Responses of soil-mantled hillslopes to transient channel incision rates

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[1] Channel incision drives hillslope morphology in humid soil-mantled landscapes. When channel incision rates change, numerous hillslope soil properties (e.g., erosion rates, soil thickness, and soil production) adjust in response to this change. Here we investigate the timescales of adjustment of hillslope soil properties when channel incision rates change in time. An idealized soil-mantled hillslope (linear sediment flux law and soil density equal to bedrock density) is investigated, and the transient evolution of this hillslope is determined analytically. This analysis reveals two dominant adjustment timescales. The longer of these two timescales determines the rate at which the entire hillslope adjusts to a change in channel incision rates and is proportional to \(4\lambda^2/(\pi^2 D)\), where \(\lambda\) is the length of the hillslope and \(D\) is the sediment diffusivity. Numerical simulations are then used to examine responses of hillslopes that involve nonlinear behavior (e.g., hillslopes that experience nonlinear sediment transport or have soil thicknesses that respond to the soil production function). Using these numerical models, we show that the ratio between the soil density \((\rho_s)\) and the density of the soil parent material \((\rho_0)\) can alter the long-term response of the hillslope such that the characteristic timescale becomes \(4\rho_s\lambda^2/(\pi^2 D\rho_0)\). In addition, we show that the adjustments of the soil erosion rate, the soil production rate, and the soil thickness have different characteristic response timescales. Hillslopes that experience sediment flux that is proportional to the depth slope product respond on longer timescales than hillslopes that experience a linear sediment flux law when channel incision rates increase. We illustrate how the spatial pattern of hillslope response to changes in channel incision rates can be used to constrain either channel incision histories or hillslope response timescales. If the hillslope response timescale is known, the pattern of hillslope disturbance can be used to constrain the celerity of an incision wave as it moves upstream through a channel. If the channel incision history is known, the hillslope response timescale may be evaluated on the basis of the spatial pattern of hillslope disturbance.


1. Introduction

[2] The evolution of soil-mantled landscapes is driven, in part, by channels at the base of hillslopes that erode through the landscape and remove the sediment that is delivered to them by the hillslopes. The relationship between channel erosion rates and hillslope evolution has been a focus of geomorphic research for over a century [e.g., Davis, 1899; Gilbert, 1877; Hack, 1960; King, 1953; Penck, 1972]. Modern understanding of the response of hillslope soils to changes in channel incision rates has followed the work of Culling [1960], who first applied a rigorous statement of mass conservation to an eroding hillslope soil.

[3] Gilbert [1909] suggested that the erosion rate of a hillslope soil is proportional to the topographic gradient, an observation that has been confirmed by a number of studies [e.g., Dietrich et al., 2003, and references therein]. When channel incision rates change, several properties of the soil adjust in response. If, for example, the channel incision rate increases, the hillslope responds by steepening near the channel. As a result, soil thins near the channel. The steepening of the hillslope near the channel leads to removal of sediment upslope, thus removing soil at that location. In this way changes in channel incision rates result in changes in soil properties that propagate away from the channel [e.g., Furbish and Fagherazzi, 2001; Mudd and Furbish, 2005; Roering et al., 2001].

[4] The adjustments of the soil to changes in channel incision rates exhibit characteristic timescales [e.g., Ahnert, 1987; Fernandes and Dietrich, 1997; Furbish and Fagherazzi, 2001; Jyotsna and Haff, 1997; Roering et al., 2001]. These timescales include the time it takes the surface topography of the hillslope to adjust to a new channel incision rate (the hillslope relaxation time [e.g., Ahnert,
Figure 1. Schematic of the soil profile, where \( v \) is the depth averaged bulk velocity of the sediment in the \( x \) direction, \( S_v \) is the mass loss rate per unit volume due to chemical weathering, \( h \) is the soil thickness, \( \eta \) is the elevation of the soil-bedrock boundary above an arbitrary datum, \( \zeta \) is the elevation of the soil surface above an arbitrary datum, \( \rho_n (L \, T^{-1}) \) is the rate of bedrock lowering due to soil production, and the overbars denote depth-averaged quantities.

1987], and the time needed for a perturbation in soil depth to propagate a specified distance upslope [e.g., Furbish and Fagherazzi, 2001].

[5] It is of fundamental importance to quantify these timescales for studies of both hillslope and fluvial geomorphology for two reasons. First, the timescale of transient hillslope response to changes in the channel incision rate provides an estimate of the period over which the resulting changes in soil properties are likely to persist on the hillslope, and thus is an estimate of the time a hillslope soil can record changes in channel incision rates. Second, the adjustment timescale of a hillslope soil responding to a change in channel incision rate must be known in order to evaluate whether it is appropriate to assume that a landscape has achieved a steady state or equilibrium condition. An equilibrium condition may be defined as the condition achieved when hillslope erosion rates have adjusted to match channel incision rates such that the surface topography of the landscape does not change in time and the landscape is lowering uniformly relative to base level [e.g., Carson and Kirkby, 1972; Gilbert, 1909; Hack, 1960; Howard, 1988].

[6] Here we extend the work of Fernandes and Dietrich [1997] and Roering et al. [2001], who examined the timescales over which hillslope soils respond to changes in channel incision rates, by including not only the response of hillslope erosion rates and surface topography but also the soil thickness. For sufficiently simple conditions, the equations that govern the evolution of hillslope soils may be solved analytically. We report such solutions and use them to elucidate how hillslope soils respond to changes in channel incision rates. Namely, these solutions allow extraction of two timescales: one that can be used to estimate the adjustment rate of the entire hillslope to changes in channel incision rates and another that can be used to estimate the rate of propagation of the effects of transient channel incision as this signal moves away from the channel. In addition, we use numerical simulations to demonstrate that if channel incision rates are perturbed the response of different hillslope properties, such as the erosion rate, the soil thickness, and the soil production rate, will respond on different characteristic timescales. These different timescales can result in situations where, for example, the erosion rate of the hillslope may be equilibrated to a change in channel incision rate but the soil thickness may still be experiencing transient behavior.

2. Mass Conservation of Eroding Hillslope Soils

[7] We begin by considering soil-mantled landscapes in which the transport of soil due to landsliding and overland flow is insignificant. We define the soil layer as the near surface material that is being mechanically disturbed by processes such as soil creep [e.g., Culling, 1963; Heimsath et al., 2002; Kirkby, 1967; Roering et al., 1999; Young, 1978], animal burrowing and disturbance [e.g., Gabet, 2000; Yoo et al., 2005], frost heave processes [e.g., Anderson, 2002], and tree throw and root growth [e.g., Gabet et al., 2003; Roering et al., 2002]. In the presence of topographic gradients, this mechanically active layer is transported downslope. Underlying this mechanically active soil layer is a mechanically undisturbed saprolite layer. Material is entrained from the inactive saprolite into the active soil layer through soil production [e.g., Carson and Kirkby, 1972; Heimsath et al., 1997]. Once soil is produced, it can either accumulate locally or be removed through either mechanical or chemical denudation processes.

[8] Consider a one dimensional hillslope that has a mantle of soil with a thickness \( h \) (L), a surface elevation \( \zeta (L) \) (Figure 1), and a soil-saprolite boundary that is at an elevation \( \eta (L) \) (Figure 1), such that the soil thickness is equal to:

\[
h = \zeta - \eta. \tag{1}
\]

A depth integrated [e.g., Mudd and Furbish, 2004; Paola and Voller, 2005] statement of mass conservation for the hillslope soil is

\[
\frac{\partial h}{\partial t} = - \frac{\partial (h \nu_x)}{\partial x} + \frac{\partial}{\partial x} \rho_v \nu_x, \tag{2}
\]

where \( \rho_s \) (M L\(^{-2}\)) is the dry bulk density of the soil, \( \nu_x \) (L T\(^{-1}\)) is the bulk velocity of the sediment in the \( x \) direction, \( \rho_v \) (M L\(^{-3}\)) is the density of the parent material, \( p_n (L \, T^{-1}) \) is the rate of bedrock lowering due to soil production, and the overbars denote depth-averaged quantities. All terms in equation (2) represent rates of change in the soil thickness, in units of length per time. The first term on the right of the
equality is the rate of change in the soil thickness due to mechanical sediment transport processes and the last term is the rate of change in the soil thickness due to soil production. Equation (2) assumes that there is no deposition or erosion at the surface of the soil (i.e., all sediment transport occurs as movement within the mechanically active soil layer), no denudation occurs by chemical processes, and that soil bulk density does not vary in space or time.

[3] The lowering of the soil-saprolite boundary is determined by the soil production rate

$$\frac{\partial \eta}{\partial t} = -p_v. \quad (3)$$

Note that by convention, the production rate is defined as positive downward. Combining equations (1)–(3), the change in the surface elevation with respect to time is

$$\frac{\partial \zeta}{\partial t} = -\frac{\partial (h_{\eta} \zeta)}{\partial x} + \left( \frac{\rho_s}{\rho_i} - 1 \right) p_v. \quad (4)$$

3. Timescales of Adjustment to Transient Channel Incision for a Simplified Soil-Mantled Hillslope

[10] A simplified version of equation (4) can be solved analytically in order to examine the basic behavior of soil-mantled landscapes as they respond to changes in channel incision rates. The value of this analysis is that it provides a way to concisely define both short and long timescales that characterize the transient response of a hillslope to a change in the channel incision rate, including how these timescales vary with hillslope position, and which are not readily accessible from direct numerical solutions of the governing equations. In addition, the analytical analysis clarifies key points introduced in the seminal work of Culling [1965], as will be described later in this section. Analytical solutions also provide an important benchmark for understanding numerical solutions and results of others [e.g., Fernandes and Dietrich, 1997; Roering et al., 2001] aimed at defining characteristic timescales of response.

[11] We first assume that the density of the saprolite is the same as the density of the soil ($\rho_s = \rho_i$), and that sediment transport is linearly proportional to slope:

$$h_{\eta} \zeta = -D \frac{\partial \zeta}{\partial x}. \quad (5)$$

where $D$ [L$^2$ T$^{-1}$] is a sediment diffusivity. It is assumed that the diffusivity is not a function of space. With these assumptions, equation (4) reduces to

$$\frac{\partial \zeta}{\partial t} = D \frac{\partial^2 \zeta}{\partial x^2}. \quad (6)$$

Culling [1965] recognized that equation (6) is analogous to the equation describing the conduction of heat in a solid, and that this equation may be solved analytically using the finite Fourier transform method [e.g., Carslaw and Jaeger, 1959; Deen, 1998].

[12] In order to solve equation (6), both initial and boundary conditions must be assigned. We consider a hillslope with a divide at the location $x = 0$ (Figure 2). No sediment passes across the divide such that the divide serves as a no flux boundary condition for the one dimensional hillslope (i.e., $\partial \zeta/\partial x = 0$ at $x = 0$). A channel is at the location $x = \lambda$ (Figure 2). The channel, as it erodes through bedrock (or sediment) and removes soil delivered to it by the hillslope, serves as the second boundary condition.

[13] The initial condition is selected such that the hillslope topography is in equilibrium with an initial channel incision rate of $I_0$ (L$^{-1}$T$^{-1}$); at equilibrium, the entire hillslope is lowering at that rate. The incision then changes to a new rate $I$ (L$^{-1}$T$^{-1}$). For simplicity, we assume that the channel incises in the vertical direction only, and that there are no random fluctuations about the rates $I$ and $I_0$. A coordinate system is then selected such that the channel at $x = \lambda$ is always at $\zeta = 0$ (this could represent a moving coordinate system [Mudd and Furbish, 2005, 2006] or a situation where erosion is matched by uplift). Using these assumptions and initial and boundary conditions, we have derived the solution to equation (6) using the finite Fourier transform method [e.g., Carslaw and Jaeger, 1959; Deen, 1998]:

$$\zeta_{\text{eq}}(x,t) = \frac{16 \lambda^2}{D \pi^3} \sum_{n=0}^{\infty} \frac{(-1)^n}{(1 + 2n)^2} \left[ (I_0 - I) e^{-\frac{\pi^2}{4 \lambda^2} (1 + 2n)^2 t} \right] \cdot \cos \left[ \frac{\pi (1 + 2n)x}{2 \lambda} \right], \quad (7)$$

where the subscript $bl$ indicates that the surface topography is measured relative to the elevation of the channel (Figure 2). Within the infinite series in equation (7) the cosine function is solely a function of position and defines the basic shape of the soil surface. This cosine function is modulated by an exponential function that varies in time; this exponential function defines the transient response of the hillslope soil. Our derivation of equation (7) differs from the derivation
Initially, the hillslope topography reflects a steady state adjustment to a channel incision rate \( I_0 \) of 0.0001 m yr\(^{-1}\). At time \( t = 0 \) the channel incision rate is increased to 0.0002 m yr\(^{-1}\).

Presented by Culling [1965] in that we include an incising channel and our initial hillslope is adjusted to a downcutting rate of \( I_0 \). In contrast, Culling [1965] presented solutions for a hillslope whose channel stayed at a constant elevation such that the hillslope was eroded to a plane, and a hillslope bounded on both sides by channels that incised at a constant rate. The significance of the new derivation is that we are able to investigate the dynamics of hillslope adjustment to changes in channel incision rates rather than the time necessary to achieve a denuded, peneplane-like surface as investigated by Culling [1965].

[14] The local erosion rate of the hillslope soil as a function of time and space can be found by differentiating equation (7):

\[
\frac{\partial \zeta}{\partial t} = -\frac{t}{\pi} (I_0 - I) \sum_{n=0}^{\infty} \frac{(-1)^n}{(1 + 2n)} e^{-\lambda^2 (1 + 2n)^2 t} \cdot \cos \left[ \frac{\pi(1 + 2n) x}{2\lambda} \right].
\]

Equation (8) is measured relative to a fixed datum (i.e., not relative to the incision rate of the channel). An example of a hillslope responding to a change in the rate of channel incision at its base as described by equations (7) and (8) is shown in Figure 3.

[15] Howard [1988] suggested that the adjustment of a hillslope to a step change in the channel incision rate can be approximated by an exponential decay function of the form

\[
F(t) = F_f + (F_i - F_f) e^{-t/\tau},
\]

where \( F \) is a time dependant function, \( F_f \) is the value of the function at its final equilibrium value, \( F_i \) is the initial value of the function, and \( \tau(T) \) is a timescale that characterizes the rate at which the transient signal decays (e.g., if \( t = 3\tau \) the function is within 5% of the final equilibrium value). Both Roering et al. [2001] and Mudd and Furbish [2005] used equation (9) to estimate hillslope response timescales, but as observed by Mudd and Furbish [2005], this approach may not be adequate to fully characterize the response timescales of soil-mantled hillslopes.

[16] Equation (9) may be used to fit values of the timescale of the hillslope response to changes in the channel incision rate \( \tau \) at different points on the hillslope, and this fit may be compared with the precise timescale of response extracted from equation (8). Figure 4 shows a comparison of the precise erosion rate predicted by equation (8) and the erosion rate based on the simple exponential response described by equation (9) as a function of time. Near the channel, the hillslope responds rapidly to the perturbation in the channel incision rate (Figure 4b), whereas at points near the divide (Figure 4a) the perturbation is only felt after some delay (as identified by Roering et al. [2001] in their Figure 8). As time increases, equation (9) can approximate the response of the hillslope, but it does not capture the shorter timescale behavior that causes a fast response near the channel and a delayed response near the divide (Figure 4).

[17] As noted by Culling [1965] and embodied in equation (7) and equation (8), the response of the hillslope can be described by a sum of harmonic functions. Each wave number (i.e., \( n = 0, 1, 2, 3, \ldots \)) in the series of harmonic functions has a different characteristic timescale, analogous to \( \tau \) in equation (9). The timescale for each harmonic function, \( \tau_n(T) \), is

\[
\tau_n = \frac{4\lambda^2}{D\pi^2(1 + 2n)^2}.
\]

As the wave number increases, the characteristic timescale decreases, such that the long-term behavior of the hillslope is dominated by the wave number \( n = 0 \), or in other words, \( \tau_0 = 4\lambda^2/D\pi^2 = 0.4053\lambda^2/D \), which is equivalent to the relaxation timescale found numerically by Roering et al. [2001] for hillslopes whose sediment flux is a linear function of slope. The effect of different wave numbers on the erosion rate of a hillslope whose channel incision rate has been perturbed is plotted in Figure 5.

[18] Because signals propagate away from the channel, the transient response of the hillslope due to the effects of higher wave numbers (i.e., \( n > 0 \)) varies as a function of distance from the channel (Figure 5). Near the channel (Figure 5b), the effect of the harmonics with higher wave number (e.g., \( n > 0 \)) combine to cause the hillslope to respond rapidly to the change in the rate of channel incision; this rapid incision is not captured by equation (9) (see Figure 4b). On the other hand, the effect of the harmonics...
with higher wave numbers is to reduce the erosion rate near the divide (Figure 5a). This relative reduction in the erosion rate due to the harmonic functions with the higher wave numbers, and therefore more rapid response timescales, is what causes the delay in the response of the erosion rate at the divide (Figure 4a).

The relative importance of individual harmonic functions, and therefore the relative importance of different response timescales (i.e., equation 10), may be determined by comparing the coefficients that multiply the time-varying exponential term in equation (8). We define a function:

\[
M_E(n, x) = \frac{(-1)^n}{1 + 2n} \cos \left[ \frac{(1 + 2n) \pi x}{2 \lambda} \right] \sec \left[ \frac{\pi x}{2 \lambda} \right].
\]  

(11)

where \( M_E \) is the ratio of the coefficients that multiply the time varying exponential term of equation (8) for a wave number \( n \) to the coefficient when the wave number, \( n \), is zero. In other words, \( M_E \) is defined by dividing the term within the summation in equation (8) with a wave number \( n \) by the term in the summation with \( n \) set to zero. By definition, \( M_E \) is equal to one when \( n = 0 \). For \( n \neq 0 \), \( M_E \) reflects the proportion of the erosion rate \( (M_E) \) signal that is contributed by the harmonic with wave number \( n \). Because each harmonic function decays over the timescale described by equation (10), \( M_E \) measures the importance of higher harmonics \( (n > 0) \) while \( t \) is less than \( \tau_n \).

The magnitude of the harmonics as a function of position is plotted in Figure 6. Recall from Figure 5 that if a given harmonic takes a negative value, this causes a delayed response in the erosion rate. This corresponds to negative values of \( M_E \), and as depicted in Figure 6 this effect is most pronounced near the divide \( (x = 0) \). Positive values of \( M_E \) indicate that fast moving signals are accelerating the response of the hillslope to the change in channel incision. This occurs in locations near the channel. Near the divide, the harmonic of \( n = 1 \) is a significant fraction of the total response of the hillslope (Figure 6). Because this harmonic is the principal cause of the delay between the time the channel incision rate is perturbed and the time the divide “feels” this signal, the time of delay can be estimated by the timescale of this harmonic. By equation (10), the delay in the response of the divide after the channel incision rate is perturbed is \( \tau_0/9 \). So, for example, in the hillslope depicted in Figures 3, 4, and 5, which has a length of 40 m and a diffusivity of 0.01 m\(^2\) yr\(^{-1}\), \( \tau_0 \) is approximately 65 ka, and \( \tau_1 \) is approximately 7 ka. Examination of Figure 4b reveals that the divide begins responding to the transient incision signal at approximately 7 ka.

4. Timescales of Hillslope Response for Soil Thickness, Soil Production, and Hillslope Erosion Rate

The analysis in sections 2 and 3 focuses on analytical solutions for idealized hillslopes where soil production and soil thickness are not considered. We now extend our analysis to include a perturbed incision rate.
analysis to include these and other factors. Returning to equations (2)–(4) to describe the soil-mantled hillslope, we assume that the soil may have a different dry bulk density than the soil (i.e., \( \rho_s \neq \rho_i \)). In addition, the soil production rate, \( p_v \), is a function of the soil bulk density \( \rho_i \). In several field locations [e.g., Heimsath et al., 1997, 2000, 2001], the soil production rate has been found to be a decreasing function of increasing soil thickness:

\[
p_v = W_0 e^{-\gamma h},
\]

where \( W_0 \) (L T\(^{-1}\)) is the nominal rate of soil production when the soil thickness is zero and \( \gamma \) (L) is a length scale that characterizes the rate of decline in the soil production rate with increasing soil thickness. Others have argued for a soil production function that peaks at some intermediate soil thickness [Ahnert, 1976; Anderson, 2002; Carson and Kirkby, 1972; Furbish and Fagherazzi, 2001; Wilkinson et al., 2005]. Here we limit our analysis to a soil production function that takes the form of equation (12).

[22] Because the soil production as described by equation (12) is a nonlinear function of soil thickness, equations (2) and (4) are best solved numerically. In addition, the relationship between slope and sediment flux may be nonlinear. Examples include sediment flux that increases asymptotically as the hillslope approaches a critical gradient [Andrews and Bucknam, 1987; Roering et al., 1999, 2001, Roering, 2004], sediment flux that varies nonlinearly with slope due to biogenic effects [e.g., Gabet, 2000; Gabet et al., 2003; Yoo et al., 2005], an effective sediment diffusivity that is a nonlinear function of soil thickness [Anderson, 2002], and sediment transport that is a function of the product of the hillslope gradient and the soil thickness [Heimsath et al., 2005].

[23] For example, Roering et al. [2001] examined the adjustment timescale of hillslopes whose sediment flux was a nonlinear function of the hillslope gradient of the form

\[
h \tau_s = -D_s \frac{\partial \chi}{\partial x} \left[ 1 - \left( \frac{1}{S_c} \frac{\partial \chi}{\partial x} \right)^2 \right]^{-1},
\]

where \( S_c \) (dimensionless) is a critical slope. They found that the hillslopes experiencing sediment flux governed by equation (13) had lower values of \( \tau \) (see equation 9) than did hillslopes where sediment transport was a linear function of slope (e.g., equation 5). The response became more rapid (e.g., \( \tau \) decreases) as hillslope gradients increased [Roering et al., 2001]. Here we investigate the timescales of hillslope response as affected by the soil production and sediment flux that is proportional to the product of soil thickness and hillslope gradient.

4.1. Overview of Numerical Simulations

[24] As we have demonstrated in section 2, changes in channel incision rates lead to signals that move away from the channels with different characteristic timescales (equation 10). We now concentrate on the timescale, \( \tau \), that may be extracted using equation (9) as described by Howard [1988] and Roering et al. [2001]. In each of our experiments, the function \( F(t) \) in equation (9) is fit using least squares minimization with three properties of the hillslope soil at the divide: the erosion rate \( (\partial \chi_{in}/\partial t) \), the soil thickness \( (h) \), and the surface topography \( (\zeta) \). We again note that when the time elapsed since the change in channel incision rate equals \( \tau \), the value of the soil property being analyzed (which could be topography, soil thickness, erosion rate, etc.) is within 5% of its final value if the new channel incision rate is steady in time. In our simulations, the most rapid incision rates (either \( I \) or \( I_0 \)) were restricted to values less than \( W_0 \) so that none of the simulated hillslopes experienced zero soil thickness (in other words, the soil production rate could always keep up with the erosion rate).

[25] Two sets of model runs were carried out. The model solves equations (2)–(4) using a finite difference scheme. In the first set of model runs, sediment transport was assumed to be a linear function of slope (equation (5)) and in the second set of model runs the sediment flux was assumed to be proportional to the product of the soil thickness and the hillslope gradient:

\[
h \tau_s = -D_s h \frac{\partial \chi_{in}}{\partial x},
\]

where \( D_s \) is a transport coefficient of units L T\(^{-1}\). We call equation (14) the depth-slope sediment flux law.

4.2. Linear Flux Law

[26] Response timescales for hillslopes whose sediment flux is linearly proportional to slope are plotted in Figure 7. Figure 7a plots the ratio between the timescale extracted by equation (9) \( (\tau) \) and the timescale \( \tau_0 \) predicted as the long-term adjustment timescale by equation (10). This plot illustrates that the ratio between the density of the soil and the density of the soil’s parent material can have a significant influence on the relaxation time of the hillslope. Figure 7b plots the ratio \( \tau \rho_s/\rho_i \tau_0 \) (this ratio is equal to \( \tau/\tau_s \), see equation (15)) for the erosion rate and the soil thickness. These curves are approximately equal to one, such that the relaxation timescale of a hillslope where the dry bulk density of the soil is different from that of its parent material can be approximated by:

\[
\tau_s = \frac{4\rho_s L^2}{\rho_i \tau^2 D},
\]
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\[ \tau_a \approx \frac{\gamma}{\rho_s \lambda^2} \]

where \( \tau_a \) is an approximate timescale of adjustment for hillslopes that experience a linear sediment flux law and experience soil production such that the hillslope is modeled using equations (2)–(4) and (5).

In Figure 8, we plot the difference between the adjustment timescale \( \tau \) extracted using equation (10) and the approximated timescale \( \tau_a \), normalized by \( \tau_a \). As observed by Furbish and Fagherazzi [2001] and Mudd and Furbish [2005], soil thickness responds on a different timescale than does the erosion rate (Figures 7b and 8a). In addition, soil production also responds on a different timescale, although the timescale of response for production more closely matches that of erosion than does the soil thickness. Our numerical experiments have shown that \( W_0 \) has a negligible influence on the timescale of hillslope response, but both the ratio between the timescale of response in the erosion rate and the timescale of response of the soil thickness is sensitive to the difference between the initial and final channel incision rates and also the length scale (\( \gamma \)) over which soil production decays (Figure 8). The length scale over which soil production decays is important because it determines how sensitive the soil production rate is to changes in soil depth.

As the difference between the initial and final incision rates grows, the discrepancy between the approximate timescale \( \tau_a \) and the timescale of response of the hillslope \( \tau \) grows for the soil production rate, the erosion rate, and the soil thickness. This effect is most pronounced for the soil thickness, such that for hillslopes where there is a large difference in the channel incision rates (e.g., \( IL_0 < 0.2 \)) the response timescale of the soil thickness can be much greater than the response timescale for the erosion rate (Figure 8a). When channel incision rates are decreased, the soil thickness responds to the change in channel incision more slowly than the surface topography, whereas when the channel incision rate increases the soil thickness adjusts more rapidly than the hillslope erosion rate. As seen in Figure 8b, the timescale of response will be more poorly predicted by \( \tau_a \) for thicker soils than for thinner soils (because \( \gamma \) sets the soil thickness for a given erosion rate).

4.3. Depth-Slope Flux Law

In the case of hillslopes where sediment flux is governed by equation (14), \( \tau_a \) is inappropriate for comparing to the timescale, \( \tau \), which is measured by equation (9), because \( D_\ell \) is of different units than \( D \). Instead, the \( D \) in equation (15) may be replaced by \( D \sim D_I h \), which is equivalent to stating that the diffusivity of the soil is depth dependant [e.g., Anderson, 2002]. Figure 9 plots the ratio of \( \tau \) to this modified \( \tau_a \) where the soil thickness is in equilibrium with the lowest channel incision rate (e.g., the slowest possible hillslope response timescale for a given model run).

Several features of response timescales for hillslopes whose sediment flux is proportional to the product of the soil thickness and the hillslope gradient contrast with hillslopes whose sediment flux is linearly proportional to slope. In general, the response timescales of hillslopes with the depth-slope sediment flux law deviate from the predicted response timescale \( \tau_a \) by a larger margin than do hillslopes that experience the linear flux law. For some hillslopes where the incision rate decreases (\( IL_0 < 1 \)) the predicted response timescale \( \tau_a \) is greater than the computed response timescale \( \tau \) although this may be an artifact of selecting the \( \tau_a \) based on the slowest possible response timescale (e.g., calculating \( \tau_a \) base on the minimum soil thickness of the model run). For the hillslopes that experience increased incision rate (\( IL_0 > 1 \)), the response timescale increases for hillslopes that experience a flux law that goes as the depth slope product. This contrasts with the hillslopes that experience a linear flux law such that signals from the channel can be stored in hillslopes that experience sediment flux that is proportional to the depth slope product for a much greater period of time than a similar hillslope that experiences a linear sediment flux law.

5. Implications of Transient Response Timescales

For a landscape that exhibits characteristics of being in a transient state, for example, a landscape that has widespread knickpoints within the fluvial network [e.g., Arrowsmith et al., 1996; Crosby and Whipple, 2006], it is in principle possible to extract information about the nature and history of this transient condition from the state of the hillslopes, including their geometry and soils [Furbish, 2003]. Examples of hillslope properties that may be usefully

Figure 7. Plots of normalized (by (a) \( \tau_0 \) and (b) \( \tau_a \)) response timescale for the soil thickness and erosion rate. Other parameters from these model runs are \( D = 0.005 \text{ m}^2 \text{ yr}^{-1}, \gamma = 0.3 \text{ m}, \lambda = 20 \text{ m}, W_0 = 0.0003 \text{ m yr}^{-1}, I_0 = -0.0001 \text{ m yr}^{-1}, \) and \( I = -0.0002 \text{ m yr}^{-1} \).
measured are topography (obtained through topographic surveys or laser altimetry [Shrestha et al., 1999]), the spatial distribution of soil thickness (obtained from soil pits or ground penetrating radar [Neal, 2004]), and the spatial distribution of soil production (obtained through cosmogenic radionuclide dating techniques [Heimsath et al., 1997]). This point can be elaborated by considering implications of the hillslope response timescale for two examples: (1) a transient landscape for which the sediment flux law has been constrained but the timing of the transient perturbation to the fluvial network is unknown and (2) the timing of the perturbation is known but the sediment flux law is uncertain.

5.1. Known Flux Law, Unknown Timing of Transient Perturbation

[32] At present there are few examples of landscapes for which the appropriate sediment flux law is known with certainty, but methods are emerging by which this important aspect of the landscape may be constrained [Anderson, 2002; Dietrich et al., 2003; Furbish, 2003, Heimsath et al., 2005]. Several authors have described methods for providing relative ages of fault scarps or terraces [e.g., Andrews and Bucknam, 1987; Nash, 1980], but in many cases absolute ages are preferable, and relative dating techniques can only yield absolute ages when degradation periods have been constrained for more than one hillslope [Nash, 2005]. If, however, the sediment flux law is known, then under ideal conditions an absolute (as opposed to relative) time elapsed since the perturbation to the channel incision rate may be estimated by measuring the erosion rate along a hillslope profile and matching this information with predictions of the hillslope response to a perturbation at its base. For example, at a location where a step change in the rate of channel incision is thought to have occurred, a point on the hillslope can be selected (for example, the divide), and the postulated history of soil production rate and soil depth can be calculated numerically (generating information similar to the time series of the erosion rate in Figure 4). The measured and calculated soil depth and soil production rate could then be matched to allow extraction of the time elapsed since the step change in incision rate.

[33] For channel incision histories more complex than a step change in incision rate, more sophisticated inversion techniques would be required to constrain the history of channel incision; these techniques are beyond the scope of this contribution. We note, however, that transient perturbations at one point in the channel network are thought to propagate upstream [e.g., Whipple and Tucker, 1999]. Consider first several basins in which the timescales of hillslope response are the same but the response of the channel network to a perturbation in the channel incision rate is different. As the changes in the channel incision rate propagate upstream these incision signals leave a record...
stored within the adjacent hillslope soils (Figure 10). The celerity at which a change in the channel incision rate propagates through the channel network will be reflected in the degree to which the perturbation of hillslope soil properties has advanced upslope away from the channel along the channel network. Thus measuring the extent to which the soil perturbation has advanced away from the channel as one moves through the drainage basin allows what is in essence a substitution of space for time, and can be indicative of the history of the celerity and magnitude of the channel incision signal (Figure 10, scenarios 1–3).

Because hillslope soils respond to changes in channel incision rates, these soils provide a window into the past; if the incision rate in the channel has changed at some time in the past, surveying hillslope soil properties as one moves away from the channel is akin to reading a recorded history of the incision history of the channel. Moreover, of fundamental importance is quantifying how long these disturbances remain imprinted on the landscape. The length of time that soil-mantled hillslopes may be used as recording devices for channel incision histories is captured by the hillslope relaxation time and the fundamental timescales addressed in sections 2–4. Thus, in some cases hillslope soils may be used to quantify changes in channel incision rates for events that occurred a million years or more in the past.

The hillslope response timescale may also be investigated based on the pattern of hillslope disturbance if the channel response to perturbation is known. In Figure 11 we show two different patterns of hillslope disturbance. The pattern of soil disturbance will reflect the speed of the hillslope response in relation to the celerity of the channel incision signal as it moves upstream if the channel incision history is the same in both basins (Figure 11). Thus different properties of a landscape as it responds to transient incision rates may be evaluated based on the existing information about the hillslope response timescale. If the response timescale is known, the pattern of hillslope disturbance can be used to understand the spatial and temporal response of channel incision rates.

Figure 10. Schematic diagram showing idealized response of soil-mantled basin to transient channel incision rates. Prior to perturbation, the erosion rate of the basin is homogeneous, and likewise, the soil production rate and soil depth are homogeneous. Scenarios 1, 2, and 3 depict adjustment of the hillslope soils in the basin to different transient behaviors of the channel after the same time elapsed from the initial perturbation.
other hand, the pattern of hillslope response may be used to evaluate the relative speed of the hillslope response (and equivalently, the hillslope response timescale) to the celerity of the channel incision signal.

5.2. Known Timing of Transient Perturbation, Unknown Flux Law

[36] In contrast to the situation described above, consider a landscape in which the sediment flux law is unconstrained but there is a known history of channel incision. Examples could include basins that drain into a body of water that both sets the base level for the basin and has a known history of relative elevation (e.g., by dated paleoshorelines or marine terraces), or a river system with dated strath terraces. As illustrated in section 4, hillslopes that have different sediment flux laws will respond differently to the same perturbation in the channel incision rate at their bases. A simple scenario consists of a step change in the channel incision rate. If the channel incision rate increases, a hillslope involving a depth-slope product flux law requires a greater time to respond to this change in incision rate than does a hillslope involving a linear sediment flux law. The implication is that if a soil property such as the soil production rate is measured at the divide at a known time following the step change in channel incision rate, then this soil property must simultaneously be compatible with the new channel incision rate and with the “correct” sediment flux law, among alternatives. That is, because different sediment flux laws have different characteristic timescales of adjustment to changes in the channel incision rate at the base of a hillslope, different sediment flux laws will produce different spatial patterns of hillslope soil properties that may be used to constrain the appropriate flux law [Furbish, 2003; Roering et al., 2004].

6. Conclusions

[37] Channel incision drives the evolution of humid soil-mantled hillslopes. When channel incision rates change, soil response signals from such changes propagate away from the channel. These signals operate on a number of time-scales. We have demonstrated that for soil-mantled hillslopes two timescales are particularly important. The first timescale, which we call \( \tau_0 \), determines how much time must pass after a change in the channel incision rate before the entire hillslope equilibrates to this new condition. For hillslopes that experience sediment flux that is linearly proportional to slope and whose soil is the same density as the material from which the soil is derived, this timescale is approximated by \( 4\lambda^2/(\rho^2D) \) where \( \lambda \) is the length of the hillslope and \( D \) is the sediment diffusivity. At a time of \( 3\tau_0 \) after a perturbation on the channel incision rate, the topography and erosion rate of the hillslope are within 5% of their final values, and the hillslope can be reasonably considered to be at steady state. An additional timescale is the time it takes a signal caused by a change in the channel incision rate to propagate from the channel to the divide. This timescale is approximated by \( \tau_0/9 \).

[38] As described by both Furbish [2003] and Roering et al. [2004], measurement of several spatially and temporally varying soil properties can be used to constrain the dynamic response of hillslopes to changes in the channel incision rate. The strongest spatial variation in soil properties can be expected to occur within the time after channel perturbation of \( 0 < t < \tau_0/9 \), when the hillslope near the channel has begun to adjust to the new channel incision rate but the divide has not yet “felt” this disturbance.

[39] On hillslopes where soil is not the same density as the material from which the soil is derived (i.e., when \( \rho_s \neq \rho_h \)), the ratio between the two densities is of first-order importance in determining the hillslope response to a change in the channel incision rate. In this case, the response timescale can be approximated by \( 4\rho_h\lambda^2/(\pi^2D\rho_s) \) if sediment flux is linearly proportional to slope. The difference in the adjustment timescales for soil thickness and soil erosion rate is sensitive to the ratio between the initial channel incision rate (\( I_0 \)) and the channel incision rate after it has been perturbed (\( I \)).

[40] For hillslopes where sediment transport is linearly proportional to slope, the difference in the adjustment
timescales for soil thickness and soil erosion rate is greater for hillslopes where the new incision rate \( I \) is a smaller proportion of the old incision rate \( I_0 \), whereas on hillslopes where the sediment flux is proportional to the depth slope product accelerated incision results in increased response timescales. Because hillslopes with different sediment flux laws respond to changes in channel incision on different timescales, it is critical to correctly identify the flux law operating on a soil-mantled landscape if one is to determine if a hillslope is approaching a steady state condition at some known time after a channel incision event. This identification of the correct flux law is also critical if one is to assess the duration that signals (such as topography, soil thickness, soil production rates, and erosion rates) from transient erosion events will be stored in hillside soils.

**Notation**

\[ D \] sediment diffusivity \( (L^2 T^{-1}) \).
\[ \eta \] elevation of soil-bedrock boundary \( (L) \).
\[ \gamma \] soil production decay length scale \( (L) \).
\[ h \] soil thickness \( (L) \).
\[ I, I_0 \] channel incision rate and initial incision rate, respectively \( (L T^{-1}) \).
\[ \lambda \] length of the hillslope \( (L) \).
\[ M_E \] ratio determining the magnitude of the time vary component of the erosion rate as a function of position and wave number.
\[ n \] wave number (dimensionless).
\[ \rho_T \] soil production rate \( (L T^{-1}) \).
\[ \rho_d \] depth averaged dry bulk density of hillslope soil \( (M L^{-3}) \).
\[ \rho_{dp} \] Dry bulk density of parent material \( (M L^{-3}) \).
\[ S_c \] critical slope for linear-critical flux law (dimensionless).
\[ t \] time \( (T) \).
\[ \tau \] decay timescale \( (T) \).
\[ \tau_a \] approximate hillslope response timescale for hillslopes where \( \rho_d \neq \rho_{dp} \).
\[ \tau_n \] decay timescale for individual harmonic functions with wave number \( n \).
\[ v_s \] depth averaged sediment velocity in the x direction \( (L T^{-1}) \).
\[ W_0 \] nominal rate of soil production \( (L T^{-1}) \).
\[ x \] distance from the divide \( (L) \).
\[ \zeta \] elevation of soil surface \( (L) \).
\[ \zeta_{rel} \] elevation of soil surface relative to the channel \( (L) \).

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**References**


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