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Citation for published version:
Del Debbio, L, Di Giacomo, A & Lucini, B 2000, 'Vortices versus monopoles in color confinement', *Nuclear Physics B - Proceedings Supplements*, vol. 94, no. 1-3, pp. 502–505. https://doi.org/10.1016/S0920-5632(01)00894-5

Digital Object Identifier (DOI):
10.1016/S0920-5632(01)00894-5

Link:
Link to publication record in Edinburgh Research Explorer

Document Version:
Early version, also known as pre-print

Published In:
Nuclear Physics B - Proceedings Supplements

Vortices versus monopoles in color confinement *

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We construct the creation operator of a vortex for $SU(2)$ pure gauge theory using the methods developed for monopoles. We interpret its vacuum expectation value as a disorder parameter for the deconfinement phase transition and find that it behaves in the vacuum in a similar way to monopoles. Results are extrapolated to the thermodynamical limit using finite-size scaling.

1. Introduction

Two kinds of topological defects were proposed by 't Hooft as possible configurations condensing in the disordered phase of $SU(N)$ Yang-Mills theories to produce confinement: Z_N vortices [1] and monopoles [2].

Monopoles have the natural topology to three dimensional space, corresponding to a mapping: $S_2 \to SU(2)$. Their condensation implies dual superconductivity of the vacuum, and gives an appealing physical picture of confinement in terms of dual Abrikosov tubes produced by dual Meissner effect [3]. Indeed, condensation of magnetic charge has been unambiguously demonstrated by numerical simulations on the lattice [4]. An operator μ carrying non-zero magnetic charge is constructed, and its vev $\langle\mu\rangle$ is measured in the confined and deconfined phases, as a candidate disorder parameter. The extrapolation to the thermodynamical limit is done by finite size scaling techniques and the result is:

$$\langle\mu\rangle \neq 0, \quad \text{for} \quad T < T_c, \tag{1}$$

$$\langle\mu\rangle = 0, \quad \text{for} \quad T > T_c \tag{2}$$

$$\langle\mu\rangle \propto (1 - T/T_c)^{\delta}, \quad \text{for } T \simeq T_c \tag{3}$$

Both T_c and the critical index of the correlation length ν are obtained, and they agree with determinations by other techniques [5]. The values of δ are:

$$\begin{array}{ll} \delta = 0.25(10) & \text{for} \quad SU(2) \\ \delta = 0.54(4) & \text{for} \quad SU(3) \end{array} \tag{4}$$

*talk presented by A. Di Giacomo

$\langle\mu\rangle \neq 0$ signals dual superconductivity.

The same disorder parameter can be defined in full QCD, and shows a similar behaviour at the deconfinement phase transition [6]. This phenomenon seems to be independent of the Abelian projection chosen to define the monopoles. The meaning of these results is that the (yet unknown) excitations, which condense in the confined phase and are weakly interacting in the dual picture, must be magnetically charged with respect to all abelian projections. Attempts to identify these excitations with the monopoles of the maximal abelian projection do not seem to work as expected [7,8].

In order to understand better the properties of the dual excitations, we investigate by similar techniques the role of vortices [9]. In 3+1 dimension vortices are string-like topological defects, associated to closed curves C. The operator $B(C)$ which creates such a vortex obeys the following algebraic relation with the Wilson loop $W(C')$, defined as the parallel transport along the curve C':

$$B(C)W(C') = W(C')B(C) \exp\left(\frac{i2\pi n_{CC'}}{N}\right) \tag{5}$$

with $n_{CC'}$ being the linking number of the curves C and C' [1].

It follows from Eq. 5 that whenever $\langle W(C')\rangle$ obeys the area law, $\langle B(C)\rangle$ obeys the perimeter law and viceversa [1]. A consequence of the area law for $\langle W(C')\rangle$ is that the Polyakov line, defined as the parallel transport along a line in the time direction closed by periodic boundary conditions (pbc), has to vanish. Instead, it can be that $\langle P(x)\rangle \neq 0$ when the Wilson loop obeys the perimeter law. We argue that the same happens for the dual loops $B(C)$. We define a disorder parameter $\langle\mu\rangle$ as the "dual Polyakov line", corresponding to the operator $B(C)$ for a curve C going through the lattice, e.g. parallel to the z axis, at fixed time and closed by pbc. In the phase where $\langle B(C)\rangle$ obeys the area law (and therefore $\langle W(C')\rangle$ obeys a perimeter law), $\langle\mu\rangle = 0$; whenever $\langle\mu\rangle \neq 0$ the perimeter law is possible for $\langle B(C)\rangle$. This observation helps in defining a disorder parameter in 3+1 dimension.

In 2+1 dimension the vortex is point-like, it can be described by a local field, and a conserved quantum number can be associated to it, which is broken in the disordered phase [1]. Instead, no conserved quantum number can be associated to vortices in 3+1 dimension.

2. Creation operator of a vortex

For the sake of definiteness, let us take for the curve C a rectangle R in the xy plane:

$$\begin{aligned} R &= \{(x,y,z) : \\ &\quad x_0 < x < x_1, y = y_0, z_0 < z < z_1\} \end{aligned} \tag{6}$$

The definition given below can be extended trivially to any curve C.

We define the vacuum expectation value (vev) $\langle B(C)\rangle \equiv \tilde{Z}/Z$, where Z is the ordinary partition function for pure Yang-Mills theories on the lattice defined by the Wilson action:

$$S[U] = \sum_{x,\mu,\nu} \mathrm{Re}\ \mathrm{Tr}\,[1 - P_{\mu\nu}] \tag{7}$$

with $P_{\mu\nu}$ is the usual plaquette. $\tilde{Z}$ is the partition function corresponding to the action $\tilde{S}$ obtained from S by the change:

$$\begin{aligned} &P_{0y}(t_0, x_0 < x < x_1, y = y_0, z_0 < z < z_1) \mapsto \\ &e^{i2\pi/N} P_{0y}(t_0, x_0 < x < x_1, y = y_0, z_0 < z < z_1) \end{aligned}$$

We study by numerical simulations, the behaviour of the "dual Polyakov line", which corresponds to the above definition with:

$$\begin{aligned} z_0 &\to -\infty \\ z_1 &\to +\infty \end{aligned}$$

in that case the change of variables becomes:

$$\begin{aligned} &P_{0y}(t_0, x_0 < x < x_1, y = y_0, z) \mapsto \\ &e^{i2\pi/N} P_{0y}(t_0, x_0 < x < x_1, y = y_0, z), \\ &\quad \text{for all } \ z \end{aligned} \tag{8}$$

For the proof that Eq. 8 really corresponds to the definition of $B(C)$ as defined by Eq. 5 and for comparison with alternative definitions in the literature, we refer to [9].

We shall also compute the correlator:

$$\Gamma(t) = \langle \mu(t_0, x_0, y_0)\, \mu(t_0 + t, x_0, y_0)\rangle \tag{9}$$

As $t \to \infty$,

$$\Gamma(t) \sim A e^{-mt} + \langle\mu\rangle^2$$

whence $\langle\mu\rangle$ can be extracted.

At finite T there is no propagation in time and $\langle\mu\rangle$ is computed directly, provided C^* boundary conditions are adopted [9,4].

3. Numerical results

We compute $\langle\mu\rangle$ by the techniques explained above, or better we compute:

$$\rho = \frac{\mathrm{d}}{\mathrm{d}\beta} \log\langle\mu\,(t_0, x_0, y_0)\rangle = \langle S\rangle_S - \langle\tilde{S}\rangle_{\tilde{S}} \tag{10}$$

which contains the same information and is less noisy [10,11,4]. The behaviour of ρ vs β for a $N_t \times N_s^3$ lattice with $N_t = 4$ and $N_s = 12, 16, 20, 24, 32$ is plotted in Fig. 1; the corresponding ρ for the creation of monopoles is plotted in the same figure for comparison. As explained in the introduction, ρ is independent within errors of the Abelian projection used to define the magnetic charges [4].

Similarly to the case of monopoles, the behaviour of ρ at large β is proportional to N_S

$$\rho \simeq -4N_s + C$$

implying that $\langle\mu\rangle = \exp\left[\int_0^\beta d\beta'\,\rho(\beta')\right]$ vanishes for $\beta > \beta_c$ as $N_s \to \infty$. At low β, ρ is bounded from below when the volume is increased, which implies $\langle\mu\rangle \neq 0$ for $\beta < \beta_c$.

Around β_c, $\langle\mu\rangle \sim (\beta_c - \beta)^\delta$. In the scaling region one expects:

$$\langle\mu\rangle \sim (\beta_c - \beta)^\delta \Phi(\frac{N_s}{\xi})$$

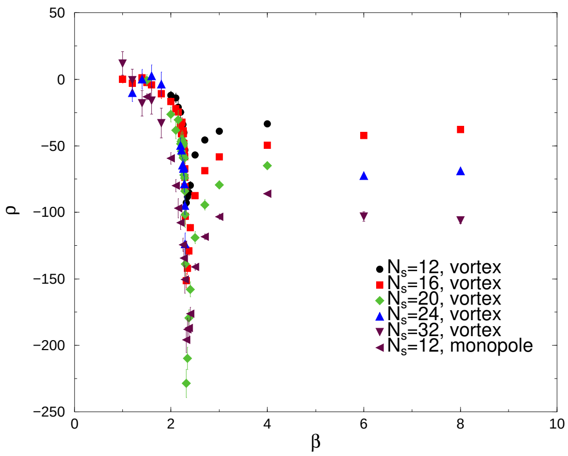

Figure 1. Comparison of ρ defined with the vortex and the monopole creation operator.

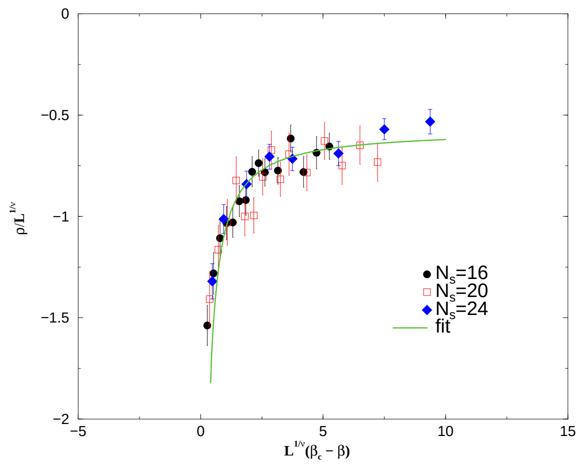

Figure 2. Plot of rescaled data.

where $\xi \sim (\beta_c - \beta)^{-\nu}$ is the correlation length.

This yields the scaling law:

$$\frac{\rho}{L^{1/\nu}} = f\left(L^{1/\nu}\left(\beta_c - \beta\right)\right) \tag{11}$$

We shall assume $f(x) = -\delta/x + c$. Requiring scaling fixes β_c, ν and δ. The quality of the scaling for $SU(2)$ is shown in Fig. 2.

The critical exponents that we obtain are:

$$\beta_c = 2.30(1), \delta = 0.5(1), \nu = 0.7(1)$$

β_c and ν are compatible with the standard determinations [5]. δ is equal within errors to the corresponding index for the monopole disorder parameter. Preliminary data for $SU(3)$ present a similar behaviour [12].

We conclude that the "dual Polyakov line" is a good disorder parameter for confinement. This is a confirmation of the argument in Ref. [1] about the area and perimeter law. A direct check is presented in Ref. [13].

4. Conclusions and outlook

A disorder parameter for confinement, related to the condensation of magnetic charges already exists [10,11], its behaviour is independent of the abelian projection used to define the monopoles, and describes confinement both in quenched [4] and unquenched QCD [6]. This means that dual superconductivity is at work as a mechanism of confinement. Assuming that there exists a dual description in terms of weakly coupled fields, they must carry magnetic charge with respect to any abelian projection.

The result of the present investigation on vortices has no direct implication on the symmetry of the dual fields. It confirms, however, the argument of Ref. [1] that $\langle B(C)\rangle$ obeys the area law when $\langle W(C')\rangle$ obeys the perimeter law and viceversa, which has also been tested in [13]. The dual Polyakov line is a good disorder parameter for confinement, in the same way as the ordinary Polyakov line is a good order parameter. An analogous analysis for $SU(3)$ is in preparation and gives similar results [12]. A measurment of the critical indices using a monopole operator in full QCD is in progress [6]. We are also trying to extend to full QCD the study of the dual Polyakov line. The dependence on N_c of both approaches is an interesting question, and deserves numerical investigation, in order to check the basic ideas of the large-N_c limit.

Acknowledgements We thank M. D'Elia, G.

Paffuti, M. Teper and A. Kovner for enlightening discussions. Financial support by the EC TMR Program ERBFMRX-CT97-0122 and by MURST is acknowledged. BL is funded by PPARC under Grant PPA/G/0/1998/00567.

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