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Wave-Current Interaction Effects on Marine Energy Converters

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Abstract

Many countries have significant interests in generating electricity using waves and tidal current technologies. In energetic areas, waves and tidal currents interact for modifying the energy resource and impacting on the design conditions. Changes to the wave climate depend on the strength of the current and the relative wave direction. SWAN simulations of the wave climate around the Orkney Islands, with and without currents, show that considerable changes in the wave climate occur near sites of interest to wave and tidal energy project developers. Using circular statistics the effect of the relative angle between the waves and the current can be investigated. Local effects can lead to 150-200% increases in wave height when the waves oppose the current. These dramatic changes lead to an increase in wave power of over 100kWm^{-1} . The complex nature of the tides in the channels also leads to large changes in wave power during the so-called *slack water* period. *Wave*

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amplification diagrams are proposed to provide a convenient summary of wave-current effects at a particular site and allow a statistical analysis to be made. When performing resource analysis and site selection work for marine energy projects, wave-current interaction must be considered.

Keywords: Marine Energy, Resource Assessment, Wave Modelling, SWAN
2010 MSC: 76D33, 62P12

1. Introduction

Currently there is significant interest in the generation of renewable electricity from the oceans, primarily by the conversion of either wave or tidal current energy. Many countries around the world have ambitious plans for exploiting the maritime energy resource to help address their long term (2050) targets for decarbonising their economies. In their 2010 annual report the International Energy Agency's implementing agreement on ocean energy systems states that, (Brito-Melo and Huckerby, 2010)

Ocean energy generation has a potential to reach 3.6 GW of installed capacity by 2020 and close to 188 GW by 2050. This represents over 9 TWh per year by 2020 and over 645 TWh per year by 2050, amounting to 0.3% and 15% of the projected EU-27 electricity demand by 2020 and 2050 respectively.

Bringing this energy to market requires understanding of the available energy resource on three distinct levels (Ingram et al., 2011). Firstly, an *early stage*, high level assessment of the resource at a geographic scale should be conducted. This is used to identify likely areas in which energy projects can be deployed. Secondly, a detailed, *project development*, study, is undertaken to

19 select the individual locations at which machines should be deployed. Finally,
20 *operational* resource monitoring and modelling is required both to forecast
21 energy production and to plan ongoing operation and maintenance work.

22 At both the *project development* and *operational* levels an understanding
23 of the interaction of the waves and currents is required. This is critical if
24 either wave energy converters or tidal current turbines are to be deployed as in
25 the first case the current will modify the shape and spectra of the waves, while
26 in the second, the unsteady fluctuations in the flow arising from the waves
27 will cause transient loading on the turbine rotor. Whilst in a few locations
28 (for example Strangford Narrows in Northern Ireland) it is possible to find
29 a site with strong tidal currents which is relatively sheltered from waves,
30 the most energetic sites being examined have both highly energetic waves
31 and strong tidal currents. In conducting resource assessments, spectral wave
32 models, e.g. SWAN (Booij et al., 1999; Ris et al., 1999), TOMAWAC (Brière
33 et al., 2007), and MIKE-21 SW (Sørensen et al., 2004) are widely used. These
34 sophisticated, third generation spectral models describe the behaviour of the
35 waves in terms of the the two-dimensional wave action density spectrum.
36 The action density spectrum, $N(\sigma, \theta) = S(\sigma, \theta)/\sigma$, is used because action
37 density is conserved in the presence of currents, whereas energy is not. The
38 evolution of the wave energy spectrum is described using the spectral action
39 balance equation which includes terms modelling the influence of bathymetry
40 and current. In this paper the structured formulation of the SWAN model
41 (van der Westhuysen et al., 2007) has been used.

42 The purpose of the study is to characterise how current at a specific loca-
43 tions modifies the wave climate. Such interactions modify the energy avail-

Figure 1: Schematics of Orkney Islands. Points indicate locations of (a) wind data, (b) wave and tidal data. (b is a closeup of a.)

44 able for conversion and, as a direct consequence, the tuning (and possibly
 45 design) of a wave energy device deployed at the location. The present study
 46 considers the highly energetic waters in the Orkney archipelago and Pentland
 47 Firth (Fig. 1). In 2009 this area was the subject of a leasing round by the
 48 United Kingdom's Crown Estate which resulted in the granting of 10 leases
 49 for a total of about 1.2GW of installed capacity (BVG Associates, 2011). By
 50 analysing the differences between SWAN predictions with and without tidal
 51 currents, locations at which there are significant changes in the wave envi-
 52 ronment are identified and the influence of current is summarised through a
 53 novel wave amplification diagram §5. Circular statistics and quantile regres-
 54 sion are performed on the simulation data to derive diagrams which explain
 55 how the effects of the tidal currents on the waves varies with the relative
 56 direction between the current and the waves.

57 2. Modelling

58 2.1. Wave Prediction Model

59 The wave field was computed using the wave prediction model, Simulating
60 Waves Nearshore (SWAN, version 40.85), developed by Delft University of
61 Technology (van der Westhuysen et al., 2007) . All parametres in this model
62 are represented using wave action density defined by the following equations.

$$N(\sigma, \theta) = S(\sigma, \theta) / \sigma, \quad (1)$$

$$S(\sigma, \theta) = 2 \int_{-\infty}^{\infty} C(\tau) e^{i\sigma\tau} d\tau, \quad (2)$$

63 where $N = N(\sigma, \theta, \mathbf{x}, t)$ is the wave action density, σ is the frequency of
64 the wave (in radians), and S is the energy density, determined based on
65 a Fourier transform of the auto-covariance function C of the sea surface
66 elevation. Wave action density is updated by solving the following action
67 balance equation:

$$\frac{\partial N}{\partial t} + \nabla_{\mathbf{x}} \cdot [(\mathbf{c}_g + \mathbf{U}) N] + \frac{\partial c_{\sigma} N}{\partial \sigma} + \frac{\partial c_{\theta} N}{\partial \theta} = \frac{S_{tot}}{\sigma}, \quad (3)$$

68 where θ is wave direction, $\mathbf{c}_g$ is group velocity, $\mathbf{U}$ is current velocity, c_{σ}
69 and c_{θ} are the propagation velocity for σ and θ spaces respectively, and S_{tot}
70 represents the sum of the dissipation and generation of the wave energy (e.g.
71 due to the effects of wind, bottom friction, white capping and so on). The
72 effect of the currents on the wave field is included by changing the wave
73 propagation velocity with the current velocity $\mathbf{U}$ as described in the above
74 equation, which can reproduce the basic effects of the current on the wave
75 fields, although it is unable to reproduce the non-linear interactions between
76 waves and currents. Wind input and wave dissipation due to wave-wave

Table 1: Computational domain for SWAN simulation.

Domain	Computational area	Grid number	Grid spacing (lon $\times$ lat)	Δt
1	W70.00 $^{\circ}$ –E10.00 $^{\circ}$, N41.00 $^{\circ}$ –N69.00 $^{\circ}$	240 $\times$ 84	0 $^{\circ}$ 20' $\times$ 0 $^{\circ}$ 20'	10min
2	W15.00 $^{\circ}$ –E5.00 $^{\circ}$, N57.00 $^{\circ}$ –N63.00 $^{\circ}$	120 $\times$ 60	0 $^{\circ}$ 6' $\times$ 0 $^{\circ}$ 6'	10min
3	W6.00 $^{\circ}$ –W2.00 $^{\circ}$, N58.00 $^{\circ}$ –N60.00 $^{\circ}$	120 $\times$ 120	0 $^{\circ}$ 2' $\times$ 0 $^{\circ}$ 1'	10min
4	W3.75 $^{\circ}$ –W2.25 $^{\circ}$, N58.50 $^{\circ}$ –N59.50 $^{\circ}$	180 $\times$ 120	0 $^{\circ}$ 0'30'' $\times$ 0 $^{\circ}$ 0'30''	3min

77 interaction and white capping were computed by the third generation mode
78 of this model with Westhuysen’s formulations (van der Westhuysen et al.,
79 2007) which is known to reproduce the development of the wave field due to
80 wind more accurately. Bottom friction was computed based on JONSWAP
81 formulations and default values were used for the other settings.

82 One-way nesting using four computational domains (Table 1) was used
83 to account for both swell and wind driven waves developed across the At-
84 lantic Ocean and also to account for deformation due to the rapidly chang-
85 ing local bathymetric conditions around the Orkney islands. Meteorological
86 re-analysis data sets from ERA-Interim (1.5 $^{\circ} \times 1.5^{\circ}$ resolution), provided
87 by European Centre for Medium-Range Weather Forecasts (ECMWF) at a
88 time-step of six hours were used for the wind input for domain 1. Wind data
89 with higher resolution, computed as described in the following section, were

90 input for domains 2–4 every hour. We modelled two cases: (case 1) where the
 91 wave field is computed without considering the effects of tidal currents and
 92 (case 2) where tidal effects are included. Tidal current velocity and tidal ele-
 93 vations were computed as described in section 2.3 and input into the domain
 94 four times every hour. Bathymetry data used in the simulation was taken
 95 from the General Bathymetric Chart of the Oceans (GEBCO) global data
 96 at a resolution of 30 arc seconds. The computations were performed for a
 97 36 day period covering 2006/July/9, 00:00–2006/August/14, 00:00 (GMT).
 98 Throughout this paper the time, t , is defined relative to the start of the
 99 computation at midnight on the 9th of July.

100 *2.2. Mesoscale Meteorological Model*

101 The Weather Research and Forecasting model (WRF-ARW, version 3.3.1),
 102 a mesoscale meteorological model developed by National Center for Atmo-
 103 spheric Research (NCAR), was used for generating high-resolution wind in-
 104 put data over the ocean. WRF computes atmospheric flow by solving equa-
 105 tions of motion for compressible and non-hydrostatic flow with the initial and
 106 boundary conditions taken from re-analysis data. We have used NCEP FNL
 107 Global Analysis data with $1.0^\circ \times 1.0^\circ$ resolution provided every six hours
 108 as the input wind data for WRF, and NCEP Real-Time Global SST anal-
 109 ysis data (RTG-SST) with $\frac{1}{12}^\circ \times \frac{1}{12}^\circ$ resolution provided every 12 hours to
 110 update the sea surface temperature. Two-domain nesting computation was
 111 performed with the domains shown in Table 2 which are slightly larger than
 112 the domains 2–3 of the wave computation. The following options were cho-
 113 sen as the physical parametrisation schemes: Single-moment six-class scheme
 114 for the microphysics (Hong and Lim, 2006); rapid radiative transfer model

Table 2: Computational conditions for WRF computation.

Domain	Computational area	Grid number	Grid spacing (lon $\times$ lat)	Δt
1	W16.20 $^{\circ}$ –E5.70 $^{\circ}$, N55.80 $^{\circ}$ –N63.70 $^{\circ}$	221 $\times$ 81 $\times$ 27	0 $^{\circ}$ 6' $\times$ 0 $^{\circ}$ 6'	30sec
2	W6.04 $^{\circ}$ –W1.76 $^{\circ}$, N57.96 $^{\circ}$ –N60.04 $^{\circ}$	216 $\times$ 106 $\times$ 27	0 $^{\circ}$ 1.2' $\times$ 0 $^{\circ}$ 1.2'	6sec

for long wave radiation (Mlawer et al., 1997); Goddard’s scheme for short
wave radiation (Chou and Suarez, 1994); Mesoscale Model (MM5) similarity
theory for the surface layer (Zhang and Anthes, 1982); NOAA land surface
model (Chen and Dudhia, 2001); Yonsei University scheme for planetary
boundary layer (Hong et al., 2006); and Kain-Fritsch scheme for cumulus
parametrisation (Kain, 2004). These settings were determined based on the
comparison of the computational results with observed data.

2.3. Tidal Current Model

Although a large number of global tidal models have been proposed (e.g.
FES2004 (Lyard et al., 2006), TPXO7.2 (Egbert and Erofeeva, 2002)) which
are able to predict tidal conditions in deep water at high accuracy, the pre-
diction by these models in coastal regions with complex shorelines is known
to be less accurate. In the present work, both the tidal current velocity and
the tidal sea surface elevation were computed using MOHID Water Mod-
elling System (Martins et al., 2001) developed at the Technical University
of Lisbon to provide input into the present wave model. MOHID solves the

continuity and momentum equations with Boussinesq approximations in hydrostatic equilibrium. The computational domain used for the tidal model is identical with domain 4 of the wave computation (see Table 1), having a 360×240 cell grid with a uniform interval of $15' \times 15'$ and a time step of 30 seconds. Boundary conditions of the tidal velocity and surface elevation were applied based on the FES2004 global tidal solution with 15 tidal constituents at $\frac{1}{8}^\circ \times \frac{1}{8}^\circ$ grid resolution, which was obtained from a hydrodynamic computation and data assimilations. While the Coriolis force was included in the computation, no wave or wind effects on the tidal current were considered.

3. Validation

For each of the models described above, a validation exercise was conducted by comparing simulated time series with observed data from met-stations and oceanographic instrument deployments. Figure 1 shows the locations used for validation: Wind measurements were taken from met-stations at Stornaway, Kirkwall, Lerwick, Fair Isle and Wick; tidal velocity measurements were used from instruments (T1 and T2) located in the Pentland Firth; wave data came from a waverider buoy (W1) located near the European Marine Energy Centre (EMEC) wave test site.

Figure 2 compares the U_{10} measured and simulated wind velocity time series at the five met-stations. U_{10} is defined as the wind velocity at 10 m above the local ground (or sea level). U_{10} is commonly reported by met-stations and weather satellites and is used as the input wind velocity in the SWAN model. In figure 2 a solid line is used to represent the simulated time series at each met-station, while dots are used for the observers data. In

Figure 2: Comparison of 10-m wind velocity U_{10} between computational results (line) and observed data (dots). Horizontal axes are the time from 2006 July 9, 00:00. Coefficients of correlation were (a) 0.853 (b) 0.739 (c) 0.801 (d) 0.834 (e) 0.625, respectively.

Figure 3: Comparisons between observation data (black) and the present computational results (blue) for tidal ellipses (left) and evolution of absolute tidal current velocity (right) at (a) T1 and (b) T2.

155 general, there is a good agreement between the overall trend and peak wind
 156 velocities at all of the locations, with the correlation coefficients varying
 157 between 0.63 and 0.85. For days 5 to 10 the wind speed at Kirkwall is under
 158 predicted by the model, whilst the model over predicts the wind speed at
 159 Lerwick and Fair Isle over the same period. Because the prevailing wind
 160 direction is from the southwest the influence of this difference on the wave
 161 field is small.

162 Comparisons of the tidal current between the present computational re-
 163 sults and observation data at T1 and T2 (see Fig.1) are presented in Fig.3.
 164 The correlation coefficients of the time series of the current velocity are 0.79
 165 and 0.71 at T1 and T2, respectively. The average difference between the mea-
 166 sured and predicted current velocity is 0.24ms^{-1} at both locations. Although
 167 the differences in the velocity are not insignificant, the overall features of the

Figure 4: Comparison of significant wave height H_s , peak period T_p , mean period T_m and mean wave direction θ_w between computational results (line) and observed data (dots) at the point W1. Horizontal axes are the time from 2006 July 9, 00:00. Buoy data is reproduced with permission of the European Marine Energy Centre Ltd.

168 velocity field, especially local variation depending on bathymetry are thought
 169 to be reasonably reproduced.

170 Figure 4 shows the comparison of significant wave height, H_s , peak, T_p ,
 171 and mean, T_m , periods and wave direction, θ_w , between observation data and
 172 the computational results in the simulation with tidal current (case 2). Two
 173 days of “spin up” are required from the beginning of the simulation before the
 174 errors from the initial conditions have washed out of the simulation and the

175 wave field is fully developed. Once the initial transients have left the domain,
176 the coefficient of correlation is 0.93 and 0.75 between the computed and
177 observed significant wave heights and mean periods, respectively, indicating
178 that an accurate hindcast has been made.

179 During days 10 to 19 and 21 to 27 days there is much less agreement
180 between the observed data and the simulation, resulting in low correlation
181 coefficients of 0.55 and 0.42 for significant wave height and wave period re-
182 spectively. During these periods, low significant wave height leads to an
183 unsteady wave spectrum, with fluctuating wave periods and direction. The
184 sea state is undeveloped and as a consequence, is not reproduced accurately
185 in the simulation. Discarding periods when the significant wave height is less
186 than 1.5m leads to correlation coefficients of at least 0.70 in all cases.

187 4. Results

188 Figure 5 shows the areas leased by the Crown Estate in the UK to devel-
189 opers wishing to exploit the wave and tidal resources in the Pentland Firth
190 and Orkney Waters in 2010. These sites represent an ambition by power
191 utilities, project developers and device developers to install 1.6GW of gen-
192 eration capacity before 2025. Two of the leased tidal sites (Westray South
193 and Cantick Head) and one of the wave sites (West Orkney South) are of
194 particular interest in the present work. At three points (Fig. 6) near to these
195 potential deployments the wave-current interactions have been examined in
196 detail. $P1$ is located close to the EMEC wave test site at the western en-
197 trance to Scapa Flow; $P2$ on the northern side of the western end of the
198 Pentland Firth just south of the Island of Hoy; and $P3$ at the northwestern

Figure 5: Sites leased for the development of wave and tidal energy projects by the UK Crown Estate in the Pentland Firth and Orkney Waters. Reproduced with permission from The Crown Estate

Figure 6: Distributions of (left) mean significant wave height with wave direction and (right) maximum tidal current velocity throughout the computational period. Areas A, B and points P1-P3 are explained in the main text.

199 end of Westray Sound. These locations are in positions where the maximum
200 current velocity exceeds 2ms^{-1} while still being subjected to the full force
201 of the Atlantic waves. The selection of these points is motivated both by
202 an analysis of the characteristics of the wave and tidal fields throughout the
203 simulation and the locations of the Crown Estate leases.

204 Figure 6 shows the mean wave height (averaged over the last 34 days of the
205 simulation) and maximum tidal current velocity over the same period from
206 the combined wave and current simulation (case 2). Eastward of the Orkney
207 islands the wave climate is dominated by the large, long period, Atlantic
208 swell waves which make the areas so suited to wave energy projects. The
209 sea to the east of the islands is sheltered from the Atlantic swell, but can be
210 subjected to local wind driven seas and lower amplitude swell from the North
211 Sea. As the tide flows between the Atlantic and the North Sea, the narrow
212 channels concentrate and accelerate the flow leading to very high tidal current
213 velocities. To the south, the Pentland Firth between the Scottish mainland
214 and the Orkneys (Figure 6 Region A) experiences the highest velocities. At
215 the western entrance to the Firth the flow is concentrated between the Island
216 of Hoy and the mainland, while at the eastern end the flow is constricted
217 between Brough Ness (on the mainland) and the island of South Ronaldsay.
218 On the southern side of the firth the Island of Stroma divides the flow between
219 the main channel and the Inner Sound. To the north, Westray Sound forms
220 a second channel which concentrates the flow. At the entrance to the Sound
221 (Figure 6 Region B) flow is funnelled between the islands of Westray and
222 Rousay and accelerated. At its southern end the island of Eday deflects the
223 flow accelerating it further. The EMEC tidal test site is located to the south

224 of Eday at the entrance to the Stronsay Firth (southeast of Region B). The
 225 third channel considered in the present work is Hoy Sound, which is located
 226 between the islands of Hoy and Mainland (the main Orkney Island). Hoy
 227 sound forms the western entrance to the large natural harbour of Scapa Flow,
 228 whose southern entrance leads into the main channel of the Pentland Firth.
 229 Figure 6 also shows the acceleration of the tidal flow around the northern
 230 islands of the archipelago, and in several other smaller inter-island channels.

231 Figure 7 shows the evolution of peak wave period, significant wave height
 232 and current velocity during a 9 hour period from 01:00 to 10:00 on day 5 of
 233 the simulation (13th of July 2006). The four phases illustrated represent the
 234 peak westward flow of tidal current (phase 1), slack water at high tide (phase
 235 2), the peak eastward flow of the tide (phase 3), and slack water period at
 236 low tide (phase 4). As the westerly tide flows into the Firth (phase 1), the
 237 flow is concentrated between the Scottish mainland and the island of Hoy; a
 238 similar concentration takes place in the Hoy and Westray Sounds and around
 239 the northern islands. During the easterly flood tide (phase 3) the Pentland
 240 Skerries and the islands of Muckle Skerry, Swona and Stroma obstruct the
 241 flow, concentrating the current in the north of the channel. The westward
 242 current in the Firth is consequently faster than the eastward current, as
 243 shown in Fig. 3 (*b-1*). During both of the “slack water” periods there are
 244 isolated regions of high current flow; in particular, the current in Hoy Sound
 245 persists as water continues to enter and leave Scapa Flow.

246 Through the simulated period the wave height from the Atlantic swell
 247 declines from about 7m to circa 4m. During phase 1, the waves are subjected
 248 to a strong current in the same direction as the direction of wave propagation.

Figure 7: Distributions of (left) peak period, (middle) significant wave height and wave direction, (right) tidal current velocity and its direction from the simulation including tidal current (case 2)

249 This lowers the wave amplitude and increases the wave length. Increasing
 250 the wave length increases the depth to which the wave motions penetrate
 251 the water column - consequently increasing the wave loading on submerged
 252 tidal turbines. During the ebb tide (phase 3) the adverse current increases
 253 the wave height and shortens the wave length. These effects are particularly
 254 pronounced in areas A (Western Pentland Firth) and B (Northern Westray
 255 Sound). Because there are still significant currents persisting at high and
 256 low water (3 hours after the maximum flood/ebb tide) the influence of the
 257 current on the waves is still visible during phases 2 and 4. This is particularly
 258 clear during the ebb tide to the south of the island of Hoy.

259 To further explore the influence of tidal current on the wave field, two
 260 separate simulations have been performed. In case 1 the wave field has
 261 been computed without the influence of tidal current and in case 2 the tidal
 262 current has been included in the simulation. The differences between these
 263 simulations (at the points $P1$, $P2$ and $P3$) on the wave field is shown in
 264 Fig. 8 as scatterplots of T_p vs. H_s at each point from the two cases. The
 265 scatter plots for case 2 show enhancement of wave height across a range
 266 of periods. These differences are particularly marked at $P2$ (Hoy) and $P3$
 267 (Westray Sound). The root mean square differences in H_s and T_p caused by
 268 tidal effects are 0.11m and 0.80s at $P1$, 0.82m and 1.66s at $P2$, and 0.39m
 269 and 1.16s at $P3$.

Figure 8: Scatter plots of peak wave period and significant wave height for the wave only simulation (top) and the combined wave and current simulation (bottom) at points P1 to P3.

4.1. Characteristics of the Wave Power Field

The wave power (or wave energy flux per metre crest length) (IEC, 2011), P , may be calculated by the following equation (assuming deep water),

$$P = \frac{\rho g^2}{64\pi} T H_s^2, \quad (4)$$

where g is the acceleration due to gravity, T_e is wave energy period and H_s is significant wave height. The wave power per metre can be computed directly from the SWAN simulations and is shown in figure 9. This figure compares the wave power per metre distributions from the two cases at the same times as phases 1–4 from Fig. 7. Because the wave power per metre varies with the square of the wave height, but only linearly with period, the wave power distribution almost corresponds directly to that of the wave height distributions from figure 9. During the flood tide the wave power is

Figure 9: Wave power per metre crest during the maximum flood tide (phase 1), high water (phase 2), maximum ebb tide (phase 3) and low water (phase 4) for the simulation excluding tidal current effects (top) and including tidal current effects (bottom).

281 reduced at the entrance to the Pentland Firth and Hoy and Westray Sounds
 282 by the tidal current. At high tide the wave power to the south of the Firth
 283 is higher than in the wave only case. During the ebb tide wave power is
 284 significantly increased in the channels where the current is flowing. This
 285 difference persists towards low water, when a localised increase in wave power
 286 can be observed to the south of the island of Hoy at the western entrance to
 287 the Pentland Firth. A difference in wave power of more than 100kWm^{-1} is
 288 observed due to the interaction of currents and waves.

289 Figure 10 represents the mean wave power per metre (averaged over the
 290 last 34 days of the simulation) computed in the two cases, together with the
 291 difference between them. On average the presence of currents increases the
 292 mean wave power per metre to the south of Hoy by a maximum of 7.9kWm^{-1} ,
 293 with a reduction in 5.5kWm^{-1} in the main channel of the Firth. The asymme-
 294 try in the differences in the mean wave power through the channel is caused
 295 by the presence of the islands at the eastern end of the Firth which deflect
 296 the ebb current northward. These differences correspond to an increase of
 297 57% and a decrease in 61% of the wave power per metre respectively.

298 5. Wave Amplification Diagram

299 We define the wave amplification factor as

$$\alpha = \frac{H_{m0} |_{tide}}{H_{m0} |_{wave}}, \quad (5)$$

300 where $H_{m0} |_{tide}$ is the spectral estimate of the significant wave height in the
 301 presence of the tidal current and $H_{m0} |_{wave}$ is the estimate in the absence of
 302 tidal effects. For deep water waves, with a Rayleigh height distribution, this

Figure 10: Mean wave power per metre for the simulations without current (left) and with current (middle), together with the difference in the mean wave power between the two cases (right).

estimate is given by

$$H_{m0} = 4.004\sqrt{m_0}, \quad (6)$$

where m_0 is the zeroth moment of the wave energy spectrum (Goda, 1985).

α can be computed for a deep water wave travelling into shallower water with a uniform current (Appendix A). Figure 11 shows comparison of wave amplification factor computed using the SWAN simulations at points $P1$ to $P3$ with that obtained using this theory. The Figure shows that the theoretical value tends to overestimate wave amplification under conditions where the small amplitude assumptions break down, due to either a very strong current velocity or a large wave height.

To represent the interaction of waves and currents at a specific location in a format which is helpful for both technology developers and project de-

Figure 11: Comparison of wave amplification factor computed in this study with that obtained by small amplification theory.

Figure 12: Wave amplification diagram: The ratio of the wave height with and without current are plotted against the relative angle between the mean wave direction and the current (solid line) together with a unit circle (dotted line).

314 velopers planning deployments, the use of a Wave Amplification Diagram is
 315 proposed. In such diagrams (Figure 12) the amplification factor, α , is plot-
 316 ted as a function of the angle between the incident current direction and the
 317 mean wave direction, ϕ .

318 When waves are in crossflow, with the current coming from $\pm 90^\circ$, no
 319 amplification is expected and the ratio should be approximately 1. When
 320 the directions of the current and wave propagation are the same (0°) the
 321 wave length should increase and the wave height decrease (Peregrine, 1976)
 322 leading to $\alpha < 1$. When the current and waves are in opposition (180°) the
 323 wave height increases and the wave length shortens, so $\alpha > 1$. For sites where
 324 there is limited wave-current interaction the analysis should show that

$$\alpha(\phi) \approx 1 \quad \forall \phi \in [0, 2\pi].$$

325 Figure 12 shows the amplification factor, computed using small amplitude
 326 wave theory (Appendix A), for a 10s period wave propagating into a uniform
 327 8ms^{-1} current in a depth of 50m. It is worth noting that the combined
 328 effects of bathymetry, wave reflection from coastlines, and concentration of
 329 tidal current by headlands and narrow channels are likely to result in wave
 330 amplification diagrams which do not have symmetry, and where there is less
 331 than 180° between the amplified and reduced regions.

332 For each of the points $P1$ to $P3$ the wave amplification factors have been
 333 computed using the ratio of the significant wave heights using (5) from the
 334 SWAN simulations. These datasets can then be investigated using circular
 335 statistical techniques (Fisher, 1993). The analysis presented has been per-
 336 formed using the *circular* statistics tool pack (Agostinelli and Lund, 2011)
 337 for the R statistical package (R Development Core Team, 2008). For each of

Table 3: Circular mean direction, $\bar{\theta}$, circular standard deviation, ν , and circular resultant magnitude, $\bar{R}$, for the relative wave-current directions for the three locations $P1$, $P2$ and $P3$ from the SWAN simulation.

point	N	$\bar{\theta}$	$\bar{R}$	ν
P1	818	313°	0.131	115°
P2	818	209°	0.341	85°
P3	818	216°	0.260	94°

the observations the relative angle between the waves and the current, θ_i , is treated as a complex unit vector, $z = \cos \theta + i \sin \theta$, and the resultant found,

$$\bar{\rho} = \frac{1}{N} \sum_{i=1}^N z_i. \quad (7)$$

The mean direction (in Radians) is given by, $\bar{\theta} = \arg \bar{\rho}$, the mean resultant length, $\bar{R}$, associated with the mean direction is defined as $\bar{R} = |\bar{\rho}|$, and the sample circular standard deviation (also in Radians) is $\nu = \sqrt{-2 \ln \bar{R}}$. These descriptive statistics (with the angles in degrees) are given in Table 3.

Because the mean direction and sample standard deviation are primarily only of use for observations drawn from a single distribution it is important to check if the data is multi-modal. Figure 13 shows circular dot-plots for the three data points with the kernel density estimate (Bai et al., 1988) shown on the same diagram. In the dot-plots the observations are discretised into 120 bins, and for each bin a single dot is plotted for each observation, with subsequent dots in the same bin stacked. In the same way that we expect continuous uni-modal random data to be drawn from the Gaussian

352 distribution, uni-modal circular data is normally modelled by the circular von
 353 Mises distribution (Fisher, 1993). In all three cases the plots show the data
 354 is multi-modal with one mode located near 320° ; for P1 the second mode is
 355 located near 140° while for P2 and P3 the second mode is located near 180° .
 356 In no cases is the data unimodal and the Rayleigh test for goodness of fit
 357 shows that all three data sets are significantly different from the von Mises
 358 distribution at the 5% level. Under these circumstances the circular mean
 359 and circular sample standard deviation will not provide good estimates of
 360 the centre and spread of the data.

361 To construct the Wave Amplification Diagrams for the three locations,
 362 quantile regression (Koenker and Hallock, 2001) has been used. Quantile
 363 regression has a number of advantages over the standard method of least
 364 squares in that as well as providing a more robust method in the presence of
 365 outliers, it enables fits to be made to specific quantiles of the data. In the
 366 present case curves are fitted to the 75^{th} , 50^{th} and 25^{th} quantiles – these are
 367 the upper-quartile, median, and lower-quartile of the data sets respectively.
 368 Linear quantile regression is based on minimising

$$\sum_{i=1}^n f(y_i - (\alpha_0 + \alpha_1 x_i))$$

369 where

$$f(y - q) = \begin{cases} \beta(y - q) & y \geq q \\ (1 - \beta)(q - y) & y < q \end{cases}$$

370 to obtain the β^{th} quantile. Cubic B-splines (with 15 degree of freedom
 371 smoothing) have been fitted using the *quantreg* tool pack for R (Koenker,

Figure 13: Circular dot-plots of the relative wave-current direction (bar) with the kernel density estimate (line) for P1, P2 and P3.

2012). In the analysis of circular data, techniques are available for Linear-Circular, Circular-Linear or Circular-Circular regression (see Fisher, 1993). In the present case, the intention is to predict the response variable as a function of angle – Linear-Circular. This case the simplest, where by unwrapping the circular prediction variable, normal regression methods can be applied. One caveat associated with unwrapping in this way is that the fitted curve may be discontinuous at the angle where the "cut" has been made. In the present analysis this approach has been used as there is not currently a package available for circular quantile regression. The observed data has been unwrapped at 265° and discontinuities can be observed in the median and upper quartile curves at this location.

Figure 14 shows a polar scatter plot of the individual observations at each of the three points with the regressed median line in black and the upper and lower quartiles as red dashed lines. The region between the upper and lower quartile lines represents the middle 50% of the observed data for each angle. Using a non-parametric analysis it can be concluded that where this range does not include the unit circle there is significant evidence to show that the wave amplification factor is different from unity.

At *P1* this analysis shows that there is a small but significant reduction in wave amplitude for wave-current angles between 270° and 90° , and a significant increase in wave height between 140° and 180° . This site (Figure 15) is characterised by low current speeds (with a maximum current speed of just over 0.5ms^{-1}) and a unimodal wave direction distribution with a median of 0° . These conditions lead to mild wave-current interactions which are consistent with those predicted by the low amplitude theory.

Figure 14: Wave amplification diagrams for $P1$, $P2$ and $P3$. Gray symbols show the individual wave amplification factors, the solid line is the median, and the dashed lines indicate the upper and lower quartiles.

Figure 15: tidal ellipses of M2, S2, K1 and O1 constituents at the spring tide plotted every one hour (left) and circular dot-plots with kernel density estimates of the wave direction (right) at P1, P2 and P3. The angles represent the directions of waves and currents measured counterclockwise from east.

397 In contrast, $P2$, (see Figure 15) has a strong tidal current of around 3ms^{-1}
398 with the mean wave direction coinciding with the major axis of the tidal el-
399 lipse ($\bar{\theta} = 354^\circ, \nu = 38^\circ$). The wave amplification diagram (Figure 14) shows
400 a small but significant reduction in amplitude between 320° and 40° and a
401 very large amplification between 160° and 200° . When the current is in exact
402 opposition to the waves the median amplification factor is almost 2. Site $P2$
403 is thus characterised as a site with very large wave-current interactions.

404 The mean wave direction at $P3$ ($\bar{\theta} = 353^\circ, \nu = 43^\circ$) also coincides with
405 the tidal flow directions, although the maximum tidal velocity of 2ms^{-1} is less
406 than that at $P2$ (Figure 15). This site shows a significant increase in wave
407 amplitude between 160° and 200° , with maximum median amplification fac-
408 tor of 1.5 (Figure 14). The site shows two regions with a significant reduction
409 in wave amplitude, one between 240° and 330° , and a second between 20° and
410 40° . The reduction in wave amplitude is not significant between 330° and
411 20° probably due to both the scatter and paucity of the data in this region.
412 $P3$ is characterised as a site where the main region of wave height reduction
413 is offset by about 50° (clockwise) from where we would expect — potentially
414 leading to challenges for a wave- or tidal-energy developer utilising the site.

415 6. Conclusion

416 The effects of the tidal currents on the wave energy resources due to
417 the wave-current interactions were investigated through the computations of
418 wave field around the Orkney Islands by comparing the computational results
419 with and without considering the tidal currents. In the present simulation,
420 wave energy increased and decreased $\pm 60\%$ at maximum due to the tidal

421 effects in the Pentland Firth, where a strong tidal current of more than 3 m/s
422 is formed. It should be noted that considerable alteration of wave energy can
423 be caused by wave-current interaction at the locations where strong currents
424 coexist with large waves.

425 The wave amplification diagram was proposed, and the relationship be-
426 tween the wave amplification factor and relative direction of currents on the
427 waves based on the circular analysis with quantile regression was discussed.
428 The basic feature that the waves are amplified by the currents with the rela-
429 tive direction of 180 degrees (opposing to the waves) and attenuated by the
430 currents with the relative direction of 0 degrees (following to the waves) is
431 seen in the diagrams although different characteristics appear in each dia-
432 gram depending on the conditions of the waves, currents and geography at
433 the locations. The wave amplification diagram characterises the effects of
434 the wave-current interaction on the wave fields and can be used for better
435 understanding the characteristics of wave energy sites.

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446 **Appendix A. Linear waves in current**

447 The amplification factor, α , for a deep water wave of period, T , propagat-
 448 ing into water of depth, d , subjected to a uniform current with velocity, U ,
 449 can be calculated as follows (Peregrine, 1976; Peregrine and Thomas, 1979):

$$\alpha = \sqrt{\left(1 + \frac{2k'h'}{\sinh 2k'h'}\right) \left(\frac{1}{k'} - u'\right) + 2u'}, \quad (\text{A.1})$$

450 where

$$u' = \frac{UT}{\lambda}, \quad h' = \frac{d\omega^2}{g}, \quad \text{and} \quad k' = \frac{kg}{\omega^2}. \quad (\text{A.2})$$

451 The wavelength, λ , angular frequency, ω , and wave number, k are defined
 452 using the normal linear wave theory definitions:

$$\lambda = \frac{g}{2\pi} T^2 \tanh\left(\frac{2\pi d}{\lambda}\right), \quad (\text{A.3})$$

453

$$\omega = \frac{2\pi}{T}, \quad (\text{A.4})$$

454 and,

$$\omega^2 = gk \tanh dk. \quad (\text{A.5})$$

455 **References**

456 Agostinelli, C., Lund, U., 2011. R package **circular**: Circular Statistics (ver-
 457 sion 0.4-3). CA: Department of Environmental Sciences, Informatics and
 458 Statistics, Ca' Foscari University, Venice, Italy. UL: Department of Statis-
 459 tics, California Polytechnic State University, San Luis Obispo, California,
 460 USA. URL: <https://r-forge.r-project.org/projects/circular/>.

- 461 Bai, Z.D., Rao, C.R., Zhao, L.C., 1988. Kernel estimators of density function
462 of directional data. *Journal of Multivariate Analysis* 27, 24–39.
- 463 Booij, N., Ris, R., Holthuijsen, L., 1999. A third-generation wave model for
464 coastal regions: 1. Model description and validation. *Journal of Geophys-*
465 *ical Research* 104, 7649–66.
- 466 Brière, C., Abadie, A., Bretel, P., Lang, P., 2007. Assessment of TELEMAC
467 system performances, a hydrodynamic case study of Anglet, France.
468 *Coastal Engineering* 54, 345–56.
- 469 Brito-Melo, A., Huckerby, J. (Eds.), 2010. Annual Report 2010: Implement-
470 ing Agreement on Ocean Energy Systems. OES-IA.
- 471 BVG Associates, 2011. Wave and tidal energy in the Pentland Firth and
472 Orkney waters: How the projects could be built. Tech. Rep. 71431. The
473 Crown Estate; 16 New Burlington Place, London W1S 2HX. United King-
474 dom.
- 475 Chen, F., Dudhia, J., 2001. Coupling an advanced land-surface/hydrology
476 model with the Penn State/NCAR MM5 modeling system. Part I: Model
477 description and implementation. *Mon. Wea. Rev.* 129, 569–85.
- 478 Chou, M.D., Suarez, M.J., 1994. An efficient thermal infrared radiation
479 parameterization for use in general circulation models. NASA Tech. Memo
480 104606, 85–.
- 481 Egbert, G.D., Erofeeva, S.Y., 2002. Efficient inverse modeling of barotropic
482 ocean tides. *J. Atmos. Oceanic Technol.* 19, 183–204.

- 483 Fisher, N., 1993. Statistical Analysis of Circular Data. Cambridge University
484 Press, Cambridge, UK.
- 485 Goda, Y. (Ed.), 1985. Random seas and maritime structures. University of
486 Tokyo Press.
- 487 Hong, S.Y., Lim, J.O.J., 2006. The WRF single-moment 6-class microphysics
488 scheme (WSM6). J. Korean Meteor. Soc. 42, 129–51.
- 489 Hong, S.Y., Noh, Y., Dudhia, J., 2006. A new vertical diffusion package
490 with an explicit treatment of entrainment processes. Mon. Wea. Rev. 134,
491 2318–41.
- 492 IEC, 2011. Marine Energy – Wave, tidal and other water current convert-
493 ers – Part 1: Terminology. Technical Specification 62600-1. International
494 Electrotechnical Commission. Geneva, Switzerland.
- 495 Ingram, D., Smith, G., Bittencourt Ferreira, C., Smith, H. (Eds.), 2011.
496 Protocols for the Equitable Assessment of Marine Energy Converters. The
497 University of Edinburgh, Edinburgh, United Kingdom.
- 498 Kain, J.S., 2004. The Kain-Fritsch convective parameterization: An update.
499 J. Appl. Meteor. 43, 170–81.
- 500 Koenker, R., 2012. R package **quantreg**: Quantile Regression (version 4.81).
501 University of Illinois at Urbana-Champaign, Champaign, Illinois, USA.
502 URL: <http://cran.r-project.org/web/packages/quantreg>.
- 503 Koenker, R., Hallock, F., 2001. Quantile regression. Journal of Economic
504 Perspectives 15, 143–56.

- 505 Lyard, F., Lefevre, F., Letellier, T., Francis, O., 2006. Modelling the global
506 ocean tides: Modern insights from FES2004. *Ocean Dynamics* 56, 394–415.
- 507 Martins, F., Leitao, P., Silva, A., Neves, R., 2001. 3D modelling in the Sado
508 estuary using a new generic vertical discretization approach. *Oceanologica*
509 *Acta*, suppl. S 24, S51–62.
- 510 Mlawer, E.J., Taubman, S.J., Brown, P.D., Iacono, M.J., Clough, S.A., 1997.
511 Radiative transfer for inhomogeneous atmosphere: RRTM, a validated
512 correlated-k model for the longwave. *J. Geophys. Res.* 102, 16663–82.
- 513 Peregrine, D., 1976. Interaction of water waves and currents. *Advances in*
514 *Applied Mechanics* 16, 9–117.
- 515 Peregrine, D., Thomas, G., 1979. Finite-amplitude deep-water waves on
516 current. *Philosophical Transactions of the Royal Society of London. Series*
517 *A, Mathematical and Physical Sciences* 292, 371–90.
- 518 R Development Core Team, 2008. *R: A Language and Environment for*
519 *Statistical Computing*. R Foundation for Statistical Computing. Vienna,
520 Austria. URL: <http://www.R-project.org>. ISBN 3-900051-07-0.
- 521 Ris, R., Holthuijsen, L., Booij, N., 1999. A third-generation wave model
522 for coastal regions: 2. Verification. *Journal of Geophysical Research* 104,
523 7667–81.
- 524 Sørensen, O.R., Kofoed-Hansen, H., Rugbjerg, M., Sørensen, L., 2004. A
525 third generation spectral wave model using an unstructured finite volume
526 technique, in: Smith, J.M. (Ed.), *Coastal Engineering 2004: Proceedings*
527 *of the 29th International Conference*, World Scientific. pp. 894–906.

- 528 van der Westhuysen, A.J., Zijlema, M., Battjes, J.A., 2007. Nonlinear
529 saturation-based whitecapping dissipation in swan for deep and shallow
530 water. *Coastal Engineering* 54, 151–70.
- 531 Zhang, D.L., Anthes, R.A., 1982. A high-resolution model of the planetary
532 boundary layer-sensitivity tests and comparisons with SESAME-79 data.
533 *J. Appl. Meteor.* 21, 1596–609.