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Coda Wave Interferometry for Accurate Simultaneous Monitoring of Velocity and Acoustic Source Locations in Experimental Rock Physics

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Key Points:

- Conventional methods for measuring changes in seismic velocity and source locations may be inaccurate, especially in a high-frequency regime
- CWI provides far greater accuracy in estimating changes in velocity and between source locations, even with high background noise levels
- A new method is presented to infer both P and S wave average velocity changes from coda wave interferometry

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Abstract

In many geoscientific, material science and engineering applications it is of importance to estimate a representative bulk seismic velocity of materials, or to locate the source of recorded seismic or acoustic waves. Such estimates are necessary in order to interpret industrial seismic and earthquake seismological data, for example in non-destructive evaluation and monitoring of structural materials, and as an input to rock physics models that predict other parameters of interest. Bulk velocity is commonly estimated in laboratories from the time-of-flight of the first-arriving wave between a source and a receiver, assuming a linear raypath. In heterogeneous media, that method provides biased estimates of the bulk velocity, and of derived parameters such as temporal velocity changes or the locations of acoustic emissions. We show that Coda Wave Interferometry (CWI) characterizes changes in the bulk properties of scattering media far more effectively on the scale of laboratory rock samples. Compared to conventional methods, CWI provides significant improvements in both accuracy and precision of estimates of velocity changes, and distances between pairs of acoustic sources, remaining accurate in the presence of background noise, and when source location and velocity perturbations occur simultaneously. CWI also allows 3D relative locations of clusters of acoustic emissions to be estimated using only a single sensor. We present a method to use CWI to infer changes in both P and S wave velocities individually. These innovations represent significant improvements in our ability to characterize the evolution of properties of media for a variety of applications.

1 Introduction

Experimental studies of wave propagation in rock cores are often performed to deduce relationships between changes in external conditions and seismic properties such as seismic velocity (Wang, 2001), anisotropy (Christensen, 1966; Sayers & Kachanov, 1995) and attenuation (Sams, Neep, Worthington, & King, 1997; Toksöz, Johnston, & Timur, 1979), and to examine the process of rock fracturing (Pyrak-Nolte, Myer, & Cook, 1990) or the distribution of acoustic emissions (Lockner, 1993; Lockner, Byerlee, Kuksenko, Ponomarev, & Sidorin, 1992). Established relationships between seismic attributes and underlying rock physical properties are particularly important for monitoring purposes in the hydrocarbon industry and in subsurface CO₂ storage projects, notably for relating effective stress changes during subsurface injection or production to changes that may

47 be observed in the seismic velocity (Arts et al., 2004; Brown, 2002; Guilbot & Smith, 2002;
48 Herwanger & Horne, 2009; Stork, Allmark, Curtis, Kendall, & White, 2018). It is there-
49 fore of great importance that models developed from laboratory experiments accurately
50 represent the response of in-situ rocks.

51 Standard methods for measuring either the velocity, or changes in the velocity of
52 a medium involve picking of first-break arrival times of seismic waves traveling between
53 a fixed source and receiver pair. The term ‘first-break’ is ambiguous, and can be taken
54 to mean the signal onset which is the time of first-arriving energy (Brillouin, 1960), the
55 arrival time of the first peak or the time of first zero-crossing (Hornby, 1998). Manual
56 picking of first-breaks is slow and may incur inconsistent user bias and error, therefore
57 there are many methods available for automatic picking of first-breaks (Boschetti, Den-
58 tith, & List, 1996; Earle & Shearer, 1994; Ervin, McGinnis, Otis, & Hall, 1983; Hatherly,
59 1982; Molyneux & Schmitt, 1999; Peraldi & Clement, 1972). Here, unless otherwise stated,
60 we use the term first-break method to mean picking the first maximum (or extremum).
61 This represents the point with the highest signal to noise ratio. The velocity of the medium
62 is then estimated using the known straight-line distance between the source and receiver.
63 For many laboratory experiments measuring such velocities, the wavelengths used are
64 on the same order as heterogeneities in the medium (e.g., pore and grain sizes). Obvi-
65 ous problems then occur: 1) the measured velocity is not sensitive to the bulk proper-
66 ties of a medium, but rather to properties along a very specific (fastest) ray path between
67 the source and receiver, resulting in a bias towards higher velocities. 2) The path fol-
68 lowed by the first-arriving energy is unlikely to be straight, so that velocity estimates
69 made using the straight-line path are biased towards lower values. 3) Biases in points
70 1 and 2 are generally unrelated so are not expected to cancel. 4) The effects of small per-
71 turbations in the medium that are not located along the specific source-receiver path can-
72 not be detected. 5) Such systematic and random errors in velocity estimation are car-
73 ried forward to any subsequent calculations, notably for example to the location of acous-
74 tic source positions. Also, the presence of attenuation and dispersion changes the shape
75 of a propagating wave (Molyneux & Schmitt, 2000), thus the determination of meaning-
76 ful velocity measurements can be problematic.

77 Weaver and Lobkis (2001) and Lobkis and Weaver (2001) showed that information
78 about a medium can be extracted from recordings of coda waves and background am-
79 bient noise. Coda waves are the multiply-scattered waves that are recorded after the ar-

80 rival of the main ballistic waves. Recordings of coda waves are far more sensitive than
81 first arrivals to changes in pore-pressure, fracture density and temperature (Snieder, Grêt,
82 Douma, & Scales, 2002; Vlastos et al., 2006; Vlastos, Liu, Main, & Narteau, 2007), due
83 to the fact that coda waves follow much longer and more complex paths, eventually sam-
84 pling the entire medium, and sampling any sub-volume of the medium multiple times.
85 There are now established methods grouped under the name *coda wave interferometry*
86 (CWI) that estimate *changes* in the velocity of the medium (rather than the absolute
87 velocity), or changes in the locations of sources or receivers using the coda (Snieder, 2006).
88 There have been several field and laboratory applications of CWI to date, including the
89 monitoring of velocity changes in ice sheets (James, Knox, Abbott, & Screaton, 2017;
90 Mordret, Mikesell, Harig, Lipovsky, & Prieto, 2016), concrete (Larose & Hall, 2009; Planès
91 & Larose, 2013), mining environments (Grêt, Snieder, & Özbay, 2006), and volcanic re-
92 gions (Sens-Schönfelder & Wegler, 2006). CWI has also been used to study earthquake
93 focal mechanisms (Robinson, Snieder, & Sambridge, 2007), earthquake separation dis-
94 tances (Robinson, Sambridge, & Snieder, 2011; Snieder & Vrijlandt, 2005), and source
95 network locations of induced micro-seismic events (Zhao & Curtis, 2019; Zhao, Curtis,
96 & Baptie, 2017). So far its implications for the interpretation of laboratory rock physics
97 experiments has been comparatively limited.

98 In this paper we test the hypothesis that Coda Wave Interferometry (CWI) can
99 provide an improvement in accuracy and precision when inferring and quantifying the
100 changes in bulk velocity and relative source locations in rock samples in laboratory set-
101 tings. We test the hypotheses that CWI provides more representative measures of bulk
102 properties, in comparison with commonly used methods in numerical and laboratory ex-
103 periments at the core-scale, and at high frequencies commonly used in a laboratory set-
104 ting.

105 First we outline the theory of Coda Wave Interferometry and how it can be used
106 in an experimental setting. Then we examine multiple samples of varying rock type and
107 heterogeneity using both numerical simulations and laboratory experiments, where changes
108 in source location and velocity are estimated using both CWI and standard methods (manually-
109 picked first breaks for velocities and multilateration for source locations). We show how
110 changes in source position and velocity can be jointly estimated by CWI when both per-
111 turbations occur simultaneously. We then demonstrate an optimization algorithm for es-
112 timating the relative locations of sources within a cluster, given the source separations

113 estimated from CWI, and show that it can be applied even in the case of having only
114 a single transducer. Following this, we test the sensitivity of CWI as well as conventional
115 methods to increasing contamination of noise. In all cases CWI is shown to out-perform
116 conventional methods.

117 Accompanying this manuscript, we provide a well-commented set of MATLAB func-
118 tions for implementing the CWI method to estimate velocity changes, and for the joint
119 estimation of velocity change and source separation. These codes use a form of CWI that
120 estimates changes relative to a moving reference seismogram, which is particularly im-
121 portant for longer deformation experiments in which scattering paths may change sig-
122 nificantly, a situation which contravenes the assumptions of standard CWI theory, and
123 requires the reference seismogram to be updated periodically. Together with the suite
124 of CWI codes made publicly available by Zhao and Curtis (2019) this allows all techniques
125 used in this paper to be implemented and reproduced.

126 **2 Coda Wave Interferometry (CWI)**

127 CWI is a method that allows small changes in velocity, the displacement of source
128 or receiver locations, or movement of scatterers to be monitored (Sens-Schönfelder & We-
129 gler, 2006; Snieder, 2006; Snieder et al., 2002). These different perturbations and their
130 effect on recorded signals are illustrated in Figure 1. First consider the effect of a veloc-
131 ity perturbation (ΔV in Figure 1a). The direct arriving wave between a source and re-
132 ceiver would only sample the perturbation once (or not at all), whereas the multiply re-
133 flected wavefield samples the perturbation many times. Therefore the change in arrival
134 times for later arriving waves (time window iv) is larger than for the first arrival (time
135 window i). The second perturbation type is a displacement of the source or receiver lo-
136 cation (Figure 1b shows a source displacement). In this case, the difference in ray paths
137 before and after the perturbation is the path between the source and the first scatter-
138 ing point (blue arrows in Figure 1b). Different paths are shortened or lengthened depend-
139 ing on the location of the first scatterer; this is reflected by the advancement and retar-
140 dation of peaks highlighted by red and blue arrows. Providing the source displacement
141 is small, the extent to which these travel times are perturbed (specifically, the variance
142 of the perturbation) is directly proportional to the displacement. The third perturba-
143 tion type is the displacement of all scattering points (yellow circles in Figure 1c): in this
144 case, all paths between scattering points are perturbed (both shortened and lengthened),

and similarly to the previous case the variance of travel time perturbations is proportional to the displacement of scattering points. All three perturbation types can be monitored by using a cross correlation of the unperturbed (u_{unp}) and perturbed (u_{per}) waveforms - the waveforms from the source recorded by the receiver before and after the change or displacement takes place.

One method to estimate the change in velocity is known as trace stretching (Sens-Schönfelder & Wegler, 2006), where the perturbed waveform is assumed to be a time-stretched version of a reference waveform; this follows if one assumes that a velocity perturbation is uniform across the entire medium, so all arriving energy is perturbed at the same temporal rate. This method also assumes no changes in the intrinsic attenuation of the medium. We stretch the time axis of the perturbed signal by a range of stretching factors (ϵ) and compute the correlation coefficient R between $u_{unp}(t)$ and the stretched version of the perturbed waveform $u_{per}(t[1 + \epsilon])$ over a given time window (t_1, t_2):

$$R^{(t_1, t_2)}(\epsilon) = \frac{\int_{t_1}^{t_2} u_{unp}(t) u_{per}(t[1 + \epsilon]) dt}{\sqrt{\int_{t_1}^{t_2} u_{unp}^2(t) dt \int_{t_1}^{t_2} u_{per}^2(t[1 + \epsilon]) dt'}}. \quad (1)$$

The optimum stretching factor ϵ_{max} that maximizes the correlation coefficient (for which $R = R_{max}$), is related to the ratio of the change in velocity ΔV to the original velocity V by

$$\epsilon_{max} = -\frac{\Delta V}{V}, \quad (2)$$

(Sens-Schönfelder & Wegler, 2006). This method requires that velocity changes are small to avoid cycle skipping in the calculation of R in Equation 1. In cases where the medium changes significantly, such as during material deformation where new scattering paths are introduced due to fracturing, it may not be appropriate to use a constant reference trace (u_{unp}) for all recorded waveforms during deformation. We therefore propose the use of a moving reference trace, where the optimum stretching factor from the initial reference trace (u_0) to any other recorded waveform during deformation (u_n) can be calculated as

$$\epsilon_{u_0 u_n} = \epsilon_{u_0 u_s} + \epsilon_{u_s u_n}, \quad (3)$$

where $\epsilon_{u_i u_j}$ is the stretching factor of trace u_j relative to u_i , $s = k \lfloor n/k \rfloor$, n is the trace number, k is the user-selected step size of the moving reference trace, and $\lfloor \dots \rfloor$ denotes a floor function, which outputs the greatest integer less than or equal to the argument. Accompanying this manuscript are a suite of MATLAB functions for implementing the

176 moving-reference stretching CWI method. Snieder (2002) derived the relationship be-
 177 tween the inferred medium velocity change from CWI, and changes in P-wave and S-wave
 178 velocities in an isotropic case:

$$179 \frac{\Delta V}{V} = \frac{\beta^3}{2\alpha^3 + \beta^3} \frac{\Delta\alpha}{\alpha} + \frac{2\alpha^3}{2\alpha^3 + \beta^3} \frac{\Delta\beta}{\beta}, \quad (4)$$

180 where α and β are the velocities of P and S waves, respectively. In an initial Poisson medium
 181 where $\alpha = \sqrt{3}\beta$, if either or both of the P or S wave velocity changes then the relation
 182 simplifies to

$$183 \frac{\Delta V}{V} = 0.09 \frac{\Delta\alpha}{\alpha} + 0.91 \frac{\Delta\beta}{\beta}, \quad (5)$$

184 and if α and β change such that the Poisson medium is preserved then

$$185 \frac{\Delta V}{V} = \frac{\Delta\alpha}{\alpha} = \frac{\Delta\beta}{\beta}. \quad (6)$$

186 The strengths of the CWI technique lie in the ability to resolve very small changes in
 187 velocity compared to standard methods. If we take the sampling interval of a recorded
 188 signal to be dt , the duration of the signal to be t_{max} , and make the conservative assump-
 189 tion that one sample interval is the smallest resolvable time difference between waveforms
 190 in the two recordings, then the maximum resolution of CWI (the smallest resolvable change
 191 in velocity that can be measured) is

$$192 \left[\frac{\Delta V}{V} \right]_{min}^{CWI} = \frac{dt}{t_{max}}. \quad (7)$$

193 The maximum resolution for measuring $\Delta V/V$ from the standard first-break method would
 194 be

$$195 \left[\frac{\Delta V}{V} \right]_{min}^{FB} = \frac{dt}{(t_0 + dt)}, \quad (8)$$

196 where t_0 is the first-break arrival time. Both equations 7 and 8 assume no background
 197 noise and hence no uncertainty in the recorded waveforms, nor ambiguity in defining a
 198 first break which can be highly uncertain in many cases. Inserting typical values for lab-
 199 oratory core scale measurements, such as those used in the experiments in the follow-
 200 ing section (sampling interval $dt = 0.04\mu s$, signal duration $t_{max} = 640\mu s$, and arrival
 201 time $t_0 = 65\mu s$), the smallest perturbations that theoretically can be detected are 0.00625%
 202 for CWI and 0.062% for the standard first break method. Hence, CWI offers an order
 203 of magnitude improvement in precision in the absence of noise. The CWI method also
 204 computes the cross-correlation function using many more data points, which should make
 205 it less susceptible to the effects of noise than a single point measure of say the first peak

for the first break estimate. We test the hypothesis that CWI provides a more accurate measure of relative velocity changes in the experiments outlined in Section 3.3.

Another advantage of using CWI is that it allows a joint estimate of both a velocity perturbation and the separation r between two source/receiver locations to be made from a single receiver. This is because velocity perturbation information is retrieved from the consistent phase information along the waveforms, whereas the source or receiver separation is related to the variance of inconsistent phase perturbations and hence to the maximum value of the cross correlation value (R_{max}) in Equation 1, and these two attributes may be observed independently. Figure 1b illustrates how the perturbations of travel times (advancement and retardation of individual peaks) relates to the displacement of the source or receiver. Snieder (2006) derives the relationship between the maximum cross-correlation and the variance of the travel time perturbations (σ_τ^2) as

$$R_{max} = 1 - \frac{1}{2}\bar{\omega}^2\sigma_\tau^2, \quad (9)$$

where $\bar{\omega}^2$ is the dominant mean-squared angular frequency in the recorded waveform which can be computed as:

$$\bar{\omega}^2 = \frac{\int_{t_1}^{t_2} \dot{u}^2(t') dt'}{\int_{t_1}^{t_2} u^2(t') dt'}, \quad (10)$$

where \dot{u} is the temporal derivative of the waveform u . When a source/receiver is displaced relative to another source/receiver by distance r , one can estimate separation r from the variance of the travel time perturbations in a range of scenarios. For isotropic sources in a two-dimensional acoustic medium:

$$\sigma_\tau^2 = \frac{1}{2\alpha^2}r^2. \quad (11)$$

For isotropic sources in a three-dimensional acoustic medium:

$$\sigma_\tau^2 = \frac{1}{3\alpha^2}r^2. \quad (12)$$

For double couple sources on the same fault plane, with the same source mechanism and in elastic media:

$$\sigma_\tau^2 = \frac{\left(\frac{6}{\alpha^8} + \frac{7}{\beta^8}\right)}{7\left(\frac{2}{\alpha^6} + \frac{3}{\beta^6}\right)}r^2, \quad (13)$$

where α and β are estimates of the P- and S-wave velocities of the medium (Snieder & Vrijlandt, 2005). These estimates of velocity represent an average for all scattering paths, assuming coda waves are evenly distributed in an isotropic medium. The type of spatial averaging that is implicit in the CWI estimate is analyzed in Section 5.

236 To summarize, the main advantages of using CWI over conventional first-break method
237 in an experimental setting (at least in theory) are that: 1) CWI is more representative
238 of changes in the bulk properties of a medium because coda waves sample the entire medium.
239 2) Coda waves sample the same area multiple times, so CWI is capable of resolving smaller
240 changes in the medium giving a theoretical order of magnitude increase in precision for
241 typical laboratory experiments. 3) CWI is generally less susceptible to the presence of
242 noise as it uses many more data points, providing more robust estimates. 4) CWI allows
243 for the separation between nearby sources to be estimated from a single receiver, even
244 in cases where medium velocity changes occur simultaneously, as the two estimates uti-
245 lize different measurements made from the correlation function in Equation 1. The source-
246 separation data are then sufficient to estimate the 3D relative locations of clusters of sources
247 using CWI with a single receiver. We now test how CWI works in practice, using nu-
248 merical simulations and laboratory experiments.

249 **3 Results**

250 **3.1 Estimating Velocity and Source Locations: Synthetic Examples**

251 Rock cores typically used for geomechanics and rock physics experiments are on
252 the scale of 3 mm to 100 mm in diameter, and seismic wave frequencies studied are on
253 the order of kHz - MHz. At these frequencies, wavelengths are similar to the scale of the
254 key heterogeneities such as pores and grains, therefore many rock samples act as strongly
255 scattering media. Most recorded waves take very complex, long paths and experience mul-
256 tiple reflections, diffractions and reflections (Sato, Fehler, & Maeda, 2012). Therefore
257 there are strong frequency dependent effects on properties derived from ultrasonic record-
258 ings at these scales (Mason & McSkimin, 1947). The complex nature of wave propaga-
259 tion through highly scattering media, such as the samples shown in Figure 2, can be stud-
260 ied using methods of digital rock physics (Madonna, Almqvist, & Saenger, 2012). First
261 a reconstructed micro-tomography (μ CT) cross-section is segmented into appropriate min-
262 eral and pore phases, and converted into velocity and density models (wave physics pa-
263 rameters used for different phases are shown in Table 1). Using finite difference meth-
264 ods (Moczo, Robertsson, & Eisner, 2007), wave propagation through the medium can
265 be simulated so that full waveforms can be generated, as though they have been recorded
266 at any point within the medium. These methods are increasingly used for estimating the
267 acoustic or elastic properties of rocks based on μ CT images (Saenger, Madonna, Osorno,

Table 1. Parameters used for finite difference wavefield simulation through the samples shown in Figure 2. Values are Voigt-Reuss-Hill averages taken from Bass (1995) and Mavko et al. (2009).

Phase	Density (kg/m ³)	Velocity (m/s)
Pore Fluid	1000	1500
Calcite	2710	6500
Plagioclase	2620	6500
Quartz	2650	5800
Potassium Feldspar	2560	6300
Biotite	3090	5260
Muscovite	2790	6460

Uribe, & Steeb, 2014; Saxena & Mavko, 2016). These methods are limited by the resolution of μ CT images, which fail to resolve sub-micron scale structures such as any microcracks that may exist.

Our aim is to understand and address problems facing core-scale experimental rock physics, especially where strong scattering occurs. To emulate these physical experiments, we simulate wave propagation using a two-dimensional, acoustic, rotated staggered-grid finite-difference solver, through three different digital rock samples: Tivoli Travertine (TT), Westerly Granite (WG) and Copp-Crag Sandstone (CS). These rock types have been selected to represent a range of types of heterogeneity, where Tivoli Travertine has high porosity with complex pore shapes and pore size distribution, Copp-Crag is a relatively homogeneous sandstone with more uniform pore shapes and pore size distribution, and Westerly Granite is the most homogeneous and exhibits little porosity. The μ CT slices and corresponding models of segmented phases for each rock type are shown in Figure 2 and are converted to wave physics models using the parameters stated in Table 1 (assuming isotropic mineralogy). The simulations do not include any effects caused by attenuation or dispersion. Each pixel is mapped to a regular grid of cells used for the finite difference method, with cell sizes of 37.5 μ m, 42 μ m and 2.9 μ m for the TT, CS and WG, respectively. The model includes reflecting boundaries to account for side wall reflections.

290 The source input pulses used are Ricker wavelets with peak frequencies of 30 MHz
291 for the TT and CS models, and 200 MHz for the smaller WG model. These frequencies
292 are significantly higher than those conventionally used in laboratory experiments, which
293 typically use peak frequencies around 1 MHz for 38 mm core diameter experiments. For
294 comparison with conventional methods, we also use a Ricker wavelet with peak frequency
295 of 1 MHz for the TT model. The simulations here are well within the high-frequency regime
296 (approximate wavelengths for each sample are labeled as λ in Figure 2). We assume a
297 point source and point receivers, much smaller than the apertures of conventional trans-
298 ducers used in laboratory experiments. We also assume perfect transducer coupling, which
299 in a laboratory setting is unknown and may be sensitive to external conditions. Accord-
300 ingly our results explore a best-case scenario at this stage of the modelling. High-contrast
301 discontinuities such as those between pores and mineral phases may cause instability prob-
302 lems on a staggered grid. To avoid these difficulties, we implement the rotated staggered
303 grid technique (Saenger & Bohlen, 2004).

304 First, we simulate a single point source located at the top of each sample and a row
305 of point receivers along the bottom (e.g., Figure 3e). Velocity is estimated at each re-
306 ceiver by manually picking the arrival time of the first peak (as well as the signal onset
307 for the TT model) and assuming straight ray paths between the known source and re-
308 ceiver locations (shown in Figure 3a, b, c and d). For the three samples, the estimated
309 velocities at each receiver show considerable variation depending on where the receiver
310 is located. For the TT model, we compare varying the source frequency (1 MHz and 30
311 MHz) as well as the method used for picking the first arrival (picking the first maximum
312 in panel a in Figure 3, and the signal onset in panel b). The strong variation in veloc-
313 ity depending on receiver position is present for both frequencies and both picking meth-
314 ods. This response is concerning as in many cases a single receiver and hence a simple,
315 non-representative velocity may be used to characterize an entire sample - from a receiver
316 at the center of the core in conventional experimental configurations (shown as dashed
317 black lines in Figure 3). Sometimes a plate-like receiver is used which spans the entire
318 base of the sample; in that case the signal recorded would be approximately equal to the
319 superposition of all the distributed transducers (Li, Schmitt, Zou, & Chen, 2018), and
320 the velocity estimated using this method is shown as a dashed green line.

321 To further explore the variation of measured velocity, a similar numerical exper-
322 iment was carried out on the three velocity models in which eikonal ray tracing was im-

323 plmented using the methods outlined by Margrave (2007). This gives an estimated ar-
324 rival time ($t[\mathbf{x}]$) for every point \mathbf{x} in the model for a fixed source location (in this case
325 the source is located at the center-top of each sample). Using these arrival times, we can
326 imagine a receiver placed at every point within and on the boundary of a model, and an
327 estimate of the velocity for that source-to-receiver path can be calculated using the stan-
328 dard travel time method assuming straight rays. Figure 4 shows the calculated veloc-
329 ity $v[\mathbf{x}]$ for all model points \mathbf{x} in each sample, again showing that measured velocity may
330 be strongly dependent on source and/or receiver locations. For Tivoli Travertine (Fig-
331 ure 4a) the variation in velocity estimates are greater than for Copp-Crag Sandstone (Fig-
332 ure 4c), and Westerly Granite (Figure 4b) has the smoothest image, reflecting the small-
333 est variation in estimated velocity $v[\mathbf{x}]$. In all cases the longer the source-to-receiver dis-
334 tance, the more stable is the result.

335 There are therefore several concerning implications of characterizing a medium with
336 velocities calculated from standard methods: 1) a measured cross-core velocity is not sen-
337 sitive to the bulk properties of a medium, but rather to the velocities along a specific ray
338 path between the point source and point receiver, as demonstrated by the variation of
339 estimated velocity with receiver position in Figures 3 and 4. Therefore, 2) the effects of
340 small perturbations in a medium that are not located on the specific source-to-receiver
341 path will not be detectable using these methods. In addition, although the results sta-
342 bilize for a more distant source and receiver pair, they are still expected to stabilize at
343 a velocity that is biased relative to the average across the sample since first-arrival travel
344 times are measured along shortest travel time ray paths.

345 The assumption that a medium is represented by a single constant ‘bulk’ velocity
346 also introduces errors into subsequent calculations, such as in the estimation of source
347 locations. This effect can be examined using a further numerical experiment. We sim-
348 ulate a series of regularly spaced sources placed on a rectilinear grid throughout each of
349 the three media, representing acoustic emissions occurring throughout the sample. We
350 measure the arrival times for each source (S) at a set of receivers (i) as t_S^i using the first-
351 break method, and use a single measured velocity through each sample (V_{med}), which
352 is assumed to be representative of the entire medium. In our implementation the exact
353 value of this velocity does not affect source locations - it only affects the estimates of the
354 source origin time (t_0). In this case it is therefore not inaccuracy in the velocity estimate
355 that will effect locations, but rather the assumption that there is a single representative

medium velocity. We estimate source locations (S_{est}) using multilateration, by implementing a grid-search through all model positions (\mathbf{x}) for each receiver location (\mathbf{x}_i) and through a range of source origin times (t_0), to find values of \mathbf{x} , and t_0 that minimize the objective function

$$\varphi(\mathbf{x}, t_0) = \sum_{i=1}^{N^i} [V_{med} \times (t_S^i - t_0) - |\mathbf{x}_i - \mathbf{x}|]. \quad (14)$$

The estimated source location S_{est} is the location \mathbf{x} that minimizes φ . Figure 5 displays the systematic error in estimated source locations S_{est} (arrowheads) compared to true locations (arrow tails) for each of the three samples. For the majority of sources in Tivoli Travertine (5a) and Copp-Crag Sandstone (5c), the resulting systematic error in source location is significant in both amplitude and direction. In Westerly Granite (5b), such errors have much smaller amplitudes. It is therefore clear that in more heterogeneous media, a single velocity is not appropriate and estimated source locations in many areas are highly inaccurate when estimated using conventional methods of multilateration assuming a single bulk velocity.

3.2 CWI and Conventional Estimates of Changes in Velocity and Source Location: Synthetic Tests

We now test CWI against conventional methods for measuring a *change* in the velocity of a medium, using finite difference numerical wavefield simulations through the three μ CT slices in Figure 2. Two slightly different velocity models for each sample are generated: one is the unperturbed medium and the other has perturbed velocities of both mineral and fluid phases equal to a -1% ($\Delta V/V = -0.01$). The simulated signals are obtained from an array of receiver positions along the bottom of the sample as used in Figure 3. The change in velocity ($\Delta V/V$) between each pair of models is estimated from these signals by CWI (using Equations 1 and 2), and using the conventional method of manual phase-picking of first-break arrivals (time of first peak) assuming straight rays. Figure 6 compares these estimates for each sample. For all samples, CWI gives more accurate (closer to the true perturbation of the model) and more precise (lower standard deviation) estimates of $\Delta V/V$, and is more robust (shows significantly less variation between different receiver locations) when compared to the first-break method. This effect is clearly dependent on the complexity of the medium: the first-break estimates for Tivoli Travertine (Figure 6a) show much stronger variation than those for Westerly Granite (Figure 6b). The CWI estimates for $\Delta V/V$, however, do not vary between samples

388 of differing complexity. Coda waves sample the entire medium rather than a specific (fastest)
389 ray path, therefore CWI is more robust to changes in receiver location. This consistency
390 of estimates shows that CWI is less dependent on sample complexity, and on receiver
391 location, and confirms the hypothesis that the multiply reflected waves used in CWI ef-
392 fectively sample the entire medium, providing more representative measures of velocity
393 changes from any source and receiver pair.

394 We also test CWI and conventional methods for estimating *changes* in source lo-
395 cations. For this test, waveforms were simulated for a cluster of sources along a fracture
396 plane in the middle of each of the three samples, and with receivers located at the bot-
397 tom and at either side of the model (experimental configuration and source cluster lo-
398 cations shown in Figure 5). The standard method of multilateration (minimizing Equa-
399 tion 14) is used to locate source positions for each source in the cluster, assuming a con-
400 stant bulk velocity which is measured with a single source and receiver placed at the top-
401 center and bottom-center of the sample respectively. CWI provides the separation be-
402 tween pairs of sources (it does not provide source locations in an absolute frame of ref-
403 erence), so Figure 7 compares separations between the estimated source locations from
404 multilateration with source separations estimated from CWI. The latter estimates are
405 from Equations 9 and 13, and an estimate of the bulk velocity of the medium (the same
406 measured velocity used in multilateration) for each sample, and separations were obtained
407 using only the top receiver (multilateration estimates require the use of all four receivers).
408 For all three media, the multilateration-method estimates are relatively scattered, par-
409 ticularly for Tivoli Travertine and Copp-Crag Sandstone. CWI estimates of the relative
410 source locations are more precise, and are more accurate up to approximately $0.2-0.4\lambda$,
411 where λ is the dominant wavelength. At larger separations cycle-skipping in the cross-
412 correlation is likely to interfere with the signals that we seek in the maximum of the cor-
413 relation function, causing estimates to tend to a constant value at larger source sepa-
414 rations. We demonstrate in Section 3.5 below how relative locations of sources can be
415 obtained using separation data from even only a single receiver, and how the working-
416 range of source separations can be increased beyond 0.4λ .

417 3.3 Experimental Examples

418 In experimental rock physics, trends in velocity are often measured to model the
419 response of seismic velocity to changes in external conditions (e.g., temperature, effec-

420 tive and differential stresses, fluid properties, etc.), conferring particular importance to
421 the interpretation of dynamic changes. This is important for a range of geophysical sce-
422 narios on a larger scale, such as monitoring subsurface fluid reservoirs or changes in rock
423 properties using time-lapse (4D) seismic methods. Here we show results of two labora-
424 tory experiments that impose changes in the external conditions of temperature and stress.
425 In the first experiment illustrated in Figure 8a, a 10 cm³ block of Halldale Sandstone was
426 heated from room temperature to an external temperature of 54°C over one hour, and
427 then left to relax to room temperature. In this experiment we do not aim for thermal
428 equilibrium, because the CWI method does not require a constant medium velocity. The
429 experiment varies temperature simply to induce a non-uniform change in velocity within
430 the medium for comparison of CWI and conventional methods. A thermocouple was at-
431 tached to an external face for continuous temperature monitoring, and two piezoelec-
432 tric transducers (PZT) were attached on opposite faces of the sample for continuous ul-
433 trasonic surveys, which were undertaken during the cooling phase back down to room
434 temperature. As the maximum temperature variation is relatively small ($\Delta 8^\circ\text{C}$), we as-
435 sume that the PZT response to temperature variation is negligible.

436 To measure P-wave velocity we use Glaser-type conical piezoelectric sensors sen-
437 sitive to displacement normal to the sensor face (McLaskey & Glaser, 2012). These laboratory-
438 standard, wide-band sensors are calibrated against theoretical displacement time history
439 and have an almost flat displacement response spectrum in the 20 kHz to 1 MHz frequency
440 band. This means that, in this frequency band, they are essentially displacement sen-
441 sors and their voltage output is linearly proportional to the surface normal displacement.
442 Aperture effects are reduced due to the relatively small 0.5 mm sensor contact area (which
443 is even higher than the resolution used in Figure 3). We used an Itasca Image pulser-
444 amplifier system with operating frequency range of 100 kHz to 1 MHz and pre-amp gain
445 of 40 dB, which switches between all transducers in an ultrasonic array, allowing each
446 to act as both a transmitter and a receiver. The amplitude of the pulse spike is 500 V
447 with approximate signal rise time of 0.3 μs and total duration of 2.8 μs , the sampling
448 period is 40 ns. The output recorded waveform at each receiver is a stack of received wave-
449 forms from 25 source pulses with a pulse repetition frequency of 20 kHz (as the pulse
450 repetition is high, we assume no loss in phase resolution).

451 The change in velocity ($\Delta V/V$) for each temperature change (ΔT) were estimated
452 using both the first-break method (manually picking the first extremum) and the CWI

453 stretching technique (plotted in Figure 9). There is a large amount of scatter in the $\Delta V/V$
454 estimates for the first break method, where there is no clear trend that can be resolved
455 above the noise. In contrast, the $\Delta V/V$ estimates using CWI form a clear and coherent
456 response to changes in temperature - a linear, negative correlation due to thermal
457 contraction. This highlights the sensitivity of standard methods to noise, and CWI's abil-
458 ity to resolve small changes in spite of the presence of noise.

459 A second experiment was carried out, illustrated in Figure 8b, where a 38 mm di-
460 ameter, 75 mm length core of a fine grained laminated carbonate was held at 45 MPa
461 effective pressure, and a differential stress was applied with a strain rate of 10^{-5}s^{-1} , un-
462 til a peak stress of 235 MPa. The stress loading history is plotted in Figure 11a, where
463 pauses in loading are periods during which the permeability of the sample was measured.
464 P wave velocity is estimated using the Glaser-type sensors described above. We measure
465 S wave velocity using sensors with PZT sensitive to displacement tangential to the sen-
466 sor face, with a central frequency of 700 kHz and a contact area of 20 mm^2 . Example
467 waveforms for this experiment are shown in Figure 10. The variation of velocity during
468 the experiment is estimated using the standard first break method for estimating P and
469 S wave velocities, and the CWI moving reference trace method (from Equations 1, 2 and
470 3) using the time window labeled in Figure 10a ($t_1 = 0.35\text{ms}$, $t_2 = 0.65\text{ms}$). In Fig-
471 ure 11a we see CWI provides a far clearer and more consistent response to external stress
472 changes compared against the change in P wave velocity estimated using first-breaks,
473 accurately mirroring the stepped stress program with far less scatter in the estimated
474 $\Delta V/V$ values, most strikingly for the earlier stress steps. First-break S wave velocities
475 exhibit a smoother response (less scatter), but also fail to mirror the stepped stress pro-
476 gram. $\Delta V/V$ estimates from CWI approximately mark the average between changes in
477 P and S wave velocities - we discuss the way in which CWI averages changes in P and
478 S wave velocities in Section 4. The higher $\Delta V_P/V_P$ in estimates from the conventional
479 method may also reflect the bias towards higher velocities, as the first arriving waves fol-
480 low only the fastest ray path. As deformation occurs, compaction is localized to specific
481 regions of the sample; if the fastest travel path samples such regions, the estimated change
482 in velocity ($\Delta V_P/V_P$) would be larger using first-breaks than estimates using CWI which
483 is more representative of the changing bulk properties of the sample.

484 As CWI uses a cross-correlation function, the method breaks down if there are very
485 large changes in the medium due to wave paths being significantly altered and (if the

medium fractures) new scattering points being introduced. This means that a single reference trace is not appropriate for CWI in such deformation experiments where the rock structure is significantly deformed. This effect can be seen in Figure 11b, where different CWI algorithms are compared. The “double wavelet” method (Snieder et al., 2002) measures delay times ($\delta\tau$) for multiple time windows down the coda: these relate to the velocity perturbation by $\Delta V/V = -\delta\tau/t$. It is clear that at later stages in the experiment (after 1 hour), the estimates of $\Delta V/V$ using the double wavelet method with a fixed reference trace (dashed purple line) are heavily distorted due to the deformation occurring within the sample. The large amount of scatter exhibited by this method highlights the problem of large changes occurring in the medium. The stretching method, without implementing a moving reference trace (dashed red line), provides more consistent estimates of $\Delta V/V$ than the double wavelet method, estimating a consistent increase in velocity. At later stages in the experiment, these estimates of $\Delta V/V$ become more scattered and the mirroring of the stepped stress program becomes less clear. For both methods, implementing the moving reference trace method (Equation 3) limits estimates to small changes in velocity, for which CWI remains accurate, to obtain an overall estimate in $\Delta V/V$ that shows a much clearer stepped response. This suggests that the moving (or periodically updated) reference trace method can account for the more extreme changes that occur in the medium. There is no prescribed value for how frequently the reference trace should be updated (k in Equation 3) as it depends on the rate of deformation and the surveying frequency, except that it should be introduced before any changes produce a half-wavelength change in the waveform in the latest time window. However, the strengths of CWI lie in the ability to resolve small changes in velocity, therefore the step size k should remain small ($k = 5$ for results shown in Figure 11b, where surveys are taken every minute).

3.4 Joint Estimation of Source Separation and Velocity Change

Since CWI estimates of the bulk velocity change ($\Delta V/V$) and source separation (r) are derived from different information (the phase and the maximum value of correlation as shown in equations 2 and 9, respectively), estimates of each can be made independently when both effects occur simultaneously. This has significant experimental advantages, as fixed source and receiver locations might no longer be necessary for continuous velocity measurements, and in deformation experiments when acoustic emissions

517 might accompany bulk velocity changes these two effects could be analyzed independently
518 - all using a single receiver.

519 We test the accuracy of these estimates using a series of finite-difference simula-
520 tions taking a central source location and changing the location by up to 1.2λ and si-
521 multaneous velocity perturbations of up to 1%. Figures 12a and b show estimates of source
522 separation (r) where no velocity perturbation occurs, and the reverse - changes in ve-
523 locity when the source remains stationary. These represent the best possible estimates
524 from CWI, as only one perturbation type occurs at a time. The additional errors asso-
525 ciated with simultaneous perturbations of r and V are shown in Figures 12c and d. We
526 see that estimates of source perturbation are barely affected by the presence of a veloc-
527 ity perturbation: the stretching method of CWI removes the effect of any velocity per-
528 turbation. However, estimates of velocity perturbation are far more sensitive to source
529 location perturbations, giving errors of 0.5% for a source displacement of around one wave-
530 length (a relatively large error given the accuracy otherwise expected from CWI). The
531 additional error appears to stem from the effect of cycle skipping in the cross-correlation
532 function when changes result in the alteration of travel times to on the order of half a
533 wavelength.

534 These results also show that in the case of simultaneous perturbations of source
535 location and velocity, source separation can be estimated much more accurately than es-
536 timates of the change in velocity. Therefore, we would expect that the 3D network of
537 relative locations of acoustic emissions that occur during deformation can be estimated
538 robustly using laboratory datasets even if velocity changes occur in the medium (Zhao
539 & Curtis, 2019; Zhao et al., 2017). This is demonstrated in the following section.

540 **3.5 Relocating relative source locations from inter-source distance**

541 Using the inter-source distances or separations between many pairs of sources, it
542 is possible to find the relative locations of a cluster of sources, provided that inter-source
543 distances are within the working range of CWI. However as we see in Figure 7, CWI
544 provides a slightly biased estimate of these separations. The relocation method solves
545 for the relative location of a cluster of sources in a probabilistic framework within which
546 it is possible to correct this bias to a significant extent (Robinson, Sambridge, Snieder,
547 & Hauser, 2013; Zhao & Curtis, 2019; Zhao et al., 2017). For one pair of events, accord-

ing to Bayes' theorem

$$P(\tilde{\delta}_t|\tilde{\delta}_{CWI}) \propto P(\tilde{\delta}_{CWI}|\tilde{\delta}_t) \times P(\tilde{\delta}_t), \quad (15)$$

where the posterior probability $P(\tilde{\delta}_t|\tilde{\delta}_{CWI})$ is the probability of the true separation having value $\tilde{\delta}_t$ given that the estimated separation from CWI is $\tilde{\delta}_{CWI}$. This is proportional to the likelihood $P(\tilde{\delta}_{CWI}|\tilde{\delta}_t)$ of having observed $\tilde{\delta}_{CWI}$ in the case that the true separation is $\tilde{\delta}_t$, multiplied by the prior probability $P(\tilde{\delta}_t)$ which describes any available information about event locations known prior to the location process. The likelihood function $P(\tilde{\delta}_{CWI}|\tilde{\delta}_t)$ describes the bias in separations estimated by CWI, and can be approximated by a Gaussian probability density function whose mean and standard deviation are described by empirical functions proposed by Robinson et al. (2011). The tilde over parameters indicates that the separation quantities are used in normalized form - they are the true values divided by the wavelength of the dominant frequency recorded in the seismogram coda.

For multiple events, Equation 15 holds for each event pair. The separation estimated from CWI for a cluster of events can be incorporated into a joint posterior function by multiplying the formulae for all available event pairs, assuming that they are independent of one another (Robinson et al., 2013):

$$P(\mathbf{e}_1, \dots, \mathbf{e}_n|\tilde{\delta}_{CWI}) = c \prod_{i=1}^n P(\mathbf{e}_i) \times \prod_{i=1}^{n-1} \prod_{j=i+1}^n P(\tilde{\delta}_{CWI,ij}|\mathbf{e}_i, \mathbf{e}_j), \quad (16)$$

where c is a constant, n is the number of events, $\mathbf{e}_i = (x_i, y_i, z_i)$ is the location of event i . Within the last term we use the locations of the i th and j th events (\mathbf{e}_i and \mathbf{e}_j) from which we can calculate their separation $\delta_{t,ij} = \|\mathbf{e}_i - \mathbf{e}_j\|_2$ (subscript 2 denotes the L-2 norm), and thus we implicitly include Equation 15. The most probable set of the event locations can be found where the joint posterior function attains its maximum. Therefore, the event locations can be estimated by solving an optimization problem. The optimization problem is converted to a minimization problem by taking the negative logarithm of Equation 16:

$$-\ln[P(\mathbf{e}_1, \dots, \mathbf{e}_n|\tilde{\delta}_{CWI})] = -\ln[c] - \sum_{i=1}^n \ln[P(\mathbf{e}_i)] - \sum_{i=1}^{n-1} \sum_{j=i+1}^n \ln[P(\tilde{\delta}_{CWI,ij}|\mathbf{e}_i, \mathbf{e}_j)]. \quad (17)$$

A uniform prior $P(\mathbf{e}_i)$ is considered in this work, so the terms containing $\ln[P(\mathbf{e}_i)]$ are constant, and the term $\ln[c]$ can be ignored in the minimization problem. Thus, the objective function becomes:

$$L(\mathbf{e}_1, \dots, \mathbf{e}_n) = - \sum_{i=1}^{n-1} \sum_{j=i+1}^n \ln[P(\tilde{\delta}_{CWI,ij}|\mathbf{e}_i, \mathbf{e}_j)]. \quad (18)$$

579 This function can be minimized using a conjugate gradient algorithm (Press, Flannery,
580 Teukolsky, & Vetterling, 1986).

581 We test this location method using the Tivoli Travertine model shown in Figure
582 2b, and source locations shown in Figure 13a, simulating a cluster of 80 acoustic emis-
583 sions around a fracture plane. We divided the events into multiple sub-clusters with 20
584 overlapping event locations, where the maximum separations in each sub-cluster remained
585 roughly within or just outside of the working range of CWI (approximately 0.5λ). The
586 separation into sub-clusters can be achieved using only the pairwise separation estimates
587 from CWI, by sorting pairs of events by estimated proximity, an optimal configuration
588 of sub-clusters can be found so that all separation values are within 0.5λ . We therefore
589 do not require knowledge of the true source locations for this step in the method.

590 For each sub-cluster, we solved for the relative event locations by minimizing Equa-
591 tion 18 using the publicly available CWI-relocation code package of Zhao and Curtis (2019),
592 taking the CWI separation estimates as inputs. We conducted the location process five
593 times with different randomly distributed initial event locations to ensure convergence
594 to the global minimum of the objective function (Equation 17). The optimizations all
595 converge to the same minimum to within trivial numerical differences. Receiver locations
596 follow the same configuration as shown in Figure 5a. Since absolute event locations re-
597 main unknown in this method, we then rotate and translate the resulting sub-clusters
598 to match locations of the overlapping sources. For comparison, we also performed the
599 conventional method for locating sources, using manual phase-picking of first-break (first
600 extremum) arrivals for multiple receivers, and multilateration (Equation 14) to estimate
601 locations of sources. The results of multilateration and CWI relocations are shown in
602 Figure 13b and c, respectively in order to cluster events.

603 We note immediately that the cluster of events from multilateration in Figure 13b
604 is rotated by 45° relative to the true locations due to velocity heterogeneity in the sam-
605 ple. Since CWI only provides *relative* locations, the cluster of CWI location in panel c
606 has been rotated to best match the results in panel b for fair comparison. The spatial
607 area of events in panel c appears to be more rectangular (like the true shape of the area
608 in panel a) than the area in panel b. Nevertheless, it is difficult to decide which of Fig-
609 ure 13b and c is better from these plots alone so Figure 14 shows the source separation
610 values of these two clusters as a function of true source separation normalized by wave-

length λ . This highlights the improvement of accuracy and precision offered by the CWI source relocation procedure. It is also important to note from Figure 14 that using the sub-cluster matching methods, the overall source network size can extend well beyond the usual working range of CWI and the source-separation bias can be largely corrected, providing there are overlapping sources between sub-clusters.

3.6 Sensitivity to Noise

In order to test the ability of CWI to estimate changes in velocity and in source or receiver location when using noise-contaminated data, we generate a synthetic record of noise which is superimposed onto the numerically simulated signals used above. We generate realistic noise as follows: 1) measure a long noise record in the Edinburgh rock physics laboratory, and process it to create a record of de-measured and de-trended seismic noise. 2) Take the Fourier Transform of the noise recording, and smooth the record in the Fourier domain to ensure there are no spectral gaps (frequency bands without noise). 3) Convolve the resulting spectrum with a sample of random Gaussian white noise so that generated noise is uncorrelated and transform back into the time domain. The resulting signal is therefore a randomly generated recording of realistic noise, which can be superimposed on the effectively noiseless waveforms generated from synthetic finite difference simulations. The signal-to-noise ratio (SNR) is calculated as $SNR = P_{signal}/P_{noise}$, where P is the average power. We add the noise at different SNR values to a range of numerically simulated signals where the velocity has been perturbed from 0 - 10% and where the source location is perturbed by 0.01λ . Estimates of the range of velocity perturbations are calculated using CWI, as well as by using conventional phase-picking methods for each level of noise contamination. For the phase-picking of first arrivals, we use automatic methods (STA/LTA method described by Earle and Shearer (1994)) as well as manually picking the time of the first extremum. These estimates are shown for low noise contamination (SNR=8) and high noise contamination (SNR=0.43) in Figure 15. The total error at each SNR value, calculated as the sum of residuals of each estimate to the true $\Delta V/V$ value is shown in Figure 16a. We find that at high SNR values, all estimates for $\Delta V/V$ show a clear response to the increasing velocity perturbation, though CWI estimates are over an order of magnitude more accurate. At low SNR values, conventional methods based on phase-picking show much more scatter in the estimates of $\Delta V/V$, whereas CWI is much more precise, and is mostly unaffected by the increased

643 contamination of noise. The first-break arrivals are of lower amplitude and are therefore
644 more susceptible to contamination by noise, whereas CWI uses the entire signal, includ-
645 ing many more data points, and is therefore more robust in the presence of noise.

646 For estimation of source separation in the presence of noise (see Figure 16b), the
647 absolute locations of sources within a small cluster were estimated by multilateration by
648 assuming a constant, isotropic P-wave velocity. However, because CWI does not provide
649 absolute source locations but instead gives the separation between two sources, r , we es-
650 timate the separation r between pairs of absolute locations from multilateration for com-
651 parison. We compare this to the r estimate from CWI for each pair of sources, and plot
652 the sum of individual residuals for all source pairs and for each method in Figure 16. We
653 find that at all SNR values CWI outperforms multilateration, particularly at high lev-
654 els of noise. These results show that CWI is a more robust way to characterize changes
655 in a medium's velocity or in relative source locations in the presence of noise. Since no
656 phase picking is necessary for CWI, this also means that less pre-processing of data is
657 required before analysis. CWI requires the computation of many cross-correlation func-
658 tions, therefore can be computationally expensive compared to conventional methods,
659 however we have demonstrated this method to offer significant improvements in both ac-
660 curacy and precision.

661 **4 Estimating Individual P and S Wave Contributions to CWI Obser-** 662 **ervations**

663 The results from CWI only provide a measure of the change in velocity and not the
664 absolute velocity itself. In itself this is not of particular concern since in many real-world
665 problems, such as those relating to the interpretation of 4D seismic data, we seek to char-
666 acterize the dynamic dependence of velocity on changes in external properties (Landrø
667 & Stammeijer, 2004). However, $\Delta V/V$ estimates from CWI are more difficult to inter-
668 pret than separate estimates of V_P and V_S that are obtainable from conventional meth-
669 ods. Given an estimate of density, estimates of V_P and V_S allow bulk and shear mod-
670 uli to be estimated, and these are parameters that appear in the majority of rock physics
671 models. CWI estimates of $\Delta V/V$ reflect a combination of P-wave and S-wave velocity
672 information due to the multiple phase conversions that occur during wave propagation.

673 To aid the interpretation of CWI $\Delta V/V$ estimates, consider the scattering model
674 presented by Snieder (2002) which assumes isotropic point scatterers inside a constant

675 velocity medium. This model represents P and S wave states as many packets of energy
 676 traveling with velocities V_P and V_S . A packet can only be in one state at a given time.
 677 When a packet of P energy travels distance a (the average distance between scatterers),
 678 it has a probability p_{PS} of converting to an S state; likewise a packet of S energy has
 679 a probability p_{SP} of converting to the P state. Over a time interval dt , a packet in the
 680 P state encounters $V_P dt/a$ scatterers, meaning that in a system with N_P and N_S pack-
 681 ets in the P and S states, the reduction in P packets due to P -to- S conversions is
 682 given by $-2p_{PS}N_P V_P dt/a$ and the increase due to S -to- P conversions is given by
 683 $p_{SP}N_S V_S dt/a$. Following from this, Snieder (2002) derives the following system of dif-
 684 ferential equations:

$$685 \quad \dot{N}_P = \frac{1}{a}(p_{SP}V_S N_S - 2p_{PS}V_P N_P), \quad (19)$$

$$686 \quad \dot{N}_S = \frac{1}{a}(2p_{PS}V_P N_P - p_{SP}V_S N_S), \quad (20)$$

687 where the dot over N_P and N_S on the left side indicates a rate of change over time. Now
 688 consider a receiver not co-located with the source, at which the time of first arriving en-
 689 ergy in the signal is comprised of only P state energy. After this time the proportions
 690 of P and S wave energy can be calculated using equations 19 and 20, and therefore so
 691 can the proportions of *changes* in P-wave velocity ($\Delta V_P/V_P$) and S-wave velocity ($\Delta V_S/V_S$).
 692 The way in which these proportions of $\Delta V/V$ vary as a function of time is shown in Fig-
 693 ure 17. For time values to be independent of the scattering properties of the medium,
 694 time is normalized by the travel time of one mean free path ($\tau_P = l_P/V_P$), where the
 695 mean free path l_P is defined as $l_P = a/(2P_{PS})$. In practice, the mean free path of a
 696 scattering medium can be estimated from the apparent attenuation of energy in recorded
 697 signals (Anugonda, Wiehn, & Turner, 2001; Obermann, Planès, Larose, Sens-Schönfelder,
 698 & Campillo, 2013). Figure 17 shows how the proportions of $\Delta V_P/V_P$ and $\Delta V_S/V_S$ change
 699 depend on the V_P/V_S ratio. At equilibrium, the proportion of $\Delta V_S/V_S$ is higher than
 700 $\Delta V_P/V_P$, even at very low V_P/V_S ratios (Figure 17a), explained by S having two states
 701 (S_1 and S_2 , which represent the two polarizations of S waves) where P only has one state.
 702 As V_P/V_S increases, so does the proportion of $\Delta V_S/V_S$ at equilibrium, as energy in S
 703 waves are traveling more slowly than P waves and so spend more time in that state be-
 704 fore encountering scatterers.

705 We can use this model to estimate the independent changes of P and S wave ve-
 706 locity. Define $q(t, \gamma)$ to be the relative contribution of $\Delta V_S/V_S$ (the red curves in Fig-

ure 17), where $\gamma = V_P/V_S$. The function q depends on time t and on the V_P/V_S ratio γ , and the relative contribution of $\Delta V_P/V_P$ (blue curves in Figure 17) is $1-q(t, \gamma)$. If P and S wave velocities change by different amounts, the measured change in velocity from CWI $[\Delta V/V]_{CWI}$ therefore varies as a function of time along the coda by

$$\left[\frac{\Delta V}{V}\right]_{CWI}(t) = [1 - q(t, \gamma)] \left[\frac{\Delta V_P}{V_P}\right] + q(t, \gamma) \left[\frac{\Delta V_S}{V_S}\right]. \quad (21)$$

For a single time window, this equation has two unknown parameters, $\Delta V_P/V_P$ and $\Delta V_S/V_S$; the value of $[\Delta V/V]_{CWI}$ can be measured and $q(t, \gamma)$ is known (from Figure 17). Measuring $[\Delta V/V]_{CWI}$ in multiple time windows along the coda therefore gives multiple equations, the same number as there are time windows. Quantities $\Delta V_P/V_P$ and $\Delta V_S/V_S$ can be estimated using an ordinary least squares inversion approach to solve the system: $d = Am$, where d is a matrix of measured values of $[\Delta V/V]_{CWI}$ for each time window, and A is matrix of $(1-q)$ and q values expected at each time window for a given V_P/V_S ratio γ . The resulting vector m contains estimates of $\Delta V_P/V_P$ and $\Delta V_S/V_S$ for a given V_P/V_S ratio, and we denote these estimates by $[\widehat{\Delta V_P/V_P}]_\gamma$ and $[\widehat{\Delta V_S/V_S}]_\gamma$, respectively. Clearly, in order to estimate the changes of V_P and V_S independently we need to be able to estimate $\gamma = V_P/V_S$.

One way to estimate γ would be to use the conventional experimental method to estimate V_P and V_S , but as we have shown herein, those methods are less accurate than CWI for subtle changes in the medium so it is desirable to find alternative methods. As Figure 17 shows, values for $q(t)$ can vary significantly depending on the V_P/V_S ratio. We can therefore refine estimates of $\Delta V_P/V_P$ and $\Delta V_S/V_S$ within a probabilistic framework, using a statistical distribution of V_P/V_S ratios rather than a single value. We illustrate this by compiling a database of 296 measured V_P/V_S ratios for dry carbonates combining data from Bakhorji (2010), Fournier et al. (2011) and Verwer, Braaksma, and Kenter (2008). This data is selected purely as a demonstration of how such a distribution could be used; in practice such a distribution should be refined as the database contains samples with a large range porosities, pore structures and measurements at different confining pressures, only some of which would be relevant for our rock type or volume of interest. From the carbonate database, we create a prior distribution of V_P/V_S ratios γ for carbonate rocks $P_{carb}(\gamma)$, shown in Figure 18a. In order to test the method we also calculate synthetic $[\Delta V/V]_{CWI}$ data using Equation 21 with a change in P wave velocity of 1%, a change in S wave velocity of 0.5%, and a V_P/V_S ratio equal to $\sqrt{3}$ ($\Delta V_P/V_P = 1\%$, $\Delta V_S/V_S = 0.5\%$, $\gamma = \sqrt{3}$), which gives $[\Delta V/V]_{CWI}$ as a function of time (Fig-

740 ure 18b). The method then proceeds as follows: using the generated $[\Delta V/V]_{CWI}$ data
 741 and the known values for $q(t, \gamma)$, we invert for $[\Delta \widehat{V}_P/V_P]_\gamma$ and $[\Delta \widehat{V}_S/V_S]_\gamma$ for a range
 742 of values of V_P/V_S ratios (γ), shown in Figure 18c. However, given the knowledge that
 743 the sample is a carbonate, not all of these values are equally likely. We should therefore
 744 weight this set of solutions by the probability P that each V_P/V_S ratio is the one in our
 745 sample - represented by the probability distribution in Figure 18a. Thus we can gener-
 746 ate probability density functions for estimates of $\Delta V_P/V_P$ and $\Delta V_S/V_S$ with the follow-
 747 ing equations:

$$748 \quad P\left(\frac{\Delta V_P}{V_P}\right) = \int_{\gamma \in R_\gamma} \delta\left(\frac{\Delta V_P}{V_P} - \left[\frac{\Delta \widehat{V}_P}{V_P}\right]_\gamma\right) \cdot P_{carb}(\gamma) d\gamma, \quad (22)$$

$$749 \quad P\left(\frac{\Delta V_S}{V_S}\right) = \int_{\gamma \in R_\gamma} \delta\left(\frac{\Delta V_S}{V_S} - \left[\frac{\Delta \widehat{V}_S}{V_S}\right]_\gamma\right) \cdot P_{carb}(\gamma) d\gamma, \quad (23)$$

750 where R_γ is the prior range of V_P/V_S ratios γ . In the case where $\Delta V_P/V_P = 1\%$ and
 751 $\Delta V_S/V_S = 0.5\%$, the resulting probability distributions for changes in P and S wave
 752 velocities are shown in Figures 18d and e. For both changes in P and S wave velocity,
 753 the method accurately estimates the velocity change. The probability distribution change
 754 in P wave velocity $\Delta V_P/V_P$ is relatively precise, with almost all estimates within $\pm 0.01\%$
 755 of the true value for velocity change. The distribution of change in S wave velocity has
 756 a wider spread, though still significant precision when compared to standard methods,
 757 with the majority of estimates within $\pm 0.03\%$ of the true velocity change. From this we
 758 can see that it is possible to estimate independent changes in P and S wave velocity us-
 759 ing CWI given the statistical distribution of V_P/V_S ratios for a rock type, and with the
 760 assumption of isotropic scattering.

761 5 Discussion

762 We have demonstrated that under the conditions examined here, using Coda Wave
 763 Interferometry for experimental applications can provide significant improvements over
 764 conventional methods, particularly in the accuracy and precision of estimates of changes
 765 in velocity and source location.

766 An important aid in the interpretation of CWI estimates is an understanding of
 767 the type of spatial average of material parameters that is implicit in CWI estimates. To
 768 examine this, a numerical experiment is conducted using the μ CT derived velocity and
 769 density models of the Tivoli Travertine (Figure 2a). The fluid velocity (initially 1500 m/s)
 770 is perturbed by a range of values (up to a +10% perturbation), and CWI is used to es-

771 estimate the velocity perturbation of the bulk medium. As the exact amount of calcite and
 772 pore fluid phases are known, as well as their properties, the change in the average prop-
 773 erties of the medium can be calculated with various averaging methods. Here we use the
 774 Voigt upper bound M_V (Voigt, 1928):

$$775 \quad M_V = \sum_{i=1}^N f_i M_i, \quad (24)$$

776 and the Reuss lower bound M_R (Reuss, 1929):

$$777 \quad \frac{1}{M_R} = \sum_{i=1}^N \frac{f_i}{M_i}, \quad (25)$$

778 where f_i is the volume fraction of the i th phase and M_i is the elastic modulus of the i th
 779 phase, M can represent the bulk modulus K or the shear modulus μ . We also use the
 780 Voigt-Reuss-Hill average (Hill, 1952) $[M_V + M_R]/2$, and the Hashin-Shtrikman bounds
 81 (Hashin & Shtrikman, 1963):

$$782 \quad K^{HS\pm} = K_1 + \frac{f_2}{(K_2 - K_1)^{-1} + f_1(K_1 + \frac{4}{3}\mu_1)^{-1}} \quad (26)$$

$$783 \quad \mu^{HS\pm} = \mu_1 + \frac{f_2}{(\mu_2 - \mu_1)^{-1} + 2f_1(K_1 + 2\mu_1)/[5\mu_1(K_1 + \frac{4}{3}\mu_1)]} \quad (27)$$

835 where the subscripts 1 and 2 refer to the two phases in the medium and the upper and
 836 lower bounds are computed by interchanging which phase is termed 1 and 2 (Mavko et
 837 al., 2009). The Reuss lower bound is equal to the Hashin-Shtrikman lower bound when
 838 one of the constituents is a liquid with zero shear modulus. We calculate the various av-
 839 erages taking the bulk and shear moduli to be $K_{calcite} = 129.53$ GPa, $\mu_{calcite} = 35$ GPa,
 840 $K_{fluid} = 2.25$ GPa, and $\mu_{fluid} = 0$. A comparison of how these different measures spa-
 841 tially average the medium is shown in Figure 19. Of the different methods used, the Reuss
 793 lower bound shows the closest estimate to the measured first break velocity in Figure
 19a, and of the CWI estimates for velocity change in Figure 19b.

794 The use of CWI estimates in current rock physics protocols is therefore possible
 795 because the appropriate information required for many rock physics models is available:
 796 the relative proportions of P and S wave velocity changes (Figure 18) is obtainable given
 797 prior knowledge of V_P/V_S ratios of the medium (based for instance on rock type as in
 798 the example above), and we can infer how CWI averages the bulk velocity change prop-
 799 erties of a medium spatially (Figure 19).

800 The method of CWI used here (Equation 1) is known as trace stretching and has
 801 some underlying assumptions and limitations. Namely it assumes that the velocity per-
 802 turbation is uniform across the entire medium so that all arriving energy is perturbed

803 at the same temporal rate, and therefore the trace is stretched linearly in time along the
 804 seismogram. Mikesell, Malcolm, Yang, and Haney (2015) provides a comparison of dif-
 805 ferent methods to estimate changes in velocity for CWI, and suggests a dynamic time
 806 warping method as a solution for inhomogeneous velocity perturbations.

807 As we have shown, CWI is able to resolve both changes in velocity and changes in
 808 source and/or receiver locations, allowing for the estimation of relative source locations.
 809 However CWI is also able to resolve another type of perturbation on which we have not
 810 focused: the average displacement of all scatterers, δ , illustrated in Figure 1c (Snieder
 811 et al., 2002). This value is related to the variance of travel time perturbations by

$$812 \sigma_{\tau}^2 = \frac{2\delta^2 t}{vl_{\star}}, \quad (28)$$

813 where l_{\star} is the transport mean free path. It would be interesting to monitor how this
 814 parameter varies during experimental rock physics and geomechanics experiments. For
 815 example, it may be possible to monitor changes in the average distance between scat-
 816 tering points, which could act as a proxy measure for inter-pore distance, itself a strong
 817 control on the time of failure (Vasseur et al., 2017). During the confining or varying of
 818 fluid pressure in an isotropic sample, scattering points would be displaced in all direc-
 819 tions, and this displacement might be measured by CWI. Similar effects occur at reser-
 820 voir scale where fluid injection or extraction can lead to seismically observable volumet-
 821 ric expansion of the reservoir. We leave this for future research.

822 Most of the numerical experiments presented here assume a high frequency regime
 823 as well as point sources and receivers. In one experiment where we lowered the frequency
 824 of by more than an order of magnitude we did not observe any significant differences in
 825 the method. Nevertheless, another area for the development of the CWI method is to
 826 investigate the dependence of CWI results over a broad range of frequencies, and using
 827 much larger aperture transducers such as those modelled by Li et al. (2018). We leave
 828 this for future research.

829 6 Conclusion

830 Conventional first-break methods based on manual phase-picking provide an es-
 831 timate of seismic velocity that is not representative of the bulk medium in a high fre-
 832 quency regime with point sources and point receivers. Such estimates of seismic veloc-
 833 ity, changes in velocity, and source location are highly variable even for a single sample,

834 and depend on the specific source/receiver path of the first arriving wave. They are there-
835 fore inadequate for characterizing the bulk properties of a rock sample, particularly those
836 with complicated pore structures approximately similar size to the wavelength of the in-
837 terrogating waves. By contrast, Coda Wave Interferometry is an effective method for coun-
838 tering these problems because coda waves sample the entire medium, and sample the same
839 regions multiple times. CWI is shown to provide an increase in precision by an order of
840 magnitude in the absence of noise, and to be a robust and accurate method for estimat-
841 ing both bulk velocity changes and perturbations of the source or receiver locations when
842 compared with standard methods in both synthetic digital rock physics and laboratory
843 experimental data. When noise is present, CWI remains far more accurate than conven-
844 tional methods, even at very low signal-to-noise ratios. Additionally, when velocity and
845 source/receiver location perturbations occur simultaneously CWI can still estimate ve-
846 locity and source separation under some conditions: source separation estimates are mostly
847 unaffected by the velocity perturbation, but velocity change estimates are much more
848 sensitive and become inaccurate in the presence of larger source perturbations, possibly
849 due to cycle-skipping. Using source separation estimates, relative locations of a cluster
850 of sources can be estimated using a single receiver, and show higher precision and ac-
851 curacy compared to conventional methods. CWI estimates a combination of changes in
852 both P and S wave velocities, and we demonstrate a model for the equilibration of the
853 contributions from P and S waves as a function of time, and show how the independent
854 changes in P and S wave velocity can be measured, given probabilistic *a priori* informa-
855 tion about the V_P/V_S ratio. Overall these results show significant potential for the use
856 of CWI to characterize changes in porous media undergoing changes in effective stress
857 and strain, and in temperature.

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865 https://github.com/JonathanSingh/cwi_codes/

866 The code package of Zhao and Curtis (2019) is available at:

867 <https://www.geos.ed.ac.uk/eip/codes.html>

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A19–A24.

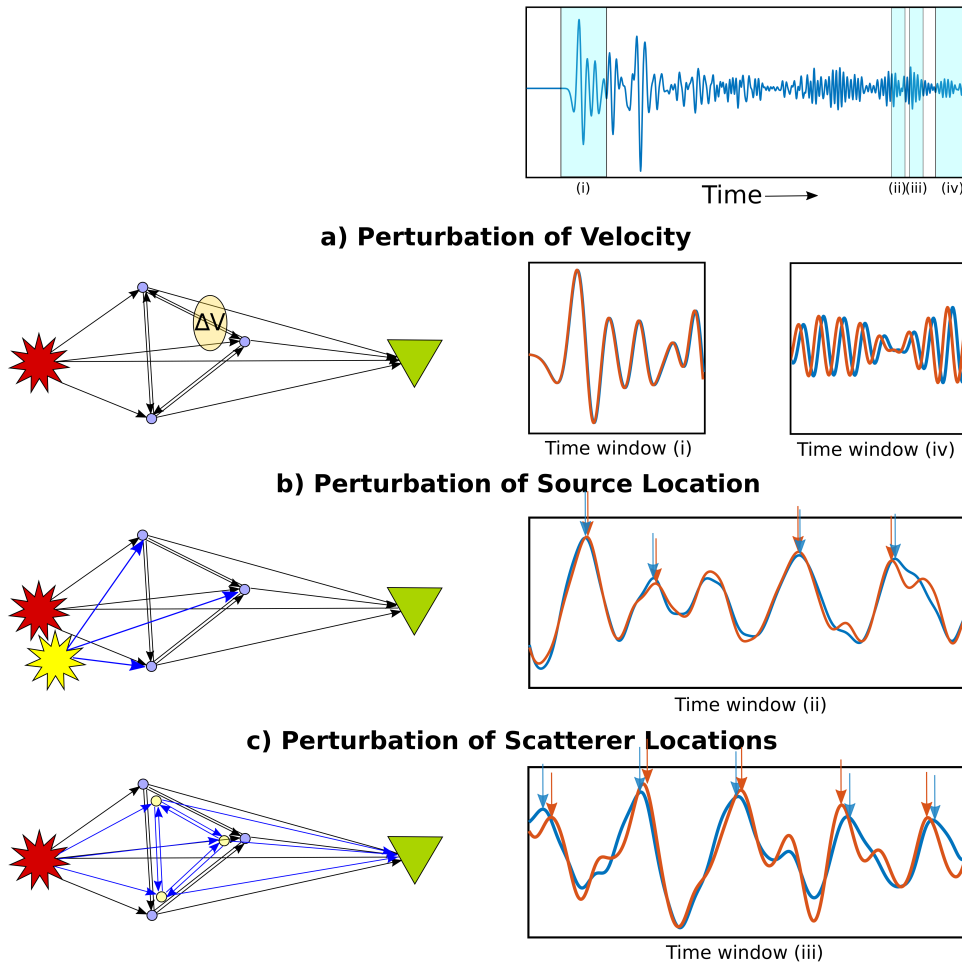
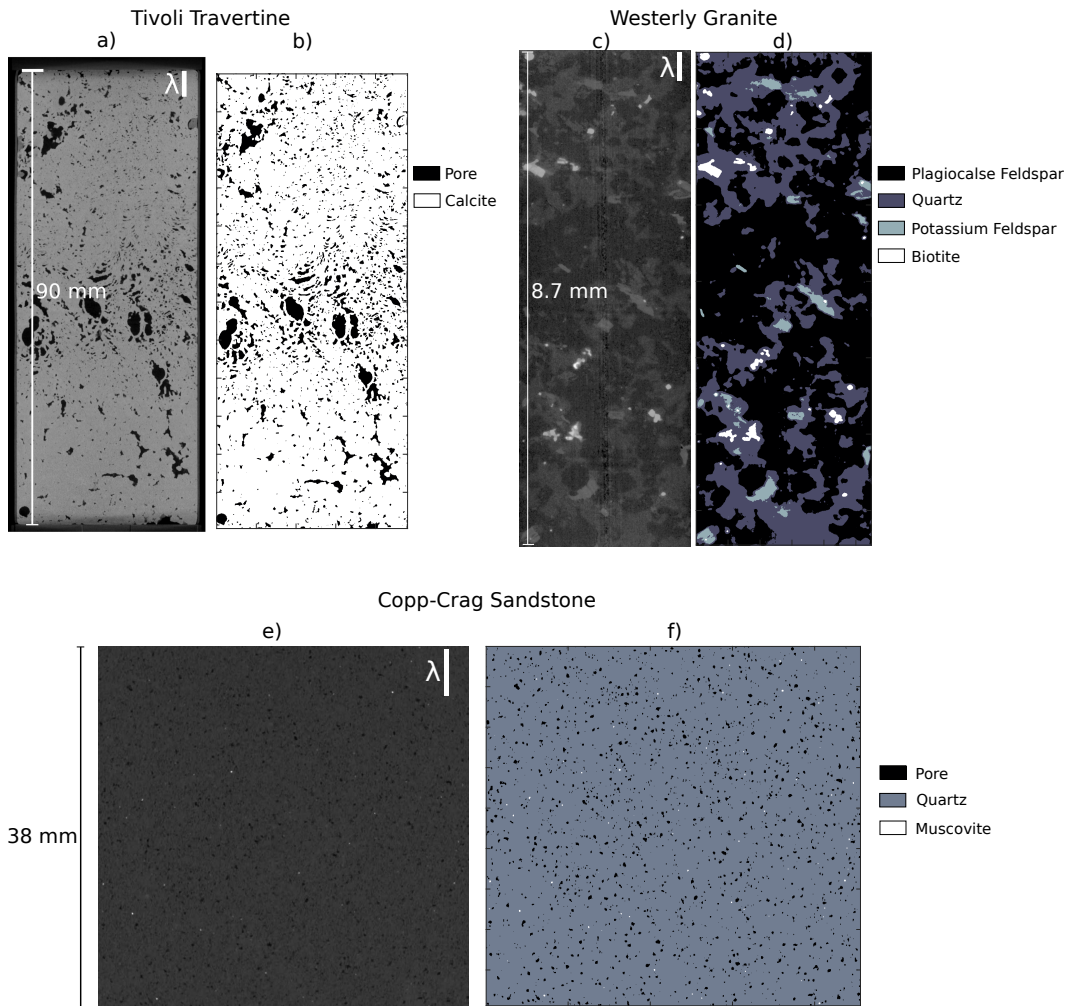


Figure 1. Illustrations of different perturbation types and their effects on coda waves. The cartoons (left) represent a scattering medium, with a source (star), receiver (triangle), and point scatterers (circles). Ray paths between the source and receiver, including multiple reverberations, are represented as black arrows. A velocity perturbation (a) is represented as a yellow ellipse, which has a velocity different to the background medium. New ray paths that are introduced due to changes in source location (b) and scatterer locations (c) are represented as blue arrows. Example recorded signals (right) at a range of time windows (i-iv) are shown before and after each perturbation takes place (blue and red, respectively). Differences in travel times of arriving energy for b) and c) are highlighted by vertical arrows.



1062 **Figure 2.** Set of X-ray μ CT slices (left images) and equivalent models of segmented phases
 1063 (right images) for three rock cores with varying heterogeneity and rock type: a) and b) Tivoli
 1064 Travertine, c) and d) Westerly Granite, e) and f) Copp-Crag Sandstone. Model sizes are:
 1065 900x2400, 1000x3000 and 900x900 pixels for Tivoli Travertine, Westerly Granite and Copp-Crag,
 1066 respectively. Approximate wavelength λ for each sample is labeled with a white bar, where the
 1067 source signals contain a peak frequency of 30 MHz for Tivoli Travertine and Copp-Crag Sand-
 1068 stone, and 200 MHz for the smaller Westerly Granite model. The properties assigned to each
 1069 material phase for wavefield simulation can be found in Table 1.

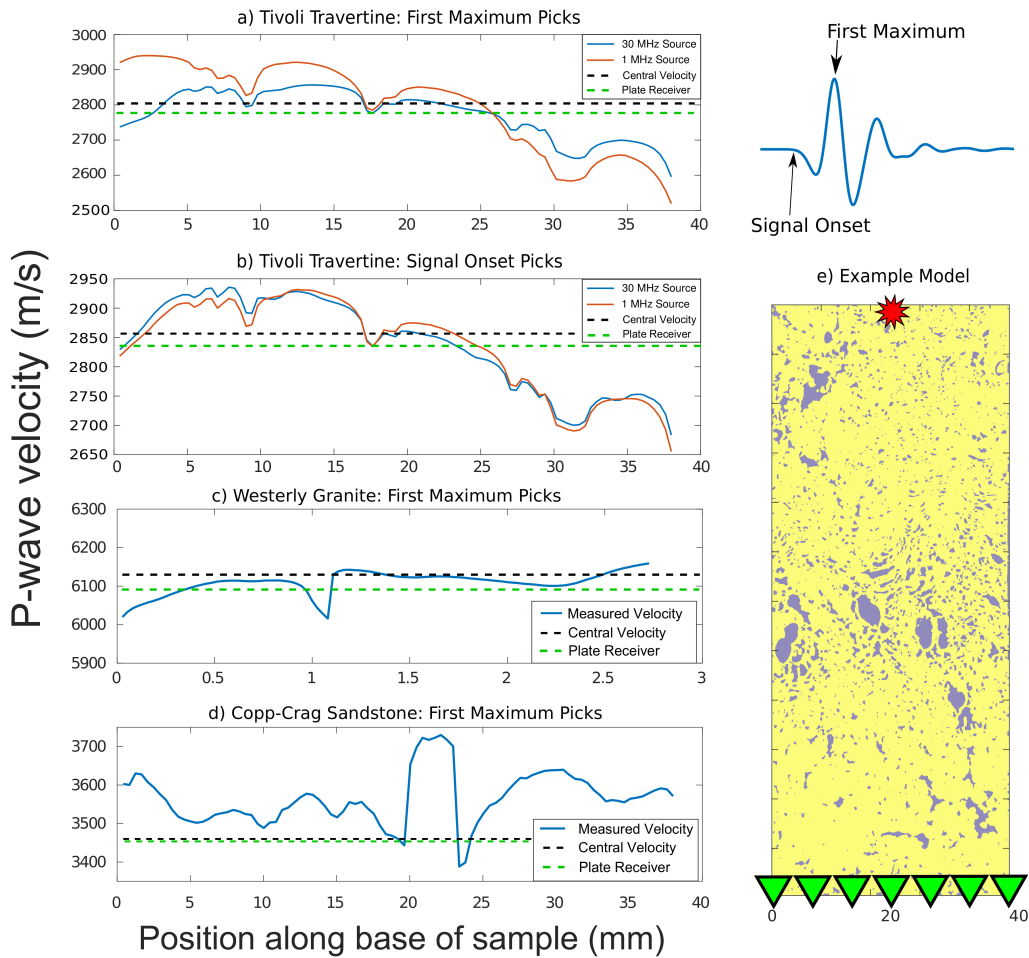
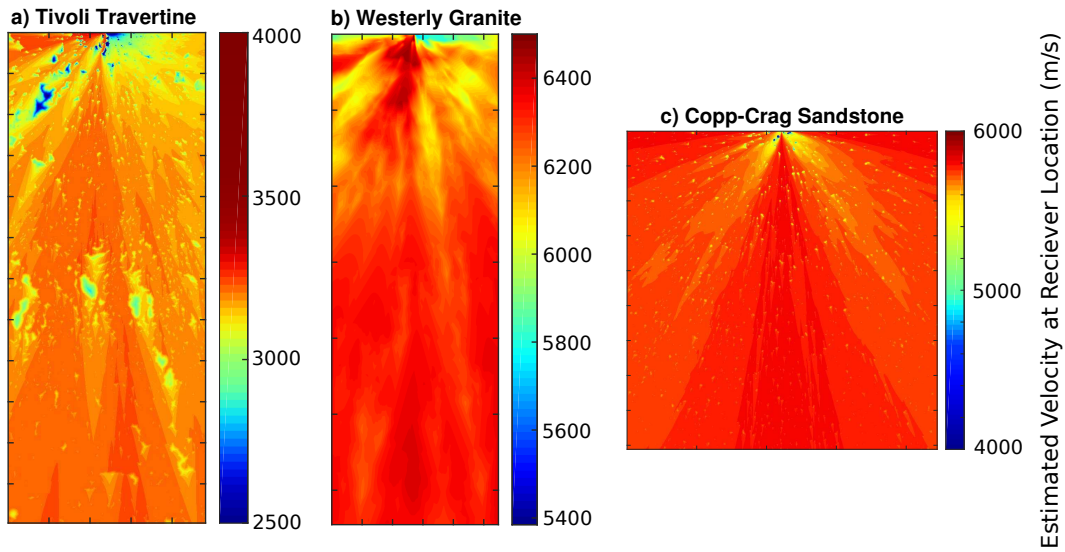
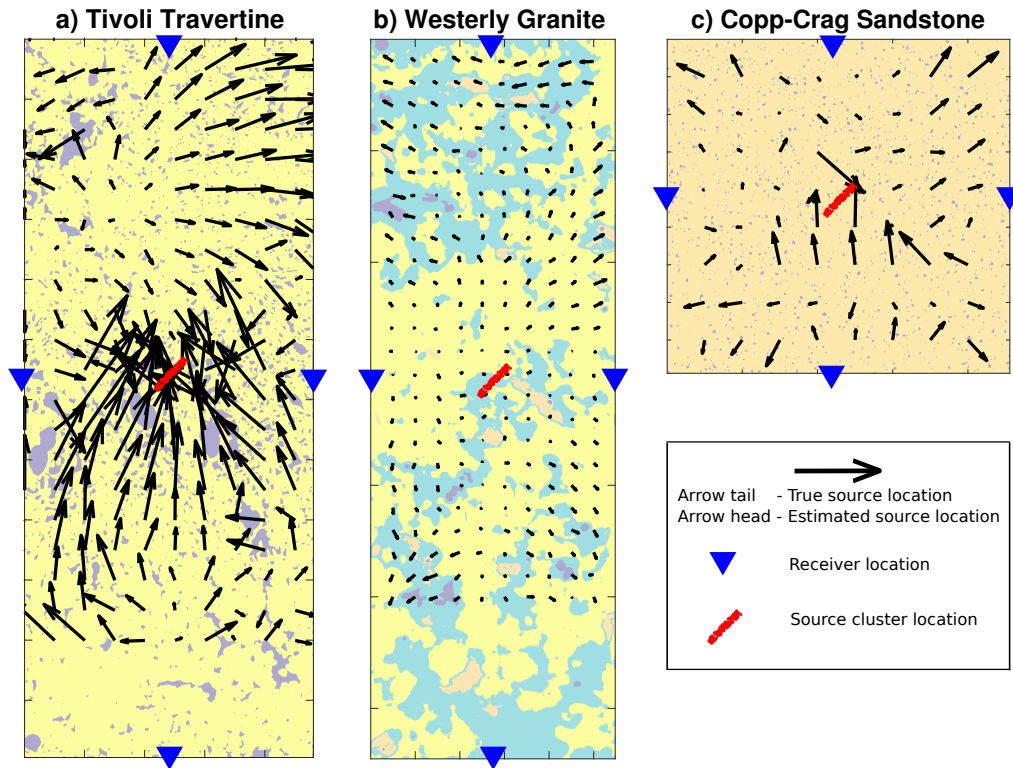


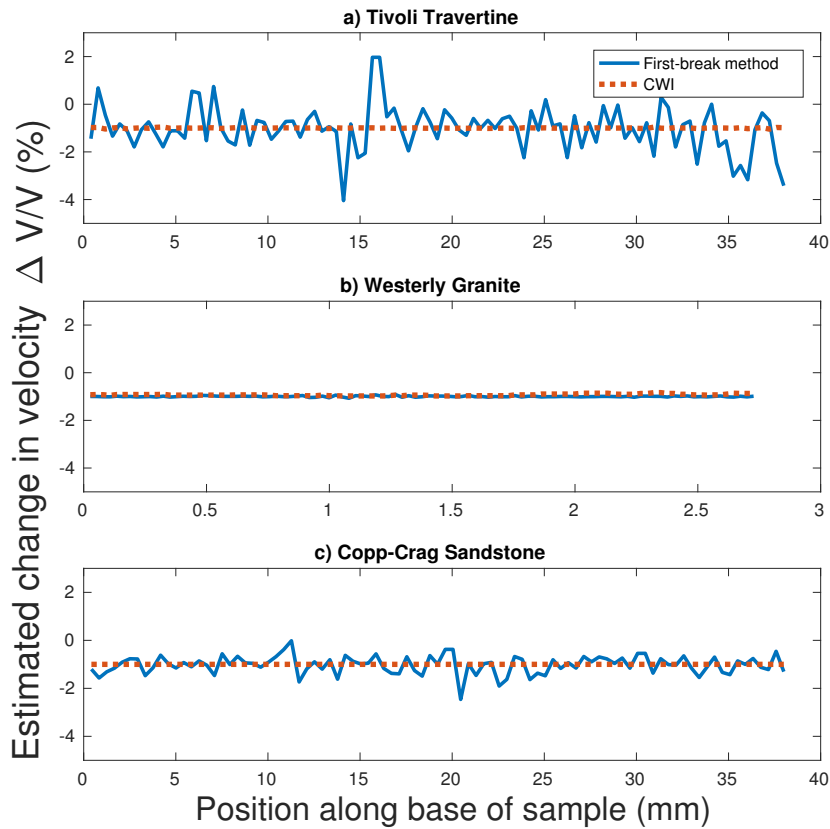
Figure 3. (a-d) Estimated seismic velocity as a function of receiver position, obtained from simulated waveforms through a μ CT digital rock sample in a model shown (e) for the Tivoli Travertine. The source (star) is fixed at the top and receivers (triangles) are distributed along the bottom. The blue curve shows velocity estimates made using first-break arrival times and straight-line source-to-receiver distances. The dashed green line represents the conventional estimate of velocity using a single receiver at the center of the core. The dashed black line represent the fastest measured velocity. Results are for a) Tivoli Travertine picking the travel time of the first maximum and using a 30 MHz (blue) and 1 MHz (red) sources, b) Tivoli Travertine picking the travel time of the signal onset and using a 30 MHz (blue) and 1 MHz (red) sources c) Westerly Granite (200 MHz source), and d) Copp-Crag Sandstone (30 MHz source). The results in panels c and d are from picking the first maximum.



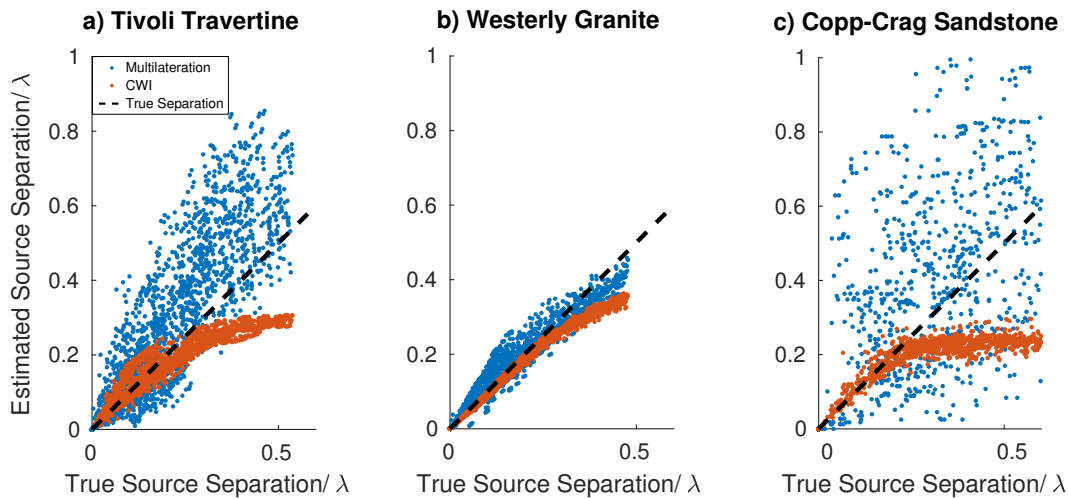
1081 **Figure 4.** The estimated velocity as if a receiver was placed at every position in the model
 1082 \mathbf{x} , using a fixed source location (centre of the top of the sample). To emulate estimates from the
 1083 first break method, an eikonal ray tracing method (Margrave, 2007) was used to calculate travel
 1084 times $t[\mathbf{x}]$, while a straight source-to-receiver ray path was used to calculate velocity $v[\mathbf{x}]$. Results
 1085 are for a) Tivoli Travertine, b) Westerly Granite, and c) Copp-Crag Sandstone.



1086 **Figure 5.** The resulting systematic errors in source location, represented as black arrows, us-
 1087 ing standard phase picking methods that assume a single representative velocity for each sample,
 1088 for a) Tivoli Travertine, b) Westerly Granite, and c) Copp-Crag Sandstone. The base of each
 1089 arrow is located at the true source positions (S_j), and estimated locations (S_{est}) are displayed at
 1090 arrow tips. The red points represent the source cluster used for the source location experiment
 1091 with results shown in Figure 7.



1092 **Figure 6.** The estimation of a relative velocity change $\Delta V/V$ for a true change in velocity of
 1093 -1% , i.e., $\Delta V/V = -0.01$. Results for a) Tivoli Travertine, b) Westerly Granite, and c) Copp-Crag
 1094 Sandstone. $\Delta V/V$ is estimated using the standard phase-picking method and Coda Wave Inter-
 1095 ferometry using each of 100 receiver locations along the base of each sample and a single source
 1096 location at the center-top of each sample.

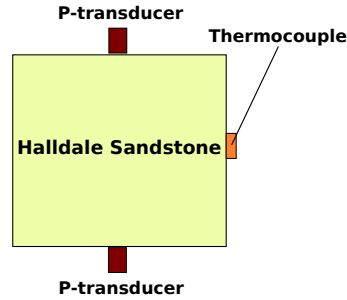


1097 **Figure 7.** A comparison of estimated inter-source separation as a function of true inter-source
 1098 separation (scaled by the wavelength λ at peak frequency) for the conventional multilateration
 1099 method (using arrival times obtained from phase picking of first arrivals) and Coda Wave Inter-
 1100 ferometry. The true source cluster locations are represented as red dots in Figures 5a, b and c. a)
 1101 Tivoli Travertine, b) Westerly Granite, and c) Copp-Crag Sandstone. The dashed line indicates
 1102 the graph locations corresponding to perfect estimates.

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a) Experiment I: Temperature

Sample Details:
 Fine grained
 Quartz rich
 $\phi = 15.1\%$
 $\rho = 2248 \text{ kg/m}^3$
 Size: 10x10x10 cm
 Saturation: Dry



b) Experiment II: Differential Stress

Sample Details:
 Core parallel with laminations
 Finely laminated carbonate
 $\phi = 10.3\%$
 $\rho = 2374 \text{ kg/m}^3$
 Length = 75 mm
 Diameter = 38 mm
 Saturated with deionized water

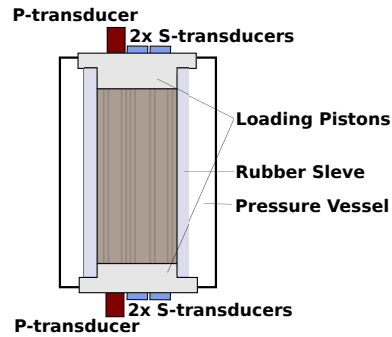
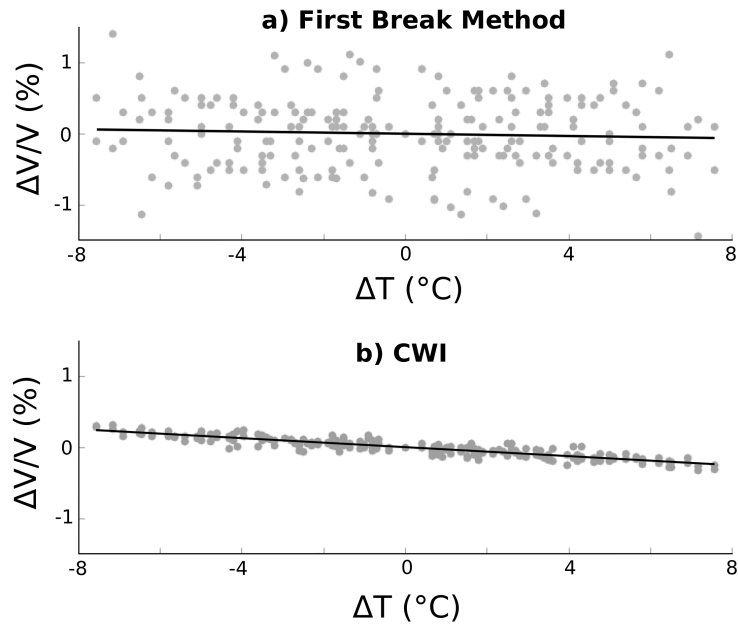
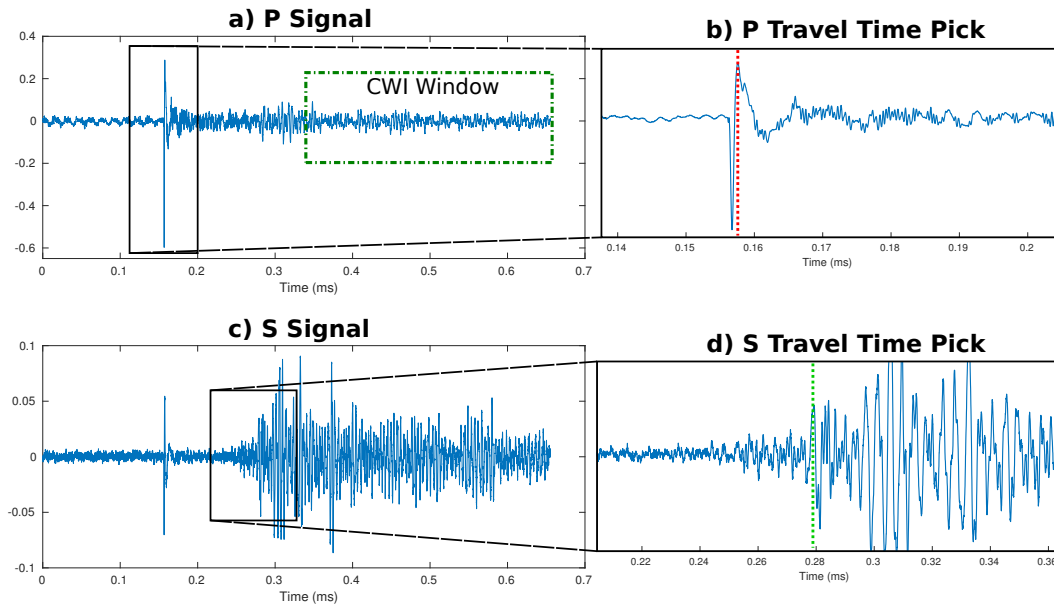


Figure 8. Schematic diagrams for the two experimental examples used for inducing a velocity change in the medium. a) Experiment I uses a variation in temperature of a cubic block of Halldale Sandstone. b) Experiment II uses varying differential stress on a finely laminated carbonate within a triaxial Hoek cell. Values for porosity (ϕ), density (ρ) and other properties of each sample are shown for each case.

1104
 1105
 1106
 1107



1108 **Figure 9.** Estimated values of percentage velocity change ($\Delta V/V$) as a function of the change
 1109 in temperature (ΔT) in a 10 cm³ sample of Halldale Sandstone, a) for the standard method of
 1110 picking arrival times, and b) for Coda Wave Interferometry. Solid lines are best-fit linear regres-
 1111 sions. The zero point on the x axis ($\Delta T = 0$) is arbitrary.



1112 **Figure 10.** Example waveforms to illustrate the picking procedure for the first break method.

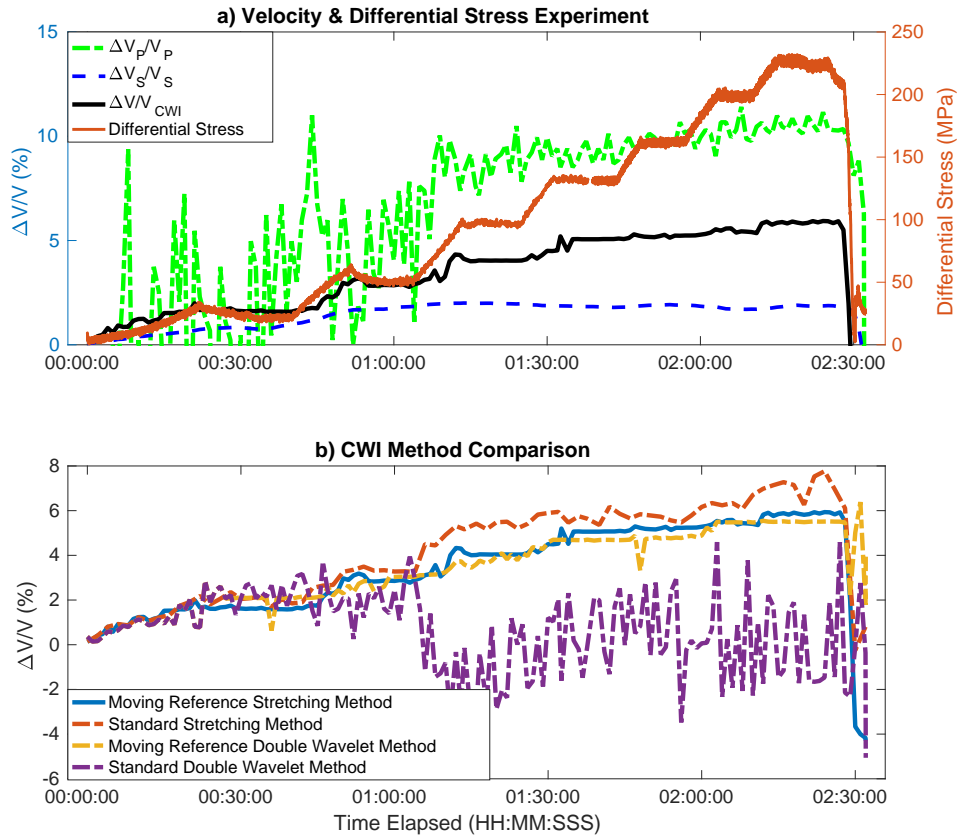
1113 a) Full recorded signal using Glaser-type sensors sensitive to displacement normal to the sen-

1114 sor face. b) First arriving waves: the first maximum is manually picked as the arrival time. c)

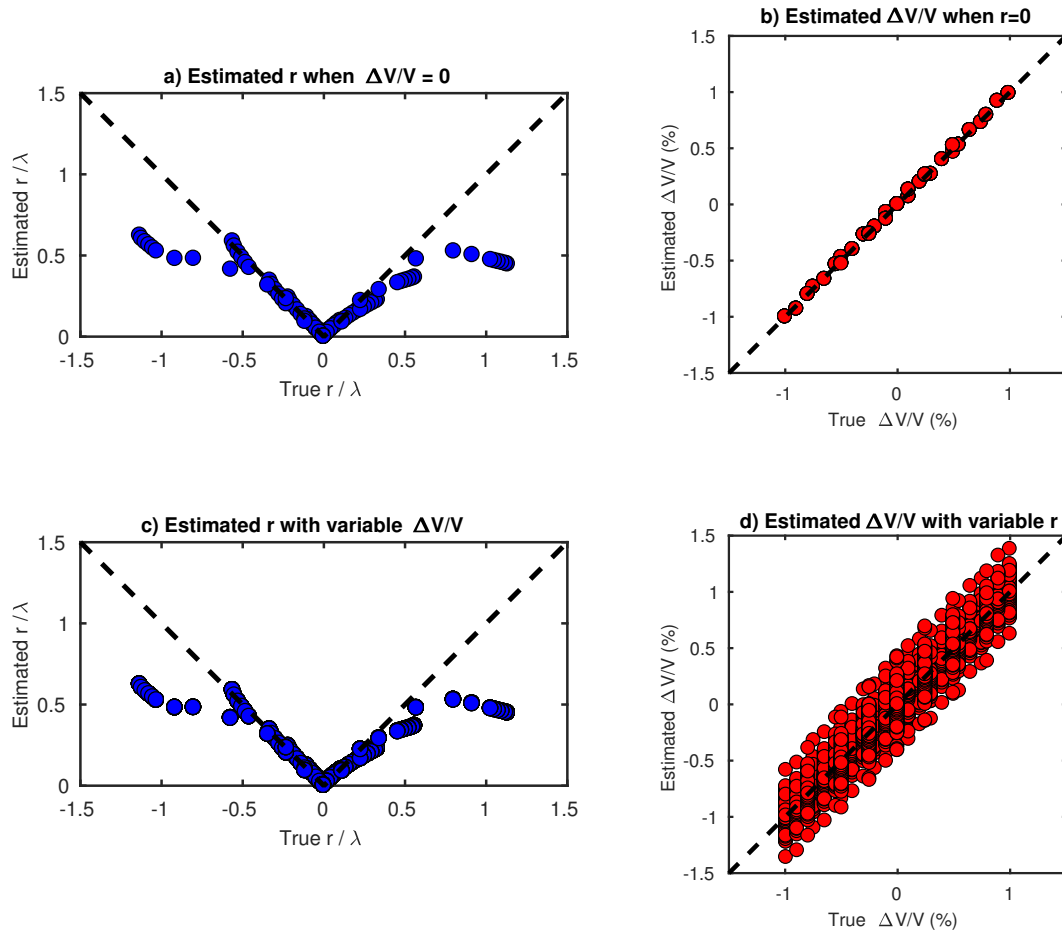
1115 Full recorded signal using S wave transducers for the source and receiver, sensitive to displace-

1116 ment tangential to the sensor face. d) Manually picked first arriving S wave maximum. The time

1117 window used for CWI is labeled in panel a.



1118 **Figure 11.** a) Velocity change of a finely laminated carbonate rock during experimental
 1119 deformation by increasing differential stress (red), with corresponding stress values labeled on
 1120 the right axis. The response of velocity ($\Delta V/V$), labeled on the left axis, is estimated by the
 1121 first-break method for P and S wave velocities (dashed lines) and by a CWI moving-reference
 1122 trace method (black). b) A comparison of CWI algorithms, showing the effect of implementing a
 1123 moving reference trace (Equation 3) for both the stretching and double wavelet methods.



1 24 **Figure 12.** Assessing the ability of CWI to estimate velocity changes $\Delta V/V$ and inter-source
 1125 separation r simultaneously in the presence of both velocity and source location perturbations. a)
 1 26 Estimated r when velocity is not perturbed. b) Estimated $\Delta V/V$ when the source location is not
 1127 perturbed. c) Estimates of r with simultaneous velocity perturbations. d) Estimates of $\Delta V/V$
 1 28 with simultaneous perturbations of source location.

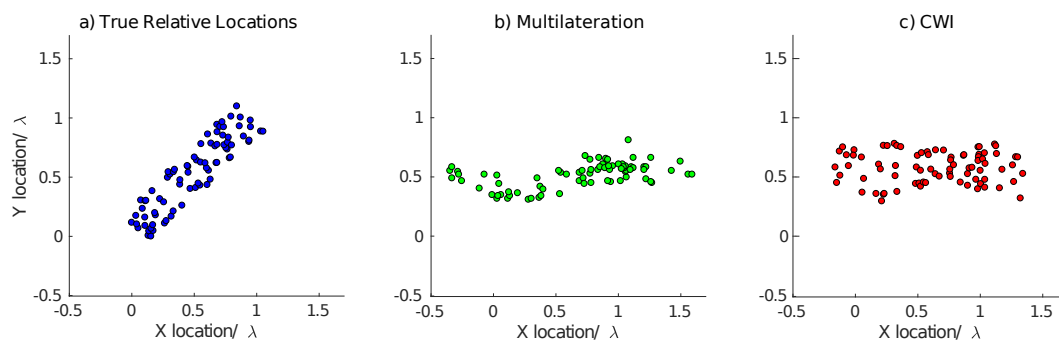


Figure 13. a) True locations of a cluster of acoustic emissions simulated in the Tivoli Travertine μ CT slice in Figure 2b. b) Estimated cluster locations using the conventional method of first-break arrival times and multilateration using the receiver geometry in Figure 5a. c) Estimated relative locations found by implementing the CWI-based optimization algorithm described in Zhao et al. (2017), using the inter-source separations estimated from CWI using the same receiver geometry (note these locations have been rotated in plane to best fit the locations in panel b for fair for comparison, as the optimization provides only relative locations).

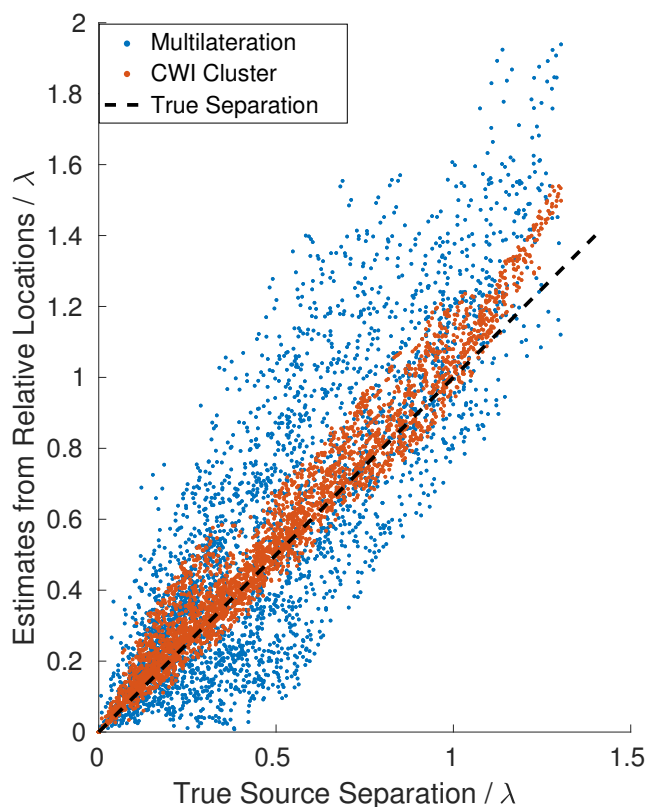
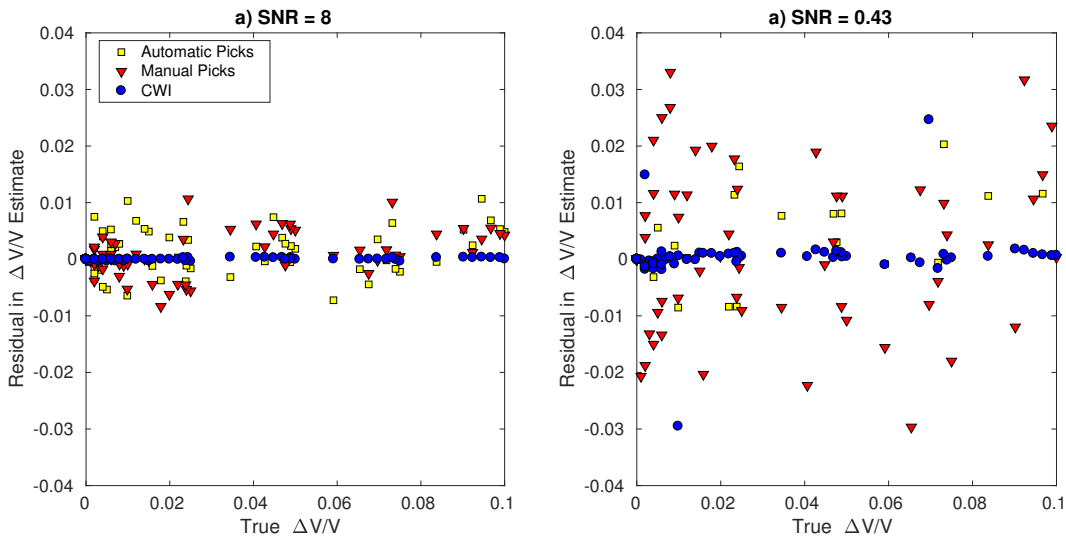
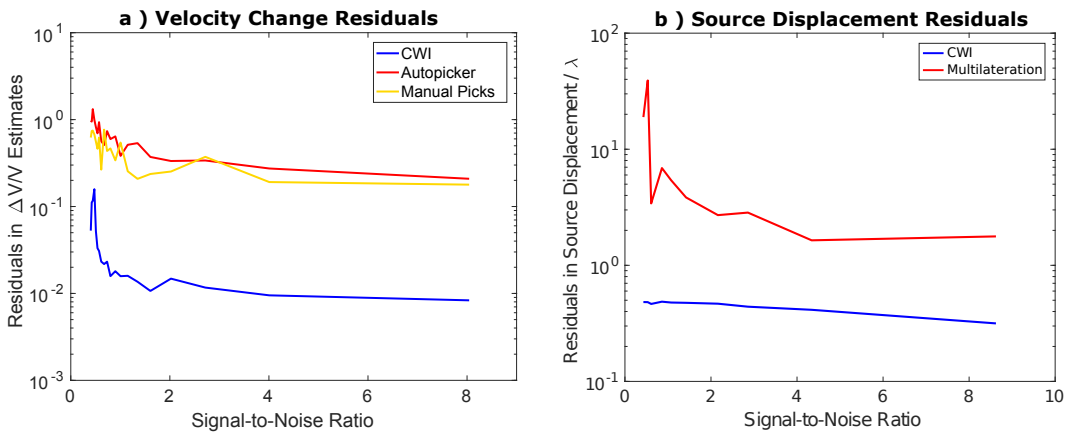


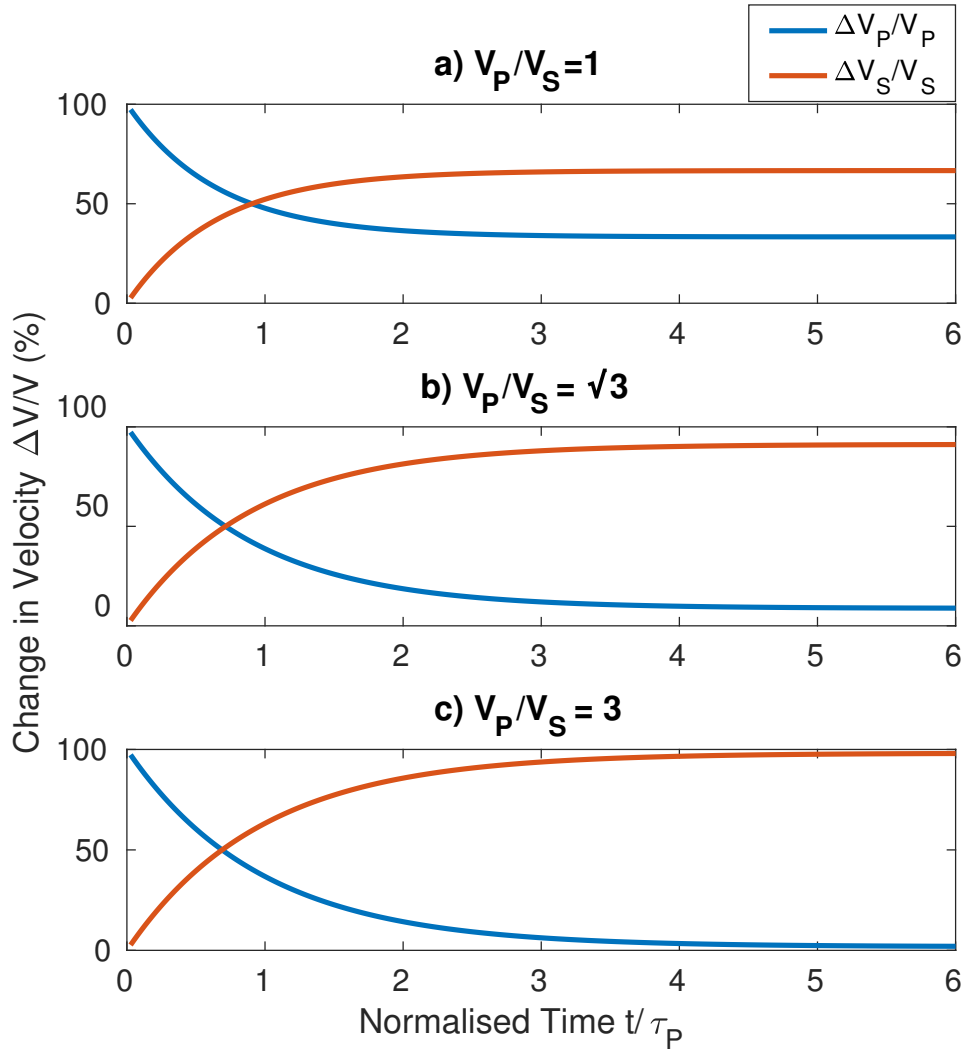
Figure 14. Source separation values from the estimated location clusters shown in Figures 13b and c, as a function of true source separation. The dashed line shows where true separation estimates would lie.



1139 **Figure 15.** Residuals of estimated $\Delta V/V$ from Coda Wave Interferometry, and from
 1140 travel times obtained by auto-picking and manual picks, estimated at a) SNR = 8 and b) SNR = 0.43
 41 and plotted as a function of the true velocity change.



1142 **Figure 16.** a) Residuals between true and estimated velocity change ($\Delta V/V$) as a function
 43 of signal-to-noise ratio. b) Residuals between true and estimated source displacement r/λ as a
 1144 function of signal ratio.



1145 **Figure 17.** Relative proportions of changes in P-wave velocity ($\Delta V_P/V_P$) and S-wave velocity
 1146 ($\Delta V_S/V_S$) which make up the change in velocity estimated from CWI ($\Delta V/V$) as a function of
 1147 time along the coda, using equations 19 and 20 taken from Snieder (2002). Multiple relations are
 1148 shown for media of varying V_P/V_S ratios: a) b) $V_P/V_S = 1$, b) $V_P/V_S = \sqrt{3}$, c) $V_P/V_S = 3$. Time
 1149 is normalized by dividing time t by the transit time of one mean free path ($\tau_P = l_P/V_P$).

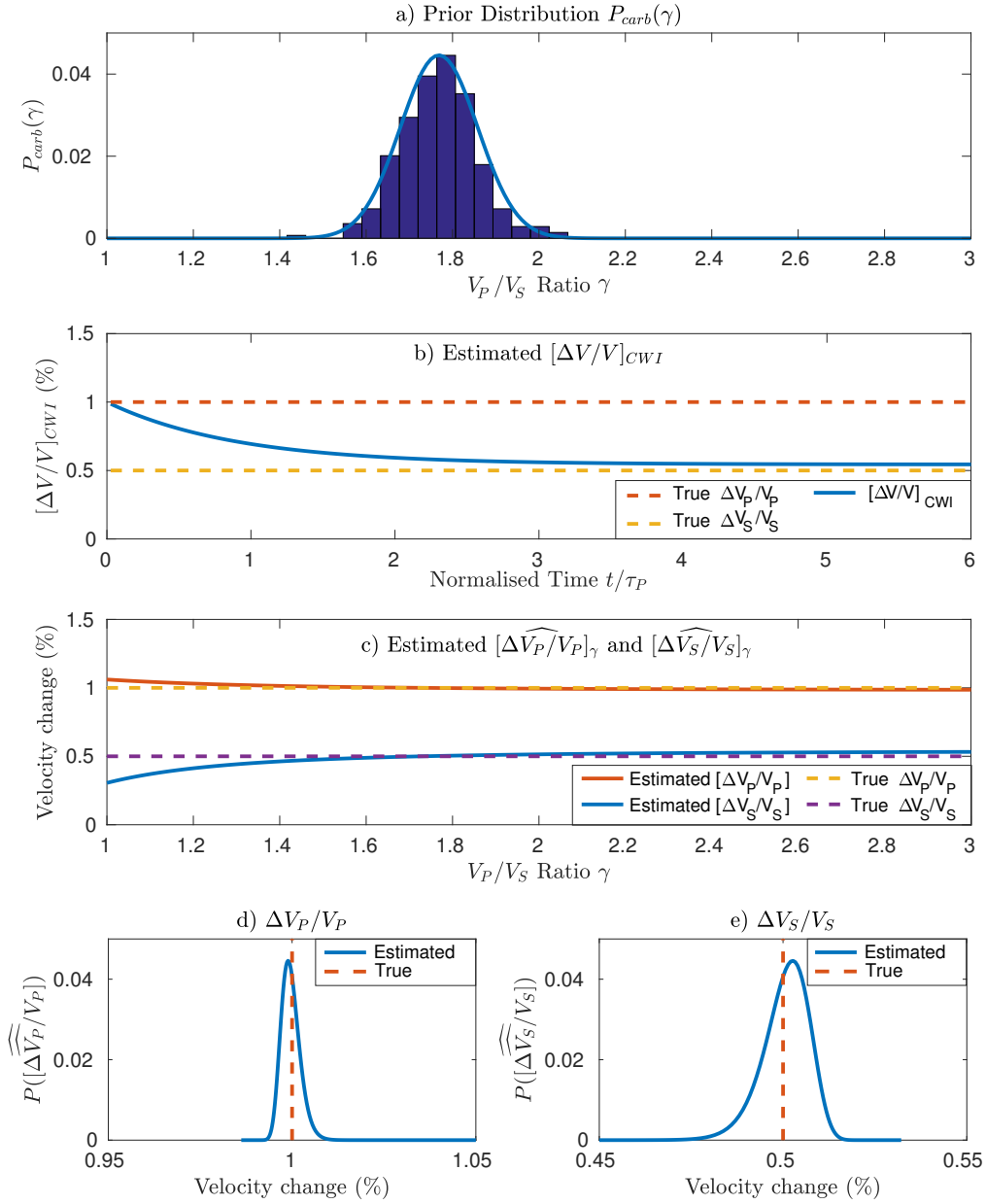
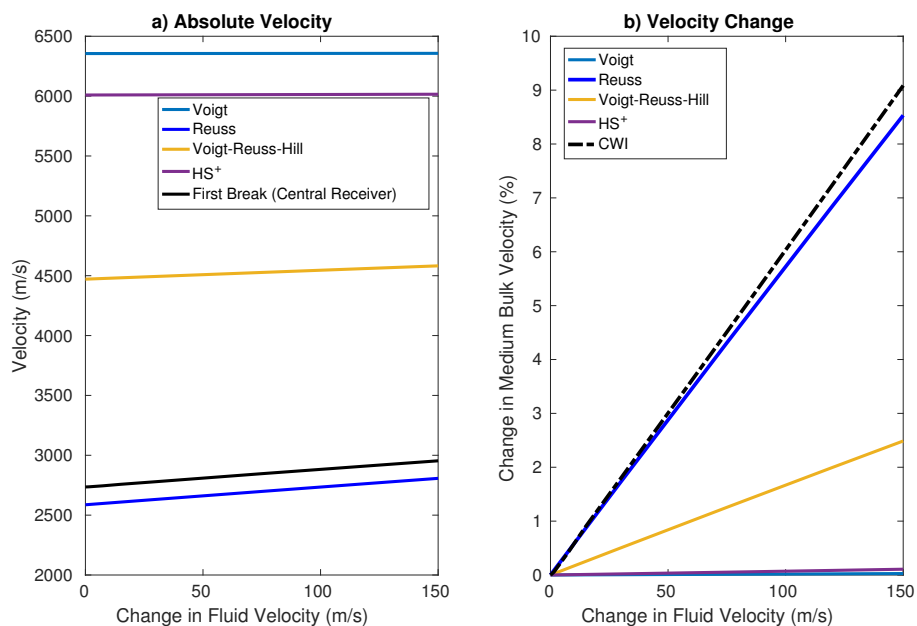


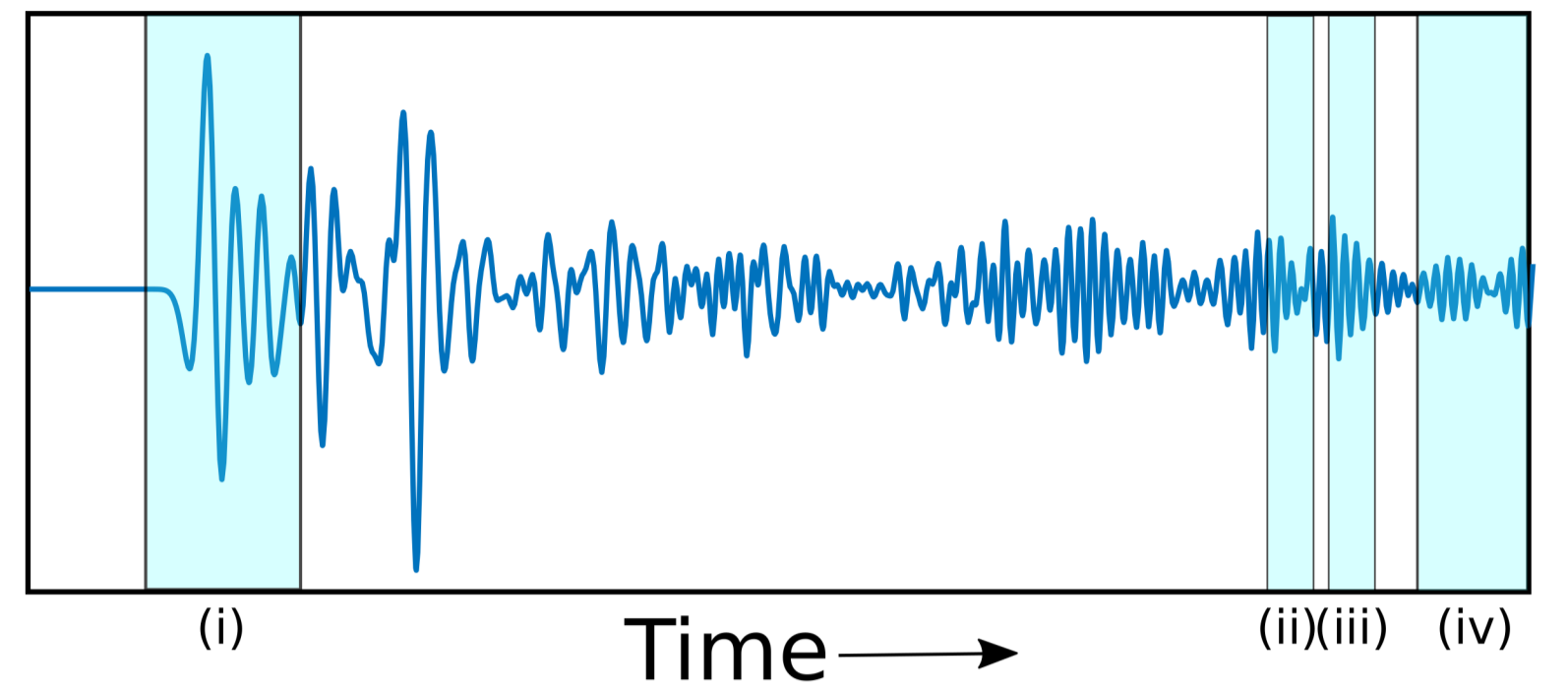
Figure 18. a) Prior distribution of V_P/V_S ratios from measured dry carbonate data compiled from Bakhorji (2010), Fournier et al. (2011) and Verwer et al. (2008). The curve shows the best fitting normal distribution function of the histogram. b) Synthetic $[\Delta V/V]_{CWI}$ data generated using Equation 21, where $\Delta V_P/V_P = 1\%$, $\Delta V_S/V_S = 0.5\%$ and $\gamma = \sqrt{3}$. c) Estimated $[\widehat{\Delta V_P/V_P}]_\gamma$ and $[\widehat{\Delta V_S/V_S}]_\gamma$ from an ordinary least squares inversion of the forward modeled $[\Delta V/V]_{CWI}$ data in panel b, as a function of the V_P/V_S ratio used in the inversion. d) and e) show the probability density functions (solid blue lines) for estimates of $\Delta V_P/V_P$ and $\Delta V_S/V_S$, where the dashed red lines represent the true changes in velocity ($\Delta V_P/V_P = 1\%$, $\Delta V_S/V_S = 0.5\%$), using samples from prior distribution in panel a and Equations 22 and 23.



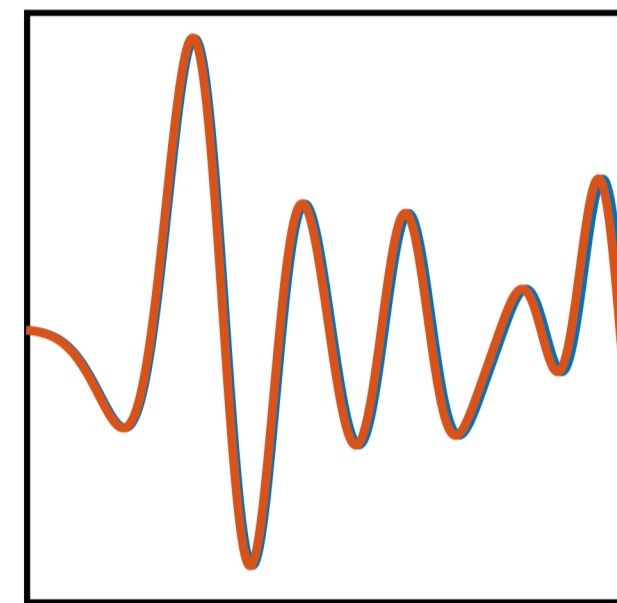
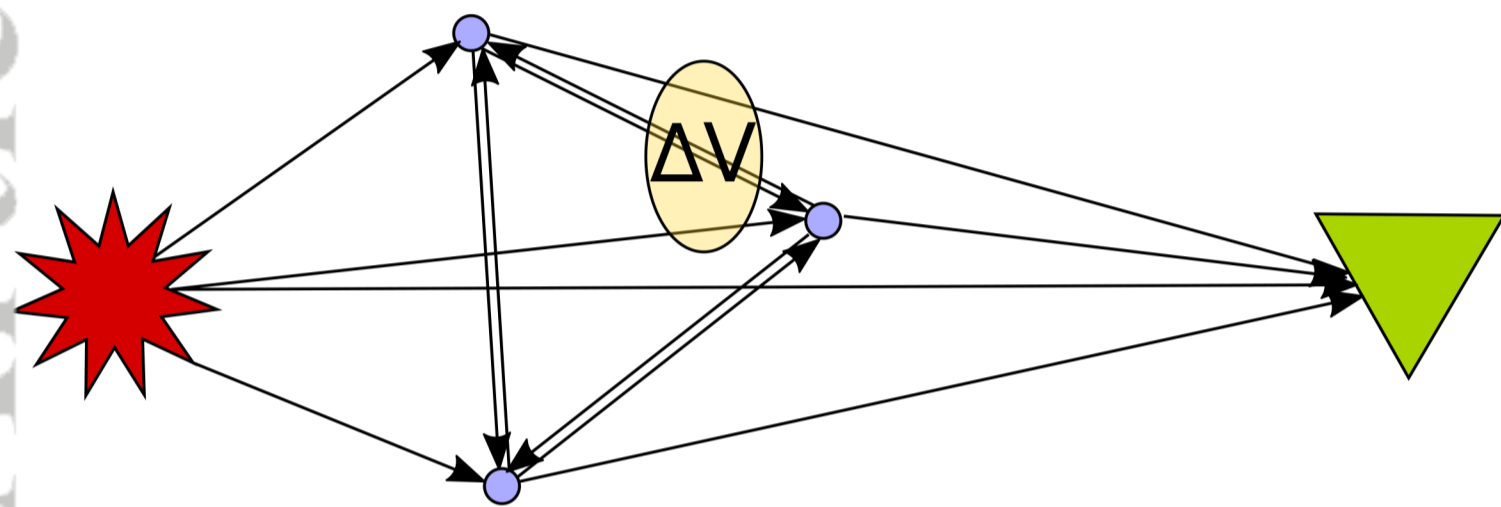
1159 **Figure 19.** a) Calculated average velocity for the Tivoli Travertine digital rock sample fol-
 1160 lowing multiple perturbation of fluid velocity. The medium velocity is calculated using a range
 1162 of bounding methods including the Voigt upper bound, Reuss lower bound, the Voigt-Reuss-Hill
 1163 average and the Hashin-Shtrikman upper bound (HS⁺), see Mavko et al. (2009). The velocity
 1164 is also estimated using the first break method on a central receiver (black). b) The change in
 1165 bulk velocity ($\Delta V/V$) as a function of fluid velocity perturbation, calculated with the multiple
 averages. The dotted black line is the estimate of velocity change ($\Delta V/V$) attained using CWI.

Figure 1.

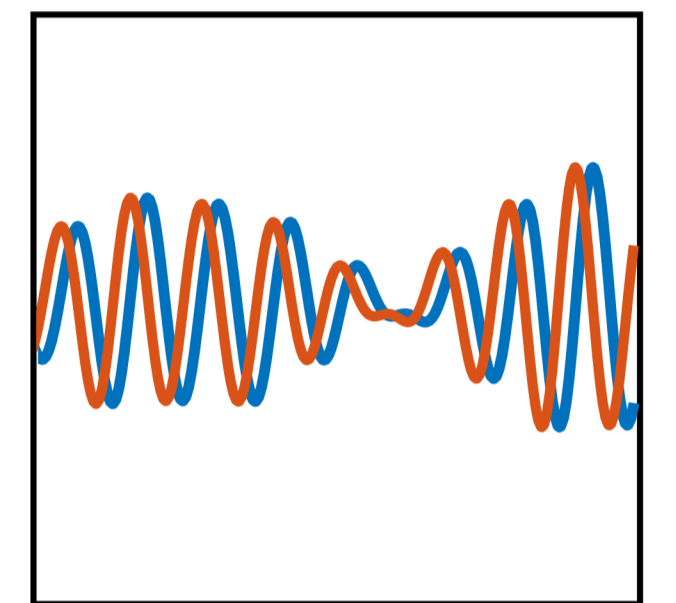
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a) Perturbation of Velocity

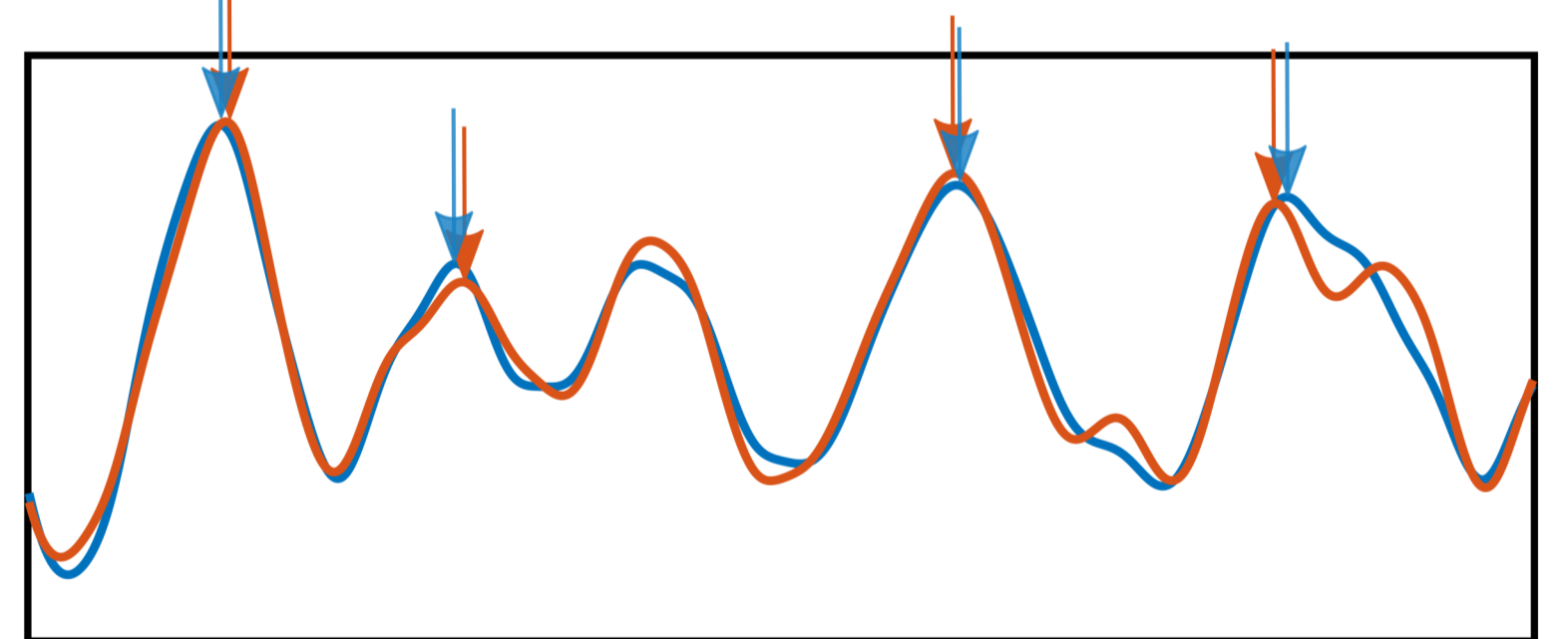
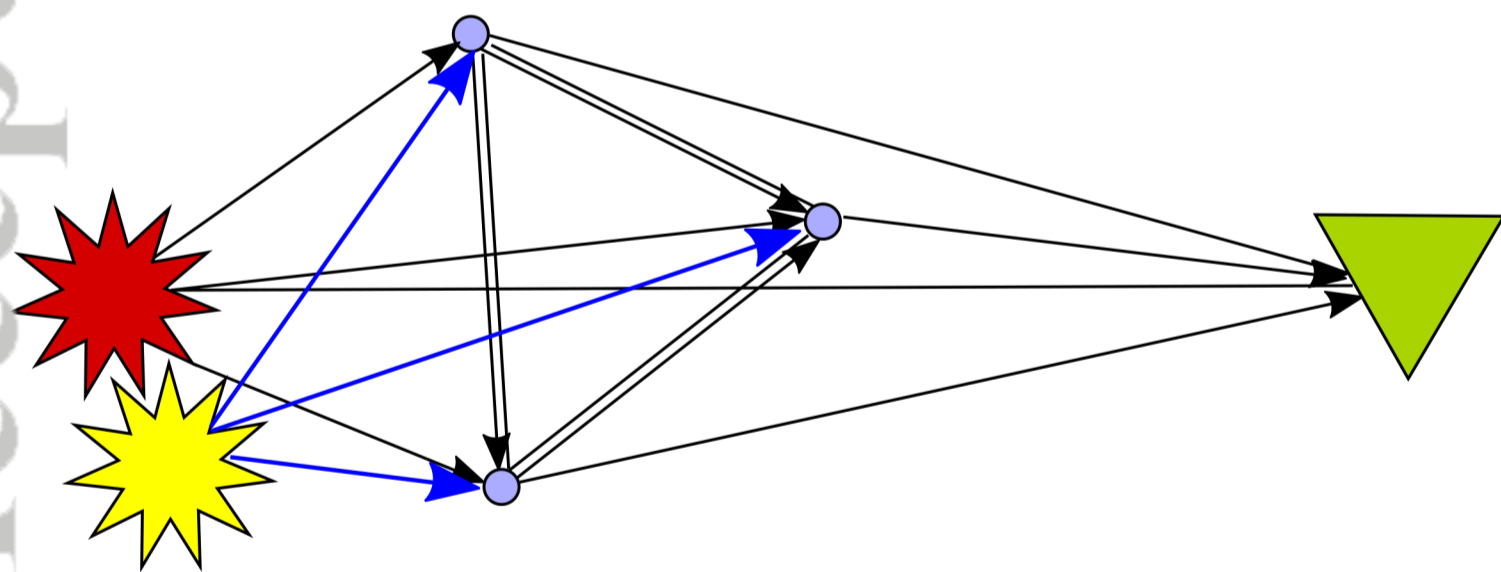


Time window (i)



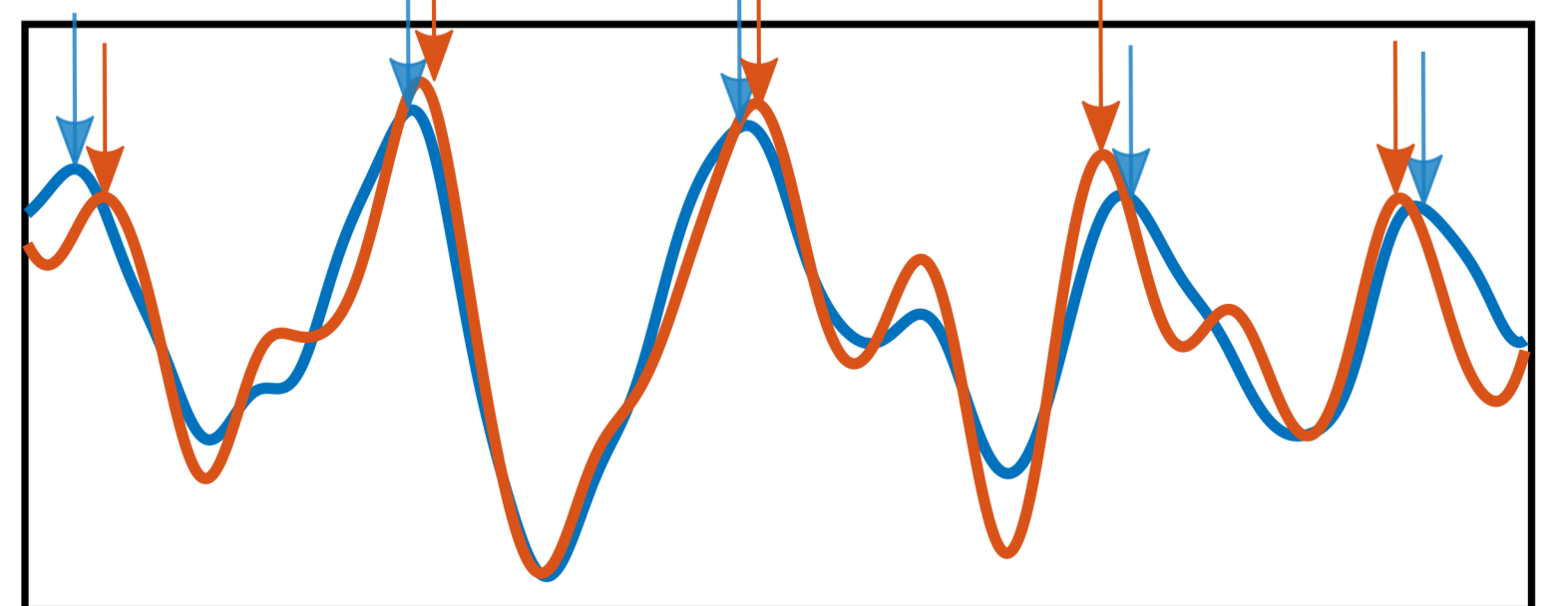
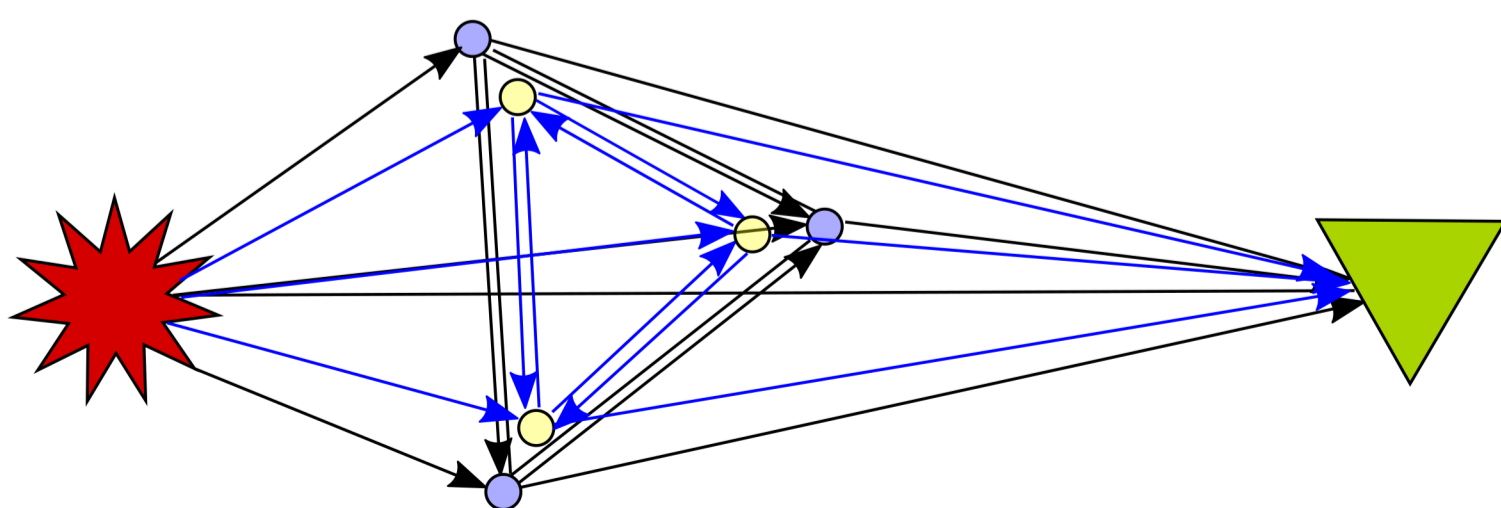
Time window (iv)

b) Perturbation of Source Location



Time window (ii)

c) Perturbation of Scatterer Locations



Time window (iii)

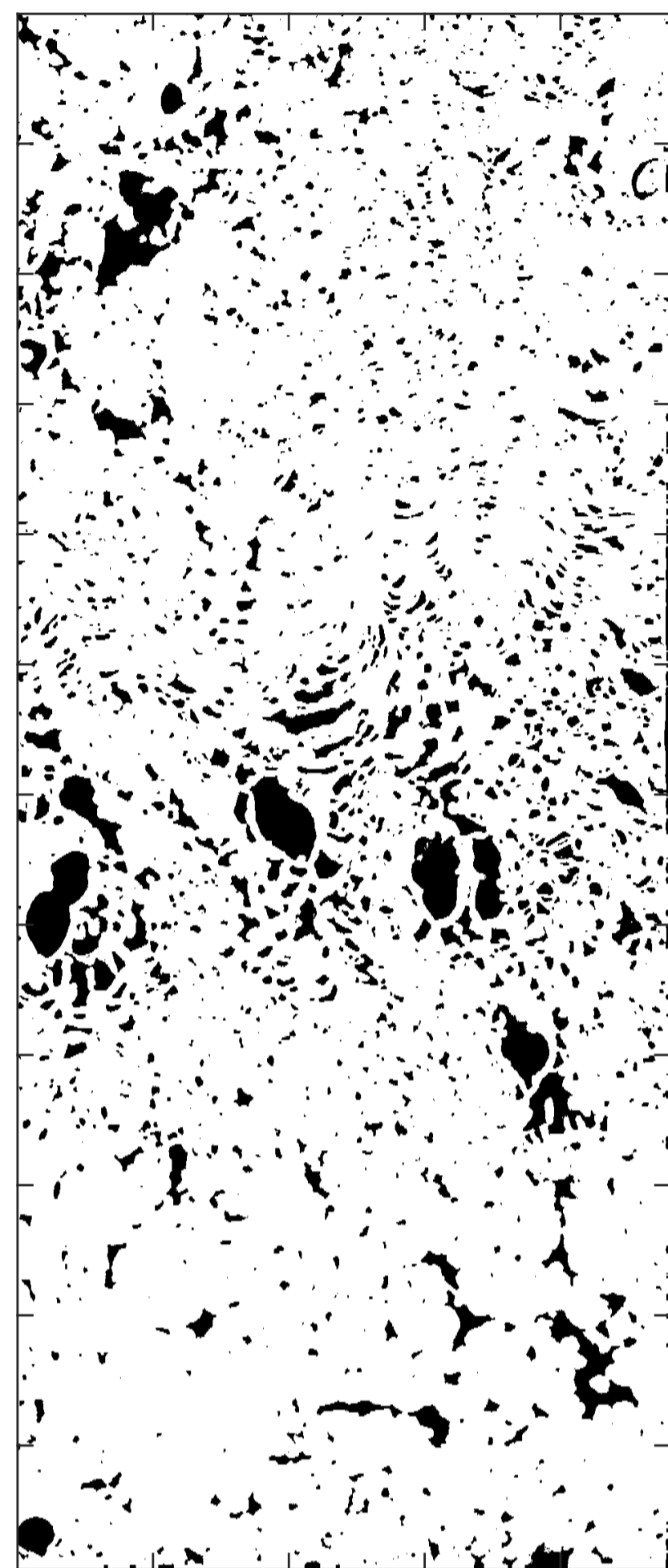
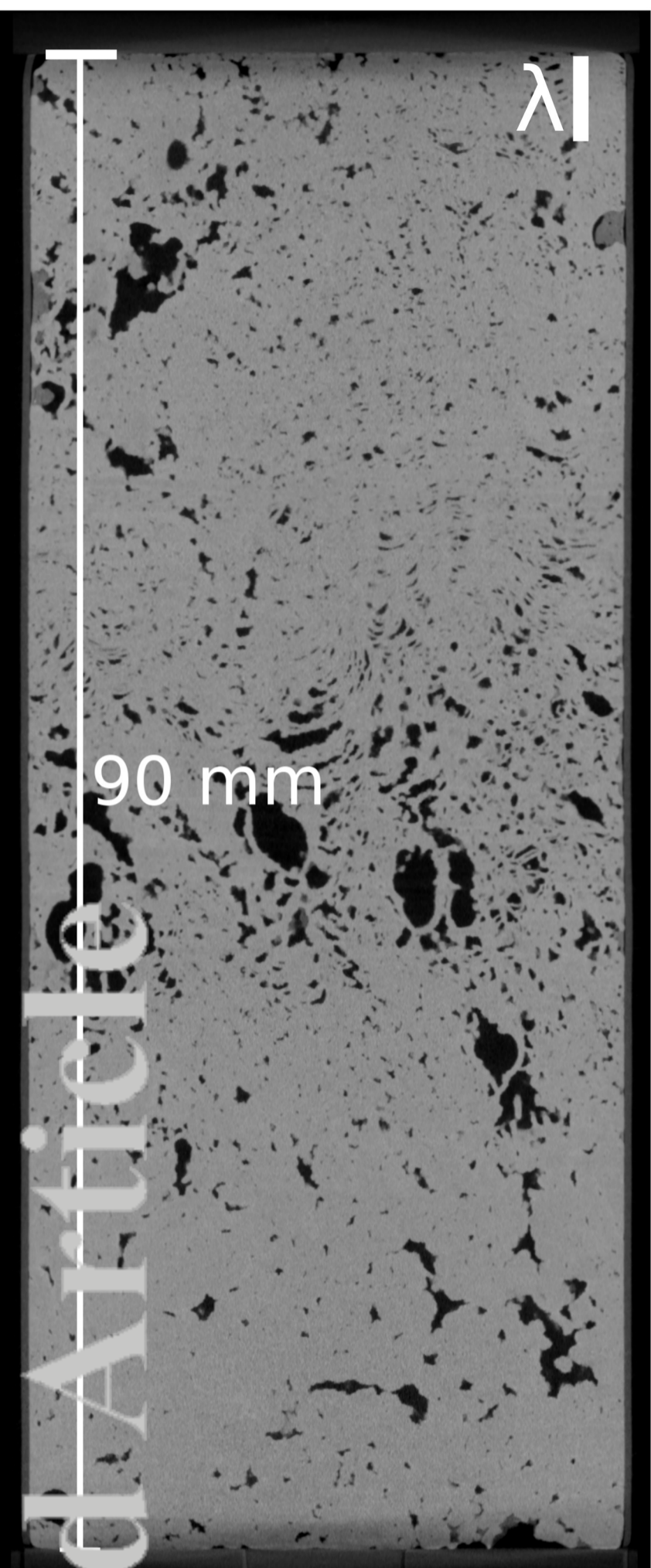
Figure 2.

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Tivoli Travertine

a)

b)

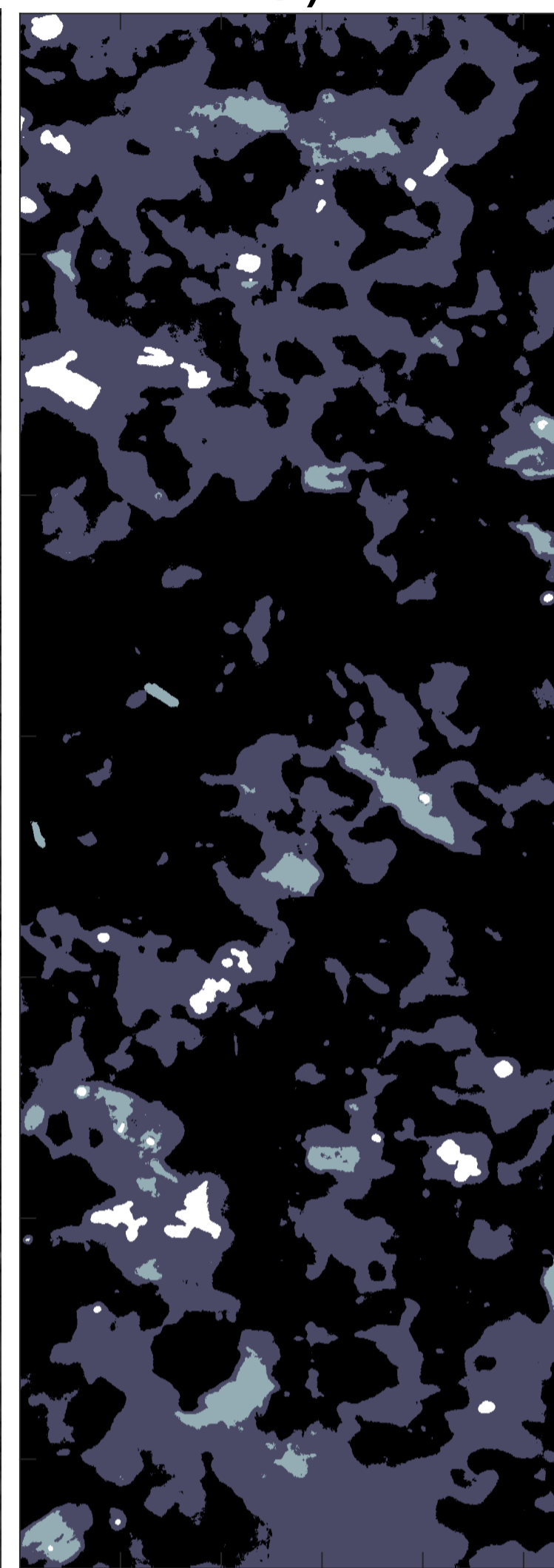
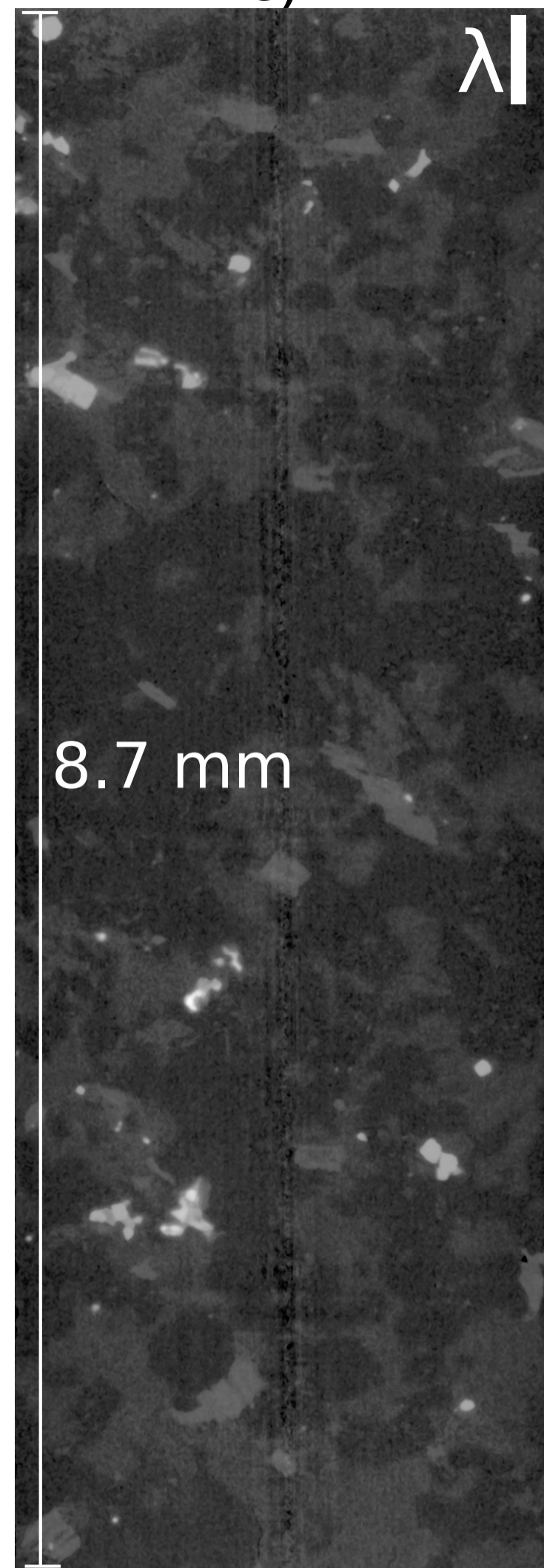


■ Pore
□ Calcite

Westerly Granite

c)

d)

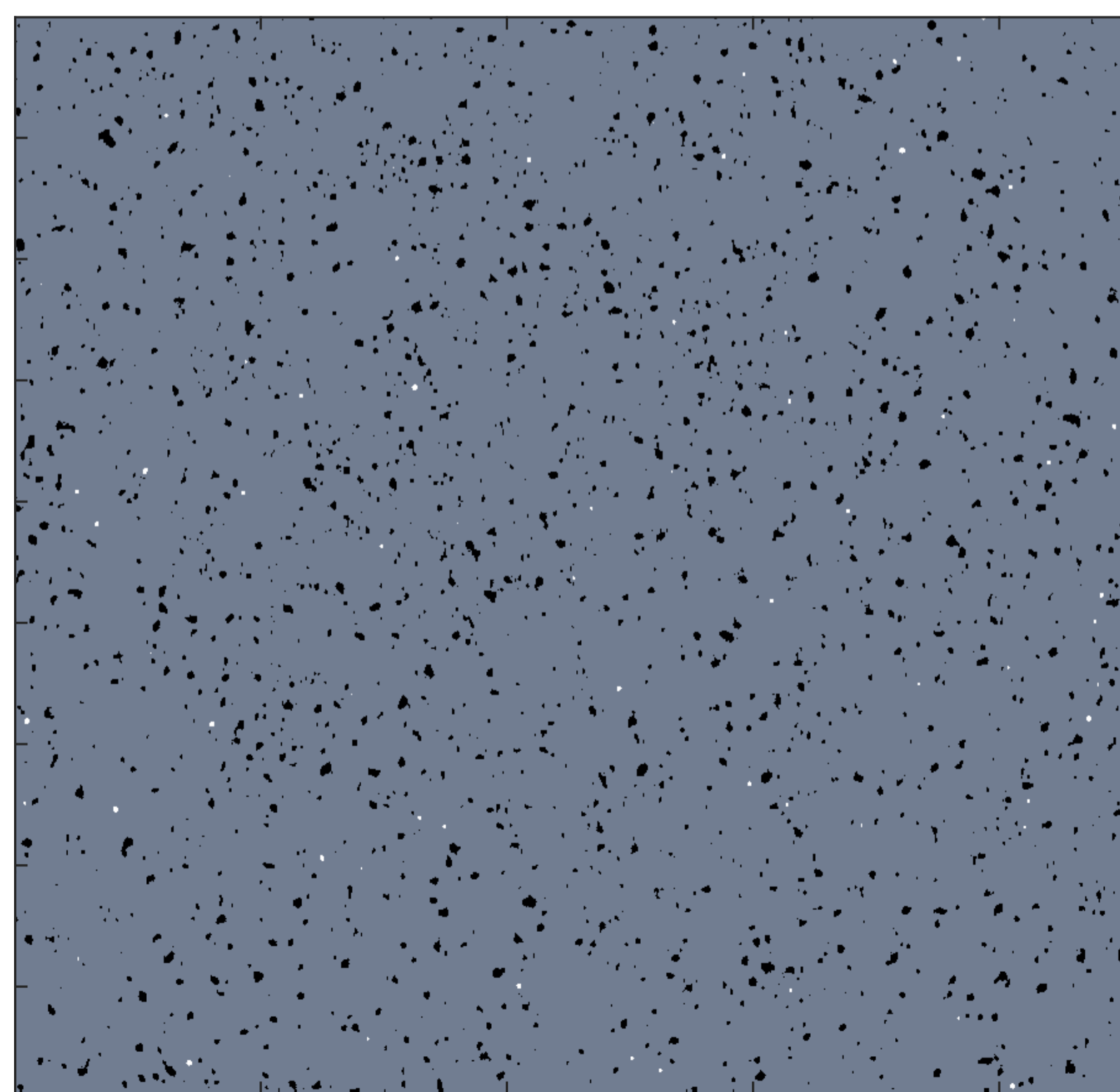
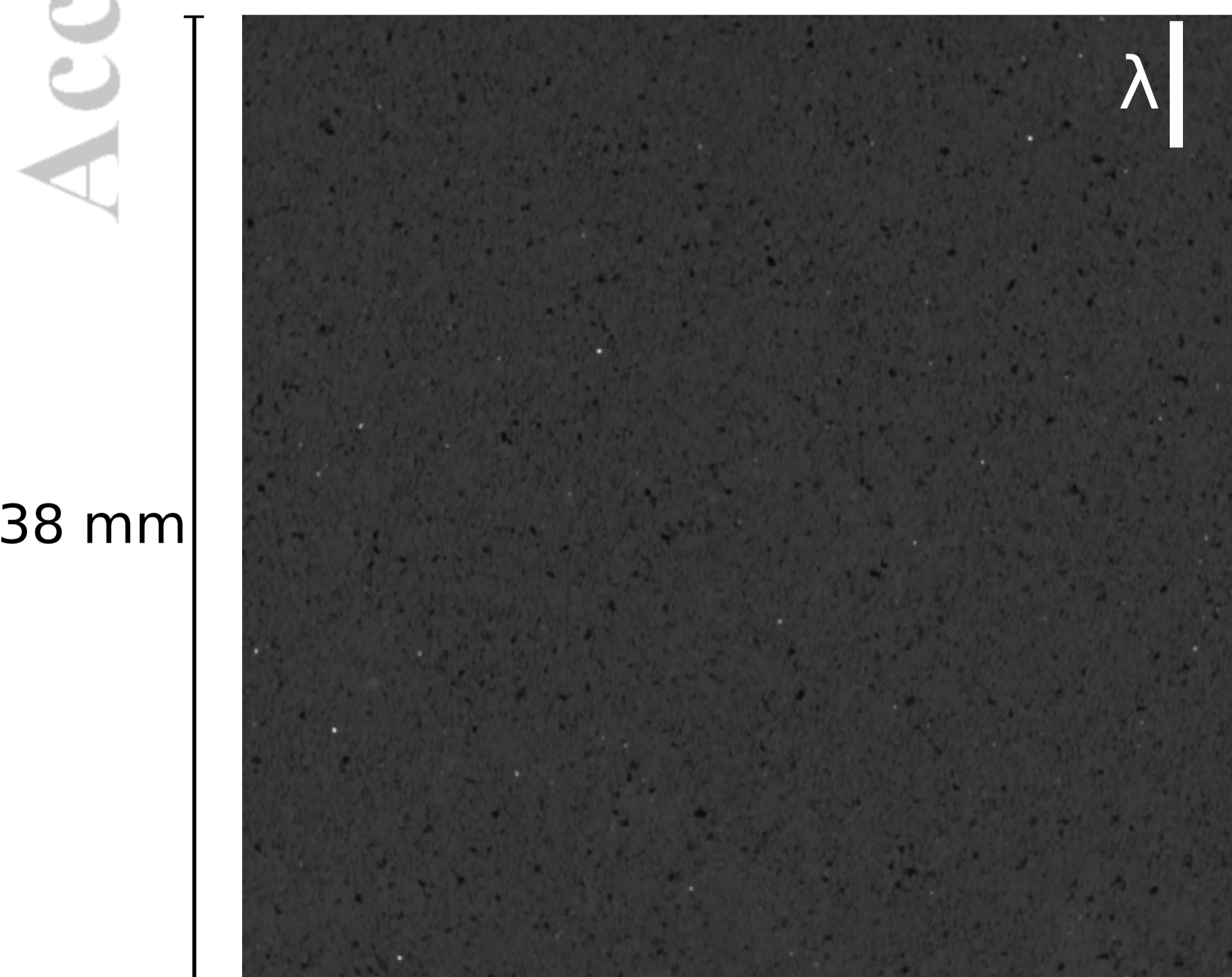


■ Plagioclase Feldspar
■ Quartz
■ Potassium Feldspar
□ Biotite

Copp-Crag Sandstone

e)

f)



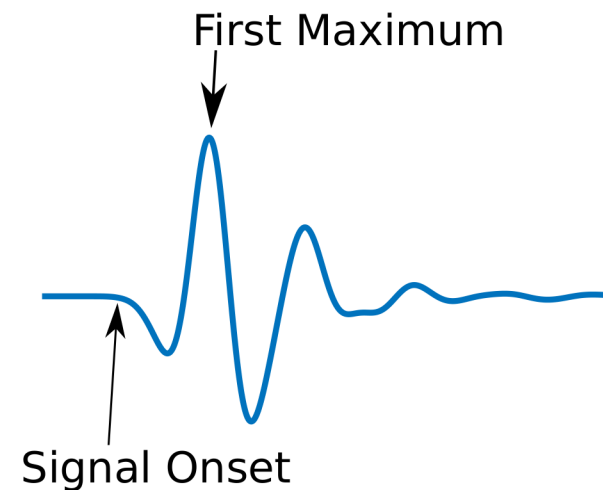
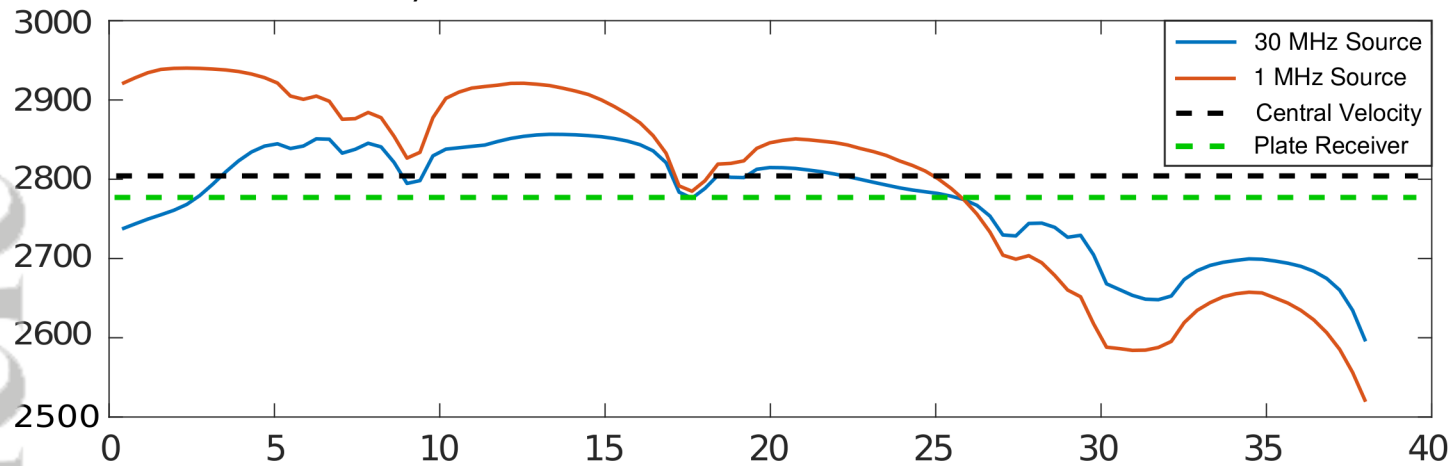
■ Pore
■ Quartz
□ Muscovite

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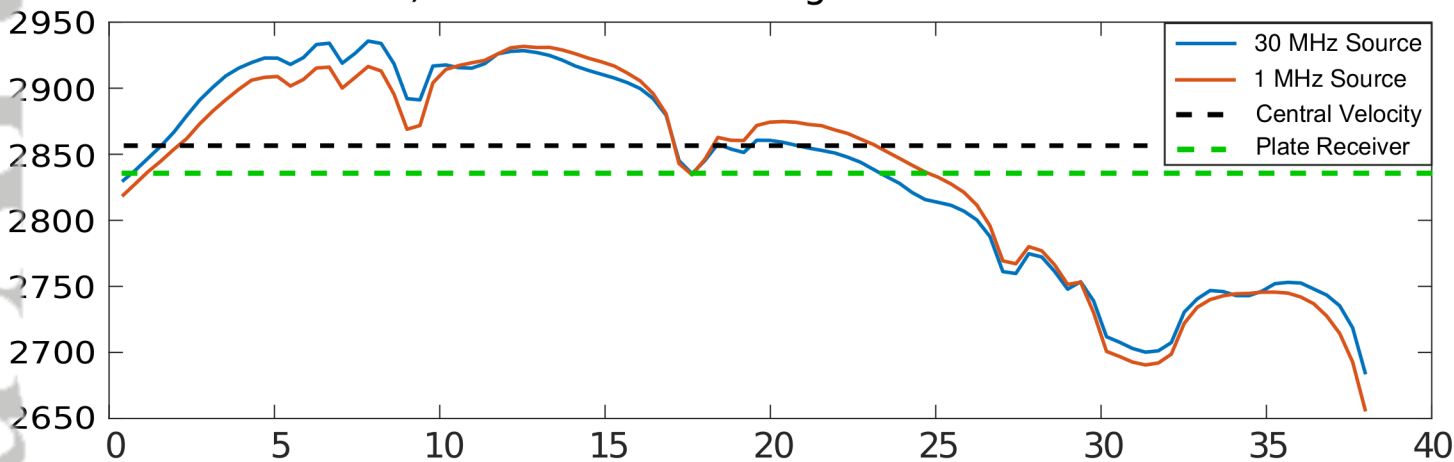
Figure 3.

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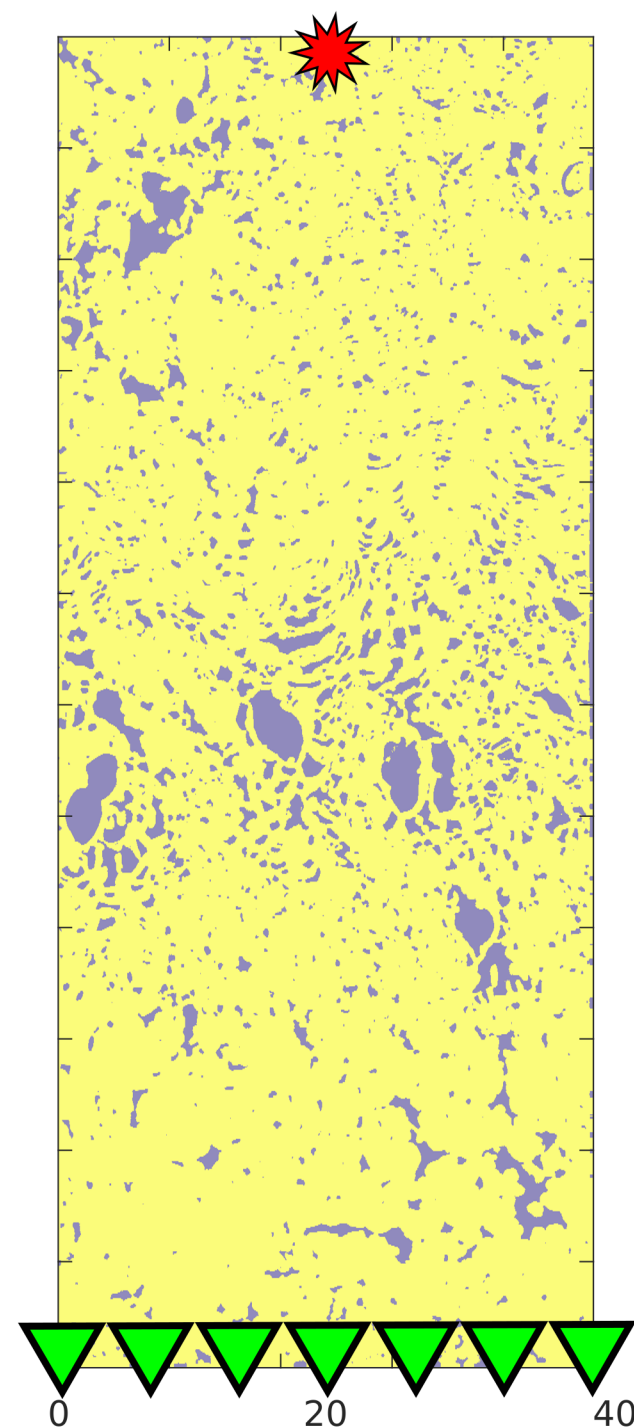
a) Tivoli Travertine: First Maximum Picks



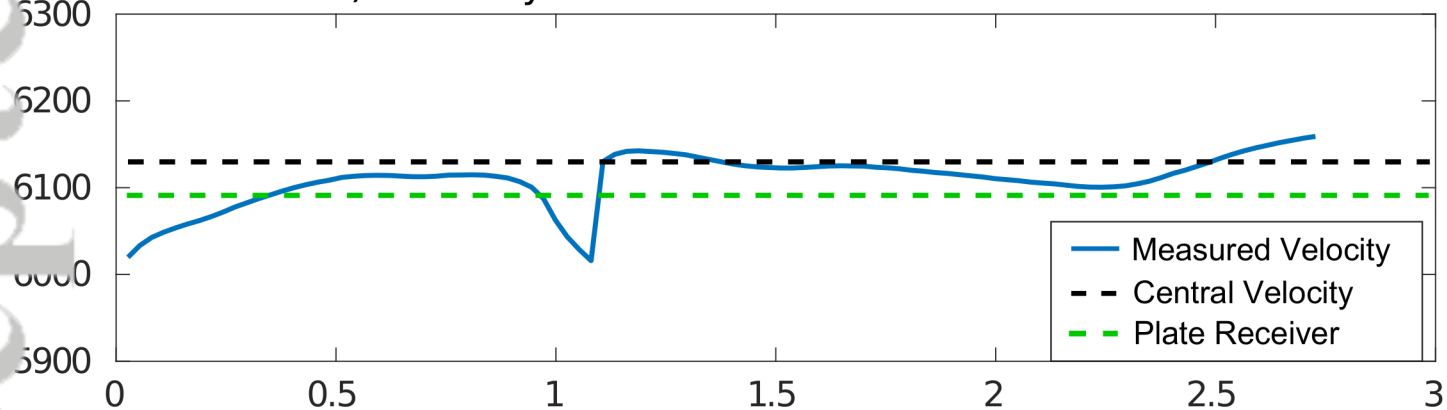
b) Tivoli Travertine: Signal Onset Picks



e) Example Model



c) Westerly Granite: First Maximum Picks



d) Copp-Crag Sandstone: First Maximum Picks

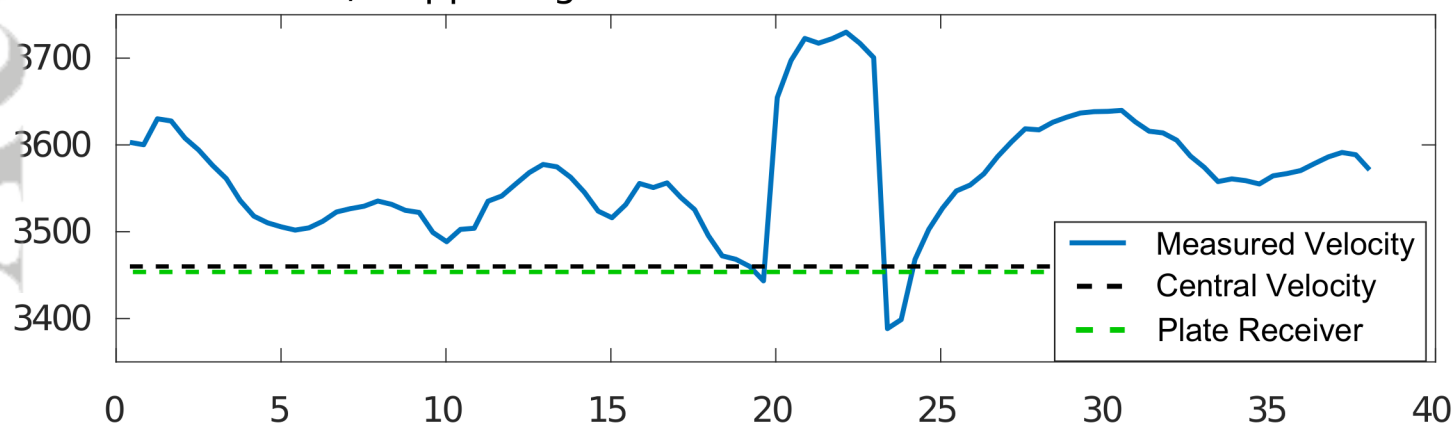
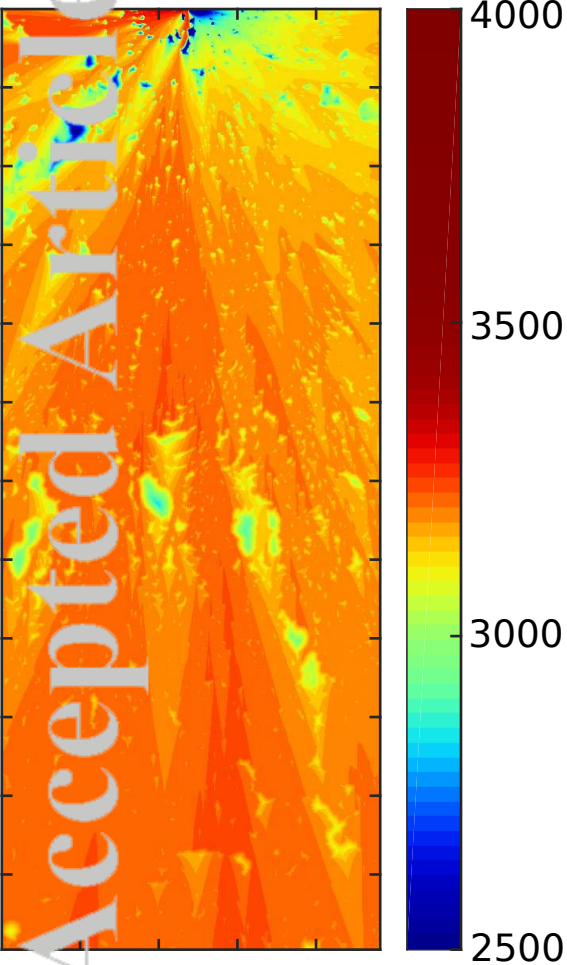


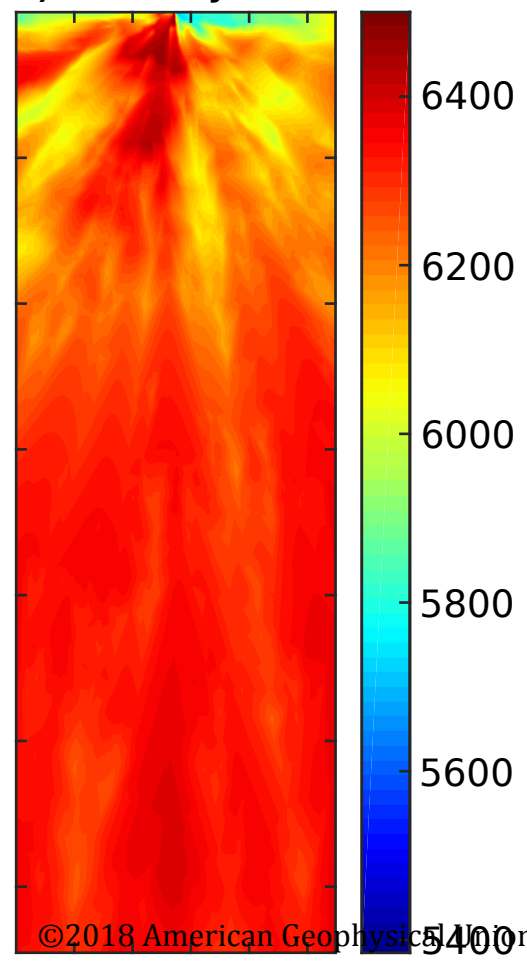
Figure 4.

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a) Tivoli Travertine



b) Westerly Granite



c) Copp-Crag Sandstone

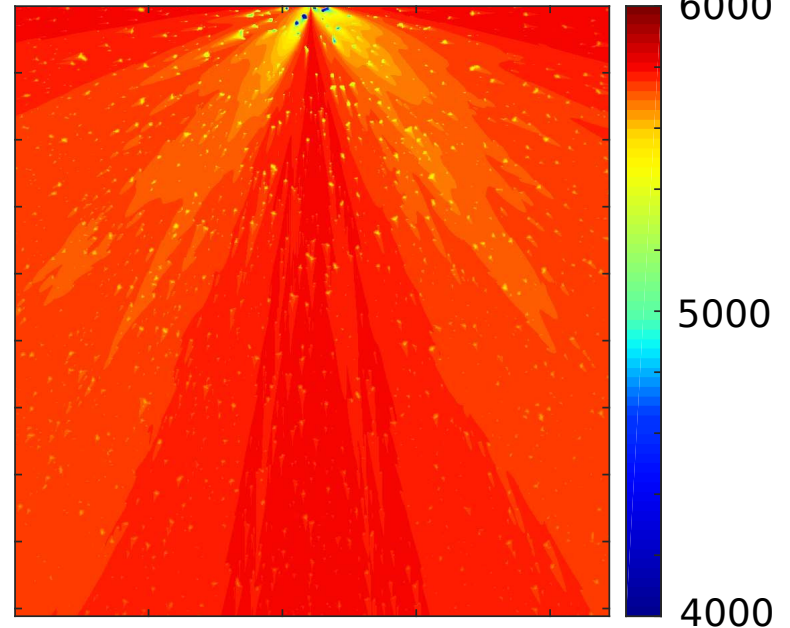
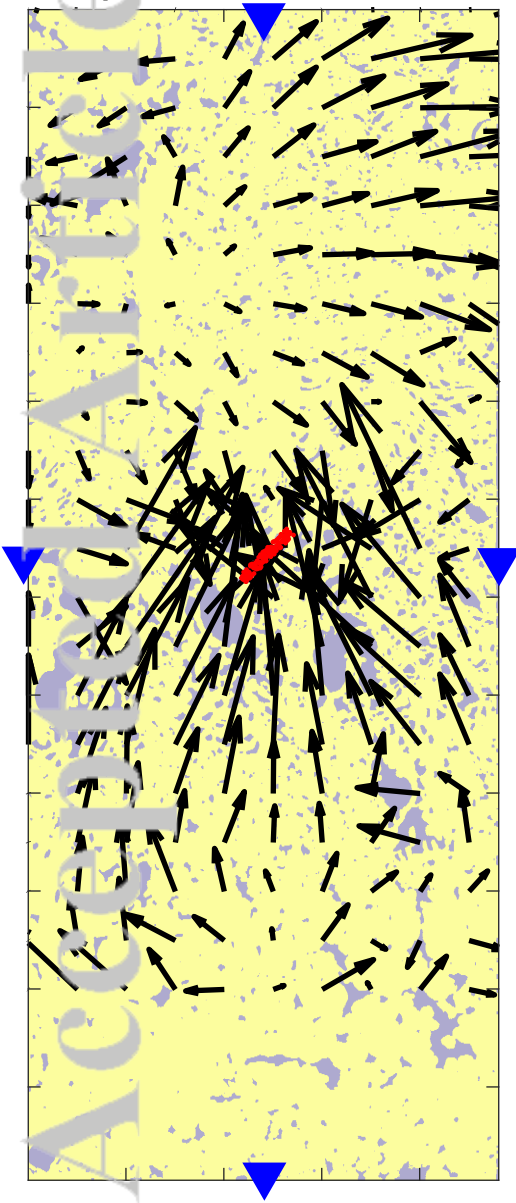


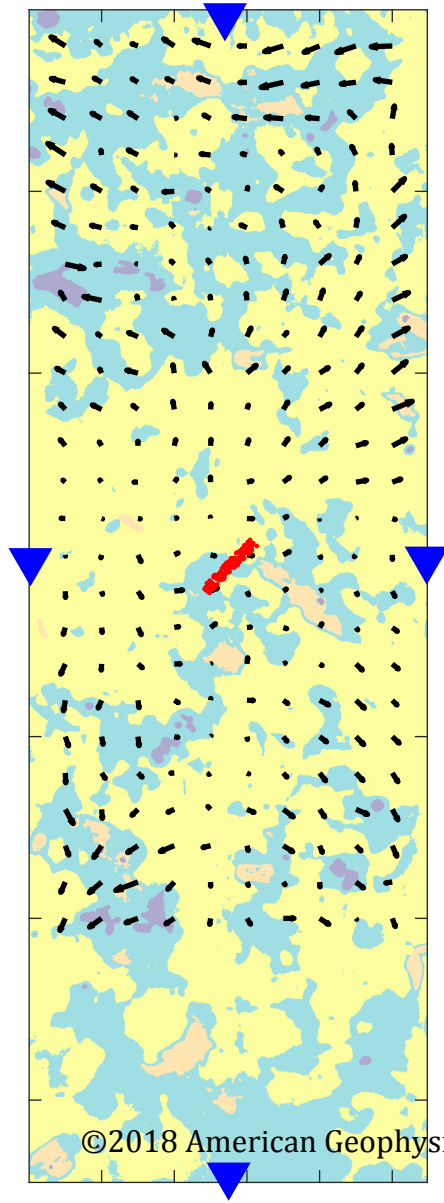
Figure 5.

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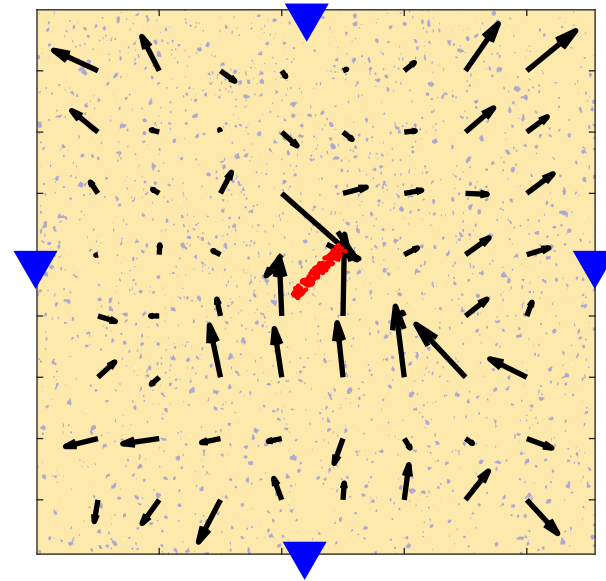
a) Tivoli Travertine




b) Westerly Granite




c) Copp-Crag Sandstone



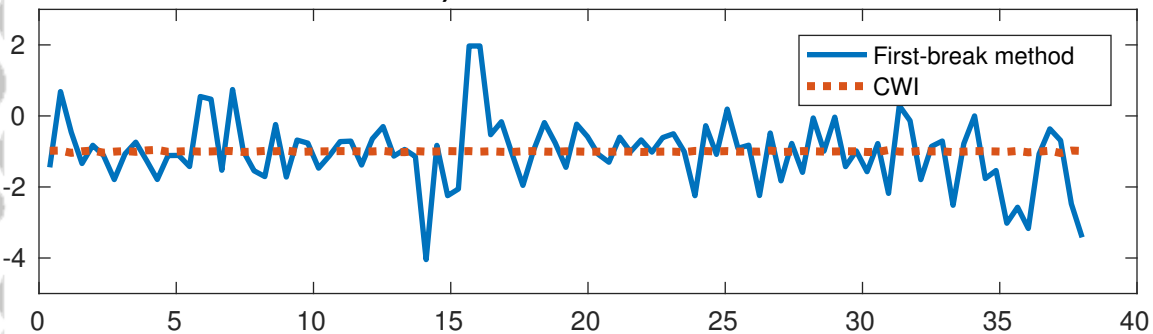

 Arrow tail - True source location
 Arrow head - Estimated source location

 Receiver location

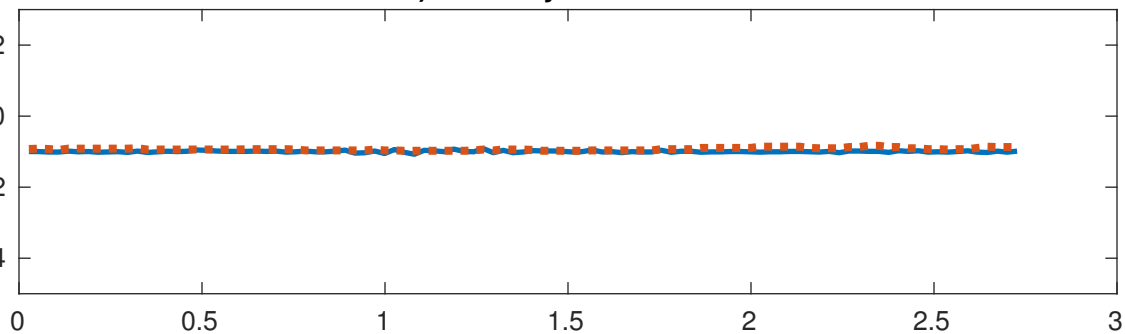
 Source cluster location

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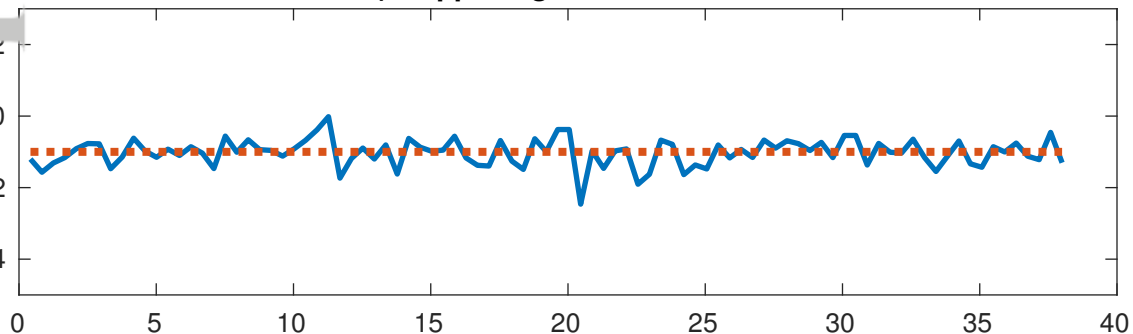
a) Tivoli Travertine



b) Westerly Granite

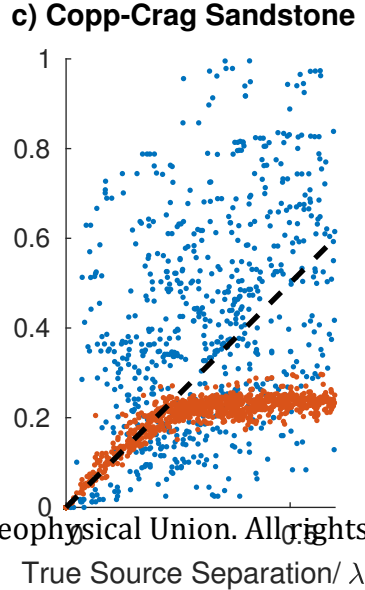
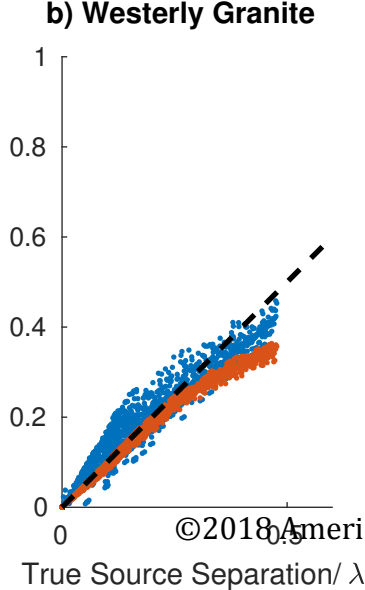
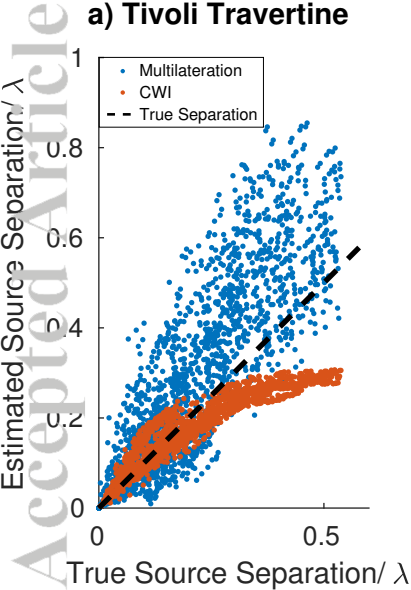


c) Copp-Crag Sandstone



Position along base of sample (mm)

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a) Experiment I: Temperature

Sample Details:

Fine grained

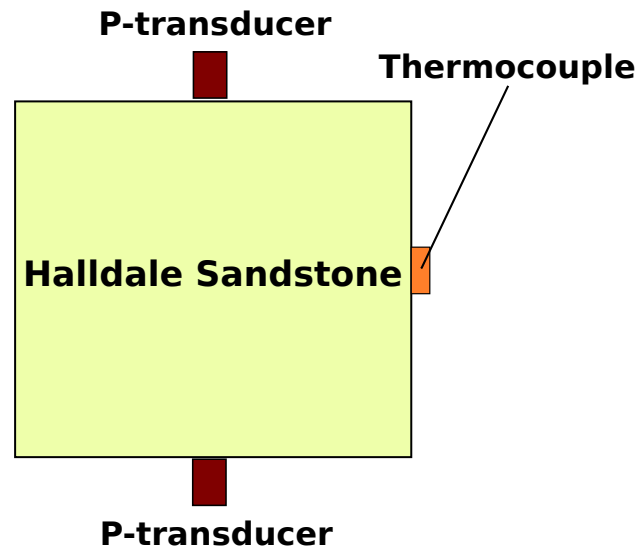
Quartz rich

$\phi = 15.1\%$

$\rho = 2248 \text{ kg/m}^3$

Size: 10x10x10 cm

Saturation: Dry



b) Experiment II: Differential Stress

Sample Details:

Core parallel with laminations

Finely laminated carbonate

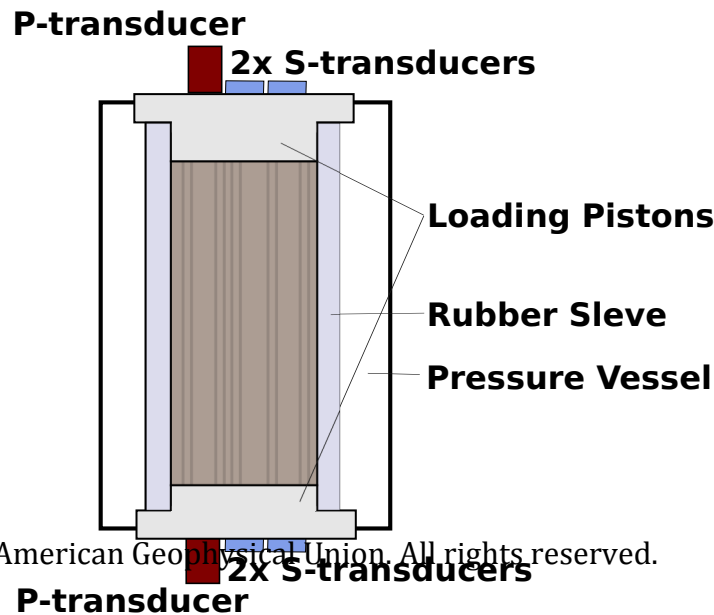
$\phi = 10.3\%$

$\rho = 2374 \text{ kg/m}^3$

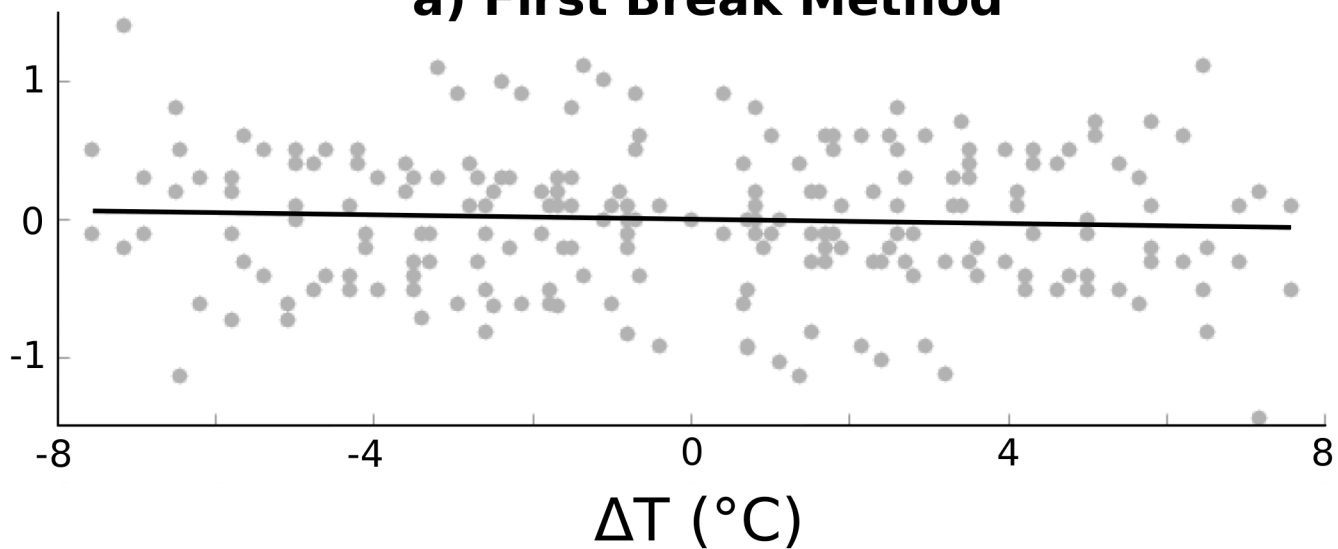
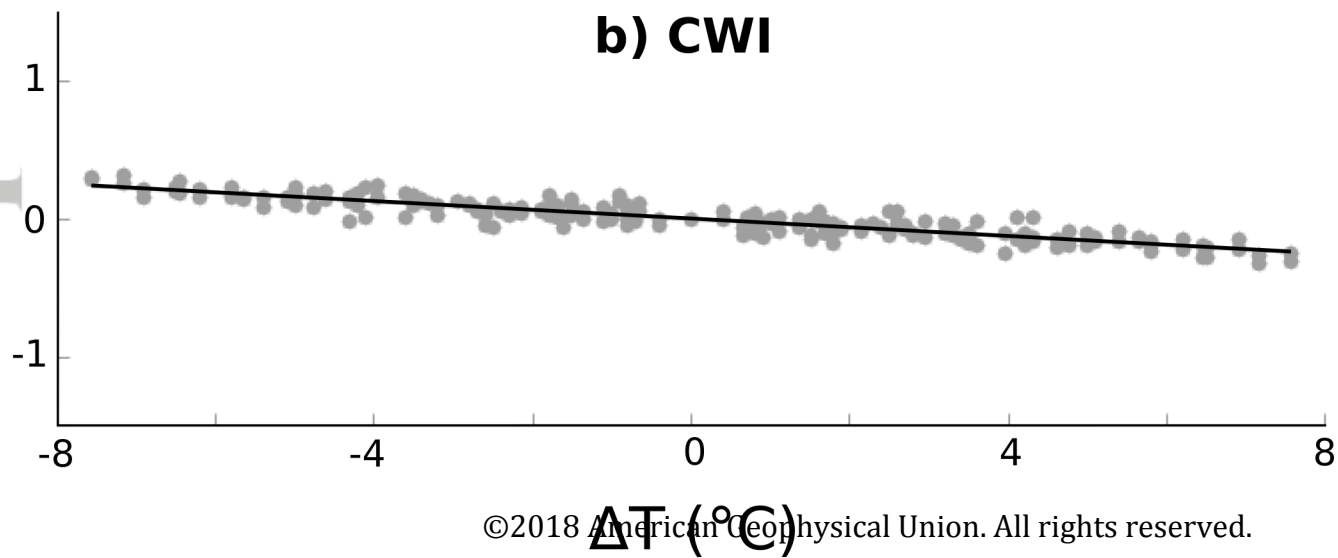
Length = 75 mm

Diameter = 38 mm

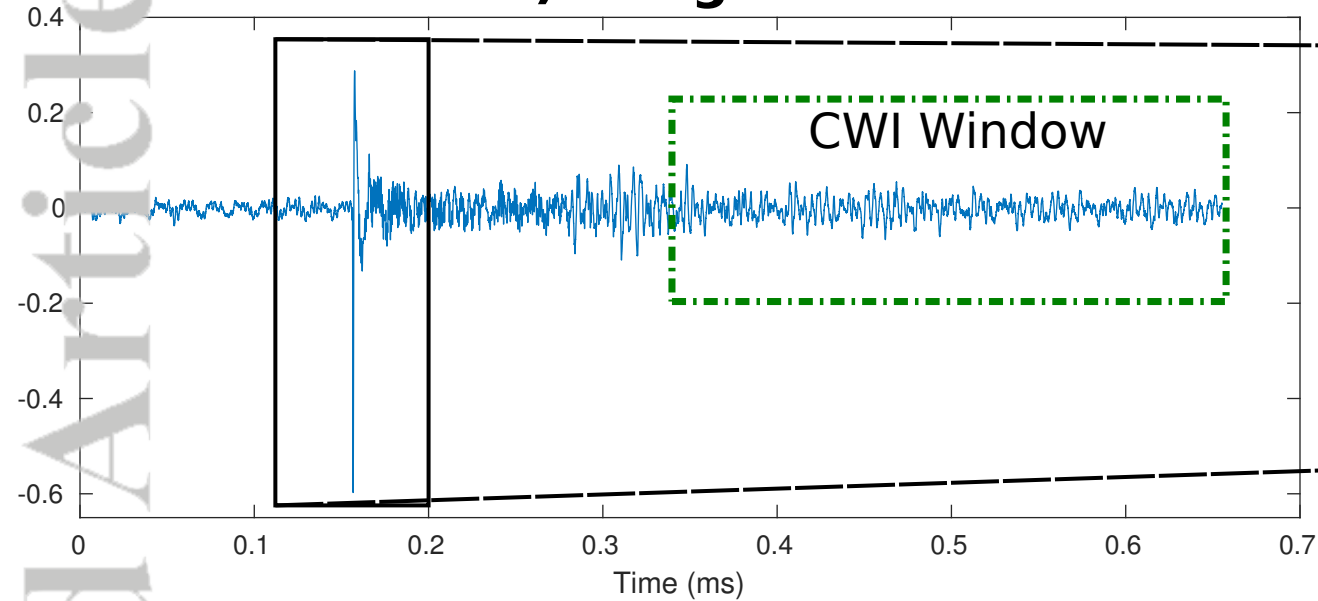
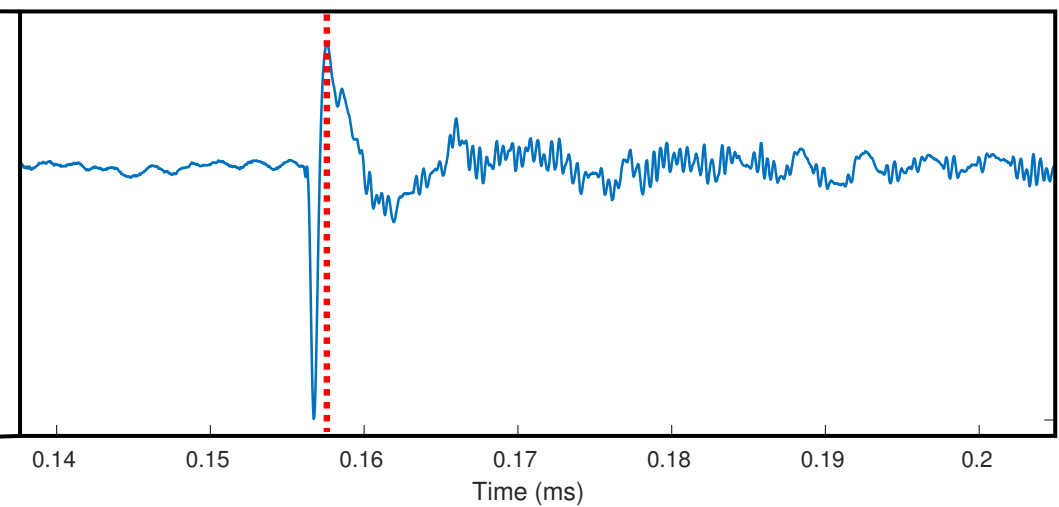
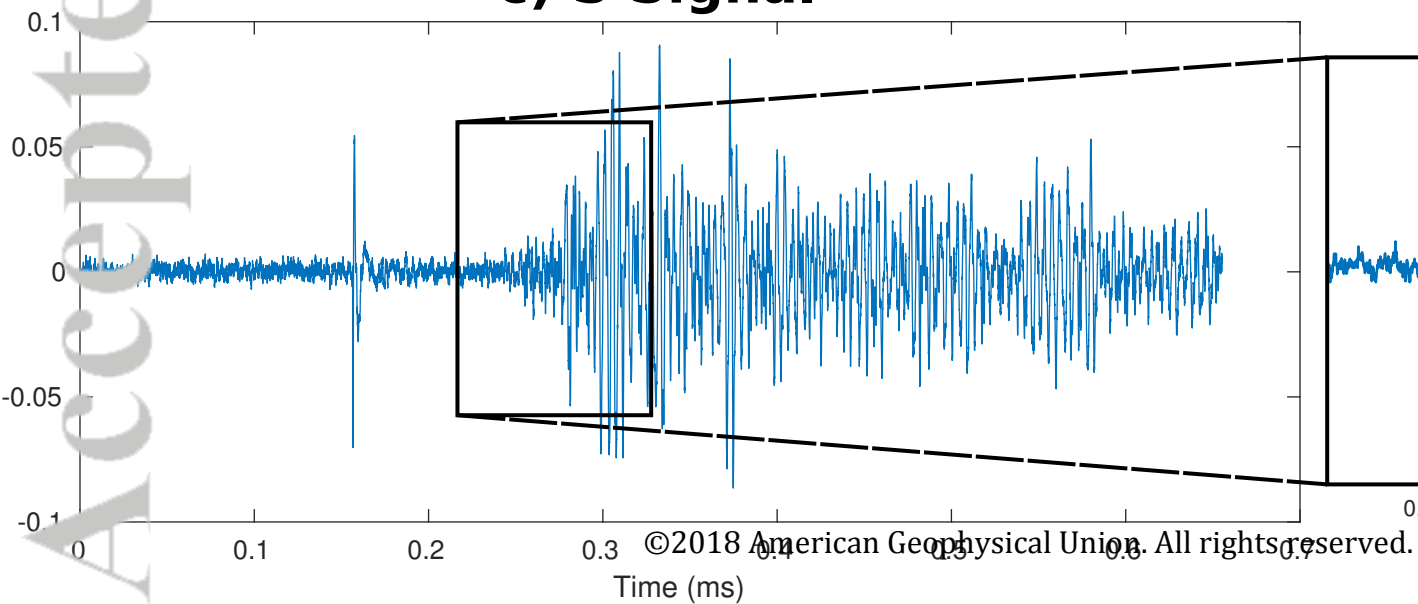
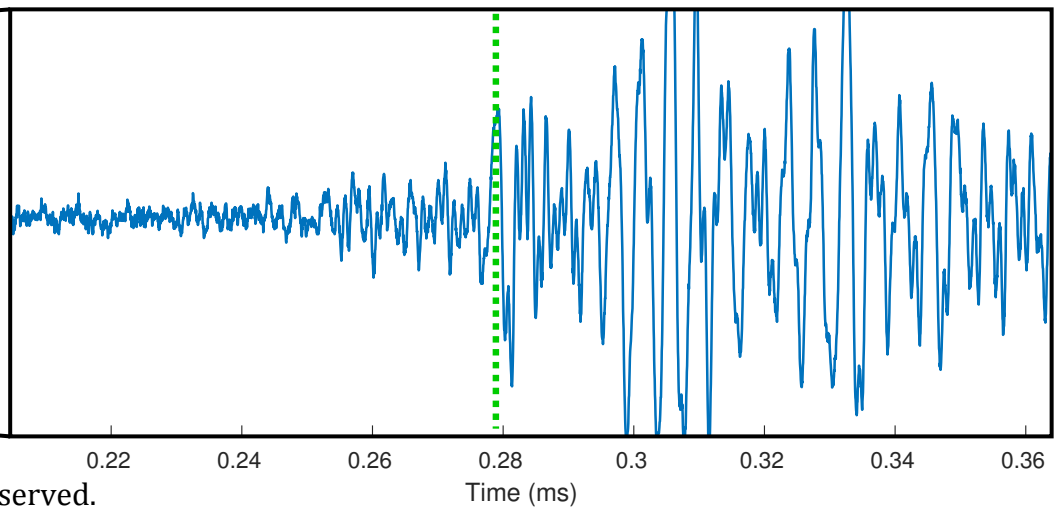
Saturated with deionized water



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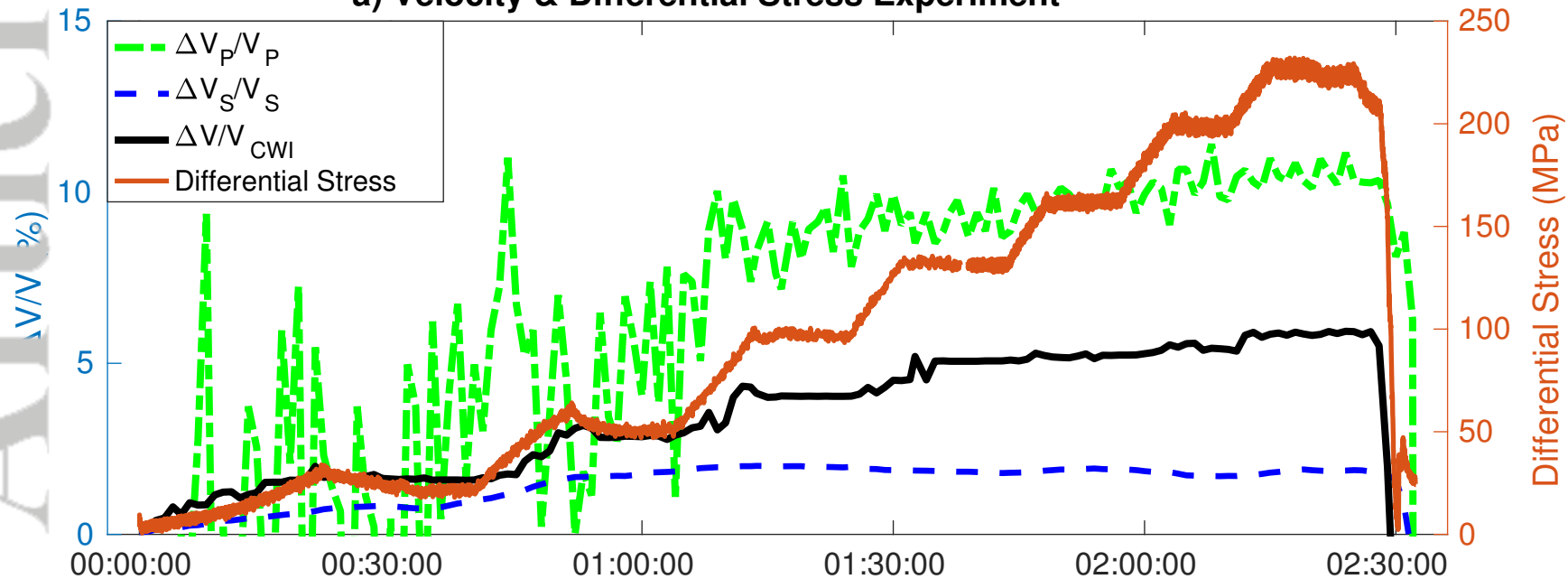
a) First Break Method**b) CWI**

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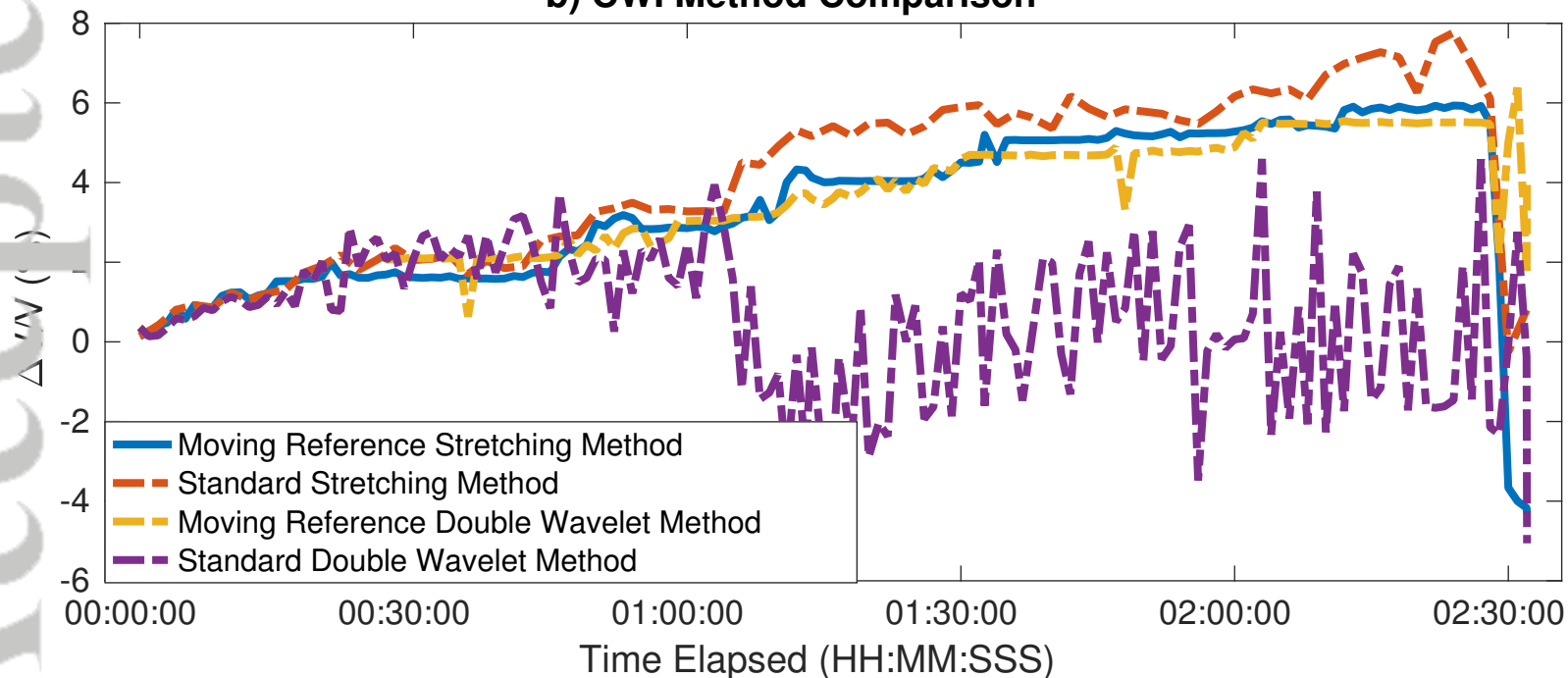
a) P Signal**b) P Travel Time Pick****c) S Signal****d) S Travel Time Pick**

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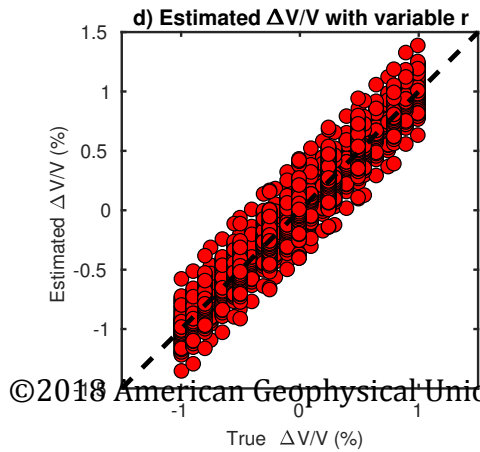
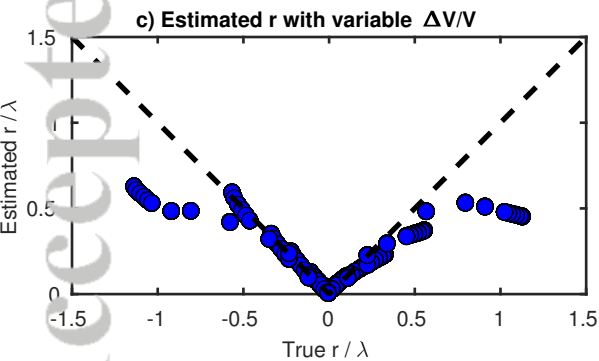
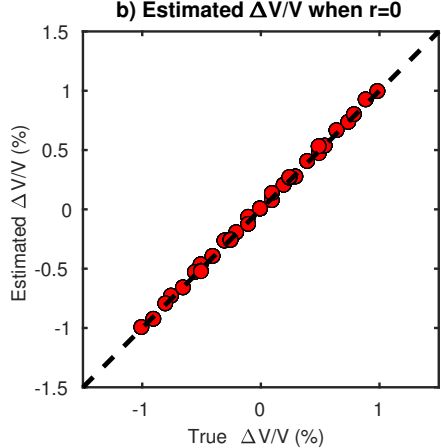
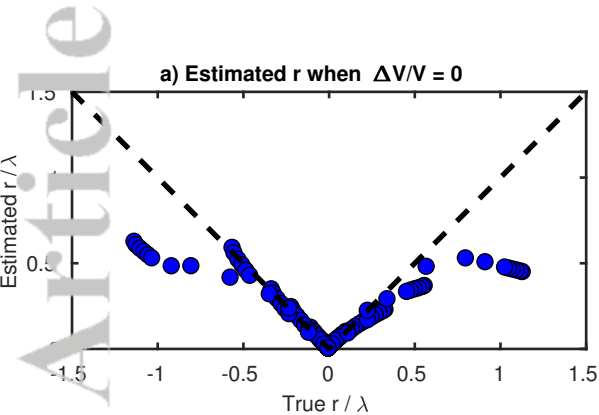
a) Velocity & Differential Stress Experiment



b) CWI Method Comparison

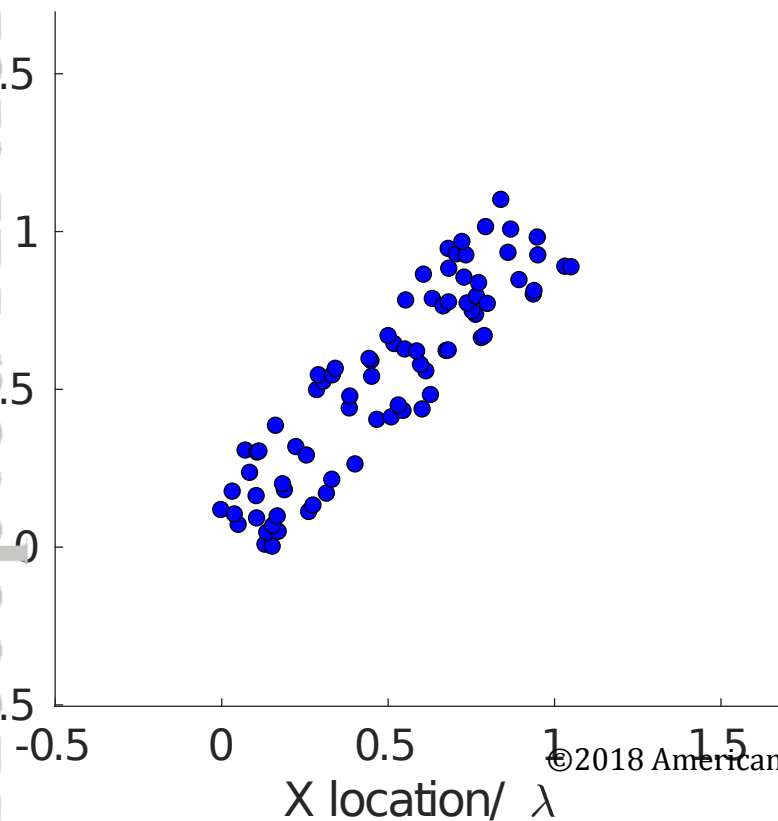


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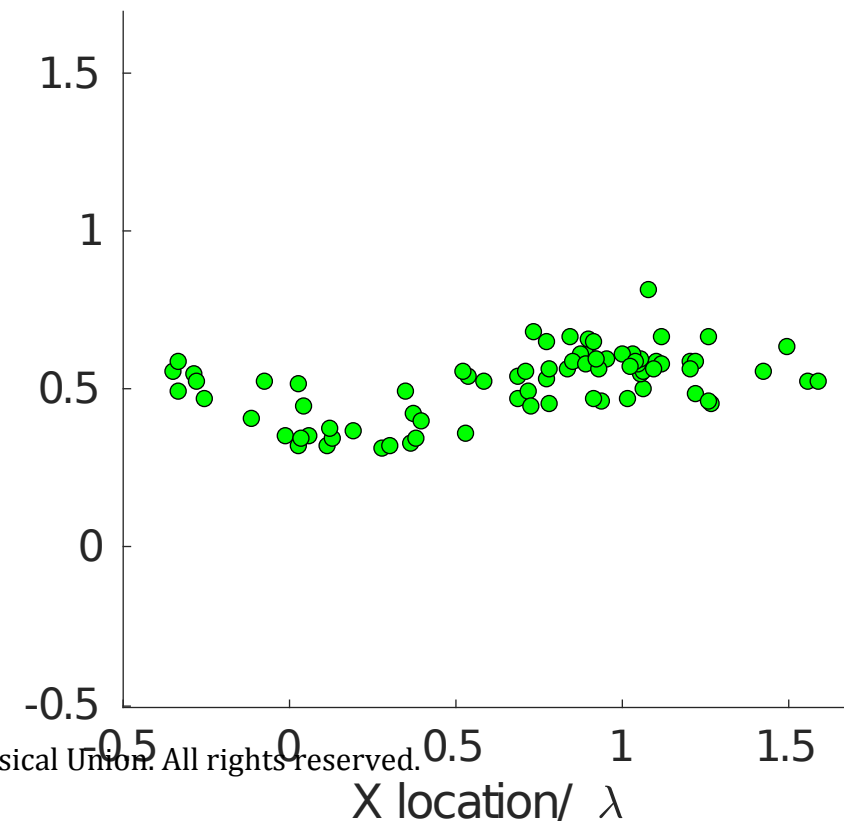


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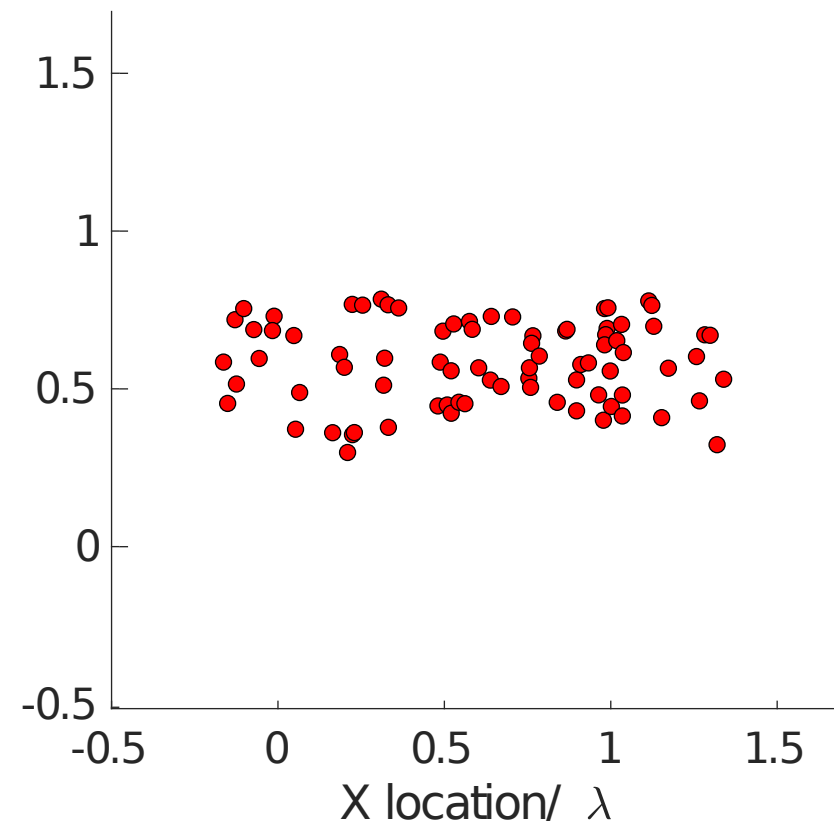
a) True Relative Locations



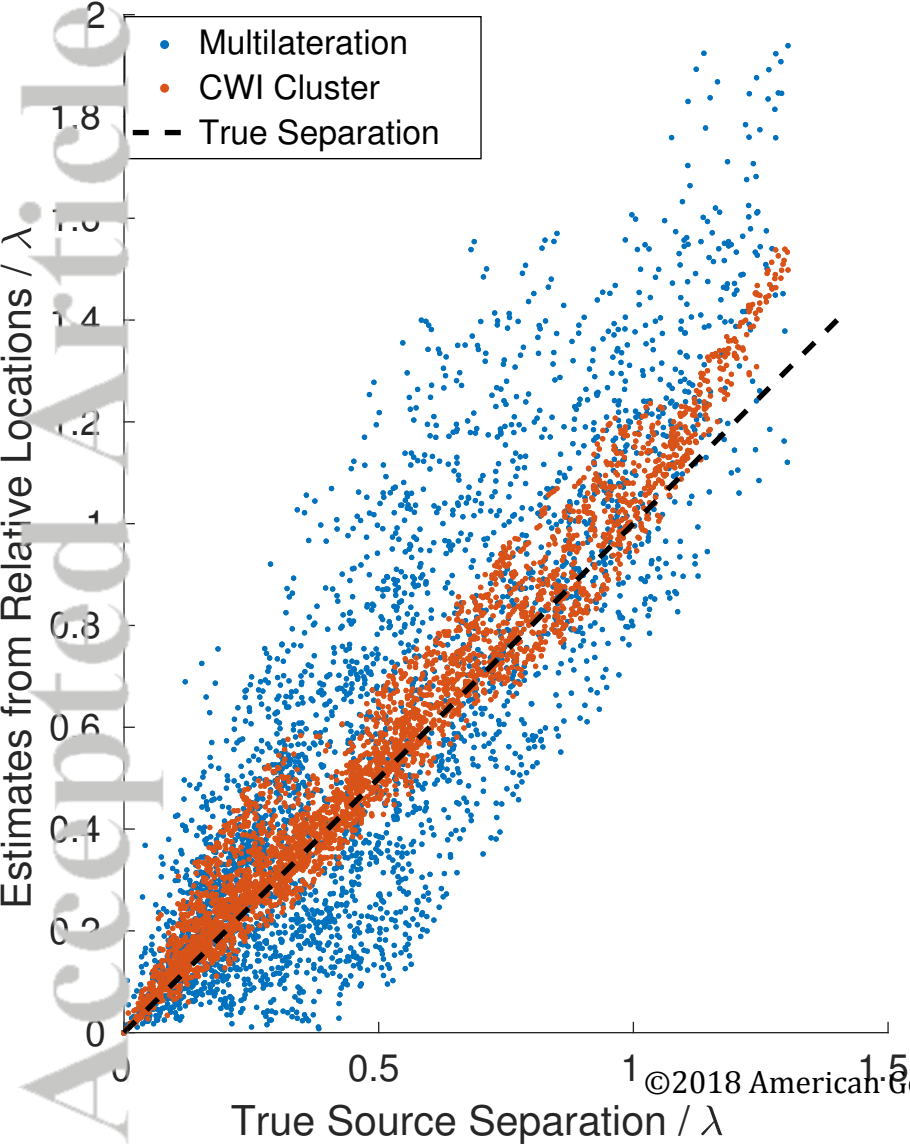
b) Multilateration



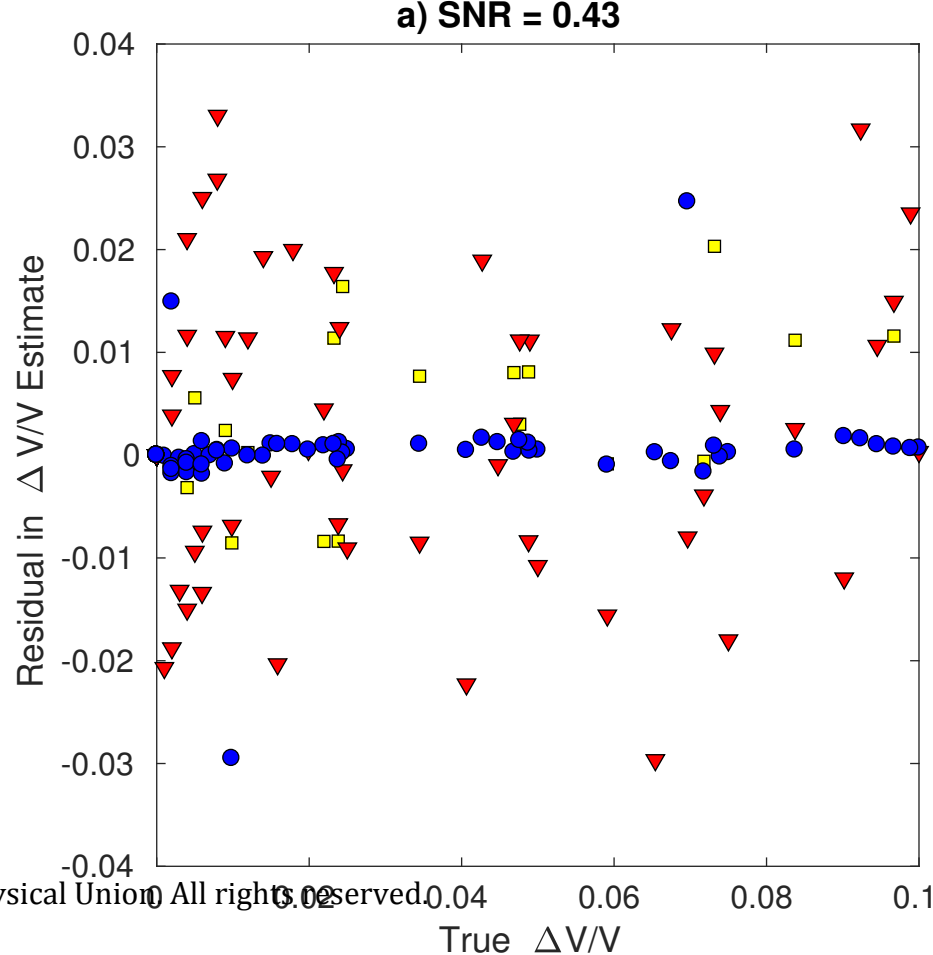
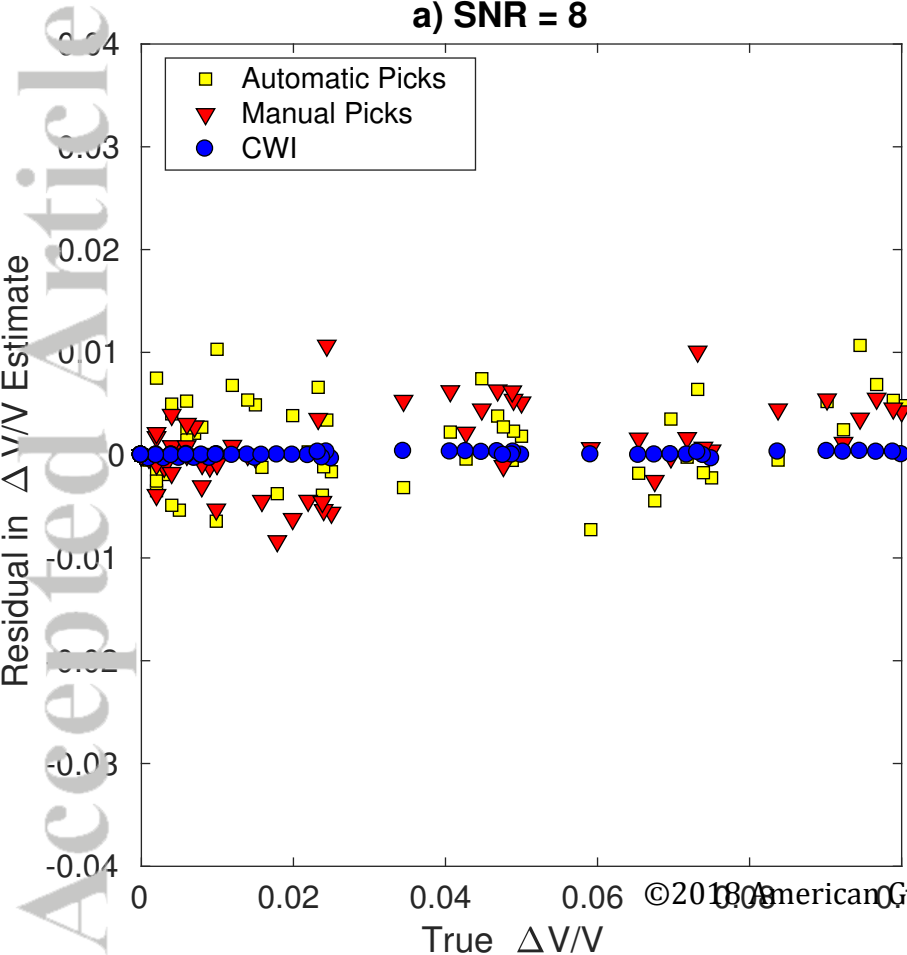
c) CWI



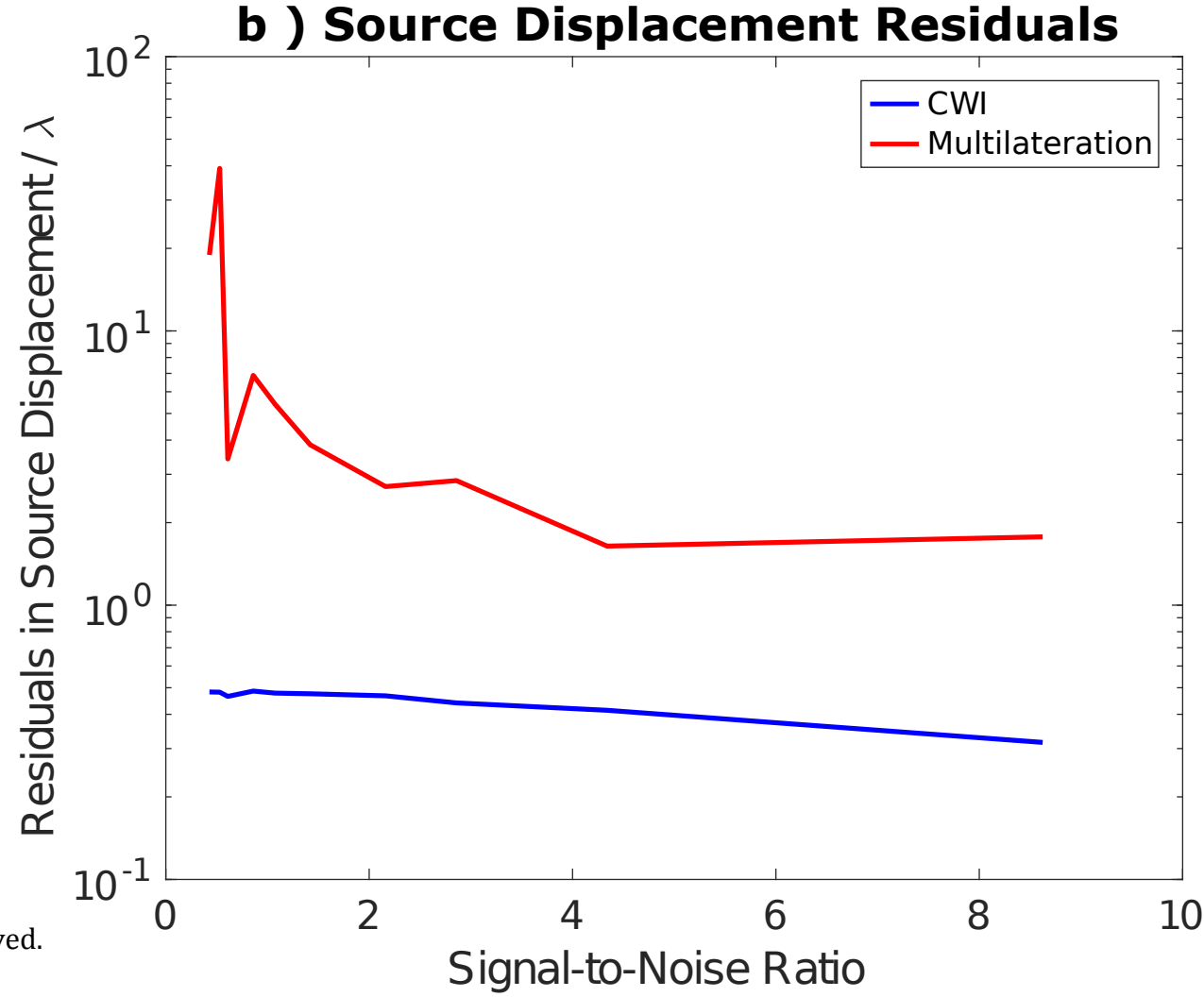
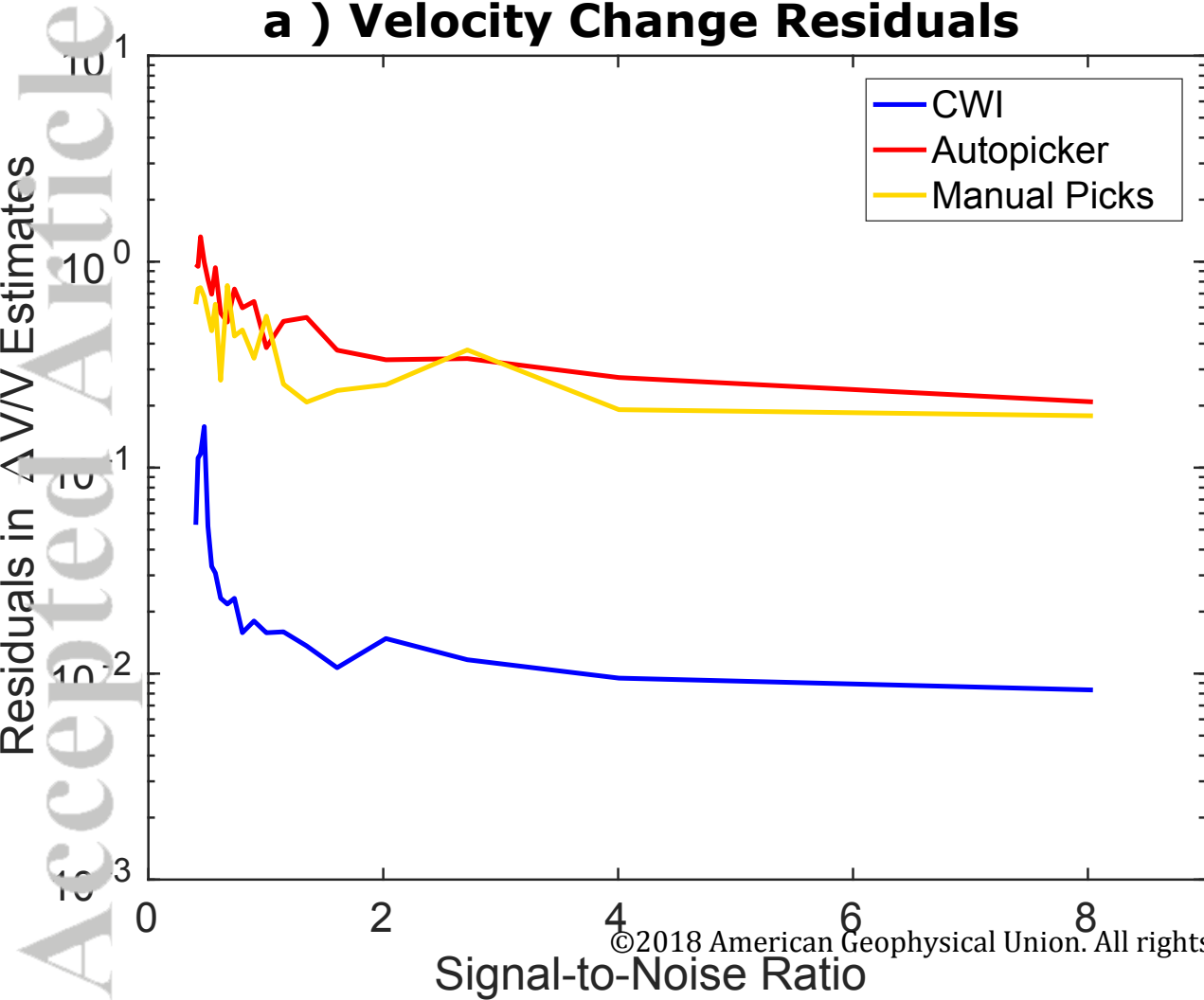
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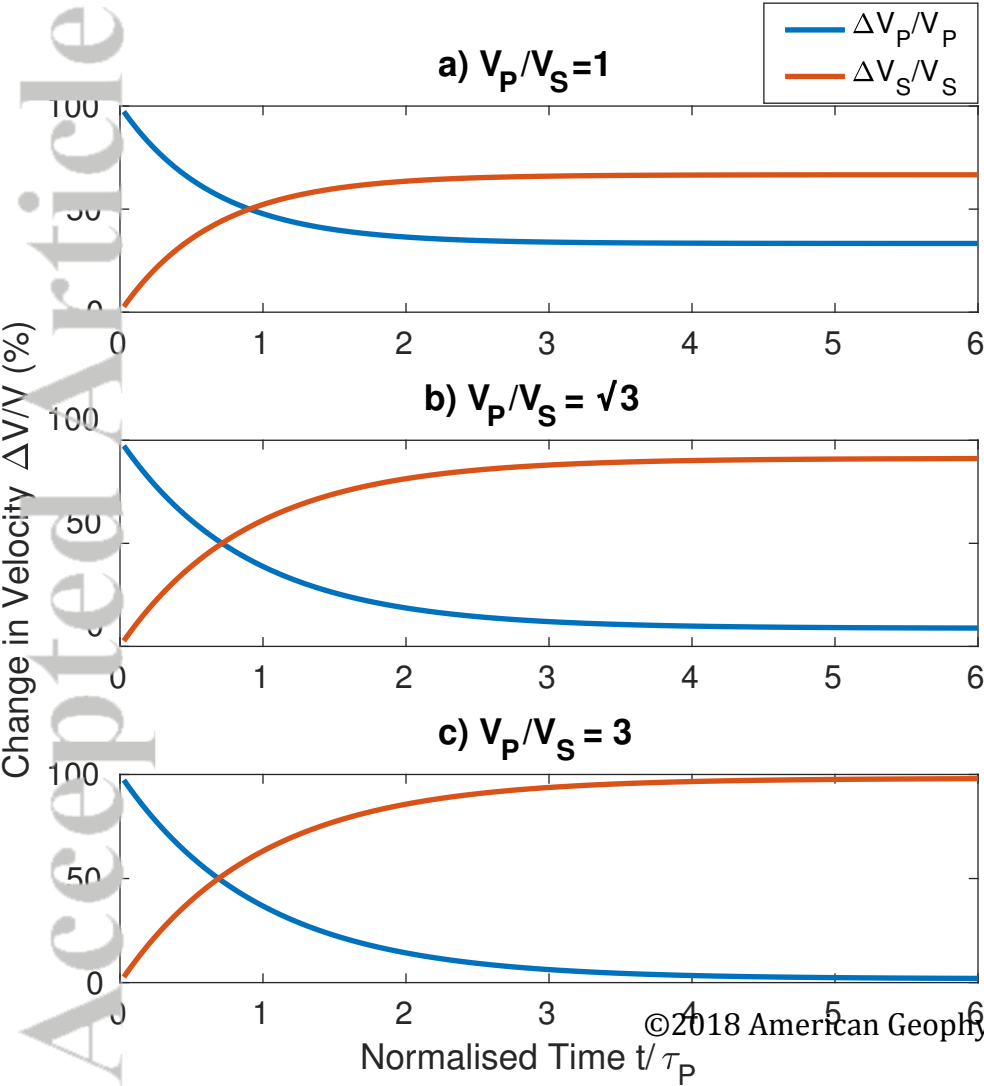
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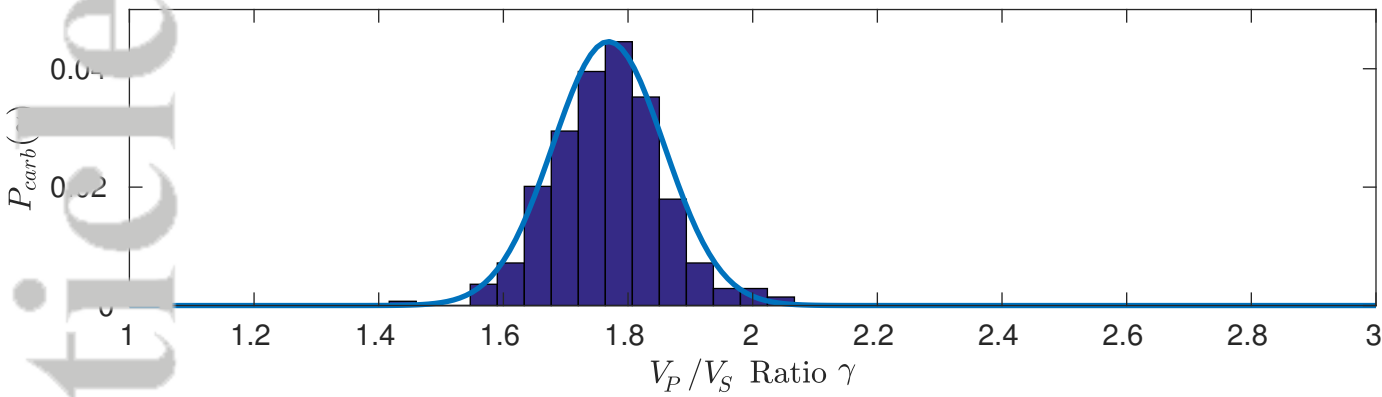


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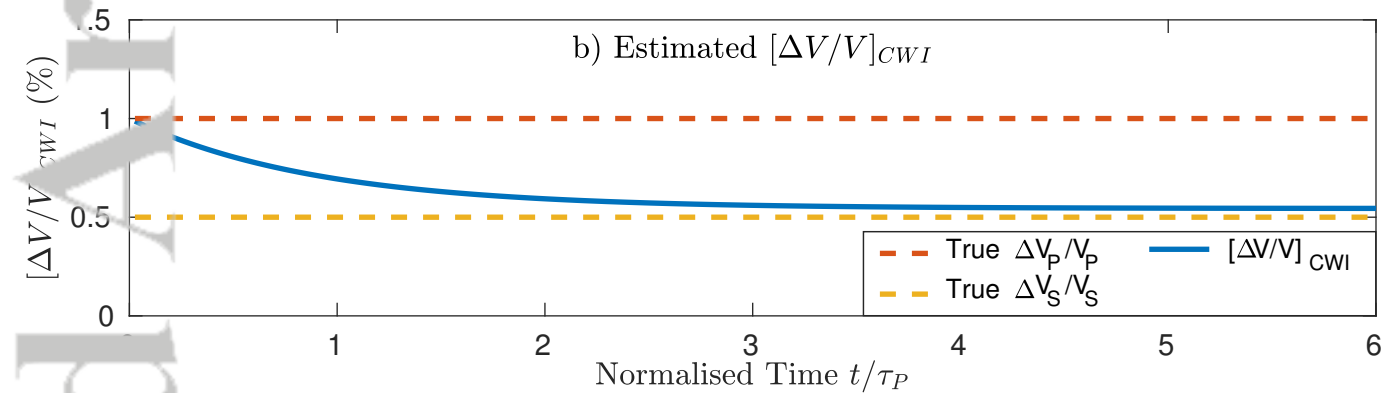


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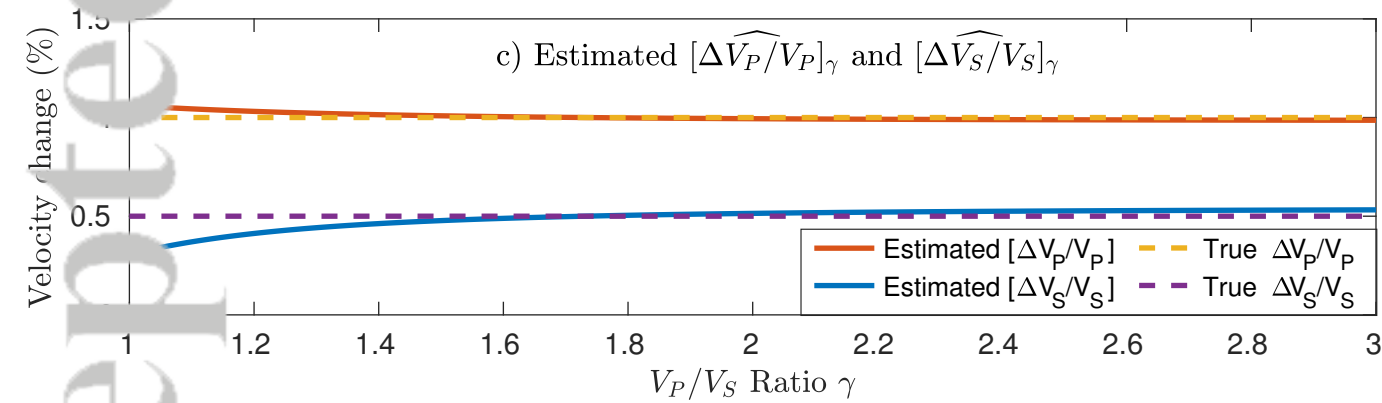
a) Prior Distribution $P_{carb}(\gamma)$



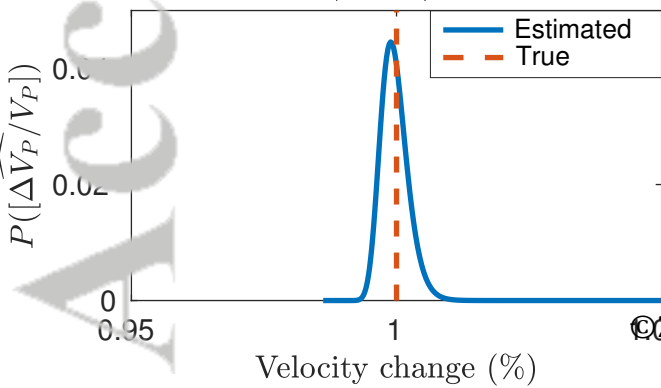
b) Estimated $[\Delta V/V]_{CWI}$



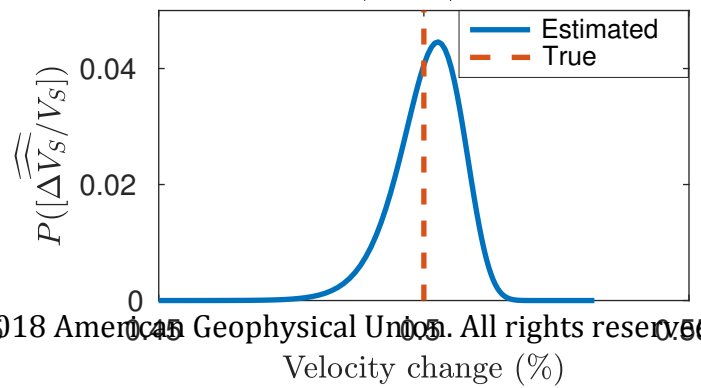
c) Estimated $[\Delta \widehat{V}_P/V_P]_\gamma$ and $[\Delta \widehat{V}_S/V_S]_\gamma$



d) $\Delta V_P/V_P$



e) $\Delta V_S/V_S$



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