_n \\ \vdots \\ \varepsilon_{(j-1)n+1} \\ \vdots \\ \varepsilon_{jn} \\ \vdots \\ \varepsilon_{(l-1)n+1} \\ \vdots \\ \varepsilon_{ln} \end{pmatrix}_{(ln)\times 1} \tag{A.1}$$

Each scalar in vector (a) in Eq. (A.1) represents the simulated value of the latent variable for an

account in a scenario. For instance, y^*_{j,n,t_s} represents the simulated latent variable for account n in scenario j at stress testing time t_s. Divide vector (a) equally into l sections, one for each scenario, with each section having n scalars. Rearrange vector (a) into a new matrix with the number of rows being the number of accounts n, and the number of columns being the number of scenarios l. That is, put the first n scalars from vector (a) into the first column, the next n scalars into the second column, the j th n scalars into the j th column, so on until the last n scalars from vector (a) are put into the last column of the new matrix. In this way, each column of the new matrix has the simulated values of the latent variable for all the accounts in the same scenario, and all columns represent all the l scenarios:

$$\begin{pmatrix} y^*_{1,1,t_s} & \cdots & y^*_{j,1,t_s} & \cdots & y^*_{l,1,t_s} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ y^*_{1,n,t_s} & \cdots & y^*_{j,n,t_s} & \cdots & y^*_{l,n,t_s} \end{pmatrix} \tag{A.2}$$

Based on Eq. (11), compare the simulated scalars in Eq. (A.2) with 0 and decide whether each of the n accounts is predicted to default or not to default in each scenario:

$$\begin{pmatrix} d^*_{1,1,t_s} & \cdots & d^*_{j,1,t_s} & \cdots & d^*_{l,1,t_s} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ d^*_{1,n,t_s} & \cdots & d^*_{j,n,t_s} & \cdots & d^*_{l,n,t_s} \end{pmatrix} \tag{A.3}$$

Subsequently, based on Eq. (14), the simulated default rate in each of the l scenarios can be computed by averaging over the simulated default behaviours of all the n accounts in each column in Eq. (A.3), and obtain the simulated default rates in all l scenarios: $\left(\hat{R}_{1,t_s} \quad \cdots \quad \hat{R}_{j,t_s} \quad \cdots \quad \hat{R}_{l,t_s}\right)$.

Appendix B. Robustness checks

We use uniform priors in this research. For robustness check, we also employ non-informative priors under a *multivariate normal* distribution. We set the prior mean for each coefficient to 0. The prior precision for each coefficient is set to 10^{-9}. As both methods employ non-informative prior and rely fully on the data, we obtain almost identical estimation results (at 6th decimal place) using the two prior distributions, as shown in Table B.1.

Table B.1

Estimation Results using the Bayesian approach with non-informative uniform and normal priors

Variables (Macro and behavioural variables are lagged 3 months)	Training sample: accounts originated during 2014 with December 2016 as the observation date	
	Uniform Prior distribution	Normal prior distribution
	Posterior mean (std.dev)	Posterior mean (std.dev)
Intercept	-14.758733 (1.2752167)	-14.758733 (1.2752167)
d_l_tbill_3m	0.195955 (0.9538694)	0.195955 (0.9538693)
d_l_unemployment_rate	-0.319338 (0.5026099)	-0.319338 (0.5026099)
d_l_CPI	-0.047746 (0.1548964)	-0.047746 (0.1548964)
d_l_consumer_confidence	0.008596 (0.0145325)	0.008596 (0.0145325)
d_l_retail_sales	23.398211 (14.0626727)	23.398211 (14.0626700)
d_l_personal_earnings	24.469273 (37.9106425)	24.469274 (37.9106147)
d_l_IPI	0.182078 (0.1334646)	0.182078 (0.1334646)
d_l_dowjones_index	1.888173 (2.0835078)	1.888173 (2.0835078)
d_l_CS_houseprice_index	0.679918 (0.2080482)	0.679918 (0.2080481)
original_debt_to_income_ratio	0.035027 (0.0060389)	0.035027 (0.0060389)
original_loan_to_value	0.004576 (0.0034240)	0.004576 (0.0034240)
l_current_actual_upb	-0.151780 (0.0859478)	-0.151780 (0.0859478)
l_current_interest_rate	1.505791 (0.1572061)	1.505791 (0.1572061)
l_remaining_months	-0.003141 (0.0011293)	-0.003141 (0.0011293)
loan_age	0.085288 (0.0347888)	0.085288 (0.0347888)
loan_age_sq	-0.001753 (0.0009265)	-0.001753 (0.0009265)
	Number of draws in MCMC = 100000 Burn-in = 200000	Number of draws in MCMC = 100000 Burn-in = 200000

Table B.2 presents the Bayesian coefficients convergence diagnostics for the Bayesian estimations using the uniform and normal prior distributions. Initial draws from the first half of the MCMC are compared with draws from the second half. The z-scores for the coefficients are within the [-2, 2] range. Based on the Geweke diagnostics of convergence results, the Markov chains converge well.

Table B.2
Bayesian coefficients convergence diagnostics

Geweke diagnostic of convergence	z-scores	
	Uniform prior	Normal prior
Intercept	1.08196	1.08196
d_l_tbill_3m	-1.11816	-1.11816
d_l_unemployment_rate	-0.70955	-0.70955
d_l_CPI	0.87389	0.87389
d_l_consumer_confidence	-0.01717	-0.01717
d_l_retail_sales	-0.80482	-0.80482
d_l_personal_earnings	-1.09172	-1.09172
d_l_IPI	-0.20158	-0.20158
d_l_dowjones_index	-0.82063	-0.82063
d_l_CS_houseprice_index	-1.91440	-1.91440
original_debt_to_income_ratio	0.94838	0.94838
original_loan_to_value	0.96615	0.96615
l_current_actual_upb	-0.63741	-0.63741
l_current_interest_rate	-1.12304	-1.12304
l_remaining_months	0.63890	0.63890
loan_age	-0.51490	-0.51490
loan_age_sq	0.32803	0.32803

Appendix C. Simulating from each macroeconomic coefficient individually as opposed to simulating from all macroeconomic coefficients simultaneously

In our stress testing exercise using the posterior distribution approach we simulate from all the macroeconomic coefficients simultaneously and also simulate from the macroeconomic variables. We now simulate from each macroeconomic coefficient individually while holding the rest of the macroeconomic coefficients at their Bayesian posterior mean estimates to disentangle the coefficient uncertainty effect of each parameter from the others. The macroeconomic variables are held at their mean values between Jan 1999 to Sept 2017 to disentangle the coefficient uncertainty effect of each coefficient from the uncertainty over the values of the macroeconomic variable itself. We take 100 draws from the marginal posterior distribution of each of the macroeconomic coefficients. The stress testing exercises are repeated 100 times each with random simulations for the coefficients and the error terms. The stress tests are applied to all the accounts in the test sample alive at the stress testing time period Oct 2017. Table. C.1 shows the VaRs and max percentiles of the simulated default rate distributions as well as the predicted number of defaults that the VaR results are based on. Individually, no coefficients show considerably and consistently dominant effect of coefficient uncertainty on VaRs compared other coefficients.

Table C.1 VaRs of the estimated DR distributions when individually simulating from each of the macroeconomic coefficients and the corresponding numbers of predicted defaults

	VaR 99%	# predicted defaults at VaR 99%	VaR 99.9%	# predicted defaults at VaR 99.9%	100^{th} percentile	# predicted defaults at 100^{th} percentile
interest rate	0.000415	17	0.000513	21	0.000562	23
unemployment rate	0.000440	18	0.000489	20	0.000538	22
CPI	0.000440	18	0.000489	20	0.000587	24
consumer confidence	0.000415	17	0.000489	20	0.000635	26
retail sales	0.000440	18	0.000513	21	0.000587	24
personal earnings	0.000464	19	0.000538	22	0.000587	24
IPI	0.000415	17	0.000513	21	0.000587	24
stock price index	0.000415	17	0.000513	21	0.000611	25
house price index	0.000440	18	0.000513	21	0.000562	23