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Some Cautionary Remarks on Abelian Projection and Abelian Dominance

L. Del Debbio[a], M. Faber[b], J. Greensite[c] *, Š. Olejník[d]

[a]Physics Dept., Univ. of Wales Swansea, SA2 8PP Swansea, UK

[b]Inst. für Kernphysik, Tech. Univ. Wien, 1040 Vienna, Austria

[c]Physics Dept., San Francisco State Univ., San Francisco, CA 94132 USA

[d]Inst. of Phys., Slovak Acad. of Sci., 842 28 Bratislava, Slovakia

Some critical remarks are presented, concerning the abelian projection theory of quark confinement.

1. Introduction

Interest in the abelian projection theory of quark confinement, proposed by 't Hooft in 1981 [1], has been increasing in recent years. Very briefly, the idea is to select an "abelian projection" gauge which reduces the gauge symmetry of an $SU(N)$ gauge theory to $U(1)^{N-1}$. This choice enables one to identify abelian gauge fields and magnetic monopoles. The theory is that abelian electric charge will then be confined by monopole condensation. In this talk I would like to make some general remarks on these ideas, and on the related concept of abelian dominance.

Most of the recent efforts in the abelian projection theory have involved lattice Monte Carlo simulations, and most of those simulations have used a particular abelian projection gauge known as the maximal abelian gauge, introduced by Kronfeld et al. [2] in 1987. On the lattice, for $SU(2)$ gauge theory, maximal abelian gauge is obtained by maximizing the quantity

$$\sum_x \sum_\mu \mathrm{Tr}[U_\mu(x)\sigma^3 U_\mu^\dagger(x)\sigma^3] \tag{1}$$

This gauge choice has the effect of making link variables as diagonal as possible, leaving a residual $U(1)$ symmetry. Having made the maximal abelian (or any other abelian projection) gauge choice, one can extract, from the full link variable $U_\mu(x)$, a diagonal matrix $A_\mu(x)$ which transforms as an abelian gauge field under the residual $U(1)$ symmetry:

$$\begin{aligned} U &= WA \\ A &= \begin{bmatrix} e^{i\theta} & \\ & e^{-i\theta} \end{bmatrix} \end{aligned} \tag{2}$$

(see also eq. (10) below). Using the A abelian gauge field, one may investigate (among other things): (i) Creutz ratios and Polyakov lines [3]; (ii) monopole densities [2]; (iii) dual London relations [4]; and (iv) expectation values of monopole creation operators [5,6]. Of particular note is the work of the Kanazawa group [3], who found that certain quantities such as the string tension, extracted from Wilson loops constructed from the abelian link variables A alone, are close to the values obtained from the full link variables, a property known as "abelian dominance." A much more extensive list of recent work can be found in ref. [7].

From these and other investigations, two conclusions have been drawn:

1. The $U(1)$ gauge field looks like that of a dual superconductor.

2. The abelian projection theory of confinement is confirmed.

The first conclusion is well supported by existing data. My remarks will be directed at the second conclusion, which has been inferred from the first.

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2. Center Dominance

To motivate the first calculations I will discuss, let me recall a theory of confinement which was popular in the late 1970's, namely the Z_N vortex condensation theory [8,9]. In this picture the vacuum state of an $SU(N)$ gauge theory is presumed to be dominated by long vortices carrying multiples of Z_N magnetic flux (a closely related picture was developed somewhat earlier by the Copenhagen group; c.f. ref. [10]). If the area of a spacelike Wilson loop is pierced by m such vortices, the value of the loop is multiplied by a factor of $\exp(2im\pi/N)$. The area law is attributed to random fluctuations in the number of Z_N vortices piercing the loop. 't Hooft introduced a singular gauge transformation operator, denoted $B(C)$, which creates a closed Z_N vortex along curve C. A necessary condition for magnetic disorder is a perimeter law behavior

$$< B(C) > \sim \exp[-\mu L(C)] \tag{3}$$

for the VEV of the vortex creation operator. It was also suggested that the confining dynamics of an $SU(N)$ gauge theory is described by an effective Z_N gauge theory [11].

The heyday of the vortex condensation theory preceded the widespread use of lattice Monte Carlo methods, but in view of the numerical work that has been done in recent years on the abelian projection theory, it is interesting to go back and conduct similar numerical experiments on the Z_N vortex theory. The general idea is that, just as maximal abelian gauge brings the link variables U of an $SU(2)$ gauge theory as close as possible to the $U(1)$ gauge fields A, so we would like to go one step further and use the remnant $U(1)$ gauge freedom to bring the A gauge field as close as possible to a Z_2 gauge field, with values $\pm I$. The procedure is to begin by fixing to maximal abelian gauge, with A the diagonal matrix shown in eq. (2). We then use the remnant $U(1)$ symmetry to maximize

$$\sum_x \sum_\mu \cos^2(\theta_\mu(x)) \tag{4}$$

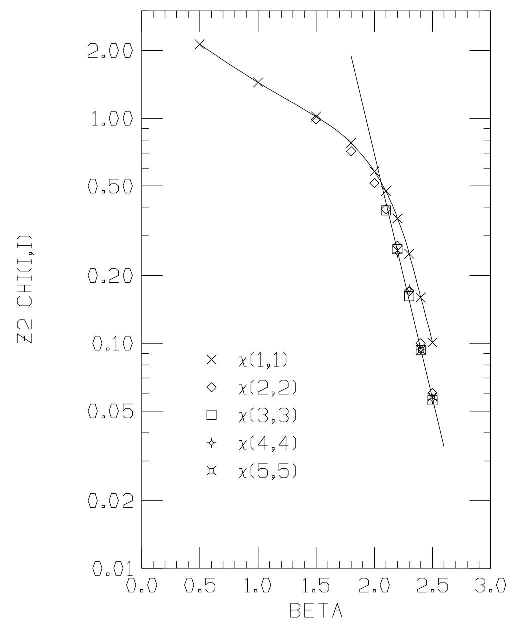

Figure 1. Creutz ratios from center-projected lattice configurations.

leaving a remnant Z_2 symmetry. This we call "Maximal Z_2 Gauge." Then define, at each link,

$$Z \equiv \text{sign}(\cos\theta) = \pm 1 \tag{5}$$

which transforms like a Z_2 gauge field under the remnant symmetry. "Center Projection" $U \to Z$, analogous to "abelian projection" $U \to A$, replaces full link variables by the center element ZI, in the computation of observables such as Wilson loops and Polyakov lines.

Results for Creutz ratios in the center projection, for maximal Z_2 gauge, are shown in Fig. 1. The data was taken for $SU(2)$ lattice gauge theory in $D = 4$ dimensions. Lattice sizes were 10^4 for $\beta \leq 2.3$, 12^4 at $\beta = 2.4$, and 16^4 at $\beta = 2.5$. The straight line is the standard scaling function for the asympototic string tension

$$\sigma a^2 = \frac{\sigma}{\Lambda^2}(\frac{6}{11}\pi^2\beta)^{102/121} \exp[-\frac{6}{11}\pi^2\beta] \tag{6}$$

with the value $\sqrt{\sigma}/\Lambda = 67$.

What is remarkable about Fig. 1 is the fact that, from $\chi(2,2)$ onwards, the data for $\chi(R,R)$

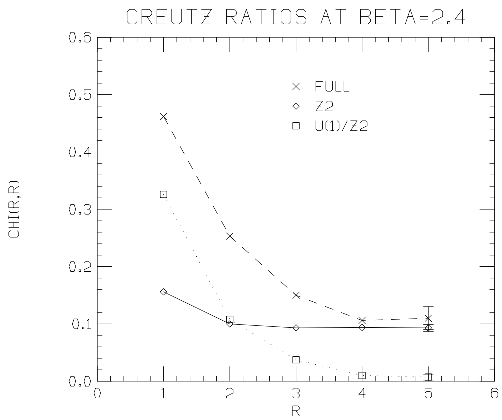

Figure 2. Creutz ratios $\chi(R,R)$ vs. R at $\beta = 2.4$, for full, center-projected, and $U(1)/Z_2$-projected lattice configurations.

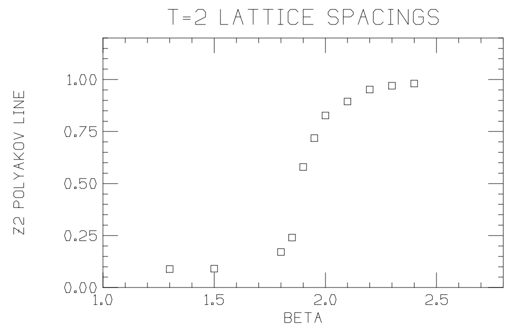

Figure 3. Polyakov lines vs. β in center projection, for a $6^3 \times 2$ lattice.

at fixed β practically fall on top of one another. This is quite different from the standard curves, either in the full theory or the usual abelian projection, where only the envelope of the $\chi(R,R)$ data fits the scaling curve. What it means is that the center projection sweeps away the short-distance, $1/r$-type potential, and the remaining linear potential is revealed almost from the beginning. This is seen quite clearly in Fig. 2, which displays the data for $\chi(R,R)$ at $\beta = 2.4$ for the full theory (crosses), the center projection (diamonds), and also for the $U(1)/Z_2$-projection (squares). The latter projection consists of the replacement $U \to A/Z$ for the link variables. We note that the center-projected data is virtually flat, from $R = 2$ to $R = 5$, which means that the potential is linear in this region, and appears to be the asymptote of the full theory. It should also be noted that abelian link variables with the center factored out, i.e. $U \to A/Z$, appear to carry no string tension at all.

We have not yet done a thorough investigation of finite temperature effects in center projection. Fig. 3, however, shows the Polyakov line on a $6^3 \times 2$ lattice in center projection. The phase transition, indicated by a sudden jump in the data, appears to set in where it is supposed to.

Z_2 vortices are the *only* field configurations in Z_2 gauge theory, and we have found that the Z_2 configurations extracted from $SU(2)$ lattice gauge theory give the full asymptotic string tension ("center dominance"). This data could certainly be taken as evidence in favor of the vortex condensation theory; in fact, various arguments in favor of the abelian projection theory can be reused to support the vortex condensation theory. This recycling of arguments requires only a slight change in terminology: replace "maximal abelian gauge," "abelian projection," and "abelian dominance," by "maximal Z_2 gauge," "center projection," and "center dominance," respectively, with the relevant topological configurations being vortices rather than monopoles. One would focus on the VEV of vortex creation operators $< B(C) >\sim e^{-\mu L(C)}$ rather than monopole creation operators $< \Phi_M >\neq 0$, and the QCD ground state would resemble a "spaghetti vacuum" of vortices, rather than a dual superconductor.

Now if abelian dominance suggests that monopole condensation is the confinement mechanism, while center dominance points instead to vortex condensation, which mechanism should one believe in? It could be argued that while abelian dominance shows the abelian link to be the crucial component of the full link, center dominance shows the center variable to be the crucial variable of the abelian link, and therefore center dominance is somehow the more basic phenomenon. We would not make this argument, however. The reason is that neither the vortex condensation theory, nor the abelian projection theory, gives an adequate explanation of the potential between static sources in higher group representations.

3. Casimir Scaling

It has been found in many lattice Monte Carlo investigations, in both $D = 3$ and $D = 4$ dimensions and for both $SU(2)$ and $SU(3)$ gauge groups, that the string tension of static sources in representation R of the gauge group is approximately proportional to the quadratic Casimir C_R of the representation [12]. This result holds with an accuracy of about 10%, from the onset of confinement to the onset of color screening. In particular, for the $SU(2)$ gauge group,

$$\sigma_j \approx \frac{4}{3} j(j+1)\sigma_{1/2} \qquad \text{intermediate region} \tag{7}$$

while asymptotically (after color-screening)

$$\sigma_j \to \begin{cases} \sigma_{1/2} & j = \text{ half-integer} \\ 0 & j = \text{ integer} \end{cases} \tag{8}$$

This "Casimir scaling" of the string tension at intermediate distances is easily derived at strong-couplings from either the Kogut-Susskind Hamiltonian or the heat-kernel action. It can also be derived in $D = 2$ dimensions, at weak couplings, for any lattice action. It is not obvious why Casimir scaling persists at weaker couplings in 3 and 4 dimensions, but probably the explanation is connected to the concept of dimensional reduction, introduced in refs. [13,14], which would allow us to infer Casimir scaling in 3 and 4 dimensions from the 2-dimensional result. In any case, approximate Casimir scaling of string-tensions up to the onset of color-screening is a numerical fact. The question is whether this fact is consistent with either the vortex condensation or the abelian projection theory.

Consider first the vortex theory, and quarks in the adjoint representation. The problem is that adjoint quarks are neutral (i.e. invariant) with respect to Z_2 gauge transformations; as a consequence, adjoint Wilson loops are unaffected by insertion or removal of a Z_2 vortex. Thus, fluctuations in the number of vortices piercing the loop cannot possibly be responsible for an area law for adjoint loops; the vortex theory is not at all compatible with Casimir scaling at intermediate distances.

There is a similar problem for the abelian projection theory. According to this theory it is the abelian charge, singled out by the unbroken Cartan subalgebra, which is confined. The adjoint representation is $j = 1$; the $m = +1$ and $m = -1$ color components have double abelian charge ($++$ and $--$, respectively) as compared to the $+/-$ abelian charge of the fundamental representation. The $m = 0$ component, however is uncharged w.r.t the $U(1)$ subgroup, *and this neutrality holds regardless of color screening.* This means that there is no apparent mechanism for string formation between $m = 0$ quark components, and therefore no reason for an area law; $\sigma_{j=1} = 0$ would be expected at all scales. Similarly, for the $j = 3/2$ representation, the $m = \pm\frac{3}{2}$ components have triple abelian charge, while the $m = \pm\frac{1}{2}$ components have single abelian charge, the same as the fundamental representation. One therefore expects that $\sigma_{3/2} = \sigma_{1/2}$ prior to color screening, from the onset of confinement.

It is really a qualitative point that is being made here. If the confining force is only sensitive to abelian charge, then the quark components with the lowest abelian charge should dominate the Wilson loops. This point is illustrated by making the same abelian projection $U \to A$ for the higher representation loops that has been used for the fundamental loops. In this illustration, one finds [15]

$$\begin{aligned} < W_j(C) > &= < \mathrm{Tr} \exp[i \oint dx^\mu A^a_\mu T^j_a] > \\ &\to < \mathrm{Tr} \exp[i \oint dx^\mu A^3_\mu T^j_3] > \\ &= \sum_{m=-j}^{j} < \exp[im \oint dx^\mu A^3_\mu] > \end{aligned}$$

so that in particular

$$< W_1^{ab.}(C) >= 1 + < e^{i \oint A^3} > + < e^{-i \oint A^3} >$$

which implies $\sigma_{j=1} = 0$ for abelian-projected configurations; and

$$< W^{ab.}_{3/2}(C) > \; = \; < e^{i\frac{1}{2}\oint A^3} > + < e^{-i\frac{1}{2}\oint A^3} > + < e^{i\frac{3}{2}\oint A^3} > + < e^{-i\frac{3}{2}\oint A^3} >$$

implying $\sigma_{j=3/2} = \sigma_{j=1/2}$. Again, these results have nothing to do with color-screening.

We conclude that confinement of *abelian* charge in an $SU(2)$ gauge theory would imply

$$\sigma_{j=1} = 0 \quad \text{and} \quad \sigma_{j=3/2} = \sigma_{j=1/2} \tag{9}$$

from the confinement scale onwards, which disagrees with existing numerical data. However, the argument presented is rather qualitative, and it is obviously desirable to check eq. (9) in a confining non-abelian gauge theory, where we are confident that confinement is indeed due to abelian monopole configurations.

4. The 3D Georgi-Glashow Model

The Georgi-Glashow model is an $SU(2)$ gauge theory with a Higgs field in the adjoint representation. It has been argued pursuasively by Polyakov [16] that in D=3 dimensions, confinement in the Higgs phase is due to 't Hooft-Polyakov monopoles, which of course are instantons in three dimensions. We are therefore able to check that confinement of abelian charge leads to eq. (9) by lattice Monte Carlo simulation.

The lattice action for the Georgi-Glashow model is [17]

$$\begin{aligned} S &= \frac{1}{2}\beta_G \sum_{plaq} \mathrm{Tr}[UUU^\dagger U^\dagger] \\ &+ \frac{1}{2}\beta_H \sum_{n,\mu} \mathrm{Tr}[U_\mu(n)\phi^\dagger(n+\mu)U^\dagger_\mu(n)\phi(n)] \\ &- \sum_n \left\{ \frac{1}{2}\mathrm{Tr}[\phi\phi^\dagger] + \beta_R \left(\frac{1}{2}\mathrm{Tr}[\phi\phi^\dagger] - 1 \right)^2 \right\} \end{aligned}$$

Define observables

$$\begin{aligned} R &= < \mathrm{Tr}[\phi\phi^\dagger] >^{1/2} \\ Q &= \frac{1}{2} < \mathrm{Tr}[U_\mu(n)\sigma^3 U^\dagger_\mu(n)\sigma^3] > \end{aligned}$$

in unitary gauge $\phi = \rho\sigma^3$. A jump in these two quantities is an indication of a transition from the symmetric phase to the Higgs phase. Our investigation has only been at fixed values $\beta_G = 2$, $\beta_R = 0.01$, and we look for the Higgs transition by varying β_H. Figure 4 locates the Higgs transition near $\beta_H = 0.45$ from a jump in Q. A similar jump in the value of the R observable is found at this point.

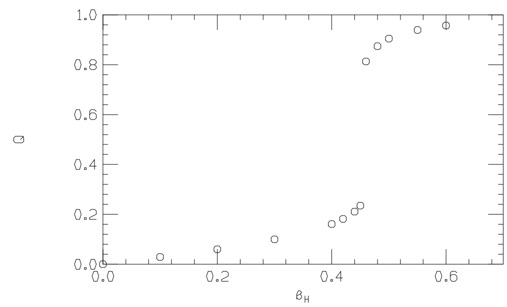

Figure 4. Q vs. β_H in the D=3 Georgi-Glashow model.

Figure 5 shows the behavior of the $\chi(2,2)$ Creutz ratios for fundamental and adjoint loops in the same range of couplings. Note that prior to the transition the adjoint value is roughly double the fundamental, while after the transition the adjoint value is almost vanishing.

Creutz ratios in the Higgs phase at $\beta_H = 0.46$, which is just past the transition, are shown in Fig. 6. The string tension for the adjoint loop is clearly consistent with zero, with Creutz ratios actually going negative at $I = 3$. The data is also consistent with $\sigma_{1/2} \approx \sigma_{3/2}$. So this example does seem to support the reasoning leading to eq. (9).

For comparison, let us return to pure Yang-Mills theory in $D = 3$ dimensions. It is found that the $\chi_j(I,I)$ ratios, extracted from loops computed in abelian-projected configurations, are similar to those shown in Fig. 6. In contrast, Casimir scaling is quite evident for ratios obtained from the full, unprojected link configurations. Space limitations prevent displaying the relevant data here, but it may be found in Figures 1 and 2 of ref. [18].

To summarize:

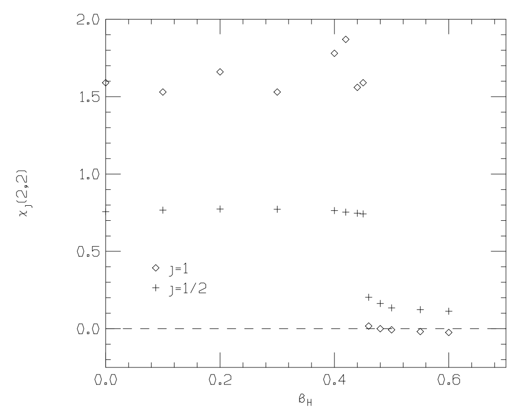

Figure 5. $\chi(2,2)$ Creutz ratios for fundamental and adjoint loops, in the D=3 Georgi-Glashow model.

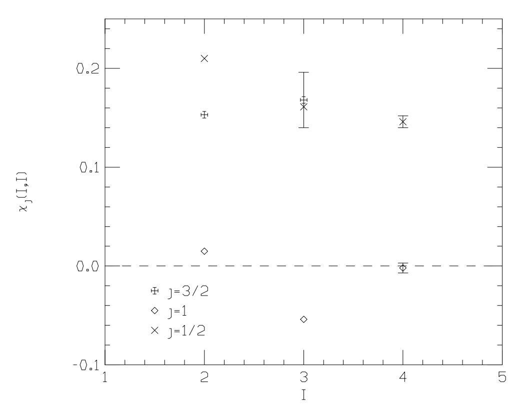

Figure 6. $\chi_j(I,I)$ vs. I. Creutz ratios for $j = \frac{1}{2}, 1, \frac{3}{2}$, just inside the Higgs phase of the D=3 Georgi-Glashow model.

1. In pure SU(2) gauge theory,

$$\sigma_{j=1} \approx \frac{8}{3}\sigma_{j=1/2} \qquad \sigma_{j=3/2} \approx 5\sigma_{j=1/2}$$

whereas,

2. In a theory where monopoles are known to drive the confinement mechanism (D=3 Georgi-Glashow, Higgs phase)

$$\sigma_{j=1} \approx 0 \qquad \sigma_{j=3/2} \approx \sigma_{j=1/2}$$

3. In pure SU(2), with abelian projection in maximal abelian gauge,

$$\sigma_{j=1} \approx 0 \qquad \sigma_{j=3/2} \approx \sigma_{j=1/2}$$

The result $\sigma_{j=1} \approx 0$ is simply due to the neutrality of the $m = 0$ adjoint quark component.

In connection with abelian-projected configurations, it should be noted that the contributions $< \exp[\pm i \oint A^3] >$ to the abelian adjoint loop,

$$< W_1^{ab.}(C) >= 1+ < e^{i \oint A^3} > + < e^{-i \oint A^3} >$$

coming from the double-charged components $m = \pm 1$, *do* have an area law, and in fact the string tension of these components is quite close to that of the full, unprojected loops. This could be taken as evidence of some form of abelian dominance [19], but that is somewhat beside the point we are making here. Confinement of the double-charged adjoint quark components, in the abelian projection picture, is not in doubt. The real issue is how the linear potential can act on the abelian neutral ($m = 0$) component, if only abelian charge is sensitive to the confining force.

5. XY-Maximal Abelian Gauge

Abelian dominance, at least for certain quantities, can be impressive. The general rule is to fix to maximal abelian gauge, do the $U \rightarrow A$ abelian projection

$$U = a_0 I + i\vec{a}\cdot\vec{\sigma} \quad \longrightarrow \quad A = \frac{a_0 I + i a_3 \sigma^3}{\sqrt{a_0^2 + a_3^2}} \qquad (10)$$

and then calculate observables with projected A configurations.

A skeptic, however, might argue that the maximal abelian gauge choice forces most of the quantum fluctuations into the diagonal part of the link. Then perhaps it is not really surprising that the diagonal component alone can reproduce various observables with reasonable accuracy; the underlying reason may be more a matter of kinematics than dynamics.

To investigate this issue, we ask the question: What if one chooses a gauge such that only links the XY-plane are as diagonal as possible, i.e.

$$\sum_x \sum_{\mu=1}^{2} \mathrm{Tr}[\sigma_3 U_\mu \sigma_3 U_\mu^\dagger] \qquad \text{is maximized} \qquad (11)$$

and then abelian project? Do we find abelian dominance? And what if we "abelian project," via eq. (10), without fixing any gauge at all? The answers are as follows:

1. Abelian-projected loops in the XY-plane exhibit abelian dominance.

Figure 7 shows a plot of Creutz ratios vs. β, extracted from abelian-projected loops in the XY-plane. It has the standard form; the envelope of $\chi(I,I)$ appears to fit a scaling curve, which is shown with the value $\sqrt{\sigma}/\Lambda = 85$.

2. There is no obvious abelian dominance in the ZT-plane. In fact, abelian loops in the ZT-plane are indistinguishable from "abelian-projected" loops *with no gauge-fixing whatever!*

Figure 8 shows loop values vs. area at $\beta = 2.4$, for abelian projected loops in the ZT-plane (diamonds). For comparison, loop values for the full, unprojected configurations are also shown (crosses). It is clear that the loop values for projected configurations drop like a stone, as compared to the full values (or compared to the projected loop values in the XY-plane). Because of the rather large error bars, Creutz ratios taken from this data are not very meaningul. What is of more significance is to compare the loop values taken from projected configurations in the

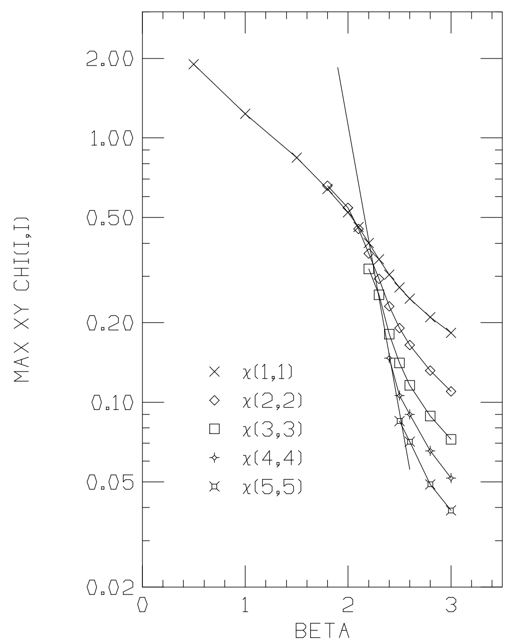

Figure 7. Creutz ratios vs. β, extracted from abelian loops in the XY-plane in XY-maximal abelian gauge

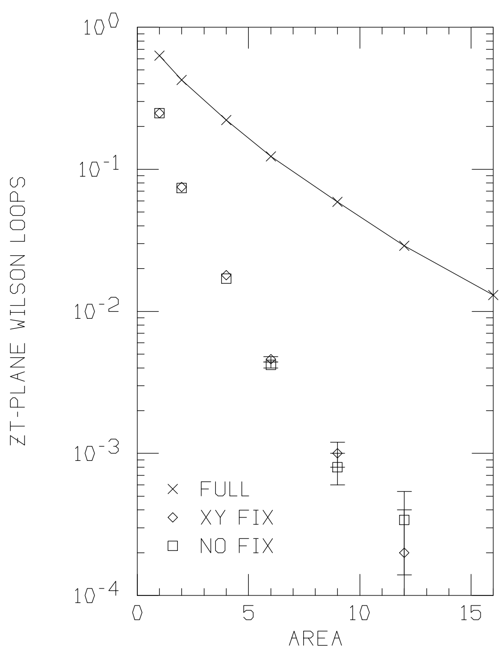

Figure 8. Loop values vs. Area at $\beta = 2.4$, extracted from loops in the ZT-plane. Crosses show the full, gauge-invariant values; diamonds are values for projected configurations in XY-maximal abelian gauge; squares are values for projected configurations with no gauge-fixing at all.

XY-maximal abelian gauge, with configurations also projected according to eq. (10) but obtained using no gauge fixing whatever (squares). It is clear that in the ZT-plane, in contrast to the XY-plane, the loop values of projected configurations are completely insensitive to the presence or absence of the XY-maximal abelian gauge fixing.

This example supports the skeptical view. Since we see abelian dominance in the plane where links are nearly diagonal, and don't see it in other directions, it suggests that abelian dominance is simply a consequence of having nearly diagonal links, and not necessarily evidence in favor of the abelian projection theory.

6. Conclusions

To summarize: we have found "center dominance" in maximal Z_2 gauge. To the extent that abelian dominance supports the abelian projection theory of confinement, center dominance supports the vortex condensation theory.

Neither theory seems to explain, even qualitatively, the existence of a linear potential between adjoint quarks up to color-screening, let alone the approximate Casimir scaling of string tensions.

We have also seen that it is possible to choose a gauge (the XY-maximal abelian gauge), in which one sees "abelian dominance" in the XY-plane, but not in the ZT-plane. What this suggests is that abelian dominance in maximal abelian gauge could be an artifact of setting links almost diagonal, rather than a definite indication of the underlying confinement mechanism.

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