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Citation for published version:

Zhou, R & Lu, Y 2018, 'A mesoscale interface approach to modelling fractures in concrete for material investigation', *Construction and Building Materials*, vol. 165, pp. 608-620.
<https://doi.org/10.1016/j.conbuildmat.2018.01.040>

Digital Object Identifier (DOI):

[10.1016/j.conbuildmat.2018.01.040](https://doi.org/10.1016/j.conbuildmat.2018.01.040)

Link:

[Link to publication record in Edinburgh Research Explorer](#)

Document Version:

Peer reviewed version

Published In:

Construction and Building Materials

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1 **A mesoscale interface approach to modelling fractures in concrete for material**
2 **investigation**

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7 **Abstract**

8 Advanced computational modelling can provide a powerful tool for material investigation and
9 characterisation. For concrete materials, appropriate description of the heterogeneity and
10 realisation of complex fractures are two challenging aspects in high fidelity numerical
11 simulations. This paper presents a new mesoscale model for concrete with the ability of
12 simulating natural evolution of fracture at the interface between the aggregates and mortar
13 matrix and without restriction to the loading conditions. To this end, a combined cohesive and
14 contact interface approach is employed. The contact-friction process at a fractured interface is
15 treated as an independent process that complements the general cohesive law, thus allowing
16 the closure of cracked surfaces and the development of residual shear resistance in a realistic
17 manner. Parametrisation is conducted to examine the effects of pertinent interface parameters
18 on the macroscopic behaviour of concrete. The modelling approach is demonstrated to be
19 capable of simulating the behaviour of concrete under a variety of loading conditions, including
20 confined and dynamic compression. The new mesoscale model provides a comprehensive
21 numerical means for investigating into the micro-mesoscale mechanisms underlying the
22 macroscopic behaviour of concrete.

23
24 **Keywords:** concrete material; heterogeneity; fracture; mesoscale model; cohesive zone;
25 contact model.

26 **1. Introduction**

27 Concrete is a non-homogeneous composite with large heterogeneities. The behaviour of
28 concrete is fundamentally affected by the fracture mechanisms, particularly at interfaces
29 between aggregates and the mortar matrix, i.e. the interfacial transition zone or ITZ. Modelling
30 of concrete is complicated because of the development of fractures, in that at the initial stage
31 concrete behaves primarily like a heterogeneous continuum solid, but when fractures grow it
32 gradually becomes discontinuous.

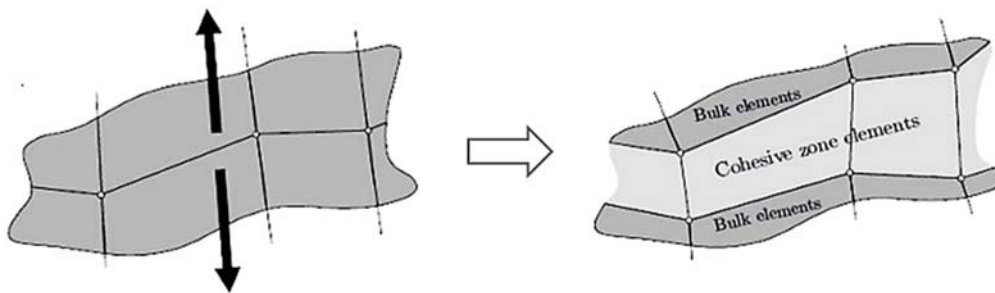
33 Modelling of concrete at the mesoscale makes it possible to describe the composition of
34 the material, and it has been a subject of continuous interest in the research community
35 concerning brittle and quasi-brittle solids (e.g. [1-3]). As summarised in [4], three distinctive
36 approaches have been employed in mesoscale modelling of concrete, namely lattice model,
37 discrete element model (DEM), and continuum finite element (FE)-based model.

38 A key factor that determines the extent to which a mesoscale model may be capable of
39 realistically representing the intrinsic failure mechanisms is the modelling of fractures. In
40 lattice models [5-6], fracture is generally represented by continually breaking the lattice
41 members, which may be beam or truss elements, when a failure criterion is met. This approach
42 is suitable for crack opening; but it cannot accommodate possible crack closure. The discrete
43 element or particle models possess inherent advantages in accommodating crack-induced
44 discontinuity; however its ability in modelling the continuum and partially damaged states of
45 concrete is subject to the equivalent description of the continuum properties, and such
46 equivalent description is difficult to generalize for different stress conditions [4].

47 Finite element-based mesoscale model is well suited for representing the intact concrete
48 as it is essentially a non-homogenous continuum. As in the general FE model of concrete,
49 cracks may be described using either a smeared or a discrete approach. However, previous

50 research has shown some well-known issues with the standard continuum elements, such as
51 the mesh size dependency and the limited deformation modes in the smeared crack approach
52 when the softening behaviour is involved [7].

53 The incorporation of cohesive interface elements within a finite element framework
54 makes it possible to follow the initiation and propagation of multiple cracks. These interface
55 lines can branch, coalesce, and eventually form new free surfaces. The mechanical properties
56 of the interface can generally be described using a cohesive law, which represents a gradual
57 loss of the strength with increasing separation and can also be related to the work of separation,
58 or fracture energy that is required for the complete formation of a free surface [8]. Figure 1
59 depicts the formation of a separation (crack) over an interface with cohesive zone elements.



60

61

Fig. 1. Cohesive elements along mesh lines (after [8])

62

63 As the macroscopic failure in concrete is much dependent on the interface between
64 aggregate and mortar, a sound representation of the mechanical properties and the fracture at
65 the ITZ is crucial for a realistic modelling of the mesoscopic damage mechanisms for concrete-
66 like materials. Therefore in the present study the focus has been placed to develop a holistic
67 interface approach to capture the complex damage process at the ITZ for any stress conditions.

68 It is generally understood that the real ITZ has a very thin thickness of 20-50 μm [9-10],
69 and it has a different mechanical property from the cement paste. Because of its thin thickness,

70 it may be reasonably represented by zero-thickness cohesive elements.

71 The adequacy of using cohesive elements for modelling the ITZ in a general mesoscale
72 model depends upon the capacity of the cohesive elements in catering to complex stress
73 conditions. A classical cohesive model is suited for modelling the interface failure involving
74 model I and mode II fractures. Applying this cohesive element model proves to work well under
75 tension-dominated loading, but it performs poorly in other loading conditions including axial
76 compression [2]. The reason is deemed to relate to the inability of the cohesive element in
77 representing the shear failure of the ITZ under a complex stress condition.

78 Some other techniques have also been developed in attempt to address the coupled
79 effect of normal and shear stresses at a cohesive interface. An interface element which
80 incorporates the interaction of cohesion, tensile strength and the friction angle in a constitutive
81 model has been proposed [7] to investigate the concrete fracture mechanism under complex
82 loading conditions. The main feature of this interface element is that it introduces a friction
83 dissipative mechanism between two potential crack surfaces into the cohesive law intrinsically.
84 By defining several loading fracture surfaces at different loading stages with shape parameters,
85 such a model can generally simulate the whole process from fracture to pure friction. However
86 some of the parameters used in the model cannot be obtained easily and some are also case-
87 dependent. Moreover, as Ruiz et al. [11] suggested, the contact and friction process should be
88 regarded as independent phenomena outside cohesive law. This is because physically fracture
89 and friction are two independent processes, and in particular the presence of friction may result
90 in a steady frictional resistance while the normal cohesive strength simultaneously weakens.
91 Thus a contact-friction algorithm is deemed to be more appropriate to represent the interaction
92 resistance at cracked surfaces.

93 In this paper, a holistic interface approach combining the cohesive mechanism with the

94 contact-friction mechanism is developed to explicitly represent the behaviour of ITZ in a
95 mesoscale concrete model. Relatively simple and explicit physical laws are employed for
96 individual mechanisms. In conjunction with the mesoscale description of the complex
97 geometric interface between the mortar and random aggregates, which allows for the fracture
98 path to develop in a more realistic manner, the combined framework provides a comprehensive
99 method to capture the detailed damage processes in concrete under all general loading
100 conditions. The application of the model for material investigations is demonstrated by
101 numerical simulation of concrete under different loading conditions in comparison with
102 experimental observations.

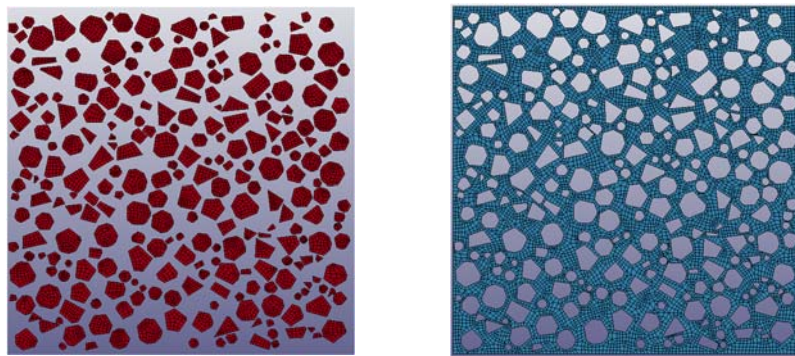
103 **2. Modelling approach for ITZ in a mesoscale framework**

104 **2.1 Overview of the mesoscale model and meso-structure generation**

105 The present study is focused on fracture modelling of concrete in a two-dimensional (2D)
106 mesoscale model framework, with a holistic interface description for the ITZ. The mesoscale
107 structure of concrete is represented by a stochastic distribution of coarse aggregates embedded
108 in the mortar matrix. The aggregates are modelled by random polygon particles, and the
109 nominal size of the individual aggregates obeys a given grading curve. The generation of the
110 mesoscale geometry follows a commonly adopted take-and-place procedure [12], satisfying
111 non-overlapping and minimum gap requirements. The density of the aggregates can be
112 controlled by specifying a volume ratio, e.g. 45% in this paper. For normal concrete, the coarse
113 aggregates are defined as those with a minimum nominal size of 4.75 mm [1]. Herein the
114 procedure is programmed using MATLAB.

115 After the generation of the mesoscale structure, the geometrical data can be brought into
116 a finite element meshing processor. In the present study, ANSYS pre-processor is used to
117 perform the FE-meshing. Figure 2 illustrates a typical mesoscale model geometry. In this figure,

118 only two material components, namely aggregates and the mortar matrix, are shown. The third
119 component, i.e. the interface transition zone (ITZ) between aggregates and mortar matrix can
120 be created subsequently. To overcome the issues with modelling the ITZ with an equivalent
121 thin layer of solid elements as mentioned earlier, herein the ITZ is explicitly modelled with a
122 combined classic zero-thickness cohesive interface element and a contact algorithm. The
123 creation of such a combined interface element will be discussed in the next section.



124
125 (a) Aggregates

(b) Mortar matrix

126 **Fig. 2.** Mesoscale model of concrete and FE mesh

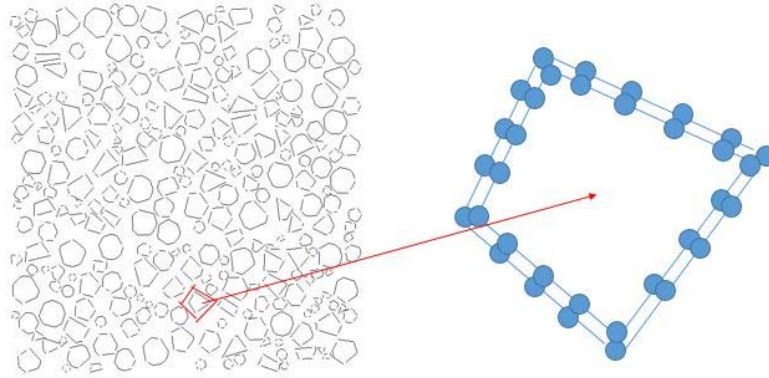
127 **2.2 Modelling of ITZ with a cohesive zone model**

128 The main advantage of using a cohesive zone model for fracture is that it can simulate the
129 gradual process of the cracking surface separation with a cohesive law. A typical cohesive law
130 relates the relative displacement of two associated points of the interface (δ) to the traction
131 force per unit area (T) that is needed for separation. Different cohesive laws may be defined
132 for the normal and tangential directions, respectively, but in most models the cohesive laws for
133 the two directions are coupled, meaning that both the normal and tangential tractions (T_n, T_t)
134 depend on both the normal and tangential opening displacements (δ_n, δ_t).

135 **2.2.1 Cohesive elements insertion**

136 In the present mesoscale model the boundaries between aggregates and mortar matrix are all

137 treated as the potential crack surfaces, and the zero-thickness elements are inserted. To achieve
138 this, a duplicate set of the nodes are required at all the interface locations. The original nodes
139 and the duplicated nodes form the two potential cracking surfaces of cohesive elements, and
140 they can separate during crack propagation. Each pair of two nodes at the same location (see
141 Figure 3) will be constrained by a separation-traction law.



142

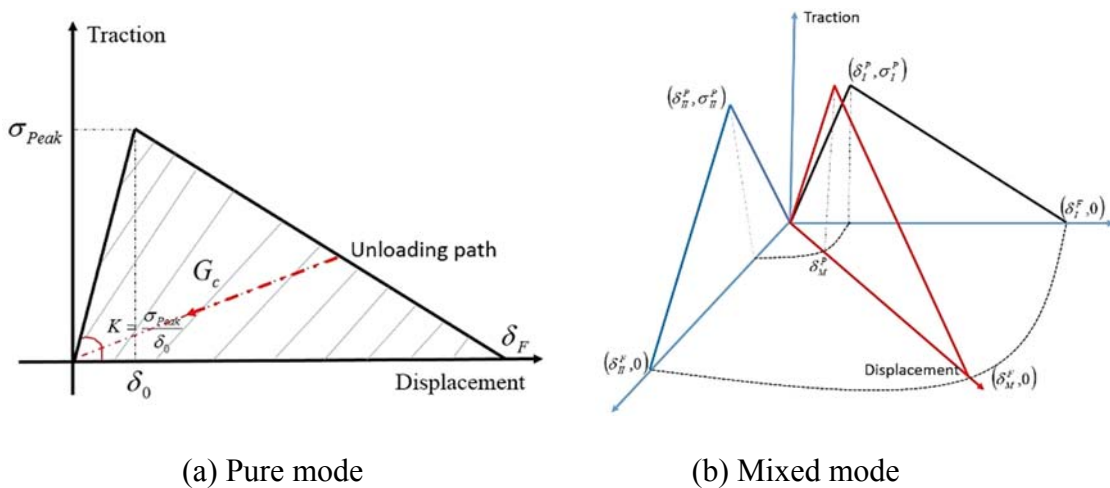
143 **Fig. 3.** Illustration of zero-thickness interface elements insertion

144 Depending on the response of the cohesive surface prior to the development into the
145 softening stage, two types of cohesive zone approaches may be considered when the cohesive
146 elements are inserted, namely intrinsic and extrinsic cohesive zone models [13]. Intrinsic
147 cohesive elements are embedded in the discretized structure at the beginning of the simulation,
148 and during the whole simulation process the mesh connectivity remains unchanged. Extrinsic
149 cohesive models, on the other hand, insert the cohesive elements adaptively into the mesh,
150 which means the cohesive elements are inserted only when the boundary stresses reach the
151 critical material strength. At this juncture, it is worth noting that in the mesoscale model the
152 crack paths will be subject to natural regularisation due to the presence of the aggregates, and
153 hence can be highly irregular. Therefore in the present study we adopt the intrinsic cohesive
154 model approach for the ITZ.

155 **2.2.2 Cohesive constitutive model**

156 The cohesive constitutive model used in the present study is a typical simple bilinear cohesive
 157 model for modelling the interface failure involving interaction between model I and model II
 158 fractures [14]. It considers the irreversible damage and allows for independent definitions of
 159 the constitutive relations for different fracture modes of tension and shear. The constitutive
 160 laws used for modelling mode I and mode II fracture are depicted by the curves in the ‘traction-
 161 δ_I ’ and ‘traction- δ_{II} ’ planes respectively, as it is shown schematically in Figure 4. Only some
 162 key parameters such as the stiffness, K_N and K_S , the peak tractions $\sigma_I^P, \sigma_{II}^P$ and the fracture
 163 energies G_{IC}, G_{IIC} in the normal and shear directions respectively, need to be specified.
 164 Generally the interface layers in a mesoscale concrete model will not be subjected to just pure
 165 Mode I or Pure Mode II loading. Therefore, a mixed-mode needs to be specified to couple the
 166 two independent models. In the present study, the representative power law [14], in which a
 167 mixed-mode initiation displacement and the ultimate mixed-mode displacement (total failure)
 168 can be correspondingly calculated, is employed.

169



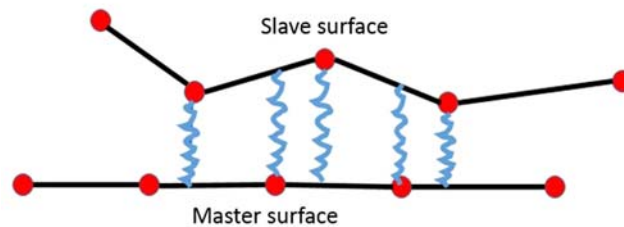
172 **Fig. 4.** Illustration of mixed-mode constitutive law for cohesive elements (after [14])

173

174 **2.3 Incorporation of contact-friction mechanism**

175 As mentioned earlier the performance of the traditional cohesive model becomes poor if the
176 interface is subject to a compressive or shear loading while a crack is developing. This is
177 deemed to be due to a lack of representation of the contact and friction mechanism. To revolve
178 this problem, in the present model a penalty-based algorithm is adopted to handle the contact
179 between two cracked surfaces of an interface element. The penalty-based algorithm proves to
180 be stable and it is also easy to implement in FEM [15].

181 A sketch of the penalty-based contact algorithm is shown in Figure 5. An equivalent
182 elastic, compression-only spring is placed in the normal direction to resist penetration. Each
183 slave node is checked for penetration through the master surface. If there is no penetration
184 nothing is done but when it does penetrate, an interface force is applied between the slave node
185 and its contact point. The magnitude of this force depends on the amount of penetration with a
186 linear relationship.



187

188 **Fig. 5.** Sketch of the penalty-based contact method

189 In the tangential direction, a friction stress is introduced according to the Coulomb
190 Friction law. Two types of friction stress limit, namely the maximum static friction and the
191 kinetic friction are both considered in present study. While the friction stress developed in the
192 kinetic stage can be easily defined by a linear relationship to the normal stress, the
193 determination of the limit static friction stress, which develops before a complete de-cohesion
194 (loss of cohesion), is not straightforward and warrants a special consideration. This will be

195 discussed in association with the combined model in what follows.

196 **2.4 Cohesive-plus-contact model**

197 As stated earlier, we simulate cohesive fracture and the contact-friction as two independent
198 mechanisms. Depending on the stress condition, an interface element may develop into a full
199 crack state without activating any contact frictional effect, or it may involve degradation of
200 cohesion and friction sliding at the same time. To enable both mechanisms to work, it is
201 important to define how friction should develop while the cohesion degrades, which indicates
202 crack opening at the same interface.

203 In the literature the treatment of the transition stage from cohesion to pure friction at an
204 interface varies, from the very beginning to a complete de-bonding of the cohesive zone.
205 Tvergaard [16] introduced the friction mechanism to a cohesive law such that the friction takes
206 effect only when de-cohesion is fully attained. This approach can successfully predict the
207 residual stress after de-bonding but cannot model the additional load-carrying capacity due to
208 the fracture roughness. Chaboche et al. [17] modified the interface law by introducing a friction
209 term from the very beginning. This model effectively treated the friction mechanism as a
210 kinematic hardening effect with a decreasing hardening modulus as the damage progresses,
211 thus it is capable of predicting the additional load capacity due to friction. More recent works
212 on this topic have focused on coupling the initiation of friction with the onset of fracture, for
213 instance [18-20].

214 A general feature among the above mentioned studies is that the friction effect between
215 two potential crack surfaces is incorporated into the cohesive law internally. This would result
216 in a more complex cohesive constitutive law and the equivalent parameters can be sensitive to
217 specific loading conditions.

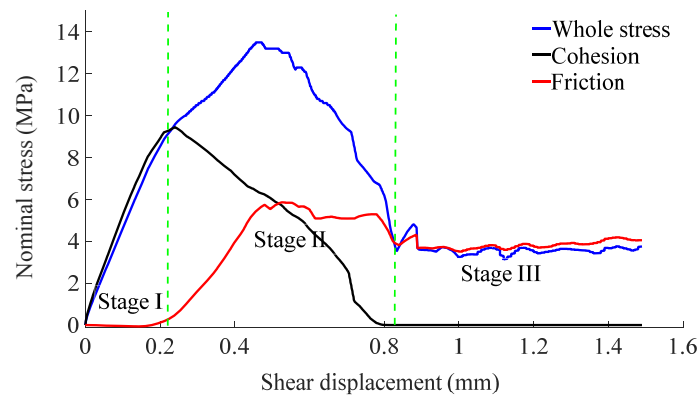
218 In the present paper, the physical process of contact and friction at a fractured interface is
219 modelled independently and it complements the general cohesive law, thus providing a
220 framework that allows the distinctive mechanisms of cohesion, contact (closure of cracked
221 surfaces) and friction to develop directly. Each mechanism has a clear physical meaning and
222 this paves the way for the determination of the relevant parameters for each mechanism in a
223 more straightforward manner. Together with a more realistic description of the topology of the
224 fracture path in the mesoscale model, this enables a realistic simulation of the complex fracture
225 and degradation process in concrete suitable for the material investigations.

226 Preliminary explorations in the present study revealed that simply adding the cohesive
227 and contact-friction components together cannot yield satisfactory results; the model could
228 easily become unstable and produce erroneous results. This is most probably because
229 traditional contacts can only introduce the frictional resistance after the complete failure of
230 cohesion, causing the stability issue and an inability to predict the additional force-transfer
231 capacity due to friction during the transition stage.

232 To overcome this problem, we propose to introduce a continuous friction mechanism
233 which starts from the beginning of loading. To reflect different degree of the friction
234 engagement at different states of the interface, the whole process is subdivided into three stages.
235 Stage 1 corresponds to the undamaged state of the cohesive element. During this stage, the
236 cohesion dominates and the friction term is negligible. Stage 2 is the process from the onset of
237 fracture until full de-cohesion. In this stage, the friction and the remaining cohesion act on the
238 same interface simultaneously. The friction term acting at this stage may be viewed as a
239 hardening effect leading to increased resistance capacity of the bulk material. It should be noted
240 that the frictional movement during this stage is not an explicit interface slide but is constrained
241 by the constitutive cohesion law that relates the friction force to the shear deformation, and

242 therefore is effectively a static friction. The pure “sliding” friction stage is defined as stage 3.
243 In this stage, the interface is fully separated thus the two contacting surfaces slide against each
244 other with a frictional law. The friction at this stage is of kinetic character.

245 Figure 6 illustrates the behaviour of the combined cohesion and friction model with three
246 distinctive stages of the response, as obtained from a representative numerical result. The
247 determination of the pertinent parameters is discussed in the next section.



248

249 **Fig. 6.** Evolution of the combining mechanism during different stages

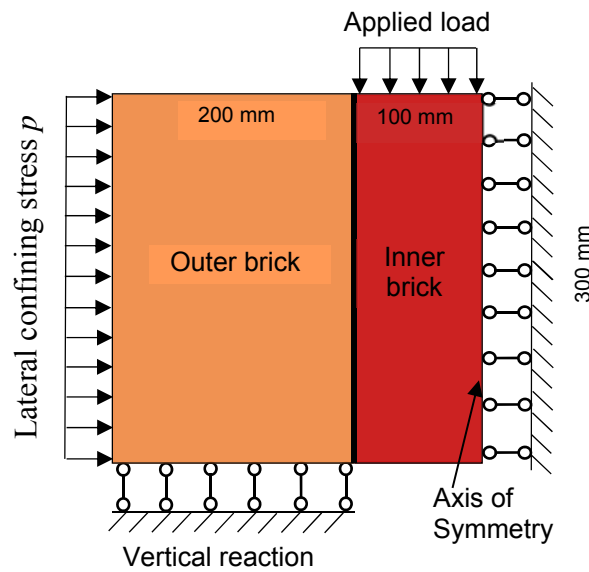
250

251 2.5 Parameter settings in the combined interface model

252 In addition to parameters which may be related to the basic material properties, such as the
253 cohesive strengths and kinetic friction coefficient, appropriate initial stiffness and the static
254 friction stress limit (*SFSL*) need be specified for the cohesive zone model. In addition, a suitable
255 mesh size should also be chosen. This section examines the sensitivity of combined cohesive
256 model to these parameters and discusses suitable values that may be adopted for the cohesive-
257 plus-contact model.

258 A classical triplet shear experiment with lateral confining pressure is modelled for this
259 investigation. The triplet experiment is commonly used in the testing of masonry materials to

260 determine the ultimate shear strength of the mortar joints. Figure 7 shows a schematic of a
 261 finite element model simulating the experimental setup. The two side blocks (bricks)
 262 supported rigidly at the lower edge, whereas the shear load is applied on the upper edge of the
 263 middle brick, which is simulated with a velocity boundary condition in the FE model. A
 264 constant normal pressure can be introduced by applying horizontal compression force on the
 265 side surfaces of the outer bricks. Considering symmetry, only half of the specimen is modelled.



266

267

Fig. 7. Numerical model of shear test

268 For the sake of simplicity, the blocks are modelled as elastic with the following properties:

269 Elastic modulus $E = 37$ GPa, Poisson's ratio $\nu = 0.2$ and mass density $\rho = 2.3 \times 10^{-3}$ g/mm³.

270 The interface layer is nonlinear and is modelled by the combined cohesive-plus-contact model.

271 The parameters used in defining the interface model are as follows: peak traction in tension

272 $\sigma_I^P = 2.7$ MPa, peak traction in shear $\sigma_{II}^P = 10.8$ MPa, fracture energy in mode-I $G_{IC} = 0.03$

273 N/mm, fracture energy in Mode-II $G_{IC} = 0.3$ N/mm and the kinetic friction coefficient $\mu = 0.7$.

274 The mass density of the interface element is assumed as the same as that for the brick elements.

275 It should be noted that these parameter values are in line with masonry but the simulation itself

276 is generic for quasi-brittle solids and is not intended to tie with any physic experiment at this
277 stage.

278 **2.5.1 Mesh size**

279 Simulations with five different mesh sizes, namely $h = 50, 25, 10, 5, 2$ mm have been conducted
280 for the current specimen of 300mm in length. To maintain consistency across all models with
281 different mesh sizes concerning the initial stiffness of the cohesive element, a sufficiently large
282 stiffness $K_N = K_S = 10^6$ MPa / mm is used for all the cases. The overall results in terms of the
283 nominal shear stress vs. shear displacement tend to converge with a mesh size no larger than 5
284 mm. A further examination of the effects of the mesh size on the two independent mechanisms
285 (cohesion and friction) indicate that the cohesion tends to attain a generally convergent result
286 when the mesh size is smaller than 25 mm, whereas the friction needs much finer mesh to
287 approach the convergence. Generally a convergent result can be obtained for both cohesion and
288 friction when the mesh size is no larger than 5 mm. Hence the mesh size 5 mm at the interface
289 layer has been chosen for the simple shear test on the cohesive plus contact model hereinafter. .

290 **2.5.2 Initial stiffness of cohesive element**

291 The initial stiffness of the cohesive elements K is crucial for an adequate behaviour of the
292 cohesive zone model. A general guideline in setting a suitable initial stiffness was proposed in
293 Turon et al. [21] as:

$$294 \quad K = \frac{\alpha E}{h_{mesh}} \quad (1)$$

295 where E is the Young's modulus for bulk element, h_{mesh} the mesh size of the bulk element and
296 α the coefficient. Ideally, α should be set infinitively large to eliminate the artificial compliance
297 due to the introduction of intrinsic cohesive element. However, an excessively large stiffness
298 for the interface element may cause other numerical problems such as spurious oscillations of

299 the traction [22]. Furthermore, large stiffness may also reduce the critical time step when an
300 explicit dynamic algorithm is used, such as with LS-DYNA as used in the present study, which
301 would increase the computing cost.

302 Five different interface stiffness parameters are examined here, namely, $\alpha = 0.05, 1, 10,$
303 50 and 100 . From the simulated nominal shear stress vs. shear displacement results, it can be
304 observed that the stiffness generally has little influence on the global stress-displacement
305 curves. Further examinations into the effect on the cohesion and friction shows that the effect
306 of the stiffness on the cohesion response is also very limited. In contrast, the friction response
307 is significantly influenced by the cohesive stiffness. As can be expected, when the cohesive
308 stiffness is set relatively small ($\alpha = 0.05$ or 1), the frictional mechanism is involved from the
309 very beginning of the shear process. On the other hand, if the cohesive stiffness is set relatively
310 large ($\alpha \geq 10$), the frictional mechanism comes into action only after a certain degree of
311 ‘separation’. It is interesting to find that when the stiffness parameter is larger than 10 the
312 friction mechanism is not engaged until the shear displacement reaches about 0.2 mm, which
313 is actually a typical threshold of fracture [23-24].

314 From the above results, it may be concluded that a relatively large value with $\alpha \geq 10$
315 needs to be employed for the cohesive stiffness in order to ensure that the friction starts from
316 the onset of fracture. Considering the computational cost, which tends to increase with the
317 increase of the cohesive stiffness as mentioned earlier 2, a value of $\alpha = 50$ is deemed to be
318 appropriate and this value is used hereafter in the present study. It is worth mentioning that
319 such a setting is consistent with the suggestions for cohesive zone models in some previous
320 studies [21].

321 **2.5.3 Friction stress limit**

322 The contact friction algorithm is introduced to the whole fracture process from the very

323 beginning. However before the complete loss of cohesion, the process is a combined de-
324 cohesion and contact, in which the nodes that are initially at the same locations are still
325 constrained by the cohesive constitutive law but they permit tangential motion with frictional
326 sliding. At this stage, the model has no response to the Coulomb's kinetic friction law because
327 there is essentially no 'free' relative slip between each pair of nodes. The friction stress that
328 develops at this stage can be very large. It is reasonable and necessary to impose a static friction
329 stress limit parameter (*SFSL*) in the contact-friction algorithm to set a limit value for the
330 maximum static frictional stress. This can be implemented by integrating a tiebreak contact
331 model with a traditional contact algorithm in the finite element framework.

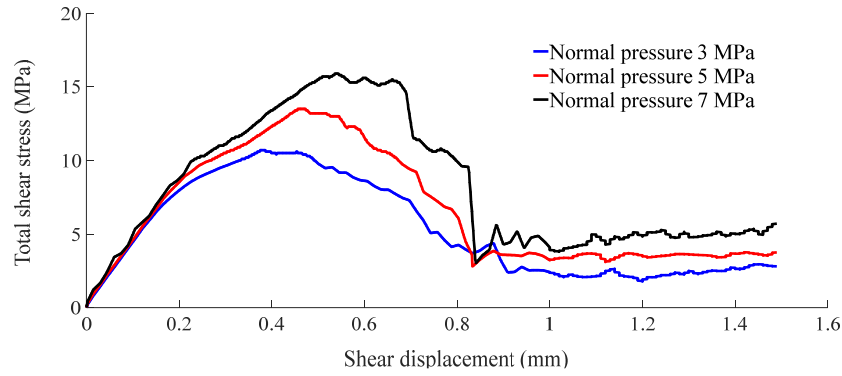
332 The static friction limit value must ensure a realistic static friction and at the same time
333 guarantee a smooth transition from the de-cohesion process to the pure friction stage. In this
334 respect this parameter should on the one hand directly relate to the shear strength of the bulk
335 material and on the other relate to the kinetic frictional coefficient of the contacting surface.
336 However, the bulk materials on the two sides of a cohesive element are generally treated as
337 simple linear-elastic material as in the present study. Based on preliminary analyses, it is
338 suggested that the static friction limit could be set as two times of the cohesion strength (in the
339 shear direction) for concrete-like materials, while a general kinetic frictional coefficient of
340 around 0.7 is adopted.

341 **3. Model performance and experimental verifications**

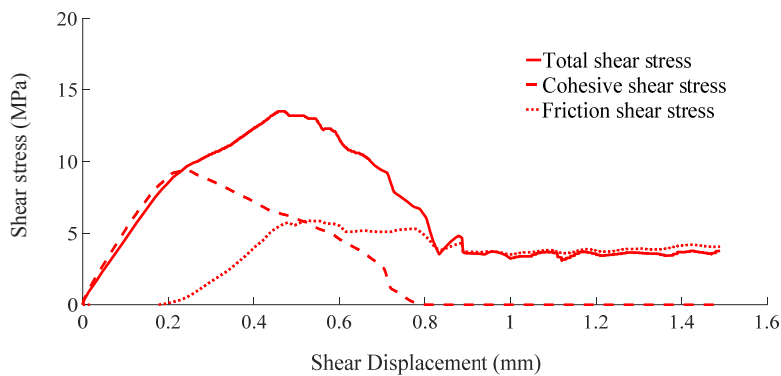
342 **3.1 Shear under different lateral pressures**

343 When the combined cohesive and contact model is subjected to a mixed loading condition of
344 shear with a certain level of normal pressure, the contact-friction effects will be involved and
345 this is expected to increase the overall shear strength. The simulated shear stress vs. shear
346 displacement curves for various levels of the normal compression pressure are shown in Figure

347 8(a). As expected, the current model successfully predicts a persistent increase in the shear
 348 strength as the normal compression increases. The residual shear strength which should be
 349 attributed to the basic friction stress effect (i.e. normal stress times the frictional coefficient) is
 350 also correctly obtained.



351
 352 (a) Total shear under three different lateral pressure



354
 355
 356 (b) Contributions to total shear under a lateral pressure of 5 MPa

357
 358 **Fig. 8.** Simulated shear stress vs. shear displacement relationships

359
 360 A further check on the evolution of cohesion and friction under a normal pressure of 5
 361 MPa is presented in Figure 8(b). The other two cases follow the same trend. As it is shown, the
 362 principle of the cohesive-plus-contact model is quite similar with the basic idea in discrete
 363 element method (DEM) modelling in which two particles are linked with cohesion and friction,
 364 following Mohr-Coulomb rule.

365 The cohesive-plus-contact model is then further checked by the shear test with a constant
366 normal compression but with various friction coefficients in the contacting surfaces, ranging
367 from 0.3 to 0.9. The results demonstrate that while the cohesive stress remains almost
368 unchanged, the total shear stress of the model is increased and the increase is in line with the
369 increase of the friction effect.

370 **3.2 Experimental verification**

371 Having examined the working principles of the proposed cohesive-plus-contact model and the
372 general effects of the key parameters, this section presents an experimental verification against
373 a representative test on masonry specimens [25] to further validate the numerical model. As
374 mentioned, the experiment was a triplet setup which is commonly used in the testing of
375 masonry materials to determine the ultimate shear strength of the mortar joints. The failure
376 mode is well controlled to the mortar joint interfacing two masonry blocks, and hence it
377 provides a well-defined benchmark for the present verification purpose.

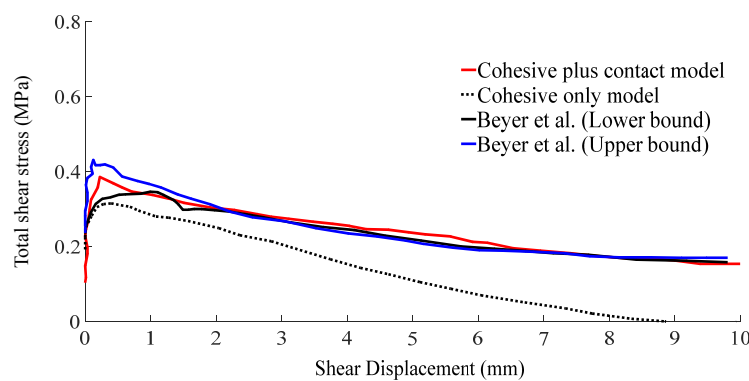
378 In the numerical model the brick parts are assumed to be elastic. The Young's modulus
379 and density of the bricks are directly taken from the experiment, with $E = 12$ GPa and $\rho =$
380 9.32×10^{-4} g/mm³. The Poisson's ratio is assumed to be 0.15 according to typical masonry
381 material properties [20]. For the material properties of mortar joint between the bricks, the two
382 key parameters, i.e. the critical (cohesive) stress and the friction coefficient, are directly
383 obtained from the experimental data, and the values are $\sigma_I^P = \sigma_{II}^P = 0.25$ MPa and $\mu =$
384 0.71, which are fitted with Mohr-Coulomb relationship. The values for Mode-I and Mode-II
385 fracture energy are taken from another shear test of masonry wall [26], i.e. $G_{IC} = 0.17$ N/mm
386 and $G_{IIC} = 2.55$ N/mm as there is no direct data available.

387 The initial stiffness of the interface can be determined according to Eq. (1). Using the
388 parameter $\alpha = 50$ and considering a mesh size 5 mm, $K_N = K_S = 1.2 \times 10^5$ MPa / mm. The

389 static friction stress limit parameter is taken as two times of the cohesion strength in shear
390 (i.e. $SFSL = 0.5$ MPa) according to the guideline discussed earlier, considering $\sigma_{II}^P = 0.25$
391 MPa and $\mu = 0.71$.

392 The comparison of the shear stress - shear displacement relationship between the
393 experimental and the numerical simulation is illustrated in Figure 9. The numerical simulation
394 results agree well with the experiments, and the predicted curves generally fit within the upper
395 and lower bounds of the experimental data. The transition from cohesion to friction regime is
396 very smooth, and the combined cohesive and friction model predicts correctly the pure friction
397 stage.

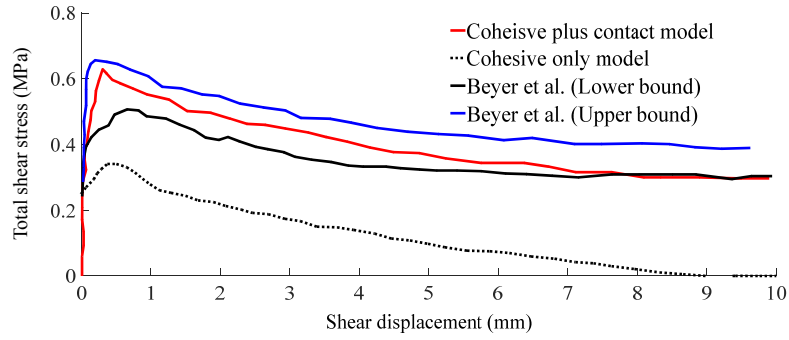
398 The results from using only the cohesive element are also shown in Figure 9. As can be
399 seen, the model fails to represent the dependency of the shear behaviour on the stress condition
400 in the normal direction. This is attributable to an inability of such a model to deal with mixed-
401 mode loading conditions, and the lack of a mechanism to maintain contact and prevent
402 penetration after the failure of the cohesive elements. The proposed approach with a
403 combination of cohesion and contact well resolves the penetration problem and allows the
404 contact-friction to develop, and the final failure mode agrees very well with the experiment.



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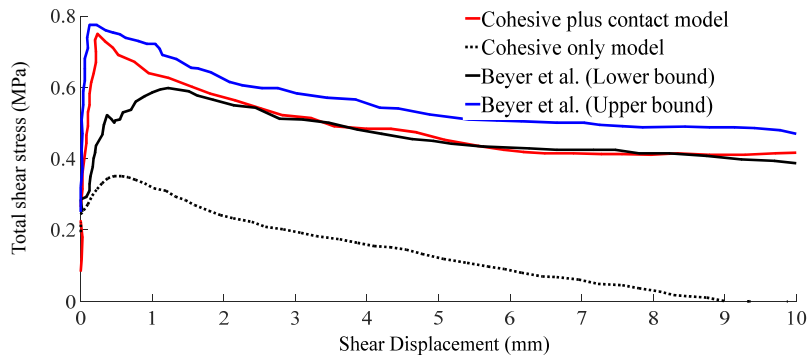
(a) Lateral pressure = 0.2 MPa



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(b) Lateral pressure = 0.4 MPa



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(c) Lateral pressure = 0.6 MPa

411 **Fig. 9.** Comparison of shear behavior with triplet tests by Beyer et al. [25] under different

412

lateral pressures

413

414 **4. Mesoscopic analysis of concrete incorporating the interface model**

415 The proposed new interface approach is subsequently applied in the mesoscale concrete model

416 framework. A standard cubic concrete specimen of 100 mm is modelled in a 2D plane stress

417 setting. The model is subjected to both tension and compression loadings to examine its ability

418 in simulating the damage evolution and the macroscopic response of concrete under different

419 stress conditions.

420 In order to be able to produce the full range of the response of concrete including the

421 softening stage, the analysis is carried using an explicit transient analysis scheme with LS-
422 DYNA [27]. The loading is applied in a displacement-controlled manner through imposing a
423 velocity boundary condition. The duration and pattern of the loading is carefully tuned such
424 that the response resembles closely a quasi-static characteristic. More details of the loading
425 scheme in the numerical simulation can be found in Tu and Lu [2].

426 **4.1 Material parameters for mesoscale concrete**

427 In the mesoscale model, the interface model with the combined cohesive and contact-friction
428 components is used to model the fracture process at the interface between aggregates and
429 mortar matrix, i.e., the ITZ.

430 The two bulk constituent materials, namely the aggregates and the mortar matrix, are
431 modelled as continuum solids. Since the aggregate material is usually much stronger than
432 mortar, it is reasonable to model the aggregates with a linear elastic material model. The mortar
433 matrix may be represented by a damage-plasticity model to account for the damage and plastic
434 deformation that may incur within the mortar matrix. Herein we use the K&C concrete damage
435 model (material #72R3 in LS-DYNA) which has been calibrated extensively, including the
436 controlling over the mesh dependency, in the literature [e.g. 2, 28]. It is also worth mentioning
437 at this juncture that it is possible to extend the present cohesive-contact scheme to all element
438 interfaces, including ITZ and mortar element, but obviously that would increase the
439 computational cost.

440 All the material parameters here are assigned with values to represent a class of normal
441 concrete with a compressive strength of 30 MPa. These material properties are determined
442 based on data collected from relevant literature [29-30]. The material properties for aggregates
443 include: $E = 60$ GPa, mass density $\rho = 2.6 \times 10^{-3}$ g/mm³, and the properties for mortar include:
444 $E = 30$ GPa, $\rho = 2.3 \times 10^{-3}$ g/mm³ and compressive strength = 45 MPa. The Poisson's ratio

445 is assumed to be 0.2 for both aggregate and mortar. The parameters used for the ITZ include
446 normal peak traction $\sigma_f^P = 2.3$ MPa, energy release rate in mode-I $G_{IC} = 0.03$ N/mm and the
447 friction coefficient $\mu = 0.7$.

448 It should be noted here that the shear properties of ITZs, including the shear strength and
449 the fracture energy in shear mode, are not precisely known in the literature. Therefore it is
450 necessary to conduct parameter studies to find proper values for them, which will be given later.

451 Assuming the macroscopic elastic modulus of concrete with a nominal compressive
452 strength 30 MPa is around 30 GPa [31], the initial stiffness of the interface can be estimated as
453 $K_N = K_S = 1.5 \times 10^6$ MPa/mm according to Eq. (1), taking $\alpha = 50$. A refined mesh $h = 1$ mm,
454 which is dictated by the need of discretising the mesoscale geometry of concrete, is adopted in
455 the mesoscale model.

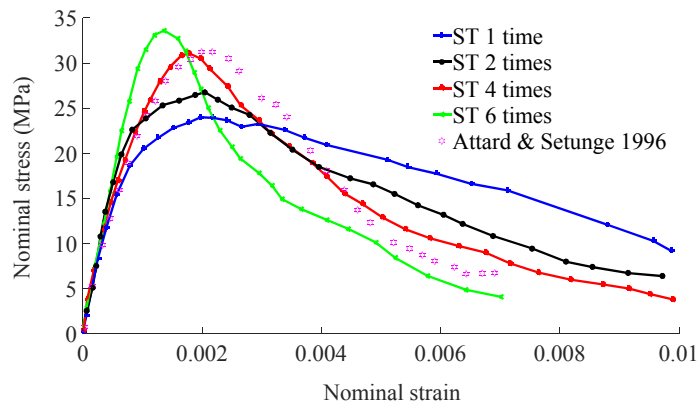
456 **4.2 Parametrisation of other important material properties**

457 For variable parameters which are difficult to determine from physical experiment, in particular
458 the shear strength of the cohesive element S , the shear fracture energy G_{II} , and the friction
459 stress limit parameter, a series of numerical simulations has been performed to examine their
460 influences by comparing the macroscopic stress-strain relationship with relevant concrete
461 experiments. Since these three parameters primarily influence model II cracking, and have little
462 influence on the tension response, the parameterisation is conducted by examining the
463 behaviour of the mesoscale model in quasi-static uniaxial compression.

464 The first parameter being examined here is the shear strength. The ratio between the shear
465 and the normal critical traction (ST ratio) for a cohesive model is of particular importance for
466 concrete. Experimental evidences [32-34] suggest that the peak strength is significantly larger
467 in pure shear mode-II than in pure mode-I owing to the interlocking of aggregate particles in
468 concrete. Swartz et al. [34] estimates the ST ratio (shear fracture strength to tensile strength) to

469 range between 3 and 6 by analytical and numerical simulations of several mixed-mode tests.

470 Figure 10 shows the global stress-strain relationships in compression produced by the
471 mesoscale model with different ST ratios. As can be seen, with an increase of the ST ratio, the
472 compressive strength increases. For the targeted concrete of 30 MPa compressive strength, it
473 appears that a ST ratio of 4 is appropriate for the cohesive material. This value is slightly lower
474 than the one chosen in [11] in which a ST value of 5 was used for a homogeneous concrete
475 model. It is worth noting that aggregate interlocking in the later contact-friction stage is largely
476 represented in the current mesoscale by the explicit inclusion of the aggregates in the model.

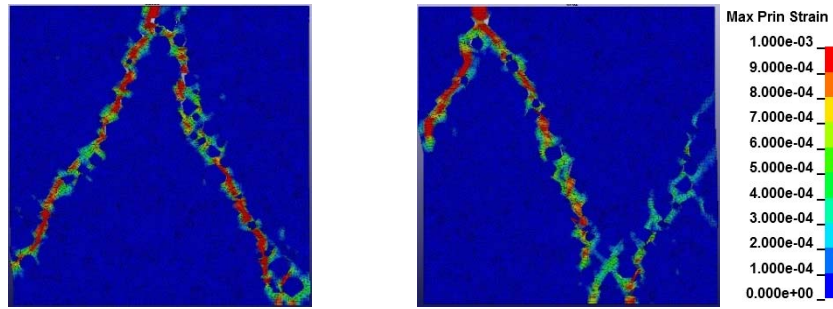


477
478 **Fig. 10.** Influence of shear strength of cohesive element for concrete under compression (SE
479 $= 10$ and $SFLS = 2\sigma_{II}^P$)

480
481 It is also interesting to observe that the cohesive shear strength also has an influence on
482 the damage patterns, which relate to the softening behaviour. As shown in Figure 11, the final
483 cracking patterns are markedly different for the lower shear strength cases (ST ratio of 1 or 2)
484 as compared to the higher shear strength case (ST ratio of 3). The final damage pattern from
485 the higher shear strength case agrees very well with experimental evidence [35] and the
486 numerical result by other researchers [29].

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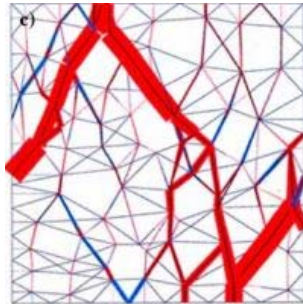


489

(a) ST = 2

(b) ST = 4

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(c) Lopez et al. [29]

(d) Fatima et al. [35]

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Fig. 11. Crack patterns (principle strain) under compression with different cohesive shear strengths

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The next factor examined is the fracture energy ratio between the Mode-II and the Mode-I for the cohesive model, herein referred to as SE ratio. Experimental observations generally suggest that the fracture energy in pure mode-II is much larger than in tension mode-I [36]. However there is a large scatter in the specific SE ratios used by different researchers, for examples, 8 to 10 by Swartz, et al. [37] but around 25 by Bažant and Pfeiffer [38].

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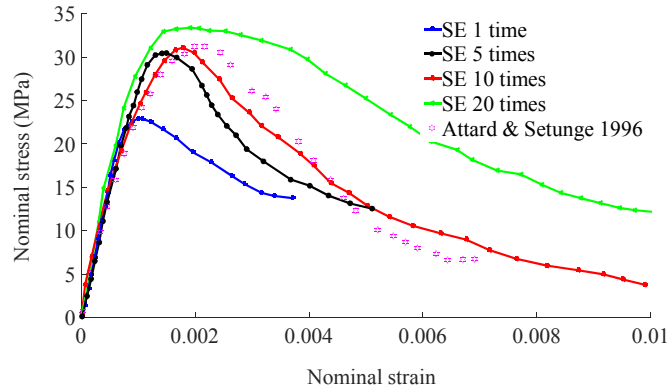
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Figure 12 shows the global stress-strain relationships for the model with different SE ratios. As can be seen, with a higher value of G_{II} , the dissipated fracture energy increases and thus modifies mostly the post-peak behaviour and shifts the transition to softening towards higher strain value. A SE value of 10 appears to be appropriate in the current mesoscale framework as it leads to the strain at peak strength of around 0.002 which is typical for normal

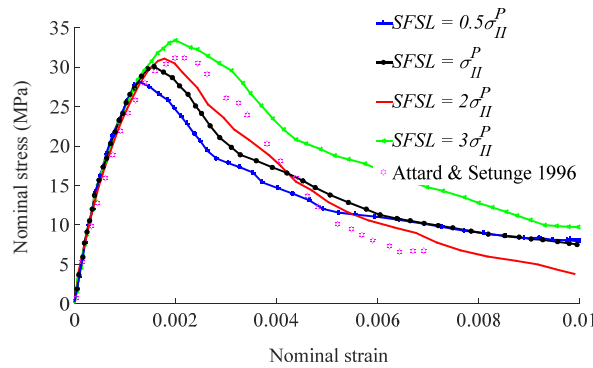
505 concrete under uniaxial compression. This choice is on the lower end of the experimental
 506 evidence as mentioned earlier, and is deemed reasonable considering the fact that the geometric
 507 aspect of the aggregate interlocking is already represented in the present mesoscale model.



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Fig. 12. Influence of shear fracture energy ($ST = 4$ and $SFSL = 2\sigma_{II}^P$)



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Fig. 13. Influence of friction limit $SFSL$ ($ST=4$ and $SE = 10$)

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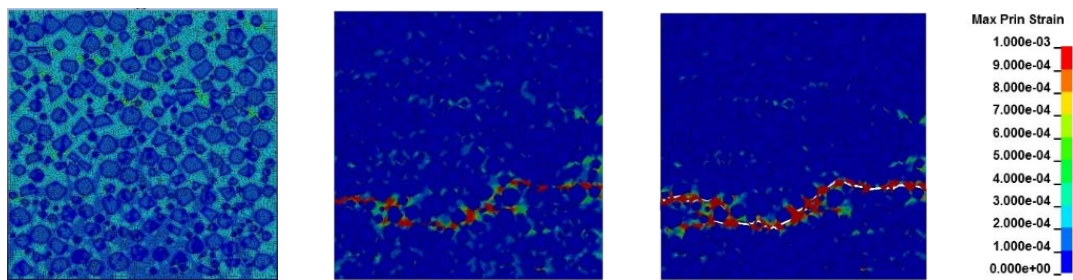
513

As discussed earlier, the static friction limit $SFSL$ is a key parameter in the cohesive-plus-
 514 contact model and it controls the static friction mechanism before de-cohesion. Preliminary
 515 numerical analysis tends to suggest a limit value of $SFSL$ equal to two times of the cohesion
 516 strength. The adequacy of this in the mesoscale model is further examined herein. Figure 13
 517 shows the nominal compressive stress-strain curves for different values of $SFSL$. The results
 518 confirm that the value of two times of cohesion is appropriate for the mesoscale concrete model
 519 as well.

520 **4.3 Simulation of uniaxial tension**

521 The cubic mesoscale numerical specimen is subjected to axial tension. Figure 14 illustrates the
522 development of the cracks. The damage is illustrated by the axial tensile strain in the range of
523 (0, 0.001), with the upper bound signifying open cracks. It can be observed that upon the peak
524 stress many micro-cracks have developed and are located mostly at the interface between
525 aggregates and mortar. As the deformation increases, concentrated macro cracks starts to
526 emerge, and this brings the specimen into the softening stage. Because of the stress relief,
527 unloading and recovery of the elastic deformation takes place in the areas outside the macro
528 crack. Many small micro-cracks stop opening further. With further increase of the applied
529 tensile deformation, the concentrated macro cracks propagate transversely, cutting through the
530 ITZ region, and finally coalesce to form virtually a single crack across the entire width of the
531 specimen. This phenomenon is a reproduction of what has been observed generally in direct
532 tension experiments.

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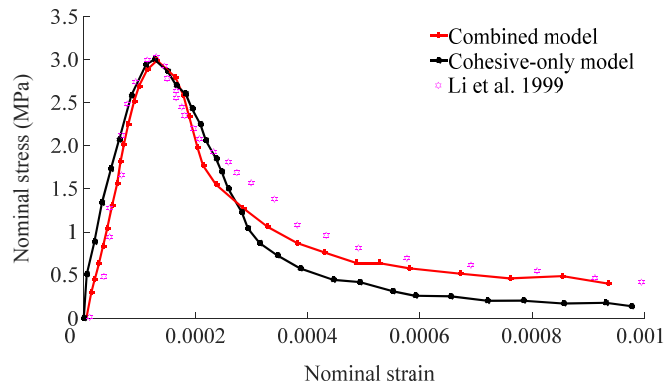
535 (a) Before peak load (b) Around peak (c) Final crack pattern

536 **Fig. 14.** Crack patterns in tension: (a) Before peak load; (b) Around peak; (c) Final pattern

537

538 The corresponding tensile stress-strain curve is given in Figure 15 in comparison with the
539 experimental data [39]. For comparison the tensile stress-strain curve produced with a model
540 without involving the contact mechanism is also presented. It can be seen that the stress-strain

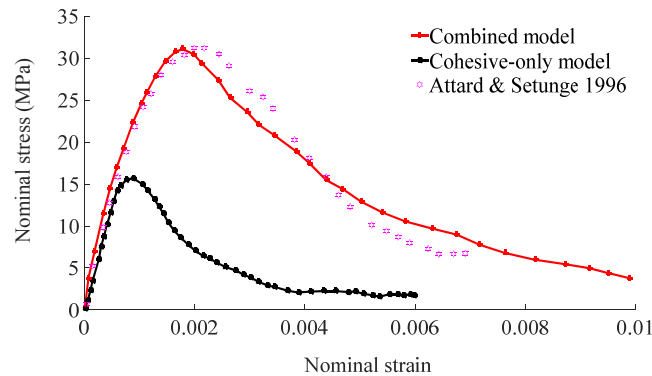
541 curve from the cohesive-plus-contact model gives almost the same result as the pure cohesive
 542 model under an axial tension. This is because almost no contact-frictional mechanism is
 543 involved in a direct tension situation. The tensile strength is around 3 MPa, as expected, with
 544 a corresponding peak strain 120 microstrain which is consistent with experimental observations.



545

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Fig. 15. Stress strain curves in tension



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Fig. 16. Stress-strain curves in uniaxial compression

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550 **4.4 Simulation of uniaxial compression**

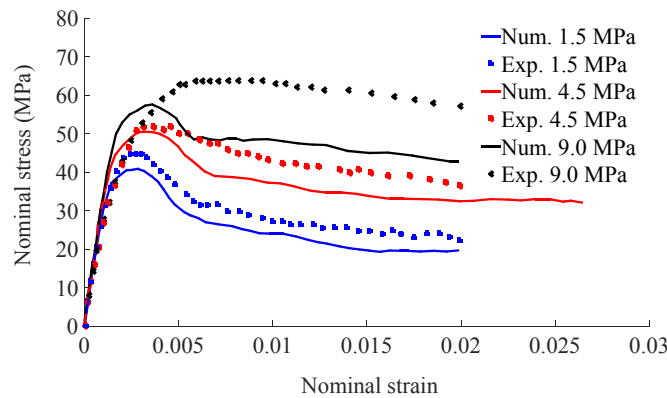
551 A perfect uniaxial compression, without any lateral constraint on the loading faces, is simulated
 552 herein. The results are compared in Figure 16. The inherent problem with the cohesion-only
 553 model becomes apparent; such a model fails to achieve satisfied either the compressive strength
 554 or the softening response. The cohesive-plus-contact model, on the other hand, predicts well

555 the experimental curve both in terms of the strength and deformation, including the softening
556 stage of the response.

557 4.5 Compression with lateral confinement

558 The compressive behaviour of concrete is known to be sensitive to the lateral confinement.
559 Generally with the increase of the lateral confinement pressure, both the compressive strength
560 and the ductility show significant enhancement.

561 Simulations of the compressive behaviour of the concrete specimen under different levels
562 of confining pressure, at 1.5, 4.5, and 9.0 MPa, respectively, have been performed. The
563 confining pressure is applied as inward lateral force on the two sides of the specimen. Figure
564 17 illustrates the nominal axial stress-strain responses of the specimens under the different
565 confining pressures. The experimental data from triaxial loading tests reported in Sfer et al. [40]
566 are selected for a direct comparison.



567

568

Fig. 17. Confinement effects

569 The simulation results agree favourably with the experimental data. As expected the
570 compressive strength of the concrete increases markedly with an increase of the confining
571 pressure. The lateral confinement also results in an increase in the ductility of the concrete
572 material, which also shows a reasonable agreement between the numerical and experimental

573 results. It can be seen that difference between the model and experimental data appear to
574 increase with the lateral confinement. This is believed to relate to the limitation of a 2D model
575 in representing the full lateral confinement effect, and may only be properly resolved when the
576 proposed cohesive plus contact approach is implemented in 3D mesoscale concrete model,
577 which should be a subject for the future study.

578 **5. Dynamic compression**

579 Classical experimental results have shown that the “apparent” dynamic compressive strength
580 of concrete increases with the increase of the strain rate, and such an increase is generally
581 defined by a Dynamic Increase Factor. However, the true mechanism underlying the occurrence
582 of the DIF is still a subject of continued debate. Various analytical and numerical studies in
583 more recent years, including an analysis using a 3D mesoscale model [41], provided detailed
584 results showing the predominant contribution of the lateral inertial confinement in the increase
585 of the dynamic compressive strength.

586 In this section the new 2D mesoscale model with cohesive-plus-contact interface for the
587 ITZ is employed to simulate the dynamic compression. With an explicit representation of the
588 ITZ, the mesoscale model is expected to describe the dynamic fracture process and its
589 influences more directly, and thus provides additional insight into the dynamic behaviour of
590 the concrete material.

591 For a comparison, a 2D homogeneous model and a 2D mesoscale model with an
592 equivalent solid ITZ layer are also analysed for the same variation range of the strain rates. To
593 facilitate a direct observation of the contribution of the structural inertial effects, all the
594 constituent materials are considered to be rate insensitive, i.e, no embedded strain rate
595 enhancement factor is adopted in the material properties in all the models. This means any
596 increase in the apparent compressive strength of the simulated test specimen is attributable to

597 the structural effect, including the inertial confinement and the dynamic interface contact
598 mechanisms, as well as the material heterogeneities.

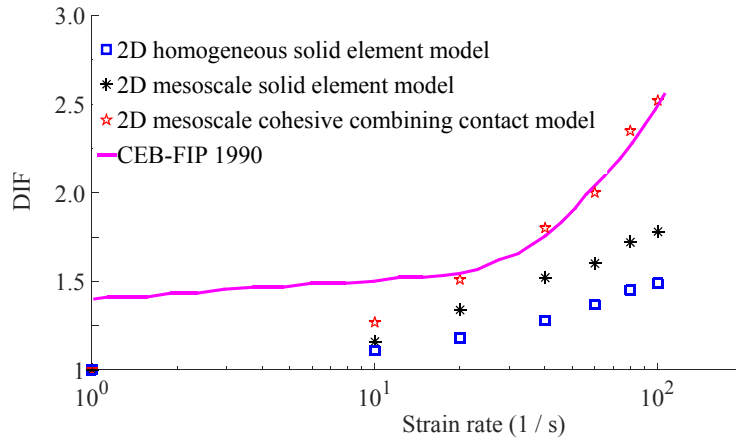
599 A similar displacement controlled loading via a velocity boundary condition as in the
600 quasi-static analysis is used in the dynamic simulation, but a higher velocity and a shorter time
601 duration is adopted in order to achieve a desirable strain rate.

602 For the current concrete specimen of 100 mm size, a strain rate range of up to 100 /s is
603 simulated. Further increased strain rate would cause excessive stress wave effect which may
604 only be minimized with the use of smaller specimens [42]. The average stress on the loading
605 and supporting faces is extracted from the numerical results, from which the peak values are
606 identified which represent the dynamic strength of the specimen. The ratio between the
607 dynamic strength and the quasi-static strength (30 MPa herein) is then obtained as the DIF for
608 each strain rate.

609 Figure 18(a) plots the DIF vs. strain rate curves based on the simulation results from the
610 three models. It can be observed that all models exhibit an increase in the nominal compressive
611 strength as the strain rate increases, despite that no strain rate enhancement has been
612 incorporated in the material constitutive model. The general trend of the simulated DIF curves
613 resembles well the curves given by the empirical formula in CEB-FIP [43]. The cohesive-plus-
614 contact model exhibits an improved agreement with the empirical curve, and this is deemed to
615 be attributable to a comprehensive representation of the various mechanisms that could affect
616 the failure process under a dynamic compression, including the contact-friction mechanism in
617 addition to the effects of the inertial confinement and the presence of the aggregates.

618 Representative damage and crack patterns under different strain rates are provided in
619 Figure 18(b). One can notice that with the increase of the loading rate, the number of micro-
620 cracks also increases. Cracks tend to propagate within the matrix phase bypassing the aggregate

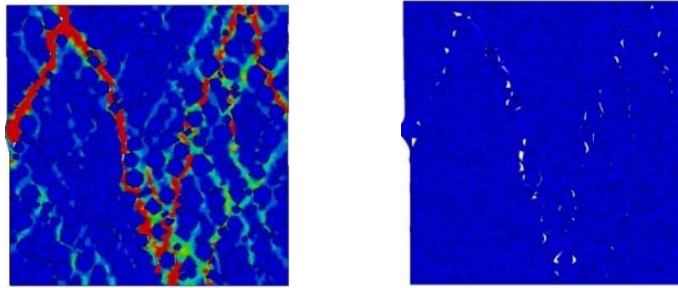
621 inclusions within the strain rate range under consideration.



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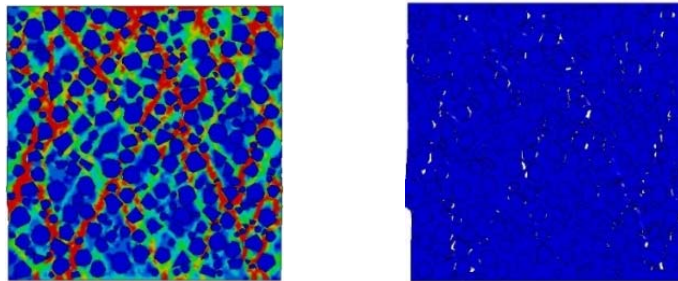
(a) DIF curves



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(b) Damage pattern under strain rate 10 /s



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(c) Damage pattern under strain rate 100 /s

628 **Fig. 18.** DIF vs. strain rate results and associated damage patterns for combined cohesive-

629

contact model

630 6. Conclusions

631 In this paper, a 2D mesoscale concrete model incorporating a combined cohesive and contact-

632 friction interface for the ITZ has been presented. The contact-friction mechanism is modelled

633 as an independent mechanism which works alongside the classical cohesive approach. The
634 general behaviour of the combined cohesive and contact-friction interface approach is first
635 verified in a triplet shear configuration with varying lateral pressure. The results show that the
636 model is capable of representing the physical process of shear resistance in the presence of
637 lateral pressure, and a smooth transition from nucleation of cracks to the pure frictional state
638 can also be realised.

639 The cohesive-plus-contact interface model is then implemented in the 2D mesoscale
640 concrete model. Adding onto a realistic description of the topology of the fracture path from
641 the mesoscale model, the interface approach enables a more realistic simulation of the complex
642 fracture and degradation process in concrete suitable for different loading conditions. A
643 parameter investigation is conducted to examine the influence of the shear properties for the
644 ITZ, for which the experimental data are generally lacking, on the macroscopic responses of
645 concrete. The model is then validated against relevant experimental results under uniaxial
646 tension, uniaxial compression, and compression with lateral confinement.

647 As an application, the model is subsequently applied in the simulation of dynamic
648 compression under high strain rates. The simulation results show a good agreement with
649 empirical data in terms of the increase of the dynamic strength with the strain rate and the
650 damage patterns.

651 The mesoscale model with the proposed cohesive-plus-contact model provides a
652 comprehensive numerical tool for investigation into the micro-mesoscale mechanisms
653 underlying the bulk properties of concrete material. It can be employed to assist in the material
654 investigation as well as characterisation of the material behaviour in complex loading
655 conditions.

656 It should be noted that in the current mesoscale model, the cohesive-plus-contact elements

657 have been employed only for the ITZ layer surrounding the aggregates. As such the model can
658 explicitly simulate fracture and fracture-induced discontinuity across the ITZ, which is known
659 to play a key role in the damage process of concrete. However, the fracture and damage within
660 concrete can extend into the mortar matrix, and can even occur within aggregates under
661 extreme loading such as high rate tension. To facilitate a full simulation of fracture process in
662 concrete, a model allowing for discontinuity and friction mechanisms to develop in all three
663 mesoscale parts, i.e. aggregates, mortar and ITZ, will be needed. This is a topic that warrants
664 further research and the progress in that direction will be reported separately.

665

666 **ACKNOWLEDGEMENTS**

667 The research reported in the paper is partly funded by the Chinese Scholarship Council and the
668 University of Edinburgh through a joint scholarship for the PhD study of the first author.

669

670 **REFERENCES**

- 671 [1] Wriggers, P., and Moftah, S. Mesoscale models for concrete: Homogenisation and
672 damage behaviour. *Finite elements in analysis and design* 2006; 42(7): 623-636.
- 673 [2] Tu, Z., and Lu, Y. Mesoscale modelling of concrete for static and dynamic response
674 analysis-Part 1: model development and implementation. *Structural Engineering and*
675 *Mechanics* 2011; 37(2): 197-213.
- 676 [3] Wang, X., Yang, Z. J, Yates, R., Jivkov, A.P., Zhang C. Monte Carlo simulations of
677 mesoscale fracture modelling of concrete with random aggregates and pores. *Constr*
678 *Build Mater* 2015; 75: 35-45.
- 679 [4] Lu Y. Modelling the dynamic response of concrete with mesoscopic heterogeneity. In
680 *Understanding the Tensile Properties of Concrete* (Ed. J. Weerheijm), Chapter 8,

- 681 Woodhead Publishing Limited. 2013:218-267.
- 682 [5] Hwang, Y. K., Bolander, J. E., Lim, Y. M. Simulation of concrete tensile failure under
683 high loading rates using three-dimensional irregular lattice models. *Mechanics of*
684 *Materials* 2016; 101: 136-146.
- 685 [6] Lilliu, G., and van Mier, J. G. 3D lattice type fracture model for concrete. *Engineering*
686 *Fracture Mechanics* 2003; 70(7): 927-941.
- 687 [7] López, C. M., Carol, I., Aguado, A. Meso-structural study of concrete fracture using
688 interface elements. I: numerical model and tensile behavior. *Materials and structures*
689 2008; 41(3): 583-599.
- 690 [8] Kregting, R. Cohesive zone models: towards a robust implementation of irreversible
691 behavior. Dissertation, Eindhoven University of Technology, The Netherlands, 2005.
- 692 [9] Nitka, M., and Tejchman, J. Modelling of concrete behaviour in uniaxial compression
693 and tension with DEM. *Granular Matter* 2015; 17(1): 145-164.
- 694 [10] Scrivener, K. L., Crumbie, A. K., Laugesen, P. The interfacial transition zone (ITZ)
695 between cement paste and aggregate in concrete. *Interface Science* 2004; 12(4): 411-421.
- 696 [11] Ruiz, G., Pandolfi, A., Ortiz, M. Three - dimensional cohesive modeling of dynamic
697 mixed - mode fracture. *International Journal for Numerical Methods in Engineering*
698 2001; 52(1-2): 97-120.
- 699 [12] Wang, Z., Kwan, A., Chan, H. Mesoscopic study of concrete I: generation of random
700 aggregate structure and finite element mesh. *Computers & Structures* 1999; 70(5): 533-
701 544.
- 702 [13] Snozzi, L., Caballero, A., Molinari, J.-F. Influence of the meso-structure in dynamic
703 fracture simulation of concrete under tensile loading. *Cement and Concrete Research*
704 2011; 41(11): 1130-1142.

- 705 [14] Gerlach, S., Fiolka, M., Matzenmiller, A. Modelling and analysis of adhesively bonded
706 joints with interface elements for crash analysis." *Proceedings of the LS-DYNA*
707 *conference*, Bamberg, 2005.
- 708 [15] Vulovic, S., Zivkovic, M., Grujovic, N. Contact Problems Based on the Penalty Method.
709 *Scientific Technical Review* 2008; 63(3-4): 2126-2132.
- 710 [16] Tvergaard, V. Effect of fibre debonding in a whisker-reinforced metal. *Materials Science*
711 *and Engineering: A* 1990; 125(2); 203-213.
- 712 [17] Chaboche, J., Girard, R., Schaff, A. Numerical analysis of composite systems by using
713 interphase/interface models. *Computational Mechanics* 1997; 20(1-2): 3-11.
- 714 [18] Koutromanos, I., and Shing, P. Cohesive crack model to simulate cyclic response of
715 concrete and masonry structures. *ACI Structural Journal* 2012; 109(3): 349-358.
- 716 [19] Sacco, E., and Toti, J. Interface elements for the analysis of masonry structures.
717 *International Journal for Computational Methods in Engineering Science and*
718 *Mechanics* 2010; 11(6): 354-373.
- 719 [20] Alfano, G., and Sacco, E. Combining interface damage and friction in a cohesive - zone
720 model. *International Journal for Numerical Methods in Engineering* 2006; 68(5): 542-
721 582.
- 722 [21] Turon, A., Davila, C. G., Camanho, P. P., Costa, J. An engineering solution for mesh size
723 effects in the simulation of delamination using cohesive zone models. *Engineering*
724 *fracture mechanics* 2007; 74(10): 1665-1682.
- 725 [22] Turon, A., Camanho, P. P., Costa, J., Dávila, C. A damage model for the simulation of
726 delamination in advanced composites under variable-mode loading. *Mechanics of*
727 *Materials* 2006; 38(11): 1072-1089.
- 728 [23] Nakamura, H., Tran, K., Kawamura, K., Kunieda, M. Crack propagation analysis due to

- 729 rebar corrosion. *Proc. The 7th International Conference on Fracture Mechanics of*
730 *Concrete and Concrete Structures (FramCoS-7)*, Korea, 2010: 921-928.
- 731 [24] Vandewalle, L. RILEM TC 162-TDF: Test and design methods for steel fibre reinforced
732 concrete. *Materials and Structures* 2000; 33(225): 3-6.
- 733 [25] Beyer, K., Abo-El-Ezz, A., Dazio, A. *Quasi-static cyclic tests on different types of*
734 *masonry spandrels*, vdf Hochschulverlag AG an der ETH Zürich, 2010.
- 735 [26] Pina-Henriques, J., and Lourenço, P. B. Masonry compression: a numerical investigation
736 at the meso-level. *Engineering computations* 2006; 23(4): 382-407.
- 737 [27] LS-DYNA keyword user's manual version 971. *Livermore Software Technology*
738 *Corporation*, 2007.
- 739 [28] Tu, Z., and Lu, Y. Evaluation of typical concrete material models used in hydrocodes for
740 high dynamic response simulations. *International Journal of Impact Engineering* 2009;
741 36(1): 132-146.
- 742 [29] López, C. M., Carol, I., Aguado, A. Meso-structural study of concrete fracture using
743 interface elements. II: compression, biaxial and Brazilian test. *Materials and Structures*
744 2008; 41(3): 601-620.
- 745 [30] Nagai, K., Sato, Y., Ueda, T. Mesoscopic simulation of failure of mortar and concrete by
746 2D RBMSM. *Journal of Advanced Concrete Technology* 2004; 2(3): 359-374.
- 747 [31] ACI. Building code requirements for structural concrete (ACI 318-08) and commentary.
748 American Concrete Institute, 2008.
- 749 [32] Cedolin, L., Bisi, G., Nardello, P. Mode II fracture resistance of concrete. *Concrete*
750 *Science and Engineering* 1999; 1(1): 37-44.
- 751 [33] Jia, Z., Castro-Montero, A., Shah, S. Observation of mixed mode fracture with center
752 notched disk specimens. *Cement and Concrete research* 1996; 26(1): 125-137.

- 753 [34] Swartz, S., and Taha, N. Crack propagation and fracture of plain concrete beams
754 subjected to shear and compression. *Structural Journal* 1991; 88(2): 169-177.
- 755 [35] Fatima, E., Jhamb, A., Kumar, R. Ceramic dust as construction material in rigid
756 pavement. *American Journal of Civil Engineering and Architecture* 2013; 1(5): 112-116.
- 757 [36] Achintha, M., and Burgoyne, C. Fracture energy of the concrete-FRP interface in
758 strengthened beams. *Engineering Fracture Mechanics* 2013; 110: 38-51.
- 759 [37] Swartz, S., Lu, L., Tang, L., Refai, T. Mode II fracture-parameter estimates for concrete
760 from beam specimens. *Experimental Mechanics* 1988; 28(2): 146-153.
- 761 [38] Bažant, Z., and Pfeiffer, P. Shear fracture tests of concrete. *Materials and Structures* 1986;
762 19(2): 111-121.
- 763 [39] Li, Q., and Ansari, F. Mechanics of damage and constitutive relationships for high-
764 strength concrete in triaxial compression. *Journal of engineering mechanics* 1999; 125(1):
765 1-10.
- 766 [40] Sfer, D., Carol, I., Gettu, R., Etse, G. Study of the behavior of concrete under triaxial
767 compression. *Journal of Engineering Mechanics* 2002; 128(2): 156-163.
- 768 [41] Lu, Y., and Zhou, R. A 3-D perspective of dynamic behaviour of heterogeneous solids.
769 *Proc., EPJ Web of Conferences*, EDP Sciences, 04038, 2015.
- 770 [42] Song, Z. H., and Lu. Y., Mesoscopic analysis of concrete under excessive high strain-rate
771 compression and implications on interpretation of test data. *Int J Impact Eng.* 2012; 46:
772 41-55.
- 773 [43] CEB-FIP Model Code 1990, Evaluation of the time dependent behavior of concrete.
774 Redwood Books, Trowbridge, Wiltshire, UK, 1993.