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Citation for published version:

Connolly, DP, Kouroussis, G, Laghrouche, O, Ho, CL & Forde, MC 2015, 'Benchmarking railway vibrations - Track, vehicle, ground and building effects', *Construction and Building Materials*, vol. 92, pp. 64–81. <https://doi.org/10.1016/j.conbuildmat.2014.07.042>

Digital Object Identifier (DOI):

[10.1016/j.conbuildmat.2014.07.042](https://doi.org/10.1016/j.conbuildmat.2014.07.042)

Link:

[Link to publication record in Edinburgh Research Explorer](#)

Document Version:

Peer reviewed version

Published In:

Construction and Building Materials

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Benchmarking Railway Vibrations - Track, Vehicle, Ground and Building Effects

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In : Construction and Building Materials. 92, p. 64–81

Keywords: *Passenger comfort, railway track dynamics, underground train, high speed rail, urban tram, critical velocity, structural vibration, railway vibration, environmental assessment, EIA, track displacements, soil parameter assessment, site investigation*

Abstract

This paper reviews, synthesises and benchmarks new understandings relating to railway vibrations. Firstly, the effect of vibrations on passenger comfort is evaluated, followed by its effect on track performance. Then ground-borne vibration is discussed along with its effect on the structural response of buildings near railway lines. There is discussion of the most suitable mathematical and numerical modelling strategies for railway vibration simulation, along with mitigation strategies. Regarding ground borne vibration, structural amplification is discussed and how vibration mitigation strategies can be implemented. There is also a focus on determining how ‘critical velocity’ and ‘track critical velocity’ are evaluated – with the aim of providing clear design guidelines related to Rayleigh wave velocity. To aid this, conventional site investigation data is reviewed and related to critical velocity calculations. The aim is to provide new thinking on how to predict critical velocity from readily available conventional site investigation data.

Introduction

Over the last 50 years there has been an increasing demand for more access to the railways for both passenger and freight carriage. One emerging way to satisfy this demand for increased capacity has been to create high speed passenger routes – thus freeing up capacity on the existing networks for classic rail serving urban conurbations and providing extra capacity for freight trains. Figure 1 illustrates the evolution of railway train speeds since the 1960’s.

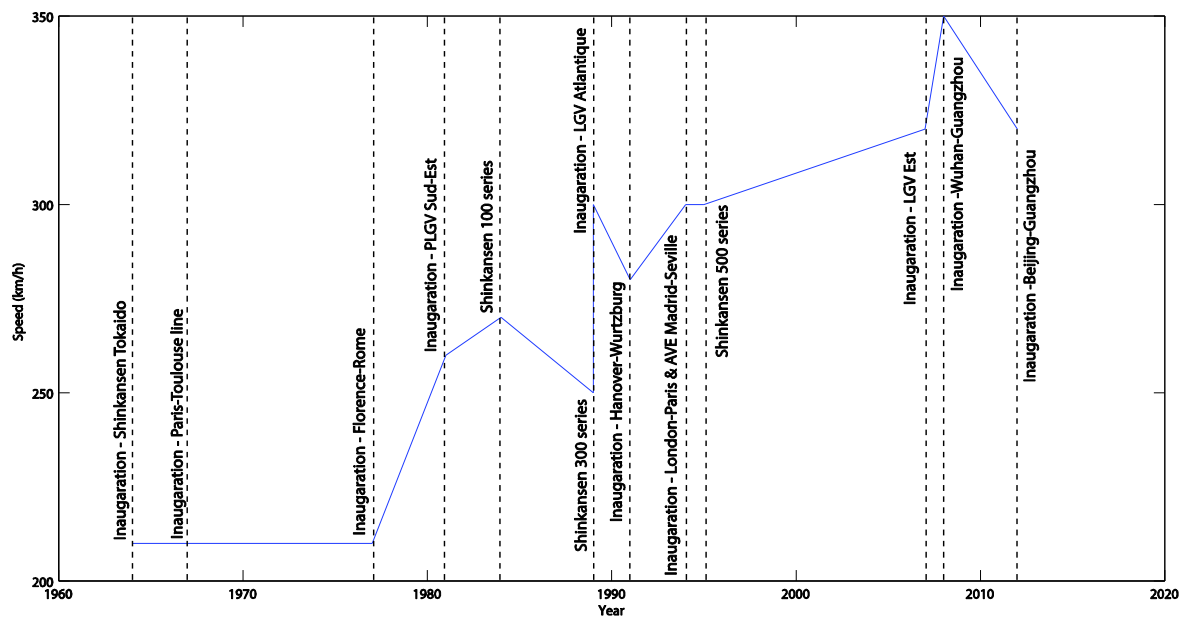


Figure 1 - Historical maximum train speed timeline (data taken from [1])

With the creation of this extra capacity, much at high speeds - railway vibration is now a growing engineering challenge due to the higher speeds and heavier loads, in close proximity to densely populated urban environments. These faster or heavier trains impart greater forces into the track and can result in elevated vibration levels within both train and track, thus effecting passenger safety, maintenance costs and passenger comfort. In addition, when these vibrations propagate outward from the track they interact with their surrounding environment, which can cause negative side effects, particularly in urbanised areas.

This paper attempts to provide a comprehensive, detailed review of the vibrations generated within the train, track, ground and nearby structures, with each component being reviewed separately. For each element, vibration generation/propagation is described, along with practical considerations, mitigation possibilities and potential modelling approaches. It aims to do so in a manner that is useful for both academics and practitioners.

Firstly, as the wheel-rail interface is the source of vibration, the mechanisms that generate vibrations are discussed, along with general wave propagation theory. Then vibration propagation within the train vehicle is reviewed with a focus on passenger comfort which is becoming increasingly important on new lines. Next the role of the track is considered with a focus on numerical modelling and common vibration mitigation procedures. The role of the ground in the transfer of vibration from track to nearby structures is also analysed with a focus on modelling and critical velocity effects. Lastly, building vibration and the generation of in-door noise is reviewed. It should be noted that this work is complementary to other state of the art reviews into railway vibration, as listed in Table 1 below.

| Topic of review paper | Reference |
|-----------------------|-----------|
|-----------------------|-----------|

| | |
|--|--------------------|
| Railway vibration - modelling approaches | [2], [3], [4] |
| Track settlement | [5] |
| Track loading conditions | [6], [7], [8], [9] |
| Vehicle dynamics | [10] |
| Track vibration mitigation | [11] |
| Tunnel vibration | [12] |
| Building vibration | [13], [14] |
| Passenger comfort | [15] |

Table 1 - Topics of previous review papers

Background

Vibration generation

Railway vibration and noise arise from the forces generated at the contact point between train wheel and the rail. These forces can be broken down into their quasi-static and dynamic components [16]. Quasi-static forces arise from the train weight and are independent of train speed. They dominate the track response and near field, at distances up to one quarter of a wavelength [17]. If the track is considered as a Euler beam resting on an elastic foundation, the quasi static deflection of a typical track is shown in Figure 2 and can be calculated analytically by

$$w(x, t) = w(x - v_0 t) = [\cos(\beta|x - v_0 t|) + \sin(\beta|x - v_0 t|)] \frac{P + m_w g}{8E_r I_r \beta^3} e^{-\beta|x - v_0 t|}$$

Where $w(x,t)$ is the track deflection at position 'x' and time 't'. ' E_r ' and ' I_r ' are the Young's modulus and second moment of area of the Euler beam respectively. The train load is of constant force 'P', moving at a constant speed ' v_0 ', and ' K_f ' is the stiffness per unit length of the Winkler foundation. β is defined as:

$$\beta = \sqrt[4]{\frac{K_f}{4E_r I_r}}$$

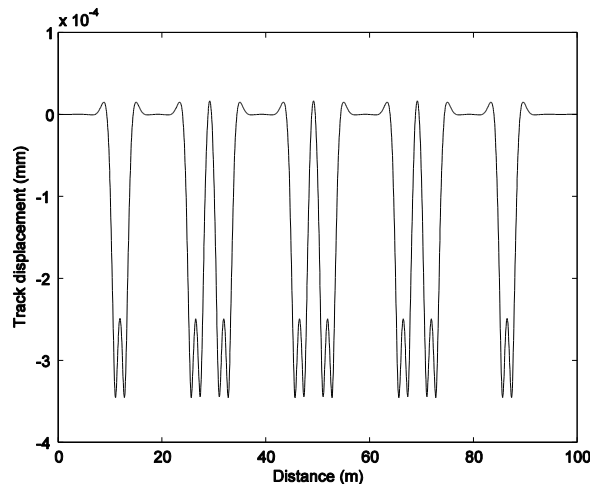


Figure 2 – Quasi-static track deflection

In contrast, dynamic excitation is speed dependent and arises from factors such as changes in stiffness due to sleeper placement, irregularities at the wheel/rail interface and the soil support conditions. Dynamic excitation from irregularities and rail joints is slowly becoming a less influential excitation mechanism due to the widespread use of continuously welded rails and improved track maintenance.

Although all railway lines generate vibration, differences between train/track features mean that the characteristics of the generated vibrations vary widely. Historically, four cases have been of particular concern:

Underground trains – generate vibrations with a higher frequency spectrum than over-ground tracks. Although noise is also generated, it is contained within the tunnel and therefore vibrations are of primary concern for structures located above the line.

High speed trains – generate elevated amplitude vibrations due to their increased speeds. Additionally, if their speed becomes comparable to the wave speed in the supporting soil then vibration levels may become magnified.

Urban tramways – Generate relatively low amplitude vibrations, however their close proximity to buildings can cause negative structural effects. Furthermore, increases in unsprung mass due to the more frequent deployment of low-floor vehicles has exacerbated vibration problems.

Freight trains – generate high amplitude, low frequency vibration (due to their low speed) that can propagate to large distances from the track.

Wave propagation

Upon generation, the vibrations caused at the wheel/rail interface propagate through the track and into the free field in the form of waves. These waves are categorised as either body waves or surface waves. Surface waves travel along a structures (i.e. soil) surface and decay exponentially with depth. Body waves propagate primarily beneath the soil surface. Wave propagation characteristics are shown in Figure 3.

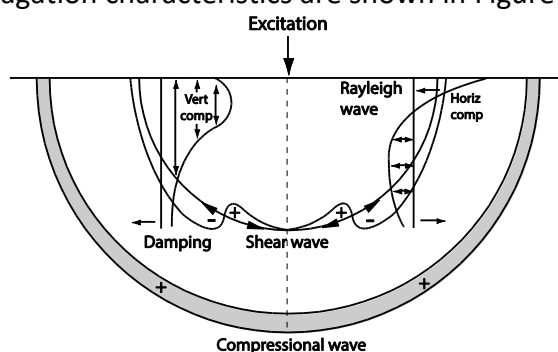


Figure 3 - Seismic wave propagation (slice view) [after 18]

Compressional waves (P-waves) propagate in a longitudinal direction and travel faster than all other types of waves. Shear waves (S-waves) propagate in a transverse direction and although they travel faster than Rayleigh waves, they always travel slower than P-waves. Rayleigh waves are the slowest type of seismic wave, the speed of which can

be approximated using numerous formulas ([19], [20], [21]). The analytical formulas to calculate P and S wave speeds are:

$$C_p = \sqrt{\frac{\lambda + 2\mu}{\rho}}$$

$$C_s = \sqrt{\frac{\mu}{\rho}}$$

Where ρ is density, λ is bulk modulus and μ is the shear modulus (λ and μ are also known as Lamé's parameters).

Although other types of waves are theoretically possible (e.g. Lamb waves in layers and Stoneley waves at interfaces), compressional, shear and Rayleigh are the most common and are the focus of this research. Furthermore, an emphasis is placed on the propagation of Rayleigh waves as they transmit approximately two thirds of the total excitation energy (Rayleigh waves \approx 67%, S-waves \approx 26%, P-waves \approx 7% [18]). Therefore they are most likely to cause negative effects in both the railway track and nearby structures.

There are a wide variety of parameters used to classify soil material. Different parameters are used in different circumstances and determined in different ways. Table 2 lists some of these parameters and their context, where S.I. is Site Investigation:

| Soil parameter | Soil type | Purpose or application | Obtained in traditional S.I. |
|----------------------------|-----------------|--------------------------------------|------------------------------|
| Liquid limit | Clay | Classification | Yes |
| Plastic limit | Clay | Classification | Yes |
| Moisture content | Granular & clay | Classification | Yes |
| Density | Granular & clay | Classification + numerical analysis | Yes |
| Particle size distribution | Granular & clay | Classification | Yes |
| Shear strength | Granular & clay | Design: foundation & slope stability | Yes |
| Over-consolidation ratio | Granular & clay | | No |
| Poisson's ratio | Granular & clay | Numerical analysis | No |
| Young's modulus | Granular & clay | Numerical analysis | No |
| Damping | Granular & clay | Numerical analysis | No |

Table 2 – Material properties

For numerical analysis wave propagation can be efficiently described using four main material properties: Density, Poisson's ratio, Young's modulus and damping. Although more traditionally measured soil characteristics such as moisture content, particle size distribution, liquid and plastic limits, consolidation ratio, etc. affect material characteristics (and thus wave propagation), their effect is usually accounted for by changes in the values of the aforementioned parameters.

The real challenge is that the conventional site investigation does not measure Density, Poisson's ratio, Young's modulus and damping. This aspect will be explored later in the paper and a proposal made to relate conventional site investigation data to required numerical analysis input data.

Review of numerical analysis input data

Density – Measured in a conventional SI - the mass divided by the unit volume of a material. Density typically increases with depth because lower soil layers tend to have experienced elevated consolidation and therefore the solid particles are more tightly packed together.

Poisson's ratio – When a material is compressed using a force in a single direction, Poisson's ratio defines the degree to which the material expands in the other two directions. This is the ratio of expansion to the contraction caused by the compression.

Sudden increases of Poisson's ratio within a soil are often due to the presence of the water table. This is particularly true for clays which when fully saturated become incompressible (i.e. $\nu \approx 0.5$). In this case the P-wave speed increases dramatically because the wave speed becomes more representative of the water rather than the soil. On the other hand the S-wave velocity remains unchanged because water has no shear strength and thus the wave speed remains representative of the soil. Changes in wave speed with respect to Poisson's ratio are shown in Figure 4. It can be noticed that Poisson's ratio also has an effect on Rayleigh wave speed. This effect is minor because the Rayleigh wave speed can never exceed the shear wave speed. Therefore Rayleigh wave speed is usually located in the range of 85-95% of the S-wave velocity.

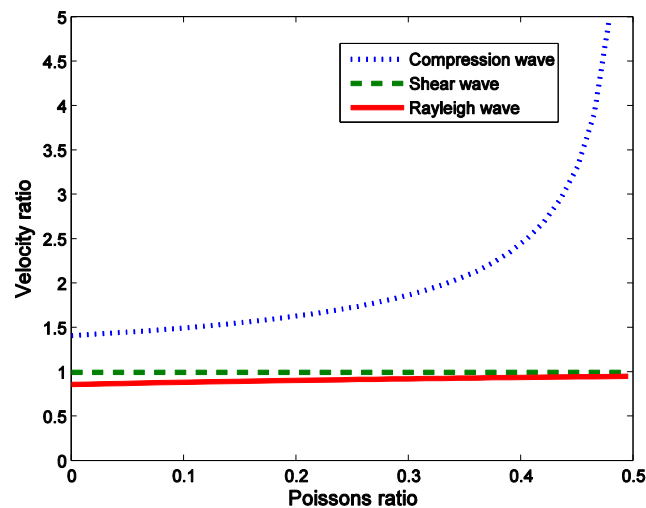


Figure 4 - The effect of Poisson's ratio on seismic wave speeds

Young's modulus – A measure of the stiffness of a material. It is calculated using the tangent modulus of the initial, linear portion of the stress-strain curve. As stiffness (of both track and subgrade) is the main criteria used for quality control during construction, Young's modulus is a highly influential parameter in the generation and propagation of railway vibration.

At large strains soils behave non-linearly because shear modulus depends highly on strain. Although large strains may occur in geotechnical engineering applications such as

pile driving, blasting or on off-shore oil rigs, in the case of ground vibration from railways, soil particle deformation is typically very small in comparison to its dimensions. The magnitude of strain experienced by the soil during train passage is therefore low (10^{-5} %) and can be modelled using ‘small strain’ theory. This allows for the soil to be considered as a linear elastic material and for the shear modulus to be considered to be equal to the ‘maximum shear modulus’.

Damping – A measure of the rate at which energy is reduced as it disperses and passes through a material. The total damping ratio is composed of geometrical and material damping and has a non-linear relationship with frequency. This frequency dependence makes damping modelling more complex for time domain modelling in comparison to frequency domain modelling. Regarding in-situ soils, material damping is typically greatest in the upper layers and reduces with depth. This is because the soil particles in the upper layers are less compacted, meaning the wave loses greater energy as it passes through the air voids. Furthermore, if a soil is saturated then it may exhibit elevated viscous damping at high frequencies. Regarding the track, damping is caused by the ballast and, if present, by a combination of rail pads, under-sleeper pads and ballast mats.

The role of the train

Vibration generated at the wheel/rail interface propagates both into the track/soil, and also vertically upwards into the train vehicle. It is important that vehicle vibration is minimised because it affects passenger comfort, which is often a primary track design requirement (especially on high speed lines). Therefore trains are typically fitted with two suspension systems that are connected via a bogie. The suspension system connecting the wheel and bogie is the first suspension system and the second suspension system connects the bogie and car body ([22]). Originally this design was used predominantly to improve passenger comfort on high speed trains, however is now becoming common among other train types such as intercity locomotives.

| Vibration Frequency | Effect on Passenger | Ref |
|---------------------|------------------------------|------------------|
| 0.8-8Hz | Reading & writing impairment | [24], [23] |
| 1.25-6.3Hz | Reading impairment | [25] |
| 0.25-0.32Hz | Motion sickness | [24] |
| | Other | [23], [24], [25] |

Table 3 – Vehicle vibration effects on passengers

Passenger comfort is often prioritised on high speed lines because the high speed market is positioned towards business and luxury customers. In comparison, for freight lines, passengers are typically not carried, so comfort is not prioritised. Therefore, particularly for business travellers, it is important that the train provides an environment conducive to work (i.e. reading and writing capabilities are not impaired) – see Table 3.

Low frequency lateral vibration in the range (0.8-8Hz) was found to interfere with reading and writing tasks by [26], with frequencies below 5 Hz proving particularly influential. Similar findings were made by [27] who found that passenger reading speed was impaired between 1.25 and 6.3 Hz, with maximum interference at 4Hz. Further studies [24] were conducted and it was found that passenger writing ability was also effected by low

frequency vibration, however the duration of vibration was also important. It was also found that the probability of spilling liquid from a hand-held cup was also affected by frequencies in the same range.

In addition to facilitating reading and writing tasks, it is important that railway travel does not interfere with sedentary tasks [28] or cause passengers to experience motion sickness. Therefore [29] analysed 4,000 commercial train passages and found that tilting trains induced higher levels of motion sickness than non-tilting trains. Additionally, it was found that vibrations in the 0.25-0.32Hz range were most influential in causing motion sickness.

A challenge with assessing passenger comfort is that results are highly dependent on the individual under investigation. Therefore the majority of studies are performed under physical test conditions and used to derive empirical exposure relationships. These experiments are often undertaken in lab conditions where the subjects experience vibrations at predetermined and discrete frequencies. Despite this, train passengers are routinely subject to a spectrum of vibration frequencies that are constantly changing. Therefore, with the end goal of reducing passenger discomfort, the vibration levels entering the train vehicle must first be modelled and then abatement measures investigated.

To model vehicle vibration, a time domain multibody modelling approach can be used. For example, [30] developed a numerical model to simulate the vibrations of a high speed train passing over a bridge. The model results were assessed using maximum acceleration criteria and a comfort index, and it was found that rail roughness had a significant effect on passenger comfort. Alternative evaluation approaches are outlined in [31], [32],[33], [34], [35], [36], [37].

Mitigation of vehicle vibration

Solutions to the reduction of vehicle vibration [38] can be divided into design solutions and maintenance solutions (Table 4).

| Mitigation of vehicle vibration | | Refs |
|--|---|-------------|
| Design solution | Reduction in unsprung mass | [38] |
| | Better wheel/axle design, braking control or higher performance materials | [39] |
| | Resilient wheels | [40] |
| | Active vibration control systems | [41] |
| Maintenance solution | Assess wheel shape | |

Table 4 - Strategies for the mitigation of vehicle vibration

The role of the track

The role of the track is to support the moving vehicles and distribute their forces into the supporting soil. A key factor is track stiffness. If it is too low then the soil and ballast experience increased deformation and the locomotives require additional power to traverse the track. Alternatively, if the track stiffness is too high then the wheel stresses are concentrated within a small area on the rail thus generating corrugation [39]. Additionally, if stiffness varies radically over track sections (e.g. transitions) then this also causes elevated track deterioration and vibrations [40].

Idealised track stiffness values are different between countries, however these are typically based on expected train types (e.g. freight or high speed). Therefore, for lines that facilitate different types of rolling stock, it is difficult to determine an ideal track stiffness. Thus, track vibration and degradation can be problematic in such cases. Two influential track imperfections (not solely caused due to track stiffness) that contribute to track vibrations are:

Wheel out-of-roundness – Wheels become out-of-round as a result of the manufacturing process, or from repeated loading at high frequency contents [41]. The most common manifestation is the formation of wheel flats, caused primarily by train braking/deceleration. As a result, a high frequency impact force is generated at every rotation when the corners of the wheel flat impact the rail. Recently, this has been highlighted as one of the key factors contributing to track vibration [42], [43].

Rail irregularities – Changes in lateral/vertical geometry, or stiffness associated with the rail. These may occur due to many factors, such as switches, welds, track settlement, ballast deterioration, ballast-slab transition zones and excessive wear [6].

As mentioned earlier, there are numerous dynamic excitation mechanisms that contribute to railway vibrations [45], many of which are outlined in Figure 5. It can be seen that the excitations associated with the locomotive are found at low frequencies and the excitations associated with the wheel, rail and track are found at higher frequencies. These higher frequencies often manifest themselves in the form of air-borne noise, while the lower frequencies generate ground vibration. Bands have been drawn on Figure 5 to illustrate a variety of perception threshold levels, in accordance with BS ISO 14837 [46]. It should also be noted that train-track resonant conditions can develop where the natural track frequencies are excited. These effects and their interaction with the underlying soil are still not fully understood, and are currently the focus of much research [47].

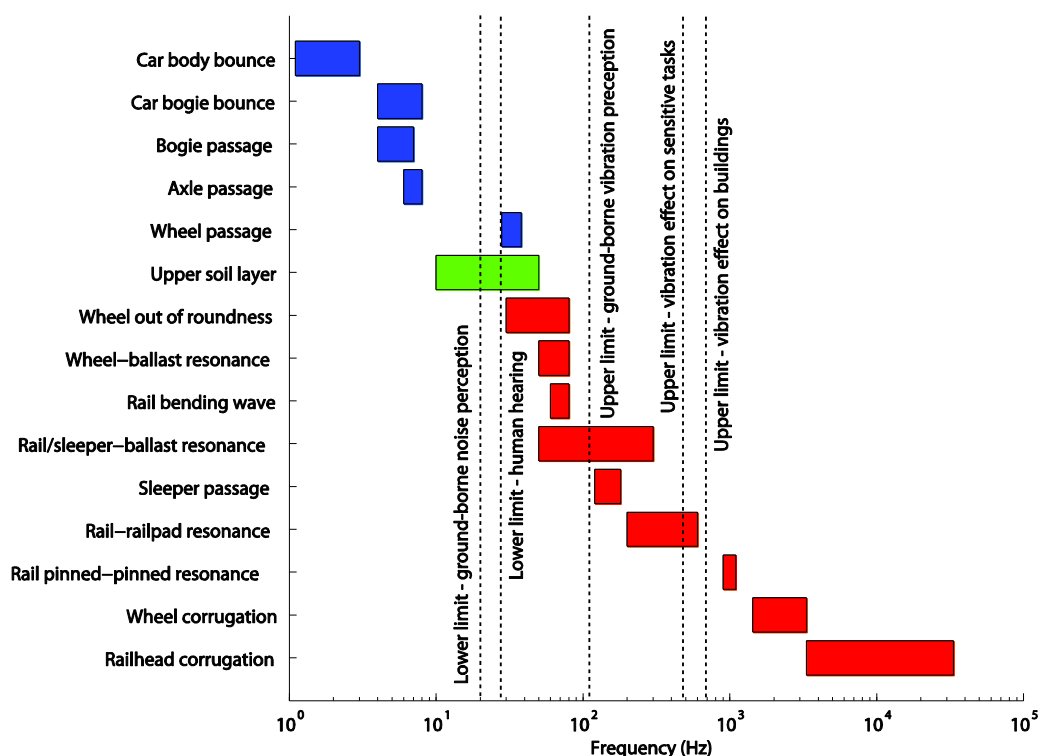


Figure 5 – Typical frequency ranges of excitation

Analytical track modelling

Train-track interaction is the source of railway vibration and therefore it is important that it is modelled accurately. If the vibration response is modelled inaccurately, then not only the track vibration will be erroneous, potential train, ground and structural vibration calculations will also contain errors. Modelling can be undertaken using either analytical or numerical models, and can be computed in either the time or frequency domain. Furthermore, depending on project scope, a moving or stationary load can also be simulated.

Analytical models are attractive because they are more computationally efficient in comparison to numerical approaches. One reason for this is that numerical models suffer from spurious reflections at truncated model edges. For non-complex geometries such as soil masses, absorbing boundaries can be used, however, for more complex track type models, such boundary conditions are challenging to implement. Therefore, when using numerical methods, the total length of track modelled must be much larger than the region of interest.

For such reasons, analytical models have been analysed widely, particularly before the advent of modern computing technology. Early approaches to track modelling used the assumption that the rail could be modelled as a single beam (typically Euler-Bernoulli) resting on a Winkler foundation ([47], [48], [49], [50], [51], [52]). This foundation was used to describe the track or a combination of the track and soil. Therefore the track was often modelled as a homogenous material.

Single layer methods were useful for investigating the low frequency characteristics of the track [9]. Therefore further investigation was undertaken to investigate the interaction between various beams and plates with underlying soil, by [53]. It was found that model accuracy was improved by replacing the Euler beam formulation with that of a Timoshenko beam [54], [55], [56] and [57]. Timoshenko beams account for additional degrees of freedom (shear forces) in comparison to Euler beams and therefore allow for higher accuracy modelling of track dynamics.

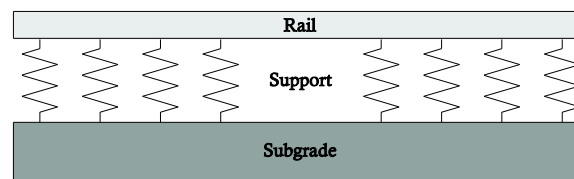


Figure 6 – Single layer track model (continuously supported)

Two layer models were also proposed and were considered an improvement over single layer models because they allowed for railpads, sleepers and ballast to be simulated [17], [58]. In the two layer model illustrated in Figure 7, the ballast and railpads are assumed to be massless, however these can be included by introducing additional layers (e.g. 3 layer models [59]). Despite this, the use of a continuous support meant that sleeper effects were not modelled. Instead the effect of the discrete sleepers was uniformly distributed over the length of the model. Although this was found to be adequate for some problems [9], and was well suited for slab track modelling, when modelling ballasted tracks it meant that sleeper passage frequencies (Figure 5) were ignored.

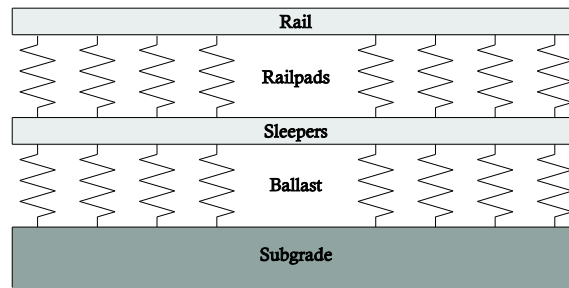


Figure 7 – Two layer track model (continuously supported)

To facilitate the inclusion of sleeper effects, discretely supported models were also developed [60], [61], [17]. Figure 8 shows a Timoshenko beam discretely supported by individual sleepers with viscous damping.

It should also be noted that the track support conditions (subgrade) have a significant effect on track stiffness and therefore have been found to affect track response. For the models previously mentioned, the track base has been assumed rigid. Although this may be suitable for tracks where the supporting material has a stiffness comparable to the track ([62]), discrepancies occur when the supporting material is soft. Therefore research has also been undertaken into modelling the subgrade as an elastic half-space [63], [64], [65], [66], [67].

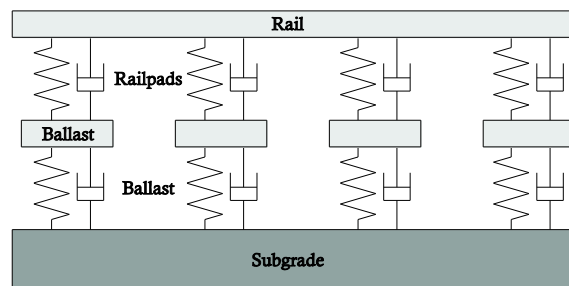


Figure 8 – Two layer track model (discretely supported)

A common method for analysing the frequency characteristics of railway tracks is to calculate the track receptance (track deflection per unit of stationary input force). This transfer function describes the propagation of energy through a system for a set range of frequencies. It is used because it provides a useful tool for investigating the natural frequencies of the track. When known, these frequencies can be dampened accordingly.

The receptance results for the various types of frequency domain track models discussed previously are shown in Figure 9. On the figure are drawn the typical upper thresholds for human (5-80Hz) and structural (30-250Hz) vibration perception. It can be seen that there are discrepancies between modelling approaches for both the human and structural response ranges, thus emphasising the need to correctly model the discrete support condition and multiple layers.

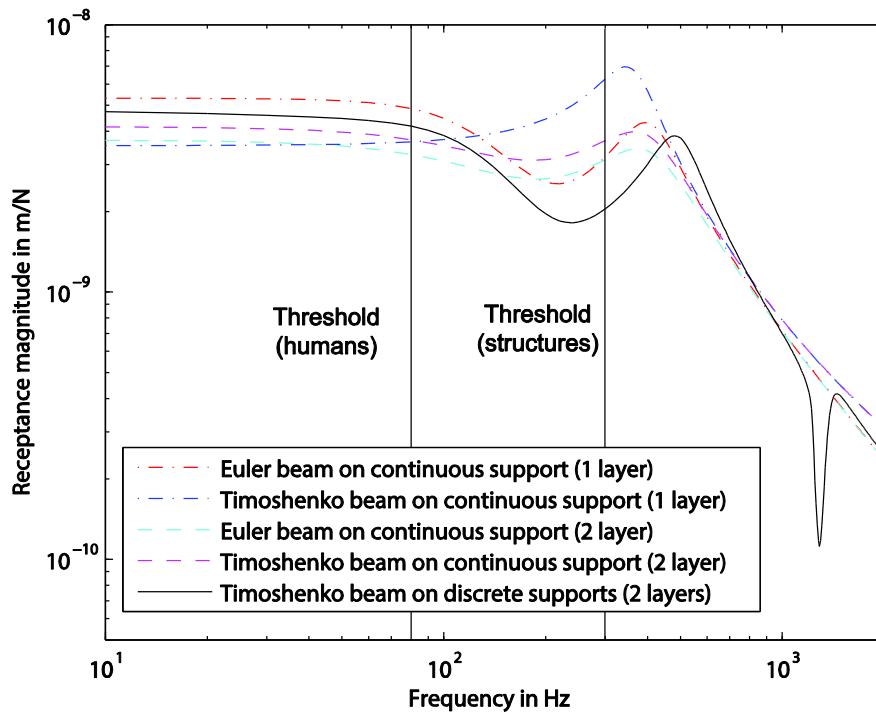


Figure 9 – Analytical track models – receptance characteristics

Although point receptances are useful for understanding general track characteristics, it is also often desirable to investigate the effect of a moving load on track response. This introduces more complexity into the analytical approach and has been studied for a moving point load, [68], [64], [47], [69], [70], [71]. In an attempt to make the excitation mechanism more realistic, the problem has also been formulated using a multi-body excitation [72], [73], [74].

Finite element track modelling

Although analytical models are well suited to predicting the response of the straightforward 2D track geometries mentioned previously, they are less well suited for including multi-body vehicle effects and track/wheel irregularities. To overcome these limitations the track has frequently been modelled using more versatile numerical methods. In particular, the finite element (FE) method has proven a highly applicable tool.

A wide range of proposed FE track models were reviewed by [9]. Similarly to the analytical approaches (Figure 6 - Figure 8), these can be divided into two dimensional, one layer, two layer [75], [76], and three or more layer models [77], [78], [79], [46], [80], [81]. Multi-body vehicle modelling approaches were then used to more realistically replicate the frequency content of excitation from a moving train ([82], [83],[21]). It was found that upgrading the approach from a basic moving point load improved model accuracy drastically [82], and therefore a fully integrated train-track system could be simulated (e.g. Figure 10). Similarly, this allowed for the investigation of vibration response due to wheel flats [84], rail irregularities [75], [85], [86], ballast-slab transition zones [87] and other defects [88].

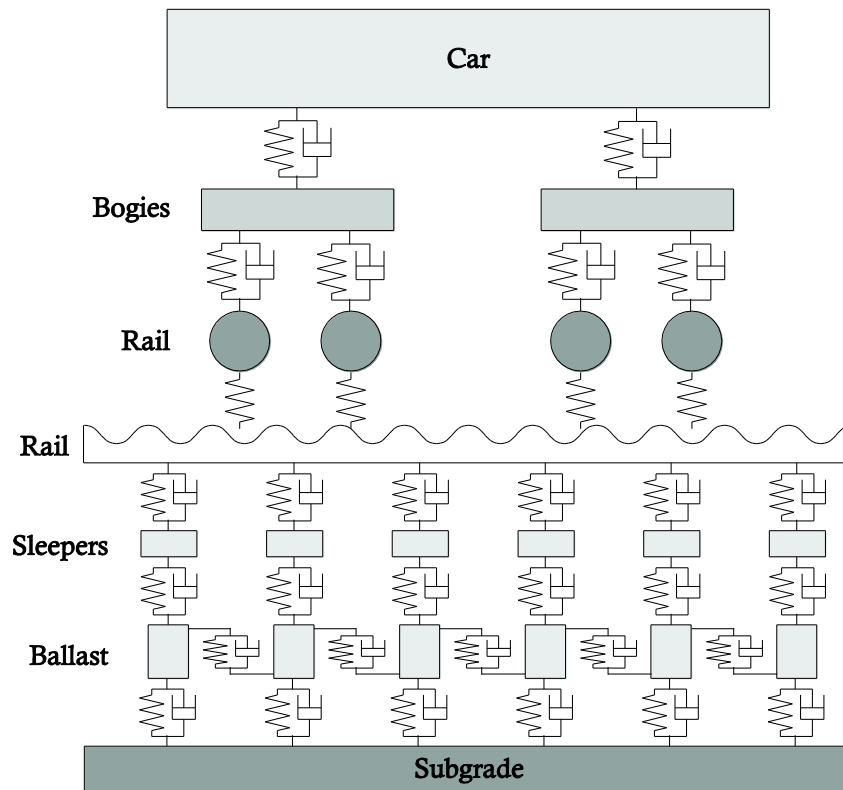


Figure 10 – Coupled train-track model

In a similar manner to analytical model development, research has been undertaken in an attempt to include subgrade stiffness (i.e. soil) contributions in track response calculations. The difference between these types of track models and the ones discussed in the ground modelling section later in this report, are that the track response is of interest, not the far field ground vibrations. Therefore the soil model is included solely to increase the accuracy of the track model. The most common approach to achieving this is to model the ground as a linear elastic half-space. This approach was undertaken by [89] to study critical velocity effects. A challenge of this approach is that computational time is increased and absorbing boundary conditions are needed to terminate the unbounded soil domain.

Another notable assumption associated with the previously mentioned models is that the track response can be reduced down to two dimensions. A challenge with this approach is that the stress distribution from the rails is then uniformly distributed over the sleeper width. This is unrepresentative of the physical problem. To overcome this, [90], [91] and [92] modelled the track components in three dimensions. An advantage of this was that none of the track material properties required tweaking to account for the 2D stress distribution approximation. To overcome some of the aforementioned challenges, it should be noted that a variety of commercial train-track modelling software is also available (e.g. Vampire, Simpack and Adams/rail).

| Model type | Significance | Ref |
|---|-----------------|---------|
| Single beam (typically Euler-Bernoulli) on Winkler Foundation | Analytical – 2D | [48-53] |

| | | |
|-----------------------------|-----------------|---------------------------|
| Single beam (Timoshenko) | Analytical – 2D | [55-58] |
| Two layer models | Analytical – 2D | [17], [59] |
| Three layer models | | [60] |
| Discretely supported models | Analytical – 2D | [61], [62], [17] |
| Elastic half-space models | Analytical – 2D | [64-68] |
| Moving point load | Analytical – 2D | [69], [65], [48], [70-72] |
| Multi-body excitation | Analytical – 2D | [73-75] |
| Two layer models | FEM – 2D | [76-77] |
| Three layer models | FEM – 2D | [78-80, 47, 81-82] |
| Three layer models | FEM – 3D | [94, 93] |

Table 5 – Summary of track modelling approaches

Ballast modelling

[9] commented that if track vibration is to be better understood, ballast behaviour must first be further investigated. This is particularly important for investigating and improving ballast maintenance. Therefore, in recent years, there has been a rapid increase in ballast modelling, with the discrete element method (DEM - [95]), proving a highly suitable approach [96-99]. This is because ballast particles are typically large (approximately 40mm) and thus cannot be realistically modelled as a continuum material. Therefore the DE approach allows for the simulation of a large body of individual ballast particles, which can be used to investigate its abrasive, compactive and crushable nature. A challenge with DE modelling is that the complex geometries and non-linear nature of individual ballast particles leads to high computational demand. Therefore, current research is focused on reducing this demand, with a goal of integrating DE ballast models within a FE track modelling framework [100-101].

Mitigation of track vibration

Track vibrations are undesirable from both passenger comfort and environmental standpoints. Additionally, as the wheel/rail interface is the source of all railway vibration, it is often efficient to isolate the vibration as close to the excitation location as possible. This is typically done using either permanent or temporary measures.

Temporary measures are approaches such as rail grinding to reduce the imperfections at the interface, thus creating a smoother contact between wheel and rail. Alternative permanent measures include the use of continuously welded rail technology which eliminates the discontinuity associated with rail joints. This approach is becoming widespread, particularly on high speed lines. Another permanent approach is to insert highly damped pads between track components [102]. For example rail pads can be inserted between rail and sleepers to reduce the vibration and stresses between components. Sleeper pads act in a similar manner, but are placed between sleepers and ballast. Both rail and sleeper pads act to dampen high frequency vibration and shift the spectrum to a lower range. This helps to reduce the overall track vibration levels, however these low frequency vibrations are more likely to resonate with structure/buildings close to the track. It should also be noted that slab tracks can be used instead of ballasted tracks,

and facilitate a reduction in vibration levels due to their increased stiffness. Additionally, floating slab tracks are often used in tunnels to provide additional isolation.

The role of the ground

The ground acts as the transmission path for vibration that can eventually reach nearby structures. Thus it is important to understand and predict the characteristics of it.

A large body of work has been, and continues to be undertaken into Elastodynamic waveform modelling – see Table 6.

| Modelling approach | Comments | Ref |
|---|---|---------------------------------|
| Elastodynamic wave propagation | Analytical expressions for a homogenous half-space excited by a stationary point load | [103-106] |
| | Analytical approach to include a moving train/track model | [107-114, 48, 115, 49, 116-122] |
| Finite difference time domain method (FDTD) | Low computational effort and high performance ABC's | [123-124] |
| Finite element modelling | Complex geometries can be modelled | [93-94, 125, 90] |
| Coupled finite element – boundary element modelling | Complex geometries can be modelled using FE and large offsets modelled using BE | [126-132] |
| 2.5D methods | Reduces computer run time | [133-134, 84, 135-136] |
| Pipe-in-pipe (PiP) models | Efficiently calculate underground vibration | [137-138] |
| Empirical approaches | For initial scoping studies | [139-141, 236] |

Table 6 - Summary of Elastodynamic railway modelling research

Such problems were first analysed by [103], and later by [104-106] to develop analytical expressions to describe wave propagation within a homogenous half-space due to a stationary point load. Although these solutions proved invaluable in the validation of many complex wave propagation models proposed thereafter, the adaption of such an analytical approach to include a moving train/track model that is representative of a railway system, is challenging. The first steps towards achieving this were undertaken by [114, 48, 115, 49, 116-122] who built upon Lamb's work to facilitate moving load simulation. Further modifications were proposed by [107-113] to tailor the approach to railways. Despite this, to enable the train-track system to be modelled analytically, many assumptions must be made to reduce model complexity, particularly with respect to model geometry and excitation mechanism. Although significant research into this area is still being pursued [142], these challenges mean that it has become more common to use numerical techniques as they are better suited to simulating the aforementioned complexities.

One of the more straightforward numerical modelling tools is the finite difference time domain method (FDTD) [143,144]. It is based on approximating a strong formulation of the

seismic wave equation using a central differencing integration scheme. The domain is usually discretised into velocity and stress components that are staggered in 2D/3D space.

The advantages of the FDTD method are that it requires relatively low computational effort. This is because the velocity/stress discretisation is well structured and therefore it is straightforward to divide the workload between multiple computer processors. Also, high performance absorbing boundary conditions have been developed (e.g. perfectly matched layers) which are more straightforward to implement within the FDTD method than for other alternative numerical methods. (E.g. [145-146]). This means that the domain sizes can be reduced, therefore further decreasing the total number of model calculations.

These strengths have led to the investigation of the suitability of FDTD for railway problems to be explored. However, the FDTD method offers reduced performance in modelling domains with complex geometries and free surfaces. Therefore it is challenging to simulate railway track components, or the coupling between wheel and rail. Despite this, it was successfully used by [123] to model a railway track and embankment, with the input force obtained directly from a time history collected experimentally from a train. Alternatively, [124] used the FDTD technique to explicitly model the railway track. In this work an unpublished method to reduce dispersion within the complex track geometries was used.

To overcome the absorbing boundary challenges faced by the FDTD method, Sheng et al ([68, 147]) proposed a semi-analytical. The governing equations for the vehicle were computed in the frequency domain while the track-soil equations were solved in the frequency-wavenumber domain. This resulted in a single transfer function for the overall system. It was found that this approach was more computationally efficient in comparison to alternative numerical methods [148]. It was also able to efficiently model track/wheel irregularity effects.

A weakness of the approach was that many assumptions were required to formulate the analytical expressions used to calculate the transfer functions. Therefore it was difficult to include complex changes to geometry or calculate structural vibration. This is important because detailed soil models are commonly used to appraise mitigation measures (e.g. wave barriers). Furthermore, the coupling between track and ground was based on the stress distribution between each sub-model, which should ideally be determined from in-situ experiments [149].

An alternative modelling tool to the FDTD method is the finite element method (FEM) , [150-152]. It utilises a weak form of the seismic wave equation and a key strength is that complex geometries can be modelled, particularly with the widespread availability of graphical user interfaces (e.g. ABAQUS, LSDYNA, ANSYS) associated with commercial software. This is useful for railway applications because track components and their connections can be modelled explicitly ([93-94]). For time domain modelling it allows for the straightforward investigation of track defects (e.g. rail irregularities and local defects [125]) and changes in stiffness (e.g. transitions from ballast to slab track [90]). Despite this, as with the FDTD method, for unbounded domains (e.g. soils) an absorbing boundary condition (ABC) is required to prevent reflections from edges of the domain.

For time domain FE modelling, solutions such as infinite elements [153-155] the combined FE-thin layer method (Fig. 13 – [156]), and the scaled boundary FE method [157] have been proposed. More recently, the usage of perfectly matched layers (PML) has been preferred due to its superior performance. PML is a series of layers that have identical material properties to the modelling domain, however they act to stretch both the real and

imaginary coordinate space. This serves to dampen wave amplitudes in a frequency independent and more efficient manner than previous ABC approaches [158,159,145].

A weakness of time domain simulation for 3D wave propagation modelling is the large computational expense required to compute the solution at every timestep. This results in computational run times of hours/days. Therefore, a commonly used alternative approach is to perform the analysis in the frequency domain. This requires that an eigenvalue problem is solved for each frequency of interest, resulting in reduced computational requirements. A weakness of the frequency domain FE approach is that the majority of time domain ABC's cannot be used in the frequency domain, thus making it difficult to prevent boundary effects.

To overcome these limitations the finite element has frequently been coupled with the boundary element method ([160-161] (Fig. 11)). The boundary element method only requires the boundary of the domain to be meshed, and uses greens functions to model wave propagation. This means that vibrations can be modelled at large offsets which is attractive for railway applications. A disadvantage of the boundary element method is that irregular geometries within the domain cannot be modelled. This makes track modelling challenging and therefore it has been coupled with the finite element method with the aim of combining the strengths of both methods. To do so, the track region and near field are typically modelled using the finite element method, while the far field is simulated using the boundary element method. Although this approach can be formulated in the time domain [126-128], a frequency domain approach has more often been used due to reduced run times [129-132]. Furthermore, when formulating the BE equations in the frequency domain, damping is modelled in a straightforward manner using a complex valued shear modulus, while in the time domain, damping is more challenging to simulate.

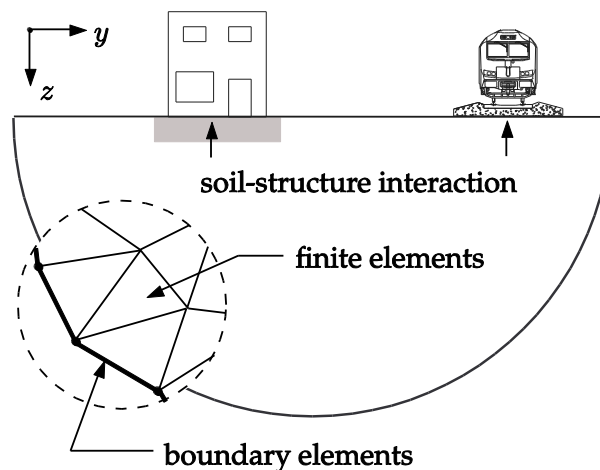


Figure 11 – The coupled BE/FE approach

A downside of the BE/FE approach is that the matrix formulation for the FE region is well structured and uniform whereas the matrices for the BE regions are sparse. Therefore it can become computationally expensive to compute the solution. Additionally, although it is novel to predict ground vibration levels at very large offsets (inside the BE region), the assessment of structural vibration is typically of greater importance in commercial projects. For areas inside the BE region, structural vibration is difficult to determine due to the difficulty of coupling a building to the surface of a BE domain [162]. Therefore, it is common for the soil and building to be modelled as two uncoupled systems [163].

One approach to reduce the computational demand of the aforementioned large 3D models is to use a 2.5D approach [133-134, 84, 135-136]. To do so the track is usually considered as invariant in the direction of train passage, thus allowing for the problem to be approximated using a 2D geometry, while accounting for 3D loading conditions (Fig 12). The solution is found in the frequency-wavenumber domain and results in much reduced computational cost in comparison to 3D models. To do so, a Fourier transformation in the direction of train passage is used:

$$\tilde{u}(x, k_y, z, \omega) = \int_{-\infty}^{+\infty} \hat{u}(x, y, z, \omega) \exp(ik_y y) dy$$

$$\hat{u}(x, y, z, \omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \tilde{u}(x, k_y, z, \omega) \exp(-ik_y y) dk_y$$

where x , y and z are the coordinate axis (perpendicular to train passage, parallel to train passage and vertical respectively). To calculate the solution, a 2D model is computed for all desired frequencies (ω) and wavenumbers (k_y), and then the 3D solution is recovered via the inverse Fourier transformation with respect to wavenumber. Large computational savings are made unless the number of wavenumbers is large.

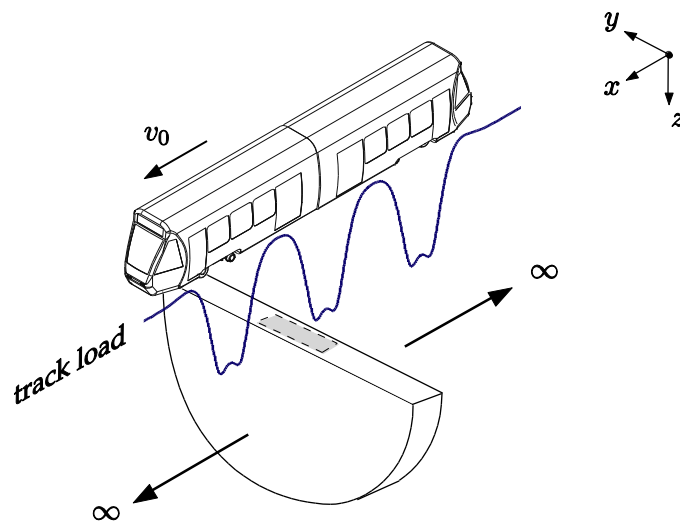


Figure 12 – The 2.5D approach

Although this approach is efficient for calculating the soil response for invariant track geometries (e.g. slab track), it is more difficult to model the stress distribution associated with discrete sleeper (i.e. ballast) tracks. One approach to overcome this limitation is to use a Floquet transform [164-165].

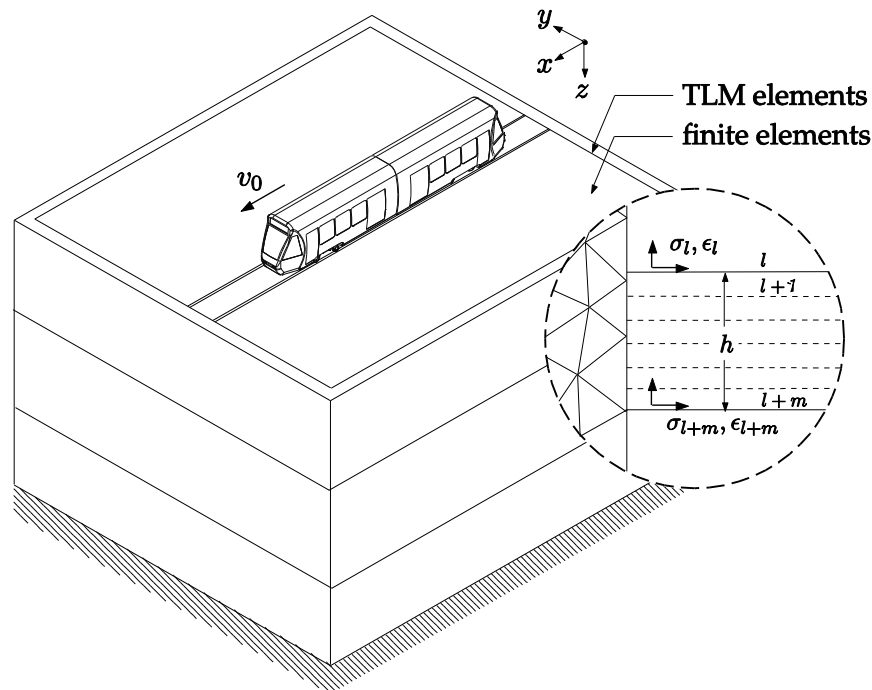


Figure 13 – The coupled FE/TLM approach

Although a broad selection of numerical approaches have been discussed to predict soil vibration propagation, it should be noted that there are many unmentioned alternative approaches. Some of these include pipe-in-pipe (PiP) models [137-138] to efficiently calculate underground vibration, empirical approaches [139-141] for initial scoping studies, and hybrid models [132], and coupled FE-meshless methods. It should also be noted that for any model to be considered valid, it must be checked against a variety of experimental results [166–168, 237].

Furthermore, one notable aspect to consider when choosing a prediction tool is the desired frequency range. If re-radiated ground-borne noise is a concern then the frequency range under consideration must be higher than for ground-borne vibration. A challenge with numerical modelling is that the maximum frequency that can be accurately resolved is related to the element discretisation. Higher frequencies require smaller element dimensions thus significantly increasing computational requirements. Therefore, sometimes analytical alternatives are attractive because they are less sensitive to changes in frequency content.

Mitigation of ground vibration

Soil vibration can be mitigated using trenches (aka wave barriers - Figure 14a). These are excavated soil sections located in the ground at distance from the track. They are typically filled using a low density material (e.g. gas cushion [169] or polyurethane[170]) to increase the reflection coefficient between the soil and the trench, thus increasing its effectiveness. Essentially, the backfill should be of a low acoustic impedance compared to the background soil. They have been successfully used on Swedish projects in Gnarp, Stockholm, Uppsala and Saffle in the 1980's [171], and Dusseldorf, Germany [171]. Extensive numerical modelling has been undertaken into their effectiveness [18, 170, 172-176, 94], and it is consistently found that trench depth should be approximately half the length of the dominant Rayleigh wavelength.

An alternative approach is to use subgrade stiffening at strategic ground locations (Figure 14b). This serves to increase the soil stiffness and thus dampen the propagation of vibration. Possible stiffening techniques include deep vibro-compaction, deep soil mixing, stone columns, grouting or vacuum consolidation. A more comprehensive comparison between techniques can be found in [177]. Despite this, it should be noted that the deployment of such ground vibration mitigation measures is financially intensive and should be avoided if possible.

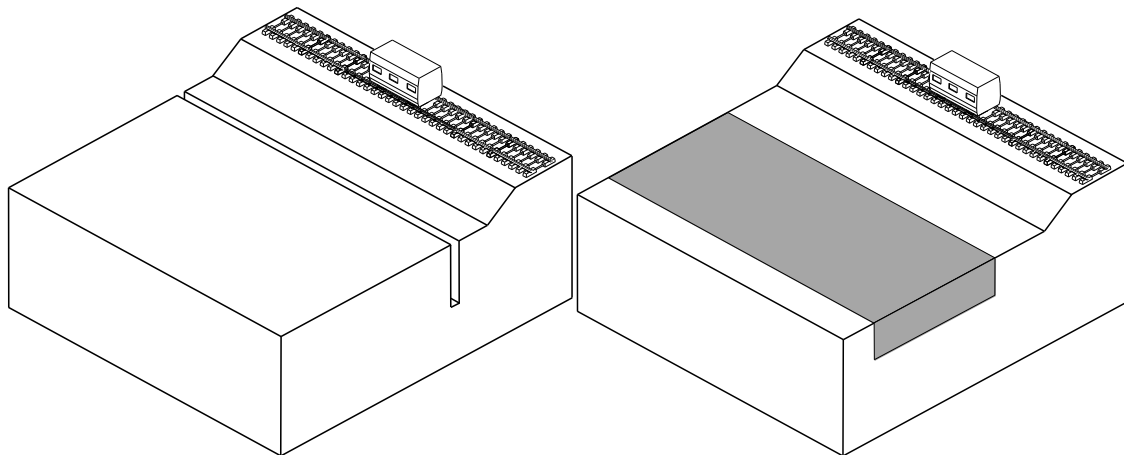


Figure 14 – Left: (a) Trench deployment, Right: (b) Subgrade stiffening

Critical velocity effects

When train speeds approach the underlying Rayleigh wave speed of the supporting soil it is possible that large increases in track vibration may occur. This is an area of increasing importance for high speed rail because as operational speeds increase, the probability of train speeds approaching this 'critical velocity' at a particular site increases. Another important aspect is the 'track critical velocity', which is often described as the minimal phase velocity of bending waves propagating in the track supported by the ballast. Despite this, it is typically found that 'track critical velocity' is many times higher than the soil Rayleigh wave velocity.

Experimental evidence of critical velocity effects has been collected on Swedish, UK and Dutch lines and is shown in Figure 15. It is clear that as the normalised speed (train velocity/Rayleigh wave velocity) increases towards a value of 1, the track displacement grows exponentially. Trains exceeding the Rayleigh wave velocity are referred to as 'trans-Rayleigh' trains. These can give rise to a 'boom' and the creation of a Mach cone [90]. This exponential growth is a function of the initial track displacement and therefore not always problematic. For example, if track displacements are very low, then although an increase in train speed may cause a threefold increase in displacement, this value may still be below the maximum safety threshold. Despite this, it is clear that under certain, and not fully understood circumstances, that track deflections can become large (e.g. >10mm [178]).

This area is the focus of extensive research activity and is challenging due to the complex coupling between track and ground structure that contains many individual Rayleigh wave speeds and resonant frequencies, which can be analysed using a dispersion

diagram [17]. Similarly, the train excitation also generates a wide spectrum of excitation frequencies that increases problem complexity.

Another unanswered question is the behaviour of track deflections above the critical velocity. As seen in Fig. 15, data is scarce related to train speeds above this value. Therefore, although numerical models have been used to extrapolate the true behaviour of the response curve, and to approximate critical velocity values using dispersion curves [17], this is still an active area of research.

One of the challenges in modelling critical velocity behaviours is that when track displacements become large, it undergoes ‘large’ shear strain. Therefore the system no longer behaves in a linear manner and thus material behaviour must be modelled using non-linear theory [179]. This increases model complexity, makes non-linear material parameter selection challenging, and increases computational cost. Furthermore, when modelling non-linear soil effects it is usually necessary to use time domain modelling approaches rather than the frequency domain.

In practical terms many railway designers will attempt to ensure that train velocity does not exceed 0.7 times the Rayleigh wave velocity.

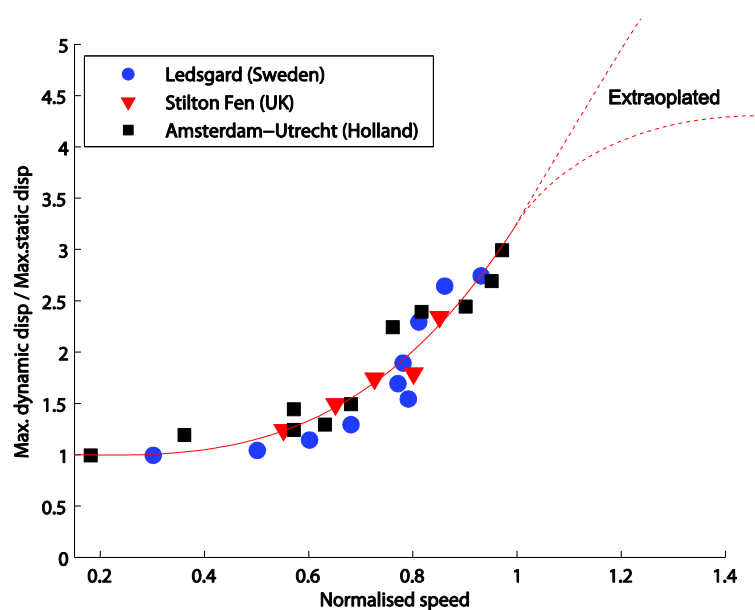


Figure 15 – Critical velocity effects (Redrawn from [178])

Mitigation of critical velocities

A key difference between mitigating common ground vibrations and vibrations due to critical velocity effects are the stakeholders involved. For common ground vibration, the levels are relatively small and the main stakeholders are the owners of buildings located close to the track. For vibration due to critical velocity effects, the key stakeholders are the railway operators and track asset managers.

Abatement measures are undertaken in the soil located beneath/within the track (active isolation) whereas for common vibration these measures are taken in the soil closer to the structure (passive isolation).

In practice, when excessive vibration levels have been encountered, the most common mitigation measures have been to reduce vehicle speed over soils with low Rayleigh wave velocities. This approach was initially undertaken at Ledsgard, Sweden where the operational line speed was reduced from 200 km/h to 130 km/h [181]. This meant that the normalised speed with respect to the Rayleigh wave speed was reduced, and therefore track deflections were reduced. A disadvantage of this approach is that it is expensive over long time scales because line capacity is reduced. Therefore, in the long-term, an engineering solution is preferable.

An engineering solution is to increase soil stiffness. Historically, one of the most commonly recorded soil properties is undrained shear strength. Therefore, to determine the necessary stiffness required to safeguard against critical velocity effects, the following equation can be used:

$$E = K_c \cdot c_u$$

Where the correlation factor K_c is dependent on plasticity index. Table 7 shows typical values for an overconsolidation ratio of between 1 and 2. In practise, it is often assumed that for design purposes, the “design critical velocity” is limited to 0.7 times Rayleigh wave velocity. Figure 16 shows design critical velocities for various undrained shear strength and plasticity index values (density=1800 kg/m³). It is observed that:

1. Higher plasticity soils can be more problematical than low plasticity soils
2. Further consideration needs to be given to soft/weak soils. For example, [182, 115] reported unexpectedly poor performance of weak soils in Sweden. This could be due a build-up in positive pore water pressure in the saturated clay – giving rise to a reduction in the effective stress and consequently a short term reduction in the encountered undrained shear strength. The latter would then reduce the Rayleigh velocity and thus the “design critical velocity”. This latter mechanism, well known in highway construction design, has not been discussed in the railway environment.

| Plasticity Index, I_p (%) | Correlation factor, K_c |
|-----------------------------|---------------------------|
| < 30 | 600 – 1,500 |
| 30 – 50 | 300-600 |
| > 50 | 175 |

Table 7 – Soil correlation factors based on plasticity index (taken from [183])

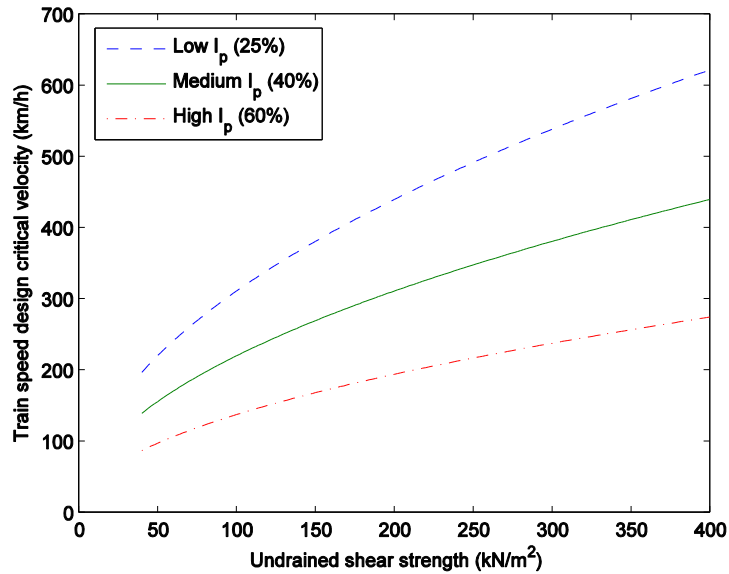


Figure 16 – Design critical velocity (saturated soil)

Note that ground improvement techniques used for critical velocity mitigation are similar to the subgrade stiffening described for common vibration abatement, but placed beneath the track, rather than at soil locations outwith the track. The purpose of this is to increase the underlying Rayleigh wave speed. At Ledsgard, lime/cement columns were placed to depths of between 7m and 13m below the track. This solution was found to significantly reduce the track deflections. Alternative solutions include stone columns, piles and the application of polyurethane [184, 185].

The role of structures

Vibrations generated within the train/track propagate through the soil before reaching man-made structures. They then cause structural vibration and in-door noise, which is a growing problem internationally [186]. The soil serves to dampen vibrations, particularly at high frequencies so the majority of the vibration spectrum arriving at these structures is below 100 Hz (Figure 5). As the natural frequencies of most buildings are located below 10Hz, the soil shifts the track vibration to a range more likely to resonate with structures. Despite this, for intercity trains and high speed trains passing on traditional at-grade, cutting or embankment tracks, noise is usually more dominant than vibration. This is partly because on these lines, due to compulsory land purchases during construction, few buildings are located within 60m of the track. Locations out-with this distance are still subject to railway noise, however vibrations are sufficiently dampened thus preventing disturbance.

In comparison, there are two cases where railway vibration is an increasing concern: (1) underground high speed lines, and (2) urban tramways. In these cases it is more common that vibrations will arrive at nearby structures with the amplitude and frequency characteristics to cause negative structural effects. Therefore, vibration

propagation/prediction for underground high speed lines, and urban tramways is the focus of this section.

Underground high speed lines are problematic because noise is confined within the tunnel, however vibration propagates to the surrounding soil and interacts with structures located above the tunnel. Although many railway tunnels are located deep below the earth's surface, buildings with deep foundations may experience elevated vibration levels. For these cases, S-waves dominate the response at distances close to the tunnel, however as distance increases, P-wave can become destructive. In comparison, for at-grade tracks, Rayleigh surface waves carry the majority of vibration energy.

The second case is that of urban tramways. Although trams move with low speeds, they can have high unsprung masses [187], and their urban setting means that buildings are located very close to the track. Therefore, there are a large number of stakeholders potentially affected.

The third case is that of freight traffic which moves with low speeds, and typically has very high unsprung mass [188,189]. This generates high amplitude, low frequency vibration (+20 dB below 20 Hz) that can propagate to large distances from the track. It is difficult to mitigate such vibration because the long Rayleigh wavelengths propagate deep within the soil.

Irrespective of the proposed train-track system, when a new line is planned it is usually implicit that a vibration assessment is undertaken. This involves a scoping assessment, and depending on its findings, is followed by a more detailed assessment at any sensitive sites.

Scoping/empirical assessment

Scoping assessments are used during (or sometimes before) the planning stage of a new line and are most frequently based upon empirical relationships and previous experience. It is typical for such empirical relationships to be the only form of vibration study undertaken on a new line as numerical modelling remains primarily confined to research.

A commonly used approach is that outlined by [139], [190] where a recipe is used to adjust a base vibration-distance curve to calculate absolute vibration levels within structures close to the line. The recipe is based on vibration levels recorded from previous train passages and provides a straightforward method for assessment. An important issue raised is that existing buildings can amplify vibrations significantly (i.e 100% increase), depending on their type. This is a significant simplification as many structural and furnishing factors may need to be taken into account for a specific structure. An alternative assessment and calculation procedure is outlined by [191] which was derived from TGV passages recorded in France and used to calculate tunnel vibrations for a UK rail project.

Similarly, [192] statistically analysed vibration levels on Swedish and Norwegian rail lines to develop a methodology for predicting high speed rail vibrations in Oslo. Also, [193] outlined an empirical model based on experimental results obtained in Sweden. The model was able to predict one second r.m.s (root mean square) values based on receiver distance, wheel force, vibration attenuation and train speed. Lastly, [194] proposed a scoping model that was based on straightforward analytical expressions and then calibrated using experimental field data.

Detailed prediction

Detailed prediction models may involve a variety of experimental and numerical methodologies. They are deployed for highly sensitive sites where either high accuracy is required, or the problem is bespoke.

For tramway vibration, indoor noise (i.e. reradiated groundborne noise) is of primary concern and when undertaking prediction it is common to use a physical experiment approach [139], 195-197]. This involves performing impact tests at the location of the potential tram line and measuring the vibration frequency response in the floors of surrounding buildings (typically vertical vibration at floor mid-spans). If the impact force is also recorded then a transfer function can be calculated. This transfer function is combined with a transfer function simulating the vibration response of the newly proposed train/track system under investigation (calculated numerically, experimentally or empirically [198]. The final transfer function represents the frequency response of the buildings to the new line, and can be used to estimate indoor noise levels.

The strength of using an experimental approach is that the exact soil wave propagation characteristics of the test site, and the coupling between soil and structure are included in the calculation. If an alternative numerical approach was used the soil properties at the test site would need to be acquired and then used to recreate a model of the test site and proposed new track [199]. As existing soil records are usually not sufficiently detailed to determine the numerical parameters required for detailed investigation, in-situ geophysical tests must be performed (although, some simple relationships between conventional SI data and numerical input data were given above). The most common acquisition method for this application is multi-channel analysis of surface waves testing (MASW), which enforces the assumption of horizontal homogenous soil layering, and challenging in the presence of high gradient topographies (i.e. building foundations). It has also been well documented that the optimisation approach performed during post-processing results in a non-unique solution [200]. Furthermore, measurement procedures related to the calculation of low frequency damping are not yet well developed.

In addition to the errors introduced during MASW testing and post-processing, there are also potential errors introduced by the numerical soil modelling approach (e.g. boundary effects, non-linear effects...etc) [201]. Therefore, in comparison to the experimental transfer function approach, numerical methods introduce unnecessary risk into the prediction process. Similar challenges arise relating to the modelling of the soil-structure interaction/coupling [202-204], which if modelled explicitly using numerical methods, requires extensive computational effort. Furthermore, it should also be noted that the time required to perform a numerical prediction is often longer than an experimental one. This is because the deployment of numerical models is computationally intense and requires more time than the experimental transfer function investigation. This effect can be magnified if high accuracy predictions are required (e.g. for a hospital, manufacturing plant or concern hall), because for this single site, the same model may need to be computed numerous times to assess the effect of input parameter uncertainty on vibration levels [205-206].

For underground high speed lines, it is possible to use a similar experimental approach, where a borehole is drilled to the depth of the proposed tunnel, and an impact performed at the base. This method is impractical if the water table location is likely to change after construction (e.g. tidal effects), or if large offset measurements are required.

Similarly, if the proposed site cannot be accessed then it may become necessary to use numerical modelling.

The numerical modelling approaches used to predicting structural vibration from underground lines [137-138], [207-208], [124], [129] are similar to those discussed in the previous soil propagation section. This is because it is common to predict soil vibration using numerical methods and then use an empirical transfer function based on the building type [139], [209-210] to calculate the structural vibration and thus indoor noise. This approach is taken because the computational demand of coupling a building to a train, track and soil model is prohibitive, with many site-specific (and thus challenging to quantify) parameters.

Numerical models remain attractive because, assuming a suitable model exists, the assessment process may be cheaper and quicker to deploy, particularly if the consultant is not based near the projects' geographical location. Despite this, to achieve high accuracy, numerical methods often require geophysical investigations to determine input parameters (e.g. soil, track and building characteristics). If these parameters can be obtained, then rather than potentially compromising the extensive effort required to simulate soil vibration by combining it with an empirical approach for building vibration, there is a desire to model the entire physical problem numerically. Thus, with the aim of superseding the current approach, much research is being undertaken to numerically predict the entire wave propagation path from train wheel to structure and its ultimate conversion into indoor noise. If the entire system can be correctly modelled then it becomes possible to better investigate mitigation measures and the effect of track changes directly on indoor noise. Despite this, if/when numerical modelling overcomes these problems; another challenge is the quantification and acquisition of additional structure-soil coupling parameters.

In an attempt to model the entire wave propagation path, [211] developed a finite element model of a building and used a harmonic load to simulate the vibrations generated by an underground train. Alternatively, [212] used a sub-modelling approach to couple a 2D building with a 3D underground train, track and soil model. This frequency domain approach allowed for the rapid calculation of the building response and included rail irregularities.

[213-214] also predicted structural vibration and in-door noise, but using a FE/BE approach. The track was modelled using the finite element method and the boundary element method was used for the soil response. To simulate the structural response a sub-structuring approach was used and the in-door acoustic response calculated using spectral finite element methods.

As vibration prediction models become more complex, achieving higher accuracy, software computational times increase. The four cornerstones of a prediction model are interlinked and are shown in Figure 17. As a counterexample, for a simple empirical model, the execution time is negligible, the usability is high meaning extensive training is not required, and parameter requirements are low meaning few input parameters require investigation prior to execution. However, these advantages are offset by the low model accuracy.

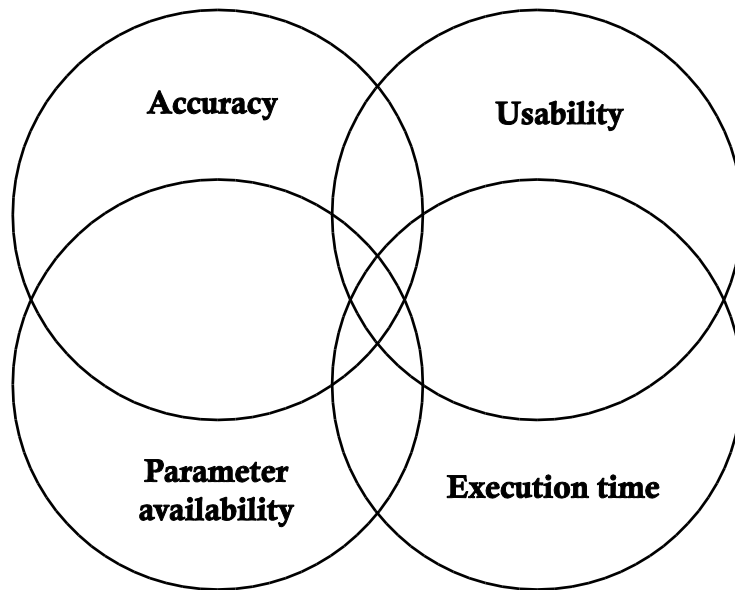


Figure 17 – The four desirable characteristics of a vibration prediction model

The effects of structural vibration

If the natural frequencies of the ground vibration and a nearby structure are similar, then the structure will vibrate. This can impact on humans dwelling within this structure [215] through three primary methods:

1. Whole body response
2. Low frequency noise from vibrating floors and walls
3. Noise/vibration from other objects, e.g. rattling of windows, doors and furniture

A large body of research has attempted to derive exposure–response relationships to determine the level of annoyance of building residents to railway vibrations [216,217]. For example, [218,188] derived relationships for freight trains, [219] analysed annoyance for passenger trains, and [220] analysed the effect of the quantity of train passages on annoyance.

Additionally, vibration is perceived differently depending on the subject’s reference value of vibration. For example, for an existing railway line that is upgraded and vibration levels are less than 25% greater than the levels previously experienced, residents are unlikely to perceive the increase [184].

There are many national and international standards which provide guidelines for acceptable vibration levels [221,188]. Vibration levels are typically calculated differently depending on country, however in general, can be divided into three broad types: Maximum running RMS (root mean squared) values, energy equivalent RMS descriptors and VDV (vibration dose values) as used in the UK [222]. These types are also subject to individual frequency weightings and directional weightings depending on country. Some of the most commonly used international metrics to predict vibration levels include: KB [223], VDV [224] and VdB [139]. Furthermore, investigations into the relationship between noise and vibration and have found that:

1. When combined, the presence of both noise and vibration can affect the human perception of each individual component.
2. Vibration has little effect on perceived noise levels [225], but that noise has an effect on perceived vibration levels [226]. This has been found in both the lab and in field testing [227].
3. No standards or guidelines currently include combined noise-vibration relationships.
4. Human perception to vibration (and noise) is highly subjective and dependent on many factors [228-229, 221].

Mitigation of structural vibration

If a structure is planned to be constructed near a railway line then rather than implement a ground abatement measure (e.g. trench), it is also possible to isolate the vibration via structural design. This approach is most commonly employed for buildings above underground railway lines [230].

Many structural abatement type approaches stem from earthquake engineering principles. The main principle is to isolate the building from the ground vibration, typically, using a spring type mechanism (aka base isolation) to alter the frequency response of the system. Despite this, isolation is complex because the soil-building response is dynamic and effected by the foundation properties, isolation method, structural flexibility and building damping [13]. Approaches to isolation include [13]:

1. Mounting the new building on springs [230–234]
2. Increasing the thickness of the lower floor [235].

Conclusions

From a synthesis of over 200 scientific papers, a number of new conclusions regarding ground borne vibrations and Rayleigh waves from classic and high speed railways have emerged:

(1) Vehicle vibration is an area that affects passenger comfort. The implications of differing vibration frequency ranges have been synthesised from the literature highlighting the effect on the passengers' ability to read and write – typically it is worst at around 4 Hz. Motion sickness occurs at even lower frequencies in the range 0.25-0.32 Hz. Mitigation strategies have then been synthesised from the literature – e.g. reduction in unsprung mass and active control; systems.

(2) It is well established that track vibrations are undesirable because they cause riding quality and safety concerns; and increase track degradation. A large volume of papers have been written on predicting track and ground borne vibrations using techniques ranging from classical mathematical analyses to finite element (FEM) and boundary element methods (BEM). The many analysis techniques have been summarised in tabular format drawing attention to 2-D, 2.5-D and 3-D analyses. An attempt has been made to present a clear and simplified set of guidelines for designers. An obvious conclusion is that the more complex 3-D models are computationally hungry.

(3) Literature refers to the 'critical velocity' of both the train and the track. Although difficult to quantify in absolute terms, it has been demonstrated that if the train exceeds

both the Rayleigh wave velocity and the ‘track critical velocity’ (> Rayleigh velocity) then the train will be subject to large amplitude vibrations and a Mach cone wave propagation pattern will develop.

(4) It is now clear that for design and operational purposes, that trains should be operated below the Rayleigh wave velocity.

(5) The review of papers demonstrates that conventional site investigations (SI) do not provide the relevant data for estimation of Rayleigh velocities. Despite this, for the first time, using work by Jamiolkowski et al. [183], a relationship was developed relating a conventional SI to input parameters for the calculation of Rayleigh wave velocity, linked to soil plasticity. From this analysis, for the first time, estimates of ‘critical velocity’ have been made, albeit these must be seen as embryonic at this stage in the state-of-the-art. The ‘critical velocity’ was calculated at 0.7 of the Rayleigh wave velocity.

(6) At the proposed ‘critical velocity’, calculated as 0.7 times the Rayleigh wave velocity, exaggerated problems due to the ‘track critical velocity’ should therefore be avoided.

(7) The ground provides the propagation path between track and nearby buildings. Structural vibration is an increasing concern in urban environments. It has been shown that structural vibrations become more noticeable when combined with noise pollution. The FRA proposes that structural building resonance can amplify ground borne vibrations by over 100%.

(8) Various vibration mitigation strategies have been identified ranging from ground improvement to wave isolation measures such as trenches backfilled with low acoustic impedance backfill.

Acknowledgements

This work was supported financially by EPSRC grant number EP/H029397/1. The authors are grateful to Heriot Watt University, Université de Mons and the University of Edinburgh for the support and resources provided to undertake this research.

References

- [1] UIC High Speed Department, “High speed lines in the world,” 2013.
- [2] H. Hung and Y. Yang, “A Review of Researches on Ground-Borne Vibrations with emphasis on those induced by trains,” *Proc. Natl. Sci. Counc.*, vol. 25, no. 1, pp. 1–16, Jun. 2000.
- [3] G. Lombaert, G. Degrande, S. François, and D. J. Thompson, “Ground-borne vibration due to railway traffic,” in *International Workshop of Railway Noise*, 2013, no. figure 1, pp. 266–301.

- [4] J. Avillez, M. Frost, S. Cawser, C. Skinner, A. El-Hamalawi, P. Fleming, and P. Shields, "Procedures for estimating environmental impact from railway induced vibration - a review," in *Proceedings of the Internoise 2012/ASME NCAD meeting*, 2012.
- [5] T. Dahlberg, "Some railroad settlement models—a critical review," *Proc. Inst. Mech. Eng. Part F J. Rail Rapid Transit*, vol. 215, no. 4, pp. 289–300, Jan. 2001.
- [6] A. M. Remennikov and S. Kaewunruen, "Progress in Structural Engineering and Materials : Structural Safety and Reliability A review of loading conditions for railway track structures due to train and track vertical interaction," *Struct. Control Heal. Monit.*, no. October 2006, pp. 207–234, 2008.
- [7] D. W. Barke and W. K. Chiu, "A Review of the Effects of Out-Of-Round Wheels on Track and Vehicle Components," *Proc. Inst. Mech. Eng. Part F J. Rail Rapid Transit*, vol. 219, no. 3, pp. 151–175, Jun. 2005.
- [8] Y. Sato, A. Matsumoto, and K. Knothe, "Review on rail corrugation studies," *Wear*, vol. 253, pp. 130–139, 2002.
- [9] K. Knothe and S. Grassie, "Modelling of railway track and vehicle/track interaction at high frequencies," *Veh. Syst. Dyn.*, vol. 22, pp. 209–262, 1993.
- [10] K. Popp, H. Kruse, and I. Kaiser, "Vehicle-Track Dynamics in the Mid-Frequency Range," *Veh. Syst. Dyn.*, no. January 2014, pp. 423–464, 2010.
- [11] Deutsche Bahn AG and Hans-Joerg Terno, "State of the art review of mitigation measures on track. Deliverable 3.1 (RIVAS)," 2011.
- [12] A. Eitzenberger, "Train-induced Vibrations in Tunnels – A Review," 2008.
- [13] D. E. H. Newland and H. E. M. Hunt, "Isolation of buildings from ground vibration - a review of recent progress," *Proc. Inst. Mech. Eng.*, vol. 205, pp. 39–52, 1991.
- [14] D. Clouteau, R. Cotteneau, and G. Lombaert, "Dynamics of structures coupled with elastic media—A review of numerical models and methods," *J. Sound Vib.*, vol. 332, no. 10, pp. 2415–2436, May 2013.
- [15] N. J. Mansfield, "Low Frequency Vibration Comfort - a literature review," *Hum. Perform.*, 2006.
- [16] G. Lombaert and G. Degrande, "Ground-borne vibration due to static and dynamic axle loads of InterCity and high-speed trains," *J. Sound Vib.*, vol. 319, pp. 1036–1066, Jan. 2009.
- [17] D. Thompson, *Railway Noise and Vibration. Mechanisms, modelling and means of control*. Elsevier Ltd, 2009, pp. 1–532.

- [18] R. Woods, Screening of surface waves in soils, *J Soil Mech Found Div ASCE*, 94 (1968), pp. 951-979
- [19] L. Bergmann, *Ultrasonics and their Scientific and Technical Applications*. Wiley, New York, 1948.
- [20] M. Rahman and T. Michelitsch, "A note on the formula for the Rayleigh wave speed," *Wave Motion*, vol. 43, no. 3, pp. 272–276, Jan. 2006.
- [21] P. G. Malischewsky, "Comparison of approximated solutions for the phase velocity of Rayleigh waves (Comment on 'Characterization of surface damage via surface acoustic waves')," *Nanotechnology*, vol. 16, no. 6, pp. 995–996, Jun. 2005.
- [22] G. Kouroussis, D. P. Connolly, and O. Verlinden, "Railway induced ground vibrations - a review of vehicle effects," *Int. J. Rail Transp.*, vol. 2, no. 2, 2014.
- [23] J. Sundström and S. Khan, "Influence of stationary lateral vibrations on train passengers' difficulty to read and write.," *Appl. Ergon.*, vol. 39, no. 6, pp. 710–8, Nov. 2008.
- [24] C. Corbridge and M. J. Griffin, "Effects of vertical vibration on passenger activities: writing and drinking," *Ergonomics*, vol. 34, no. 10, pp. 1313–1332, 1991.
- [25] H. Suzuki, "Research trends on riding comfort evaluation in Japan," *Proc. Inst. Mech. Eng. Part F J. Rail Rapid Transit*, vol. 212, no. 1, pp. 61–72, Jan. 1998.
- [26] M. G. Pollard and N. J. A. Simons, "Passenger comfort-the role of active suspensions," *J. Automob. Eng.*, vol. 198D, no. 11, pp. 161–175, 1984.
- [27] M. J. Griffin, "Effects of horizontal whole-body vibration on reading," *Appl. Ergon.*, vol. 25, no. 3, pp. 165–169, 1994.
- [28] M. S. Khan and J. Sundstrom, "Effects of vibration on sedentary activities in passenger trains," *Low Freq. Noise Vib. Act. Control*, vol. 26, no. 1, pp. 43–55, Mar. 2007.
- [29] H. Suzuki, H. Shiroto, and K. Tezuka, "Effects of Low Frequency Vibration on Train Motion Sickness Vibration," *Q. Rep. RTRI*, vol. 46, no. 1, pp. 35–39, 2005.
- [30] M. H. Kargarnovin, D. Younesian, D. Thompson, and C. Jones, "Ride comfort of high-speed trains travelling over railway bridges," *Veh. Syst. Dyn.*, vol. 43, no. 3, pp. 173–199, 2005.
- [31] W. Yoo, C. Lee, W. Jeong, and S. Kim, "Development and application of new evaluation system for ride comfort and vibration on railway vehicles," *J. Mech. Sci. Technol.*, vol. 19, no. 7, pp. 1469–1477, Jul. 2005.

- [32] A. Stribersky, F. Moser, and W. Rulka, "Structural dynamics and ride comfort of a rail vehicle system," *Adv. Eng. Softw.*, vol. 33, no. 7–10, pp. 541–552, Jul. 2002.
- [33] Y. Wu and Y. Yang, "Steady-state response and riding comfort of trains moving over a series of simply supported bridges," *Eng. Struct.*, vol. 25, no. 2, pp. 251–265, Jan. 2003.
- [34] J. Yau, Y. Yang, and S. Kuo, "Impact response of high speed rail bridges and riding comfort of rail cars," *Eng. Struct.*, vol. 21, no. 9, pp. 836–844, Sep. 1999.
- [35] J. H. Lee, B. S. Jin, and Y. Ji, "Development of a Structural Equation Model for ride comfort of the Korean high-speed railway," *Int. J. Ind. Ergon.*, vol. 39, no. 1, pp. 7–14, Jan. 2009.
- [36] M. Nejlaoui, Z. Affi, A. Houidi, and L. Romdhane, "Analytical modeling of rail vehicle safety and comfort in short radius curved tracks," *Comptes Rendus Mécanique*, vol. 337, no. 5, pp. 303–311, May 2009.
- [37] K. Karakasis, D. Skarlatos, and T. Zakinthinos, "A factorial analysis for the determination of an optimal train speed with a desired ride comfort," *Appl. Acoust.*, vol. 66, no. 10, pp. 1121–1134, Oct. 2005.
- [38] G. Wilson, H. Saurenman, and J. Nelson, "Control of ground-borne noise and vibration," *J. Sound Vib.*, vol. 87, no. 2, pp. 339–350, 1983.
- [39] A. Pita, P. F. Teixeira, and F. Robuste, "High speed and track deterioration: the role of vertical stiffness of the track," *Proc. Inst. Mech. Eng. Part F J. Rail Rapid Transit*, vol. 218, no. 1, pp. 31–40, Jan. 2004.
- [40] T. Dahlberg, "Railway Track Stiffness Variations – Consequences and Countermeasures," vol. 8, no. 1, pp. 1–12, 2010.
- [41] J. Nielsen and A. Johansson, "Out-of-round railway wheels-a literature survey," *Proc. Inst. Mech. Eng. Part F J. Rail Rapid Transit*, vol. 214, no. 2, pp. 79–91, Jan. 2000.
- [42] J. Nielsen, A. Mirza, S. Cervello, A. Frid, R. Muller, B. Nelain, and P. Ruest, "Train Induced Ground Vibration – Optimised Rolling Stock Mitigation Measures and their Parameters (RIVAS)," 2013.
- [43] J. Nielsen, A. M. Mirza, P. Ruest, P. Huber, S. Cervello, R. Muller, and B. Nelain, "Guideline for Design of Vehicles Generating Reduced Ground Vibration (RIVAS)," 2013.
- [44] N. Varandas, P. Hölscher, M.a.G. Silva, Dynamic behaviour of railway tracks on transitions zones, *Comput Struct*, 89 (13–14) (2011), pp. 1468-1479
- [45] M. Heckl, G. Hauck, and R. Wettschureck, "Structure-Borne Sound and Vibration From Rail Traffic," *J. Sound Vib.*, vol. 193, no. 1, pp. 175–184, May 1996.

- [46] British Standards Institution, "Mechanical vibration — Ground-borne noise and vibration arising from rail systems — BS ISO 14837 Part 1 : General guidance," 2005.
- [47] R. Ferrara, G. Leonardi, and F. Jourdan, "A Two-Dimensional Numerical Model to Analyze the Critical Velocity of High Speed Infrastructure The 2D Numerical Model," in *Proceedings of the fourteenth international conference on civil, structural and environmental engineering computing*, 2013, pp. 1–16.
- [48] J. Kenney, "Steady-state vibrations of beam on elastic foundation for moving load," *J. Appl. Mech.*, vol. 76, pp. 359–364, 1954.
- [49] L. Fryba, *Vibration of Solids and Structures Under Moving Loads*. Groningen, The Netherlands: Noordhoff International Publishing, 1972, pp. 1–524.
- [50] S. Iwnicki, *Handbook of Railway Vehicle Dynamics*. 2006, pp. 1–552.
- [51] S. G. Newton and R. a Clark, "An investigation into the dynamic effects on the track of wheelflats on railway vehicles," *Arch. J. Mech. Eng. Sci. 1959-1982 (vols 1-23)*, vol. 21, no. 4, pp. 287–297, Jun. 1979.
- [52] C. Cai, Y. K. Cheung, and H. C. Chan, "Dynamic response of infinite continuous beams subjected to a moving force - An exact method," *J. Sound Vib.*, vol. 123, no. 22, pp. 461–472, 1988.
- [53] S. Chonan, "The elastically supported Timoshenko beam subjected to an axial force and a moving load," *Int. J. Mech. Sci.*, vol. 17, no. 9, pp. 573–581, 1975.
- [54] L. Auersch, "Dynamic interaction of various beams with the underlying soil – finite and infinite, half-space and Winkler models," *Eur. J. Mech. - A/Solids*, vol. 27, pp. 933–958, Sep. 2008.
- [55] A. Suiker, R. de Borst, and C. Esveld, "Critical behaviour of a Timoshenko beam-half plane system under a moving load," *Arch. Appl. Mech.*, vol. 68, no. 3–4, pp. 158–168, Apr. 1998.
- [56] Y. Chen, "Response of an Infinite Timoshenko Beam on a Viscoelastic Foundation To a Harmonic Moving Load," *J. Sound Vib.*, vol. 241, no. 5, pp. 809–824, Apr. 2001.
- [57] M. H. Kargarnovin, D. Younesian, D. J. Thompson, and C. J. C. Jones, "Response of beams on nonlinear viscoelastic foundations to harmonic moving loads," *Comput. Struct.*, vol. 83, no. 23–24, pp. 1865–1877, Sep. 2005.
- [58] M. Kargarnovin, "Dynamics of Timoshenko beams on Pasternak foundation under moving load," *Mech. Res. Commun.*, vol. 31, no. 6, pp. 713–723, Dec. 2004.
- [59] T. Wu and D. J. Thompson, "On the parametric excitation of the wheel/track system," *J. Sound Vib.*, vol. 278, no. 4–5, pp. 725–747, Dec. 2004.

- [60] X. Lei and J. G. Rose, "Track vibration analysis for railways with mixed passenger and freight traffic," *Proc. Inst. Mech. Eng., Part F J. Rail Rapid Transit*, 2008.
- [61] D. Mead, "Free wave propagation in periodically supported infinite beams," *J. Sound Vib.*, vol. 11, pp. 187–197, 1970.
- [62] B. Croft, "The development of rail-head acoustic roughness."
- [63] G. Kouroussis, G. Gazetas, I. Anastasopoulos, C. Conti, and O. Verlinden, "Discrete modelling of vertical track–soil coupling for vehicle–track dynamics," *Soil Dyn. Earthq. Eng.*, vol. 31, no. 12, pp. 1711–1723, Dec. 2011.
- [64] K. Knothe and Y. Wu, "Receptance behaviour of railway track and subgrade," *Arch. Appl. Mech.*, vol. 68, no. 7–8, pp. 457–470, Aug. 1998.
- [65] S. Patil, "Natural Frequencies of a Railroad Track," *J. Appl. Mech.*, vol. 54, no. 2, pp. 299–304, 1987.
- [66] Z. G. Cao, Y. Q. Cai, H. L. Sun, and C. J. Xu, "Dynamic responses of a poroelastic half-space from moving trains caused by vertical track irregularities," *Int. J. Numer. Anal. Methods Geomech.*, vol. 35, pp. 761–786, 2011.
- [67] Y. Cai, Z. Cao, H. Sun, and C. Xu, "Effects of the dynamic wheel–rail interaction on the ground vibration generated by a moving train," *Int. J. Solids Struct.*, vol. 47, no. 17, pp. 2246–2259, Aug. 2010.
- [68] X. Sheng, C. J. C. Jones, and D. J. Thompson, "A theoretical model for ground vibration from trains generated by vertical track irregularities," *J. Sound Vib.*, vol. 272, no. 3–5, pp. 937–965, May 2004.
- [69] S. Timoshenko, *The collected papers of Stepan Prokopovitch Timoshenko 1878-1972*. London ; New York: McGraw-Hill, 1953, pp. 1–642.
- [70] M. Shamalta and A. V Metrikine, "Analytical study of the dynamic response of an embedded railway track to a moving load," *Arch. Appl. Mech.*, vol. 73, pp. 131 – 146, 2003.
- [71] M. J. M. M. Steenbergen and A. V Metrikine, "The effect of the interface conditions on the dynamic response of a beam on a half-space to a moving load," *J. Mech.*, vol. 26, pp. 33–54, 2007.
- [72] L. Sun, "Dynamic displacement response of beam-type structures to moving line loads," *Int. J. Solids Struct.*, vol. 38, no. 48–49, pp. 8869–8878, Nov. 2001.
- [73] A. V Metrikine, "Stability of a two-mass oscillator moving on a beam supported by a visco-elastic half-space," *Int. J. Solids Struct.*, vol. 42, no. 3–4, pp. 1187–1207, Feb. 2005.

- [74] J. C. O. Nielsen and T. J. S. Abrahamsson, "Coupling of physical and modal components for analysis of moving non-linear dynamic systems on general beam structures," *Int. J. Numer. Methods Eng.*, vol. 33, pp. 1843–1859, 1992.
- [75] X. Sheng, C. J. C. Jones, and D. J. Thompson, "A comparison of a theoretical model for quasi-statically and dynamically induced environmental vibration from trains with measurements," *J. Sound Vib.*, vol. 267, pp. 621–635, 2003.
- [76] X. Lei and N. Noda, "Analyses of dynamic response of vehicle and track coupling system with random irregularity of track vertical profile," *J. Sound Vib.*, vol. 258, pp. 147–165, 2002.
- [77] J. Nielsen and A. Igeland, "Vertical Dynamic Interaction Between Train and Track Influence of Wheel and Track Imperfections," *J. Sound Vib.*, vol. 187, no. 5, pp. 825–839, Nov. 1995.
- [78] G. Kouroussis, O. Verlinden, and C. Conti, "A two-step time simulation of ground vibrations induced by the railway traffic," *Proc. Inst. Mech. Eng. Part C J. Mech. Eng. Sci.*, vol. 226, no. 2, pp. 454–472, Oct. 2012.
- [79] Y. Q. Sun and M. Dhanasekar, "A dynamic model for the vertical interaction of the rail track and wagon system," *Int. J. Solids Struct.*, vol. 39, pp. 1337–1359, 2002.
- [80] S. H. Ju, J. R. Liao, and Y. L. Ye, "Behavior of ground vibrations induced by trains moving on embankments with rail roughness," *Soil Dyn. Earthq. Eng.*, vol. 30, no. 11, pp. 1237–1249, Nov. 2010.
- [81] W. Zhai, "Two simple fast integration methods for large-scale dynamic problems in engineering," *Int. J. Numer. Methods Eng.*, vol. 39, no. 1994, pp. 4199–4214, 1996.
- [82] W. M. Zhai, Q. C. Wang, Z. W. Lu, and X. S. Wu, "Dynamic effects of vehicles on tracks in the case of raising train speeds," *Proc. Inst. Mech. Eng. Part F J. Rail Rapid Transit*, vol. 215, no. 2, pp. 125–135, Jan. 2001.
- [83] G. Kouroussis, O. Verlinden, and C. Conti, "On the interest of integrating vehicle dynamics for the ground propagation of vibrations: the case of urban railway traffic," *Veh. Syst. Dyn.*, vol. 48, no. 12, pp. 1553–1571, Dec. 2010.
- [84] P. A. Costa, R. Calcada, and A. S. Cardoso, "Influence of train dynamic modelling strategy on the prediction of track-ground vibrations induced by railway traffic," *Proc. Inst. Mech. Eng. Part F J. Rail Rapid Transit*, vol. 226, no. 4, pp. 434–450, Jan. 2012.
- [85] R. G. Dong, S. Sankar, and R. V. Dukkipati, "A finite element model of railway track and its application to the wheel flat problem," *Arch. Proc. Inst. Mech. Eng. Part F J. Rail Rapid Transit 1989-1996 (vols 203-210)*, vol. 208, no. 16, pp. 61–72, Jun. 1994.
- [86] S. H. Ju, H. Lin, C. Hsueh, and S. Wang, "A simple finite element model for vibration analyses induced by moving vehicles," *Online*, no. April, pp. 1232–1256, 2006.

- [87] L. Auersch, "The excitation of ground vibration by rail traffic: theory of vehicle-track-soil interaction and measurements on high-speed lines," *J. Sound Vib.*, vol. 284, pp. 103–132, Jun. 2005.
- [88] M. Banimahd, J. Kennedy, P. Woodward, and G. Medero, "Behaviour of train – track interaction in stiffness transitions," *Proc. Inst. Civ. Eng.*, vol. 1, pp. 1–10, 2010.
- [89] Y. Bezin, S. D. Iwnicki, M. Cavalletti, E. De Vries, F. Shahzad, and G. Evans, "An investigation of sleeper voids using a flexible track model integrated with railway multi-body dynamics," *J. rail rapid transit*, vol. 223, pp. 597–607, 2009.
- [90] A. El Kacimi, P. K. Woodward, O. Laghrouche, and G. Medero, "Time domain 3D finite element modelling of train-induced vibration at high speed," *Comput. Struct.*, vol. 118, pp. 66–73, Mar. 2013.
- [91] J. Wang, X. Jin, and Y. Cao, "High-speed maglev train-guideway-tunnel-soil modelling of ground vibration," *Proc. Inst. Mech. Eng. Part F J. Rail Rapid Transit*, vol. 226, no. 3, pp. 331–344, Oct. 2011.
- [92] J. Wang, X. Jin, Y. Cao, and X. Du, "Numerical Simulation of High-Speed Maglev Vehicle-Guideway-Tunnel-Soil System," *Int. J. Comput. Methods Eng. Sci. Mech.*, vol. 13, no. 2, pp. 93–107, Mar. 2012.
- [93] D. Connolly, A. Giannopoulos, and M. C. Forde, "Numerical modelling of ground borne vibrations from high speed rail lines on embankments," *Soil Dyn. Earthq. Eng.*, vol. 46, pp. 13–19, Mar. 2013.
- [94] D. Connolly, A. Giannopoulos, W. Fan, P. K. Woodward, and M. C. Forde, "Optimising low acoustic impedance back-fill material wave barrier dimensions to shield structures from ground borne high speed rail vibrations," *Constr. Build. Mater.*, vol. 44, pp. 557–564, Jul. 2013.
- [95] C. O'Sullivan, *Particulate Discrete Element Modelling: A Geomechanics Perspective*. 2011, p. 576.
- [96] W. L. Lim and G. R. McDowell, "Discrete element modelling of railway ballast," *Granul. Matter*, vol. 7, no. 1, pp. 19–29, Jan. 2005.
- [97] M. Lu and G. R. McDowell, "The importance of modelling ballast particle shape in the discrete element method," *Granul. Matter*, vol. 9, no. 1–2, pp. 69–80, Jul. 2006.
- [98] S. Lobo-Guerrero and L. E. Vallejo, "Discrete Element Method Analysis of Railtrack Ballast Degradation during Cyclic Loading," *Granul. Matter*, vol. 8, no. 3–4, pp. 195–204, Mar. 2006.
- [99] A. Suiker, "Dynamic Behaviour of a Layer of Discrete Particles, Part 2: Response To a Uniformly Moving, Harmonically Vibrating Load," *J. Sound Vib.*, vol. 240, no. 1, pp. 19–39, Feb. 2001.

- [100] F. Dubois, M. Renouf, P. Taforel, and C. Voivret, "On the use of an high performance hybrid FEM/DEM modelling approach for an improved simulation of railway track," in *RTSE International Workshop - Ballast: Issues and Challenges*, 2013.
- [101] H. Huang and S. Chrismer, "Discrete element modeling of ballast settlement under trains moving at 'Critical Speeds'," *Constr. Build. Mater.*, vol. 38, pp. 994–1000, Jan. 2013.
- [102] P. A. Costa, R. Calçada, and A. S. Cardoso, "Ballast mats for the reduction of railway traffic vibrations. Numerical study," *Soil Dyn. Earthq. Eng.*, vol. 42, pp. 137–150, Nov. 2012.
- [103] H. Lamb, "On the propagation of tremors over the surface of an elastic solid," *Philos. Trans. R. Soc. London. Ser. A, Contain. Pap. a Math. or Phys. Character*, vol. 203, pp. 1–42, Jul. 1904.
- [104] J. D. Achenbach, "Lamb 's problem for solids of general anisotropy," *Science (80-.)*, vol. 24, pp. 227–242, 1996.
- [105] J. D. Achenbach, *Wave Propagation in Elastic Solids*. North Holland, 1984, p. 440.
- [106] T. G. Gutowski and C. L. Dym, "Propagation of ground vibration: A review," *J. Sound Vib.*, vol. 49, no. 2, pp. 179–193, 1976.
- [107] V. V Krylov, *Noise and Vibration from High-Speed Trains*. Thomas Telford Ltd, 2001, pp. 1–435.
- [108] V. V. Krylov, "Vibrational impact of high-speed trains. I. Effect of track dynamics," *J. Acoust. Soc. Am.*, vol. 100, no. 5, p. 3121, 1996.
- [109] H. Dieterman and A. Metrikine, "The Equivalent stiffness of a half-space interacting with a beam. Critical velocities of a moving load along the beam," *Eur. J. Mech. - A/Solids*, vol. 15, no. 1, pp. 67–90, 1996.
- [110] G. Degrande and G. Lombaert, "An efficient formulation of Krylov's prediction model for train induced vibrations based on the dynamic reciprocity theorem," *J. Acoust. Soc. Am.*, vol. 110, no. 3, pp. 1379–1390, 2001.
- [111] H. Grundmann and S. Lenz, "Nonlinear interaction between a moving SDOF system and a Timoshenko beam / halfspace support," *Arch. Appl. Mech.*, vol. 72, pp. 830–842, 2003.
- [112] A. Karlstrom and A. Bostrom, "An analytical model for train-induced ground vibrations from railways," *J. Sound Vib.*, vol. 292, pp. 221–241, Apr. 2006.
- [113] H. Takemiya, "Simulation of track–ground vibrations due to a high-speed train: the case of X-2000 at Ledsgard," *J. Sound Vib.*, vol. 261, no. 3, pp. 503–526, Mar. 2003.

- [114] I. Sneddon, *Fourier Transforms*. McGraw-Hill, New York, USA, 1951.
- [115] D. C. Gakenheimer and J. Miklowitz, "Transient Excitation of an Elastic Half Space by a Point Load Traveling on the Surface," *J. Appl. Mech.*, vol. 36, no. 3, pp. 505–515, 1969.
- [116] J. Luco and R. Apsel, "On the greens functions for a layered half-space - part 1," *Bull. Seismol. Soc. Am.*, vol. 73, no. 4, pp. 909–929, 1983.
- [117] B. Alabi, "A parametric study on some aspects of ground-borne vibrations due to rail traffic," *J. Sound Vib.*, vol. 153, no. 1, pp. 77–87, 1992.
- [118] A. T. de Hoop, "The moving-load problem in soil dynamics - the vertical displacement approximation," *Wave Motion*, vol. 36, pp. 335–346, 2002.
- [119] M. C. M. Bakker, M. D. Verweij, B. J. Kooij, and H. A. Dietermana, "The traveling point load revisited," *Wave Motion*, vol. 29, pp. 119–135, 1999.
- [120] W. Liao, T. Teng, and C. Yeh, "A method for the response of an elastic half-space to moving sub-Rayleigh point loads," *J. Sound Vib.*, vol. 284, no. 1–2, pp. 173–188, Jun. 2005.
- [121] H. Grundmann, M. Lieb, and E. Trommer, "The response of a layered half-space to traffic loads moving along its surface," *Arch. Appl. Mech. (Ingenieur Arch.)*, vol. 69, no. 1, pp. 55–67, Feb. 1999.
- [122] M. Lieb and B. Sudret, "A fast algorithm for soil dynamics calculations by wavelet decomposition," *Arch. Appl. Mech. (Ingenieur Arch.)*, vol. 68, no. 3–4, pp. 147–157, Apr. 1998.
- [123] M. Katou, T. Matsuoka, O. Yoshioka, Y. Sanada, and T. Miyoshi, "Numerical simulation study of ground vibrations using forces from wheels of a running high-speed train," *J. Sound Vib.*, vol. 318, pp. 830–849, Dec. 2008.
- [124] R. M. Thornely-Taylor, "The Prediction Of Vibration , Groundborne And Structure-Radiated Noise From Railways Using Finite Difference Methods – Part I - Theory," in *Proceedings of the Institute of Acoustics*, 2004, vol. 26, pp. 1–11.
- [125] G. Kouroussis, O. Verlinden, and C. Conti, "Free field vibrations caused by high-speed lines: Measurement and time domain simulation," *Soil Dyn. Earthq. Eng.*, vol. 31, no. 4, pp. 692–707, Apr. 2011.
- [126] P. Galvin and J. Domínguez, "Experimental and numerical analyses of vibrations induced by high-speed trains on the Córdoba–Málaga line," *Soil Dyn. Earthq. Eng.*, vol. 29, no. 4, pp. 641–657, Apr. 2009.
- [127] J. O'Brien and D. Rizos, "A 3D BEM-FEM methodology for simulation of high speed train induced vibrations," *Soil Dyn. Earthq. Eng.*, vol. 25, pp. 289–301, Apr. 2005.

- [128] P. Galvín and a. Romero, "A 3D time domain numerical model based on half-space Green's function for soil–structure interaction analysis," *Comput. Mech.*, Nov. 2013.
- [129] X. Sheng, C. Jones, and D. Thompson, "Prediction of ground vibration from trains using the wavenumber finite and boundary element methods," *J. Sound Vib.*, vol. 293, no. 3–5, pp. 575–586, Jun. 2006.
- [130] L. Andersen and S. R. K. Nielsen, "Reduction of ground vibration by means of barriers or soil improvement along a railway track," *Soil Dyn. Earthq. Eng.*, vol. 25, no. 7–10, pp. 701–716, Aug. 2005.
- [131] L. Auersch, "The Influence of the Soil on Track Dynamics and Ground-Borne Vibration," in *Noise and Vibration Mitigation for Rail Transportation Systems, Proceedings of the 9th International Workshop on Railway Noise*, T. Maeda, P.-E. Gautier, C. E. Hanson, B. Hemsworth, J.T. Nelson, B. Schulte-Werning, D. Thompson, and P. de Vos, eds., Vol. 99, 2008, pp. 122–128.
- [132] H. Verbraken, G. Lombaert, and G. Degrande, "Verification of an empirical prediction method for railway induced vibrations by means of numerical simulations," *J. Sound Vib.*, vol. 330, no. 8, pp. 1692–1703, Apr. 2011.
- [133] S. François, M. Schevenels, P. Galvin, G. Lombaert, and G. Degrande, "A 2.5D coupled FE–BE methodology for the dynamic interaction between longitudinally invariant structures and a layered halfspace," *Comput. Methods Appl. Mech. Eng.*, vol. 199, no. 23–24, pp. 1536–1548, Apr. 2010.
- [134] Y. Yang and H. Hung, "A 2.5D finite/infinite element approach for modelling viscoelastic bodies subjected to moving loads," *Int. J. Numer. Methods Eng.*, vol. 51, no. 11, pp. 1317–1336, Aug. 2001.
- [135] X. Bian, W. Jin, and H. Jiang, "Ground-borne vibrations due to dynamic loadings from moving trains in subway tunnels," *J. Zhejiang Univ. Sci. A*, vol. 13, no. 11, pp. 870–876, Nov. 2012.
- [136] Y. B. Yang, F. Asce, and H. H. Hung, "Soil Vibrations Caused by Underground Moving Trains," *J. Geotech. Geoenvironmental Eng.*, vol. 134, pp. 1633–1644, 2008.
- [137] S. Gupta, M. Hussein, G. Degrande, H. E. M. Hunt, and D. Clouteau, "A comparison of two numerical models for the prediction of vibrations from underground railway traffic," *Soil Dyn. Earthq. Eng.*, vol. 27, no. 7, pp. 608–624, Jul. 2007.
- [138] M. F. M. Hussein and H. E. M. Hunt, "A numerical model for calculating vibration due to a harmonic moving load on a floating-slab track with discontinuous slabs in an underground railway tunnel," *J. Sound Vib.*, vol. 321, no. 1–2, pp. 363–374, Mar. 2009.
- [139] Federal Railroad Administration, "High-Speed Ground Transportation Noise and Vibration Impact Assessment. U.S. Department of Transportation," 2012.

- [140] D. P. Connolly, G. Kouroussis, A. Giannopoulos, O. Verlinden, P. K. Woodward, and M. C. Forde, "Assessment of railway vibrations using an efficient scoping model," *Soil Dyn. Earthq. Eng.*, vol. 58, pp. 37–47, Mar. 2014.
- [141] F. Rossi, A Nicolini "A simple model to predict train-induced vibration - theoretical formulation and experimental validation.", *Environ Impact Asses*, 23 (3) (2003), pp. P.
- [142] Salvador, J. Real, C. Zamorano, A. Villanueva, A procedure for the evaluation of vibrations induced by the passing of a train and its application to real railway traffic, *Math Comput Model*, 53 (1–2) (2011), pp. 42-54 305-322,
- [143] J. Virieux, "P-SV wave propagation in heterogenous media: Velocity-stress finite-difference method," *Geophysics*, vol. 51, no. 4, 1986.
- [144] R. W. Graves, "Simulating Seismic Wave Propagation in 3D Elastic Media Using Staggered-Grid Finite Differences," *Bull. Seismol. Soc. Am.*, vol. 86, no. 4, pp. 1091–1106, 1996.
- [145] U. Basu, "Explicit finite element perfectly matched layer for transient three-dimensional elastic waves," *Int. J. Numer. Methods Eng.*, vol. 77, pp. 151–176, 2009.
- [146] A. Giannopoulos, S. Melling, and D. P. Connolly, "A Second Order PML Implementation for FDTD Seismic Modelling," in *Near Surface Geoscience 2012 – 18th European Meeting of Environmental and Engineering Geophysics*, 2012.
- [147] X. Sheng, C. J. C. Jones, and M. Petyt, "Ground Vibration Generated By a Load Moving Along a Railway Track," *J. Sound Vib.*, vol. 228, no. 1, pp. 129–156, Nov. 1999.
- [148] X. Sheng, C. J. C. Jones, and M. Petyt, "Ground Vibration Generated By a Harmonic Load Acting on a Railway Track," *J. Sound Vib.*, vol. 225, no. 1, pp. 3–28, Aug. 1999.
- [151] X. Sheng, "Ground vibrations generated from trains (PhD thesis)," University of Southampton, 2001.
- [150] G. Liu and S. Quek, *The Finite Element Method - A practical course*. Oxford, UK: Butterworth-Heinemann, 2003, pp. 1–365.
- [151] M. Petyt, *Introduction to finite element vibration analysis*. Cambridge University Press, 2010, p. 518.
- [152] I. Smith and D. Griffiths, *Programming the finite element method*. NY: Wiley, 1997, pp. 1–648.
- [153] P. Bettess and O. C. Zienkiewicz, "Diffraction and refraction of surface waves using finite and infinite elements," *J. Numer. Methods Eng.*, vol. 11, no. 8, pp. 1271–1290, 1977.
- [154] P. Bettess, *Infinite elements*. Penshaw press, 1992.

- [155] R. Astley, "Infinite elements for wave problems: a review of current formulations and an assessment of accuracy," *Int. J. Numer. Methods Eng.*, vol. 49, pp. 951–976, 2000.
- [156] J. Barbosa, J. Park, E. Kausel, Perfectly matched layers in the thin layer method, *Comput Methods Appl Mech Eng*, 217–220 (April) (2012), pp. 262-274
- [157] T. Ekevid, N. Wiberg, Wave propagation related to high-speed train A scaled boundary FE-approach for unbounded domains, *Comput Methods Appl Mech Eng*, 191 (2002), pp. 3947-3964
- [158] J. P. Berenger, "A perfectly matched layer for the absorption of electromagnetic waves," *J. Comput. Phys.*, vol. 114, no. 2, pp. 185–200, Oct. 1994.
- [159] U. Basu, "Perfectly matched layers for time-harmonic elastodynamics of unbounded domains: theory and finite-element implementation," *Comput. Methods Appl. Mech. Eng.*, vol. 192, pp. 1337–1375, Mar. 2003
- [160] J. Domínguez, *Boundary elements in dynamics*. WIT Press, 1993, p. 450.
- [161] E. Çelebi, S. Firat, and İ. Çankaya, "The effectiveness of wave barriers on the dynamic stiffness coefficients of foundations using boundary element method," *Appl. Math. Comput.*, vol. 180, no. 2, pp. 683–699, Sep. 2006.
- [162] P. Fiala, S. Gupta, G. Degrande, and F. Augusztinovicz, "A Numerical Model for Re-radiated Noise in Buildings," pp. 115–121, 2008.
- [163] M. Villot, C. Guigou, P. Jean, and N. Picard, "Definition of appropriate procedures to predict exposure in buildings and estimate annoyance (RIVAS)," 2012.
- [164] D. Clouteau, M. Arnst, T. M. Al-Hussaini, and G. Degrande, "Freefield vibrations due to dynamic loading on a tunnel embedded in a stratified medium," *J. Sound Vib.*, vol. 283, no. 1–2, pp. 173–199, May 2005.
- [165] D. Clouteau, R. Othman, M. Arnst, and H. Chebli, "A numerical model for ground-borne vibrations from underground railway traffic based on a periodic finite element-boundary element formulation," *J. Sound Vib.*, vol. 293, pp. 645–666, 2006.
- [166] G. Degrande and L. Schillemans, "Free Field Vibrations During the Passage of a Thalys High-Speed Train At Variable Speed," *J. Sound Vib.*, vol. 247, no. 1, pp. 131–144, Oct. 2001.
- [167] Connolly DP. *High speed rail dataset*; 2014. www.davidpconnolly.com
- [168] G. Kouroussis, D. P. Connolly, M. C. Forde, and O. Verlinden, "Train speed calculation using ground vibrations," *Proc. Inst. Mech. Eng. Part F J. Rail Rapid Transit*, vol. 0, no. 0, pp. 1–18, Dec. 2014.

- [169] H. Kanda, H. Ishii, and O. Yoshioka, "Field Measurement and its analysis of 'Hybrid Vibration Isolation Wall' using gas cushions," *Transp. Res. Board Annu. Meet.*, 2006.
- [170] A. Alzawi and M. Hesham El Naggar, "Full scale experimental study on vibration scattering using open and in-filled (GeoFoam) wave barriers," *Soil Dyn. Earthq. Eng.*, vol. 31, no. 3, pp. 306–317, Mar. 2011.
- [171] K. R. Massarsch, "Vibration Isolation using Gas-filled Cushions," in *Geo-Frontiers*, 2005, pp. 1–22.
- [172] S. Kattis, D. Polyzos, and D. Beskos, "Modelling of pile wave barriers by effective trenches and their screening effectiveness," *Soil Dyn. Earthq. Eng.*, vol. 18, no. 1, pp. 1–10, Jan. 1999.
- [173] Y. Yang and H. Hung, "A parametric study of wave barriers for reduction of train-induced vibrations," *Int. J. Numer. Methods Eng.*, vol. 40, pp. 3729–3747, 1997.
- [174] G. Gao, G. Shi, S. Feng, and C. Qiu, "3D analysis of in-filled trench as passive barriers for ground vibration isolation," *Sci. China Ser. G Physics, Mech. Astron.*, vol. 51, no. 10, pp. 1573–1585, Aug. 2008.
- [175] A. Karlstrom and A. Bostrom, "Efficiency of trenches along railways for trains moving at sub- or supersonic speeds," *Soil Dyn. Earthq. Eng.*, vol. 27, no. 7, pp. 625–641, Jul. 2007.
- [176] S. H. Ju, "Three-Dimensional Analyses of Wave Barriers for Reduction of Train-Induced Vibrations," no. July, pp. 740–748, 2004.
- [177] P. Coulier, G. Degrande, A. Dijckmans, J. Houbrechts, G. Lombaert, W. Rucker, L. Auersch, M. R. Plaza, V. Cuellar, D. Thompson, A. Ekbald, and A. Smekal, "Scope of the parametric study on mitigation measures on the transmission path (RIVAS - Deliverable D4.1)," 2011.
- [178] B. Picoux, R. Rotinat, J. P. Regoin, and D. Le Houédec, "Prediction and measurements of vibrations from a railway track lying on a peaty ground," *J. Sound Vib.*, vol. 267, no. 3, pp. 575–589, Oct. 2003.
- [179] P. A. Costa, R. Calcada, A. S. Cardoso, and A. Bodare, "Influence of soil non-linearity on the dynamic response of high-speed railway tracks," *Soil Dyn. Earthq. Eng.*, vol. 30, pp. 221–235, 2010.
- [180] C. Madshus, S. Lacasse, A. Kaynia, and L. Harvik, "Geodynamic Challenges in High Speed Railway Projects," in *Geotechnical Engineering for Transportation Projects*, 2004, pp. 192–215.
- [181] G. Holm, B. Andreasson, P. Bengtsson, A. Bodare, and H. Eriksson, "Mitigation of Track and Ground Vibrations by High Speed Trains at Ledsgard, Sweden. Report 10. Svensk Djupstabilisering," 2002.

- [182] C. Madshus, "High-Speed Railway Lines on Soft Ground: Dynamic Behaviour At Critical Train Speed," *J. Sound Vib.*, vol. 231, no. 3, pp. 689–701, Mar. 2000.
- [183] M. Jamiolkowski, R. Lanellotia, S. Marchetti, R. Nova, and E. Pasqualini, "Design parameters for soft clays," in *In Proc. 7th European Conference Soil Mechanics and Foundation Engineering*, 1979, pp. 27–57.
- [184] P. K. Woodward, J. Kennedy, G. M. Medero, and M. Banimahd, "Maintaining absolute clearances in ballasted railway tracks using in situ three-dimensional polyurethane GeoComposites," *Proc. Inst. Mech. Eng. Part F J. Rail Rapid Transit*, vol. 226, no. 3, pp. 257–271, Sep. 2011.
- [185] P. K. Woodward, J. Kennedy, G. M. Medero, and M. Banimahd, "Application of in situ polyurethane geocomposite beams to improve the passive shoulder resistance of railway track," *Proc. Inst. Mech. Eng. Part F J. Rail Rapid Transit*, vol. 226, no. 3, pp. 294–304, Sep. 2011.
- [186] D. Stiebel, R. Muller, E. Bongini, A. Ekbald, G. Coquel, and A. A. Alguacil, "Definition of reference cases typical for hot-spots in Europe with existing vibration problems (RIVAS - Deliverable D1.5)," 2012.
- [187] G. Kouroussis, O. Verlinden, and C. Conti, "Contribution of vehicle / track dynamics to the ground vibrations induced by the Brussels tramway," in *International Conference on Noise and Vibration Engineering (ISMA2010)*, 2010, vol. 2010, pp. 3489–3502.
- [186] C. E. Hanson, D. A. Towers, and L. D. Meister, "Transit noise and vibration (Federal Railroad Administration)," 2006.
- [187] R. A. Hood, R. J. Greer, M. Breslin, and P. R. Williams, "The calculation and assessment of ground-borne noise and perceptible vibration from trains in tunnels," *J. Sound Vib.*, vol. 193, no. 1, pp. 215–225, 1996.
- [188] CargoVibes (FP7); 2013.
- [189] RIVAS – Railway Induced Vibration Abatement Solutions (FP7); 2013.
- [190] Hanson CE, Towers DA, Meister LD. Transit noise and vibration (Federal Railroad Administration); 2006.
- [191] R.A. Hood, R.J. Greer, M. Breslin, P.R. Williams, The calculation and assessment of ground-borne noise and perceptible vibration from trains in tunnels, *J Sound Vib*, 193 (1) (1996), pp. 215-225
- [192] C. Madshus, C. Bessason, and L. Harvik, "Prediction model for low frequency vibration from high speed railways on soft ground," *J. Sound Vib.*, vol. 193, no. 1, pp. 195–203, 1996.

- [193] M. Bahrekazemi, "Train-Induced Ground Vibration and Its Prediction (PhD thesis)," Stockholm University, 2004.
- [194] F. Rossi and A. Nicolini, "A simple model to predict train-induced vibration: theoretical formulation and experimental validation," *Environ. Impact Assess. Rev.*, vol. 23, no. 3, pp. 305–322, May 2003.
- [195] J. Nelson and H. Saurenman, "A prediction procedure for rail transportation groundborne noise and vibration," *Transp. Res. Rec. J. Transp. Res. Board*, no. 1143, pp. 26–35, 1987.
- [196] E. Bovey, "Development of an impact method to determine the vibration transfer characteristics of railway installations," *J. Sound Vib.*, vol. 87, no. 2, pp. 357–370, 1983.
- [197] C. With and A. Bodare, "Prediction of train-induced vibrations inside buildings using transfer functions," *Soil Dyn. Earthq. Eng.*, vol. 27, no. 2, pp. 93–98, Feb. 2007.
- [198] H. Kuppelweiser and A. Ziegler, "A tool for predicting vibration and structure-borne noise immissions caused by railways," *J. Sound Vib.*, vol. 193, no. 1, pp. 261–267, 1996.
- [199] C. With, A. V. Metrikine, and A. Bodare, "Identification of effective properties of the railway substructure in the low-frequency range using a heavy oscillating unit on the track," *Arch. Appl. Mech.*, vol. 80, no. 9, pp. 959–968, Aug. 2009.
- [200] M. Schevenels, G. Lombaert, G. Degrande, and S. François, "A probabilistic assessment of resolution in the SASW test and its impact on the prediction of ground vibrations," *Geophys. J. Int.*, vol. 172, no. 1, pp. 262–275, Jan. 2008.
- [201] H. E. M. Hunt and M. F. M. Hussein, "Vibration from railways: can we achieve better than +/-10dB prediction accuracy?," in *14th International Congress on Sound and Vibration*, 2007.
- [202] J. P. Wolf and C. Song, "Some cornerstones of dynamic soil – structure interaction," *Eng. Struct.*, vol. 24, pp. 13–28, 2002.
- [203] S. C. Dutta and R. Roy, "A critical review on idealization and modeling for interaction among soil–foundation–structure system," *Comput. Struct.*, vol. 80, no. 20–21, pp. 1579–1594, Aug. 2002.
- [204] S. G. Shah, C. H. Solanki, and J. A. Desai, "Soil structure interaction analysis methods State of art Review," *Int. J. Civ. Struct. Eng.*, vol. 2, no. 1, pp. 176–204, 2011.
- [205] O. Bewes, "Managing uncertainty in groundborne noise and vibration predictions," in *ANC conference*, 2012, no. May.

- [206] S. Jones, K. Kuo, M. Hussein, and H. Hunt, "Prediction uncertainties and inaccuracies resulting from common assumptions in modelling vibration from underground railways," *Proc. Inst. Mech. Eng. Part F J. Rail Rapid Transit*, vol. 226, no. 5, pp. 501–512, Mar. 2012.
- [207] S. Jones, "Ground Vibration from Underground Railways: How Simplifying Assumptions Limit Prediction Accuracy (PhD thesis)," University of Cambridge, 2010.
- [208] L. Andersen and C. J. C. Jones, "Coupled boundary and finite element analysis of vibration from railway tunnels—a comparison of two- and three-dimensional models," *J. Sound Vib.*, vol. 293, no. 3–5, pp. 611–625, Jun. 2006.
- [209] J. Lermo, E. Ovando, and M. Limaymanta, "Empirical transfer functions to build soil classification maps for seismic design," no. 1998, 2008.
- [210] L. Auersch, "Building Response due to Ground Vibration — Simple Prediction Model Based on Experience with Detailed Models and Measurements," *Int. J. Acoust. Vib.*, vol. 15, no. 3, pp. 101–112, 2010.
- [211] T. Balendra, K. Chua, K. Lo, and S. Lee, "Steady-state vibration of subway-soil-building system," *J. Eng. Mech.*, vol. 115, no. 1, pp. 145–162, 1989.
- [212] M. Hussein, H. E. M. Hunt, K. Kuo, P. A. Costa, and J. Barbosa, "The use of sub-modelling technique to calculate vibration in buildings from underground railways," *Proc. Inst. Mech. Eng. Part F J. Rail Rapid Transit*, Nov. 2013.
- [213] P. Fiala, G. Degrande, and F. Augusztinovicz, "Numerical modelling of ground-borne noise and vibration in buildings due to surface rail traffic," *J. Sound Vib.*, vol. 301, no. 3–5, pp. 718–738, Apr. 2007.
- [214] A. B. Nagy, P. Fiala, F. Márki, F. Augusztinovicz, G. Degrande, S. Jacobs, and D. Brassensx, "Prediction of interior noise in buildings generated by underground rail traffic," *J. Sound Vib.*, vol. 293, no. 3–5, pp. 680–690, Jun. 2006.
- [215] M.G. Smith, I. Croy, M. Ogren, K. Persson Waye, "On the influence of freight trains on humans: a laboratory investigation of the impact of nocturnal low frequency vibration and noise on sleep and heart rate," *PLoS One*, 8 (2) (2013), p. e55829
- [216] Yano T. *Community response to Shinkansen noise and vibration: a survey in areas along the Sanyo Shinkansen line*; 2004. p. 1837–41.
- [217] R. Klæboe, "Vibration in dwellings from road and rail traffic—Part II: exposure–effect relationships based on ordinal logit and logistic regression models," *Appl Acoust*, 64 (1) (2003), pp. 89–109
- [218] C. Sharp, J. Woodcock, G. Sica, E. Peris, A.T. Moorhouse, D.C. Waddington, "Exposure–response relationships for annoyance due to freight and passenger railway vibration exposure in residential environments," *J Acoust Soc Am*, 135 (1) (2014), pp. 205–212

- [219] Zapfe J, Saurenman H, Fidell S. Ground-bourne noise and vibration in buildings caused by rail transit (TCRP Project D-12). *Transport Res Board*, December; 2009. p. 1–208.
- [220] A. Gidlöf-Gunnarsson, M. Orgen, T. Jerson, E. Ohrstrom, Railway noise annoyance and the importance of number of trains, ground vibration, and building situational factors, *Noise Heal*, 14 (59) (2012), pp. 190-201
- [221] P. Elias and M. Villot, “Review of existing standards, regulations and guidelines , as well as laboratory and field studies concerning human exposure to vibration. Deliverable D1.4. (RIVAS),” 2011.
- [222] D.C. Waddington, J. Woodcock, E. Peris, J. Condie, G. Sica, A.T. Moorhouse, et al., Human response to vibration in residential environments, *J Acoust Soc Am*, 135 (1) (2014), pp. 182-193
- [223] Deutsches Institut fur Normung. DIN 4150-2 – human exposure to vibration in buildings. International Standards Organisation; 1999.
- [224] M.J. Griffin, A comparison of standardized methods for predicting the hazards of whole-body vibration and repeated shocks, *J Sound Vib*, 215 (4) (1998), pp. 883-914
- [225] H. Howarth and M. Griffin, “The annoyance caused by simultaneous noise and vibration from railways,” *J. Acoust. Soc. Am.*, vol. 89, no. 5, pp. 2317–2323, 1991.
- [226] P. Paulsen and J. Kastka, “Effects of combined noise and vibration on annoyance,” *J. Sound Vib.*, vol. 181, no. 2, pp. 295–314, 1995.
- [227] H. Findeis and E. Peters, “Disturbing effects of low-frequency sound immissions and vibrations in residential buildings,” *Noise Heal.*, vol. 6, no. 23, pp. 29–35, 2004.
- [228] G. Sica, E. Peris, J. S. Woodcock, A. T. Moorhouse, and D. C. Waddington, “Design of measurement methodology for the evaluation of human exposure to vibration in residential environments.,” *Sci. Total Environ.*, Jul. 2013.
- [229] E. Peris, J. Woodcock, G. Sica, A. T. Moorhouse, and D. C. Waddington, “Annoyance due to railway vibration at different times of the day.,” *J. Acoust. Soc. Am.*, vol. 131, no. 2, pp. EL191–6, Feb. 2012.
- [230] R. A. Walker, “Buildings on springs,” *Nature*, vol. 2011, no. 5051, pp. 794–796, 1966.
- [231] R. A. Walker, *Buildings on springs. International Series of Monographs in Civil Engineering*, vol. 2. Pergamon, Oxford, 1969.
- [232] J. P. Talbot and H. E. M. Hunt, “The effect of side-restraint bearings on the performance of base-isolated buildings,” *Proc. Inst. Mech. Eng. Part C J. Mech. Eng. Sci.*, vol. 217, no. 8, pp. 849–859, Jan. 2003.

- [233] T. Boxoen, D. Risbey, and C. Weber, "Train Induced Vibration Isolation of theskyvue , Sydney Australia," no. August, 2010.
- [234] J. P. Talbot, "On the Performance of Base-Isolated Buildings: A Generic Model (PhD thesis)," 2001.
- [235] N. Zhao, M. Sanayei, J. A. Moore, J. A. Zapfe, and E. M. Hines, "Mitigation of Train-Induced Floor Vibrations in Multi-Story Buildings Using a Blocking Floor," in *Structures Congress*, 2010, pp. 1–11.
- [236] D.P. Connolly, G. Kouroussis, P.K. Woodward, A. Giannopoulos, O. Verlinden, M.C. Forde, Scoping prediction of re-radiated ground-borne noise and vibration near high speed rail lines with variable soils, *Soil Dyn Earthq Eng*, 66 (2014) (2014), pp. 78-88,
- [237] Connolly, DP, Kouroussis, G, Fan, W, Percival, M, Giannopoulos, A, Woodward, PK, et al. An experimental analysis of embankment vibrations due to high speed rail. In: *Proceedings of the 12th International Railway Engineering Conference*; 2013.