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Inequality and Risk-Taking Behaviour

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Abstract

This paper investigates social influences on attitudes to risk and reanalyses how risk taking varies with relative position and inequality. Individuals with low initial wealth, about to participate in a tournament with richer opponents, may take fair gambles even though they are risk averse in both consumption and tournament rewards. It is shown that this risk taking decreases in the inequality of initial endowments, but in contrast it increases in the inequality of tournament rewards.

Keywords: risk preferences, relative concerns, tournaments, inequality.
JEL codes: C72, D31, D62, D63, D81.

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1 Introduction

There is a long tradition of treating risk attitudes as exogenous and fixed. However, there is now much empirical evidence that choices under uncertainty are subject both to systematic variation and to social influence. For example, recent research by Falk et al. (2015) finds that risk taking is higher in countries with higher inequality. Further, there is substantial evidence that risk taking is influenced by relative position, with those who are behind others in tournament situations willing to take on more risk, both in sports (Genakos and Pagliero, 2012) and in finance (Brown et al., 1996; Dijk et al., 2014).[1]

Theoretical explanations of social influence on risk attitude are scarce but include Robson (1992, 1996), Becker et al. (2005), Ray and Robson (2012). However, one of the central predictions of the existing literature is that risk taking is increasing in wealth equality. This is both counter-intuitive and lacks empirical support. Existing models also suggest that the highest level of risk taking should be by those in middle of the wealth distribution. Again this runs against the evidence noted above for risk taking by those at the back of the field.

This paper tries to reconcile theory with the evidence by analysing the role of reward inequality in a tournament setting. A large population starts with different levels of wealth and compete for multiple, ranked rewards. These can be interpreted as representing different levels of status or different matching outcomes. In this strategic situation, an individual's indirect utility function can be convex in initial wealth and thus for standard theoretical reasons he will be willing to take fair gambles before the tournament. Importantly, these implied risk attitudes are not fixed but rather vary with the degree of competition, which itself is determined by two different forms of inequality - inequality in initial endowments and inequality in the tournament rewards. For example, the gap between the best and worst rewards could be small or large. This is the first study to study systematically the effect of reward inequality on risk taking.

Specifically, I show that under a simple symmetry condition the lowest ranked in society will be risk loving. Thus, it can explain why those who are behind others would be willing to take on more risk. Further, under appropriate regularity assumptions on the utility function related to the concept of prudence, I find that risk-taking behaviour is increasing in inequality of final *rewards*, even though it is decreasing in the inequality of initial wealth.

Finally, I consider the maximum level of sustainable wealth equality. Robson (1992) introduces the concept of a stable distribution of wealth, a distribution such that there is no incentive to gamble. Concentrating on the most equal stable distribution, I show that it depends on the distribution of rewards, with more equal distributions of rewards supporting a more equal distribution of wealth. Thus, in contrast to earlier findings by Becker et al. (2005) and Ray and Robson (2012), the most equal stable distribution of wealth can be arbitrarily equal, if rewards are sufficiently equally distributed.

[1] A wider experimental literature on social influence on decision making under uncertainty is surveyed in Trautmann and Vieider (2012).

The basic intuition for risk taking is that an individual who has an endowment that is low relative to his rivals can expect only a low reward from participating in the tournament, even if his initial wealth is high in absolute terms. Thus, the marginal value of doing better in the tournament can be arbitrarily high - the individual is "desperate". Consequently, the individual's indirect utility will be convex in present wealth, giving an incentive to gamble. More generally, either an increase in inequality of rewards or a decrease in inequality of endowments will increase the competitiveness of the tournament and increase the incentive to gamble.

This model can provide a theoretical mechanism which would support the apparent positive empirical relationship between inequality and risk-taking behaviour, but the causation flows in a different way than is normally assumed. High reward inequality induces greater risk-taking behaviour which increases the minimum level of wealth inequality that is compatible with stability. Thus, wealth inequality and risk taking are jointly caused by another factor - reward inequality. It remains true that, as with the previous literature, greater wealth inequality, considered in itself, reduces risk taking. Nonetheless, the overall relationship between risk taking and inequality of wealth can be positive if differences in reward inequality across societies are greater than cross-country differences in initial wealth inequality.

This paper is certainly not the first to consider the relationship between risk taking and relative concerns. However, while there a wider literature on status and relative concerns, the number of works considering the effect on risk taking is quite small, including Robson (1992, 1996), Harbaugh and Kornienko (2000), Cole et al. (2001), Becker et al. (2005), Ray and Robson (2012). This paper differs from this existing literature in two main ways. First, as noted above, existing models suggest that risk taking should be increasing in equality, a result that seems to run counter to intuition and to evidence. In particular, a recent and comprehensive cross-country study of risk attitudes is found in Falk et al. (2015) who examine data on 80,000 subjects from 76 countries surveyed using a common methodology. Risk taking was elicited both by a mixture of quantitative questions, a series of five binary choices between a fixed lottery and varying sure payments, and a self-assessment question. They find that such risk taking is higher in more unequal countries. Second, previous theoretical work considers only inequality in wealth but not inequality in rewards.

Fang and Noe (2016) also consider how tournaments affect risk taking but in a somewhat different framework. Hopkins and Kornienko (2010) introduces the distinction between endowment and reward inequality but, as with the vast majority of work on relative concerns, do not consider risk taking. The previous study closest to the current work is Robson (1996). He considers a model where men care about relative wealth because of the possibility of polygyny: high relative wealth means that a man can attract multiple partners. This gives men an incentive to gamble. In current terminology, men face greater reward inequality than women. But the general relationship between reward inequality and risk taking is not explored.

2 A Status Tournament

The base model is similar to that found in Frank (1985), Hopkins and Kornienko (2004) and Becker, Murphy and Werning (BMW) (2005), but here is modified to allow for reward inequality to vary. A large population of agents compete in a tournament with a range of ranked rewards that could represent either different levels of status or of marriage opportunities. Agents make a strategic decision over how to allocate their endowment between performance in the tournament and private consumption. As BMW first discovered, this situation can lead to individuals being willing to take fair gambles if they are offered before the tournament. This is because the utility function implied by equilibrium behaviour in the tournament can be convex in initial endowments, even though an individual has preferences that are concave in both consumption and rewards. The model is solved backwards. This section analyses the tournament stage of the game. The next section looks at the implied incentives to take gambles prior to the tournament.

I assume a continuum of agents. The game begins with each being allocated a different endowment of wealth z with endowments being allocated according to the publicly known distribution $G(z)$ on $[\underline{z}, \bar{z}]$ with $\underline{z} > 0$. The distribution $G(z)$ is twice differentiable with strictly positive density $g(z)$.

Next, and before the tournament, individuals may have an incentive to gamble with their wealth. It is assumed that a range of fair gambles are offered each in the form of a continuous density over a bounded interval. As Ray and Robson (2012) suggest, these gambles could be lotteries in the common meaning of the term or, more generally, entry into risky occupations or making risky investments.

One would expect that gambles are taken until the market clears in the sense that the distribution of wealth is such that no-one wishes to gamble further. Stable wealth distributions that give no incentive to gamble are characterised in Section 4. However, in this section I analyse the tournament taking place with the initial wealth distribution $G(z)$. The point is that it is the anticipation of taking part in a tournament when the distribution of wealth is not stable which gives the incentive to take risks. It is thus necessary to model the hypothetical possibility of playing the tournament under the initial non-stable wealth distribution in order to understand risk attitudes. Thus, in the rest of this section $G(z)$ refers to the initial distribution of endowments.[2]

In the tournament itself, agents make a simultaneous decision on how to divide their wealth z between performance x and consumption c, with $x + c = z$. Performance has no intrinsic utility, but rewards s are awarded on the basis of performance, with the best performer receiving the highest reward, and in general, one's rank in performance determining the rank of one's reward. A specific interpretation in BMW and Hopkins and

[2]If gambles are taken then the tournament may be contested with a distribution that is different from the initial distribution $G(z)$, even if $G(z)$ determined those risk attitudes. This is not a major concern because, first, the focus here is on risk attitudes not the tournament itself; second, the distribution is arbitrary and so it does not matter if it changes, because the resulting stable distribution retains suitable properties - stable distributions are continuous (Robson, 1992) and smooth (see the online appendix).

Kornienko (2004) is that x represents expenditure on conspicuous consumption, and s is the resulting status. An alternative, first due to Cole et al. (1992), is that s represents the quality of a marriage partner achieved. Relating this to evolutionary considerations, the range of rewards in a society which permits a high degree of polygyny would be wider than in a society in which strict monogamy is enforced. Whatever the interpretation, what is important here is that there is a schedule of rewards or status positions available, which are assigned by performance in the tournament. In contrast, regular consumption c is supplied by a competitive market at a constant price.

In any case, it is assumed that all individuals have the same preferences over consumption c and status or rewards s,

$$U(c, s) \tag{1}$$

where U is a strictly increasing, strictly concave, three times differentiable function with $U_c, U_s > 0$, and $U_{cc}, U_{ss} < 0$. So, agents are risk averse with respect to both consumption and status. I also assume that $U_{cs} \geq 0$, so that the case of additive separability $U_{cs} = 0$ and status and consumption being positive complements $U_{cs} > 0$ are both included. As BMW stress, it is when $U_{cs} > 0$, strict complementarity between rewards and consumption, that the results on risk taking are strongest. Note that x does not appear in the utility function and thus represents a pure cost to the individual. The amount spent on x could represent conspicuous consumption, labour effort or resources devoted to fighting or lobbying.

The order of moves is, therefore, the following:

1. Agents receive their endowments z. Because the distribution of wealth $G(z)$ is common knowledge, they therefore know their relative position in the field of competitors.

2. Agents are offered fair gambles which they are free to accept or to reject.

3. Agents commit a part x of their after gambling wealth z to performance in the tournament.

4. Each agent receives a reward s, the value of which is determined by performance in the tournament.

5. Agents consume their remaining endowment $c = z - x$ and their reward s, receiving utility $U(c, s)$.

To this point, the model is identical to that of BMW (and very similar to that of Hopkins and Kornienko, 2004). However, here I follow Hopkins and Kornienko (2010) in assuming that the rewards or status positions of value s have an arbitrary publicly known distribution with a twice differentiable distribution function $H(s)$ on $[\underline{s}, \bar{s}]$, with $\underline{s} > 0$, and strictly positive density $h(s)$. BMW assume that $H(s)$ is fixed as a uniform distribution on [0,1]. As they point out, for the existence of equilibrium, this represents a harmless normalisation. However, this clearly prevents the major exercise here: identifying the change of behaviour arising from changes in the distribution of rewards.

Rewards or status are assigned assortatively according to rank in performance, with the highest performer receiving the highest reward and the lowest performer the lowest reward. Let $F(x)$ be the distribution of choices of performance. One's position in this distribution will determine the award achieved. Precisely, an individual who chooses a performance level x will receive a reward

$$S(x, F(\cdot)) := H^{-1}\left(\theta F(x) + (1-\theta)F^{-}(x)\right) \tag{2}$$

where $F^{-}(x) = \lim_{\xi \uparrow x} F(\xi)$ and for some $\theta \in (0,1)$. The role of the parameter θ is to break potential ties that would occur if a mass of agents were to choose the same level of performance. For example, if a mass of agents chose the same performance, this rule would be consistent with the corresponding range of rewards being equally distributed amongst those agents.[3] However, if all contestants choose according to a continuous strictly increasing strategy $x(z)$, then, first, $F(x) = F^{-}(x)$ for all x, and, second, $F(x(z)) = G(z)$. Together, this implies, $H(s) = F(x) = G(z)$, one holds the same rank in endowments, performance and in reward achieved, or

$$S(x, F(x)) = H^{-1}(F(x)) = H^{-1}(G(z)) := S(z). \tag{3}$$

We can call $S(z)$ the reward or status function, as in a monotone equilibrium, it represents the relationship between initial endowment and the reward or status achieved.

Importantly, the reduced form equilibrium utility given a monotone equilibrium performance function $x(z)$ will then be

$$U(z) = U(z - x(z), S(z)). \tag{4}$$

We will see that this function $U(z)$ can be convex, even given our concavity assumptions on $U(c, s)$. Therefore, agents would accept a fair gamble over their endowment, if such a gamble was offered before the tournament.

If all agents follow a monotone strategy $x(z)$, then an individual with endowment z should choose $x(z)$. If she considers deviating to a different level of performance $x(\hat{z})$, she will have no incentive to do so if

$$-x'(\hat{z})U_c(z - x(\hat{z}), S(\hat{z})) + S'(\hat{z})U_s(z - x(\hat{z}), S(\hat{z})) = 0. \tag{5}$$

Setting $x(\hat{z}) = x(z)$ and rearranging, we have

$$x'(z) = \frac{U_s(z - x(z), S(z))S'(z)}{U_c(z - x(z), S(z))}. \tag{6}$$

The solution to the above differential equation with boundary condition,

$$x(\underline{z}) = 0 \tag{7}$$

[3] Note that $F(x)$ and $F^{-}(x)$ are only distinct when a positive mass of agents choose the same performance $\hat{x}$. Denote $\bar{r} = F(\hat{x})$ and $\underline{r} = F^{-}(\hat{x})$ then the average value of rewards ranked between $\underline{r}$ and $\bar{r}$ is $v = \int_{\underline{r}}^{\bar{r}} H^{-1}(r)\, dr / (H(\bar{r}) - H(\underline{r}))$ and by the mean value theorem there is a $\theta \in (0,1)$ such that $H^{-1}(\theta F(x) + (1-\theta)F^{-}(x)) = v$.

will determine the equilibrium strategy. The proof (in the Appendix) shows that despite the possibility of the equilibrium utility function $U(z)$ being convex, individual utility is pseudoconcave in performance x so that the first order condition (5) above does represent a maximum.

Proposition 1. *There exists a unique solution $x(z)$ to differential equation (6) with boundary condition (7). This is the unique symmetric equilibrium to the tournament.*

Having established the framework of the tournament, the next step is to proceed in solving backwards. The next section considers the risk attitudes of agents who are about to participate in the tournament.

3 Implied Risk Attitudes

The main focus of this paper is to examine the risk attitudes implied by participation in the status tournament. As described in the previous section, an individual with initial endowment z will anticipate equilibrium utility $U(z) = U(z - x(z), S(z))$, where $x(z)$ is the equilibrium choice of performance and $S(z) = H^{-1}(G(z))$ is the reward function. If this function is convex for some range of endowments z, then individuals with endowments in that range would take fair bets if such bets were offered to them prior to the tournament. The analysis in this section focuses on the question as to when in fact this function will be convex.

We have by the envelope theorem $U'(z) = U_c(z - x(z), S(z))$ and

$$U''(z) = U_{cc}(z - x(z), S(z))(1 - x'(z)) + U_{cs}(z - x(z), S(z))S'(z), \tag{8}$$

on $(\underline{z}, \bar{z})$, where $x'(z)$ is as given in the differential equation (6). Perhaps more usefully, to clarify the different potential effects on risk attitudes, one can decompose the expression (8) into (suppressing arguments)

$$U''(z) = U_{cc} + U_{cs}S'(z) - x'(z)U_{cc} = U_{cc} + S'(z)\left(U_{cs} - \frac{U_{cc}U_s}{U_c}\right) \tag{9}$$

which not only separates the negative and positive elements, but also the traditional and non-traditional parts. The first part U_{cc} is negative and reflects risk aversion towards regular consumption. The second and third give the competitive aspect which is positive. By inspection one can immediately see that $U''(z)$ will be positive, even though the traditional term $U_{cc} < 0$, if either $x'(z)$ or $S'(z)$ is sufficiently large.[4] Note that $S'(z) = g(z)/h(S(z))$. Thus, BMW's result that equality in endowments would lead agents to be willing to accept lotteries follows quite directly. If the distribution of endowments $G(z)$ is strongly

[4]One might think that risk taking would be related to $S''(z)$, specifically whether the distribution of rewards is convex. But, because of the envelope theorem, $U'(z)$ does not depend on $S'(z)$ and so $U''(z)$ does not depend on $S''(z)$ but on $S'(z)$.

unimodal, then its density $g(z)$ will have a very high value at and around its mode. Thus, BMW's focus on wealth alone predicts risk taking typically will be highest at middling levels of wealth.

Turning to the impact of rewards on risk attitudes, one can immediately see that the effect will largely be opposite to that of endowments. An increase in inequality of rewards will tend to lower its density $h(s)$ which lead to more risk taking, because as noted above risk taking is increasing in $S'(z) = g(z)/h(S(z))$. Reward inequality thus leads to risk taking, endowment inequality to risk aversion.

However, the problem in obtaining unambiguous global results on risk attitudes is that changes in either rewards or endowment inequality have additional effects. First, there are wealth effects, having a higher or lower reward or endowment in itself may change one's risk attitude. Second, changes in inequality affect everyone's incentives to compete in the tournament, and higher performance in the tournament means lower consumption. This tends to increase risk aversion through the conventional channel, lowering U_{cc} if (plausibly) U_{ccc} is positive.

I start with a basic characterisation result. Because risk attitudes depend both on the distribution of rewards and of initial endowments, a natural benchmark is where the two distributions are equal, so that $S(z) = H^{-1}(G(z)) = z$ and $S'(z) = 1$. It is also relatively a plausible case as if both distributions are unimodal (as is typical for many empirical distributions) then this assumption would be approximately correct. What is important about equal distributions for the results below is that it implies that $S'(z) = 1$. Thus, differences in means or scale between the distributions would not change the result. However, previewing results from the next section, if the distribution of endowments were significantly more unequal than the distribution of rewards so that $S'(z) < 1$, risk taking would be less likely to hold. Unfortunately, even with equal distributions, it is not possible to obtain a global result on risk attitudes except for the two special cases considered here - precisely because of the opposing effects of endowments and rewards.

Proposition 2. *Let the distribution of endowments $G(\cdot)$ and the distribution of rewards $H(\cdot)$ be identical. Assume either that preferences are separable so that $U_{cs} = 0$ or that preferences are Cobb-Douglas so that $U(c, s) = c^\alpha s^\beta$, then there is at most one such point $\hat{z}$ such that $U''(\hat{z}) = 0$. If there is such a crossing point $\hat{z}$, then $U''(z) > 0$ for $z \in (\underline{z}, \hat{z})$ and $U''(z) < 0$ for $z \in (\hat{z}, \bar{z})$. If preferences are symmetric so that $U_c(y, y) = U_s(y, y)$, then, the poorest individuals will be risk loving. That is, there will be a $\hat{z} \in (\underline{z}, \bar{z}]$ such that $U''(z) > 0$ on $(\underline{z}, \hat{z})$.*

This result implies that, under these utility specifications, there are only three possible configurations for risk preferences. Either everyone is risk taking or all are risk averse, or the poor are risk taking and the rich are risk averse (see the function U_A in Figure 1 in Section 3.1 below for example). Example 1 below shows that all three such configurations are possible. In the last case, then the mass of middle-ranked individuals will be risk loving with respect to losses, and risk averse with respect to gains, which is reminiscent of prospect theory. The final part of the proposition is a sufficient condition for low status

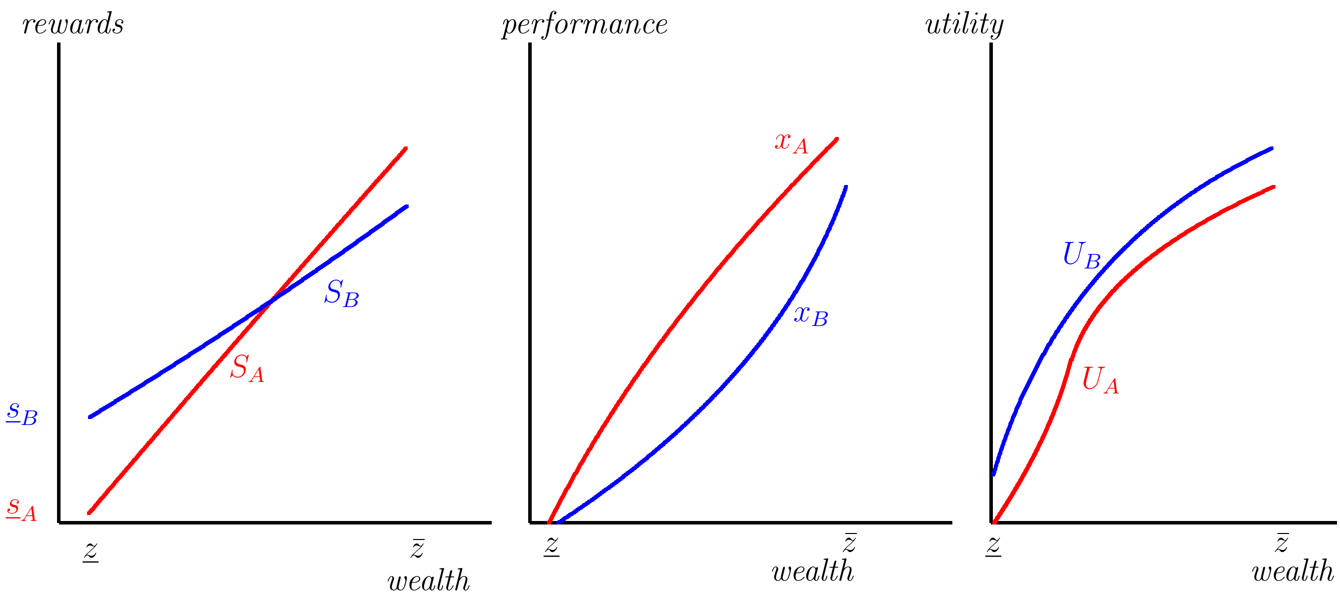

Figure 1: Illustration of Proposition 3: A's reward function S_A is steeper than S_B leading to A's equilibrium utility function U_A being convex at low levels of wealth. Typically, U_A is lower and performance x_A is higher with more unequal rewards.

individuals to be risk loving - that is, to rule out the less interesting case where all are risk averse.[5]

It is worth remarking that our finding that the poorest can be the most risk taking is contrast to the results of BMW, Robson (1992) and Ray and Robson (2012) who find support for the Friedman-Savage conjecture that risk taking is greatest at middle incomes. However, an empirical study of demand for lottery tickets in the United States, one example of risk taking, found that demand is highest at low incomes (Clotfelter et al., 1999). Haisley et al. (2008) in a laboratory study find that demand for lottery tickets increases when subjects perceived their incomes to be relatively low.

3.1 Effects of Greater Inequality: Rewards vs Endowments

Let us now move to a principal question in this paper, the relationship between risk attitudes and inequality in rewards. Specifically, it is possible to show that making rewards more unequal leads to more risk-taking behaviour. With more inequality, low-ranked competitors face lower rewards and all face higher incentives to compete in the tournament. Both factors encourage risk taking. It is also important to verify BMW's claim that in contrast increases in the dispersion of initial wealth should reduce the desire to gamble. The basic thrust of their claims are supported in a new result given below.

[5]Another sufficient condition for risk taking (but which does not require the two distributions to be equal) is that the minimum reward is sufficiently low.

Suppose that the distributions of rewards differ across two societies A and B for exogenous reasons, giving rise to the distributions $H_A(s)$ and $H_B(s)$ respectively. I then see how these differences affect risk attitudes. Some notion of a distribution being more dispersed than another is needed. I use a strong version of the dispersive order. Specifically, I say that a distribution H_A is strictly larger in the dispersive order than a distribution H_B, or $H_A >_d H_B$ if

$$h_A(H_A^{-1}(r)) < h_B(H_B^{-1}(r)) \text{ for all } r \in [0, 1]. \tag{10}$$

The original definition of this stochastic order (Shaked and Shanthikumar, 2007, pp148-9) has the same condition but with a weak inequality, and on (0,1). A simple example of distributions satisfying this stronger condition would be any two uniform distributions where one distribution has support on a strictly longer interval than the other (see Hopkins and Kornienko (2010) for further examples and discussion). This is consistent with a form of mean preserving spread on rewards. For example, two uniform distributions having the same mean but with H_A having a wider support would be suitable.

Further, to see how risk attitudes vary with changes in initial endowments and in rewards some additional assumptions on the third derivatives of the utility function are necessary. This should not be surprising. In the standard theory of risk attitudes, such assumptions are often necessary for comparative statics. For example, for an individual to have declining absolute risk aversion (DARA), the third derivative of the utility function must be positive ("prudence"). Thus, I introduce a set of assumptions which are related to having DARA with respect to consumption risk.[6]

A1: $U_{ccc}(c, s) \geq 0$ and $U_{ccs}(c, s), U_{css}(c, s) \leq 0$.

A2: The ratio $U_{cc}(c, s)/U_c(c, s)$ is non-decreasing in c and in s; equivalently $U_{ccc}(c, s)U_c(c, s) - U_{cc}^2(c, s) \geq 0$ and $U_{ccs}(c, s)U_c(c, s) - U_{cs}(c, s)U_{cc}(c, s) \geq 0$.

A3: The ratio $U_{cc}(c, s)/U_{cs}(c, s)$ is non-decreasing in c; equivalently $U_{ccc}(c, s)U_{cs}(c, s) - U_{cc}(c, s)U_{ccs}(c, s) \geq 0$.

Note that all these properties are satisfied by Cobb-Douglas, $c^\alpha s^\beta$, and CES, $(c^\rho + s^\rho)^{1/\rho}$, utility functions for $\alpha, \beta, \rho \in (0, 1)$. In terms of the economic interpretation of these conditions, clearly, A1 represents "prudence", that is a positive third derivative, with respect to consumption. Similarly, A2 mirrors the DARA assumption for risk preferences with a single variable (that in current notation $-U''(z)/U'(z)$ is decreasing in z). That is, conventional absolute risk-aversion with respect to consumption U_{cc}/U_c approaches risk neutrality (becomes less negative) as consumption or rewards rise. A3 is similar in that risk aversion with respect to consumption does not grow in absolute size compared to the cross term U_{cs} as consumption rises.

[6]These assumptions are principally used in obtaining comparative statics results on the level of absolute risk aversion $AR(z)$ and were chosen for reasons of plausibility and to generalise Cobb-Douglas and CES preferences. Since DARA implies that low wealth individuals will be the most risk averse, these assumptions if anything make the results below more difficult to show.

Greater dispersion in rewards will lead to a steeper reward function, see for example panel 1 of Figure 1. Consider the expression (8), then one can see there will be a direct and positive effect on $U''(z)$ from the rise in the slope of the reward function $S'(z)$. Second, low-ranked competitors will get lower rewards ($S_A(z) < S_B(z)$ for low endowment levels), increasing the marginal value of rewards U_s. But there are further effects through the competitive response - competitors will put more resources into performance and both performance $x(z)$ and its slope $x'(z)$ will rise. Unfortunately these multiple changes make finding a global result very difficult as higher performance and thus lower consumption typically increases risk aversion, so for many the overall effect will be ambiguous. The poor will definitely become more risk loving, as the consumption of the poorest is tied down by the boundary condition (7) which is unchanged. Thus, it is possible to obtain the following result: greater inequality of rewards causes the poor to be more risk taking. This is shown both in terms of the sign of the second derivative $U''(z)$ and in terms of the implied level of absolute risk aversion, $AR(z) = -U''(z)/U'(z)$.

Proposition 3. *Assume A1-A3 and that* $U''_B(z) \leq 0$ *on* $(\underline{z}, \underline{z} + \epsilon)$ *for some* $\epsilon > 0$.
(a) Suppose that the distribution of rewards in society A is strictly more dispersed than in B, $H_A >_d H_B$*, and the minimum reward is lower* $\underline{s}_A \leq \underline{s}_B$*. Then the poor in A are more risk loving. That is, there will be a* $\hat{z} \in (\underline{z}, \bar{z}]$ *such that* $U''_A(z) > U''_B(z)$ *and* $AR_A(z) < AR_B(z)$ *for all* $z \in (\underline{z}, \hat{z})$.
(b) Suppose that the distribution of initial endowments in society A is strictly more dispersed than in B, $G_A >_d G_B$*, and the minimum endowment is lower* $\underline{z}_A \leq \underline{z}_B$*. Then the poor in A are more risk averse. That is, there will be a* $\hat{r} \in (0, 1]$ *such that* $U''_A(G_A^{-1}(r)) < U''_B(G_B^{-1}(r))$ *and* $AR_A(G_A^{-1}(r)) > AR_B(G_B^{-1}(r))$ *for all* $r \in (0, \hat{r})$.

See Figure 1 for an illustration of the first result. In the first panel the reward function $S(z)$ is plotted against wealth. The dispersion order implies that A's reward function S_A is everywhere steeper than S_B. The second panel illustrates typical results on how performance responds to the greater level of competition implied by greater inequality of rewards. While there are no such results in this paper, Hopkins and Kornienko (2010) already have shown that greater dispersion of rewards induce higher performance and a decrease in utility for most, and sometimes for all, individuals. See also Example 1 below. The third panel shows the main result of Proposition 3: higher reward inequality can change the risk attitudes of those at the bottom from risk averse to risk taking.

The intuition for the second result is that a greater concentration of endowments makes a tournament more competitive as competitors are more evenly matched. Consider, for example, a foot race with a very diverse field. There the fast runners will not have to exert themselves too much in order to win the race ahead of their slower rivals. Mathematically, the $S'(z)$ term in the expression for risk aversion (8) is equal to $g(z)/h(S(z))$ so that a lower endowment density $g(z)$ directly leads to lower risk taking. Lower ranked agents will also have lower wealth in the more unequal society - this will also make them more risk averse, given our DARA-like conditions in Assumptions A1-A3. Further, just as in the previous section, there are other factors to consider, because this change in inequality will also change competitive behaviour in the tournament. Nonetheless, as BMW supposed, greater dispersion of wealth can lower individuals' willingness to gamble. Note that as the comparison is across endowment distributions with different supports, risk attitudes are compared at constant ranks - a method that is discussed at much greater length in Hopkins and Kornienko (2010).

3.2 Cobb-Douglas

In this section, for concreteness we look at Cobb-Douglas preferences, for which closed form solutions for equilibrium behaviour and preferences are possible. Suppose $U(c, s) = c^{\alpha} s^{\beta}$. Let $\gamma = \beta/\alpha$. Then,

$$x'(z) = \gamma \frac{S'(z)}{S(z)}(z - x) \tag{11}$$

with again $x(\underline{z}) = 0$. This differential equation has an explicit solution, so one can calculate

$$x(z) = z - \frac{\underline{s}^{\gamma}\underline{z} + \int_{\underline{z}}^{z} S^{\gamma}(t)dt}{S^{\gamma}(z)},\ c(z) = \frac{\underline{s}^{\gamma}\underline{z} + \int_{\underline{z}}^{z} S^{\gamma}(t)dt}{S^{\gamma}(z)},\ U(z) = \left(\underline{s}^{\gamma}\underline{z} + \int_{\underline{z}}^{z} S^{\gamma}(t)dt\right)^{\alpha}.$$

Thus, with some manipulation,

$$U'(z) = \alpha c^{\alpha-1}(z) S^{\beta}(z),\ U''(z) = c^{\alpha-2}(z) S^{\beta-1}(z) \left(\beta c(z) S'(z) - \alpha(1-\alpha) S(z)\right). \tag{12}$$

When $G(\cdot) = H(\cdot)$ so that $S(z) = z$, and given the boundary condition $c(\underline{z}) = \underline{z}$, we have risk taking at the bottom if $\beta > \alpha(1-\alpha)$. That is, the parameter β just has to be not too small relative to α; for example, if $\alpha = 1/2$ then β must be only bigger than 1/4.

Example 1. Suppose rewards are uniform on $[\varepsilon, 1-\varepsilon]$ and wealth is uniform on $[1, 5]$ and $\alpha = \beta$ so that $\gamma = 1$. We have then

$$S(z) = \varepsilon + \frac{1-2\varepsilon}{4}(z-1)$$

and

$$U(z) = \left(\varepsilon + \int_1^z \varepsilon + \frac{1-2\varepsilon}{4}(t-1)dt\right)^{\alpha} = \left(\frac{(z-1)^2 + 2\varepsilon(-1 + 6z - z^2)}{8}\right)^{\alpha}$$

Take, for example, $\alpha = 0.4$. With a relatively equal distribution of rewards/status S_B, for example with $\varepsilon = 0.25$, all agents are risk averse. However, rewards are more unequal in A with $\varepsilon = 0.1$. Then, $U_A(z)$ is convex on [1,2.44) and is concave on (2.44, 5]. See Figure 1. That is, take an individual with an endowment of about 2.5, then that individual will be risk loving with respect to losses and risk averse with respect to gains. Note that A's equilibrium utility $U_A(z)$ is everywhere lower than $U_B(z)$ and equilibrium performance $x_A(z)$ is everywhere higher ($x_B = (z-1)/2$ and $x_A = (z^2-1)/(2z-1)$). We can also verify that more dispersed wealth makes agents more risk averse. Keeping rewards dispersed with $\varepsilon = 0.1$ but making wealth also more dispersed, so for example wealth is now uniform on $[0.5, 5.5]$, utility will return to being concave at all wealth levels.

4 Reward Equality and Stable Wealth Distributions

We have seen that anticipated participation in a tournament can give individuals an incentive to take gambles. BMW, following Robson (1992), consider distributions of wealth that in contrast are *stable* in the sense that given such a distribution, no agent wishes to gamble and therefore the distribution of wealth does not change. Stable distributions can also be seen as clearing the market for gambling. If the initial wealth distribution was not stable, then there would be an incentive to gamble until the redistribution of wealth resulting from gambling made it stable.[7]

Note that there will generally be many wealth distributions that are stable. Thus, BMW focus on the stable wealth distribution (which they call the "$*$ allocation") that induces risk neutrality at all levels of wealth.[8] Distributions that are less dispersed than the stable distribution will induce gambling (something that we formalise below). Thus, this stable distribution represents an upper bound on sustainable equality of wealth.[9] So, let us call it the *most equal stable distribution* or MESD. I start with formal definitions.

Definition 1. A stable wealth distribution is a distribution of wealth $G(z)$ such that, for a given distribution of rewards, equilibrium utility $U(z)$ is concave for all z in the support of the distribution $G(z)$.

Definition 2. Consider the set of stable distributions for a fixed average wealth level μ. The most equal stable distribution MESD is a stable wealth distribution $G^*(z)$ such that $U''(z) = 0$ for all z in its support.

The first result in this section gives an important reason it should indeed be called the most equal stable distribution. Any distribution that is even locally more equal is unstable. In contrast, any distribution such that $U''(z)$ is strictly negative could be subject to an increase to equality without becoming unstable.

Proposition 4. *Any distribution of wealth $\hat{G}(z)$ that is locally less dispersed than the MESD $G^*(z)$ is not stable. That is, if $\hat{g}(z) > g^*(z)$ on an interval (z_1, z_2) but $\hat{g}(z) = g^*(z)$ on $[\underline{z}, z_1]$ then $\hat{U}''(z) > 0$ on $(z_1, z_1 + \epsilon)$ for some $\epsilon > 0$.*

Now, if one sets the expression for $U''(z)$ in (8) to zero, this leads to the following differential equation (suppressing arguments)

$$S'(z) = \frac{U_c U_{cc}}{U_s U_{cc} - U_c U_{cs}} = \phi(c(z), S(z)) \tag{13}$$

[7]This approach is not entirely satisfactory in that it does not provide an explicit strategic analysis of the decision to gamble. That is, it does not specifically analyse the game implied by all players choosing gambles, knowing others are making that choice. However, it is the approach taken in the literature following from Robson (1992). Further, a strategic analysis would be technically challenging, though see Cole et al. (2001) and Fang and Noe (2015) for some work in this direction.

[8]They show that marginal utility having a constant value λ, or $U'(z) = U_c(c(z), S(z)) = \lambda$ in current notation, is also a solution to the problem of a utilitarian social planner.

[9]Another reason to focus on this particular stable distribution is given in Ray and Robson (2012) who show that in dynamic model the steady-state distribution of wealth will be equivalent to the most equal stable distribution.

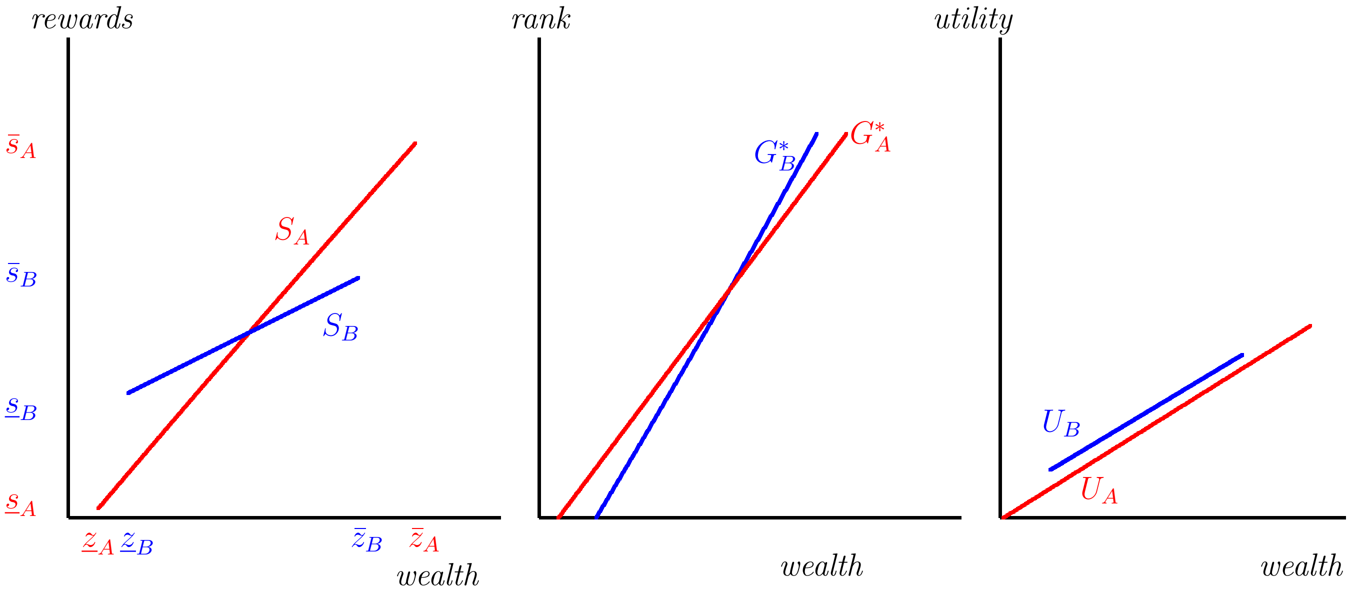

Figure 2: Illustration of Proposition 5: A's reward function S_A is steeper than S_B. To maintain risk neutrality, the MESD in A, G_A^*, must be more dispersed than the MESD in B, G_B^*, with minimum wealth $\underline{z}_A$ being lower and maximum wealth $\bar{z}_A$ being higher than in B.

with boundary condition $S(\underline{z}) = \underline{s}$.[10] Using this differential equation (13) and the differential equation (6) for equilibrium performance, one can write a new differential equation for equilibrium consumption,

$$c'(z) = \frac{U_c U_{cs}}{U_c U_{cs} - U_s U_{cc}} = \psi(c(z), S(z)). \tag{14}$$

Given the boundary condition (7) for equilibrium performance, the boundary condition for the above equation will be $c(\underline{z}) = \underline{z}$. A solution of the two equations simultaneously will provide the MESD. Specifically, the MESD $G^*(z)$ is defined as $G^*(z) = H(S^*(z))$, where $S^*(z)$ is the solution to the equation (13).

One can draw the following comparative statics result. The MESD moves with the distribution of rewards. If rewards become more (less) equal, the minimum level of wealth inequality falls (rises) in the sense of second order stochastic dominance.

In what follows, it is assumed that there are different distributions of rewards in societies A and B, $H_A(s)$ and $H_B(s)$ respectively. Under each distribution of rewards, we calculate $S_i^*(z)$ for $i = A, B$, the associated reward function that induces risk neutrality at all wealth levels. I find that a greater dispersion in rewards necessitates a greater dispersion in wealth in order to maintain risk neutrality. An example is illustrated in Figure 2.

Proposition 5. *Assume that the distribution of rewards H_A is more dispersed than H_B, $H_A >_d H_B$, that the minimum reward is lower or $\underline{s}_A < \underline{s}_B$, that the maximum reward is*

[10] Results on differentiability and uniqueness of $S(z)$ can be found in the supplementary materials.

higher $\bar{s}_A > \bar{s}_B$, assume the mean reward and mean initial endowments are the same in A and B, and assume A1-A3. Then, A's MESD wealth distribution, G^*_A, is more dispersed in terms of second order stochastic dominance than the MESD in B, G^*_B.

This has an important implication. If we consider a sequence of distributions of rewards each progressively more equal than the previous, then the corresponding distributions of wealth would also become progressively more equal. Thus, despite the earlier results of BMW and Ray and Robson (2012), it is thus possible to sustain an equal society, even in the presence of status competition, provided there is an equality in terms of status rewards in society.

Finally, one can combine Propositions 3 and 5 to arrive at the following corollary that is extremely important for understanding the empirical implications of these theoretical results. Define the overall wealth distribution to be any combination of the initial wealth distribution $G(z)$ and its corresponding MESD $G^*(z)$. For example, the young in society would have their initial endowments, whilst the distribution of wealth amongst old, those who have already gambled, is determined by the MESD. Then, consider two societies where there is no difference between them in terms of average endowment or average reward, but levels of inequality can vary.

Corollary 1. *(a) Take two societies A and B that have the same initial distribution of endowments $G(z)$ but A's rewards are more dispersed, $H_A >_d H_B$. Hence, the MESD in A is more unequal in terms of second order stochastic dominance by Proposition 5 and there is more risk taking in A by Proposition 3. Thus, the cross-society correlation between risk taking and inequality of overall wealth is positive.*
(b) Take two societies A and B that have the same distribution of rewards $H(s)$ but A's initial endowments are more dispersed, $G_A >_d G_B$. The MESD in A and B will be the same and there is more risk taking in B by Proposition 3. Thus, the cross-society correlation between risk taking and inequality of overall wealth is negative.

Simply put, this suggests that if differences in reward inequality across societies are greater than differences in inequality in initial endowments, then risk taking can be more common in societies with greater wealth inequality. But, if this is reversed, so that differences in wealth inequality are bigger, then there can be a negative relationship between risk taking and wealth inequality across countries.

4.1 Cobb-Douglas

Assume Cobb-Douglas preferences $U(c, s) = c^\alpha s^\beta$, then the differential equations (13) and (14) become respectively

$$S'(z) = \frac{\alpha(1-\alpha)S(z)}{\beta c(z)};\ c'(z) = \alpha.$$

This implies that performance and consumption are linear in wealth, specifically $x(z) = (1-\alpha)(z-\underline{z})$ and $c(z) = \alpha z + (1-\alpha)\underline{z}$. This in turn can be used to solve for $S^*(z)$:

$$S^*(z) = A[c(z)]^{(1-\alpha)/\beta} = A(\alpha z + (1-\alpha)\underline{z})^{(1-\alpha)/\beta},$$

where A is a constant of integration. One can check that this implies $U(z) = A^\beta c(z)$ which is linear as required.

Example 2. Assume that rewards are distributed uniformly on $[\varepsilon, 1-\varepsilon]$. Assume further that $\alpha = \beta = 1/2$ (of course, this means that the utility function is not strictly concave, but as we will see it makes everything conveniently linear). Then, given mean wealth of $1/2$, the unique distribution $G^*(z)$ that solves for S^* is

$$G^*(z) = \frac{(1-\varepsilon)z - \varepsilon/2}{1-2\varepsilon}.$$

That is, it is uniform on $[\varepsilon/(2(1-\varepsilon)), (2-3\varepsilon)/(2(1-\varepsilon))]$. We have

$$S^*(z) = (1-\varepsilon)z + \varepsilon/2,\ U^*(z) = \frac{\varepsilon/2 + (1-\varepsilon)z}{\sqrt{2(1-\varepsilon)}}.$$

Clearly, a decrease in ε makes the distribution of rewards more dispersed. It will also make the equilibrium distribution of wealth $G^*(z)$ more dispersed. Equally, a more equal distribution of rewards, implies a more equal stable distribution of wealth. Indeed, as ε approaches $1/2$, then both the distribution of rewards and the distribution of wealth become entirely concentrated at $1/2$.

5 Conclusions

This paper has reexamined the link between inequality and risk taking behaviour. While risk taking is predicted to be decreasing with initial wealth inequality, it is also predicted to be increasing with inequality in rewards. I also give conditions under which those with low initial wealth, those at the back of the field before the tournament, will be risk taking.

The idea that low ranked agents may have an incentive to gamble has an apparent similarity to the idea of "gambling for resurrection", in which agents who are near to bankruptcy have an incentive to gamble because any downside losses would be truncated. See, for example, Gollier et al. (1997). However, none of the results in this paper depend on any such mechanism. Here agents will take fair bets, even though they will have to suffer the downside in full.[11] Clearly, if limited liability were a possibility, then the incentive to gamble would be increased.

Should such risk taking be encouraged? This paper has no formal results on welfare. The main reason is that results in this direction already exist. As is well known, in

[11]It is true that the reward level cannot fall below the minimum $\underline{s}$ and in that sense there is a form of limited liability. The main results here do not depend on this.

tournaments like this all competitors can be made better off by a social planner simply imposing the rewards assortatively, which prevents wasteful competition. The implications of this policy for risk taking are straightforward: without the tournament there would be no additional competitive incentive to take risk, and risk attitudes would be determined by underlying preferences which are assumed risk averse.

A remaining puzzle is what exactly is predicted relationship between inequality and risk taking. This paper shows that greater inequality in wealth reduces risk taking but also that greater reward inequality increases it. The results on stable wealth distributions provide a condition under which overall relationship between inequality and risk taking will be positive. If differences in reward inequality across societies are greater than differences in inequality in initial endowments, then risk taking will be greater in societies with greater wealth inequality.

There are particular grounds for hope for empirical work if reward inequality can be separately identified. The current model requires a distribution of rewards or status outcomes that is exogenous and independent of the distribution of wealth. Marriage arrangements are one example of how rewards could vary in this way. Further, while the underlying causation for these differing customs may be economic, such institutions change slowly and most individuals would plausibly take them as fixed, and thus an analyst can hope to treat them as exogenous. That is, if long run social arrangements ("culture") cause risk taking that results in inequality in wealth, then there is a hope for meaningful empirical testing. This is a fascinating possibility which merits further investigation.

Appendix

Proof of Proposition 1: This proof follows that of Proposition 1 of Hopkins and Kornienko (2004). A sketch is as follows. Given $U_{cs} \geq 0$, best replies are (weakly) increasing in z. Given the tie breaking rule (2), a symmetric equilibrium strategy must in fact be strictly increasing. If the equilibrium strategy is strictly increasing then it can be shown to be continuous and, furthermore, differentiable. Thus, it satisfies the differential equation (6). This has a unique solution by the fundamental theorem of differential equations.

It is also worth emphasising that the first order condition (5) is a maximum (despite the equilibrium utility function $U(z)$ potentially being convex), as if all others adopt the proposed equilibrium strategy, an agent's utility is pseudoconcave in x for each individual. That is, $U(z - x, H^{-1}(F(x)))$ is increasing in x for x less than the equilibrium choice $x(z)$ and is decreasing in x for x greater than $x(z)$. That is, utility $U(z - x, H^{-1}(F(x)))$ is pseudoconcave in performance x. To show this, if all agents adopt a strictly increasing strategy $x(z)$ then an individual's utility can be written as $U(z-x, H^{-1}(F(x)))$ and $\partial U/\partial x = -U_c(z-x, H^{-1}(F(x)))+U_s(z-x, H^{-1}(F(x)))f(x)/h(\cdot)$. Then, one has $\partial^2 U/\partial x \partial z = -U_{cc} + U_{cs} f(x)/h(\cdot) > 0$. Take $\breve{x} < x(z)$ and let $\breve{z}$ be such that $x(\breve{z}) = \breve{x}$, so that $\breve{z} < z$. Hence, for any $\breve{x} < x(z)$, $dU(z - \breve{x}, H^{-1}(F(\breve{x})))/dx \geq dU(\breve{z} - \breve{x}, H^{-1}(F(\breve{x})))/dx = 0$. Thus, utility is increasing in x for x below the equilibrium choice $x(z)$. A similar argument can establish that it is decreasing in x for x above $x(z)$.

Finally, the boundary condition (7) must hold as the agent with lowest wealth $\underline{z}$ in a symmetric equilibrium has status $S(\underline{z}) = \underline{s}$ and thus chooses performance x to maximise $U(\underline{z} - x, \underline{s})$. Clearly, the optimal choice of performance for the agent with wealth $\underline{z}$ is zero. □

Proof of Proposition 2: If $G(\cdot) = H(\cdot)$ then $S(z) = z$ and $S'(z) = 1$. Consider any point $\hat{z}$ such that $U''(\hat{z}) = 0$. Then, in the additively separable case, applying $S'(z) = 1$ to (8), one has $U''(\hat{z}) = U_{cc}(1 - x'(\hat{z})) = 0$. Thus, it holds that $1 - x'(\hat{z}) = c'(\hat{z}) = 0$. If $U'''(\hat{z}) < 0$, then there can only be a single crossing of the zero line by $U''(z)$ and from above. One has $U'''(\hat{z}) = U_{cc}c''(\hat{z})$ given $c'(\hat{z}) = 0$. Given $c'(\hat{z}) = 0$ and $U''(\hat{z}) = 0$, one has $c''(\hat{z}) = -U_{ss}/U_c > 0$ so that $U_{cc}c''(\hat{z}) < 0$. The result is established.

Turning to the Cobb-Douglas case, one can see that after some calculation that $U''(z) = 0$ if $c(z)/z = \alpha(1-\alpha)/\beta$. One can then calculate $U'''(\hat{z})$, given $U''(\hat{z}) = 0$, as $c^{\alpha-2}(\hat{z})\hat{z}^{\beta-1}(\beta c'(\hat{z}) - \alpha(1-\alpha))$. Further, if $U''(\hat{z}) = 0$, then $c'(\hat{z}) = 1 - \beta/\alpha \times c(z)/z = \alpha$, so that $U'''(\hat{z}) < 0$ if $\alpha + \beta < 1$, which holds by assumption because $U(c,s) = c^\alpha s^\beta$ is assumed concave.

Finally, given the initial condition that $x(\underline{z}) = 0$, then one has

$$U''(\underline{z}^+) = U_{cc}(\underline{z}, \underline{z}) \left(1 - \frac{U_s(\underline{z}, \underline{z})}{U_c(\underline{z}, \underline{z})}\right) + U_{cs}(\underline{z}, \underline{z}) = U_{cs}(\underline{z}, \underline{z}) > 0,$$

with the last step following from the symmetry assumption. □

Proof of Proposition 3: One has from (7) that $c(\underline{z}) = \underline{z}$, so that from (8) it follows that the right derivative of $U''(z)$ at $\underline{z}$ is

$$U''(\underline{z}^+) = U_{cc}(\underline{z}, \underline{s})(1 - x'(\underline{z}^+)) + U_{cs}(\underline{z}, \underline{s})S'(\underline{z}^+) \tag{15}$$

and from (6) that the right derivative of $x(z)$ at $\underline{z}$ is

$$x'(\underline{z}^+) = \frac{U_s(\underline{z}, \underline{s})S'(\underline{z})}{U_c(\underline{z}, \underline{s})}. \tag{16}$$

It can be calculated that

$$\frac{\partial x'(\underline{z}^+)}{\partial \underline{s}} = \frac{U_{ss}U_c - U_{cs}U_s}{U_c^2} S'(\underline{z}^+) < 0. \tag{17}$$

This implies that $x'(\underline{z}^+)$ is monotone in $\underline{s}$.

(a) First, consider the difference in dispersion of rewards. The second derivative (from the right) of the utility function for the poorest agent is $U''(\underline{z}^+) = U_{cc}(\underline{z}, \underline{s})(1 - x'(\underline{z}^+)) + U_{cs}(\underline{z}, \underline{s})S'(\underline{z}^+)$ and the first is $U'(\underline{z}^+) = U_c(\underline{z}, \underline{s})$. Because $c(\underline{z}) = \underline{z}$ and for the moment holding $S(\underline{z}) = \underline{s}$ constant, the only way that either $U''(\underline{z}^+)$ or $AR(\underline{z}^+)$, i.e. $AR(z)$ evaluated at $\underline{z}$ using right derivatives, can change is in terms of S' and x'. The dispersive order, by its definition (10), implies that $h_A(H_A^{-1}(r)) < h_B(H_B^{-1}(r))$ for $r \in [0, 1]$. Now,

$S'(z) = g(z)/h(S(z)) = g(z)/h(H^{-1}(r))$. Thus, given $g(z)$ is the same, the dispersive order implies that $S'_A(z) > S'_B(z)$ for all $z \in [\underline{z}, \bar{z}]$. It is easy to verify that if $S'(\underline{z}^+)$ is higher then so will $x'(\underline{z}^+)$ as given in (16). Thus, both $S'(\underline{z}^+)$ and $\underline{(}z^+)$ are also higher leading to an increase in $U''(\underline{z}^+)$ and a decrease in $AR(\underline{z}^+)$, holding $\underline{s}$ constant.

Second, consider the difference in $\underline{s}$. One has, keeping $S'(\underline{z}^+)$ constant,

$$\frac{\partial U''(\underline{z}^+)}{\partial \underline{s}} = U_{ccs}(1 - x'(\underline{z}^+)) + U_{css}S'(\underline{z}^+) - \frac{\partial x'(\underline{z}^+)}{\partial \underline{s}} U_{cc},$$

which, given A1, is certainly negative where $x'(\underline{z}^+) < 1$. From (17), $x'(\underline{z}^+)$ is monotone in $\underline{s}$. Thus, as noted, there must be a value s_0 such that if $\underline{s} = s_0$ then $x'(\underline{z}^+) = 1$. If, as assumed, $\underline{s}_B$ is such that $U''(\underline{z}^+) \leq 0$, then $\underline{s}_B > s_0$ and $x'(\underline{z}^+) \leq 1$. If also $\underline{s}_A > s_0$, then it follows that in A $U''(\underline{z}^+)$ will be greater than in B, as $U''(\underline{z}^+)$ is monotone in $\underline{s}$ on $(s_0, \underline{s}_B)$. If $\underline{s}_A \leq s_0$, then $U''(\underline{z}^+)$ is greater in A. Turning to $AR(z)$, one has suppressing arguments,

$$\frac{\partial AR(\underline{z}^+)}{\partial \underline{s}} = -\frac{((1 - x')U_{ccs} + U_{ccs}S' - U_{cc}\partial x'/\partial \underline{s})\, U_c - U_{cc}\,(U_{cc}(1 - x') + U_{cs}S')}{U_c^2}, \quad (18)$$

which is positive given assumption that $U''(\underline{z}^+) < 0$ so that $x' \leq 1$, the earlier finding that $\partial x'/\partial \underline{s} < 0$, and A1. Thus, the effect on $U''(\underline{z}^+)$ and $AR(\underline{z}^+)$ of a decrease in $\underline{s}$ is positive and negative respectively.

(b) We again consider $U''(\underline{z}^+)$ as given in (15) and show that $U''(\underline{z}_A^+) < U''(\underline{z}_B^+)$. First, by the dispersive order we have $g_A(\underline{z}_A) = g_A(G_A^{-1}(0)) < g_B(G_B^{-1}(0))$ and so $S'(\underline{z}_A^+) = g_A(\underline{z}_A)/h(\underline{s}) < g_B(\underline{z}_B)/h(\underline{s}) = S'(\underline{z}_B^+)$ and thus the greater dispersion in itself decreases $U''(\underline{z}^+)$. Second, we have to consider the effect of the change in wealth as by assumption $\underline{z}_A \leq \underline{z}_B$. The effect from wealth will also be negative if the following holds

$$U_{ccc}(1 - x'(\underline{z}^+)) + U_{ccs}S'(\underline{z}^+) - \frac{\partial x'(\underline{z}^+)}{\partial \underline{z}} U_{cc} > 0 \quad (19)$$

From (16) it can be calculated that, holding $S'(\underline{z}^+)$ constant,

$$\frac{\partial x'(\underline{z}^+)}{\partial \underline{z}} = \frac{U_{sc}U_c - U_{cc}U_s}{U_c^2} S'(\underline{z}^+) > 0. \quad (20)$$

However, the possibility that $U_{ccs} < 0$ means that the inequality (19) may not hold. But if $U''(\underline{z}^+) \leq 0$ then $x'(\underline{z}^+) < 1$ and $S'(\underline{z}^+) \leq -(1 - x'(\underline{z}^+))U_{cc}/U_{cs}$. Thus, the left hand side of (19) is greater or equal than

$$(1 - x'(\underline{z}^+))(U_{ccc} - U_{ccs}\frac{U_{cc}}{U_{cs}}) - \frac{\partial x'(\underline{z}^+)}{\partial \underline{z}} U_{cc}.$$

Given A3, it follows that $U_{ccc} - U_{ccs}U_{cc}/U_{cs} \geq 0$ and the inequality (19) holds. Thus, both the effect from lower wealth and higher dispersion lead $U''(\underline{z}^+)$ to decrease. $U''_A(G_A^{-1}(r))$

will be lower on an interval $(0, \hat{r})$ by continuity. Turning to $AR(z)$, we have suppressing arguments

$$\frac{\partial AR(\underline{z}^+)}{\partial \underline{z}} = \frac{(x'-1)(U_{ccc}U_c - U_{cc}^2) + U_c U_{cc}\partial x'/\partial \underline{z} + S'(U_{cc}U_{cs} - U_{ccs}U_c)}{U_c^2}, \tag{21}$$

which is negative given (20), and A2. Thus, the decrease in $\underline{z}$ raises $AR(\underline{z}^+)$. □

Proof of Proposition 4: Second, start with a distribution of wealth $G^*(z)$ such that $U''(z) = 0$ on its support $[\underline{z}, \bar{z}]$. Then suppose there is a local decrease in dispersion of wealth so that $\hat{g}(z) > g^*(z)$ on an interval (z_1, z_2) but $\hat{g}(z) = g^*(z)$ on $[\underline{z}, z_1]$. Thus because $g^*(z)$ is unchanged on $[\underline{z}, z_1]$ we have $\hat{x}(z_1) = x(z_1)$ and $\hat{U}''(z_1) = U''(z_1) = 0$. However, the increase in $\hat{g}$ on (z_1, z_2) requires (generically) that $\hat{g}'(z_1) = \hat{G}''(z_1) > G^{*\prime\prime}(z_1)$. Since by definition $S(z) = H^{-1}(G(z))$, and $H(s)$ is unchanged, we have $\hat{S}''(z_1) > S''(z_1)$. Differentiating $x'(z)$ as given in (6), one obtains that $x''(z)$ is increasing in $S''(z)$ (but its other arguments $x'(z_1)$, $S'(z_1)$, $S(z_1)$ and $x(z_1)$ are unchanged). Thus, $\hat{x}''(z_1) > x''(z_1)$. Differentiating $U''(z)$ a further time, it is easy to verify that $U'''(z)$ is increasing in both $x''(z)$ and $S''(z)$ - again its other arguments are unchanged. So we have $\hat{U}'''(z_1) > U'''(z_1) = 0$. So $\hat{U}''(z) > 0$ on $(z_1, z_1 + \epsilon)$ for some $\epsilon > 0$. So, as claimed, the wealth distribution $\hat{G}(z)$ is not stable. □

Proof of Proposition 5: By the dispersive order we have $h_A(H_A^{-1}(r)) < h_B(H_B^{-1}(r))$. Together with our other assumptions on minimum and maximum rewards, it implies that $H_A(s)$ and $H_B(s)$ are single crossing, with a unique reward $\hat{s}$ such that $H_A(\hat{s}) = H_B(\hat{s}) = \hat{r}$.

I first establish that $\underline{z}_A < \underline{z}_B$, the minimum MESD wealth level is lower in A. Suppose not so that $\underline{z}_A \geq \underline{z}_B$. Then as solutions $(c(z), S(z))$ to the differential equation system cannot cross on the (c, S) plane, given our initial conditions that $c_A(\underline{z}_A) = \underline{z}_A \geq \underline{z}_B = c_B(\underline{z}_B)$ we have $c_A > c_B$ for a fixed level of S. Thus, given $\partial\phi(c, s)/\partial c < 0$, as shown in (23), we would have $S'_A(z) < S'_B(z)$ at any potential point of crossing of $S_A(z)$ and $S_B(z)$ (graphed alone as a function of z). Since we have $S_A(\underline{z}_A) = \underline{s}_A < \underline{s}_B = S_B(\underline{z}_B)$, S_A would never in fact cross S_B so that $S_A(z) < S_B(z)$ everywhere.

If indeed $S_A(z) < S_B(z)$ everywhere, I show that the implied wealth distributions, $G_A^*(z) = H_A(S_A(z))$ and $G_B^*(z) = H_B(S_B(z))$ do not have the same mean, which is a contradiction. We look at the inverse distribution functions $G_i^{-1}(r)$ on $(0, \hat{r})$. Since we have $G_A^{-1}(0) = \underline{z}_A \geq \underline{z}_B = G_B^{-1}(0)$, at the first crossing $G_A^{-1}(r)$ must cross $G_B^{-1}(z)$ from above, but, because $S'(z) = g(z)/h(S(z))$, setting $z = G^{-1}(r)$, we have from (13),

$$g_i(G_i^{-1}(r)) = h_i(H_i^{-1}(r))\phi(c(G_i^{-1}(r)), H_i^{-1}(r))$$

for $i = \{a, p\}$. Now, $h_A(\cdot) < h_B(\cdot)$ by the dispersive order. Further,

$$\frac{\partial\phi(c, s)}{\partial c} = \frac{-U_c U_{cs} U_{cc}^2 + U_{cc}^3 U_s - U_c^2(U_{ccc}U_{cs} - U_{cc}U_{ccs})}{(U_s U_{cc} - U_c U_{cs})^2} < 0, \tag{22}$$

(this follows from the assumptions A1 and A3) but also we have from A1,

$$\frac{\partial\phi(c, s)}{\partial s} = \frac{U_c^2(U_{css}U_{cc} - U_{ccs}U_{cs}) + U_{cc}^2(U_s U_{cs} - U_c U_{ss})}{(U_s U_{cc} - U_c U_{cs})^2} > 0.$$

Now, because we have $c_A > c_N$ and for $r \in (0, \hat{r})$, $H_A^{-1}(r) < H_B^{-1}(r)$, it follows that $g_A(G_A^{-1}(r)) < g_B(G_B^{-1}(r))$. But the slope of $G_i^{-1}(r)$ is $1/g_i(G_i^{-1}(r))$. So a $G_A^{-1}(r)$ crossing $G_B^{-1}(z)$ from above is not possible. Finally, for $r \geq \hat{r}$ we have, $H_A(s) < H_B(s)$, so that given $S_A(z) < S_B(z)$ we have $H_A(S_A(z)) = G_A^*(z) < G_B^*(z) = H_B(S_B(z))$. So, if $S_A(z) < S_B(z)$ everywhere, then $G_A^{-1}(r) > G_B^{-1}(r)$ and $G_A^*(z) < G_B^*(z)$ everywhere, and the two distributions cannot have the same mean.

So, we have $\underline{z}_A < \underline{z}_B$, the minimum MESD wealth level is lower in A. Given that solutions $(c(z), S(z))$ to the differential equation system cannot cross on the (c, S) plane, given our initial conditions $c_A(\underline{z}_A) = \underline{z}_A$, $S_A(\underline{z}_A) = \underline{s}_A$ and $c_B(\underline{z}_B) = \underline{z}_B$, $S_B(\underline{z}_B) = \underline{s}_B$ respectively, one has either $c_A < c_B$ for a given level of S or $c_A > c_B$. If the latter, then by the above argument $S_A(z)$ and $S_B(z)$ would never cross, so it must be that $c_A < c_B$ at a fixed level of s. Turning to solutions $S_A(z)$ and $S_B(z)$ graphed as a function of z alone, points of crossing of $S_A(z)$ and $S_B(z)$ are possible. However, because $c_B > c_A$ and $\phi(c, s)/\partial c < 0$ (as shown previously), then $S_A'(z) > S_B'(z)$ at any such crossing. Thus, there is at most one crossing where $S_A(z) = S_B(z)$. There must be a crossing by the above argument that if $S_A(z) < S_B(z)$ everywhere, the implied wealth distributions cannot have the same mean. Hence there is a unique crossing.

But this also implies that the inverses of $S_A(z)$ and $S_B(z)$ are also single crossing. That is, the two functions $G_A^{-1}(H_A(s)) = S_A^{-1}(s)$ and $G_B^{-1}(H_B(s)) = S_B^{-1}(s)$ are single crossing, with $G_A^{-1}(H_A(\hat{s})) = \hat{z} = G_B^{-1}(H_B(\hat{s}))$. But if the inverse of the distribution functions are single-crossing then so are distribution functions $G_A^*(z)$ and $G_B^*(z)$ with clearly $G_A^*(z) > G_B^*(z)$ on $(\underline{z}_A, \hat{z})$ and $G_A^*(z) < G_B^*(z)$ on $(\hat{z}, \bar{z}_A)$. Single crossing of this form with an equal mean implies second order stochastic dominance (Wolfstetter, 1999, Proposition 4.6). □

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Supplementary Content: Online Appendix. This provides some additional results that characterise further the Most Equal Stable Distribution (MESD) introduced in Section 4 of the main paper.

Proposition A. *First, any MESD $G^*(z)$ is differentiable and without gaps in its support. Second, for a given distribution of rewards $H(s)$, there is a unique solution $(c^*(z), S^*(z))$ to the simultaneous differential equation system (c', S') as defined by (13) and (14), such that $U(z) = U(c^*(z), S^*(z))$ is linear in z for all $z \in [\underline{z}, \bar{z}]$ so that $U''(z) = 0$ for all wealth levels in $(\underline{z}, \bar{z})$. Third, assume A1-A3. Then, for fixed mean wealth μ, there is a unique distribution of wealth $G^*(z)$ such that $H^{-1}(G^*(z)) = S^*(z)$.*

Proof: First, suppose $G^*(z)$ is not differentiable at some point $\hat{z}$ then, without loss of generality, assume that $\lim_{z \downarrow \hat{z}} g(z) = g(\hat{z}^+) > g(z)$ for all z in some interval $(z_0, \hat{z})$. Then, $S'(\hat{z}^+) > S'(\hat{z}^-)$. Thus, because $U''(z)$ is increasing in $S'(z)$ (see (8)) and is otherwise continuous, one has that $U''(\hat{z}^-) < U''(\hat{z}^+)$. Clearly, it is not possible for both $U''(\hat{z}^+) = 0$ and $U''(\hat{z}^-) = 0$. Thus, the first result is proved. Now, suppose that $g(z) = 0$ on some interval $[z_1, z_2]$ on the interior of $[\underline{z}, \bar{z}]$. Because it has been shown that $G^*(z)$ is differentiable, $S'(z_2) = g(z_2)/h(H^{-1}(G(z_2))) = 0$. Then, as $S'(z_2) = 0$ and thus $x'(z_2^+) = 0$, one has $U''(z_2^+) < 0$. Thus, this distribution does not induce $U''(z) = 0$ on its support.

Second, the definition of the differential equation (13), the solution $(c^*(z), S^*(z))$ implies that $U''(c^*(z), S^*(z)) = 0$. Thus, $U(z)$ is linear as $U'(z) = U_c > 0$. Such a solution must exist by the fundamental theorem of differential equations because both (13) and (14) are continuously differentiable and bounded. The solution is unique for a given initial condition, that is, for a given minimum wealth level $\underline{z}$. That is, there is a family of distributions $\langle G_i \rangle$ that each satisfy $H^{-1}(G_i(z)) = S^*(z)$, each corresponding to a different level of minimum wealth $\underline{z}_i$.

Third, I prove that in this family, average wealth μ is strictly increasing in $\underline{z}$. The equation system (c', S') as defined by (13) and (14) is autonomous, that is a function of c and S alone and only a function of z through c and S. It thus follows by fundamental theory of differential equations, that two solution curves $(c(z), S(z))$ cannot cross on the (c, S) plane. So, given two solutions with initial conditions $(\underline{z}_i, \underline{s})$ and $(\underline{z}_j, \underline{s})$ for some $\underline{z}_i < \underline{z}_j$, it follows that $c_j > c_i$ for any given value of S. Now consider the two associated solutions for rewards, $S_i(z)$ and $S_j(z)$ on the (z, S) plane. I claim there is no value of z such that $S_i(z) = S_j(z)$. Suppose not, then because $S_i(\underline{z}_i) = S_j(\underline{z}_j) = \underline{s}$ and $\underline{z}_j > \underline{z}_i$, at the first such crossing S_j must cross S_i from below. But from (13) one has,

$$\frac{\partial \phi(c, s)}{\partial c} = \frac{-U_c U_{cs} U_{cc}^2 + U_{cc}^3 U_s - U_c^2 (U_{ccc} U_{cs} - U_{cc} U_{ccs})}{(U_s U_{cc} - U_c U_{cs})^2} < 0, \tag{23}$$

(this follows from A1 and A3) and as $c_j > c_i$, we have $S_i' > S_j'$ at such a point of crossing. Thus, such a crossing is not possible and so, given distinct initial values of endowments $\underline{z}_j > \underline{z}_i$, it must hold that $S_i(z) < S_j(z)$ for all z. Hence, for a fixed $H(s)$, we have $G_j(z) = H^{-1}(S_j(z)) < H^{-1}(S_i(z)) = G_i(z)$. That is, $G_j(z)$ stochastically dominates $G_i(z)$ and $\mu_j > \mu_i$. This implies, that for any given level of average wealth μ, there exists a unique $\underline{z}$ such that the mean of $G^*(z)$ is μ. $\square$