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Remo Pedreschi
Professor of Architectural Technology, University of Edinburgh, UK

Dimitris Theodossopoulos
Lecturer in Architectural Technology, University of Edinburgh, UK

Institution of Civil Engineers

The double-curvature masonry vaults of Eladio Dieste

R. Pedreschi PhD, CEng, MICE and D.Theodossopoulos Dip.Civ.Eng, PhD

The Uruguayan engineer Eladio Dieste developed an innovative construction method for wide-span roof structures. Known as Gaussian vaults, their double-curved geometry is based on the catenary resulting in mainly axial compressive forces. Whereas most thin wide-span roofs have been built using concrete, Dieste used brick, and unlike traditional masonry vaults, they are only one brick-layer in thickness. Typically the vaults have a low rise, the span-to-rise ratio is normally 8–10 and buckling is the likely mode of failure. Dieste used the curved surface of the vaults to resist buckling and developed design procedures to ensure their safety. In the present paper a brief background to Dieste's work is presented including his methods of analysis and the application is considered with reference to one of his larger projects, the warehouse at the docks in Montevideo, with a span of 45 m. Through an iterative mathematical procedure, Dieste formulated the critical loads of catenary arches into graphs. The method is compared with a finite-element study, which also considers the elastic deformations under self-weight, asymmetric loading owing to wind and ultimate failure owing to buckling.

1. INTRODUCTION

Throughout the 20th century engineers have been responsible for the development of many structural forms and innovative uses of materials that soon became noted for architectural qualities as well as their efficient and resourceful materials and construction technology. The Swiss engineer Robert Maillart (1872–1940) for example early in the 20th century demonstrated that careful and intuitive structural design could combine economy of means with great elegance in the construction of slender concrete arch bridges.[1] Later in the century the Italian engineer Pier Luigi Nervi (1891–1978) made innovations in ferro-cement and construction techniques that led to many long-span structures with a fine filigree quality, such as the Palazzo dello Sport in Rome.[2] Eduardo Torroja (1895–1961) created many innovative structures, perhaps most notably the asymmetric barrel vaults of Fronton Recoletos,[3] but he also worked in the dissemination of shell design by forming the Institute for Shell and Spatial Structures. In his influential book *Philosophy of Structure*[4] he discussed the role of structure in architecture, based on his belief that the visual and architectural qualities of structures should never be ignored in design.

The Uruguayan engineer Eladio Dieste (1917–2000) is a less well-known figure, but is nevertheless deserving of equal recognition. Dieste graduated in engineering from the University of Montevideo in 1943 with an ambition: 'I am very passionate about the possibility of understanding reality by means of a physical-mathematical language'.[5] At this time in Uruguay, although still defined as a developing country, there was a strong intellectual and cultural community. Dieste was clearly influenced by this and sought to apply his engineering skills to meet the needs of his country. Throughout his work, he looked for solutions that suited the limited economic resources of Uruguay and he constantly tried to 'contemplate each problem independently, keeping in mind the conditions of our circumstances and environment'.[6]

Although many of the techniques and innovations in shell construction and prestressed concrete pioneered by European engineers were finding their way to South America, Dieste turned to a traditional material, brickwork and developed new forms of construction that satisfied the needs of modern building: accuracy, efficiency in materials, prefabrication, reliability in performance and analytical rigour. More than this, he sought for a form of architectural expression that would counter the perception in the use of brick as a 'poor man's' substitute for concrete. In his own words, 'For architecture to be truly constructed the materials should not be used without a deep respect for their essence and consequently their possibilities.'[7] Dieste saw brick as a material that could be used in overtly modern forms that belied its traditional origins. Dieste also had a strong belief in the relationship between structure and architecture and its expression, this belief being legible throughout his work and his writings: 'The resistant virtues of structures that we make depend on their form; it is through their form that they are stable and not because of an accumulation of materials. There is nothing more noble and elegant from an intellectual viewpoint than this; resistance through form.'[7]

Dieste's structures are interesting from a range of perspectives: for the purely technical; as an exploration of a structural philosophy; for the beauty of their architectural expression; and for their often ingenious construction techniques. Dieste's innovations include single- and double-curvature vaults, known respectively as free-standing barrel vaults and Gaussian vaults. His firm, Dieste y Montañez, designed and constructed many such structures across South America, over 1·5 million square metres of building. The present paper is concerned with the most sophisticated of his innovations, the Gaussian vault, a very thin shallow vault of a catenary cross-section, with a typical span-to-rise ratio of 10. The thinness of the vault over relatively long spans renders the vault sensitive to buckling. The development of these structural forms is discussed, and in particular the analytical method Dieste developed for calculating their buckling resistance. Dieste's method is compared with a finite-element (FE) study of one of his most notable projects, the J. Herrera y Obes Warehouse in Montevideo (1977–79).[5,8] The FE study also considers additional aspects not covered in Dieste's analysis such as asymmetric windload and deformation.

2. THE DEVELOPMENT OF THE GEOMETRY OF THE VAULTS

Like most engineers of the period Dieste was familiar with reinforced concrete shell construction, on which he worked early in his career. His interest in brick developed from his collaboration with the Spanish Architect Antonio Bonet on the design of a private house (Casa Berlinghieri) where he replaced the intended concrete shell with a thin brick vault. From this point onwards Dieste studied the contemporary application of structural brickwork leading to the development of a range of thin brick vaulting systems. These construction systems were derived from structural principles associated with the geometry of the catenary.

The catenary describes the form that a suspended cable will adopt owing to its self-weight. It is a form-active geometry, where all the forces are axial tension (Fig. 1). Inverting the cable will describe the geometry of a similarly form-active arch structure where the forces are axial compression owing to the self-weight. Catenary geometry can be found using physical models, seen in the work of Antoni Gaudí, Frei Otto and Heinz Isler among others,[9] particularly for three-dimensional surfaces. Other methods of defining a catenary include graphic methods, see Zalewski and Allen[10] or mathematical forms using the catenary equation (equation (1)).

$$y = \frac{\boldsymbol{T}_0}{w_0}\left(\cosh\frac{w_0 x}{\boldsymbol{T}_0} - 1{\cdot}0\right) \tag{1}$$

where w_0 is the self-weight of the cable and $\boldsymbol{T}_0$ is the force in cable at mid-point.

Dieste constructed a large number of barrel vault structures using the catenary cross-section. Typically such vaults have span-to-rise ratios of 4–5 and the compressive stresses owing to self-weight are low. Dieste took advantage of the significant depth of these vaults to act as long-span beams often with large cantilever spans, called free-standing barrel vaults. (Figs 2 and 3). These spans were possible only through the development of innovative prestressing techniques, which Dieste invented. More information on these remarkable structures is given in Pedreschi[5] and Anderson.[8] The low span-to-rise ratios keeps the stresses below the level that would cause concerns for buckling. It also, however, limits the practical span between abutments, otherwise excessively high vaults would result. If the span-to-rise ratios increase then the stresses in the vault increase (Fig. 4); even at span-to-rise ratios of 10 the axial compressive stresses are still low. The slenderness of the vault has also, however, increased by a factor of 6. The problem becomes one of resisting buckling.

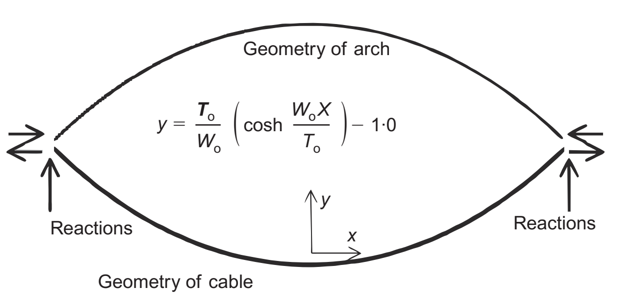

Fig. 1. Geometry of the catenary


Fig. 2. Free-standing barrel vault, Salto Uruguay (V. del Amo)


Fig. 3. Free-standing barrel vault (V. del Amo)

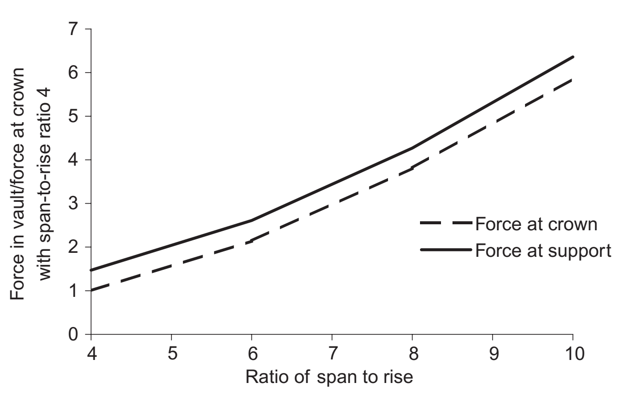

Fig. 4. Influence of span-to-rise ratio on the forces in a centenary arch

The method adopted by Dieste is to ‘resist through form’ and hence he created an innovative doubly curved undulating surface with maximum undulation at the mid-span of the vault, effectively increasing the moment of inertia and hence increasing the buckling resistance (Fig. 5). The geometry of the Gaussian vault is defined by a series of catenary curves of varying rises. The name ‘Gaussian’ is taken from the mathematician Karl Gauss (1777–1855), noted for his description of the geometry of curved surfaces. The curves share a common springing point, defined normally by the walls of a building. Each curve can be seen as being contained within an imaginary vertical plane whose baseline straddles the springing points. If this plane moves along the axis of the building and the rise of the catenary increases then a curved surface is defined with maximum undulation along the central axis of the building reducing to no undulations at the springing points. Every transverse section between the springing points has a catenary geometry.

These structures have been used in many buildings in spans up to 50 m. The thickness of the vault is kept to a minimum, always only one brick or masonry unit with a topping of 30 mm coarse sand and cement. Reinforcement is placed in the joints between the bricks and a light mesh is incorporated in the topping. The formwork for the vault is a major element in its own right and is only economic if used repeatedly. Typically the vaults appear as a series of waves (Fig. 6). Depending on the weather, the formwork can be struck as soon as 24 h after completing the vault. In large projects the formwork for one wave is constructed as a single element, sitting on rails, and is moved along the building from vault to vault (Fig. 7).

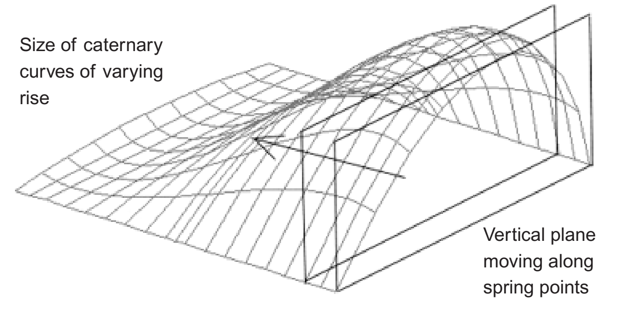

Fig. 5. Geometry of Gaussian vault

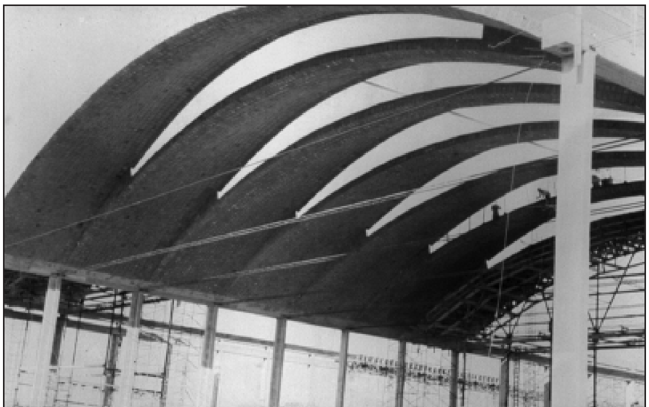

Fig. 6. Gaussian vault under construction (Dieste Archive)

Fig. 7. Formwork for Gaussian vault

Through many projects Dieste was able to develop and perfect the techniques to design and construct the vaults. Each project provided insights into the next (Fig. 8), and the growing confidence and experience led to many large projects such as the Fruit Market, Porto Alegre, Brazil. The total project is in excess of 50 000 m^2 and includes the Growers Pavilion, 290 m long with a span of 47 m.

3. METHODS OF ANALYSIS

The critical condition for the Gaussian vault is one of buckling. As stated earlier, Dieste was keen to exploit theoretical methods of analysis in the design of structures and he developed appropriate design methodologies. He presented these procedures in two short books.[11,12] The following description of the problem of the elastic instability of Gaussian vaults of double curvature and the calculation methods has been prepared from a translation of the original Spanish by the second author of the present paper. Initially the instability of single-curvature catenary arches of constant section under their own self-weight dead load is examined by forming the equation of the thrust line and evaluating the critical load by means of an iterative solution.

A catenary arch AB of a total length $S = 2\ell$ is considered (Fig. 9). The arch is considered to buckle following the dotted line and the critical load $\boldsymbol{q}_{\text{cr}}$ will be evaluated. At a generic point D at a length $\overrightarrow{\text{CD}} = x$ from the apex C, y is the ordinate of the thrust line, ρ is the curvature and φ is the hoop angle (varies

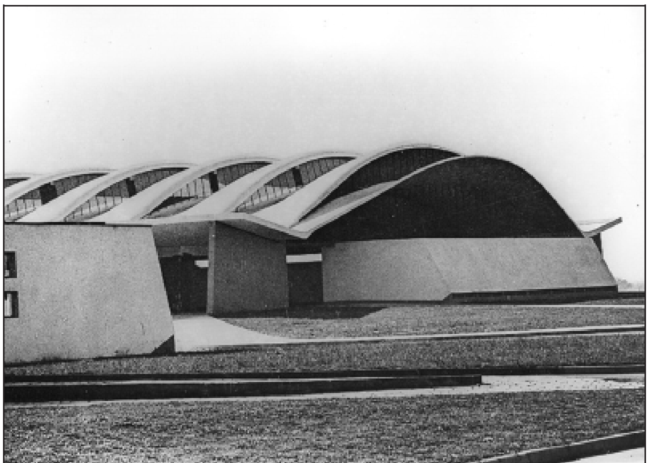

Fig. 8. Gaussian vaults, Porto Alegre, Brazil (Dieste Archive)

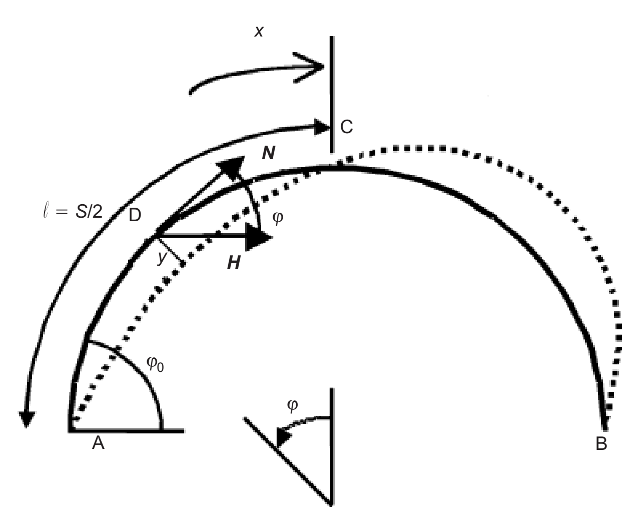

Fig. 9. Buckling of catenary arch under self-weight[12]

from 0 at the apex to φ_0 at the springing of the vault) and it is assumed that $y \ll \ell$.

If the axial force at D is $\boldsymbol{N}$, the equivalent moment owing to the offset y is then $M = +\boldsymbol{N}y$. If ρ_c is the radius of curvature at the apex and $\boldsymbol{q}$ the distributed load per unit length of the arch, then the thrust, $\boldsymbol{H} = \rho_c \boldsymbol{q}$. In a catenary

$$\boldsymbol{N}\cos\varphi = \boldsymbol{H} \Leftrightarrow \boldsymbol{N} = \frac{\rho_c}{\cos\varphi}\boldsymbol{q} \tag{2}$$

From the geometry of the arch,

$$x = \overrightarrow{CD} = \rho_c \frac{\sin\varphi}{\cos\varphi}$$

and

$$\frac{x^2 + \rho_c^2}{\rho_c^2} = \frac{1}{\cos^2\varphi}$$

therefore

$$\frac{\rho_c}{\cos\varphi} = \sqrt{x^2 + \rho_c^2}$$

and consequently

$$\boldsymbol{N} = \boldsymbol{q}\sqrt{x^2 + \rho_c^2} \tag{3}$$

The radius of curvature of the undeformed arch ρ_0 at point D is

$$\rho_0 = \frac{\rho_c}{\cos^2\varphi} = \frac{x^2 + \rho_c^2}{\rho_c} \tag{4}$$

The simplified equation of the thrust line for a curved beam can be expressed as

$$\frac{y}{\rho_0^2} + y'' = -\frac{M}{EI} \Leftrightarrow$$
$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -\frac{\boldsymbol{q}}{EI}\, y\sqrt{x^2 + \rho_c^2} - \frac{\rho_c^2 y}{(x^2 + \rho_c^2)^2} \tag{5}$$

where EI is the flexural stiffness of the arch or curved beam. If the following property, γ is defined as

$$\gamma = \frac{1}{\tan\varphi_0} \tag{6}$$

then $\rho_c = \gamma\ell$. Also, if $v = \overrightarrow{AD}$ then $x = \ell - v$ and if $u = v/\ell$, then equation (7) becomes

$$\frac{\mathrm{d}^2 y}{\mathrm{d}u^2} = -\chi y\sqrt{\gamma^2 + (1-u)^2} - \frac{\gamma^2 y}{[\gamma^2 + (1-u)^2]^2} \tag{7}$$

with $u \in [0, 2]$ and

$$\chi = \frac{\boldsymbol{q}\ell^3}{EI} \tag{8}$$

The problem then can be summarised as: for a given arch (defined by its length ℓ and a value for γ that represents the springing angle φ_0 in equation (6)) χ can be evaluated and therefore the critical load $\boldsymbol{q}_{cr,}$ equation (8). In a manner similar to the instability problem of axially loaded columns, the quantity χ is obtained from the boundary conditions in equation (7), which are $y = 0$ at the locations $u = 0$ and $u = 2$ (bases) and at $u = 1$ (apex). This approximation is usually true as it was assumed that $y \ll \ell$, $y' \ll 1$ and $\rho_0 - y \cong \rho_0$.

The differential equation (7) is integrated by means of a numerical/graphical method. A value for γ is chosen and then for every χ a value for y at the support B is calculated (y_B), which in general should not be 0. The values of y_B are then plotted and the roots of the equation $y_B(\chi)$ are evaluated graphically. The thrust lines for the first three roots are illustrated in Fig. 10. Dieste observed that for the solutions corresponding to

(*a*) χ_1 (Fig. 10(a)): the corresponding minor value for $\boldsymbol{q}_{cr}$ is not correct as it represents buckling where y is either entirely positive or negative—that is, the original length increases; this is incompatible with the flexural buckling assumptions that the length remains constant
(*b*) χ_2 (Fig.10(b)): this value gives an acceptable shape
(*c*) χ_3 (Fig. 10(c)): although mathematically correct, it is more probable the arch may have already buckled under the lower load corresponding to χ_2.

As a result, the value for χ is χ_2. Dieste evaluated values of χ for every γ following this procedure and formed a series of curves shown in the diagram in Fig. 11, discussed below.

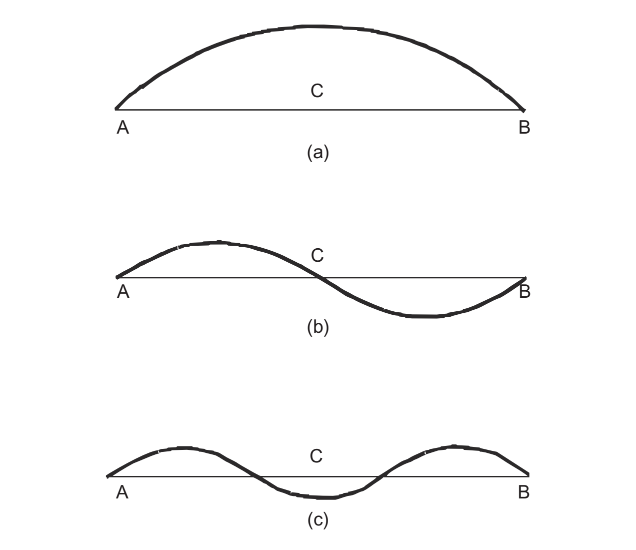

Fig. 10. Buckling modes for vaults[12]

The foregoing analysis assumes constant moment of inertia, I. In the Gaussian vault I varies between maximum at crown and minimum at support. If the masonry is assumed to be made of solid units, the moment of inertia I in each cross-section can be evaluated from the expression

$$I = \frac{\ell_s t h^2}{8} + \frac{b t^3}{12} \tag{9}$$

where ℓ_s is the length of the cross-section, t is the thickness of the shell and h is the amplitude of the undulation (Fig. 12).

$$\upsilon = \frac{I_{\text{crown}}}{I_{\text{support}}} \tag{10}$$

The charts in Fig. 11 combine χ, φ_0 and ν. Modifying the procedure established for uniform arches, q_{cr} can be calculated from equation (8): I is the value at the supports, γ in equation (6) results from the average springing angle φ_0 of all the directrices and a family of curves can then be calculated (Fig. 11) in terms of the variable ν that is used to define the change in the cross-section (equation (10)).

Dieste also developed an alternative method for calculating the critical buckling load using virtual displacements and successive approximations. This method is more rigorous and takes more detailed account of the variable cross-section geometry of the vaults, and predicts slightly higher buckling loads.[12]

4. THE GAUSSIAN VAULTS AT THE J. HERRERA Y OBES (JHO) WAREHOUSE, MONTEVIDEO DOCKS (1977–1979)

The design and construction of the Gaussian vault will be considered in more detail with reference to the above project. The warehouse was originally constructed with load-bearing brick walls and a steel barrel vault roof. The roof was destroyed by a major fire in 1977. Dieste's firm won the competition to rebuild the warehouse. Unlike most of the other entries, which proposed complete demolition and reconstruction, Dieste recommended retention and repair of the walls and a new Gaussian vault for the roof. The overall dimensions are 79 m by 46 m. The roof consists of a series of 14 discontinuous vaults (Fig. 13). The geometry of each vault can be determined by applying the catenary equation (equation (1)) at various sections as previously described. The cross-section of the vault at mid-span is shown in Fig. 14 and glazing is installed in the discontinuity between the vaults (Fig. 15). The underside of the vault diffuses the natural light to provide pleasant ambient lighting conditions. The inside distance between the side walls varied by as much as 300 mm along the length of the building. To maximise the efficient use of the formwork this variation was taken up by an in situ concrete edge beam, which itself varied to provide a consistent distance between the springing points of the vaults. The vaults are sensitive to horizontal movement at the reactions and the beam was also

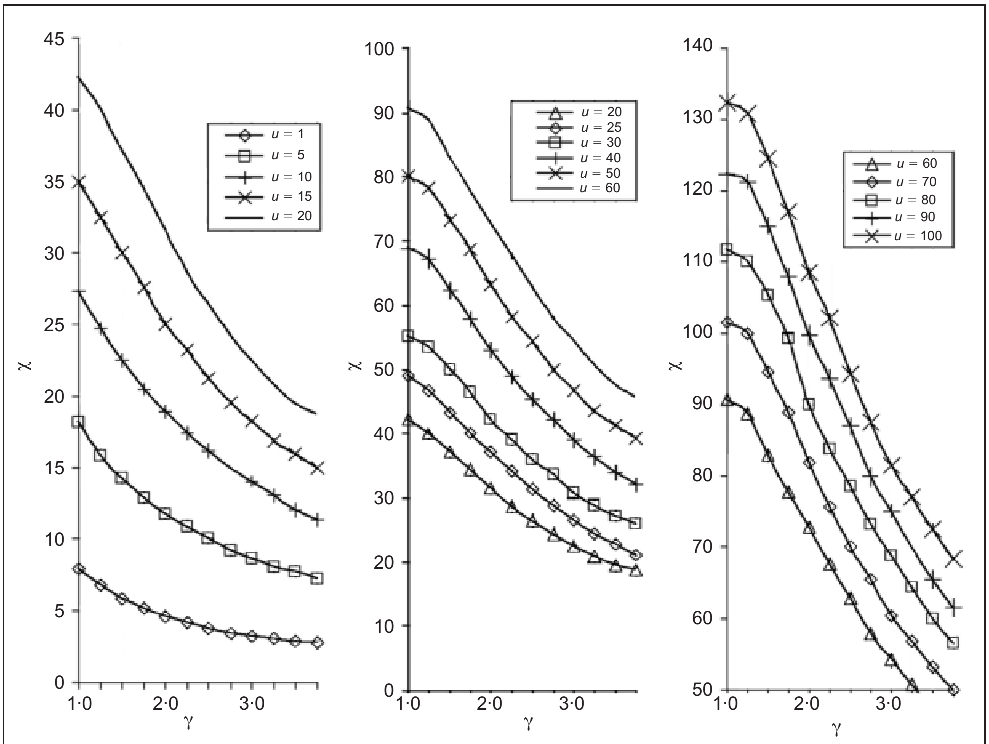

Fig. 11. Design charts for buckling of catenary arches[12]

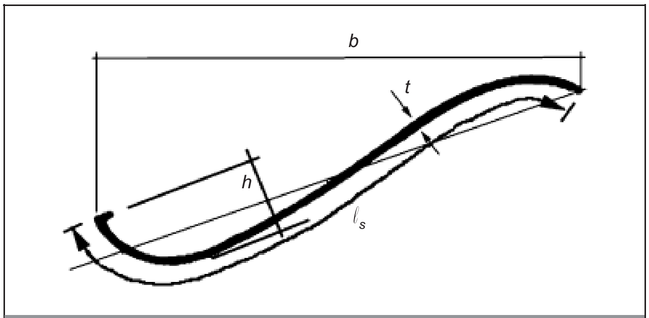

Fig. 12. Second moment of area of cross-section of Gaussian vault


Fig. 13. The JHO warehouse at Montevideo Docks (Dieste Archive)

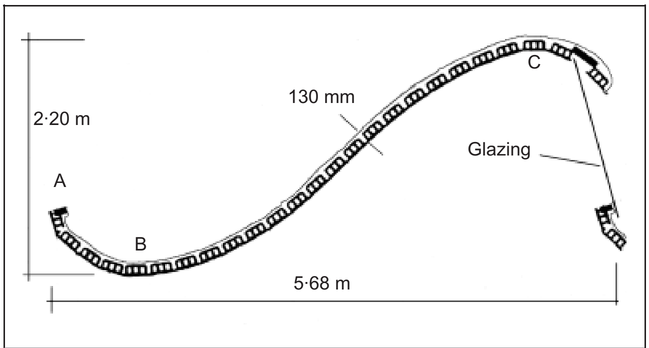

Fig. 14. Cross-section of JHO vault at crown

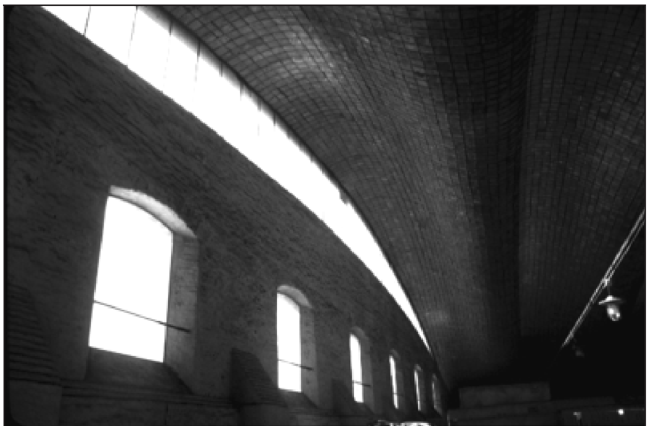
Fig. 15. Junction between gable and the vault, JHO Warehouse

necessary to provide a stiff lateral support and anchorage for the pre-tensioned tie rods, used to contain the lateral thrust from the vaults. Each vault spans 44·74 m and is 5·68 m wide, with the span-to-rise ratio varying between 7 and 10. The vault is constructed using a single layer of extruded hollow clay blocks 100 mm deep, known as 'ticholos' and topping to provide an overall thickness of 130 mm. Reinforcement is placed both transversally and longitudinally within the joints between the units. The joints are filled with a 1:2·5 cement–sand mortar. The sand used is coarse, similar in grading to sharp concrete sand and the mortar has a higher compressive strength than conventional bricklaying mortar, typically 20 N/mm^2. Dieste expresses the form of the vaults at the gables, using glazing to separate the vault from the walls, making clear that the vault does not rely on these walls for stability (Fig. 15). Owing to the early striking times of the vault the most significant loading condition occurs during transfer of self-weight as the formwork is lowered while the masonry is still developing strength.

The critical buckling load for the vault can be predicted using Dieste's methods. The geometric properties are: average springing angle, $\varphi_0 = 25^\circ$, second moment of area of the vault at the supports, $I_{\text{support}} = 0{\cdot}00104\ \text{m}^2$ and $I_{\text{crown}} = 0{\cdot}0706\ \text{m}^2$. Therefore, $u = 68$, and from Fig. 11, $\chi = 78$. Dieste used a value of 7000 N/mm^2 for the elastic modulus.[12] He obtained the value of elastic modulus from tests on prototype vaults. The value is consistent with other studies of the elastic modulus for brickwork with relatively low-strength bricks.[13] The critical buckling load is obtained from equation (8) using I_{support}. The buckling load q_{cr} is 45·44 kN/m. The self-weight of the masonry is taken as 16 kN/m^3 and, if the cross-section of the vault is considered as 5·68 m by 0·13 m, produces a linear load of 11·8 kN/m. The factor of safety against collapse is, therefore, 3·85.

5. FE STUDY OF JHO WAREHOUSE

Dieste's method was compared with the results of a finite-element analysis (FEA). A model of the vault was constructed using the FE package Abaqus[14] and used to study the vault under static loads and buckling. Self-weight and asymmetric wind loads were applied to the model. The geometry of the vault was taken from the setting-out drawings for the formwork supplied by Dieste y Montañez. A single vault was modelled in its entirety using four node shell elements of (S4R5) type. The nodes used to generate the FE mesh were the same as those used by Dieste in the construction of the vault's crown, 29 uniformly spaced nodes between the apex and the support and 18 nodes in the transverse direction. The spacing of the transverse nodes was refined in the area of the lowest catenary, where variation in stresses was likely to be greatest. Pinned conditions were applied at the supports. Although brickwork is a non-linear, anisotropic material for the purposes of the FE study it was considered as homogeneous and linear elastic; this assumption is justified for the masonry as the stresses are low in relation to the compressive strength of the masonry. Figures for the compressive strength of the actual brickwork are not available. The compressive strength of brickwork is greatly influenced by the brick strength, the mortar grade orientation of the applied compressive forces and the shape of the prism. The first author of the present paper has carried out extensive tests on brickwork[13] and it is unlikely the

the compressive strength is less than 8 MPa. In order to compare with Dieste's own methods it was important to use the same elastic properties, namely elastic modulus 7000 N/mm^2 and Poisson ratio 0·15.

The behaviour of the vault under self-weight was analysed. This corresponds to the striking of the formwork when the load is transferred from the formwork to arching action of the vault. It is the most onerous loading condition, being applied after only one to two days of curing. The stiffening effect of the topping was ignored in the analysis as it has low elastic modulus at this stage of construction and is primarily intended to provide weathertightness. The analysis shows that the stresses are compressive and low in comparison with the likely compressive strength of the masonry, the maximum stress, 1·48 N/mm^2 occurring at the springing. Fig. 16 presents the variation of axial force along three separate sections following the directix, corresponding to the lower free edge, section A, the lowest catenary curve, section B and the highest catenary curve, section C (Fig. 14). The catenary section with greatest rise, section C, has a near uniform axial force, gradually increasing towards the supports, verifying the catenary behaviour of the vaults. Along section A, the lower side edge of the vault, the compressive stresses are greater than either section C or B and gradually decrease towards the support. Along section B, the lowest catenary, the stresses are least at the crown and then increase towards the support. The stresses at the support are slightly lower for section A than section B, although the situation at the crown is reversed. If considered as independent catenaries then section B, the shallowest curve, should have the greater axial stress through its length. Table 1 compares the results of the FE analysis with the compressive stresses calculated assuming independent catenaries for the three sections, A, B and C, using equation (1). The calculated force at the crown for section A, the free edge, is considerably lower than predicted by the FE model while the calculated force at the crown of section B, the lowest rise, is greater than predicted by the FE model. There is clearly a redistribution of forces around the crown of the vault, which illustrates the influence of the transverse curvature of the vault as it stiffens the shallowest section at the crown; while section A, with a free edge, has less support, the forces at the crown are considerably greater than predicted by the catenary analysis. The calculated results for section C, the section with the greatest rise, are very close to the results from the FEA. The FEA can also provide the deflections that Dieste's theory does not predict. At the crown the deflection varies across the vault with a maximum of 14·2 mm at the outer edge of the cross-section, or 1/3160 of the span, to a minimum deflection at the highest point, (where the catenary section is the deepest), of 7·7 mm, Fig. 17. The vault, therefore, twists slightly along the directrix with a relative deflection of 6·5 mm across the crown.

A buckling analysis under self-weight was performed by applying the elastic instability process in Abaqus to the FE model. The program resolves the eigenvalue buckling problem by performing a linear perturbation analysis and estimates the critical buckling loads of stiff structures—that is, those structures that carry their design loads primarily by axial or membrane action, rather than by bending action.[14] The response of the model to instability is defined by its linear elastic stiffness in the base state, ignoring non-linear material behaviour. The analysis produces the eigenvalues for the loading conditions and these coincide with the factor of safety against buckling. In this case, the eigenvalues for the first three modes under self-weight are 4·37, 4·54 and 8·76 (where the vault is considered to buckle along its main axis) and the first buckling mode is shown in Fig. 18. These values validate the analysis by Dieste's methods, which predicted a factor of safety of 3·85.

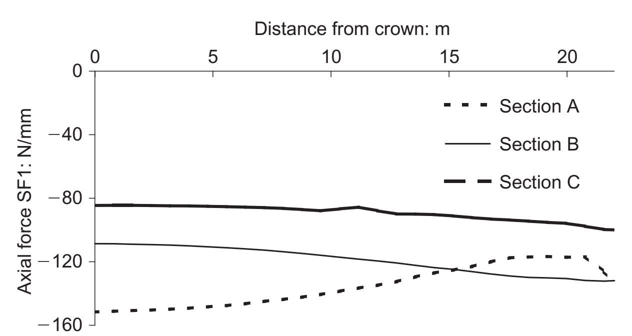

Fig. 16. Variation of axial force along vault

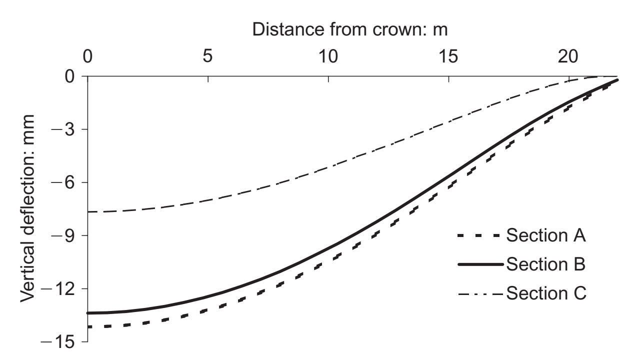

Fig. 17. Variation of deflection along vault owing to self-weight

Section	Rise of catenary: m	Forces at crown: kN		Forces at support: kN	
		Catenary equation	Finite element	Catenary equation	Finite element
A	4·707	113·2	151	123	137
B	4·201	123·8	108	132	132
C	6·507	83·0	85	97	100

Table 1. Comparison of forces in vault between catenary equation and FEA

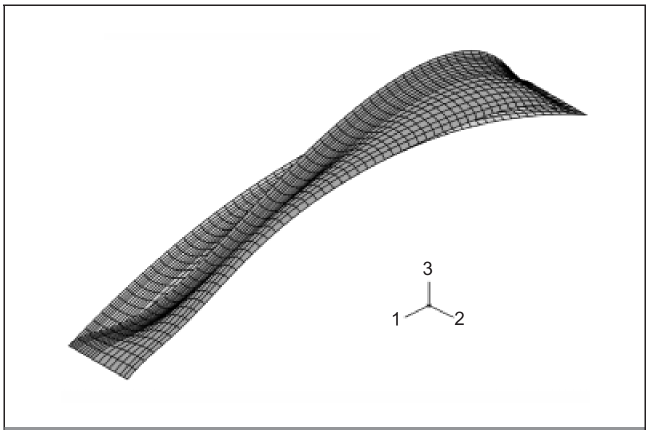

Fig. 18. Buckling of vault under self-weight

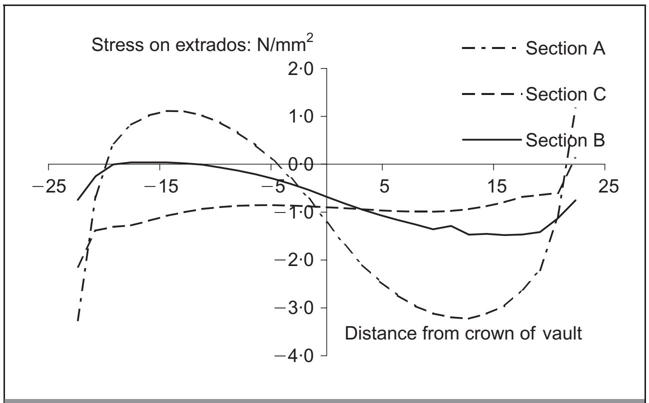

Fig. 20. Hoop stress on the extrados of the vault owing to asymmetric load

A further condition of asymmetric load was studied. The Gaussian vault takes its form from its own self-weight and the internal forces are primarily axial. Asymmetric load creates non-axial forces and bending along the directrix. In Uruguay the temperature very rarely falls below zero and snow loading is not considered in structural design. Wind load is the predominant load condition. The design wind speed for coastal locations in Uruguay is 158 km/h (similar to the design wind speeds in Scotland). Considering wind load on a shallow vault it is important to use a loading pattern that generates the most critical deformations. Melbourne[15] suggests the use of external pressure coefficients C_{pe} of −0·5 and +0·4 to the windward and leeward sections respectively of shallow curved vaults. A conservative loading pattern based on the design wind speed for Uruguay is a net upward pressure of 0·6 kN/m^2 on one side of the vault and a net downward pressure of 0·5 kN/m^2 on the other side. The FEA includes the self-weight of the vault. The deflections along section B, the lower catenary, are presented in Fig. 19. The vault is pushed to one side with maximum deflections occurring at approximately one quarter of the span from each support. Thus half of the vault is moving upwards and the windward side of the vault is being pushed downwards. The deflections are still relatively low, given the span of the vault, 66 mm, or a span/deflection ratio of 675. The stresses along the directix for sections A, B and C are presented in Fig. 20. Sections B and C are in compression along their full length but some tension occurs along the lower free edge on the leeward face of the vault, which concentrates only over a small portion of the vault around the crown. These small tensile stresses would be carried by the reinforcement in the joints and the light steel mesh in the topping. Neither the reinforcement nor the topping was included in the analysis. A buckling check for these conditions was also undertaken and the eigenvalue for the first mode is 3·69. The buckling failure is shown in Fig. 21.

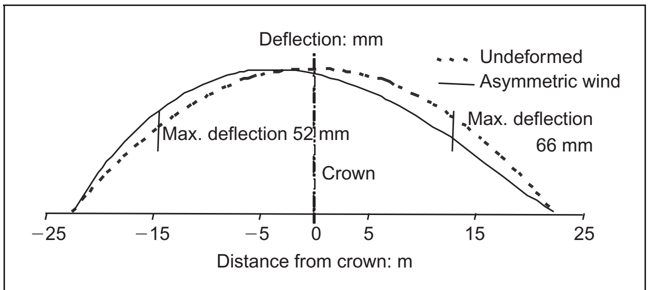

Fig. 19. Deflection along section B owing to asymmetric wind loads

6. SUMMARY AND CONCLUSION

Eladio Dieste developed and used the Gaussian vault in many projects during a long career. The undulating geometry of the vault is intended to resist buckling, while providing maximum efficiency in use of materials. The catenary geometry of the vaults ensures that the brickwork is under axial compression owing to its own self-weight. One of the longest-span vaults, the warehouse at Montevideo docks has been studied in the present paper. Using Dieste's own methods a factor of safety against elastic buckling failure of 3·85 was obtained. Using FEA and the same elatic properties a factor of safety of 4·37 against buckling was obtained—slightly higher that Dieste's.

The FEA has also validated the use of the catenary geometry as the stresses under self-weight are quite close to those predicted using the catenary equation although some redistribution of stresses occurred in the lower catenary around the crown of the vault. The vault tends to twist around the crown although the deflections are low. The vault was also analysed under conditions of asymmetric wind load. The vault tends to deform sideways with some twisting and the deflections are greater than under self-weight only but they are still comparatively low. Under this extreme loading condition the stresses in the vaults remain primarily in compression except towards the lower free edge on the leeward side of the vault. The factor of safety against elastic buckling is 3·69 and is still within the acceptable limits.

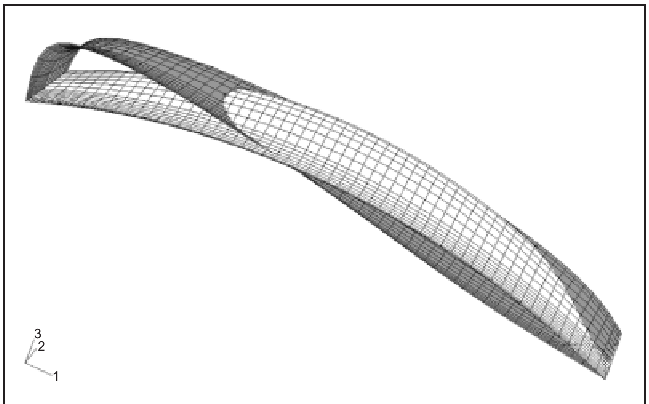

Fig. 21. Buckling of vault under asymmetric load

On the basis of the analysis it is difficult to envisage further refinement to the vault. Using masonry the minimum thickness is a function of the masonry unit and the vault is only one unit in thickness. The vault could be made shallower or the undulation at the crown reduced, leading to a comparatively small saving in materials. However this is likely to lead to a marked reduction in factor of safety below an acceptable limit for this type of construction.

The work of Eladio Dieste is visually exciting and rightly regarded for its architectural importance. However it is much more than this. The slenderness of the vault, the use of the doubly curved surface as a device for both stability and expression, the use of brickwork in an entirely non-traditional manner, the development of analysis methods, supported with practical experience and observation are the products of a truly outstanding and visionary engineer.

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