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A Neuromechanical Model of Larval Chemotaxis

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Synopsis

Larval Drosophila move up attractive chemical gradients, and down aversive ones. Although their movement is often characterised as a series of runs and directed turns, it can also be modelled as a continuous modulation of turning extent by the detected change in stimulus intensity as the animal moves through the gradient. We show that a neuromechanical model of peristaltic crawling and spontaneous bending in the larva can be adapted to produce taxis behaviour by the simple addition of a local segmental reflex to modulate transverse viscosity (or ‘bendiness’) proportionally to the intensity change detected in the head. Altering the gain produces weaker or stronger, negative or positive taxis, with behavioural statistics that qualitatively match the larva.
Introduction

For many animals, oriented movement along sensory gradients (taxis) is an important behaviour to locate key resources such as food or mates. Research into the underlying mechanisms often focusses on how taxis can be generated by the animal’s nervous system, e.g., through a combination of inherent locomotion patterns for propulsion, and decisions to alter direction based on sensory information. An alternative view is to consider the whole animal, embedded in its environment, as a closed loop dynamical system that can maintain a consistent output but also be biased by inputs. Importantly, in this view, the biophysical system is not just the mechanical ‘plant’ used by the animal to execute its actions but can be a crucial part of establishing the right dynamics, by exploiting physical interactions. Similarly, this view stresses the role of the output in shaping the sensory input, potentially in just such a way as to provide the requisite input when it is needed for control.

Larval Drosophila exhibit a typical, and widely studied, taxis behaviour: moving up attractive chemical gradients, and down aversive ones (Gomez-Marin et al., 2011; Gershow et al., 2012; Khurana and Siddiqi, 2013; Gomez-Marin and Louis, 2014). They also orient with respect to light gradients, gravity and even electrical fields (Gepner et al., 2015); and in the absence of any clear stimulus directionality, perform exploratory behaviour, with apparently spontaneous changes of direction interrupting approximately straight runs (Lahiri et al., 2011; Berni, 2015). We have previously suggested that taxis could be controlled through a simple mechanism that couples the change in experienced stimulus strength directly to the amplitude of oscillation in heading direction (Wystrach et al., 2016). In the larva, this oscillation is the result of bending in the head and body segments, which alters the direction of propulsion caused by the peristaltic crawling. The larva senses odour primarily through its dorsal organ on the head (Cobb, 1999), which is thus actively propelled through the gradient. In our abstracted model, we assumed the animal maintains a constant forward speed, and has a regular left/right oscillation in heading direction. The sensed change in odour concentration due to forward or lateral movement alters the subsequent oscillation amplitude. For example, an increase in concentration, indicating motion up the gradient, decreases the oscillation so the larva maintains this heading direction; whereas a concentration decrease causes it to make larger bends and hence turn back towards the source. Notably, in this control mechanism, neither the sensing or its effect on the output is lateralised, so the ‘perception’ of the gradient only emerges from the animal’s underlying motor pattern, and subsequently shapes this pattern, in a tightly coupled feedback loop.

Although this model suggests there may be no neural correlate of ‘decisions to turn’ in the larva, it nevertheless assumed the underlying locomotor pattern is produced by inherent control, e.g., a central pattern generator (CPG) circuit for the lateral oscillation.
More recently, we have used a biomechanical model of the larva’s segmented body to explore how peristaltic crawling and lateral bending might emerge without explicit neural generation of the underlying patterns (Loveless et al., in review). We treat the segments as discrete point masses interacting via damped translational and torsional springs, and show that using simple local reflexes to enhance the passive mechanics of the system leads to the emergence of coordinated peristaltic crawling (forward or backward) and spontaneous (chaotic) body bending. The resulting system, given frictional interaction with a substrate, produces behaviour that closely resembles exploratory crawling in the larva.

In this paper, we augment this biomechanical model with a taxis reflex, following the same simple control concept as the previous abstracted model. That is, we use the change of sensory input at the head segment to influence, with a particular gain, the amplitude of the ongoing bend in each segment. The production of bends remains a purely emergent property of the mechanics without any neural control. We show that this is sufficient to produce positive or negative taxis in the simulated agent, depending on the sign of the gain, and stronger or weaker taxis depending on the size of the gain. We discuss similarities and differences from taxis behaviour in real larva.

**Methods**

**Neuromechanical model**

The work presented here builds on an existing model of larval crawling, described in detail in (Loveless et al., in review). We here provide an overview of the model, but refer readers to that paper for detailed specification (see appendix S1), justification and analysis. The model describes the motion of the midline of the larval body in the plane using 12 points that represent the boundaries between body segments and the head and tail extremities. Each point is treated as an identical mass, and each is linked to adjacent points with linear translational and torsional springs in parallel with linear dampers in a Kelvin-Voigt arrangement (Fung, 1993), to represent elasticity in the soft cuticle of the larva (Figure 1). The current body configuration can thus be expressed as a set of axial stretches $q_i$ (the difference in length of the $i$th segment from its length at equilibrium) and transverse bending angles between segments $\phi_i$. The head and tail are additionally linked, representing the effect of the incompressible internal coelemic fluid of the larva, by maintaining a constant overall length. Without friction, the passive mechanics of this system produce axial and transverse standing waves, i.e., coordinated motions of the segments that resemble peristalsis and body bends.

We assume there is sliding friction between the larval body and the substrate, and energy is also lost to viscous friction within the body during both axial compression/extension
and transverse bending. We allow power flow due to muscle activation, controlled by two reflexes (figure 1). The first is local to each segment and acts to overcome the effects of friction, by activating whenever a segment is compressing, producing a muscle tension that amplifies the compression. The axial tension $Q_i$ in the $i$'th segment produced by this reflex is given in terms of the local stretch rate $\dot{q}_i$ and local axial reflex gain $b_i$ as

$$Q_i = \begin{cases} -b_i, & \text{if } \dot{q}_i < 0 \\ 0, & \text{if } \dot{q}_i \geq 0 \end{cases}$$ (1)

The second neural circuit is a mutual inhibition between non-adjacent segments (the head and tail are treated as adjacent) that restricts the active compression to a small number of segments at any time. This enables acceleration of the center of mass relative to the substrate, i.e., peristaltic crawling (Ross et al., 2015; Alexander, 2003; Loveless et al., in review). Due to energetic coupling of axial and transverse motion, through the body mechanics, the model also produces spontaneous body bending, which reorients the crawling direction, i.e., turning occurs without any explicit neural control.

This emergent turning produces a ‘random’ (actually a deterministically chaotic) exploration of the 2D plane. To convert the resulting exploration into taxis, we note that the larva should travel in a roughly straight line while going up an attractive gradient, or while going down an aversive gradient, and that the larva should tend to reorient when it is going down an attractive gradient, or going up an aversive gradient. In other words, the extent of the body bending should be linked to the changing perceptual experience. We can affect bending by altering the effective physics of the body: specifically, we reason that an effective increase in transverse viscosity should lead to a damping of transverse motion and a corresponding tendency towards straight line motion, while an effective decrease in transverse viscosity should reduce the damping of transverse motion and therefore tend to encourage bending and reorientation. We thus posit the following local segmental reflex for transverse motion:

$$\tau_i = c_i \tanh(\beta \dot{\phi}_i) \tanh(\gamma P(t))$$ (2)

where $\tau_i$ is the torque applied about the $i$'th segment boundary, $P(t)$ is the time-varying perception input (defined below), $\beta$ is an angular velocity gain, $\gamma$ is a perception gain, and $c_i$ is an overall reflex gain for the $i$'th segment boundary. We limit the choice of parameters by setting $\beta > 0$ and $c_i > 0$, so that the sign of the torque is determined by $\gamma$.

The rationale of our reflex model can be understood by splitting it into two factors. The first factor $c_i \tanh(\beta \dot{\phi}_i)$ represents (saturating) feedback of the local angular velocity. The torque resulting from this first factor only should act in the same direction as the angular velocity, opposing the torque produced by transverse friction, and thereby reducing
the effective transverse viscosity of the body. Following this line of reasoning, we choose to set the local reflex gain $c_i$ to be dependent upon the local transverse viscosity $\eta_{t,i}$

$$c_i = c \frac{\eta_{t,i}}{\max_i \eta_{t,i}}$$  \hspace{1cm} (3)

where normalisation by $\max_i \eta_{t,i}$ is intended to nondimensionalise the transverse viscosity, allowing the new global reflex gain parameter $c$ to have dimensions of torque. In practise we set $c = 1$ for simplicity, leaving

$$c_i = \frac{\eta_{t,i}}{\max_i \eta_{t,i}}$$  \hspace{1cm} (4)

so that the reflex torque becomes

$$\tau_i = \frac{\eta_{t,i}}{\max_i \eta_{t,i}} \tanh(\beta \dot{\phi}_i) \tanh(\gamma P(t))$$  \hspace{1cm} (5)

The second factor $\tanh(\gamma P(t))$ acts to modulate the change in effective transverse friction based on perception. If this factor is positive, the overall effect of the reflex is to reduce effective transverse viscosity, while if it is negative the effect is reversed and effective transverse viscosity increases. If the overall feedback torque is small (e.g. if $\beta$, $\gamma$, or $c$ are relatively close to zero) then the perception input will have a small effect on the transverse viscosity, while if the overall feedback torque is large there is the possibility for the perception input to have a large effect on the transverse viscosity. This provides an interpretation of the perception gain $\gamma$ as a preference parameter – if the absolute value of $\gamma$ is large, the model larva should show a strong behavioural response to the stimulus, while for small $\gamma$ the behavioural response may be weak. Furthermore, following our arguments above, if $\gamma > 0$ we expect an aversive response, while for $\gamma < 0$ we expect an attractive response. We note a further expectation that the strength of the behavioural response when $\gamma \gg 1$ may be diminished due to saturation of the second factor in Eq. 2 and a corresponding inability to distinguish small changes in perception – in this case, the second factor can only detect whether the head is travelling up or down gradient, but not how strong the local gradient is.

**Perceptual model**

We model the larva as moving within an exponential stimulus field centred on the origin of our cartesian coordinate frame

$$S = A_s e^{-\lambda_s ||r||}$$  \hspace{1cm} (6)

where $A_s$ is a parameter which sets the absolute intensity of the stimulus field, and $\lambda_s$ sets the rate at which the stimulus decays away from the origin, while $r$ is the radius.
vector from the origin to the site of measurement of the stimulus. In practice, we take \( \mathbf{r} \) to be the radius vector from the stimulus source to the model larva’s head, since this is the location of most of the sensory organs involved in taxis.

The early stages of sensory processing in the larva tend to respond strongly to changes in stimulus intensity more than to the absolute stimulus intensity, and show a normalised response across a range of absolute stimulus intensities. Following (Davies et al., 2015), we model the output of the early stages of sensory processing as a “perception” signal given by

\[
P = \frac{1}{S} \frac{dS}{dt} \quad (7)
\]

Denoting the coordinates of the larva’s head as \( \mathbf{r} = [x, y]^T \) and the linear velocity of the larva’s head as \( \dot{\mathbf{r}} = [\dot{x}, \dot{y}] \), this becomes

\[
P = \frac{1}{S} \left( \frac{\partial S}{\partial x} \dot{x} + \frac{\partial S}{\partial y} \dot{y} \right) = \frac{1}{S} \nabla S \cdot \dot{\mathbf{r}} \quad (8)
\]

Substituting the expression for the exponential stimulus field given above, we find

\[
\nabla S = -\frac{A_s \lambda_s}{\sqrt{x^2 + y^2}} [x, y]^T \quad (9)
\]

so that

\[
P = -\lambda_s \frac{x \dot{x} + y \dot{y}}{\sqrt{x^2 + y^2}} = -\lambda_s \frac{\mathbf{r} \cdot \dot{\mathbf{r}}}{\parallel \mathbf{r} \parallel} = \lambda_s \parallel \dot{\mathbf{r}} \parallel \cos \theta \quad (10)
\]

where \( \theta \) is the angle between the head’s velocity vector \( \dot{\mathbf{r}} \) and the vector pointing from the head position to the origin \( -\mathbf{r} \). As expected, \( P \) is positive when the larva is travelling up the sensory gradient \( (\parallel \theta \parallel < \pi/2) \), negative when the larva is travelling down the sensory gradient \( (\parallel \theta \parallel > \pi/2) \), and has a magnitude which depends on both the speed at which the head is travelling and on the bearing of the head to the odor source. Note that due to normalisation of the sensory input, the absolute intensity parameter \( A_s \) has no effect on the perception signal.

Substituting this definition of \( P \) into to the transverse reflex equation 2, we have

\[
\tau_i = \frac{\eta_{i,\lambda}}{\max_i \eta_{i,\lambda}} \tanh(\beta \dot{\phi}_i) \tanh(\gamma \lambda_s \parallel \dot{\mathbf{r}} \parallel \cos \theta) \quad (11)
\]

which makes it clear that there is some redundancy in the parameters \( \gamma \) and \( \lambda_s \), so that we may set \( \lambda_s = 1 \) without loss of generality. This leaves two free parameters in the transverse reflex model, \( \beta \) and \( \gamma \). We choose to set \( \beta = 1000 \gg 1 \) so that the factor \( \tanh(\beta \dot{\phi}_i) \) saturates to \( \pm 1 \), matching our binary-valued axial reflex (Equation 1). This leaves us to explore possible values of the perception gain / preference parameter \( \gamma \).

We set all mechanical parameters and the parameters for the axial reflex circuits to
the values chosen in our previous paper (Loveless et al., in review), with the exception of the transverse viscosity $\eta_{t,i}$ which has been set to twice its previous value. This choice was made in order to more clearly demonstrate the action of the transverse reflex on the model behaviour.

**Results**

In figure 3 we show some representative trajectories generated by our taxis model (see also supplementary videos). In each case the larva starts at the peak of the gradient. For positive gains it crawls away, eventually exiting the space. For negative gains, it consistently loops back towards the peak, with the extent of excursions decreasing as the strength of the gain is increased.

To analyse the behaviour of our taxis model, we generated $N = 1000$ trajectories for each of the cases $\gamma = \pm 200$. As expected the larvae with $\gamma = -200$ showed a strong approach behaviour, remaining localised near the peak of the stimulus field (Fig 4A), while larvae with $\gamma = 200$ showed strong avoidance behaviour, following fairly direct paths away from the peak (Fig 4B).

We first quantified the paths of the simulated larvae using the same methods we applied to the unbiased exploratory paths presented in our previous paper Loveless et al. (in review). Paths in the approach group ($\gamma = -200$) had a high tortuosity and fractal dimension (mean tortuosity = 0.72, mean fractal dimension = 1.51) relative to the avoidance group ($\gamma = 200$, mean tortuosity = 0.11, mean fractal dimension = 1.22; Fig 4C), indicating that approach paths tended to be plane-filling and less linear than avoidance paths (Benhamou, 2004). In accordance with these results, the mean-squared displacement measured within the avoidance group followed an approximately quadratic growth across the duration of the entire experimental trial, typical of rectilinear motion, whereas the approach group showed an initial quadratic growth followed by linear and then sub-linear growth (Fig 4D). We previously found that our model of unbiased exploration produced initially quadratic growth of the mean-squared displacement, followed by asymptotic linear growth (Loveless et al., in review), as has also been observed during unbiased exploration in the real larva (Jakubowski et al., 2012; Gunther et al., 2016). This suggests that our modelled taxis reflex can be interpreted, at the population level, as biasing an ongoing deterministic anomalous diffusion process into either superdiffusive ($\gamma > 0$, quadratic growth) or subdiffusive ($\gamma < 0$, sub-linear growth) regimes.

The distribution of body bending angles (Fig 4E) shows that the modelled larvae take on “straighter” configurations during avoidance behaviour (mean = $4.6 \times 10^{-3}$ deg, variance = 75.5 deg$^2$, kurtosis = 13.17), and tend to take on larger curvatures during approach behaviour (mean = 0.11 deg, variance = 853.52 deg$^2$, kurtosis = 1.94), in accordance with the rationale for our taxis reflex model and in agreement with the shape
of paths taken by the two groups. During both avoidance and approach, the body bend distribution is symmetric (avoidance skewness = 0.02, approach skewness = 0.07) and centred around 0 (avoidance mean = 4.6 × 10^{-3} deg, approach mean = 0.11 deg), demonstrating that larvae are not, on aggregate, biased towards bending either to the left or the right.

The run length distribution of both avoidance and approach groups is well fit by an exponential, but avoidance behaviour is biased towards longer runs when compared to approach behaviour (avoidance time constant = 44.49 seconds, approach time constant = 10.67 seconds; Fig 4F), as is also seen in the real larva during thermotaxis (Luo et al., 2010).

We also analysed the behaviour of our model using three measures which are common in the extant literature on taxis behaviour. These measures depend upon the bearing angle between our larva’s centre of mass velocity and the local gradient of the stimulus field.

First, we examined the overall distribution of bearing angles for our simulated larvae (Fig 4G). Similar to results for the real larva (Wystrach et al., 2016), the bearing distribution for avoidance behaviour was unimodal, symmetric, and centred on 180°, and fell to zero outside of the range [90°, 270°], corresponding to travel directly away from the stimulus peak. In contrast, the bearing distribution for approach behaviour was trimodal, with a pair of large, symmetric peaks centred close to 90° and 270° and a single shallow peak at 180°. This corresponds to a large amount of time spent “spiralling” towards the stimulus peak, with the peak located directly to the left or right of the animal.

Next, we computed the probability density of turns (defined to occur at the onset of a body bend > 20°) across absolute bearing angle (Fig 4H). Both approach and avoidance behaviours showed a monotonic increase in turn probability as absolute bearing increased from 0° (bearing towards stimulus peak) to 180° (bearing away from stimulus peak). Similar to the real larva (Davies et al., 2015), the turn probability for approach behaviour in our model showed a roughly sigmoid shape, with greater probability assigned to intermediate bearings (∼ 90°) and less to large bearings (∼ 180°) than during avoidance behaviour, which followed a roughly exponential distribution.

Finally, we measured the probability of the larva turning to the left rather than right side, across bearing angles (Fig 4I). The model larvae exhibiting approach behaviour showed a strong bias for turning towards the stimulus peak (probability of left turn < 0.5 for bearing ∈ (0°, 180°), probability of left turn > 0.5 for bearing ∈ (180°, 360°)), which is similar to results for the real larva (Davies et al., 2015). Model larvae exhibiting avoidance behaviour showed a much weaker bias, though perhaps surprisingly, in the same direction. For real larvae this curve appears flat for aversive behaviour.
Discussion

Behaviour emerges from the coupling of brains and bodies. We have combined a model of segmented larval biomechanics with a simple, non-lateralised reflex and shown this can produce directed taxis up or down a sensory gradient. The key mechanism is to modulate spontaneous mechanically-driven bending by adjusting transverse viscosity in each segment proportionally to the immediately perceived change in stimulus intensity. By altering the gain factor, the behaviour produced can be stronger or weaker attraction (with negative gain) or stronger or weaker aversion (with positive gain). That is, if an increase in the sensory signal is coupled to a damping of transverse motion, the larva will tend to go straight when going up a gradient and reorient more when going down it, ultimately leading it towards the sensory source, and vice versa for the opposite signal-damping coupling.

We note that, for the values of negative gain examined here, this mechanism tends to produce ‘orbiting’ behaviour with the sensory source predominantly at around 90 degrees to the larva, while for positive gain larvae display dispersive behaviour with the source remaining behind the larva. Similar distributions of bearing angles are observed for real larva (Wystrach et al., 2016). Indeed, our model also qualitatively reproduces the distribution of run durations observed during approach and avoidance (Luo et al., 2010), as well as the experimentally observed distributions of turn probability and left-turn probability over bearing angle (Davies et al., 2015).

Several of these characteristics have been captured in previous models of larval chemotaxis. In Davies et al. (2015), the probability of transition from a ‘run’ to a ‘turn’ was altered by the change in odour intensity, and a similar approach coupled to a more realistic model of olfactory sensory neuron responses in Schulze et al. (2015). In Wystrach et al. (2016) it was proposed that the distinction between runs and turns be replaced by a continuum of smaller or larger oscillations in heading direction, and shown that this could replicate many aspects of the behaviour without requiring ‘decisions’ to turn. The current model is in the spirit of this latter approach, but dispenses with any need to posit an underlying CPG to generate body bends, as these emerge spontaneously from the inherent dynamics of forward crawling in coupled segments (Loveless et al., in review). As such, if ‘turns’ are identified with larger bends, then their chaotic generation can replace the probabilistic approach of the earlier models and the distinction between these explanations becomes less marked.

Although the presented model uses direct control of torque to counter or reinforce the effects of transverse viscosity, a biologically plausible means to achieve this effect would be a neural reflex circuit that couples the muscles on one side of a body segment to proprioceptive sensory neurons on the opposite side (see figure 2). The effective transverse viscosity will be increased if muscles on one side of the body contract while the opposite
side is shortening, while the effective viscosity will be decreased if muscles contract while the opposite side is lengthening. The EL neurons reported in (Heckscher et al., 2015) appear to be a possible substrate for this function. We then need only add a general signal (to all segments) that modulates the action of this local reflex. Neurons that run throughout the ventral nerve cord and connect the brain to all segments have been identified (Fushiki et al., 2016). It is also of interest that a small set of neurons in the brain’s premotor subesophageal zone appear to directly affect the production of large reorientations (high head angular speeds) in larva, with the same effects observed for taxis in odour, light and temperature gradients (Tastekin et al., 2015). Finally, we note that the action of the local reflex could be enhanced by the presence of reciprocal inhibition, acting to relax the muscles on one side of the body while those on the other side contract – such inhibitory pathways are a common feature in the spinal reflexes of jointed animals, including humans (Purves et al., 2004).

Acknowledgements

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References


Figures

Figure 1: (from Loveless et al. (in review)) The larva is represented by a mechanical model (top) of its midline with 11 segments. The boundaries between segments are treated as point masses linked by linear translational and torsional springs in parallel with linear dampers in a Kelvin-Voigt arrangement Fung (1993). The controller (bottom) uses positive feedback within each segment to counteract friction and distant inhibition between non-adjacent segments to create a coherent peristaltic wave of contraction capable of driving locomotion. The mechanics produce spontaneous body bending without any explicit neural control.

Figure 2: Taxis is achieved by increasing (decreasing) transverse viscosity during favourable (unfavourable) stimulus conditions, thus leading to decreased (increased) bend/turn amplitude (top panel). This could be achieved through descending chemosensory innervation of a segmentally localised contralateral reflex acting to modulate the effective local transverse viscosity (bottom panel, see text).

Figure 3: Changing the perception gain / preference parameter $\gamma$ causes the model larvae to exhibit increasingly attraction. Representative trajectories are shown for $\gamma = 1000, 200, 100$ (aversive behaviour, top row, left-to-right) and $\gamma = -100, -200, -1000$ (attractive behaviour, bottom row, left-to-right). Trajectories start at the green circle, end at the red circle (larvae doing negative taxis escaped the area shown), and are superimposed on a map of the log stimulus intensity.
Figure 4: Analysis of positive ($\gamma = -200$) and negative ($\gamma = 200$) taxis behaviour in $N = 1000$ simulated larvae. A: centre of mass trajectories for the simulated larvae during positive taxis. B: trajectories during negative taxis. Larvae started from almost identical configurations but with random orientation. C: larval paths show higher tortuosity and fractal dimension during positive taxis (blue/dark) compared to negative taxis (red/light; horizontal lines = mean tortuosity, vertical lines = mean dimension), indicating rectilinear motion during negative taxis and plane-filling motion during positive taxis. D: mean-squared displacement shows asymptotic quadratic growth during negative taxis and absence of growth during positive taxis. E: distribution of body bends. F: distribution of run durations during negative and positive taxis. Run lengths were calculated as duration between successive crossings of a threshold body bend ($20^\circ$), indicated by vertical lines in panel D. G: probability distribution of bearings to stimulus source. H: turn probability as a function of absolute bearing angle. I: probability of left turn as function of bearing angle.