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Determination of a steady velocity field in a rotating frame of reference at the surface of the Earth's core

R. G. Davis* and K. A. Whaler
Department of Geology and Geophysics, The Grant Institute, West Mains Road, University of Edinburgh, Edinburgh EH9 3JW, *UK*

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SUMMARY

We relax the steady-motions theorem by solving for a steady velocity field at the surface of the core in a frame of reference drifting at a linear rate with respect to an observer fixed in the mantle frame of reference. We make the frozen-flux approximation, and compare the misfit of the secular variation (SV) predicted by the drifting velocity field with that from a steady velocity field fixed to the mantle frame of reference. The decrease of the misfit to the geomagnetic SV across the period 1960–80 is substantial, but is marginal across the interval 1930–60. The drift rate changes sign at the 1970 geomagnetic 'jerk' epoch, indicating a change in phase speed between the mantle and core flow. The marginal decrease in misfit prior to 1960 is inadequate to fit the SV data, necessitating a more complex drift function or perhaps a fully time-dependent flow. The results suggest that the SV is driven by deep-seated convection rather than from the core–mantle boundary.

Key words: Core–mantle boundary, flow imaging, geomagnetism.

INTRODUCTION

Early calculations of the core-surface flow from magnetic observations by Kahle, Vestine & Ball (1967) suffered from what are now well known, but not fully resolved, ambiguities. The problem of determining core flow was shown to be non-unique by Roberts & Scott (1965), who also introduced the frozen-flux approximation. Backus (1968) framed the problem in a mathematical context, and since then a number of non-uniqueness-reducing assumptions have been developed. The three most widely used are that the flow is toroidal (Whaler 1980), tangentially geostrophic (Hills 1979; Le Mouël, Gire & Madden 1985; Backus & Le Mouël 1986) or steady (Gubbins 1982; Voorhies & Backus 1985). The interested reader is directed to the recent reviews by Bloxham & Jackson (1991) and Whaler and Davis (1996) for a discussion of the frozen-flux approximation and flow at the core surface.

The steady-flow assumption requires a time-scale over which the flow can be approximated as steady: if the time-scale is too short the flow is not uniquely resolved, and if it is too long the assumption is unlikely to be valid. Voorhies & Backus (1985) formulated a determinant condition that must be satisfied by the magnetic data in order for the assumption to reduce the non-uniqueness. Bloxham & Jackson (1991) compared its magnitude with the inferred errors from point estimates of the field continued downwards to the core surface. They found that, with real data, approximately 50 years were required in order for the data to resolve the velocity field above the amplitude of the randomly generated noise they added to the magnetic signal. Clearly, 50 years is a long time over which to assume a steady flow, given that decade fluctuations in length-of-day (LOD) are due to exchanges of angular momentum between the core and mantle (Bloxham & Jackson 1991), and is perhaps falling into the lower end of the time-scale over which magnetic diffusion cannot be neglected (Bloxham & Gubbins 1987; Gubbins & Bloxham 1987).

Computations of a steady flow can be justified on the grounds that the flow is driven ultimately either by deep-seated convection with a turn-over time of thousands of years or from the core–mantle boundary (CMB) due to horizontal temperature gradients: either way these are stable driving forces on a much longer time-scale than the secular variation (SV), and hence there may be a large-scale steady component. Bloxham (1992) calculated the 'steady part of the SV' over a 150 year time-series of the main field (MF) using a non-linear advection calculation, and was able to fit over 90 per cent of the variance of the time-dependent main field. From a pragmatic point of view, the steady-flow assumption provides us with a method of determining core-surface flow.

We seek to improve the fit to SV data by relaxing the steady-flow assumption, which can only describe linear trends in SV and MF time-series. Although the core fluid 'feels' the effects of the Earth's rotation through the Coriolis force, it is not yet clear if the coupling between the core and mantle is so tight as to lock features of the flow to either thermal,

*Now at: Institute of Physics, University of Bayreuth, W-8580 Bayreuth, Germany.

topographic or electrical conductivity heterogeneities at the CMB. Thus, we examine a suggestion of Voorhies (1984) to allow the flow to be steady in a frame of reference drifting with respect to the mantle, and examine whether such a flow improves the fit to the magnetic data over that of a steady flow stationary in the mantle frame of reference. This is similar to the approach of Zhang & Busse (1990), who set the solution in a rotating frame.

Previously, we solved for a series of flows that were steady over decade-long time-scales in the period 1900–80 in the frozen-flux approximation (Davis & Whaler 1992), and found reasonable fits to the SV within the uncertainty estimates assumed over the later part of the time-series. We could not fit SV even with unconstrained flows over the time-interval encompassing the 1913 magnetic jerk epoch (Ducruix, Gire & Le Mouël 1983), and so have chosen to use only the last 50 years of the time-series of SV available from the time-dependent field model of Bloxham & Jackson (1992). In the following, we formulate the method, present results for the period 1930–1980 and end with conclusions and a discussion.

STEADY FLOW AT THE SURFACE OF THE CORE IN A DRIFTING FRAME OF REFERENCE

The starting point of most core flow inversions, the radial component of the induction equation in the frozen-flux approximation (Roberts & Scott 1965), is

$$\frac{\partial B_r}{\partial t} + B_r \nabla_{\mathrm{H}} \cdot \mathbf{q} + \mathbf{q} \cdot \nabla_{\mathrm{H}} B_r = 0, \tag{1}$$

where B_r is the radial component of the MF, $\mathbf{q}$ is a velocity vector for fluid flow, $\mathbf{q} = (u_r, u_\theta, u_\phi)$ in spherical polar coordinates, in the outer liquid core, and $\nabla_{\mathrm{H}} = \nabla - \mathbf{r}(\mathbf{r} \cdot \nabla)$ where $\mathbf{r}$ is a unit vector normal to the core surface. We assume that the CMB is spherical, and that the mantle is a perfect insulator (see Benton & Whaler 1983). We neglect magnetic diffusion in the core [but see Voorhies (1993) and Voorhies & Nishihama (1994), who have included the effects of a conducting mantle and magnetic diffusion].

As the SV is derived from differencing the MF, the best method for finding $\mathbf{q}$ is to integrate (1) with respect to time and fit the MF, dispensing with SV. Steady flows have been obtained by solving a linearized version of the time-dependent problem (Voorhies 1986a, b) and the full non-linear problem (Bloxham 1989). Here we assume that the main field is perfectly known and fit the SV at a number of epochs, as this substantially reduces the computer time taken and the basic method can be adapted to test various relaxations of the steady-motions theorem.

Assuming that the core fluid is incompressible, and that the radial component of the velocity vector vanishes at the surface of the core, we estimate the velocity field $\mathbf{q}$ at the surface of the free stream, which advects the radial component of the MF (B_r), in order to obtain the radial component of SV ($\partial B_r/\partial t$). We substitute spherical harmonic expansions for MF, SV and for the velocity, i.e.

$$\mathbf{q} = \nabla \times \left[\mathbf{r}c \sum_{l,m} t_l^m Y_l^m(\theta, \phi) \right] + \nabla_{\mathrm{H}} \left[rc \sum_{l,m} s_l^m Y_l^m(\theta, \phi) \right] \tag{2}$$

(where t_l^m and s_l^m are the toroidal and poloidal velocity coefficients, $Y_l^m(\theta, \phi)$ are spherical harmonics of degree l and order m, and c is the radius of the core), into (1). After multiplying by the complex conjugate of the SV spherical harmonic and then integrating over the CMB, we obtain the matrix equation

$$\dot{\mathbf{g}}(t) = \mathbf{A}(t)\mathbf{m}(t), \tag{3}$$

where $\dot{\mathbf{g}}(t)$ is a column vector of SV coefficients, $\mathbf{A}(t)$ is an equations-of-condition matrix relating the MF, SV and velocity field, $\mathbf{m}(t)$ is a column vector of toroidal and poloidal velocity coefficients, and t is time. For details of the derivation see Whaler (1986), Bloxham (1988b) and Jackson & Bloxham (1991).

Now consider a steady velocity field represented by a vector of toroidal and poloidal velocity coefficients, $\mathbf{m}_s$, and a transformation of them into a drifting frame of reference. The transformation is a *phase* translation, so that the velocity field is spatially time-dependent but the total kinetic energy is constant with respect to time. The physical interpretation of such a transformation might be as a weak coupling between the core and mantle: the flow drifts past like a wave but the flow is weakly coupled and unaffected by the mantle to zeroth order. A time-dependent velocity field represented by the spherical harmonic coefficients $\mathbf{m}_t(t)$ can be computed from

$$\mathbf{m}_t(t) = \mathbf{v}_p(t) + \mathbf{R}(t)\mathbf{m}_s, \tag{4}$$

where $\mathbf{v}_p(t)$ is the phase speed which has a single component ordered with the toroidal, solid-body flow component t_1^0, and $\mathbf{R}(t)$ is a square matrix which rotates the velocity field in the azimuthal direction. The matrix $\mathbf{R}(t)$ has columns arranged in the usual sequence for velocity spherical harmonics $(t_1^0, {}^c t_1^1, {}^s t_1^1, t_2^0, {}^c t_2^1 \ldots | s_1^0, {}^c s_1^1, {}^s s_1^1, s_2^0, {}^c s_2^1 \ldots)$, has identical elements for toroidal t_l^m and poloidal s_l^m coefficients, and diagonal elements given by

$$R(t)_{ii} = 1 \quad m = 0,$$

$$R(t)_{ii} = \cos(m\Delta\phi(t)) \quad m > 0,$$

and non-zero non-diagonal elements with the same index i as coefficients with $m > 0$, and with the j index for toroidal and poloidal coefficients given by

$$R(t)_{ij} = -\sin(m\Delta\phi(t)) \quad {}^c t_l^m, {}^c s_l^m \quad j = i + 1,$$

$$R(t)_{ij} = \sin(m\Delta\phi(t)) \quad {}^s t_l^m, {}^s s_l^m \quad j = i - 1,$$

where m is the harmonic order of the velocity spherical harmonic coefficient and $\Delta\phi(t)$ is the angular displacement of the drifting frame of reference at time t. Substituting eq. (4) into eq. (3) gives

$$\dot{\mathbf{g}}(t) = \mathbf{A}(t)[\mathbf{v}_p(t) + \mathbf{R}(t)\mathbf{m}_s]. \tag{5}$$

We will not solve this equation for the velocity coefficients $\mathbf{m}_s$, as $\mathbf{R}$ is time-dependent and non-linear, depending on the cosine and sine of the angular displacement. Instead, rearrange eq. (5) as

$$\dot{\mathbf{g}}(t) - \mathbf{A}(t)\mathbf{v}_p(t) = \mathbf{A}(t)\mathbf{R}(t)\mathbf{m}_s; \tag{6}$$

we can then compute $\mathbf{m}_s$ from a prescribed $\mathbf{v}_p(t)$. An alternative and more numerically efficient method is to transform the input spherical harmonic models of the MF, SV and SV uncertainties into the translating frame, thus avoiding the matrix multiplication $\mathbf{A}(t)\mathbf{R}(t)$. The calculations used the following algorithm:

(1) rotate the MF models through $\Delta\phi = \int V(t)\,dt$, where $V(t)$ is the first element of $\mathbf{v}_p(t)$;
(2) compute the induced SV, $\dot{\mathbf{g}}_{ind}(t) = \mathbf{A}(t)\mathbf{v}_p(t)$;
(3) rotate the SV and subtract $\dot{\mathbf{g}}_{ind}(t)$;
(4) compute the velocity field estimate $\hat{\mathbf{m}}_s$.

With the steady velocity field computed in the drifting frame, we can either compute the predicted SV in the drifting frame and transform it back to the mantle frame of reference, or transform the velocity coefficients back to the mantle frame of reference and compute the predicted SV in the mantle frame from the time-dependent velocity field.

We estimate the velocity field $\hat{\mathbf{m}}_s$ using regularized least squares (Gubbins 1983; Whaler 1986):

$$\hat{\mathbf{m}}_s = (\mathscr{A}^T\mathbf{C}_e^{-1}\mathscr{A} + \lambda\mathbf{C}_m^{-1})^{-1}\mathscr{A}^T\mathbf{C}_e^{-1}\dot{\mathbf{G}}\,, \quad (7)$$

where $\dot{\mathbf{G}}$ and $\mathscr{A}$ have subvectors $\dot{\mathbf{g}}_i$ and submatrices $\mathbf{A}_i$ for each epoch i (see Whaler & Clarke 1988), $\mathbf{C}_e$ is the SV covariance matrix, $\mathbf{C}_m$ is an *a priori* covariance matrix, and λ is a damping parameter. The diagonal elements of $\mathbf{C}_e$ used the SV variances, and the off-diagonal elements were set to zero. The damping parameter λ controls the relative weight attached to fitting the data or generating a smooth model.

As we have introduced another unknown into the velocity field, we simplify the calculation by assuming that $\mathbf{v}_p$ is time-independent, and we use a minimization algorithm,

$$V_{k+1} = V_k + \gamma_k \delta V, \quad (8)$$

where γ is the fractional gradient of misfit and δV is the phase-velocity increment.

The regularization condition used minimizes the second spatial derivative of the horizontal component of the flow,

$$\oint_{cmb} (\nabla_H^2 u_\theta)^2 + (\nabla_H^2 u_\phi)^2\, d\Omega \quad (9)$$

(Bloxham 1988b), which gives *a priori* covariance matrix $\mathbf{C}_m$ elements proportional to l^{-5} for large harmonic degree l. We define, for later use, the solution norm

$$S_n = (\mathbf{m}_s^T\mathbf{C}_m^{-1}\mathbf{m}_s)^{1/2}. \quad (10)$$

The residual norm minimized is given by

$$R_n = [(\dot{\mathbf{G}} - \dot{\mathbf{G}}')^T\mathbf{C}_e^{-1}(\dot{\mathbf{G}} - \dot{\mathbf{G}}')]^{1/2} \quad (11)$$

(where $\dot{\mathbf{G}}'$ is a vector of predicted SV spherical harmonic coefficients). The misfit σ is defined as

$$\sigma = R_n/N^{1/2}, \quad (12)$$

where N is the total number of SV coefficients. The rms flow speed is defined as the square root of the integral over the CMB of the velocity squared:

$$q_{rms} = \frac{1}{2\sqrt{\pi}}\left(\oint_{cmb} \mathbf{q}\cdot\mathbf{q}\, d\Omega\right)^{1/2}. \quad (13)$$

The time-dependent model of Bloxham & Jackson (1992) was used to compute spherical harmonic models of MF, SV, and SV uncertainties, truncated at spherical harmonic degree and order 14. The triangle rule for Gaunt and Elssaser integrals (Bullard & Gellman 1954; Whaler 1986) gives the truncation of the velocity field at $l_{vel} = l_{sv} + l_{mf} = 28$. With the covariance matrix derived from eq. (8), it was safe to truncate the velocity field much earlier, at $l_{vel} = 14$; convergence of the velocity expansion was checked, i.e. higher degree and order coefficients were effectively zero.

RESULTS

Tables 1 and 2 summarize the results for velocity fields calculated over the period 1930–80. The flows are calculated over 10 years and use 11 models of the MF, SV and SV uncertainties. The damping parameter for these solutions has been adjusted to produce reasonable values of induced SV at the CMB and rms flow speed. Table 1 shows the results for purely steady flows fitted over consecutive 10 year intervals. The variation in S_n, increasing with time, except for the epoch 1970–80, reflects on the one hand the variation in SV uncertainties which are larger further back in time, and on the other the complexity of the time dependence of the SV (Bloxham 1987; Bloxham & Jackson 1989, 1992). Table 2 shows the results for the drifting velocity solutions: the solution norm and rms speed for each individual flow are independent of time.

The misfit for the steady and drifting flows for each 10 year interval can be compared directly, as their solution norms are approximately equal—note that the misfit calculated here is not a weighted misfit reflecting the number of parameters used to fit the SV model. If the flows were fully time-dependent and the coefficients were independent of each other at each epoch, then the weighted misfit could be computed from the covariance matrix of the model and its resolution matrix (Gubbins & Bloxham 1985). However, comparing statistically the fit of time-dependent flow and steady flow for a time-series of SV has been postponed because of the large computational effort required to calculate the resolution matrix. In addition, a straightforward application of the analysis would overestimate the misfit for the drifting solutions presented here, as the velocity coefficients at each epoch are non-linearly related to each other.

A comparison of the misfit between the steady and drifting flows shows an improvement in the relative fit to the SV coefficients with the introduction of the drifting frame of reference; in particular, the fit is much improved for the

Table 1. Statistics for five steady flows calculated over the intervals shown in the first column, with a constant damping parameter λ, and the misfit, model norm and rms speed in the mantle frame of reference.

Model time span	σ	S_n (km yr^{-1})	q_{rms} (km yr^{-1})
1930–40	1.56	503	9.4
1940–50	1.60	647	10.1
1950–60	1.77	869	11.23
1960–70	2.27	1291	12.86
1970–80	2.81	1264	14.8

Table 2. Statistics for five flows steady in a drifting frame of reference calculated over the intervals shown in the first column, and the drift speed, the misfit, model norm and rms speed in the mantle frame of reference.

Model time span	V (km yr^{-1})	σ	S_n (km yr^{-1})	q_{rms} (km yr^{-1})
1930–40	−30.13	1.23	573	9.2
1940–50	−75.78	1.22	672	9.9
1950–60	−120.2	1.45	840	11.2
1960–70	−119.6	1.70	1215	9.31
1970–80	+104.6	2.01	1268	11.9

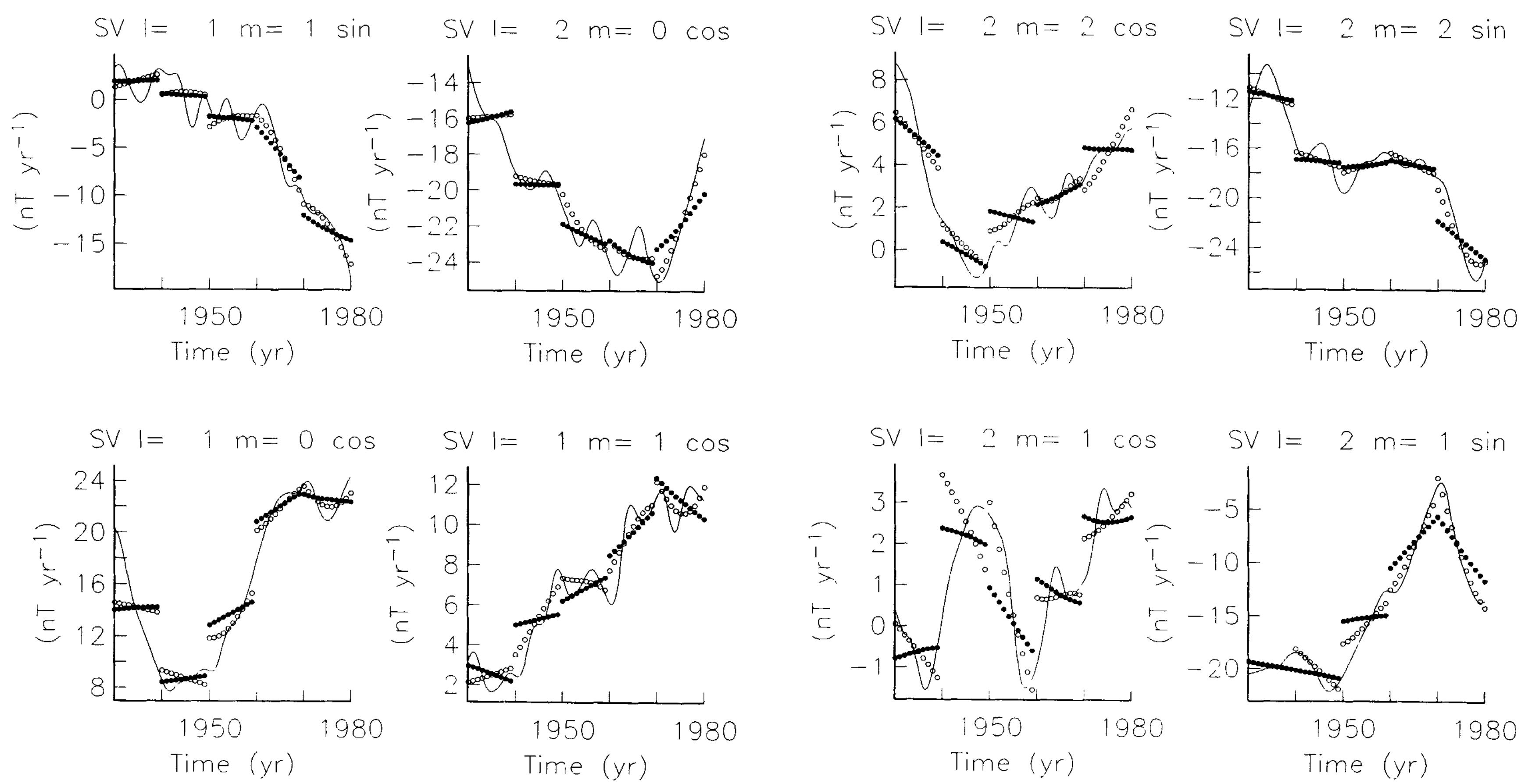

Figure 1. Time-series of predicted SV spherical harmonic coefficients from the steady flows shown in Table 1, the drifting flows shown in Table 2 and the time-dependent model of Bloxham & Jackson (1992). The symbols are: time-dependent model, continuous curve; steady flow, solid circles; drifting flow, open circles. The degree l and order m of each coefficient is given at the top of each plot.

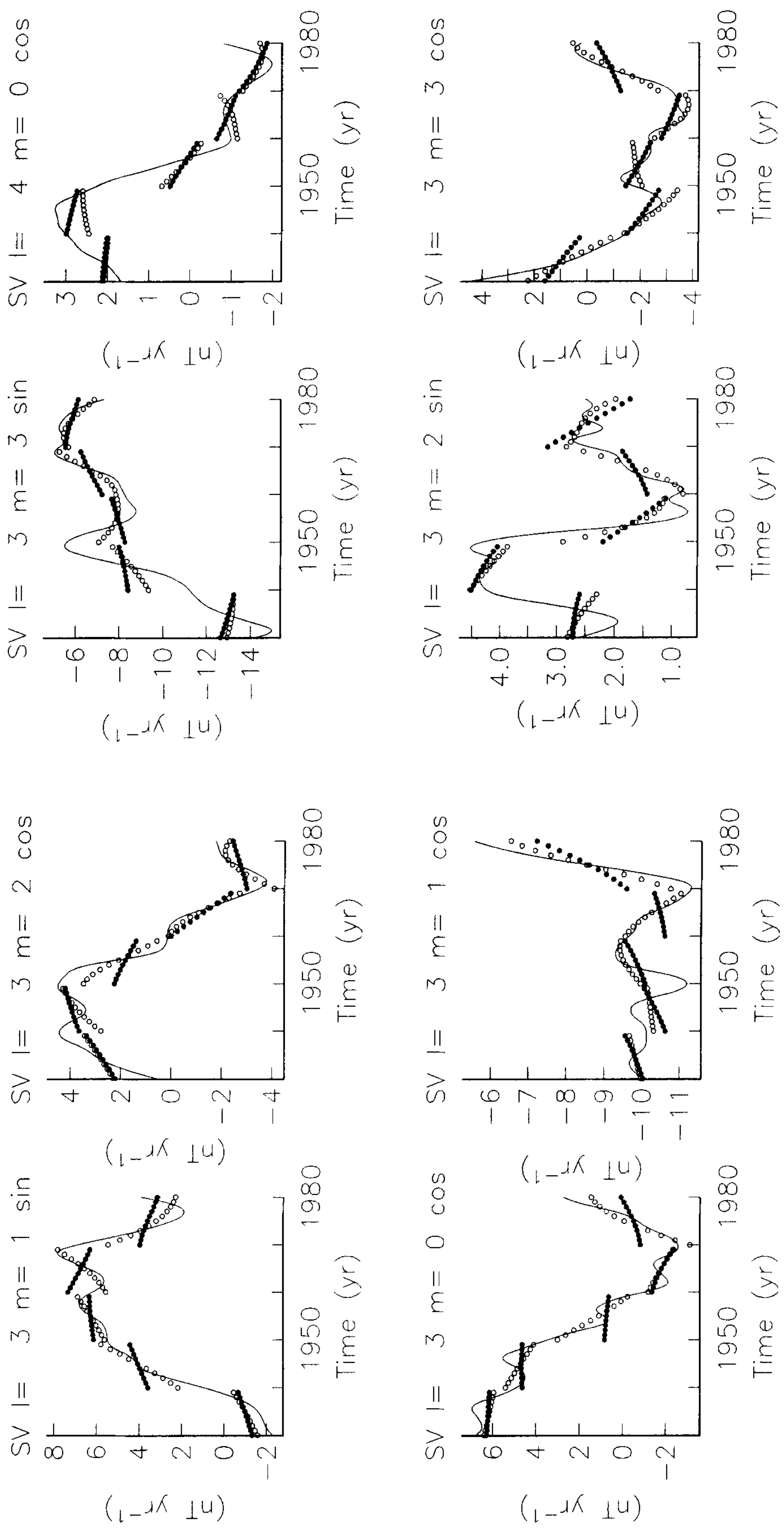

Figure 1. (*Continued.*)

two flows covering the interval 1960 to 1980. Table 2 also shows the sign and magnitude of the drift speed. It should be emphasized that the drift speed is constant over each 10 year interval, and hence the azimuthal angular displacement of the flow pattern with respect to the mantle frame of reference is a linear function of time. The trend of the drift speed in Table 2 as a function of time is negative (giving westward drift of the velocity pattern with respect to the mantle frame of reference), except for the 1970–80 epoch.

Fig. 1 shows time-series of the first 16 predicted SV spherical harmonic coefficients for both the steady and drifting velocity solutions compared with the time-dependent model. The degree l and order m of the spherical harmonic and whether it is cosine or sine are shown at the top of each plot. Regardless of the spherical harmonic, and the time interval of the velocity model, the steady flows can only follow the average trend of the time-dependent model. For the interval 1960–80, the drifting-flow models follow the change in SV much better than the steady-flow ones. In coefficients where the time dependence is strong, for example $\dot{g}_2^0$, $\dot{g}_2^1$ and $\dot{h}_3^2$, the simple linear dependence of the drift velocity does not produce good agreement. Evidence of the 1970 geomagnetic jerk can be seen in some of the spherical harmonic series of SV coefficients; Courtillot & Le Mouël (1988) show that the jerk is dominated by order-one coefficients, $\dot{h}_2^1$ and $\dot{h}_3^1$. In Fig. 1 it is most clearly seen in the SV coefficents $\dot{g}_2^0$, $\dot{h}_2^1$, $\dot{g}_3^0$ and $\dot{g}_3^1$. We interpret the change in sign of the drift speed between the flows covering the intervals 1960–70 and 1970–80 as indicative of the 1970 jerk. The dynamics behind the physical mechanism causing the change in drift speed, however, is a matter of speculation.

For the drifting velocity solutions, it is evident that the flows fitted over the interval 1930–60 are not much improved over their steady-flow counterparts, perhaps indicative of the requirement of time-dependent drift speed, or even fully time-dependent flow. For the interval 1940–50, some of the discrepancies may be due to a change in flow direction beneath the North Atlantic (Bloxham 1989). However, the gradients of the time-series of some of the predicted coefficients are actually reversed so their signs are those of the time-dependent model: for example 1930–40, $\dot{g}_1^0, \dot{g}_2^1$; 1940–50, $\dot{h}_2^1$; 1950–60, $\dot{g}_2^2$. The time-series of others are merely improved: for example 1930–40, $\dot{g}_3^3$; 1940–50, $\dot{g}_1^1, \dot{h}_3^1, \dot{g}_3^3$; 1950–60, $\dot{g}_1^0, \dot{g}_2^1, \dot{h}_2^1, \dot{h}_3^1, \dot{g}_3^2, \dot{h}_3^3$, but a few are made considerably worse: 1940–50, $\dot{g}_2^1$; 1950–60, $\dot{g}_3^3$.

Figs 2(a) and (b) show time-series of the SV of the magnetic components X, Y and Z at two permanent magnetic observatories on the surface of the Earth. These are shown by Bloxham & Jackson (1992) in their comparison of the time-dependent model with the magnetic field data from permanent observatories, and so the reader can compare the predictions of the velocity models and the raw data. These time-series again demonstrate that the steady-flow models can only follow the average trend of the magnetic change, regardless of the station or magnetic field components, but allowing a drifting frame of reference improves the situation. There is a good improvement in the fit to the Y component, particularly in the interval 1950–80 and where there is a strong magnetic jerk in the signal at about 1970, shown in Fig. 2(a). The X and Z components are more difficult to follow with a steady flow, but allowing the frame of reference to drift improves the 1970–80 fit.

Fig 3 shows the flow pattern at the surface of the core as a vector plot of the total flow component; the patterns for 1940–50 and 1960–70 were so similar to that for 1950–60 that they were omitted. The flow in the northern hemisphere is very similar in the three flows, with the gyre changing strength through the sequence. The flow below the southern Eastern Pacific changes direction around 1950–60, and in the southern hemisphere there is strong westward flow in the Atlantic. The flow below the Indian Ocean is weak in 1930–40, strengthens in 1950–60, then weakens again in 1970–80. These flows compare well with those found by other authors, for example the steady flows of Bloxham (1989, 1992) and Voorhies (1993), the geostrophic flows of Jackson *et al.* (1993) and the toroidal flows of Lloyd & Gubbins (1990): see the review by Bloxham & Jackson (1991).

CONCLUSIONS AND SUMMARY

We have demonstrated the improvement in the fit to spherical harmonic models of the SV of a steady velocity field drifting at a uniform rate with respect to the mantle frame of reference in comparison to a steady velocity field fixed to the mantle frame across part of the geomagnetic record. There is still much time dependence in the SV signal that could be accounted for by a more complicated time dependence of the velocity field.

The rationale behind the assumption of a steady flow in a drifting frame is that, if the mechanism driving the core fluid motion, which in turn causes the SV, is stable on a time-scale longer than decades, then the flow will be steady. In addition, changes in the axial component of the rotation of the Earth are communicated to the core via some coupling mechanism at the CMB, and it is not yet clear how vigorous the core flow is and how tightly the fluid motion is coupled to the mantle. The most favoured mechanisms for driving the fluid motion responsible for the SV are deep-seated thermal and/or compositional convection (Gubbins & Roberts 1987) and convection driven by lateral variation in heat-flux at the CMB (Kohler & Stevenson 1990; Bloxham & Jackson 1990; Zhang & Gubbins 1992, 1993). If the latter mechanism dominates, then one would expect the flow to be locked to the mantle frame, and any time dependence would be on the time-scale of chemical and physical fluctuations at the CMB. If deep-seated convection drove the SV and the flow were de-coupled from the mantle, then the flow would drift in a frame of reference with respect to the mantle, and changes in LOD could be accounted for by changes in the axial components of the flow. Thus, further testing of the assumption of steady flow in a drifting frame of reference may allow us to discriminate between the two modes of driving from the geomagnetic record. The improved fit to the SV coefficients across 1960–80 is tentative evidence of a model of SV driven from below rather than from the CMB: we expect to reduce the misfit to a satisfactory size by using a time-dependent drift speed over this time interval, and will also test the change in drift speed found here across the jerk in 1970. It is not clear if the poor fit across 1950–60 and in particular across 1930–50 reflects a non-constant drift rate or some more complicated time dependence of the velocity field. Future investigations will solve for a drift rate that is an arbitrary function of time and extend the calculations back in time to 1840.

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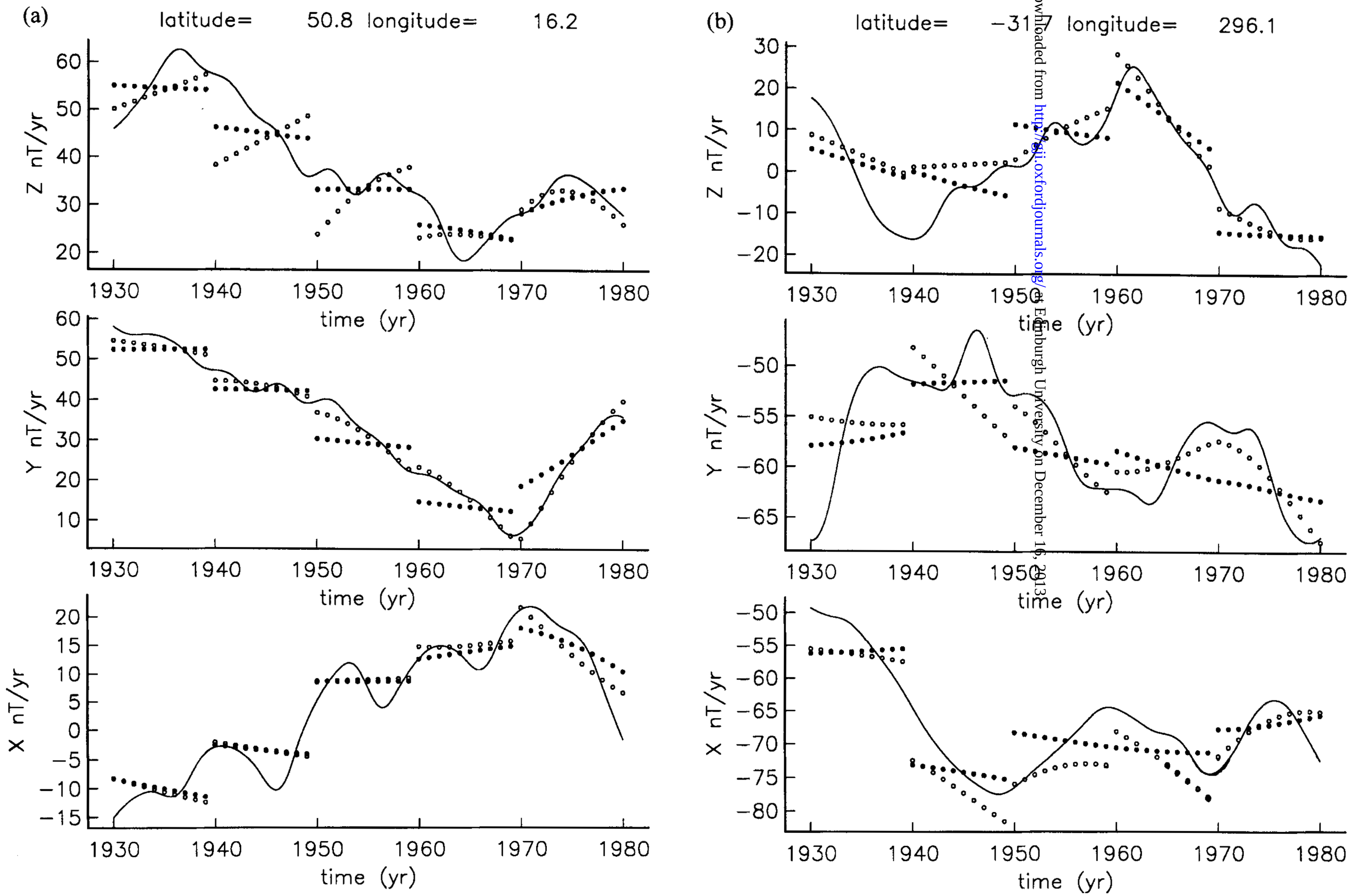

Figure 2. Time-series of the SV of the three orthogonal surface magnetic components *X*, *Y* and *Z* for the steady flows shown in Table 1, the drifting flows in Table 2, and the time-dependent model of Bloxham & Jackson (1992). The locations of the sites are the permanent magnetic stations at (a) Hurbanova and (b) Pilar. See Bloxham *et al.* (1989) and Bloxham & Jackson (1992) for listings of magnetic stations and time-series of magnetic data. The symbols are: smooth continuous curve, time-dependent model of Bloxham & Jackson (1992); solid circles, steady flows; open circles, drifting flow.

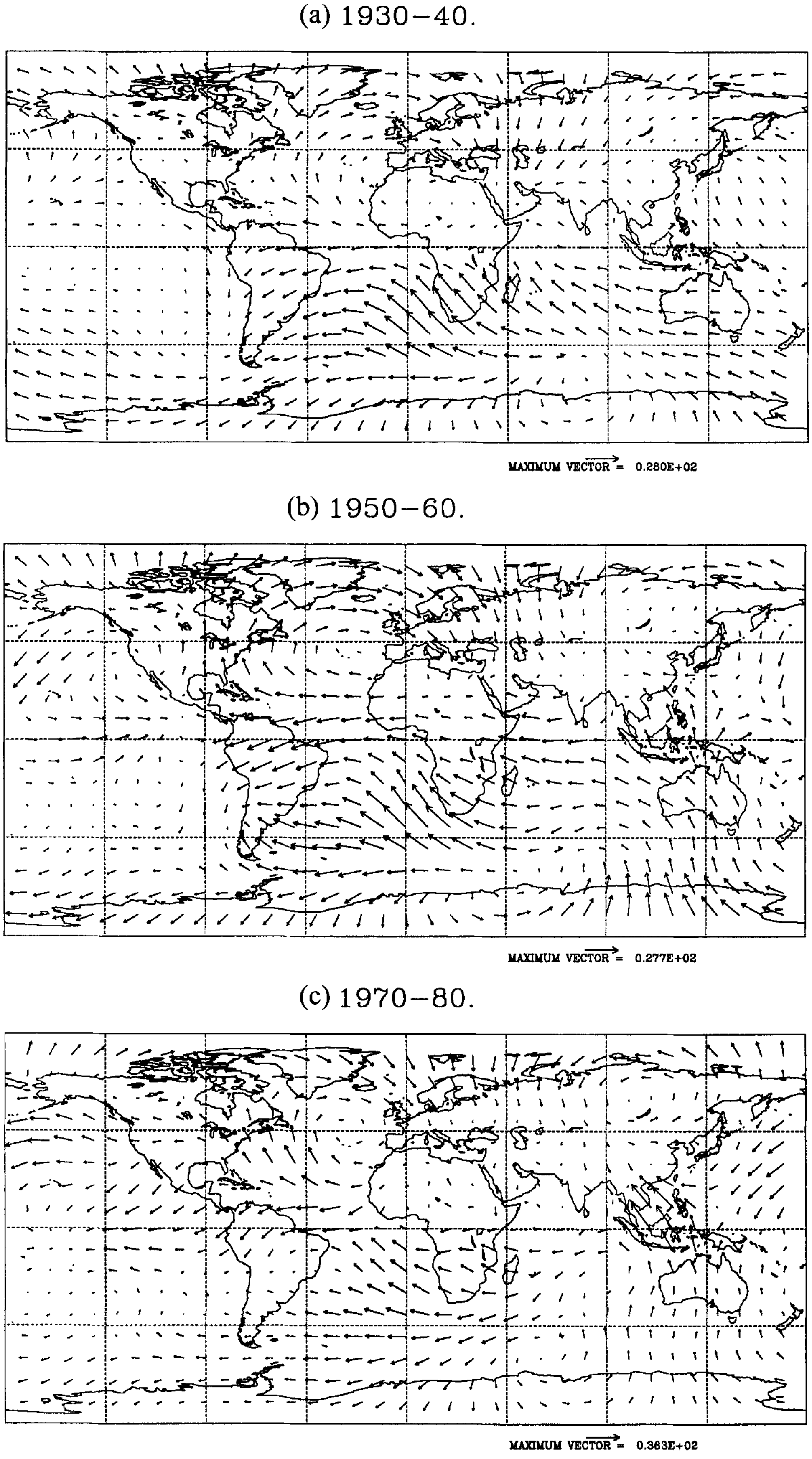

Figure 3. Vector maps of the total surface flow at the surface of the core in cylindrical equidistant plots with reference vectors in km yr^{-1}: (a) 1930–40, (b) 1950–60 and (c) 1970–80.

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