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# On the Influence of Non-Linear Phenomena on Perceived Interactions in Percussive Instruments

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**Abstract.** In this paper, we investigate the hypothesis that perceived impact strength is strongly influenced by the non-linear behavior produced by large deformations in percussive instruments. A sound corpus is first generated from a physical model that simulates non-linear vibrations of a thin plate. The effect of non-linear phenomena on the perceived strength is further quantified through a listening test. The aim of this study is to improve the expressive potential of synthesizers of percussive sounds through the development of signal transformation models. Future work will focus on the modeling of sound morphologies that correspond to non-linear behavior and the development of a transparent control strategy.

**Keywords:** Sound-Synthesis, Non-Linear Behavior, Auditory Perception, Impact Sounds, Control

## 1 Introduction

Digital sound synthesis enables the exploration of large sound spaces. One challenge is to find perceptually consistent control strategies to explore these spaces and hereby improve user expressivity.

For a synthesizer to be "expressive", it has been conjectured that the mapping between the inputs and outputs of the device must be transparent [1]. Transparency is here defined as an indicator of the psychophysiological distance, in the mind of the user and the audience, between the inputs and outputs of the instruments. Therefore, the control parameters and their link with the user's actions must be defined intuitively to allow for proper expression within a synthesizer. This has been investigated in several studies [2] [3] [4].

In the context of environmental sound synthesis, Conan proposed an intuitive control for continuous interaction sounds [5]. This work is a follow-up of the investigations of Aramaki et al. on the synthesis of percussive sounds with a control of the attributes of the impacted object (material, shape and size) [6][7][8][9]. These studies have led to the development of a sound synthesis environment with an intuitive control of the evoked action (impact, rolling, scratching, rubbing) and object (material, size and shape) for solid interactions. Inspired by the ecological approach for auditory events [10][11][12], this environment has been developed within the action-object paradigm, which states that the sound is the result of an action on an object and that invariant structures can be independently associated to the actions and the objects. This synthesizer constitutes an experimental research environment for the PRISM laboratory that can be adapted for various uses, such as music practice, video games [13], or more generally sound design.

A dynamic mapping of the user's gesture with the synthesis parameters has been set up in specific cases to improve expressive control of the synthesizer, e.g. from squeaks to self-oscillating transitions for continuous interactions with a touch pad [14]. The perceived material has also been mapped with the strength of the impacts, and a drumstick has been instrumented to capture the gesture of the user. However, no work has so far been done to define the perceptual expectations related to the strength of the impact in a more general situation.

The objective of this study is to define expectations regarding the evolution of the sound radiated by an object when it is struck with different strengths. To this end, we study the effects of non-linear behavior on the radiated sound. The idea is to apply transformations to the signal for a transparent mapping of the strength of an impact for percussive sounds. In the context of virtual music instruments, this parameter could be linked to the velocity, a central parameter for expressiveness of percussive instruments on midi controllers.

Such an idea has been investigated in the musical context for years. Most virtual instruments offer dynamic behavior and adapt the sound rendering to the velocity of touch on the keyboard, especially for the highest values. A simple and convincing effect is to add brightness for high velocity values. For physical systems, this effect is due to the behavior of the impactor, whose deformation varies non-linearly with the pressure exerted [15]. For instance, the deformation of piano hammers follows a non-linear law related to their composition (wooden core surrounded by felt) [16][17]. Also, rules of thumb are proposed in various publications for a transparent mapping between the sound synthesis parameters and the user's gesture. One example is the mapping of the modulation index with the impact velocity in the context of FM [18], or the shapes of harmonic distortion laws to replicate a "physical" behavior [19]. In the context of environmental sound synthesis, one can also mention Warren and Verbrugge's investigations on sound morphologies responsible for breaking event recognition [20].

The proposed approach is to model the physical system and its non-linear behavior in order to synthesize realistic sounds for various configurations. The relevance of these phenomena with respect to perceived strength of the impact can then be evaluated through listening tests. Finally, we aim to model sound morphologies responsible for the evocation of high intensity impacts.

Physical modelling allows the generation of realistic sounds, and the mapping of the user's gesture is obvious and transparent since the synthesis parameters are physically meaningful. Moreover, the ever-growing computing capacities of commercially-available hardware makes it possible to consider real-time synthesis for increasingly complex models, opening the possibility of designing virtual instruments with this type of synthesis [21][22][23]. However, the inherent constraints of a physical model limit the possibilities of sound creation. Indeed, the design of such a virtual instrument is based on modelling the behavior of an existing element rather than on the free description of a sound. The positioning of this study is to use physical models to generate a sound corpus that is further analyzed to extract and model the sound morphologies (signal model) corresponding to the phenomena studied. In this way, we free ourselves from the constraints of physical modeling by allowing the generation of metaphorical sounds. On the other hand, this type of approach, as presented by Rocchesso et al. [24], is complex because it implies three levels of research: auditory perception, physics-based sounds modeling, and expressive parametric control.

This paper presents a preliminary study for the development of perceptually controlled synthesis processes. We propose here to demonstrate that non-linear phenomena play a major role in the perception of the impact strength in the case of thin plates.

## 2 Review of Notable Non-Linear Behavior for Everyday Sounding Objects

Our perceptual expectations are driven by real sounding objects that we are used to hearing and manipulating. In this section, we briefly review the non-linear behaviors that can induce a notable transformation of the sound radiated by an object impacted with different strengths.

For this purpose we focus on percussive musical instruments, which sound timbre changes according to the excitation. The choice of the impactor (hammer, mallet, drum stick, brush, plectrum, fingers, hand palm) and the gesture (force profile, impact position) define the quantity of energy and its distribution on the resonator modes. In general, the impactor hardens as the intensity of the impact increases, inducing a brighter sound.

After excitation, the sound radiated by the instrument can be modified in case of interactions between the resonator and other elements. For example, the specific timbre of the tanpura and sitar is caused by the interaction of the strings with the bridge and the sympathetic strings. These interactions can lead to a muted sound (ghost notes, cymbal choke, palm mute), pitch modifications (natural and pinched harmonics, slide, bend, ghost notes, udu drum) and the appearance of other partial tones over time (slapping, string buzz, tanpura and sitar double bridge, snare wire, rattling elements in a cajon or a mbira).

Geometric non-linearities in the resonator itself may also occur for large amplitude deformations, resulting in time varying mode frequencies, harmonic distortion and mode coupling. These phenomena are particularly prevalent in the case of gong and cymbal crashing (wave turbulences). They can also be heard through timbre variations in steel drums, or pitch bending of strings and membranes due to large amplitude vibrations. Beyond such high amplitude vibrations, plastic deformations or ruptures can be observed.

These observations can be transposed to objects of everyday life. For example, the force with which someone knocks on the door may be recognized through the impactor (what part of the hand, and with which hardness), and the interaction with surrounding walls and rattling elements. Thin structures that admit large elastic deformations (e.g. metal sheets) behave similarly to cymbals and gongs. Weaker flexible structures deform (cans, plastic bottles), fibrous materials creak then crack (wood, composite), while more fragile structures crack then break (glass slides). Also, paper sheets, aluminum foil, plastic bags, fabrics, loose membranes and strings do not vibrate much due to their low stiffness. Their behavior is strongly non-linear, and the radiated sound is not tonal.

To sum up, the dynamic behavior of an impacted object may be due to the non-linear behavior of the object or non-linear interactions with other elements. A study has already been initiated to define a morphological invariant of non-linearity related to interactions [25]. In this paper, the objective is to check whether non-linear phenomena related to large vibrational amplitudes of an object are central to the perceived intensity of the impact.

## 3 Methods

We hypothesized that the perception of the strength of an impact is correlated with the occurrence of audible non-linear phenomena in the sound radiated by an object for large vibration amplitudes. It is assumed here that the strength of the impact is expressed by the object's ability to resist this impact. This implies that the perceived strength depends on the structural characteristics of the impacted object. To verify this hypothesis, an experiment was conducted in which subjects were to judge the strength of the impact and the object's ability to resist this impact by answering the following question: *In this test, we ask you to evaluate the strength of the impact and the "suffering" of the object for each sound* (In French, *Vous devrez évaluer la force de l'impact et la "souffrance" de l'objet pour chacun des sons*). The subjects were told that the "suffering" of an object reflects its difficulty to bear the deformation produced by the excitation. The degree of "suffering of the object" was used as an indicator for the perceived weakness of the object, supposed to be correlated with the occurrence of audible non-linear phenomena.

We chose to focus on the behavior of thin plates to test our hypothesis because it generates characteristic and easily recognizable sounds for large deformations.

This is a first approach to identify sound morphologies linked to audible non-linear phenomena, that can be used to define a signal transformation model that can be applied to different sound textures.

### 3.1 Synthesis of the Stimuli

**Thin plate model.** The stimuli were synthesized by numerical solving the Von Karman system, a widely used model of nonlinear vibration of plates at moderate amplitudes with a quite compact form [22].

$$u_{tt} = -\frac{D}{\rho H}\Delta\Delta u + \frac{1}{\rho H}\mathcal{L}(\phi, u) - 2\sigma_0 u_t + 2\sigma_1 \Delta u_t + \frac{e}{\rho H}F(t) \tag{1a}$$

$$\Delta\Delta\phi = -\frac{EH}{2}\mathcal{L}(u, u) \tag{1b}$$

Where $u(x, y, t)$ is the transverse plate deflection, $\phi(x, y, t)$ is often referred to as the Airy stress function, $F(t)$ is the excitation force, $e = \delta(x - x_i, y - y_i)$ the 2D Dirac function, $\sigma_0 = 0.1$ rad$\cdot$ s$^{-1}$ and $\sigma_1 = 0.001$m$^2\cdot$s$^{-1}$ are the damping coefficients, $H$ is the plate thickness, in m. The material is defined as steel, with a density of $\rho = 7800$ kg$\cdot$m$^{-3}$, $D$ is set as a function of Young's modulus $E = 210$ GPa and Poisson's ratio $\nu = 0.3$ as

$$D = \frac{EH^3}{12(1 - \nu^2)}. \tag{2}$$

In Cartesian coordinates, the nonlinear operator $\mathcal{L}$ is defined by

$$\mathcal{L}(f, g) = \partial_x^2 f \partial_y^2 g + \partial_y^2 f \partial_x^2 g - 2\partial_x\partial_y f \partial_x\partial_y g \tag{3}$$

for any two arbitrary functions $f(x, y, t)$ and $g(x, y, t)$.

We set the length and width $L_x = 0.6$ m and $L_y = 0.7$ m, and we synthesized the sounds for three different thicknesses $H_1 = 1$ mm ; $H_2 = 2$ mm ; $H_3 = 3$ mm.

The boundary conditions were set free for $x = 0$ and $x = L_x$

$$u_{xx} + \nu u_{yy} = u_{xxx} + (2 - \nu)u_{xyy} = 0, \tag{4}$$

and simply supported for $y = 0$ and $y = L_y$

$$u = u_{yy} = 0. \tag{5}$$

These boundary conditions have been chosen to increase the occurrence of wave turbulences and limit frequency variations.

We chose to model the impact by a raised cosine rather than by a mallet model in order to be able to control the brightness independently of the strength of the impact.

$$F(t) = \begin{cases} A * (-\cos(2\pi t/T) + 1) & 0 \leq t < T \\ 0 & \text{otherwise} \end{cases}$$

A constant impact duration $T$ is assumed for any amplitude, which corresponds to an impactor with a linear behavior. Thus, the influence of geometric non-linearities can be assessed without the non-linear behavior of the impactor interfering with this measurement. $T$ was set to 2 ms and the amplitude A was linearly varied over 10 levels from 100 N to 1000 N.

**Finite Difference Scheme.** Time and space are discretized. We note $u^n_{l,m}$ the transverse displacement at the $n^{th}$ time step and the grid point ($x = h*l; y = h*m$), with $h$ the spacing between two grid points, $k = 1/f_e$ the time step ($f_e = 44100$ Hz).

We used the following conservative scheme, as defined in [26]

$$\delta_{tt}\boldsymbol{u} = -\frac{D}{\rho H}\delta_{\Delta,\Delta}\boldsymbol{u} + \frac{1}{\rho H}l(\mu_{t\cdot}\boldsymbol{\phi},\boldsymbol{u}) - 2\sigma_0\delta_{t\cdot}\boldsymbol{u} + 2\sigma_1\delta_{t-}\delta_{\Delta}\boldsymbol{u} + \frac{\boldsymbol{J}}{\rho H}F \tag{6a}$$

$$\mu_{t+}\delta_{\Delta,\Delta}\boldsymbol{\phi} = -\frac{EH}{2}l(\boldsymbol{u}, e_{t+}\boldsymbol{u}) \tag{6b}$$

with

$$e_{t+}u^n_{l,m} = u^{n+1}_{l,m}, \qquad e_{t-}u^n_{l,m} = u^{n-1}_{l,m} \tag{7a}$$

$$e_{x+}u^n_{l,m} = u^n_{l+1,m}, \qquad e_{x-}u^n_{l,m} = u^n_{l-1,m} \tag{7b}$$

$$e_{y+}u^n_{l,m} = u^n_{l,m+1}, \qquad e_{y-}u^n_{l,m} = u^n_{l,m-1} \tag{7c}$$

$$\mu_{t+} = (e_{t+} + 1)/2 \tag{7d}$$

$$\delta_{t\cdot} = (e_{t+} - e_{t-})/2k \tag{7e}$$

$$\delta_{t-} = (1 - e_{t-})/k \tag{7f}$$

$$\delta_{tt} = (e_{t+} - 2 + e_{t-})/k^2 \tag{7g}$$

$$\delta_{\Delta} = (-4 + e_{x+} + e_{x-} + e_{y+} + e_{y-})/h^2 \tag{7h}$$

$$\begin{aligned}\delta_{\Delta,\Delta} = (20 + 2[&e_{x+}e_{y+} + e_{x+}e_{y-} + e_{x-}e_{y+} + e_{x-}e_{y-}] \\ &- 8[e_{x+} + e_{x-} + e_{y+} + e_{y-}] + e^2_{x+} + e^2_{x-} + e^2_{y+} + e^2_{y-})/h^4\end{aligned} \tag{7i}$$

$$\delta_{xx} = (e_{x+} - 2 + e_{x-})/h^2 \tag{7j}$$

$$\delta_{yy} = (e_{y+} - 2 + e_{y-})/h^2 \tag{7k}$$

$$\mu_{x-,y-} = (e_{x-} + 1)(e_{y-} + 1)/4 \tag{7l}$$

$$\delta_{x+y+} = (e_{x+} - 1)(e_{y+} - 1)/h^2 \tag{7m}$$

$$l(f,g) = \delta_{xx}f\delta_{yy}g + \delta_{yy}f\delta_{xx}g - 2\mu_{x-,y-}(\delta_{x+,y+}f\delta_{x+,y+}g) \tag{7n}$$

$J$ is the interpolation operator. $J_{N_x/2+1,N_y/2+1} = \frac{1}{h^2}$ for the node in the middle of the plate (an even number of elements in length $N_x$ and width $N_y$ is defined), $J_{l,m} = 0$ for the other nodes.

More details are available for the implementation of the scheme in [27].

**Loudness Equalization.** A total of 30 different sounds were synthesized (3 levels for the thickness of the plate, 10 levels for the strength of the impact). The samples were normalized (their maximum value is set to 1) as follows:

$$s^n_{norm} = \frac{s^n}{\max(|s^n|)} \tag{8}$$

with $s^n$ the original signal and $s^n_{norm}$ the normalized signal at the time step $n$.

Finally, we proceeded to a loudness equalization. There is no model for the loudness of complex sounds, so it is necessary to probe perception with pairwise comparisons. Comparing all pairs with an adaptive procedure is too time-consuming for a pre-test (435 pairs, with about 1 minute testing time per pair). Furthermore, the task is complex because the sounds to be evaluated have a different temporal evolution. In general, sounds corresponding to a strong non-linear behavior have less energy at low frequencies and are damped more quickly. A simplified procedure consists in choosing the median sample ($H = 2mm$ ; $A = 500N$) as a reference and compare it with the other sounds. This limits the comparison to 29 pairs and minimizes the difference between the sounds to be compared.

The resulting stimuli are available for online listening [3].

### 3.2 Participants and Procedure.

Fourteen participants (3 female, 11 male), aged 22 to 50, were tested in this experiment. They all had normal audition and gave consent to participate in the experiment.

Participants were asked to evaluate the strength of the impact and the "suffering" ("souffrance" in French) of the object for each stimulus by moving two sliders on a scale without markers. The 30 sounds were randomly presented through headphones (Sennheiser HD650) in a quiet environment.

### 3.3 Results

ANOVAs were conducted for the perceived strength of the impact and the perceived "suffering" of the object. Factors were the thickness (1 mm, 2 mm, 3 mm) and the impact strength amplitude (100 N, 200 N, 300 N, 400 N, ... , 1000 N). Results are displayed in fig.1&2.

## 4 Discussion

There is a direct correlation between the perception of the impact strength and the "suffering" of the object. The evaluation of the strength and the "suffering" are almost identical for all the sounds, although the perceived impact strength is slightly higher in general. This result is also revealed in the feedback from the different participants who said they answered almost the same value for both parameters for most sounds. This reflects the fact that the presence of non-linearities due to large object deformations is used as a perceptual cue to assess the impact strength, which is consistent with the initial hypothesis. This observation is reinforced by the significant differences between the changes in the evolution of the perceived strength regarding the excitation amplitude $A$ for each

[3] https://cloud.prism.cnrs.fr/index.php/s/tx67ywnVvMg21jC

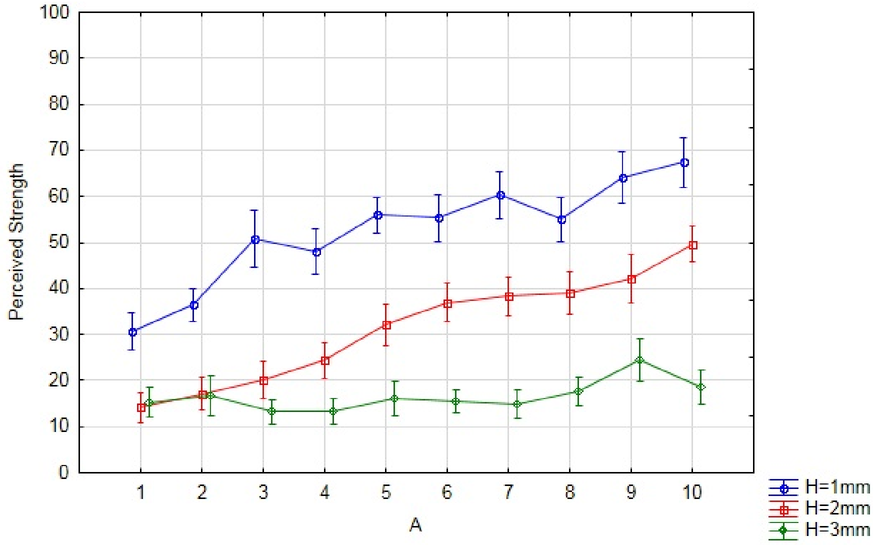


**Fig. 1.** Representation of responses for the perceived strength of the impact for all samples (percentage of cursor range). The level of the excitation $A$ is presented on the horizontal axis, and each curve represents a level for the plate thickness. Dots denote the average value for all participants, vertical bars denote +/- standard errors.

high $H$ level (the interaction effect $A$x$H$ is significant $p = 0.00000$). Indeed, the perceived strength for $H_3$ (almost no non-linear phenomena for the whole range) is relatively constant regarding the level of the impact strength. Conversely, we observe a significant evolution of the perceived strength regarding $A$ for the smallest thicknesses $H_1$ and $H_2$, which correspond to sounds that present a progressive appearance of non-linear phenomena when the impact strength increases.

Also, the upper part of the scale remained unused for most participants. Several participants told that they were expecting stronger amplitudes of excitation that would damage or brake the plate. On the other hand, we notice that the strength is never perceived as zero, unlike the "suffering" of the object, and that the value of the perceived strength is always higher than the value of the "suffering" of the object. This is consistent with the idea that the "suffering" of the object only begins when non-linear phenomena begin to appear, unlike the strength that always has to be different from 0 for a sound to occur.

## 5 Conclusion & Perspectives

The purpose of this study was to investigate perceptual expectations regarding the evolution of the sound radiated by an object with respect to the strength of the impact it undergoes. To this end, we sought to evaluate the effect of non-linear phenomena on the perceived impact strength. This paper focuses on

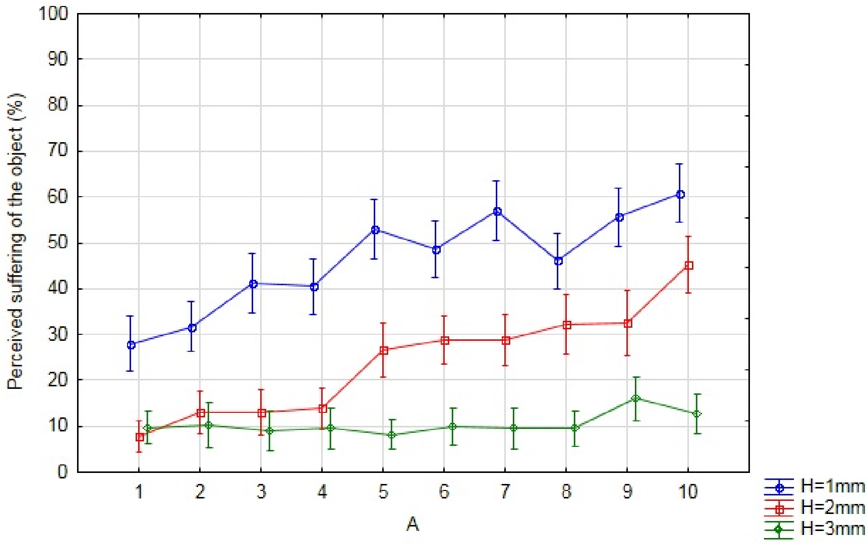


**Fig. 2.** Representation of responses for the perceived "suffering" of the object for all samples (percentage of cursor range). The level of the excitation amplitude is presented on the horizontal axis, and each curve represents a level for the plate thickness. Dots denote the average value for all participants, vertical bars denote +/- standard errors.

the study of thin plates for moderate vibration amplitudes. A listening test was conducted to evaluate the impact strength and the "suffering" of the object for 30 sounds synthesized by physical modeling of the system, corresponding to 3 plates of different thicknesses impacted with an excitation amplitude ranging from 100 N to 1000 N. The evaluation results show that the perceived impact strength is directly correlated with the occurrence of non-linear phenomena in the case of thin plates.

Further, the deformations are perceived as small for the present sound corpus, since only the lower section of the evaluation scale is used by the subjects. This result encourages us to extend this experiment by modelling the effects of plastic deformations and rupture on the sound radiated by an object.

The next step is to propose signal transformation models corresponding to the different non-linear phenomena, and perceptually relevant controls to improve the expressive potential of percussive sound synthesizers.

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