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An Improved Two-Stage Optimization-Based Framework for Unequal-Areas Facility Layout

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Abstract The unequal-areas facility layout problem is concerned with finding the optimal arrangement of a given number of non-overlapping indivisible departments with unequal area requirements within a facility. We present an improved optimization-based framework for efficiently finding competitive solutions for this problem. The framework is based on the combination of two mathematical optimization models. The first model is a nonlinear approximation of the problem that establishes the relative position of the departments within the facility, and the second model is an exact convex optimization formulation of the problem that determines the final layout. Aspect ratio constraints on the departments are taken into account by both models. Our computational results show that the proposed framework is computationally efficient and consistently produces competitive, and often improved, layouts for well-known instances from the literature as well as for new large-scale instances with up to 100 departments.

Keywords Facility Layout; Nonlinear Optimization

1 Introduction

The unequal areas facility layout problem (FLP) is concerned with finding the optimal arrangement of a given number of non-overlapping indivisible departments with unequal area requirements within a facility. The objective is to minimize the total expected cost of flows inside the facility, where the cost incurblack for each pair of departments is taken as the rectilinear distance between the centroids of the departments times the projected flow between them. This flow may reflect different quantities such as transportation costs, the construction of a material-handling system, the costs of laying communication wiring, or simply adjacency preferences. The problem has two sets of constraints: department area requirements, and department location requirements (such as departments not overlapping, lying within the facility, being fixed to a particular location within the facility, or forbidden from specific parts of the facility). We assume that the facility and the departments are all rectangular. Since the height and width of the departments can vary, finding their optimal rectangular optimal shapes is also part of the problem. The ratios height/width and width/height, called aspect ratios, also pose a challenge because departments with low aspect ratios are more desirable in real-world applications, but a constraint on the largest ratio allowed generally makes the problem harder. See [17,9] for some applications related to the FLP, and the surveys [15,33] for overviews of the FLP literature.

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Two types of approaches for finding provably optimal solutions for the FLP have been proposed. Graph-theoretic approaches (see [14]) assume that the desirability of locating each pair of facilities adjacent to each other is known. Mathematical programming formulations have objective functions based on a weighted sum of the centroid-to-centroid distances between departments. Exact mixed integer programming formulations were first proposed in [29,30], and FLPs with up to 12 departments have been solved to global optimality [32,10,11,28,37]. However, approaches for realistically sized problems do not guarantee optimality. These include nonlinear optimization, genetic algorithms, tabu search, simulated annealing, fuzzy logic; see [3,16,20,15,23,26,27] as well as the survey papers mentioned above.

Our contribution is an improved two-stage nonlinear optimization-based framework for efficiently finding high-quality solutions for medium- and large-scale FLP instances. We improve on the results in [16] while following the spirit of the framework originally proposed in [3] as was done in [16] for FLP, and in [5] for multi-floor layout. The framework used here is made of the combination of two nonlinear optimization models. The first model is a relaxation of the layout problem that establishes the relative positions between pairs of departments. The second model then computes a feasible layout satisfying these positions, and because the relative positions of the departments are fixed (as determined by the first model), the second model is a convex optimization problem.

The novelty in this paper, with respect to the approach in [16], is in the formulation of the first model. First, we propose a more precise first-stage model that treats the departments as rectangles instead of approximating them by circles. This allows us to enforce the aspect ratio constraints exactly at the first stage, rather than having to indirectly approximate them as in [16]. Second, we use a simpler objective function than that in [16], and this gives an important improvement in the running time. For example, running the framework in [16] on the classical Armour-Buffa 20-department instance requiblack about 17 s per run; we can perform 20 runs of our framework in the same time on a comparable computer. Combining this new first stage with the convex second stage of [16], we observe that we are usually able to improve on the results of [16]. Moreover, we present results on several sets of medium-size instances from the literature and on new instances with up to 100 departments. We observe that our proposed framework consistently computes competitive solutions in fewer than 100 s.

This paper is organized as follows. Section 1.1 gives the notation used and the exact nonlinear optimization formulation of FLP that is the basis for the proposed framework. Section 2 describes the new first-stage model that we propose. Section 3 explains the second-phase stage, which is essentially the same as in [16]. Section 4 presents our results, and Section 5 concludes the paper.

1.1 Notation

We label the departments $i = 1, ..., N$, where N is the total number of departments. The problem data consists of the area A_i for department i, the nonnegative costs c_{ij} per unit distance between departments i and j (assumed to be symmetric, i.e., $c_{ij} = c_{ji}$), and for each department i, bounds $w_i^{\min}$ and $w_i^{\max}$ on the width, and $h_i^{\min}$ and $h_i^{\max}$ on the height. The variables are the width and height of each department, denoted w_i and h_i respectively for department i, and the position of that department expressed by the coordinates of its center (x_i, y_i). (This model assumes that the center of the facility is located in the origin.) Finally, the parameter $\beta \geq 1$ is set by the user and is an upper bound on the aspect ratios height/width and width/height of the departments.

An exact nonlinear optimization formulation of the FLP is as follows:

$$\min_{x_i,y_i,h_i,w_i} \quad \sum_{1\le i<j\le N} c_{ij}(|x_i-x_j|+|y_i-y_j|), \tag{1}$$

$$\text{s.t.} \quad x_i+\frac{1}{2}w_i \le \frac{1}{2}w_F, \quad \text{and} \quad \frac{1}{2}w_i - x_i \le w_F, \quad \text{for } i=1,\ldots,N, \tag{2}$$

$$y_i+\frac{1}{2}h_i \le \frac{1}{2}h_F, \quad \text{and} \quad h_i - y_i \le h_F, \quad \text{for } i=1,\ldots,N, \tag{3}$$

$$w_ih_i = A_i, \quad \text{for } i=1,\ldots,N, \tag{4}$$

$$\max\left\{\frac{w_i}{h_i},\frac{h_i}{w_i}\right\} \le \beta, \quad \text{for } i=1,\ldots,N, \tag{5}$$

$$w_i^{\min} \le w_i \le w_i^{\max}, \quad \text{for } i=1,\ldots,N, \tag{6}$$

$$h_i^{\min} \le h_i \le h_i^{\max}, \quad \text{for } i=1,\ldots,N, \tag{7}$$

$$|x_i-x_j| \ge \frac{1}{2}(w_i+w_j) \quad \text{or} \quad |y_i-y_j| \ge \frac{1}{2}(h_i+h_j), \quad \text{for all } 1\le i<j\le N. \tag{8}$$

This formulation minimizes the total cost of the flow between departments. Constraints (2)–(3) ensure that departments are located inside the facility, and constraints (4) enforce the area requirement of each department. Constraints (5) guarantee that for every department, the aspect ratio is no more than β. Constraints (6)–(7) are width and height upper and lower bounds for each department. Finally, constraints (8) prevent the overlapping of departments.

Two aspects of the FLP make it difficult: a) the constraints (8) have a disjunctive nature; and b) in the aspect ratio constraints (5) small values of β are usually desirable, but as β approaches 1, the department shapes are closer to squares, which is a significant restriction. Indeed, for some instances there may not exist any layout at all where all the departments are square. Moreover, as β approaches 1, the relative position given by the first-stage model often generates an infeasible second-stage model. Our first stage addresses both of these aspects.

2 New First-Stage Model

The objective of the first stage is to obtain a good estimate of the desiblack relative position of the departments. To achieve this, we relax constraints (8) and penalize overlapping in the objective function via a penalty function. Specifically, we use the squablack Euclidean distance between departments i and j

$$D_{ij}^2 = (x_i-x_j)^2+(y_i-y_j)^2$$

and the penalty term $f(D_{ij}^2)$, where $f(t)=\frac{1}{t}-1,\ t>0$. The function f has three barrier-like properties:

$$f \text{ is convex in } t,\ \lim_{t\to 0^+} f(t)=+\infty, \text{ and } \lim_{t\to+\infty} f(t)=-1.$$

Thus, $f(D_{ij}^2)\to+\infty$ as the centers of departments start to overlap. This barrier was originally used for VLSI floorplanning in [13], further tested in that context in [2], and used in convexified form in [3] for FLP and [24] for floorplanning. However, the current work is the first time that this barrier has been used without convexification or other modifications in a two-stage framework for the FLP.

The second novel aspect of our first stage is the use of rectangles, instead of circles, to represent the departments. This exact representation allows us to enforce the aspect ratio constraints exactly in the first stage, rather than ignoring them (as in [3]) or indirectly approximating them (as in [16]). This is the main motivation for using rectangles in the first stage, instead of approximating the departments using circles. Specifically, we define the (squablack) target distance between departments i and j in terms of their heights and widths:

$$T_{ij}^2 = \frac{1}{4}\left((w_i+w_j)^2+(h_i+h_j)^2\right),$$

see Fig. 1, and we penalize overlap using the scaled squablack Euclidean distance:

$$f\left(\frac{D_{ij}^2}{T_{ij}^2}\right). \tag{9}$$

The purpose of the target distance T_{ij} is not to specify precisely the distance between the departments. Rather $D_{ij}/T_{ij} \approx 1$ indicates that some of the borders of the rectangles are close, whether or not the rectangles are overlapping by a small amount. We opt for a simple choice of T_{ij} that is not precise, because an exact measure of the overlap would require a more complicated formula.

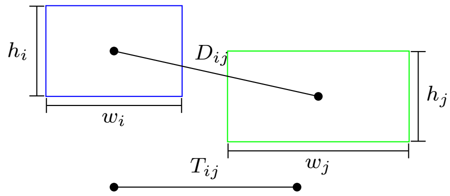

Fig. 1 Target distance for departments i and j

The new first-stage model is thus:

$$
\begin{aligned}
\min_{x_i,y_i,h_i,w_i} \quad & \sum_{1\le i<j\le N} \left(c_{ij} D_{ij}^2 + K \frac{T_{ij}^2}{D_{ij}^2} - 1 \right) && (10)\\
\text{s.t.} \quad & x_i + \frac{1}{2} w_i \le \frac{1}{2} w_F, \quad \text{and} \quad \frac{1}{2} w_i - x_i \le w_F, \quad \text{for } i = 1,\dots,N, && (11)\\
& y_i + \frac{1}{2} h_i \le \frac{1}{2} h_F, \quad \text{and} \quad h_i - y_i \le h_F, \quad \text{for } i = 1,\dots,N, && (12)\\
& w_i h_i \ge A_i, \quad \text{for } i = 1,\dots,N, && (13)\\
& \beta w_i - h_i \ge 0, \quad \text{for } i = 1,\dots,N, && (14)\\
& \beta h_i - w_i \ge 0, \quad \text{for } i = 1,\dots,N, && (15)\\
& w_i^{\min} \le w_i \le w_i^{\max}, \quad \text{for } i = 1,\dots,N, && (16)\\
& h_i^{\min} \le h_i \le h_i^{\max}, \quad \text{for } i = 1,\dots,N. && (17)
\end{aligned}
$$

The objective function combines the total connectivity cost between departments, and the penalty term (9) for overlap. To appropriately balance these terms, we scale the penalty term by the parameter:

$$K = \alpha \sum_{1\le i<j\le N} c_{ij}, \quad \text{with} \quad 0 < \alpha \le 1. \tag{18}$$

Unlike the first-stage models in [3] and [16], this objective function is not convex; nevertheless we can compute local optimal solutions and if we appropriately tradeoff the connectivity cost with the overlap penalty, these solutions will give information about relative positions between departments that are passed on to the second stage via the constraints (29), and contribute to the quality of the layouts computed by the second stage model.

Constraints (11)–(12) are the same as (2)–(3). Constraints (13) are the convexified version of constraints (4). Constraints (5) are equivalent to (15) and (14) because

$$\max\left\{ \frac{w_i}{h_i}, \frac{h_i}{w_i} \right\} \le \beta \quad \Leftrightarrow \quad \beta \ge \frac{w_i}{h_i} \quad \text{and} \quad \beta \ge \frac{h_i}{w_i}.$$

Figure 2 shows a typical local optimum for the first-stage model applied to the well-known Ba12 benchmark problem. Note that because this is the first stage and our objective is not to find a feasible layout, but rather information about the relative positions of departments, we allow departments to possibly overlap, and also to possibly be partially (at most 10% of their width or height) outside the facility. Strict non-overlap and containment in the facility are enforced in the second stage (Section 3).

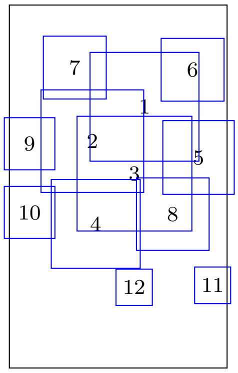

Fig. 2 Example of First-Stage Optimal Solution for Ba12

3 Second-Stage Model

The second-stage model produces a feasible layout satisfying the conditions described in Section 1.1. This convex optimization model can be efficiently solved and provides a unique optimal layout:

$$
\begin{aligned}
\min_{u_{ij},v_{ij},x_i,y_i,h_i,w_i} \quad & \sum_{1\le i<j\le N} c_{ij}(u_{ij}+v_{ij}), && (19)\\
\text{s.t.} \quad & u_{ij} \ge x_j - x_i, \quad \text{and} \quad u_{ij} \ge x_i - x_j, \quad \text{for all } 1 \le i < j \le N, && (20)\\
& v_{ij} \ge y_j - y_i, \quad \text{and} \quad y_{ij} \ge y_i - y_j, \quad \text{for all } 1 \le i < j \le N, && (21)\\
& x_i + \frac{1}{2}w_i \le \frac{1}{2}w_F, \quad \text{and} \quad \frac{1}{2}w_i - x_i \le w_F, \quad \text{for } i = 1, \dots, N, && (22)\\
& y_i + \frac{1}{2}h_i \le \frac{1}{2}h_F, \quad \text{and} \quad h_i - y_i \le h_F, \quad \text{for } i = 1 \dots N, && (23)\\
& w_i h_i \ge A_i, \quad \text{for } i = 1, \dots, N, && (24)\\
& \beta w_i - h_i \ge 0, \quad \text{for } i = 1, \dots, N, && (25)\\
& \beta h_i - w_i \ge 0, \quad \text{for } i = 1, \dots, N. && (26)\\
& w_i^{\min} \le w_i \le w_i^{\max}, \quad \text{for } i = 1, \dots, N, && (27)\\
& h_i^{\min} \le h_i \le h_i^{\max}, \quad \text{for } i = 1, \dots, N. && (28)\\
& \text{No-overlap constraints for all } 1 \le i < j \le N. && (29)
\end{aligned}
$$

Some comments about this model are in order. First, the variables u_{ij}, v_{ij} together with constraints (20) and (21) arise from using a standard technique to linearize the rectilinear distance. Because the costs c_{ij} are non-negative, the minimum is attained at $u_{ij} = x_i - x_j$ or $u_{ij} = x_j - x_i$, and similarly for variable v_{ij} and the difference in the y coordinates.

Second, constraints (29) are selected by applying the following rule to the first-stage solution. For each pair of departments i, j, let $\Delta x = |x_i - x_j|$ and $\Delta y = |y_i - y_j|$ according to the result of the first-stage. For example, let us assume that, in the first-stage solution, i is to the left of j, i.e., $x_i < x_j$, and i is above j, i.e., $y_i > y_j$. If $\Delta x > \Delta y$ then we add a constraint that separates these two departments horizontally (see Fig. 3) by adding the no-overlap constraint $x_j - x_i \ge \frac{1}{2}(w_i + w_j)$. If $\Delta x \le \Delta y$ then we separate the departments vertically by adding the constraint $y_i - y_j \ge \frac{1}{2}(h_i + h_j)$.

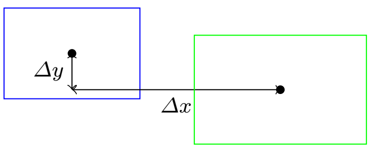

Fig. 3 Determining the Relative Position of Two Departments

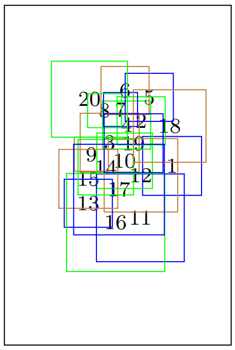

Fig. 4 AB20 first-stage layout with $\alpha = 0.01$

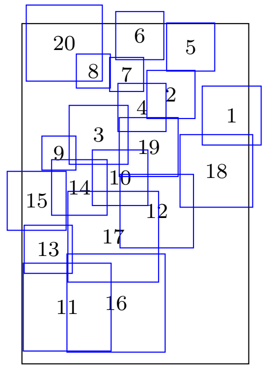

Fig. 5 AB20 first-stage layout with $\alpha = 0.2$

4 Computational Results

We test the proposed framework using the nonlinear optimization solver SNOPT 7.2-8 accessed via the modeling language AMPL for the first-stage model and the solver CPLEX 12.5.1.0 also accessed via AMPL for the second-stage model. The tests were performed on a dual core Intel(R) Xeon(R) X5675 @ 3.07 GHz with 8 Gb of memory. We provided a initial solution for the measure of the approximating rectangles: $w_i = 2\sqrt{A_i\beta}$ and $h_i = 2\sqrt{A_i/\beta}$. The centroids of the N rectangles (x_i, y_i) were initially placed at regular intervals around a circle, as $x_i = 100\cos(2\pi(i-1)/N)$ and $y_i = 100\sin(2\pi(i-1)/N)$.

4.1 Choosing the Value of the Parameter α

We first discuss how to choose the value of α, which plays an important role in the quality of the solutions. If $\alpha = 0$ then the centers of the departments will overlap, and their relative positions will not be clear. If α is too large, the departments will be pushed against the walls of the facility and the relative positions will be distorted. We need to choose α between a value for which we start to distinguish the department's centroids (Fig. 4) and a value for which the departments approach the boundaries of the facility (Fig. 5).

Our experiments showed that an exhaustive search of the selected range of α is not necessary. For example, for the well-known Armour-Buffa 20-department (AB20) instance with aspect ratio 5, running our framework for 20 choices of α chosen at random uniformly in $[0.01, 2.0]$ yields a best-layout with a cost of 3016.3 in a running time of 14.6 s (the best solution known to date has a cost of 3009 [16]). For 2000 values of α, we obtained a cost of 2806.4 in 1411 s, corresponding to a improvement of 7.0% for an increase in running time of 2 orders of magnitude. The results for intermediate numbers of α-values are given in Table 1.

Number of α-values	Best Cost	Time	Line-by-line Improvement	Cumulative Improvement
20	3016.3	16.4 s	—	—
100	2939.0	73.4 s	2.6%	2.6%
500	2858.5	370.2 s	2.7%	5.2%
1000	2858.5	759.0 s	0%	5.2%
1500	2829.2	1074.5 s	1.0%	6.2%
2000	2806.4	1411.3 s	0.8%	7.0%

Table 1 Results for AB20 with varying numbers of α-values

Based on these results, we tended to choose small number of α-values for our experiments, and always obtained competitive, and often improved, results as reported in Sections 4.3 and 4.4.

4.2 Test Instances

We compare our results with those from Jankovits et al. [16] (referblack to as JLAV) when available, and also with the best results in the literature. Table 2 lists the instances used. These include classical benchmarks, and also the recently published SC30 and SC35 from a real-world problem [26]. Instances JLAV-30A to JLAV-30E are instances with 30 departments first published in [16]. The AnVi-xx instances are newly generated for this paper; they can be downloaded at `http://www.miguelanjos.com/flplib`. The empty space is defined as the difference between the available area (Height $\times$ Width) and the sum of the department areas, as a percentage of the available area. The flow density is defined as the percentage of the pairs of departments with positive flow interactions with respect to the number of possible pairs.

Problem name	Number of depts	Height	Width	Empty space	Flow density	Source
O9	9	13	12	0%	41.7%	[7]
O10	10	13	12.7	0.06%	35.6%	[7]
vC10	10	51	25	0.0%	26.7%	[36]
FO10	10	13	12.7	0.06%	20.0%	[28]
BM12	12	8	6	33.3%	25.8%	[7]
Ba12	12	10	6	18.3%	89.4%	[4]
Ba14	14	9	7	3.2%	62.6%	[4]
AB20	20	30	20	0.0%	64.7%	[1]
Tam30	30	45	40	11.1%	67.4%	[31, 35]
JLAV-30A	30	14	13	0.0%	75.9%	[16]
JLAV-30B	30	20	10	0.0%	72.4%	[16]
JLAV-30C	30	25	14	0.0%	77.2%	[16]
JLAV-30D	30	16	16	6.3%	74.3%	[16]
JLAV-30E	30	20	12	5.0%	78.2%	[16]
SC30	30	15	12	9.4%	11.5%	[26]
SC35	35	16	15	20.0%	9.1%	[26]
AnVi-50	50	21	18	1.6%	29.6%	This paper
AnVi-70	70	27	20	1.9%	29.1%	This paper
AnVi-80	80	26	22	0.3%	30.3%	This paper
AnVi-100	100	31	25	4.3%	31.3%	This paper

Table 2 Test Instances and their Properties

4.3 Results for Smaller Instances

In Table 3, we report our best results on the instances with fewer than 20 departments. We set a maximum aspect ratio 3 for instance BM12, as done in [16], but do not add 4% of empty space, as done in [26]. For instances Ba12 and Ba14 we used a minimum side length of 1, except for department number 14 for which the minimum is 0 (i.e., no shape restriction), as done in [23].

We also report the best results from [16] and from elsewhere in the literature. These instances have been extensively studied, and the global optimum is known for most of them. We observe that we consistently improve on the results in [16]. The last column in Table 3 shows the cost difference (as a percentage) between our solutions and the best solutions known. The results known for these instances are better than ours, however their methods are computationally more expensive. The computational time for our approach varies from 53 s for the nine departments instance to 645 s for the 14 departments instance, for 1000 different α values.

4.4 Results for Medium and Large Instances

Table 4 presents detailed results for the AB20 instance. This is a well-known and extensively studied instance known to be difficult, in part because it has no empty space. We are able to improve on the previous best-known results for every aspect ratio except 6. The improvement is sometimes large: 25.7% for an aspect ratio of 10 and 46.3% for an aspect ratio of 3. The computational time never exceeded 1250 s for 1000 α values tested. Figure 6 shows the best layout obtained with aspect ratio 3.

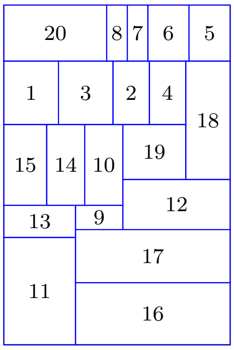

Fig. 6 Best AB20 solution with aspect ratio 3

Table 5 compares our results for SC30 and SC35 with those for SEQUENCE [26] and GA/LP [23]. (There are no JLAV results for these instances.) SEQUENCE obtains better solutions on average, but it takes about 24 h of computational time for an aspect ratio of 5. However, with less computational time, for instance SC35 with aspect ratio 3, our framework obtained a better solution. GA/LP [23] obtained the best values 3371.0 and 3385.5, respectively for SC30 and SC35, with aspect ratio 5. The running times for GA/LP are around 6 h and 8 h respectively for these instances, while we spent less than 1600 s and 2500 s, respectively, for 1000 α values tested. Here, we could even improve on the SEQUENCE results for SC35 by 0.6% and 7.7% for aspect ratios of 2 and 3 respectively.

Table 6 gives results for instance Tam30 with aspect ratios from 2 to 10. We improved on the previous best results for all aspect ratios except 5, for which the best result was obtained by GA/LP [23]. Table 7 gives results for Tam30 with 3 fixed departments. To the best of our knowledge, this case has been tested in a comparable manner only by JLAV. We improve on the JLAV results for the aspect ratios presented in Table 7. We obtained a significant cost blackuction with only 10 choices of α, taking less than 60 s. Table 8 gives the results for the JLAV instances with various aspect ratios. Our approach consistently improved on theirs, often significantly, and requiblack less time: only around 30 s and 10 choices of α.

The last set of tests use 10 values of α for each of the instances with between 50 and 100 departments; see Table 9. These results show that our technique can compute layouts efficiently even for very large instances, as the running times were 70 s, 150 s, 230 s, and 760 s, respectively, for AnVi-50, AnVi-70, AnVi-80, and AnVi-100.

Problem	JLAV [16]	Best Known Solution	Our Approach	Cost Difference
O9	298.3	235.95 [28]	259.0	9.8%
vC10	29 193.3	19 350.46 [8]	24 746.7	27.9 %
O10	320.07	238.27 [28]	262.9	10.3%
FO10	35.71	29.41 [28]	31.8	8.1%
BM12	185.65	142 [26]	164.1	15.6%
Ba12	12 222.3	8021 [23]	10 826.4	35.0%
Ba14	7 416.0	4 686.8 [23]	6 186.4	32.0%

Table 3 Results for small instances

Aspect Ratio	JLAV	GA/LP [23]	Our Approach	Cost blackuction
10	—	3 758.7	2 793.7	25.7%
9	—	—	2 869.1	—
8	3 014.2	—	2 842.6	5.7%
7	2 979.3	4 718.8	2 829.3	5.0%
6	2 708.0	—	2 781.3	-2.7 %
5	3 009.0	5 023.7	2 858.5	5.0 %
4	2 960.5	5 196.3	2 919.1	1.4%
3	—	5 400.0	2 899.8	46.3%

Table 4 Results for AB20 instance

Problem	Aspect ratio	SEQUENCE [26]	GA/LP [23]	Our approach	Cost blackuction
SC30	5	3 706.83	3 370.98	4 342.8	-28.8%
	4	4 165.83	——	4 363.2	-4.7%
	3	4 332.87	——	4 564.2	-5.3%
	2	4 790.43	——	5 413.7	-13.0 %
SC35	5	3 247.48	——	3 655.6	-12.6%
	4	3 604.12	3 385.48	3 770.6	-11.4%
	3	4 332.87	——	3 999.0	7.7%
	2	4 839.45	——	4 808.6	0.6%

Table 5 Aspect ratio sensitivity analysis for SC30 and SC35

Aspect ratio	Kim et al.[18]	JLAV	GA/LP [23]	Our approach	Cost blackuction
10	—	24 098	—	20 489.8	15.0%
9	—	23 924	—	20 391.8	14.8 %
8	—	23 420	—	20 514.1	12.4 %
7	—	23 974	—	20 505.0	14.5 %
6	—	23 770	—	20 528.6	13.6 %
5	—	24 916	19 009.90	20 523.8	- 8.0 %
4	—	25 000	—	20 658.9	17.4 %
3	—	—	—	20 751.6	—
2	21 560.6	—	—	20 745.2	3.8%

Table 6 Results for Tam30

Aspect ratio	JLAV	Our approach	Cost blackuction
10	24 017	21 293.4	12.8 %
9	24 477	21 965.7	11.4 %
8	23 994	21 328.6	12.5%
7	24 200	21 124.6	15.6%

Table 7 Results for Tam30 with 3 fixed departments

5 Conclusion

We have presented an improved optimization-based framework for efficiently finding competitive solutions for unequal-area facility layout. It combines two nonlinear optimization models: the first establishes the relative position of the departments within the facility, and the second determines the final layout. Both models include aspect-ratio constraints on the departments. Our results show that our framework is computationally efficient. It consistently produces competitive, and often improved, layouts for well-known instances from the literature as well as for new instances with up to 100 departments.

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	JLAV-30A			JLAV-30B		
Aspect ratio	[16]	Our approach	Cost blackuction	[16]	Our approach	Cost blackuction
10	9445	8699.8	7.9%	10511	9539.6	9.2%
9	9591	8510.1	11.3%	10532	9771.2	7.2%
8	9312	8401.4	9.8%	10506	9762,5.6	7.1%
7	9320	8471.7	9.1%	10414	9768.1	6.2%
6	9504	8733.6	8.1%	10604	9671.0	8.8%
5	9544	8577.4	10.1%	10424	9930.5	4.7%
4	9509	8770.2	7.8%	10199	9843.9	3.5%

	JLAV-30C			JLAV-30D		
Aspect Ratio	[16]	Our approach	Cost blackuction	[16]	Our approach	Cost blackuction
10	16 079	14 052.6	12.6%	11 358	10 216.5	10.1%
9	16 142	14 135.1	12.4%	11 228	10 154.6	9.6%
8	15 904	14 564.7	8.4%	10 937	10 089.2	7.8%
7	15 961	13 869.3	13.1%	10 903	10 063.3	7.7%
6	15 748	14 343.3	8.9%	11 054	10 004.0	9.5%
5	15 759	14 703.2	6.7%	11 014	10 265.4	6.8%
4	15 438	14 594.0	5.5%	11 055	10 284.8	7.0%

	JLAV-30E		
Aspect ratio	[16]	Our approach	Cost blackuction
10	12 444	11 846.0	4.8%
9	13 118	11 632,3	11.3%
8	13 160	11 739.1	10.8%
7	12 871	11 802.0	8.3%
6	12 642	11 722.3	7.3%
5	12 815	11 650.3	9.1%
4	12 465	11 837.3	5.0%

Table 8 Comparison of results for JLAV-30A to JLAV-30E

Aspect ratio	AnVi-50	AnVi-70	AnVi-80	AnVi-100
6	17714.2	42902.4	63717.4	117493.5
5	17727.0	43432.1	63744.1	117791.9
4	17927.4	42927.5	64509.2	117253.6

Table 9 Best layouts computed for new large instances

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