Representationalism in measurement theory – Structuralism or Perspectivism?

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6 Representationalism in Measurement Theory. Structuralism or Perspectivalism?

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1 Introduction

Paradigm shifts, conceptual revolutions, or even just multiple alternative models of ostensibly the same natural phenomenon, system, or entity pose a severe challenge to traditional scientific realism. A standard scientific realist expects that our theories and models correspond to the relevant features of the natural world they are meant to represent, or that they at least aim to do so. As far as the standard realist is concerned, at most one such model will correspond to the way the world actually is; so how can more than one model enjoy predictive and other empirical successes?

In this chapter I look at two contemporary forms of scientific realism, each of which departs in crucial respects from the standard scientific realist: structural realism and perspectival realism.[1] Both take seriously the challenge of a plurality of models and theories, but they wish to retain key elements of scientific realism, such as a commitment to a correspondence between scientific representations and the world, and to the idea that science makes progress. Despite these shared commitments to realism, perspectival and structural realism offer substantially different responses to the challenges that arise from a plurality of models. After laying out the differences between the two views in section 2, I use models of measurement as a type of scientific representation to illustrate the strengths and weaknesses of structural and perspectival realism. I conclude that, at least for meta-sciences like measurement theory, structural and perspectival realism might be complementary.

2 Realism: Structural and Perspectival

Both structural realists (Worrall 1989; Ladyman 1998; French 2014) and perspectival realists (Giere 2006; Massimi 2012; Teller 2017; Teller, this volume) want to address challenges arising from the plurality of scientific representations of ostensibly the same phenomenon or subject matter while maintaining a broadly realist outlook on science.[2] The plurality of models is a challenge for the requirement of literalness of scientific representation endorsed by traditional scientific realists. The requirement of literalness is typically articulated from the syntactic view of theories and amounts to the claim that theoretical terms do not in general differ in their semantics from observational terms. The debate I focus on here, by contrast, is played out against the backdrop of the semantic view of theories, which takes *models* as paradigmatic scientific representations. Whereas a traditional realist would be inclined to hold that a successful representation of a phenomenon means that there is a close correspondence between elements of the model and elements of the phenomenon represented, both structural and perspectival realists recognize that models contain features that do not readily correspond to features in the phenomenon represented. The fact that sometimes more than one model can be used to represent the same phenomenon provides particularly strong evidence for this lack of literal correspondence between model and phenomenon. The question for both structural and perspectival realists is how to respond to the plurality of models while retaining a commitment to realism. Structural realists focus on the *commonalities* among different representations and models, whereas perspectivalists emphasize the *differences* between models.

Structural realists suggest that we should focus on what is common to competing (successful) representations and that this commonality is structural. While each model will differ from the others in some way, all models of the phenomenon will have certain structural similarities. Our task is to identify these structural similarities, which is often done by finding transformations between models that leave particular features invariant. According to structural realism, what we learn about the world from these different models is confined to the structural similarities they share. Some structural realists want to take this epistemic view further and conclude that the world itself contains nothing but structure, but for present purposes I shall be concerned only with epistemic structural realism, not ontic structural realism (for the distinction, see Ladyman 1998).

For a plurality of models of ostensibly the same entity/system, this will mean that structuralists will only take features present in *all* models as *representational*, that is, only those features that are shared between the different models will count as relevant to the question of truth-qua-correspondence, whereas features pertaining only to some models will be regarded as artifacts of the representation. Structuralists will further add that what is shared between the different models are *structural* features, which are contrasted with haecceitistic or quidditistic differences among models. Structural features typically include relations among the elements of the model which remain invariant even as we "swap" or "replace" particular elements. For example, representations of particles that differ only with respect to *which* particle in an ensemble of identically prepared particles has a given property will be regarded as only having haecceitistic difference. Such haecceitistic differences, according to the structuralists, are not up for evaluation with respect to which of them "gets it right"; instead they are merely artifacts of representation. Models that differ from the original model only in assigning different "labels" to the particles are structurally similar to the first one, and there is nothing to choose between them.[3] Structural realism reduces the plurality of representations by treating many representations as equivalent.

The relational character of structure is contrasted, on the one hand, with haecceitistic and quidditistic differences and, on the other, with nature and ontology. In both cases, the idea seems to be that the same relational structure may underpin different conceptions of the nature of the phenomenon in question, or be instantiated in models with haecceitistic or quidditistic differences. Unsurprisingly, perhaps, structuralists have focused much of their attention on the highly mathematized models used in the physical sciences. For such models, it is comparatively easy to give a characterization of the structure of the representation, and the abstraction involved in the mathematical representation makes it easier to see how the same relations can be used in otherwise different theories and models. The main challenge for structuralism is to develop a notion of structure that is both substantive enough to be controversial while also being a plausible candidate for what is in fact preserved across different theories/models.

Perspectivalists take a rather different approach to the plurality of models. Instead of focusing on the commonality among different models, perspectivalists regard each model as a complementary perspective on the same phenomenon. Unlike the structuralist, who limits what we should take as corresponding to the world in our models to what is (structurally) common to them, the perspectivalist takes differences between models as (potentially) informative about the world. Perspectivalists reject the idea that we ever approach the natural world independent of taking a particular perspective. Representation is inevitably perspectival; there is no view from nowhere (Giere 2006; van Fraassen 2008).

Moreover, not all perspectives are easily compatible. Notoriously, water is described as a viscous fluid by fluid dynamics and as a collection of particles by statistical mechanics.[4] These two descriptions seem to be in direct conflict, attributing contradictory properties to the same entity. Traditional realists would be inclined to insist that at least one of these models must be mistaken: it simply does not correspond to the nature of water. Structural realists might try to retreat to merely structural features of each model, but it is not obvious how that is going to resolve the difficulty in this case. Perspectivalists, by contrast, would like to retain both models as offering important insights into the nature of water. Neither is to be given up in favor of the other. Instead both models say something true about water, something that would be lost if we chose only one perspective as the uniquely true perspective. Whether there is nonetheless room for realism from a perspectivalist standpoint will depend on how perspectivalists can characterize the relationship between the two or more apparently inconsistent models (Giere 2009; Massimi 2018b).

Unlike structural and traditional realism, then, perspectivalism seems to be committed to a form of unavoidable pluralism. One question for perspectivalists is whether this pluralism is confined to our knowledge and representation of the world or whether it extends to the world, that is, whether the plurality of perspectives reveals that the world itself is somehow ontologically pluralistic. Especially the latter view seems to be difficult to reconcile with core commitments to realism. Realists are typically committed to a realist semantics for scientific representation, an optimistic epistemic outlook on scientific success and progress, and a picture of the world as uniquely structured in a certain way (Psillos 1999). This final point matters if we are to make sense of realism as being committed to the idea that our scientific theories correspond to what the world is like in its own right. If ontology is radically pluralistic, perhaps in the sense that entities only exist insofar as they are represented in a certain way, then this would seem to undermine a basic commitment of scientific realism. Perspectivalists in the philosophy of science do not typically wish to embrace this radical ontological departure from standard scientific realism (Massimi 2012).

The main challenge for perspectival realism, then, is to make sense of the idea that each perspective captures something true about the phenomenon in question while maintaining that these perspectives shed light on the same phenomenon or entity. This claim suggests a notion of perspectival truth that requires clarification and defense, since it seems difficult to reconcile the pluralism inherent in perspectivalism with the idea that claims about the world are either true or false. Some claims, it seems, would be true according to one perspective, yet false according to another (see Massimi 2018a for a qualified defense of perspectival truth). Moreover, something needs to be said about why it is that the different perspectives contribute something epistemically valuable to inquiry, while nonetheless remaining distinct and possibly irreconcilable. Even if the pluralism is confined to our knowledge or representation of the world, most realists would also be uncomfortable with the idea that our knowledge is always confined to perspectival knowledge only (Chakravartty 2010).

Perspectival and structural realism, then, differ in their approach to scientific representation. To assess the strength and weaknesses of each as realist approaches to scientific representation, I will now turn to measurement theory. Measurement theory addresses the question how numerical representations of empirical attributes and phenomena of interest are possible. Any form of realism about such representations will want to insist that there are some constraints on which representations qualify as adequate representations of the relevant attributes.

Measurement theory is of particular interest for the comparison of perspectival and structural realism for two reasons. First, measurement theory is not a first-order science in the manner of physics or biology; its subject matter is not a specific class of phenomena or aspect of the natural world. Measurement theory, at least as it is understood today, is a meta-science that studies the mathematical formalism used to represent measurements. What we can learn from it may hence be quite different from the conclusions we draw from case studies of models in particular sciences. Second, since measurement theory explicitly deals with a certain type of scientific representation, it seems especially appropriate to ask what structural and perspectival realists might have to say about it. In the next section I will present some problems for a literalist reading of measurement representations, which I interpret as being akin to traditional scientific realism. In sections 4 and 5 we will see how structural and perspectival realism can be combined to provide a better understanding of measurement representations.

3 Representationalism and Literalism in Measurement Theory

Measurement theory was not always a meta-science. Especially in the first half of the 20th century, the study of measurement and quantities was considered part of physical theorizing. Many important contributions to measurement theory were made by physicists, often as part of working out the foundations of physics (Tolman 1917; Campbell 1920; Bridgman 1927). The idea behind these approaches was that measurement theory was supposed to give an accurate account of physical quantities. Physical quantities were thought to be unique in permitting numerical representation, and the question was which features of these attributes made them numerically representable.

Early axiomatizations of measurement focused on the idea that quantitative attributes were numerically representable *because* they were additive (Helmholtz 1887/2010; Hölder 1901). We can both order objects of a domain by length (from shortest to longest) and concatenate objects in the domain in such a way that the combined object has the "sum" of the lengths of the two concatenated objects. Lengths, masses, and other paradigmatic physical magnitudes can be "added" in (almost) the way numbers are added. The natural conclusion for many thinkers was that quantities can be given numerical representations in virtue of being additive. Additivity was thereby made into a necessary condition for being a quantitative attribute. These early axiomatizations for quantities contained two types of axioms: axioms governing the ordering of objects and axioms governing additivity. These axioms were thought to constrain how numbers could be assigned to objects, or perhaps they were understood as something like conditions for the possibility of numerical assignment.

This approach to the question of how numerical representations of attributes are possible is characterized by a form of "literalism": it is possible to represent attributes numerically if and only if there is a direct correspondence between features of the attribute and features of the numbers. Moreover, one such feature, additivity, was selected as a necessary and sufficient condition for all numerical representation. For physical attributes, additivity had to be demonstrated empirically, by finding suitable concatenation operations for objects instantiating the attribute in question. The apparent direct correspondence between the operation of placing rods end to end, or placing weights in the same pan of a beam balance, and arithmetical addition operations on numbers was understood to be the key to the numerical representation of attributes like length and mass.

While additivity seems to fit nicely as a criterion for some paradigmatic physical quantities like mass and length, it does not fit neatly for all physical quantities. There are two types of problems. First, not all physical attributes seem to be additive in the sense that combining objects with different magnitudes of these attributes results in an increased magnitude of the attribute that could be interpreted as the sum of the two contributing magnitudes. Density and temperature are typical examples of this. Mass density is understood as mass per volume. Both mass and volume are additive quantities and hence fall squarely into the physical measurement paradigm. But while the masses and volumes of appropriately concatenated objects will increase in such a way as to form the sums of the respective masses and volumes, the same is not true for density. Fluids of different densities will typically form uniform density layers (e.g., when trying to mix honey and milk) instead of combining or produce a mixture of intermediate density somewhere between the two starting densities. Similarly, if we mix two fluids of different temperature, say coffee and milk, the resulting fluid does not have a temperature that is the "sum" of the two contributing temperatures but instead an intermediate temperature.[5]

The second type of problem is due to the operationalism built into many versions of the additivity paradigm as a result of its commitment to literalism. Additivity of an attribute is linked to the availability of a concatenation operation for objects instantiating the attribute, which means this approach rules out attributes for which no concatenation operation is available and attributes for which no *unique* concatenation operation is available.

Concatenation operations do not seem to be available for temporal intervals (except perhaps for the special case of adjacent intervals), yet we do think that time is numerically representable and indeed in some sense additive. The problem here is simply that we cannot manipulate events and intervals as easily as we can manipulate certain kinds of physical objects. Even in the case of physical objects, our ability to concatenate them is limited: we assume that the masses of planets behave in a manner comparable to that of pebbles, even though we cannot concatenate the former in the same way we concatenate the latter.

On the other hand, some quantities seem to have more than one "natural" concatenation operation. Compare, for example, electrical resistance in series and parallel circuits. Resistors connected in series yield additive resistance in a straightforward way: the total resistance in the circuit is just the sum of the resistance of each resistor. Resistors connected in parallel do not yield additive resistance, but yield the reciprocal of resistance: the total reciprocal resistance is the sum of the reciprocal resistance of each resistor. Neither parallel nor series circuits are more natural than the other, yet in both cases we seem to end up with an additive quantity: resistance and its reciprocal. The two quantities seem so closely connected that even distinguishing them seems somewhat misleading. Instead it looks like there are just two different ways of concatenating resistors, and either way of doing it yields a total resistance measure that is additive. There is no unique way of combining resistors in a circuit that yields an additive representation; instead there are two.

A similar sort of problem can be generated for the case of length. While we ordinarily assume that the natural way to concatenate lengths is to place rods end to end in a straight line, Brian Ellis (1966) showed that placing rods at right angles to each other also yields an additive representation of length, just not the one we find familiar. While Ellis's example might seem contrived, it is very difficult to say why we should prefer our standard concatenation of length to his unconventional one, other than sheer familiarity. The concern for the additivity paradigm is that the straightforward link between a natural concatenation operation and a numerical representation of the attribute featuring the addition operation breaks down.

The additivity paradigm is motivated by a form of *literalism* about numerical representations of quantities: quantitative attributes are numerically representable because, under concatenation, objects with that attribute behave like numbers with respect to addition. Numbers correspond to objects, and addition between numbers corresponds to concatenation between objects. If there is either no plausible way of concatenating the relevant objects (e.g., temporal intervals or planets) or if there is more than one plausible way of doing so (e.g., rods or resistors), then this literal interpretation becomes doubtful. There is no longer a unique, natural correspondence between the manipulation of objects (and thereby indirectly the magnitudes of quantities) and the numbers.

The literalism of the additivity paradigm is, hence, rather restrictive. While there are some physical quantities that satisfy the strict requirements of additivity (at least in a limited domain), even among physical quantities there are problem cases. For sciences other than physics, the problem is far more severe: in sciences like psychology, no attributes of interest seem to have additive structure or be amenable to concatenation. Unsurprisingly, psychologists like S. S. Stevens (1946) rejected the additivity paradigm and proposed instead that measurement simply meant the numerical representation of attributes according to some rule or other. This notion of measurement strikes many as too weak and too easily achieved (see Michell 1999 for a detailed critique of this and related notions of measurement in psychology). The question is, therefore, whether it is possible to free measurement representations from the shackles of literalism without giving up on the idea that numerical representations of attributes reflect something about the nature of the attributes thus represented.

The radical literalism of the additivity paradigm is akin to the view standard scientific realists take with respect to scientific representation in general. The standard realist expects that features of the representation correspond (literally!) to features of the phenomenon or entity represented and, conversely, they require of a representation that it captures the features of the represented entity. A close correspondence between features of the representation and features of the represented entity is what makes for successful scientific representation. This is the reasoning behind the additivity paradigm as well. Numerical representations are additive and, hence, we want to be entitled to infer that attributes represented numerically are also additive. Conversely, if a representation were to lack key features of an attribute, such as its additivity, the representation would be inadequate. For standard realists, this kind of literalism is part of what it means to be a realist.

In the following section, I will look at the representational theory of measurement (RTM), which arose in response to the problems with the additivity paradigm. I shall first show that RTM looks like a form of structural realism about representation. In section 5, we will see that this structural realism needs to be combined with perspectivalism.

4 Structural Realism in the Representational Theory of Measurement

Today measurement theory is a mathematical framework that describes the conditions under which numerical (and, more generally, mathematical) representations of attributes are possible. The most developed framework of this kind is the representational theory of measurement, which describes measurement as a representation of empirical relational structures by numerical relational structures (Krantz, Suppes, Luce, and Tversky 1971, 9). Even contemporary alternatives to representationalism, for example (Domotor and Batitsky 2008), share this highly mathematical character and do not proceed from within a particular science. A great advantage of RTM is that it describes a range of different types of structures axiomatically and shows what type of numerical representations are possible for these structures. The representationalist theory thereby incorporates a key feature of Stevens's permissive approach to measurement, namely the idea that *different* features of numerical structures can be used to establish a mapping between an empirical structure and a numerical structure. There is no need for such mappings to be confined to additive structures. Additive structures become merely a type of empirical structure that can be numerically represented.

To provide such an axiomatic framework, RTM first provides axioms for various types of structures.[6] A structure is here simply a set with relations and operations defined on it. The exact nature of the relations and operations is specified by the axioms. Crucially for RTM, both numerical structures (e.g., the real numbers, ordering, and addition) and empirical structures (e.g., a set of weights when ordered and concatenated using a beam balance) might satisfy these axioms. By characterizing structures in this abstract, axiomatic fashion, RTM lays the foundation for showing how a mapping from the empirical structure[7] to the numerical structure is possible. Such a mapping will typically be a *homomorphism*, that is, a structure-preserving map. As we represent an empirical structure using a numerical structure, the numerical structure will reflect *structural* features of the empirical structure. According to RTM, this preservation of structure is the key to understanding measurement representations. Much of RTM then proceeds to show, in a mathematically rigorous way, what kinds of representations are possible for different types of empirical structures.

To do so, first a representation theorem and then a uniqueness theorem are proved. The former demonstrates that if an empirical structure satisfies the axioms for a particular structure, for example an additive extensive structure, then there is a structure-preserving mapping from the empirical relational structure to a suitable numerical structure (suitable insofar as the numerical structure will also satisfy the axioms for additive extensive structures), such that certain conditions are satisfied. For additive extensive structures the following two conditions are satisfied: (i) the mapping is such that the ordering of objects in the empirical domain is reflected in the order of the numbers assigned to the objects: $a \prec b$ iff $f(a) < f(b)$; and (ii) the mapping is such that the concatenated object $a \circ b$[8] is mapped to the sum of the numerical values for a and b: $f(a \circ b) = f(a) + f(b)$.

The uniqueness theorem then shows how unique this mapping from the empirical structure to the numerical structure is by demonstrating how other mappings satisfying the same two conditions are related to our original mapping f. It turns out that for additive extensive structures, any mapping f' such that $f' = \alpha f$ for some real value $\alpha > 0$ will satisfy the two conditions given above. So once it has been established that one such homomorphic mapping from the empirical structure to the numerical structure is possible, many more such mappings are also possible, differing from the first one only by multiplication by a positive factor α. In measurement practice this is often taken to mean that we can change the unit of measurement, for example, from centimeters to inches, without losing any important information. The representational theory of measurement thereby shows which numerical representations are equivalent in the sense of being mere notational variants of each other.

While the preceding example illustrates the features of mappings for additive extensive structures, the same general method is applicable to other structures as well. Indeed, this is what most of the rest of *Foundations of Measurement* concerns itself with: various types of structures are axiomatically characterized and then shown to be representable by numerical structures to varying degrees of uniqueness. Whereas earlier axiomatizations of measurement had focused on capturing what was necessary for establishing the additive character of an attribute, RTM instead begins from the idea that measurement involves an axiomatic characterization of a measurement structure but does not put any constraints on the features such a structure might have. Once a measurement structure has been axiomatically characterized, we can then ask what kind of numerical representation of such a structure might be possible (the representation theorem) and how unique such a representation will turn out to be (the uniqueness theorem). Mass is numerically representable because massive objects stand in empirical relations of ordering and concatenation, that is, it satisfies the axioms for additive extensive structures. Temperature, on the other hand, is numerically representable because relations of congruence and betweenness hold between differences in temperature; temperature satisfies the axioms for absolute difference structures.[9] The features that make possible a representation of an attribute by numbers are structural features, as is clear from the fact that the mapping between them is a homomorphic mapping: a mapping that preserves structure.

The axiomatic characterizations of RTM are distinctively structural: the axioms characterize structures, that is, sets with relations and operations defined on them. This structural characterization turns out to be more abstract than the literalist construal of attributes as additive. An additive extensive structure, for example, is characterized by axioms describing a set with an ordering relation and a binary operation that satisfy certain conditions. The binary operation does not have to be addition, even though numerical addition satisfies the axioms. But other binary operations, like multiplication, work just as well. A consequence of this axiomatic approach is that even though the numerical structure used to represent a particular attribute may be additive in the sense of involving the addition operation, the attribute thus represented might lack a concatenation operation or might lack a unique concatenation operation. RTM can thereby explain some of the anomalies encountered under the additivity paradigm. Length and electrical resistance have additive extensive structures because they satisfy the abstract axioms specifying such structures. It turns out that they can do so in different ways depending on the empirical set-up chosen, but since the mapping is not thought to hold between a particular concatenation operation and numerical addition, instead holding in virtue of the satisfaction of the axioms, these cases are no longer anomalies under the new paradigm.

Moreover, since RTM describes a wide range of different structures, only some of which are characterized by axioms involving binary operations, RTM can allow for the numerical representation of attributes like temperature and other "intensive" quantities. RTM thereby avoids the constraints placed on numerical representation by the additivity paradigm.

The resolution of the anomalies and the inclusion of non-additive attributes is made possible by the move to a structural characterization of the target of measurement representations. Instead of literalism, which committed the additivity paradigm to the claim that measurable attributes must be additive like numbers, representationalism allows for a variety of ways in which attributes can have structures that satisfy specific axioms. Since the representation theorem shows that structures satisfying the axioms are representable by certain numerical structures (because it is possible to construct a structure-preserving map from the empirical to the numerical structure), the structural characterization is key to the representational theory. RTM assumes that what makes numerical representations possible is a structural similarity between numbers (and the relations and operations defined on them) and attributes, like mass or temperature (and the empirical relations and operations available for collections of objects instantiating them). Moreover, to decide when two numerical representations should count as notational variants of each other, RTM asks whether the two representations preserve the same structure. This is done through the uniqueness theorem, which compares homomorphic mappings to one another.[10] Structure that is invariant across different mappings is considered an objective feature of the attribute in question.

RTM can therefore be described as a form of structural realism about representation: structural commonalities among representations of the same attribute are indicative of objective or genuine features of the attribute, whereas features that vary in different representations (such as a change in unit) are to be regarded as conventional artifacts. Like structural realism, RTM assumes that there is a clear distinction between elements of the representation that correspond to features of the represented attribute and elements of the representation that are due to convention only; moreover, the features that correspond to features of the attribute are *structural* features only. Structural features are here once again *relations*, in contrast to haecceitistic features. Structural correspondence, as demonstrated through structure-preserving mappings, makes for successful representation for RTM.

5 Keeping Things in Perspective

RTM seems to be committed to a form of structural realism about numerical representation, and insofar as RTM is the most developed framework for measurement representations, this would suggest that structural realism provides an adequate account of numerical representation. Before drawing that conclusion, however, we must ask how widely accepted the representationalist paradigm for measurement theory is. Few would argue with the claim that the representationalist theory of measurement, especially as presented in *Foundations of Measurement*, constitutes the most thorough formal treatment of "measurement structures" and their numerical representation. Nonetheless, representationalism is not without critics, with a main line of criticism being whether representationalism truly amounts to a theory of *measurement* (Savage and Ehrlich 1992). This criticism seems even more pertinent given that RTM has not had as much of an impact on research practice in fields like psychology as might have been expected (Cliff 1992). While there are several aspects of measurement practice that seem to receive relatively little attention on the representationalist view, for the purposes of perspectivalism the most interesting criticism concerns the question of how we decide whether a given attribute satisfies the axioms for a particular measurement structure.

The application of the representationalist framework in any given scientific context requires three steps: one conceptual, the other two mathematical (Luce et al. 1990, 201). The first step (i) is to determine *whether* an attribute of interest satisfies the axioms for a given measurement structure. Once this has been established, the representationalist framework can then be used to show (ii) *that* a numerical representation of the attribute is possible and (iii) *how unique* that representation is. Steps (ii) and (iii) are important for establishing which scale type is appropriate for the attribute and which inferences can be drawn from the representation. The representationalist theory of measurement provides detailed proofs of representation and uniqueness theorems for wide range of axiomatically characterized measurement structures, which ensures that steps (ii) and (iii) are clearly justified in setting up a numerical representation. However, RTM has very little to say about the very first step.

To establish that an attribute of interest can be numerically represented, we need to know whether we have reason to believe that the attribute satisfies the axioms for some measurement structure. If such reasons can be found, then RTM simply provides steps (ii) and (iii). Whether such reasons can be found, however, will depend both on empirical observations and theoretical assumptions. Some of the theoretical assumptions are "inductive." Suppose we have found a means to concatenate objects systematically for a finite range of magnitudes for a given quantity and that these concatenations do indeed yield "sums." We might then wish to extend the assumption that magnitudes of this quantity are additive beyond the range for which we are able to carry out empirical concatenations. This is the type of assumption that leads us to conclude that mass satisfies the axioms for additive extensive structures, even though we have only concatenated a limited number of massive objects and even though some types of massive objects, like planets, eschew concatenation altogether. Another type of theoretical assumption concerns the dependence of quantities on other quantities, a situation commonly exploited in "indirect" or "non-fundamental" measurement. In these cases, the structure of one of the attributes is inferred from its nomic relationship to other attributes whose structure is presumed to be known. This situation is common for many measurements in physics, for example, for the measurement of temperature using the relationship between pressure, temperature, and volume. But how can we establish that such a nomic relationship indeed holds, without being able to measure each quantity independently?[11]

Much of the dispute in sciences like psychology and in other fields concerns precisely the question whether we are justified in assuming that a particular attribute indeed satisfies the axioms for a particular measurement structure. Reasons to support such a claim are never free from theoretical assumptions of the sort mentioned and can hence be contested. This theory dependence opens the door for a more perspectivalist reading of measurement representations.

The perspectivalist reading begins from the observation that an axiomatic structure by itself does not represent anything in particular. To be a representation of a particular attribute or empirical structure, the axiomatic structure needs to be interpreted. This interpretation connects aspects of the phenomenon of interest to the axioms characterizing that abstract structure. For measurement structures the interpretation will involve characterizing the phenomenon or attribute of interest as having a certain structure (van Fraassen 2008). Interpretations like these, as we have just seen, can be contested because they make theoretical assumptions.

Philosophers sometimes seem to think of interpretation as the task of finding a suitable empirical interpretation for an otherwise unspecified axiomatic structure. But while this highlights the way in which axioms leave their interpretations unspecified, this is not the way most scientists encounter the problem. Quite the reverse. Scientists typically start from an attribute or phenomenon they wish to represent numerically. The question is, *which* structural representation is appropriate? Ostensibly the same attribute (e.g., utility or temperature) or phenomenon (e.g., light or water) is given different structural characterizations in the context of different theories. For example, Bradford Skow has argued that thermodynamics only provides very weak reasons for thinking that temperature has a metric structure (either an absolute difference or a ratio scale structure), whereas statistical mechanics provides strong reasons for thinking that temperature has metric structure (Skow 2011). Which (measurement) structure we are justified in ascribing to temperature, then, depends on our theory of temperature.

This type of theory dependence looks like the theory dependence of other types of scientific representation. Recall the models of water as a fluid and as a collection of particles we briefly discussed at the beginning. Traditional realists will suggest that at most one of these models correctly represents water, whereas perspectivalists suggest that each offers an informative perspective on water. Similarly, for the case of temperature, a traditional realist will be inclined to suggest that statistical mechanics provides the correct account of what temperature is and, hence, the correct assignment of structure to temperature. By contrast, a perspectivalist will insist that we take seriously both the thermodynamic and statistical mechanical perspectives on temperature. Either way, the structural realism embedded in RTM does not tell us whether to go with traditional realism or perspectivalism on this point, since RTM does not deal with the question of how to decide which structure to ascribe to particular attributes.

Perspectival realists differ in their responses to the problem of inconsistent models (compare, for example, the difference between Giere 2006 and Massimi 2018b). In the case of different measurement structures ascribed to temperature, it is tempting to conclude that, since statistical mechanics is a more fundamental theory than thermodynamics, we should simply go with the structure ascribed to temperature by statistical mechanics. This reading seems even more compelling when we remember that in this case the two structures ascribed to temperature are not strictly speaking incompatible. After all, the metric structure ascribed to temperature by statistical mechanics is simply stronger than the mere ordinal structure implied by thermodynamics: an attribute that possesses metric structure also possesses ordinal structure. Semantically, then, we should accept the structural realists' claim that the structure of the numerical representation corresponds to the structure of the attribute. Which representation is adequate is then a question of what structure the attribute actually has. Structural realism here looks very much like traditional realism in its commitment to a correspondence between features of the representation and features of the represented attribute. The only difference between structural realism and traditional realism is that structural realism restricts this correspondence to structure only.

Perspectival realism has a different contribution to make, however. As Massimi (2012) has argued, perspectivalism contributes to the realist quest by supplying the relevant notion of justification. To be realists, not only do we need to have a realist semantics of the relevant representations, but we also need a realist epistemology that distinguishes justified from unjustified beliefs about the phenomena and entities in question.

The justification (as opposed to the aptness) of a belief, according to Massimi, is a matter of coherence with a given scientific perspective.

This notion of perspectival justification is relevant for the case of the attribution of measurement structures. The literalist additivity paradigm proceeded from the assumption that the availability of an empirical concatenation operation was both necessary and sufficient for an attribute to qualify as a quantity. No other justification could be given for representing an attribute numerically. The additivity paradigm thereby provided a universal criterion for quantitativeness, with no room for different theoretical approaches. On the representational theory, this requirement has been given up, but at the cost of leaving open how we should justify ascribing a specific structure to an attribute. Perspectival realism (of the epistemic variety) fills in the gap. Measurement structure is ascribed to an attribute from within a scientific perspective, such as thermodynamics or statistical mechanics. The ascription of a particular measurement structure to a given attribute has to cohere with the relevant theoretical background commitments and beliefs. While many scientists speculated that temperature might have a metric structure even before the advent of statistical mechanics (see Skow 2011 for discussion), it is only from the perspective of statistical mechanics that such an ascription is justified.

Since RTM is silent on the question of how to justify the attribution of measurement structures to attributes, the structural realism at work in RTM is insufficient to satisfy the realist quest for measurement representations. Structural realism only provides the realist semantics for measurement representations, since it specifies which features of the numerical representation correspond to features of the attribute. Perspectival realism is needed to complement this picture, since perspectival realism provides a notion of epistemic justification for attributions of measurement structures that makes sense of the different attributions of measurement structures by different theories.

6 Conclusion

In this chapter, I looked at structural realism and perspectival realism initially as two competing responses to the plurality of scientific representations. My focus has been on representations of measurement. I argued that literalism about measurement representations, which corresponds to a traditional form of realism about representation, is inadequate. The representational theory of measurement, which provides a thoroughgoing account of numerical representations, implies structural realism about measurement representations. While RTM avoids some of the difficulties with the literalist reading, it needs to be supplemented with epistemic perspectival realism to account for the theory dependence of our justifications for ascribing particular measurement structures to attributes of interest.

There is a broader, more speculative lesson we might learn from this case. At least in some cases, structural realism and perspectival realism are not competing realist accounts of representation but instead offer complementary pieces in the realist quest. Structural realists do well in supplying a more appropriate notion of correspondence between representation and represented phenomenon by freeing representation from literalism. Perspectival realists, on the other hand, provide an account of the justification of using different representations of ostensibly the same phenomenon or attribute. This is important since, even after all equivalent (numerical) representations have been explained by structural realism, different scientific theories still ascribe different structures to the same attribute.

Notes

1. There are non-realist versions of both structuralism and perspectivalism, which I do not have room to discuss explicitly in this chapter. These views share the respective structural and perspectival outlook on scientific representation (discussed in the next section) but without any commitment to a form of scientific realism.
2. As with any view in philosophy, there are in-house disputes among structural realists and perspectival realists. Since my main purpose here is to contrast the two different approaches, I shall set aside the finer points of disagreement in each camp.
3. The question of whether haecceitistic differences matter in the context of quantum particles is discussed in, for example, French (1989) and Huggett (1999).
4. For discussions of this example, see Morrison (1999) and Teller (2001).
5. Quantities that behave like density and temperature have sometimes been called "intensive quantities," in contrast to extensive quantities like mass and volume. The distinction between intensive and extensive quantities is not always clearly defined, nor is it uncontested. Tolman (1917, 239) defined intensive quantities to be non-additive; Suppes (1951), by contrast, replaces the additivity demand with the idea that extensive quantities are quantities that can be represented on ratio scales, whereas intensive quantities are only representable on weaker scales.
6. For example, RTM describes a structure as a (closed) additive extensive structure, by providing a set of characteristic axioms (Krantz et al. 1971, 73):

   Let A *be a nonempty set,* $\succsim$ *a binary relation on* A*, and* $\circ$ *a closed binary operation on* A*. The triple* $\langle A, \succsim, \circ \rangle$ *is a* closed extensive structure *iff the following four axioms are satisfied for all* a, b, c, d, $\in$ A:

   1. *Weak order:* $\langle A, \succsim \rangle$ *is a weak order, i.e.,* $\succsim$ *is a transitive and connected relation.*
   2. *Weak associativity:* $a \circ (b \circ c) \sim (a \circ b) \circ c$.
   3. *Monotonicity:* $a \succsim b$ iff $a \circ c \succsim b \circ c$ iff $c \circ a \succsim c \circ b$
   4. *Archimedean: If* $a \succ b$*, then for any* $c, d \in A$*, there exists a positive integer* n *such that* $na \circ c \succsim nb \circ d$*, where* na *is defined inductively as:* $1a = a$, $(n + 1)a = na \circ a$.

7. Traditionally these structures are understood as domains of concrete objects and "empirical," that is, observable qualitative relations among them. This interpretation reflects the empiricist and operationalist commitments of the

founders of RTM, but it is not the only available interpretation. Instead the structures can be understood, for example, as sets of space-time points with relations among them (Field 1980).

8. As before, the concatenated object might be understood as two physical objects combined operationally (e.g., by placing two rods end to end). Crucially, it is a mapping between an empirical or physical domain and a numerical (or mathematical) domain.
9. I will return to the question of how we are justified in ascribing a particular structure to an attribute in section 5.
10. The technical details of this comparison are a bit too elaborate to be included in the discussion here. For relevant literature, see Luce, Krantz, Suppes, and Tversky (1990), especially chap. 20.
11. For a detailed discussion of this problem in the case of temperature, see Chang (2004).

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