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A General Approach Toward Green Resource Allocation in Relay-Assisted Multiuser Communication Networks

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***Abstract*—The rapid growth of energy consumption due to the strong demands of wireless multimedia services, becomes a major concern from the environmental perspective. In this paper, we investigate a novel energy-efficient resource allocation scheme for relay-assisted multiuser networks to maximize the energy efficiency (EE) of the network by jointly optimizing the subcarrier pairing permutation formed in one-to-many/many-to-one manner, subcarrier allocation, as well as the power allocation altogether. By analyzing the properties of the complex mixed-integer nonlinear programming (MINLP) problem, which is generally very difficult to solve in its original form, we transform the problem into an equivalent convex problem by relaxing the integer variables using the concept of *subcarrier time sharing*, and by applying a successive convex approximation (SCA) approach. Based on the dual decomposition method, we derive an optimal solution to the joint optimization problem. The impact of different network parameters, namely number of subcarriers and number of users, on the attainable EE and spectral efficiency (SE) performance of the proposed design framework is also investigated. The numerical results are provided to validate the theoretical findings and to demonstrate the effectiveness of the proposed algorithm for achieving higher EE and SE than the existing schemes.**

***Index Terms*—Green communications, resource allocation, energy efficiency (EE), multiuser, cooperative relaying.**

I. Introduction

Recently, there has been tremendous growth in the number of mobile users and applications due to the introduction of Android and iPhone devices. Therefore, installation of new base stations (BSs) are required to support such high-data-rate and ubiquitous services which facilitate the people to use social networks, listen to an audio, read books and watch videos [1]. Consequently, the energy consumption significantly increases, particularly at the BSs, which leads to environmental pollution and hazards [2]. Currently, the information and communication technology (ICT) sector consumes about 4.7% of the global energy [3], [4] and it releases approximately 1.7% of the total CO_2 into the atmosphere. Further, the impact of ICT is predicted to be 4 Gt (gigatonnes) of CO_2 by 2030. Hence, improving the energy efficiency (EE) of the communication networks becomes of paramount importance in realizing 5G radio access solutions. Therefore, cellular network demands more attention towards designing energy-aware architectures and resource allocation techniques that not only extend the network's lifespan but also provide significant energy savings under the umbrella of green communications [4].

Cooperative relaying has emerged as a revolutionary technique for greatly improving the EE without deploying new BSs [5]. The two main relaying techniques are the amplify-and-forward (AF) and decode-and-forward (DF) [6]. Although the AF relaying protocol has the advantage of lower implementation complexity, the DF relaying protocol performs better than the AF when the channel quality of the source-to-relay link is good enough. Another advantage of the DF protocol is that the source and the relay nodes may be operated in different channel coding schemes. In this paper, we will focus on DF protocol. In many current systems, a majority of mobile users may be operated in geographically dispersed areas, leading to poor non-line-of-sight (NLOS) performance. Hence, the reliable communication for high data rates between the BS and the destination nodes close to the cell boundary may be not guaranteed. In this scenario, the key concept of cooperative transmission can be used to multiuser cellular wireless networks to assist transmissions between both ends. Relays in such a situation have emerged as a promising approach toward the potential enhancements in channel quality by overcoming a significant loss of signal strength along the propagation path in NLOS environments. Besides, introducing a relay in existing networks is considered to be an attractive technology to extend the network capacity and/or coverage distribution. Thus, the deployment of relays in multiuser cellular wireless networks is currently a promising option for emerging standards such as the IEEE 802.16j mobile multihop relay (MMR) network.

Resource allocation schemes, which play a vital role in the performance optimization of wireless communication networks are usually used for solving either the spectral-efficiency (SE) maximization problem of the network or the power consumption minimization problem under a minimum total

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network throughput/individual user rate constraint [7]–[12]. Vandendorpe *et al.* [7], [8] optimized the power allocation for enhancing the sum rate of DF-relaying orthogonal frequency division multiplexing (OFDM) transmission under sum and individual power constraints. The utility-based dynamic resource allocation algorithm was investigated in [10] for half-duplex (HD) relay-aided OFDM access system in order to maximize the average utility of all users with multiservice, while Sindhu *et al.* [11] jointly optimized power and subcarrier for improving the system throughput in multiuser two-way HD AF relay networks. In [13], the decoding performance of the relays and the destination was analyzed using a Markov chain for a two-path channel and closed-form expressions for outage probability were obtained by a joint consideration of all possibilities over the two hops. The work [13] has been extended in [14] for a cooperative multi-path channel (MPC) and the system outage probability was analyzed and derived in closed-form expressions. In general, the existing research works [7]–[12] have focused on throughput maximization and thus, it cannot deliver the EE maximization solution. However, due to the enormous growth of energy consumption in next-generation wireless communications, the global warming and operation costs are increasing. Therefore, it is required to balance the tradeoff between the SE and EE by allocating the available resources in future wireless communications and to revisit the design of the existing cellular networks from the green communication perspective. Further, it is of utmost importance to investigate the resource allocation algorithms that not only achieve the high system throughput but also improve the system EE.

Energy-efficient resource allocation schemes have been extensively studied in [15]–[21]. A pricing-based power allocation scheme was investigated in [15] for multiuser AF relay network for improving the EE. The average energy efficiency (EE) of multiuser multiantenna cellular networks was investigated in [16] under the hard-core point process (HCPP)-distributed BSs. The beamforming vectors for the source and the relay nodes were designed in [17] for maximizing the network lifetime. The power allocation policy was studied in [18] for an EE-SE tradeoff. The authors in [19] proposed an energy-efficient resource scheduling solution for downlink transmission in multiuser orthogonal frequency-division multiple access (OFDMA) networks under imperfect channel state information (CSI). With a similar goal, Zarakovitis [20] extended the work [19] for multicarrier under perfect CSI knowledge and designed the joint subcarrier and power allocation problem subject to a total power constraint for downlink multiuser OFDMA system. Energy-efficient joint antenna-subcarrier-power allocation scheme was investigated for downlink multiuser OFDM distributed antenna systems (OFDM-DASs) while ensuring the minimum sum rate for each user. However, the previous existing works [15]–[18] only optimized the transmit power for enhancing the EE of the network, whereas the resource allocation problem in [19] and [20] was optimized only in downlink scenario for maximizing EE. As well, most of the above reported works have not considered the joint resource allocation optimization in a dual-hop cooperative network. Besides, all of the existing works have considered one-to-one subcarrier mapping in uplink and downlink phase which limits the system throughput. To the best of the authors' knowledge, a unified resource allocation scheme considering subcarrier pairing permutation obtained in one-to-many/many-to-one way, power optimization and subcarrier allocation altogether for multiuser DF relay network has not been explored from green communication perspective yet.

In light of aforementioned discussions, in this paper, we consider a multiuser multicarrier DF relay network. The relay node operates in a half-duplex DF mode with two transmission phases. In contrast with existing works [7]–[12], wherein the throughput in OFDM relay networks was maximized by optimizing either of the following: $i)$ subcarrier allocation, $ii)$ subcarrier pairing, where the signal received at the relay over one subcarrier is re-transmitted on a different subcarrier, $iii)$ power allocation over different subcarriers at each transmitting node, or $iv)$ power allocation and subcarrier assignment, and the works on energy-efficient resource allocation [15]–[21], the uniqueness of this paper is to develop a general approach for improving the EE, in which subcarrier pairing permutation is performed in one-to-many/many-to-one manner before assigning it to a particular user pair. A unified resource allocation scheme considering subcarrier pairing permutation, power optimization and subcarrier allocation altogether for multiuser DF relay networks has not yet been explored from a green communication perspective. Therefore, this paper proposes joint optimization of subcarrier pairing permutation, subcarrier allocation and power allocation altogether for multiuser multicarrier DF relay networks for improving the EE. The main contributions of this paper are highlighted as follows:

- In contrast to [15]–[21], the EE maximization (EEM) problem in context of a multiuser multicarrier DF relay network subject to limited transmit power, subcarrier pairing permutation[1], and subcarrier allocation constraints, is formulated as a fractional programming problem with the goal of finding the optimal subcarrier pairing, power allocation and subcarrier allocation for improving the EE. The resultant problem is a non-convex mixed-integer nonlinear programming (MINLP) problem [22] and therefore it is in generally intractable. The objective function denoting the EE metric is fractional and nonlinear, which complicates the problem further.
- To make the problem tractable, we apply a successive convex approximation (SCA) method [23] and introduce a variable transformation in addition to relaxing the integer variables using the concept of subcarrier time sharing. Further, we prove that the relaxed problem is quasi-concave. By introducing Dinkelbach method [24], we resolve the fractional objective function and propose an energy-efficient iterative algorithm for finding the optimal solution to the joint optimization problem.

[1]Note: Since a DF protocol is applied at the relay node, the subcarriers can be paired in a one-to-many/many-to-one fashion, i.e., a single subcarrier of multiple access (MA)/uplink phase can pair with a single or many subcarrier of broadcast (BC)/downlink phase and vice-versa. However, each subcarrier pair assigns to only a single user pair.

- Additionally, we employ SE maximization (SEM) algorithm and EEM algorithm without (w/o) subcarrier pairing and subcarrier allocation (SP-SA). Extensive simulation results are used to demonstrate the merits and benefits of the proposed algorithms and to show the effects of many network design choices such as increasing the number of subcarriers, on the EE and SE.

The organization of this paper is as follows. The network architecture and signal models are described in Section II, followed by the problem formulation in Section III. Then, the EE resource allocation algorithm is presented in Section IV. Numerical results are depicted in Section V. Finally, we conclude this study with Section VI.

II. Network Architecture and Preliminaries

In this section, the network architecture and signal models pertaining to the multiuser multicarrier DF relay network system are presented.

A. Network Architecture

Consider a multi-pair relay-assisted network consisting of a single-antenna relay node (R), which works as a BS, and $2M$ single-antenna users, as depicted in Fig 1. Without loss of generality, the M groups are formed by pairing the $2M$ users, where user in each group intends to share information with the help of the relay node. The users in the m^{th} group are represented as $(1, m)$ and $(2, m)$, where $m \in \{1, \ldots, M\}$, respectively. A total number of N_S subcarriers are available in each hop for signal transmission. It is assumed that there is no direct channel between the users in each group due to impairments such as heavy path-loss and shadowing, and the relay has perfect channel state information (CSI) of each link. In practice, the CSIs of the links can be estimated at the relay node by exploiting the channel reciprocity between forward and backward transmissions through orthogonal pilot signals, which are simultaneously sent by multiple source and destination nodes in some dedicated beacon time slots. In general, the CSI estimation could be very accurate if the training period is sufficiently long. Further, the channels are considered as Rayleigh frequency-flat fading in nature. Moreover, the relay network is operated in a half-duplex mode. Hence, the relay node does not simultaneously receive and transmit signals. That is, the relay receives signals from the source nodes during the MA phase, while it transmits the re-encoded signals to the destination nodes during the BC phase. For implementation, there could be two radio-frequency (RF) chains (one for transmitting and the other for receiving) that share the same single antenna alternatively in the two different phases, or each RF chain is associated with a single antenna exclusively.

B. MA/Uplink Phase

Define the channel coefficient from m^{th} uplink user to the relay node R on subcarrier j as $h_{1,m}^{(j)}$. Then, in the uplink phase, the received signal at the relay on the j^{th} subcarrier, for $j \in \{1, \ldots, N_S\}$, can be given by

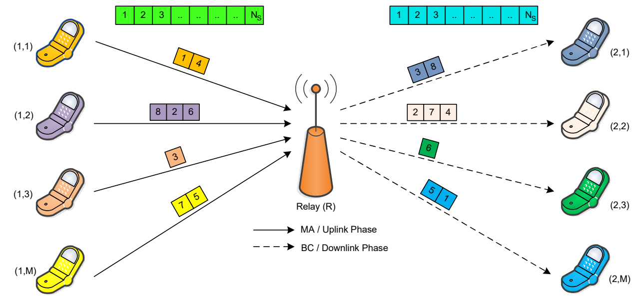


Fig. 1. Structure of multiuser relay-assisted network.

$$y_R^{(j)} = \sum_{m=1}^{M} h_{1,m}^{(j)} \sqrt{P_{1,m}^{(j)}} s_{1,m}^{(j)} + n_R^{(j)} = \mathbf{h}_1^{(j)} \mathbf{P}_1^{(j)} \mathbf{s}_1^{(j)} + n_R^{(j)} , \tag{1}$$

where, $\mathbf{h}_1^{(j)} = \left[h_{1,1}^{(j)}, \ldots, h_{1,M}^{(j)}\right]$, $\mathbf{P}_1^{(j)} = diag\left(\sqrt{P_{1,1}^{(j)}}, \ldots, \sqrt{P_{1,M}^{(j)}}\right)$ and $\mathbf{s}_1^{(j)} = \left[s_{1,1}^{(j)}, \ldots, s_{1,M}^{(j)}\right]^T$. $P_{1,m}^{(j)}$ denotes the transmit power of the m^{th} uplink user on the j^{th} subcarrier, $s_{1,m}^{(j)}$ indicates the data symbol transmitted by the m^{th} uplink user on the j^{th} subcarrier with unit transmission power, i.e., $\mathbb{E}\left[\left|s_{1,m}^{(j)}\right|^2\right] = 1$, and $n_R^{(j)}$ represents the zero-mean complex additive white Gaussian noise (AWGN) with variance $\sigma_R^{(j)^2}$.

Further, the signal-to-interference-plus-noise ratio (SINR) for the uplink phase at the relay node for the m^{th} uplink user on the j^{th} subcarrier, can be given from (1), as follows

$$\Gamma_{1,m}^{(j)} = \frac{P_{1,m}^{(j)} \left|h_{1,m}^{(j)}\right|^2}{\sum\limits_{l=1, l\neq m}^{M} P_{1,l}^{(j)} \left|h_{1,l}^{(j)}\right|^2 + \left(\sigma_R^{(j)}\right)^2} , \tag{2}$$

C. BC/Downlink Phase

In the DL phase, the relay transmits signal to all the downlink users. Since DF protocol is applied at the relay node, the sufficiently good enough uplink channel conditions allow the sophisticated DF relay node to successfully decode the received signal, i.e., $\hat{s}_{1,m}^{(k)} = s_{1,m}^{(k)}$, for $k \in \{1, \ldots, N_S\}, \forall m$. Therefore, the signal received at the m^{th} downlink user on the k^{th} subcarrier can be written as follows

$$\mathbf{y}_{2,m}^{(k)} = h_{2,m}^{(k)} \sqrt{P_{2,m}^{(k)}} \hat{s}_{1,m}^{(k)} + h_{2,m}^{(k)} \sum_{l=1, l\neq m}^{M} \sqrt{P_{2,l}^{(k)}} \hat{s}_{1,l}^{(k)} + n_{2,m}^{(k)} , \tag{3}$$

where, $h_{2,m}^{(k)}$ can be defined in a similar way as $h_{1,m}^{(j)}$ for downlink channel, $P_{2,m}^{(k)}$ represents the transmit power at the relay node for the m^{th} downlink user on the k^{th} subcarrier, and $n_{2,m}^{(k)}$ denotes the complex AWGN for the m^{th} downlink user on the k^{th} subcarrier with $\mathcal{CN}\left(0, \sigma_{2,m}^{(k)^2}\right)$.

From (3), the SINR for the m^{th} downlink user on the k^{th} subcarrier, can be written as

$$\Gamma_{2,m}^{(k)} = \frac{P_{2,m}^{(k)} \left|h_{2,m}^{(k)}\right|^2}{\sum\limits_{l=1,l\neq m}^{M} P_{2,l}^{(k)} \left|h_{2,l}^{(k)}\right|^2 + \left(\sigma_{2,m}^{(k)}\right)^2}, \qquad (4)$$

D. Physical Layer Modelling

On account of (1) and (3), we define $\boldsymbol{\mathcal{P}_1}\left[\mathbf{H}_{1_{N_S\times M}}\right] = \left[P_{1,m}^{(j)}\right]$ and $\boldsymbol{\mathcal{P}_2}\left[\mathbf{H}_{2_{M\times N_S}}\right] = \left[P_{2,m}^{(k)}\right]$, and $\mathbf{H}_{1_{N_S\times M}} = \left[h_{1,m}^{(k)}\right]$ and $\mathbf{H}_{2_{M\times N_S}} = \left[h_{2,m}^{(k)}\right]$ from the physical layer. Furthermore, we represent the subcarrier pairing policy by $\boldsymbol{\mathcal{S}_\mathcal{P}}\left[\mathbf{H}_{1_{N_S\times M}}, \mathbf{H}_{2_{M\times N_S}}\right] = [\Lambda_{j,k}]$, where $\Lambda_{j,k} \in \{0,1\}$ denotes the binary variable for the subcarrier pairing such that $\Lambda_{j,k} = 1$ if the subcarrier j in the uplink phase pairs with k in the downlink phase and $\Lambda_{j,k} = 0$, otherwise. In addition, we define the subcarrier allocation policy by $\boldsymbol{\mathcal{S}_\mathcal{A}}\left[\mathbf{H}_{1_{N_S\times M}}, \mathbf{H}_{2_{M\times N_S}}\right] = \left[\Omega_{m,(j,k)}\right]$, with $\Omega_{m,(j,k)} \in \{0,1\}$ the binary subcarrier allocation index, i.e., $\Omega_{m,(j,k)} = 1$ when the paired subcarrier (j,k) is allocated to the m^{th} uplink-downlink user pair and $\Omega_{m,(j,k)} = 0$, otherwise.

Since a DF protocol is applied at the relay node, we need to ensure that the aggregate communication rates of the two hops for each user pair are balanced. Due to one-to-many/many-to-one mapping of subcarriers, it is hard to balance the sum rate of both hops for each user pair. A time division approach is adopted for signal transmission over paired subcarriers. If the j^{th} subcarrier in the uplink phase is paired with a set of subcarriers $\mathcal{K}_{m,j}$, $\mathcal{K}_{m,j} \subseteq N_S$, in the downlink phase and assigned to the m^{th} user pair, then the m^{th} user transmits its signals in the first hop using j^{th} subcarrier with time slot $1/\left|\mathcal{K}_{m,j}\right|$, where $\left|\mathcal{K}_{m,j}\right|$ denotes the number of subcarriers in a set $\mathcal{K}_{m,j}$. Similarly, if a set of subcarriers $\mathcal{J}_{m,k}$, $\mathcal{J}_{m,k} \subseteq N_S$, in the uplink phase is paired with the k^{th} subcarrier in the downlink phase and assigned to m^{th} user pair, then the m^{th} user pair receives the signals on the k^{th} subcarrier with the time slot $1/\left|\mathcal{J}_{m,k}\right|$, where $\left|\mathcal{J}_{m,k}\right|$ is the number of subcarriers in a set $\mathcal{J}_{m,k}$. Thus, the average achievable sum-rate for the m^{th} user pair can be defined using (2) and (4) as follows

$$\mathcal{R}_m\left(\boldsymbol{\mathcal{P}_1}, \boldsymbol{\mathcal{P}_2}, \boldsymbol{\mathcal{S}_\mathcal{P}}, \boldsymbol{\mathcal{S}_\mathcal{A}}\right) = \qquad (5)$$
$$\frac{1}{2}\sum\nolimits_{j=1}^{N_S}\sum\nolimits_{k=1}^{N_S} \Lambda_{j,k}\Omega_{m,(j,k)} \min\left\{\begin{array}{l} \frac{1}{\left|\mathcal{K}_{m,j}\right|}\log_2\left(1+\Gamma_{1,m}^{(j)}\right), \\ \frac{1}{\left|\mathcal{J}_{m,k}\right|}\log_2\left(1+\Gamma_{2,m}^{(k)}\right) \end{array}\right\},$$

where $1/2$ comes from the fact that transmission takes place in two phases. Moreover, the total achievable throughput is $\mathcal{R}_T\left(\boldsymbol{\mathcal{P}_1}, \boldsymbol{\mathcal{P}_2}, \boldsymbol{\mathcal{S}_\mathcal{P}}, \boldsymbol{\mathcal{S}_\mathcal{A}}\right) = \sum\limits_{m=1}^{M} \mathcal{R}_m\left(\boldsymbol{\mathcal{P}_1}, \boldsymbol{\mathcal{P}_2}, \boldsymbol{\mathcal{S}_\mathcal{P}}, \boldsymbol{\mathcal{S}_\mathcal{A}}\right)$. Before optimizing power allocation, subcarrier pairing and subcarrier allocation from green communications perspective, we define the EE as follows:

Definition 1: EE is defined as the ratio of instantaneous maximum achievable throughput over the corresponding power consumption of the network, given by

$$\eta_{EE}\left(\boldsymbol{\mathcal{P}_1}, \boldsymbol{\mathcal{P}_2}, \boldsymbol{\mathcal{S}_\mathcal{P}}, \boldsymbol{\mathcal{S}_\mathcal{A}}\right) = \qquad (6)$$
$$\frac{\mathcal{R}_T\left(\boldsymbol{\mathcal{P}_1}, \boldsymbol{\mathcal{P}_2}, \boldsymbol{\mathcal{S}_\mathcal{P}}, \boldsymbol{\mathcal{S}_\mathcal{A}}\right)}{\sum\limits_{m=1}^{M}\sum\limits_{j=1}^{N_S}\sum\limits_{k=1}^{N_S} \Lambda_{j,k}\Omega_{m,(j,k)}\left(P_{1,m}^{(j)} + P_{2,m}^{(k)}\right) + 2MP^C + \xi_R P^C},$$

where $\xi_R > 1$. The power consumption model is formulated in a linear fashion, where P^C denotes the circuit and processing power of each user. Because of large signaling processing at the relay node, the static power consumption at the relay node must be higher than the user terminal, and thus the value of static power consumption for the relay node is considered to be $\xi_R P^C$.

Remark 1: The EE can be reckoned as the number of data bits successfully delivered to the $(2,m)^{th}$ downlink user by $(1,m)^{th}$ uplink user via the relay node (in *bits per Joule*) and it is equivalent to the throughput weighted by the inverse of the sum of transmitting and processing powers, respectively.

III. Resource Allocation Problem Formulation

In this section, we formulate the optimization problem that maximizes the network EE; and provide insights on its convexity, accompanied by transformations to result into its convex-form.

A. Primal EE Maximization (EEM) Problem

In this subsection, we formulate the primal optimization problem that intends to maximize the networks EE by optimizing the allocation policies for power and subcarrier, respectively. Further, the problem targets to remain bounded by the network's power regulation while simultaneously satisfying the suppression of channel interferences. Unlike the previous works [15]–[21], the performance of multiuser DF relay network can be significantly improved by performing the subcarrier pairing permutation in one-to-many/many-to-one fashion, and by jointly optimizing the subcarrier and power allocation. Thus, the optimization problem can be formulated as

(OP1) *To obtain:*	$\boldsymbol{\mathcal{P}_1^\star}\left[\mathbf{H}_{1_{N_S\times M}}\right], \boldsymbol{\mathcal{P}_2^\star}\left[\mathbf{H}_{2_{M\times N_S}}\right]$, $\boldsymbol{\mathcal{S}_\mathcal{P}^\star}\left[\mathbf{H}_{1_{N_S\times M}}, \mathbf{H}_{2_{M\times N_S}}\right], \boldsymbol{\mathcal{S}_\mathcal{A}^\star}\left[\mathbf{H}_{1_{N_S\times M}}, \mathbf{H}_{2_{M\times N_S}}\right]$
such that:	$\max\limits_{\boldsymbol{\mathcal{P}_1}, \boldsymbol{\mathcal{P}_2}, \boldsymbol{\mathcal{S}_\mathcal{P}}, \boldsymbol{\mathcal{S}_\mathcal{A}}} \quad \eta_{EE}\left(\boldsymbol{\mathcal{P}_1}, \boldsymbol{\mathcal{P}_2}, \boldsymbol{\mathcal{S}_\mathcal{P}}, \boldsymbol{\mathcal{S}_\mathcal{A}}\right)$
subject to:	$(C.1)\ \sum\limits_{m=1}^{M}\sum\limits_{j=1}^{N_S}\sum\limits_{k=1}^{N_S}\Lambda_{j,k}\Omega_{m,(j,k)}\left(P_{1,m}^{(j)}+P_{2,m}^{(k)}\right)\leqslant P_{max};$ $(C.2)\ 1 \leqslant \sum\limits_{j=1}^{N_S}\Lambda_{j,k} \leqslant N_S, \quad \forall k;$ $(C.3)\ 1 \leqslant \sum\limits_{k=1}^{N_S}\Lambda_{j,k} \leqslant N_S, \quad \forall j;$ $(C.4)\ \sum\limits_{m=1}^{M}\Omega_{m,(j,k)} = 1, \quad \forall (j,k);$ $(C.5)\ \Lambda_{j,k} \in \{0,1\},\ \Omega_{m,(j,k)} \in \{0,1\}, \quad \forall m,j,k;$ $(C.6)\ P_{1,m}^{(j)} \geqslant 0,\ P_{2,m}^{(k)} \geqslant 0, \quad \forall m,j,k,$

where P_{max} is the maximum available transmit power budget for the network. Further, constraint $(C.1)$ bounds the network's transmit power regulation, $(C.2)$ and $(C.3)$ mandates that each subcarrier in the uplink phase can be paired with any number of subcarriers in the downlink phase and vice-versa, and $(C.4)$ ensures each paired subcarrier is allocated to one and only one user pair. In the proposed framework, each user and the relay node are equipped with a single antenna. We can easily extend the design framework to the scenario with multiple antennas. In this scenario, the channels of the two hops convert into MIMO channels. Therefore, the SINRs can be computed similar to (2) and (4) after skillfully designing beamforming weights. Practically, the higher number of antennas can provide better interference suppression ability to the systems, however, it also increases the higher computational processing. In result, it requires more static power consumption, which leads to an EE performance tradeoff.

Remark 2: The optimization problem (**OP1**) is an amalgamation problem because $\left\{P_{1,m}^{(j)}, P_{2,m}^{(k)}\right\}$ are continuous variables, whereas, $\left\{\Lambda_{j,k}, \Omega_{m,(j,k)}\right\}$ are discrete (binary) variables. The solution to such a problem is to carry through an *exhaustive search* (ES) over all the user pairs and subcarriers, such that while maximizing the EE for each subcarrier permutation and allocation, power is allocated. Despite the fact, the optimal solution obtained via ES is the actual optimal solution to the primal problem (**OP1**), it is evident that, ES is inapplicable in real-time scenario due to its exorbitant complexity of the order $\mathcal{O}\left(M^{N_S!}\right)$.

B. Transforming the Problem into Convex Form

The optimization problem (**OP1**) is a non-convex MINLP problem in nature, and thus there is no standard technique to solve such optimization problem. As first step in transforming the non-convex MINLP problem into convex one, we introduce the auxiliary variable $\Upsilon_m^{(n)}$, $n = j = k = 1, \ldots, N_S$, and reformulate the equivalent optimization problem as follows:

(OP2) *To obtain:*	$\boldsymbol{\mathcal{P}}_\mathbf{1}^\star\left[\mathbf{H}_{1_{N_S\times M}}\right], \boldsymbol{\mathcal{P}}_\mathbf{2}^\star\left[\mathbf{H}_{2_{M\times N_S}}\right]$, $\boldsymbol{\mathcal{S}}_{\boldsymbol{\mathcal{P}}}^\star\left[\mathbf{H}_{1_{N_S\times M}}, \mathbf{H}_{2_{M\times N_S}}\right], \boldsymbol{\mathcal{S}}_{\boldsymbol{\mathcal{A}}}^\star\left[\mathbf{H}_{1_{N_S\times M}}, \mathbf{H}_{2_{M\times N_S}}\right]$
such that:	$\max\limits_{\substack{\boldsymbol{\mathcal{P}}_\mathbf{1},\boldsymbol{\mathcal{P}}_\mathbf{2}\\ \boldsymbol{\mathcal{S}}_{\boldsymbol{\mathcal{P}}},\boldsymbol{\mathcal{S}}_{\boldsymbol{\mathcal{A}}},\boldsymbol{\Upsilon}}} \hat{\eta}_{EE}\left(\boldsymbol{\mathcal{P}}_\mathbf{1}, \boldsymbol{\mathcal{P}}_\mathbf{2}, \boldsymbol{\mathcal{S}}_{\boldsymbol{\mathcal{P}}}, \boldsymbol{\mathcal{S}}_{\boldsymbol{\mathcal{A}}}\right)$ (bits/Joule/Hz)
subject to:	$(C.1) - (C.6)$; $(C.7)\ \frac{1}{\lvert\mathcal{K}_{m,n}\rvert}\log_2\left(1+\Gamma_{1,m}^{(n)}\right) \geqslant \Upsilon_m^{(n)},\ \forall m,n;$ $(C.8)\ \frac{1}{\lvert\mathcal{J}_{m,n}\rvert}\log_2\left(1+\Gamma_{2,m}^{(n)}\right) \geqslant \Upsilon_m^{(n)},\ \forall m,n.$

where $\boldsymbol{\Upsilon} = \left\{\Upsilon_m^{(n)}\right\}$, $\forall m, n$ and $\hat{\eta}_{EE}\left(\boldsymbol{\mathcal{P}}_\mathbf{1}, \boldsymbol{\mathcal{P}}_\mathbf{2}, \boldsymbol{\mathcal{S}}_{\boldsymbol{\mathcal{P}}}, \boldsymbol{\mathcal{S}}_{\boldsymbol{\mathcal{A}}}\right)$ is defined as

$$\hat{\eta}_{EE}\left(\boldsymbol{\mathcal{P}}_\mathbf{1}, \boldsymbol{\mathcal{P}}_\mathbf{2}, \boldsymbol{\mathcal{S}}_{\boldsymbol{\mathcal{P}}}, \boldsymbol{\mathcal{S}}_{\boldsymbol{\mathcal{A}}}\right) = \frac{\frac{1}{2}\sum\limits_{m=1}^{M}\sum\limits_{j=1,n=j}^{N_S}\sum\limits_{k=1}^{N_S}\Lambda_{j,k}\Omega_{m,(j,k)}\Upsilon_m^{(n)}}{\sum\limits_{m=1}^{M}\sum\limits_{j=1}^{N_S}\sum\limits_{k=1}^{N_S}\Lambda_{j,k}\Omega_{m,(j,k)}\left(P_{1,m}^{(j)}+P_{2,m}^{(k)}\right)+2MP^C+\xi_R P^C}, \tag{7}$$

The problem (**OP2**) is still non-convex. To make it more tractable, we introduce the continuous variables $\left\{\tilde{\Lambda}_{j,k}, \tilde{\Omega}_{m,(j,k)}\right\} \in [0,1]$. Physically, meaning that a subcarrier can now be shared among multiple users, instead of just one, however the power allocated to each subcarrier can control the interference produced by sharing of a subcarrier by multiple users. Furthermore, we also apply change of variables such that $\tilde{P}_{1,m}^{(j)} = \ln P_{1,m}^{(j)}$ and $\tilde{P}_{2,m}^{(k)} = \ln P_{2,m}^{(k)}$. The optimization problem (**OP2**) can be equivalently rewritten as

(OP3) *To obtain:*	$\tilde{\boldsymbol{\mathcal{P}}}_\mathbf{1}^\star\left[\mathbf{H}_{1_{N_S\times M}}\right], \tilde{\boldsymbol{\mathcal{P}}}_\mathbf{2}^\star\left[\mathbf{H}_{2_{M\times N_S}}\right]$, $\tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{P}}}^\star\left[\mathbf{H}_{1_{N_S\times M}}, \mathbf{H}_{2_{M\times N_S}}\right], \tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{A}}}^\star\left[\mathbf{H}_{1_{N_S\times M}}, \mathbf{H}_{2_{M\times N_S}}\right]$
such that:	$\max\limits_{\substack{\tilde{\boldsymbol{\mathcal{P}}}_\mathbf{1},\tilde{\boldsymbol{\mathcal{P}}}_\mathbf{2}\\ \tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{P}}},\tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{A}}},\boldsymbol{\Upsilon}}} \frac{\frac{1}{2}\sum\limits_{m=1}^{M}\sum\limits_{j=1}^{N_S}\sum\limits_{k=1}^{N_S}\tilde{\Lambda}_{j,k}\tilde{\Omega}_{m,(j,k)}\Upsilon_m^{(j)}}{\tilde{P}_{total}}$ (bits/Joule/Hz)
subject to:	$(C.1)\ \sum\limits_{m=1}^{M}\sum\limits_{j=1}^{N_S}\sum\limits_{k=1}^{N_S}\Lambda_{j,k}\Omega_{m,(j,k)}\left(e^{\tilde{P}_{1,m}^{(j)}}+e^{\tilde{P}_{2,m}^{(k)}}\right) \leqslant P_{Total};$ $(C.2)\ 1 \leqslant \sum\limits_{j=1}^{N_S}\tilde{\Lambda}_{j,k} \leqslant N_S,\quad \forall k;$ $(C.3)\ 1 \leqslant \sum\limits_{k=1}^{N_S}\tilde{\Lambda}_{j,k} \leqslant N_S,\quad \forall j;$ $(C.4)\ \sum\limits_{m=1}^{M}\tilde{\Omega}_{m,(j,k)} = 1,\quad \forall (j,k);$ $(C.5)\ \tilde{\Lambda}_{j,k} \in [0,1],\ \tilde{\Omega}_{m,(j,k)} \in [0,1],\quad \forall m,j,k;$ $(C.6)\ e^{\tilde{P}_{1,m}^{(j)}} \geqslant 0,\ e^{\tilde{P}_{2,m}^{(k)}} \geqslant 0,\quad \forall m,j,k;$ $(C.7)\ \frac{1}{\lvert\mathcal{K}_{m,n}\rvert}\log_2\left(1+\tilde{\Gamma}_{1,m}^{(n)}\right) \geqslant \Upsilon_m^{(n)},\quad \forall m,n;$ $(C.8)\ \frac{1}{\lvert\mathcal{J}_{m,n}\rvert}\log_2\left(1+\tilde{\Gamma}_{2,m}^{(n)}\right) \geqslant \Upsilon_m^{(n)},\quad \forall m,n.$

where $\tilde{P}_{total} = \sum\limits_{m=1}^{M}\sum\limits_{j=1}^{N_S}\sum\limits_{k=1}^{N_S}\tilde{\Lambda}_{j,k}\tilde{\Omega}_{m,(j,k)}\left(e^{\tilde{P}_{1,m}^{(j)}}+e^{\tilde{P}_{2,m}^{(k)}}\right)+ 2MP^C + \xi_R P^C$, $\tilde{\Gamma}_{1,m}^{(n)} = \frac{e^{\tilde{P}_{1,m}^{(n)}}\left|h_{1,m}^{(n)}\right|^2}{\sum\limits_{l=1,l\neq m}^{M} e^{\tilde{P}_{1,l}^{(n)}}\left|h_{1,l}^{(n)}\right|^2+\left(\sigma_R^{(n)}\right)^2}$ and $\tilde{\Gamma}_{2,m}^{(n)} = \frac{e^{\tilde{P}_{2,m}^{(n)}}\left|h_{2,m}^{(n)}\right|^2}{\sum\limits_{l=1,l\neq m}^{M} e^{\tilde{P}_{2,l}^{(n)}}\left|h_{2,l}^{(n)}\right|^2+\left(\sigma_{2,m}^{(n)}\right)^2}$. The problem still remains non-convex due the fractional form of the objective function and the constraints $(C.7)$ and $(C.8)$ in (**OP3**). Therefore, using SCA technique [22], we transform the constraints $(C.7)$ and $(C.8)$ into convex. Further, we convert the objective function into a subtractive form using a standard techniques like Dinkelbach's method [24]. The two step approach is mathematically shown as below:

Step 1: Applying SCA technique and imposing a lower bound as

$$\frac{1}{\lvert\mathcal{K}_{m,n}\rvert}\log_2\left(1+\tilde{\Gamma}_{1,m}^{(n)}\right) \geqslant \frac{1}{\lvert\mathcal{K}_{m,n}\rvert}\left(\frac{\alpha_{1,m}^{(n)}}{\ln(2)}\log\tilde{\Gamma}_{1,m}^{(n)}+\beta_{1,m}^{(n)}\right); \tag{8}$$

where $\alpha_{1,m}^{(n)}$ and $\beta_{1,m}^{(n)}$, $\forall m,n$, are the coefficients determined in the following way:

$$\alpha_{1,m}^{(n)} = \frac{\xi_{1,m}^{(n)}}{1+\xi_{1,m}^{(n)}}\,; \tag{9}$$

$$\beta_{1,m}^{(n)} = \log_2\left(1+\xi_{1,m}^{(n)}\right) - \alpha_{1,m}^{(n)}\log_2\left(\xi_{1,m}^{(n)}\right)\,, \tag{10}$$

for any $\xi_{1,m}^{(n)} > 0$. Further, equality in (8) is satisfied when $\alpha_{1,m}^{(n)} = \tilde{\Gamma}_{1,m}^{(n)}/\left(1+\tilde{\Gamma}_{1,m}^{(n)}\right)$ and $\beta_{1,m}^{(n)} = \log_2\left(1+\tilde{\Gamma}_{1,m}^{(n)}\right) - \alpha_{1,m}^{(n)}\log_2\left(\tilde{\Gamma}_{1,m}^{(n)}\right)$; and the equality holds *iff* $\left(\alpha_{1,m}^{(n)},\beta_{1,m}^{(n)}\right) = (1,0)$ and $\tilde{\Gamma}_{1,m}^{(n)} \to \infty$. In a similar fashion, we can also find the lower bound on $\frac{1}{|\mathcal{J}_{m,n}|}\log_2\left(1+\tilde{\Gamma}_{2,m}^{(n)}\right) \geqslant \frac{1}{|\mathcal{J}_{m,n}|}\left(\frac{\alpha_{2,m}^{(n)}}{\ln(2)}\log\tilde{\Gamma}_{2,m}^{(n)}+\beta_{2,m}^{(n)}\right)$ and update the coefficients $\alpha_{2,m}^{(n)}$ and $\beta_{2,m}^{(n)}$, $\forall m,n$.

Step 2: Applying Dinkelbach's Method [24]

$$\mathcal{F}\left(\tilde{\boldsymbol{P}}_{\mathbf{1}},\tilde{\boldsymbol{P}}_{\mathbf{2}},\tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{P}}},\tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{A}}},\boldsymbol{\Upsilon}\right)=\frac{1}{2}\sum_{m=1}^{M}\sum_{j=1}^{N_S}\sum_{k=1}^{N_S}\tilde{\Lambda}_{j,k}\tilde{\Omega}_{m,(j,k)}\Upsilon_m^{(j)} - \Psi\left(\sum_{m=1}^{M}\sum_{j=1}^{N_S}\sum_{k=1}^{N_S}\tilde{\Lambda}_{j,k}\tilde{\Omega}_{m,(j,k)}\left(e^{\tilde{P}_{1,m}^{(j)}}+e^{\tilde{P}_{2,m}^{(k)}}\right) + 2MP^C+\xi_R P^C\right), \tag{11}$$

where Ψ is a non-negative parameter that works as a network penalty or price paid for the resource allocation policies.

Remark 3: When $\Psi \to 0$, this directly implies the price/penalty[2] paid for the resource allocation is zero, and the problem degenerates to a sum-rate maximization problem, whereas when $\Psi\to\infty$, no resource allocation policy is good enough to maximize the networks EE.

Using (8) and (11), we can transform the optimization problem (**OP3**) into the following form

(OP4) *To obtain:*	$\tilde{\boldsymbol{\mathcal{P}}}_{\mathbf{1}}^{\star}\left[\mathbf{H}_{1_{N_S\times M}}\right], \tilde{\boldsymbol{\mathcal{P}}}_{\mathbf{2}}^{\star}\left[\mathbf{H}_{2_{M\times N_S}}\right]$, $\tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{P}}}^{\star}\left[\mathbf{H}_{1_{N_S\times M}},\mathbf{H}_{2_{M\times N_S}}\right], \tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{A}}}^{\star}\left[\mathbf{H}_{1_{N_S\times M}},\mathbf{H}_{2_{M\times N_S}}\right]$
such that:	$\max\limits_{\substack{\tilde{\boldsymbol{P}}_{\mathbf{1}},\tilde{\boldsymbol{P}}_{\mathbf{2}},\\ \tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{P}}},\tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{A}}},\boldsymbol{\Upsilon}}} \mathcal{F}\left(\tilde{\boldsymbol{P}}_{\mathbf{1}},\tilde{\boldsymbol{P}}_{\mathbf{2}},\tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{P}}},\tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{A}}},\boldsymbol{\Upsilon}\right)$ (bits/Joule/Hz)
subject to:	$(C.1)-(C.6)$; $(C.7)\ \frac{1}{|\mathcal{K}_{m,n}|}\left(\frac{\alpha_{1,m}^{(n)}}{\ln(2)}\log\tilde{\Gamma}_{1,m}^{(n)}+\beta_{1,m}^{(n)}\right)\geqslant\Upsilon_m^{(n)},\ \forall m,n;$ $(C.8)\ \frac{1}{|\mathcal{J}_{m,n}|}\left(\frac{\alpha_{2,m}^{(n)}}{\ln(2)}\log\tilde{\Gamma}_{2,m}^{(n)}+\beta_{2,m}^{(n)}\right)\geqslant\Upsilon_m^{(n)},\ \forall m,n.$

The problem (**OP4**) is convex if the coefficients $(\boldsymbol{\alpha},\boldsymbol{\beta})=\left\{\left(\alpha_{1,m}^{(n)},\beta_{1,m}^{(n)}\right),\left(\alpha_{2,m}^{(n)},\beta_{2,m}^{(n)}\right)\right\}$ and the subcarrier pairing and allocation variables $\tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{P}}}$ and $\tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{A}}}$ and the network price Ψ are given. The concavification of the transformed problem (**OP4**) can be ascertained by the following lemma.

[2]In this paper, the terms "penalty" and "price" are used interchangeably.

Lemma 1: For any fixed subcarrier permutation and allocation policies $\left(\tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{P}}}^{\star}\left[\mathbf{H}_{1_{N_S\times M}},\mathbf{H}_{2_{M\times N_S}}\right],\tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{A}}}^{\star}\left[\mathbf{H}_{1_{N_S\times M}},\mathbf{H}_{2_{M\times N_S}}\right]\right)$, the optimization problem (**OP4**) is concavified for given fixed coefficients $\left(\alpha_{1,m}^{(n)},\beta_{1,m}^{(n)}\right)$, $\left(\alpha_{2,m}^{(n)},\beta_{2,m}^{(n)}\right)$ and price Ψ.

Proof: After applying change of variables in the optimization problem (**OP2**), the objective function obtained in (**OP4**) can be rewritten as

$$\mathcal{F}\left(\tilde{\boldsymbol{P}}_{\mathbf{1}},\tilde{\boldsymbol{P}}_{\mathbf{2}},\tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{P}}},\tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{A}}},\boldsymbol{\Upsilon}\right)=\frac{1}{2}\sum_{m=1}^{M}\sum_{j=1}^{N_S}\sum_{k=1}^{N_S}\tilde{\Lambda}_{j,k}\tilde{\Omega}_{m,(j,k)}\Upsilon_m^{(j)} - \Psi\left(\sum_{m=1}^{M}\sum_{j=1}^{N_S}\sum_{k=1}^{N_S}\tilde{\Lambda}_{j,k}\tilde{\Omega}_{m,(j,k)}\left(e^{\tilde{P}_{1,m}^{(j)}}+e^{\tilde{P}_{2,m}^{(k)}}\right) + 2MP^C+\xi_R P^C\right). \tag{12}$$

Since $\alpha_{1,m}^{(n)}\geqslant 0$, $\beta_{1,m}^{(n)}\geqslant 0$, $\alpha_{2,m}^{(n)}\geqslant 0$, $\beta_{2,m}^{(n)}\geqslant 0$ and $\Psi\geqslant 0$, the objective function obtained in (12) and the constraints $(C.1)$, $(C.7)$ and $(C.8)$ are actually an expression derived by the summation of linear terms and concave terms. Thus, establishing its concavity. ■

IV. Energy-Efficient Resource Allocation Algorithm

The optimization problem (**OP4**) remains NP-hard to solve if the subcarrier pairing and allocation variables $\tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{P}}}$ and $\tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{A}}}$ are not given. The NP-hardness of (**OP4**) can be proven by performing polynomial reduction from Subset Sum (SS), that remains a widely celebrated NP-hard problem, defined as follows:

Definition 1 (Subset Sum (SS)): Given a set of natural numbers $\mathcal{W}=\{w_1,w_2,...\}$ and a positive integer $V<\sum\limits_{w_j\in\mathcal{W}} w_j$, does there exist a subset $\mathcal{N}\in\mathcal{W}$, where $\mathcal{N}$ denotes the set of N_S subcarriers, such that $\sum\mathcal{N}=V$?
The answer to the SS is objective either YES (SS instance satisfiable) or NO (SS instance not satisfiable), respectively.

Decision version[3] *of EEM problem* - For a given set of a subcarrier combinations and b non-decreasing rate power functions, and $a>b$, does there exist a set of subcarrier and user permutations with total transmit power constraint, such that every user's request is satisfied and no subcarrier is allocated to more than a single user pair?

Theorem 1: The optimization problem (**OP4**) is NP-hard to solve.

Proof: See Appendix A. ■

Due to the well-established fact that when the number of subcarriers goes to infinity, the duality gap between the primal and dual problem in a multicarrier system approaches zero [25]. Therefore, we aim to resolve (**OP4**) via solving its dual problem. To elaborate on this, the following definition and theorem are given.

[3]Since the hardness in terms of the computational complexity of optimization and decision versions are same, although the answers are different, we refer to the decision version.

Definition 2 (Duality Gap): The duality gap may be defined as the difference in the optimal solution obtained via the optimization problem (**OP4**) and the dual problem (described as (**DP1**) later in this paper).

Theorem 2: The duality gap between the primal and dual problem tends to be zero, for a sufficiently large number of subcarriers N_S.

Proof: The proof is provided in Appendix B. ■

Henceforth, we focus on solving the dual problem to jointly obtain the optimal solution.

A. Dual Problem Formulation

For fixed coefficients $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$, the Lagrangian function for the transformed optimization problem (**OP4**) is given by (13) at the top of the next page, where $\lambda, \boldsymbol{\mu} = \{\mu_m^{(n)}\}$ and $\boldsymbol{\nu} = \{\nu_m^{(n)}\}$, $\forall\, m, n$ are the Lagrangian multipliers associated with the constraints $(C.1)\,, (C.7)$ and $(C.8)$, respectively. Therefore, the Lagrangian dual function can be written as

(DP1) *To obtain:*	$\tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{A}}}^{\star}\left[\mathbf{H}_{1_{N_S\times M}}, \mathbf{H}_{2_{M\times N_S}}\right], \tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{P}}}^{\star}\left[\mathbf{H}_{1_{N_S\times M}}, \mathbf{H}_{2_{M\times N_S}}\right]$
such that:	$\max\limits_{\tilde{\boldsymbol{P}}_1, \tilde{\boldsymbol{P}}_2, \tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{P}}}, \tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{A}}}, \boldsymbol{\Upsilon}} \quad \mathcal{L}\left(\tilde{\boldsymbol{P}}_1, \tilde{\boldsymbol{P}}_2, \tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{P}}}, \tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{A}}}, \boldsymbol{\Upsilon}, \lambda, \boldsymbol{\mu}, \boldsymbol{\nu}\right)$
subject to:	$(C.2) - (C.5)\,.$

Hence, the dual optimization problem (**DP2**) can be formulated as

(DP2) *To obtain:*	$\tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{A}}}^{\star}\left[\mathbf{H}_{1_{N_S\times M}}, \mathbf{H}_{2_{M\times N_S}}\right], \tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{P}}}^{\star}\left[\mathbf{H}_{1_{N_S\times M}}, \mathbf{H}_{2_{M\times N_S}}\right]$
such that:	$\min\limits_{\lambda, \boldsymbol{\mu}, \boldsymbol{\nu}} \max\limits_{\substack{\tilde{\boldsymbol{P}}_1, \tilde{\boldsymbol{P}}_2, \\ \tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{P}}}, \tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{A}}}, \boldsymbol{\Upsilon}}} \mathcal{L}\left(\tilde{\boldsymbol{P}}_1, \tilde{\boldsymbol{P}}_2, \tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{P}}}, \tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{A}}}, \boldsymbol{\Upsilon}, \lambda, \boldsymbol{\mu}, \boldsymbol{\nu}\right)$
subject to:	$(C.2) - (C.5)\,.$

The dual problem (**DP2**) can be resolved in an iterative fashion by decomposing the problem into two parts, firstly a master problem that aims to update the Lagrangian multipliers and secondly, the subproblem that helps in attaining the optimal resource allocation policy.

B. Subproblem Solution

The subproblem solution can be obtained by resolving two individual problems, where the first one aims to obtain the optimal power allocation policy, whereas the second deals with the subcarrier permutation and allocation policies[4].

1) Optimal Power Allocation Policies $\left(\tilde{\boldsymbol{\mathcal{P}}}_1^{\star}, \tilde{\boldsymbol{\mathcal{P}}}_2^{\star}\right)$*:* For fixed Lagrangian multipliers, we can obtain the optimal power allocation policy by using the standard Karush-Kuhn-Tucker (KKT) conditions, which are first-order indispensable and competent conditions for optimality, stating that the gradient is equal to zero at the optimal points, henceforth, for given subcarrier pairing and allocation policy, i.e. $\{\Lambda_{j,k}, \Omega_{m,(j,k)}\} = 1$, we can write the power update equations for the user and the relay node at the $(t+1)^{th}$ iteration as follows:

$$\tilde{P}_{1,m}^{(j)}(t+1) = \left[\ln \frac{\mu_m^{(j)} \alpha_{1,m}^{(j)}}{(\Psi + \lambda) \left|\mathcal{K}_{m,j}\right| \ln(2)}\right]^+ \quad ; \qquad (14)$$

$$\tilde{P}_{2,m}^{(k)}(t+1) = \left[\ln \frac{\nu_m^{(j)} \alpha_{2,m}^{(j)}}{(\Psi + \lambda) \left|\mathcal{J}_{m,k}\right| \ln(2)}\right]^+ \quad ; \qquad (15)$$

where $[x]^+ = \max\{0, x\}$. The auxiliary variable $\tilde{\Upsilon}_m^{(j)}$ can be updated using subgradient method [22] as follows:

$$\tilde{\Upsilon}_m^{(j)}(t+1) = \left[\tilde{\Upsilon}_m^{(j)}(t) - \omega(t) N_S \left(\frac{1}{2} - \left(\mu_m^{(n)} + \nu_m^{(n)}\right)\right)\right]^+ , \qquad (16)$$

where $\omega(t)$ is the diminishing step size at the t^{th} iteration.

Remark 4: It is to be noted that a remodelled water-filling solutions are obtained for the optimal power allocation policies in (14) and (15) that not only depends on the Lagrangian multiplier λ associated with the total transmit power, but also on the prevailing penalty Ψ paid for the total network's resource utilization. The submission of λ and Ψ can be treated as a water-filling level which has to be adjusted in order to meet the total transmit power constraint $(C.1)$.

2) Optimal Subcarrier Allocation and Pairing Policies $\left(\{\tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{P}}}^{\star}, \tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{A}}}^{\star}\}\left[\mathbf{H}_{1_{N_S\times M}}, \mathbf{H}_{2_{M\times N_S}}\right]\right)$*:* For given optimal power allocation policies $\left(\tilde{\boldsymbol{\mathcal{P}}}_1^{\star}\left[\mathbf{H}_{1_{N_S\times M}}\right], \tilde{\boldsymbol{\mathcal{P}}}_2^{\star}\left[\mathbf{H}_{2_{M\times N_S}}\right]\right)$, the dual problem (**DP2**) can be transformed as follows:

(DP3) *To obtain:*	$\tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{A}}}^{\star}\left[\mathbf{H}_{1_{N_S\times M}}, \mathbf{H}_{2_{M\times N_S}}\right], \tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{P}}}^{\star}\left[\mathbf{H}_{1_{N_S\times M}}, \mathbf{H}_{2_{M\times N_S}}\right]$
such that:	$\max\limits_{\tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{P}}}, \tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{A}}}} \sum\limits_{m=1}^{M}\sum\limits_{j=1}^{N_S}\sum\limits_{k=1}^{N_S} \tilde{\Lambda}_{j,k}\tilde{\Omega}_{m,(j,k)}\mathcal{A}_{m,(j,k)} + \mathcal{K}\left(\tilde{\boldsymbol{\mathcal{P}}}_1^{\star}, \tilde{\boldsymbol{\mathcal{P}}}_2^{\star}, \tilde{\boldsymbol{\Upsilon}}^{\star}, \lambda, \boldsymbol{\mu}, \boldsymbol{\nu}\right)$
subject to:	$(C.2) - (C.5)\,,$

where $\mathcal{A}_{m,(j,k)}$ and $\mathcal{K}\left(\tilde{\boldsymbol{\mathcal{P}}}_1^{\star}, \tilde{\boldsymbol{\mathcal{P}}}_2^{\star}, \tilde{\boldsymbol{\Upsilon}}^{\star}, \lambda, \boldsymbol{\mu}, \boldsymbol{\nu}\right)$ are defined in (17) and (18) shown on the following page, It is evident that the maximization of EE only depends on (17) for a given power allocation policy, whereas (18) remains constant. Furthermore, $\mathcal{A}_{m,(j,k)}$ can be summarised as the sum of two terms, firstly, the sum-rate achieved, and secondly, the penalty paid for the maximum achievable EE.

Remark 5: The linear program in (**DP2**) can be categorically characterized as a *modified linear pairing-assignment problem* [26]. This formulation allows us to take non-integer relaxed values for $\tilde{\Lambda}_{j,k}$ and $\tilde{\Omega}_{m,(j,k)}$. However, in this paper, we prove that the optimal subcarrier allocation and pairing policies of the relaxed problem always procures an integer solution, that is nothing but, $\Lambda_{j,k}$ and $\Omega_{m,(j,k)}$, provided certain conditions are satisfied, the definition and theorem explain the details below.

[4]The subcarrier pairing and assignment policies can also be decoupled on the similar grounds.

$$\mathcal{L}\left(\tilde{\boldsymbol{P}}_{\mathbf{1}},\tilde{\boldsymbol{P}}_{\mathbf{2}},\tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{P}}},\tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{A}}},\boldsymbol{\Upsilon},\lambda,\boldsymbol{\mu},\boldsymbol{\nu}\right)=\mathcal{F}\left(\tilde{\boldsymbol{P}}_{\mathbf{1}},\tilde{\boldsymbol{P}}_{\mathbf{2}},\tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{P}}},\tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{A}}},\boldsymbol{\Upsilon}\right)-\lambda\left(\sum_{m=1}^{M}\sum_{j=1}^{N_S}\sum_{k=1}^{N_S}\Lambda_{j,k}\Omega_{m,(j,k)}\left(e^{\tilde{P}_{1,m}^{(j)}}+e^{\tilde{P}_{2,m}^{(k)}}\right)-P_{Total}\right)$$
$$-\sum_{m=1}^{N}\sum_{n=1}^{N_S}\mu_m^{(n)}\left(\Upsilon_m^{(n)}-\frac{1}{|\mathcal{K}_{m,n}|}\left(\frac{\alpha_{1,m}^{(n)}}{\ln(2)}\left(\tilde{P}_{1,m}^{(n)}+\log\left(\left|h_{1,m}^{(n)}\right|^2\right)-\log\left(\sum_{l=1,l\neq m}^{M}e^{\tilde{P}_{1,l}^{(n)}}\left|h_{1,l}^{(n)}\right|^2+\sigma_R^{(n)^2}\right)\right)+\beta_{1,m}^{(n)}\right)\right)$$
$$-\sum_{m=1}^{N}\sum_{n=1}^{N_S}\nu_m^{(n)}\left(\Upsilon_m^{(n)}-\frac{1}{|\mathcal{J}_{m,n}|}\left(\frac{\alpha_{2,m}^{(n)}}{\ln(2)}\left(\tilde{P}_{2,m}^{(n)}+\log\left(\left|h_{2,m}^{(n)}\right|^2\right)-\log\left(\sum_{l=1,l\neq m}^{M}e^{\tilde{P}_{2,l}^{(n)}}\left|h_{2,l}^{(n)}\right|^2+\sigma_{2,m}^{(n)^2}\right)\right)+\beta_{2,m}^{(n)}\right)\right), \tag{13}$$

$$\mathcal{A}_{m,(j,k)}=\left(\varphi\tilde{\Upsilon}_m^{(j)^\star}\right)-(\Psi+\lambda)\left(e^{\tilde{P}_{1,m}^{(j)^\star}}+e^{\tilde{P}_{2,m}^{(k)^\star}}\right); \tag{17}$$

$$\mathcal{K}\left(\tilde{\boldsymbol{\mathcal{P}}}_{\mathbf{1}}^{\star},\tilde{\boldsymbol{\mathcal{P}}}_{\mathbf{2}}^{\star},\boldsymbol{\Upsilon}^{\star},\lambda,\boldsymbol{\mu},\boldsymbol{\nu}\right)=-\Psi(2M+\xi_R)P^C+\lambda P_{max} \tag{18}$$
$$-\sum_{m=1}^{N}\sum_{n=1}^{N_S}\mu_m^{(n)}\left(\Upsilon_m^{(n)^\star}-\frac{1}{|\mathcal{K}_{m,n}|}\left(\frac{\alpha_{1,m}^{(n)}}{\ln 2}\left(\tilde{P}_{1,m}^{(n)^\star}+\log\left(\left|h_{1,m}^{(n)}\right|^2\right)-\log\left(\sum_{l=1,l\neq m}^{M}e^{\tilde{P}_{1,l}^{(n)^\star}}\left|h_{1,l}^{(n)}\right|^2+\sigma_R^{(n)^2}\right)\right)+\beta_{1,m}^{(n)}\right)\right)$$
$$-\sum_{m=1}^{N}\sum_{n=1}^{N_S}\nu_m^{(n)}\left(\Upsilon_m^{(n)^\star}-\frac{1}{|\mathcal{J}_{m,n}|}\left(\frac{\alpha_{2,m}^{(n)}}{\ln 2}\left(\tilde{P}_{2,m}^{(n)^\star}+\log\left(\left|h_{2,m}^{(n)}\right|^2\right)-\log\left(\sum_{l=1,l\neq m}^{M}e^{\tilde{P}_{2,l}^{(n)^\star}}\left|h_{2,l}^{(n)}\right|^2+\sigma_{2,m}^{(n)^2}\right)\right)+\beta_{2,m}^{(n)}\right)\right),$$

Definition 2 (Total-Unimodularity): A matrix X with full row rank, is defined as totally unimodular *iff* (1) one and all square sub-matrices of X follows $|X| = \{-1, 0, +1\}$ and (2) all the entries of X are integers.

Theorem 3: For any linear program having constraints of the form $Ax = v$, will always have an integer optimal solution if the constraint matrix A is totally-unimodular and the vector represented by v is an integer, respectively.

Proof: The proof is relegated in Appendix C. ■

With the obtained optimal power allocation policies $\left(\tilde{\boldsymbol{\mathcal{P}}}_{\mathbf{1}}^{\star}\left[\mathbf{H}_{1_{N_S\times M}}\right],\tilde{\boldsymbol{\mathcal{P}}}_{\mathbf{2}}^{\star}\left[\mathbf{H}_{2_{M\times N_S}}\right]\right)$, we can obtain the optimal subcarrier allocation and pairing policies in two steps as follows:

Step 1: Optimal Subcarrier Allocation Policy $\left(\tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{A}}}^{\star}\right)$

Under a given $\tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{P}}}$, the dual problem (**DP3**) can be reformulated as

(DP4)	
To obtain:	$\tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{A}}}^{\star}\left[\mathbf{H}_{1_{N_S\times M}},\mathbf{H}_{2_{M\times N_S}}\right]$
such that:	$\max\limits_{\tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{A}}}}\ \sum_{m=1}^{M}\sum_{j=1}^{N_S}\sum_{k=1}^{N_S}\tilde{\Lambda}_{j,k}\tilde{\Omega}_{m,(j,k)}\mathcal{A}_{m,(j,k)} + \mathcal{K}\left(\tilde{\boldsymbol{\mathcal{P}}}_{\mathbf{1}}^{\star},\tilde{\boldsymbol{\mathcal{P}}}_{\mathbf{2}}^{\star},\boldsymbol{\Upsilon}^{\star},\lambda,\boldsymbol{\mu},\boldsymbol{\nu}\right)$
subject to:	$(C.4)$ & $(C.5)$.

For a given subcarrier pairing policy, the optimal subcarrier allocation policy can be obtained by maximizing $\mathcal{A}_{m,(j,k)}$ as follows:

$$\Omega_{m,(j,k)}^{\star}=\begin{cases}1, \text{ for } m=\arg\max\limits_{m}\ \mathcal{A}_{m,(j,k)};\\ 0, \text{otherwise.}\end{cases} \tag{19}$$

Step 2: Optimal Subcarrier Pairing Policy $\left(\tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{P}}}^{\star}\right)$

For obtained optimal subcarrier and power allocation policies, the dual problem (**DP3**) can be transformed as below:

(DP5)	
To obtain:	$\tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{P}}}^{\star}\left[\mathbf{H}_{1_{N_S\times M}},\mathbf{H}_{2_{M\times N_S}}\right]$
such that:	$\max\limits_{\tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{P}}}}\ \sum_{m=1}^{M}\sum_{j=1}^{N_S}\sum_{k=1}^{N_S}\tilde{\Lambda}_{j,k}\tilde{\Omega}_{m,(j,k)}^{\star}\mathcal{A}_{m,(j,k)}^{\star} + \mathcal{K}\left(\tilde{\boldsymbol{\mathcal{P}}}_{\mathbf{1}}^{\star},\tilde{\boldsymbol{\mathcal{P}}}_{\mathbf{2}}^{\star},\boldsymbol{\Upsilon}^{\star},\lambda,\boldsymbol{\mu},\boldsymbol{\nu}\right)$
subject to:	$(C.2)$, $(C.3)$ & $(C.5)$.

where $\mathcal{A}_{m,(j,k)}^{\star}=\max\limits_{m}\mathcal{A}_{m,(j,k)}\,\forall\,(j,k)$. Further, the optimal subcarrier pairing policy can be obtained by solving the problem (**DP5**). Furthermore, to get a better insight of the total power dissipation, we study the impact of static power P^C with the following theorem.

Theorem 4: With an increase in static power P^C, we observe (i) For given resource allocation policies $\left(\tilde{\boldsymbol{\mathcal{P}}}_{\mathbf{1}}^{\star},\tilde{\boldsymbol{\mathcal{P}}}_{\mathbf{2}}^{\star},\tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{P}}}^{\star},\tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{A}}}^{\star}\right)$, the maximum achievable optimal $EE^\star$ in (**OP4**) strictly decreases.
(ii) The instantaneous optimal transmitting power $\left(\tilde{\boldsymbol{\mathcal{P}}}_{\mathbf{1}}^{\star},\tilde{\boldsymbol{\mathcal{P}}}_{\mathbf{2}}^{\star}\right)$ strictly increases.

Proof: The proof is provided in Appendix D. ■

C. Master problem Solution

We apply the sub-gradient method in the master problem of dual decomposition technique to update the Lagrange multipliers as in (20)-(22) shown on the top of the next page, where ξ_1, ξ_2 and ξ_3 are positive constant step sizes that are

$$\lambda(t+1) = \left[\lambda(t) + \xi_1(t)\left(\sum_{m=1}^{M}\sum_{j=1}^{N_S}\sum_{k=1}^{N_S}\tilde{\Lambda}^{\star}_{j,k}\tilde{\Omega}^{\star}_{m,(j,k)}\left(e^{\tilde{P}^{(j)^{\star}}_{1,m}} + e^{\tilde{P}^{(k)^{\star}}_{2,m}}\right) - P_{Total}\right)\right]^{+}; \quad (20)$$

$$\mu^{(n)}_m(t+1) = \left[\mu^{(n)}_m(t) + \xi_2(t)\left(\Upsilon^{(n)^{\star}}_m - \frac{1}{|\mathcal{K}_{m,n}|}\left(\frac{\alpha^{(n)}_{1,m}}{\ln 2}\left(\tilde{P}^{(n)^{\star}}_{1,m} + \log\left(\left|h^{(n)}_{1,m}\right|^2\right)\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\left. - \log\left(\sum_{l=1,l\neq m}^{M} e^{\tilde{P}^{(n)^{\star}}_{1,l}}\left|h^{(n)}_{1,l}\right|^2 + \sigma^{(n)^2}_R\right)\right) + \beta^{(n)}_{1,m}\right)\right)\right]^{+}; \quad (21)$$

$$\nu^{(n)}_m(t+1) = \left[\nu^{(n)}_m(t) + \xi_3(t)\left(\Upsilon^{(n)^{\star}}_m - \frac{1}{|\mathcal{J}_{m,n}|}\left(\frac{\alpha^{(n)}_{2,m}}{\ln 2}\left(\tilde{P}^{(n)^{\star}}_{2,m} + \log\left(\left|h^{(n)}_{2,m}\right|^2\right)\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\left. - \log\left(\sum_{l=1,l\neq m}^{M} e^{\tilde{P}^{(n)^{\star}}_{2,l}}\left|h^{(n)}_{2,l}\right|^2 + \sigma^{(n)^2}_{2,m}\right)\right) + \beta^{(n)}_{2,m}\right)\right)\right]^{+}, \quad (22)$$

being optimized to obtain the accelerated convergence rate.[5]

D. Update of Fixed Coefficients and Ψ

To ameliorate network's EE, we need to strictly improve the lower bound performance of the system that directly depends on the coefficients $\left(\alpha^{(n)}_{1,m}, \beta^{(n)}_{1,m}\right)$ and $\left(\alpha^{(n)}_{2,m}, \beta^{(n)}_{2,m}\right)$, which are successfully updated using the following theorem.

Theorem 5: If the coefficients $\alpha^{(n)}_{q,m}$ and $\beta^{(n)}_{q,m}$, $q = \{1, 2\}$, are updates as follows

$$\alpha^{(j)}_{q,m}(t+1) = \frac{\hat{\Gamma}^{(j)}_{q,m}(t)}{1 + \hat{\Gamma}^{(j)}_{q,m}(t)}; \quad (23)$$

$$\beta^{(j)}_{q,m}(t+1) = \log_2\left(1 + \hat{\Gamma}^{(j)}_{q,m}(t+1)\right) - \alpha^{(j)}_{q,m}(t+1)\log_2\left(\hat{\Gamma}^{(j)}_{q,m}(t+1)\right), \quad (24)$$

for the optimal power allocation policy $\left(\tilde{\boldsymbol{P}}^{\star}_{\mathbf{1}}, \tilde{\boldsymbol{P}}^{\star}_{\mathbf{2}}, \tilde{\boldsymbol{\mathcal{S}}}^{\star}_{\boldsymbol{\mathcal{P}}}, \tilde{\boldsymbol{\mathcal{S}}}^{\star}_{\boldsymbol{\mathcal{A}}}, \boldsymbol{\Upsilon}^{\star}\right)$ of the optimization problem (**OP4**) at the t^{th} iteration, then the optimal value of $\mathcal{F}\left(\tilde{\boldsymbol{P}}_{\mathbf{1}}, \tilde{\boldsymbol{P}}_{\mathbf{2}}, \tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{P}}}, \tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{A}}}, \boldsymbol{\Upsilon}\right)$ is monotonically increased.

Proof: The proof is similar to [15, Appendix C]. ■

Lastly, we need to update the penalty factor Ψ for the optimal resource allocation policy. This can be done in two steps, firstly, we propose a theorem for its update procedure, and secondly, we provide two theorems that establishes its convergence.

Theorem 6: If the optimal resource allocation policy $\left(\tilde{\boldsymbol{P}}^{\star}_{\mathbf{1}}, \tilde{\boldsymbol{P}}^{\star}_{\mathbf{2}}, \tilde{\boldsymbol{\mathcal{S}}}^{\star}_{\boldsymbol{\mathcal{P}}}, \tilde{\boldsymbol{\mathcal{S}}}^{\star}_{\boldsymbol{\mathcal{A}}}, \boldsymbol{\Upsilon}^{\star}\right)$ in the optimization problem (**OP1**) *w.r.t.* $\Psi^{\star}$ satisfies the following balance equation

$$\mathcal{R}_T\left(\tilde{\boldsymbol{P}}^{\star}_{\mathbf{1}}, \tilde{\boldsymbol{P}}^{\star}_{\mathbf{2}}, \tilde{\boldsymbol{\mathcal{S}}}^{\star}_{\boldsymbol{\mathcal{P}}}, \tilde{\boldsymbol{\mathcal{S}}}^{\star}_{\boldsymbol{\mathcal{A}}}, \tilde{\boldsymbol{\Upsilon}}^{\star}\right) - \Psi^{\star}\left(\sum_{m=1}^{M}\sum_{j=1}^{N_S}\sum_{k=1}^{N_S}\tilde{\Lambda}^{\star}_{j,k}\tilde{\Omega}^{\star}_{m,(j,k)} \times\left(e^{\tilde{P}^{(j)^{\star}}_{1,m}} + e^{\tilde{P}^{(k)^{\star}}_{2,m}}\right) + 2MP^C + \xi_R P^C\right) = 0, \quad (25)$$

then $\Psi^{\star}$ will become the optimal penalty for the resources being allocated.

Proof: The proof is similar to [15, Appendix D]. ■

Theorem 7: At the $(l+1)^{th}$ iteration, if the penalty factor Ψ is updated for the local maximizer $\left(\tilde{\boldsymbol{P}}^{\star}_{\mathbf{1}}(l), \tilde{\boldsymbol{P}}^{\star}_{\mathbf{2}}(l), \tilde{\boldsymbol{\mathcal{S}}}^{\star}_{\boldsymbol{\mathcal{P}}}(l), \tilde{\boldsymbol{\mathcal{S}}}^{\star}_{\boldsymbol{\mathcal{A}}}(l), \boldsymbol{\Upsilon}^{\star}(l)\right)$ of (**OP1**) for the penalty $\Psi(l)$ as

$$\Psi(l+1) = \frac{\mathcal{R}_T\left(\tilde{\boldsymbol{P}}^{\star}_{\mathbf{1}}(l), \tilde{\boldsymbol{P}}^{\star}_{\mathbf{2}}(l), \tilde{\boldsymbol{\mathcal{S}}}^{\star}_{\boldsymbol{\mathcal{P}}}(l), \tilde{\boldsymbol{\mathcal{S}}}^{\star}_{\boldsymbol{\mathcal{A}}}(l), \tilde{\boldsymbol{\Upsilon}}^{\star}(l)\right)}{\left\{\sum_{m=1}^{M}\sum_{j=1}^{N_S}\sum_{k=1}^{N_S}\tilde{\Lambda}^{\star}_{j,k}(l)\tilde{\Omega}^{\star}_{m,(j,k)}(l) \times\left(e^{\tilde{P}^{(j)^{\star}}_{1,m}(l)} + e^{\tilde{P}^{(k)^{\star}}_{2,m}(l)}\right) + 2MP^C + \xi_R P^C\right\}}, \quad (26)$$

then the penalty factor $\Psi(l)$ monotonically increases with l.

Proof: The proof is similar to [15, Appendix E]. ■

E. Iterative EEM Algorithm

In this subsection, we describe in detail the iterative resource allocation algorithm for maximizing the networks instantaneous achievable EE. For this, we first initialize the penalty factor $\Psi(x) = 0.0001$, step sizes $\xi_m = 0.0001$, $\forall m \in \{1, 2, \ldots, 10\}$, lower bound coefficients $\{\alpha^{(j)}_{1,m}, \beta^{(j)}_{1,m}\} = \{1, 0\}$ and $\{\alpha^{(j)}_{2,m}, \beta^{(j)}_{2,m}\} = \{1, 0\}$. Lastly, initialize the subcarrier pairing permutation and allocation matrices $\tilde{\boldsymbol{\Lambda}}$ and $\tilde{\boldsymbol{\Omega}}$ while satisfying the constraints $(C.2)$ – $(C.5)$. For given subcarrier pairing permutation and allocation, we can find the optimal power allocation policy $\left(\tilde{\boldsymbol{\mathcal{P}}}^{\star}_{\mathbf{1}}\left[\mathbf{H}_{1_{N_S\times M}}\right], \tilde{\boldsymbol{\mathcal{P}}}^{\star}_{\mathbf{2}}\left[\mathbf{H}_{2_{M\times N_S}}\right]\right)$ using (14) and (15) and the auxiliary variable from (16). For optimally allocated power and given subcarrier pairing, the optimal subcarrier allocation policy $\tilde{\boldsymbol{\mathcal{S}}}^{\star}_{\boldsymbol{\mathcal{A}}}\left[\mathbf{H}_{1_{N_S\times M}}, \mathbf{H}_{2_{M\times N_S}}\right]$ is computed using (**DP4**), while the optimal subcarrier pairing permutation policy $\tilde{\boldsymbol{\mathcal{S}}}^{\star}_{\boldsymbol{\mathcal{P}}}\left[\mathbf{H}_{1_{N_S\times M}}, \mathbf{H}_{2_{M\times N_S}}\right]$ can be found by solving (**DP5**) for the obtained power and subcarrier allocation. Lastly we

[5]The proof of convergence for the sub-gradient method for constant step sizes is provided in [27].

update the Lagrange multipliers $(\lambda, \boldsymbol{\mu}, \boldsymbol{\nu})$ using (20)-(22). If the convergence is reached, we update the penalty factor Ψ using (26). This procedure is repeated until convergence. The iterative EEM algorithm for finding the optimal solution is presented in flowchart as shown in Fig. 2.

Sometimes, it is preferable to consider individual node power (INP) constraints in wireless networks, particularly when each node is operated on a different power budget. The proposed design framework can be easily extended to accommodate this scenario by replacing the constraint ($C.1$) in the optimization problem (**OP1**) with the following transmit power constraints:

$$\sum_{j=1}^{N_S}\sum_{k=1}^{N_S}\Lambda_{j,k}\Omega_{m,(j,k)}P_{1,m}^{(j)} \leq P_{m,max}, \quad m = 1,\ldots,M\,; \tag{27}$$

$$\sum_{m=1}^{M}\sum_{j=1}^{N_S}\sum_{k=1}^{N_S}\Lambda_{j,k}\Omega_{m,(j,k)}P_{2,m}^{(k)} \leq P_{R,max}, \tag{28}$$

where $P_{m,max}$ and $P_{R,max}$ are the maximum allowable transmit power values for the m^{th} source node and the relay node, respectively. This new optimization problem can be solved in a similar way as in the total transmit power constraint case, although it now requires the update of $M+1$ Lagrangian multipliers in the master problem due to the $M+1$ imposed INP constraints.

Further, the complexity of the proposed algorithm can be explained as follows. Since M user pairs are operating on N_S subcarriers in each hop, thus we need to solve MN_S^2 subproblems. Further, the optimal power allocation policy $\left(\tilde{\boldsymbol{\mathcal{P}}}_{\mathbf{1}}^{\star}\left[\mathbf{H}_{1_{N_S\times M}}\right], \tilde{\boldsymbol{\mathcal{P}}}_{\mathbf{2}}^{\star}\left[\mathbf{H}_{2_{M\times N_S}}\right]\right)$ is found under a total power constraint with a complexity of $\mathcal{O}\left(K^3+1\right)$, where K denotes the number of power levels that can be taken by the user and relay node on each subcarrier, respectively. Moreover, the subcarriers are allocated for a given $\Omega_{j,k}$, thus each maximization in (**DP4**) has a complexity of $\mathcal{O}\left(M\right)$, hence the total subcarrier allocation complexity becomes $\mathcal{O}\left(MN_S^2\right)$. Furthermore, the optimal subcarrier pairing in (**DP5**) has a complexity of $\mathcal{O}\left(N_S^2\right)$. The complexity of updating a dual variable is $\mathcal{O}\left((2N)^{\varrho}\right)$ (for example, $\varrho = 2$ if the ellipsoid method is used). Thus, the total complexity for updating dual variables is $\mathcal{O}\left(7(2N)^{\varrho}\right)$. Let us suppose if the dual objective function $g\left(\lambda, \boldsymbol{\mu}, \boldsymbol{\nu}\right)$ converges in Z iterations, then total complexity for the optimal scheme is $\mathcal{O}\left(7(2N)^{\varrho}N_S^2 Z\left(2M + MK^3+1\right)\right)$, whereas, that of a one-to-one mapping scenario would be $\mathcal{O}\left(5(2N)^{\varrho}N_S^2 Z\left(M(K^3+2)+N_S\right)\right)$, respectively.

Remark 6: It is to be noted that the complexity analysis reveals a very important point. The complexity of the proposed EEM algorithm is much lower than the ES algorithm. However, the average EE and SE performance of the proposed algorithm is identical to that of the ES.

V. Numerical Results

In this section, we conduct extensive computer simulations to verify the benefits of the energy-efficient design and the performance of the proposed resource allocation algorithms.

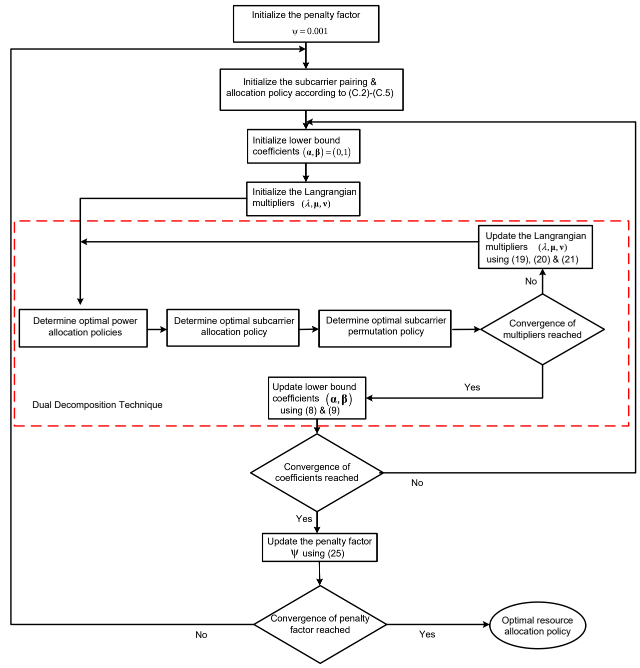


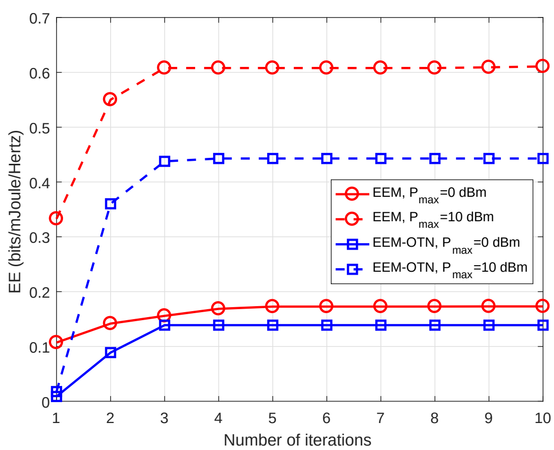
Fig. 2. Summary of the proposed energy-efficient iterative resource allocation algorithm.

A practical path-loss model recommended by the Third-Generation Partnership Project (3GPP), given by $131.1 + 42.8 \times \log_{10}(d)$ dB (d: distance in km), is employed in our simulations [28]. We consider both the Rayleigh fading and the log-normal shadowing effects described by $\sim \mathcal{CN}(0,1)$ and $\sim \ln \mathcal{N}(0, 8dB)$, respectively. The circuit and processing power dissipation per antenna at each node is assumed to be 14 dBm, respectively [15] along with $\xi_R = 2$. The adjacent subcarriers frequency spacing is 12 kHz and thermal noise density is set to be -174 dBm/Hz. In the proposed EEM algorithm, the maximum number of inner and outer iterations, I_{inner} and I_{outer}, are set as 10, while the value of the convergence tolerance is 10^{-5}. The initial value of the penalty factor Ψ is 0.001. The distances from all transmit users to the relay node and from the relay node to all receive users are denoted by d_{SR} and d_{RD}, respectively. The simulation parameters' settings are summarized in Table I. For the performance comparison analysis, we also simulate the following algorithms:

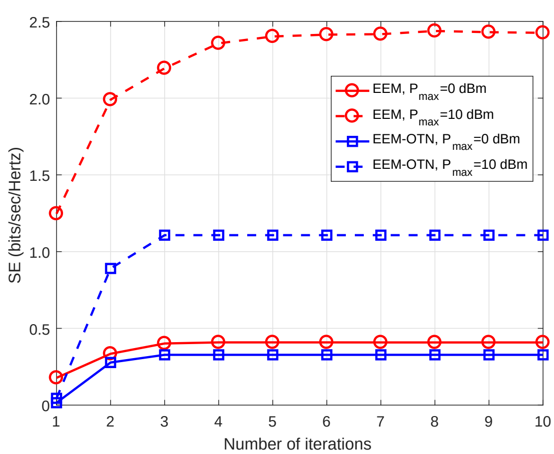
- Optimal ES: The globally optimal solution can be found by solving of the problem $(\mathbf{OP1})$ using an ES algorithm which performs an exhaustive search over all variables [26].
- SEM: The SEM algorithm is also simulated to evaluate the performance of the multiuser relay-assisted network.
- EEM without (w/o) SP-SA: Without considering the SP-SA, the optimal solution of the problem $(\mathbf{OP1})$ can be obtained for performance comparison.
- EEM with one-to-one subcarrier pairing (EEM-OTO): To attain the optimal solution of the problem $(\mathbf{OP1})$ with joint optimization of one-to-one subcarrier pairing, e.g., a single subcarrier of the MA phase can pair only with a single subcarrier of the BC phase and vice-versa, subcarrier allocation, and power allocation.

TABLE I
SIMULATION PARAMETERS

Simulation Parameter	**Values**
Number of user pairs, M	$2, 5, 10$
Number of subcarriers, N_S	$10, 16, 32$
Subcarrier bandwidth	12 kHz
Thermal noise density	-174 dBm/Hz
Log-normal shadowing	$\sim \ln \mathcal{N}(0, 8\,\text{dB})$
Distance from the source to the relay, d_{SR}	200 m
Distance from the relay to the destination, d_{RD}	200 m
Path-loss model	$131.1 + 42.8 \times \log 10(d)\ dB$
Maximum number of iterations, I_{max}	10
Step size, ϵ_i	0.001
Tolerance value	10^{-5}.
ξ_R	2



(a) EE versus Number of iterations

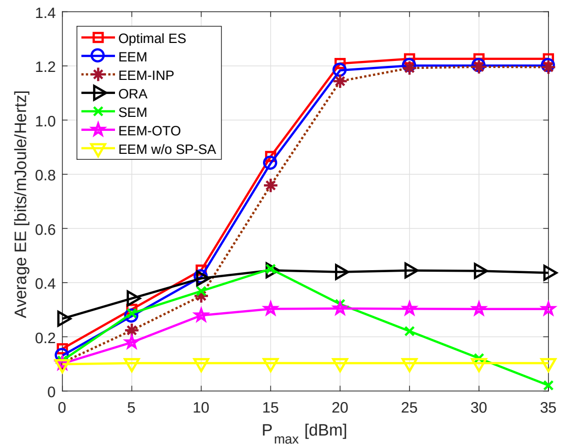


(b) SE versus Number of iterations

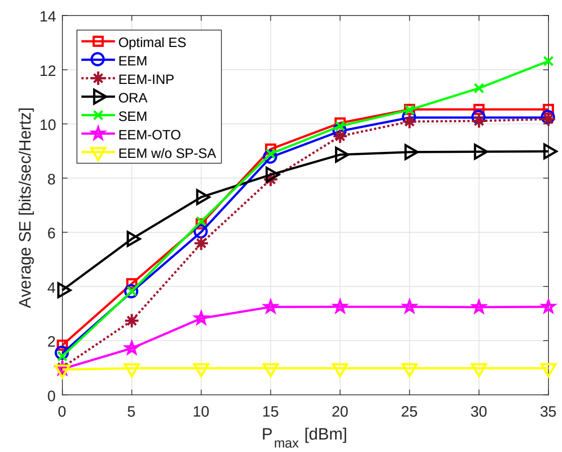
Fig. 3. Convergence behavior of iterative algorithms.

A. Convergence Performance of Algorithms

Fig. 3 illustrates the EE and SE performance of the proposed EEM algorithms versus the iteration number for a single channel realization with $M = 2$, $N_S = 32$, $d_{SR} = d_{RD} = 200$ m, and $P_{max} = \{0, 10\}$ dBm. As shown in this figure, the EE and the SE performance of the EEM algorithms are monotonically increased with iteration numbers and the proposed algorithms converge within a fewer iterations, generally fewer than 5. The attaining EE and SE performance of the EEM algorithm outperforms EEM-OTO.



(a) Average EE versus P_{max}



(b) Average SE versus P_{max}

Fig. 4. Performance comparison of our proposed EEM algorithm with other existing algorithms.

B. Performance Comparison With Other Algorithms

Fig. 4 shows the average EE and SE performance results of different algorithms for $M = 2$, $N_S = 16$, and $d_{SR} = d_{RD} = 200$ m. As a benchmark, we compare the performance of our proposed algorithm with that of the ES algorithm, which may apply to a small-scale problem for attaining the optimal solution within a reasonable computation time, and the orthogonal resource allocation (ORA) algorithm. As seen in Fig. 4, the average EE or SE performance of the proposed EEM algorithm is identical to that of the optimal ES algorithm. Moreover, both the EEM and SEM algorithms exhibit identical average EE performance in limited power budget, i.e., $P_{max} \leq 10$ dBm. However, when the power budget becomes rich, i.e., $P_{max} > 10$ dBm, in Fig. 4(a),

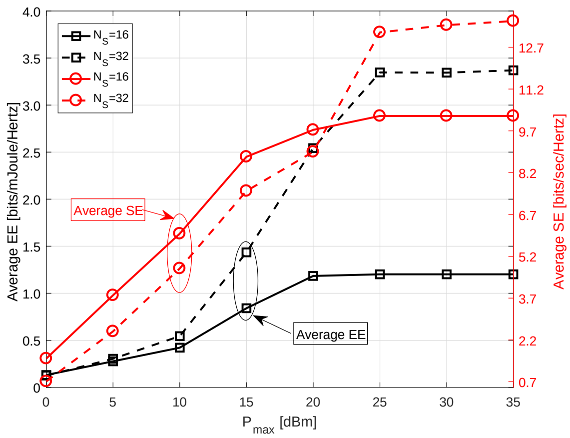


Fig. 5. Average EE and SE, and the effect of an increasing number of subcarriers, N_S.

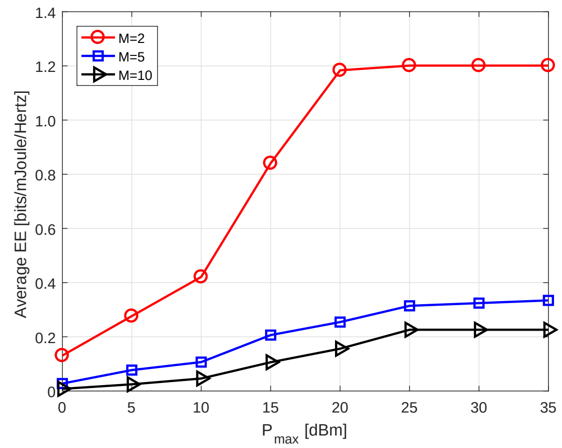


Fig. 6. Average EE and the effect of an increasing number of user pairs, M.

the average EE performance of the EEM algorithm increases rapidly and it becomes steady after $P_{max} > 20$ dBm, whereas that of the SEM algorithm quickly decreases as P_{max} increases at the cost of SE wherein each user utilizes maximum power to improve their sum rate without concerning the EE. From the results depicted in Fig. 4(b), we can observe that the average SE of the EEM and SEM algorithms increases significantly for $P_{max} < 20$ dBm and after that the performance of the EEM algorithm saturates, while the performance of the SEM algorithm increases with the increase of P_{max} because each user utilizes maximum transmit power in order to enhance the sum rate at the cost of a degradation in the average EE. The proposed resource allocation algorithm with the INP constraints, named as EEM-INP, is also simulated and compared. For a fair comparison with the total power constraint scenario, we set $P_{i,max} = P_{R,max} = \frac{P_{max}}{M+1}$. When $P_{max} \leq 20$ dBm, both the average EE and SE performances of the EEM-INP algorithm are slightly worse than those using the total power constraint. However, the performance gap gradually decreases to zero as P_{max} increases. It can also be seen from Fig. 4 that the ORA algorithm outperforms all the other algorithms when the power budget is low, i.e., $P_{max} \leq 10$ dBm. However, in the high power regime, the proposed EEM algorithm gives better SE and EE performance compared to ORA algorithm. Furthermore, the average EE or SE performance of the EEM-OTO algorithm improves before $P_{max} \leq 10$ dBm and after that it remains constant. However, the EEM-OTO algorithm performs better than that of the EEM w/o SP-SA due to unskillful utilization of the available subcarriers.

C. Effect of Different Number of Subcarriers on the attainable EE and SE

In this example, we study the effect of increasing the number of subcarriers, N_S, on the attainable average EE and SE. For this purpose, we plot average EE and SE versus P_{max} as shown in Fig. 5, where $M = 2$ and $d_{SR} = d_{RD} = 200$ m. It can be observed that the average EE and SE performance of the proposed EEM algorithm increases significantly as P_{max} increases and it remains constant after $P_{max} = 25$ dBm. The effect of increasing N_S is very limited in low transmit power regime, i.e., $P_{max} \leq 15$ dBm . However, when $P_{max} > 15$ dBm, where the transmit power budget becomes rich, the average EE and SE performance of the EEM algorithm increases swiftly as N_S increases, as expected due to frequency diversity.

D. Effect of Different User Pairs on the attainable EE

The effect of increasing the number of user pairs, M, on the attainable average EE performance is shown in Fig. 6 for different P_{max}, where $N_S = 16$ and $d_{SR} = d_{RD} = 200$ m. It is noticeable that the average EE performance of the EEM algorithm deteriorates upon increasing M, as expected due to the increase in the static power consumption.

VI. Conclusion and Future Scope

In this paper, we studied how to maximize the EE of the relay-assisted network by jointly optimizing the subcarrier pairing, subcarrier allocation, and the power allocation altogether. The original problem was a non-convex MINLP problem and thus, we converted the problem into an equivalent solvable convex problem by relaxing the integer variables and by applying a SCA approach. Furthermore, we proposed a novel iterative resource allocation algorithm to obtain the optimal subcarrier pairing, subcarrier allocation and power allocation solution via dual decomposition. In addition, we compared the performance of the proposed algorithm with that of the SEM and other algorithms, and analyzed the impact of various network parameters on the performance tradeoff between the EE and SE. To recap, an increase of number of subcarriers, N_S, can improve both the average EE and SE due to higher frequency diversity. Simulation results validated the theoretical findings and demonstrated that the proposed algorithm significantly outperforms other existing schemes in the literature. The inclusion of multiple antennas, imperfect CSI, and successive interference cancellation will be considered in our future work.

Appendix A
Proof of Theorem 1

Firstly, we demonstrate that an arbitrary instance of SS is transformed into a special instance of EEM optimization problem in polynomial time. Let us consider, an arbitrary instance of SS with a set of natural numbers $\mathcal{W} = w_j$ and a target V. Correspondingly, we formulate a two-user EEM instance as follows. For every w_j, we construct a subcarrier j with a rate-power function identical for both users. In other words, the power for rate 0 is 0, the power for rate w_j is $\frac{P}{|\mathcal{W}|}$, the power for rate $w_j + \frac{1}{|\mathcal{W}|}$ is P, and is strictly increasing.

Now we claim that, considering two user pairs, if there is a subcarrier allocation and pairing with rate requests for both the users as V and $\sum_{j=1}^{b} w_j - V$, and the total power consumption is not exceeded, then SS has a satisfying solution.

Let us consider, SS has a solution such that sum of subset Q is exactly V. If each subcarrrier in Q is allocated to a single user pair and rest of the subcarriers to other users. Further, loading each subcarrier with rate w_j, then it would be an optimally satisfying solution to the proposed EEM algorithm. However, in case wherein Q doesn't exist, then no subset can sum to V exactly. As w_j represents a natural number, the difference of sum between any subset and V must not be less than 1. Furthermore, $|W|-1$ remains the maximum number of subcarriers a user can be allocated. Consequently, one of the two users has to load at least one of the subcarrier assigned to it with rate higher than $w_j + \frac{1}{|\mathcal{W}|}$, which directly implies that the maximal individual power consumption is higher than P, thus making the total power consumption higher than P.

Appendix B
Proof of Theorem 2

We can reformulate the objective function in (**OP4**) as follows:

$$\mathcal{F}\left(\tilde{\boldsymbol{P}}_{\mathbf{1}}, \tilde{\boldsymbol{P}}_{\mathbf{2}}, \tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{P}}}, \tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{A}}}, \boldsymbol{\Upsilon}\right) =$$
$$\sum_{j=1}^{N_S}\sum_{k=1}^{N_S}\Bigg(\frac{1}{2}\sum_{m=1}^{M}\tilde{\Lambda}_{j,k}\tilde{\Omega}_{m,(j,k)}\Upsilon_m^{(j)} -$$
$$\Psi\sum_{m=1}^{M}\tilde{\Lambda}_{j,k}\tilde{\Omega}_{m,(j,k)}\left(e^{\tilde{P}_{1,m}^{(j)}} + e^{\tilde{P}_{2,m}^{(k)}}\right) - \frac{2\Psi(M+1)P^C}{N_S^2}\Bigg)\;;$$
$$\triangleq \sum_{j=1}^{N_S}\sum_{k=1}^{N_S}\Pi_{j,k}\left(\tilde{\boldsymbol{P}}_{\mathbf{1}}, \tilde{\boldsymbol{P}}_{\mathbf{2}}, \tilde{\boldsymbol{\Upsilon}}\right)\,, \tag{B.1}$$

where $\left\{\tilde{\boldsymbol{P}}_{\mathbf{1}}, \tilde{\boldsymbol{P}}_{\mathbf{2}}, \tilde{\boldsymbol{\Upsilon}}\right\} \in \mathbb{W}^M$ and $\Pi_{j,k}(\cdot) : \mathbb{W}^M \to \mathbb{R}$ may not be necessarily convex by nature. Similarly, we can reformulate the constraints $(C.1) - (C.8)$ as $\sum_{j=1}^{N_S}\sum_{k=1}^{N_S}\Theta_{j,k}\left(\tilde{\boldsymbol{P}}_{\mathbf{1}}, \tilde{\boldsymbol{P}}_{\mathbf{2}}, \tilde{\boldsymbol{\Upsilon}}, \boldsymbol{\alpha}, \boldsymbol{\beta}\right) \leqslant 0$, where $\Theta_{j,k}(\cdot) : \mathbb{W}^M \to \mathbb{R}^7$, and we define the following transformation

(TF1) *To obtain:*	$\tilde{\boldsymbol{\mathcal{P}}}_{\mathbf{1}}^{\star}\left[\mathbf{H}_{1_{N_S\times M}}\right], \tilde{\boldsymbol{\mathcal{P}}}_{\mathbf{2}}^{\star}\left[\mathbf{H}_{2_{M\times N_S}}\right]$
such that:	$\max\limits_{\tilde{\boldsymbol{\mathcal{P}}}_{\mathbf{1}}, \tilde{\boldsymbol{\mathcal{P}}}_{\mathbf{2}} \tilde{\boldsymbol{\Upsilon}}} \sum_{j=1}^{N_S}\sum_{k=1}^{N_S}\Pi_{j,k}\left(\tilde{\boldsymbol{P}}_{\mathbf{1}}, \tilde{\boldsymbol{P}}_{\mathbf{2}}, \boldsymbol{\Upsilon}\right)$
subject to:	$\sum_{j=1}^{N_S}\sum_{k=1}^{N_S}\Theta_{j,k}\left(\tilde{\boldsymbol{P}}_{\mathbf{1}}, \tilde{\boldsymbol{P}}_{\mathbf{2}}, \boldsymbol{\Upsilon}, \boldsymbol{\alpha}, \boldsymbol{\beta}\right) \leqslant x.$

where x denotes a variable $x \in \mathbb{R}^7$. Further, the optimization problem can be reformulated as (**OP5**) by substituting $x = 0$ in **TF1** and also, the perturbation function $\curlyvee(Q)$ can be defined by substituting $x = Q$ in **TF1**, where Q is a perturbation vector. From [25], when time sharing condition is satisfied, duality gap approaches to zero. Further, it concludes that time sharing condition is satisfied if the optimal policy of (**OP5**) is a concave function of the constraints. Hence, if $\curlyvee(Q)$ is a concave function of Q, then duality gap approaches to zero. Therefore, we apply the following three steps to prove the theorem.

Step 1: To obtain the time sharing property

If $\left\{\tilde{\boldsymbol{P}}_{\mathbf{1}}^{\star}, \tilde{\boldsymbol{P}}_{\mathbf{2}}^{\star}, \boldsymbol{\Upsilon}^{\star}\right\}$, $m = 1, 2$, denotes the optimal solution of (**OP5**), given by $\curlyvee(Q_1)$ and $\curlyvee(Q_2)$, then there always exist a solution $\left\{\tilde{P}_{1,3}^{(j)^\star}, \tilde{P}_{2,3}^{(k)^\star}, \Upsilon_3^{(j)^\star}\right\}$ such that

$$\sum_{j=1}^{N_S}\sum_{k=1}^{N_S}\Theta_{j,k}\left(\tilde{P}_{1,3}^{(j)^\star}, \tilde{P}_{2,3}^{(k)^\star}, \Upsilon_3^{(j)^\star}, \boldsymbol{\alpha}, \boldsymbol{\beta}\right) \leqslant \triangle Q_1 + (1-\triangle)Q_1\,; \tag{B.2}$$

$$\sum_{j=1}^{N_S}\sum_{k=1}^{N_S}\Pi_{j,k}\left(\tilde{P}_{1,3}^{(j)^\star}, \tilde{P}_{2,3}^{(k)^\star}, \Upsilon_3^{(j)^\star}\right) \geqslant \triangle\sum_{j=1}^{N_S}\sum_{k=1}^{N_S}\Pi_{j,k}\left(\tilde{P}_{1,1}^{(j)^\star}, \tilde{P}_{2,1}^{(k)^\star}, \Upsilon_1^{(j)^\star}\right) + (1-\triangle)\sum_{j=1}^{N_S}\sum_{k=1}^{N_S}\Pi_{j,k}\left(\tilde{P}_{1,2}^{(j)^\star}, \tilde{P}_{2,2}^{(k)^\star}, \Upsilon_2^{(j)^\star}\right)\,, \tag{B.3}$$

where $0 \leq \triangle \leq 1$.

Step 2: To prove the concavity of $\curlyvee(Q)$

For given $\triangle$, we can manifest Q_3 that satisfies $Q_3 = \triangle Q_1 + (1-\triangle)Q_2$. If $\left\{\tilde{P}_{1,1}^{(j)^\star}, \tilde{P}_{2,1}^{(k)^\star}, \Upsilon_1^{(j)^\star}\right\}$, $\left\{\tilde{P}_{1,2}^{(j)^\star}, \tilde{P}_{2,2}^{(k)^\star}, \Upsilon_2^{(j)^\star}\right\}$ and $\left\{\tilde{P}_{1,3}^{(j)^\star}, \tilde{P}_{2,3}^{(k)^\star}, \Upsilon_3^{(j)^\star}\right\}$ are the optimal solutions controlled by the constraints $\curlyvee(Q_1)$, $\curlyvee(Q_2)$ and $\curlyvee(Q_3)$, then by applying time sharing property we get $\left\{\tilde{P}_{1,3}^{(j)^\star}, \tilde{P}_{2,3}^{(k)^\star}, \Upsilon_3^{(j)^\star}, \boldsymbol{\alpha}.\boldsymbol{\beta}\right\}$ satisfying (B.2) and (B.3). Therefore, $\left\{\tilde{P}_{1,3}^{(j)^\star}, \tilde{P}_{2,3}^{(k)^\star}, \Upsilon_3^{(j)^\star}\right\}$ becomes the optimal solution of $\curlyvee(Q_3)$, giving

$$\sum_{j=1}^{N_S}\sum_{k=1}^{N_S}\Pi_{j,k}\left(\tilde{P}_{1,3}^{(j)^\star}, \tilde{P}_{2,3}^{(k)^\star}, \Upsilon_3^{(j)^\star}\right) \geqslant \sum_{j=1}^{N_S}\sum_{k=1}^{N_S}\Pi_{j,k}\left(\tilde{P}_{1,3}^{(j)^\star}, \tilde{P}_{2,3}^{(k)^\star}, \Upsilon_3^{(j)^\star}\right) \geqslant \triangle\Pi_{j,k}\left(\tilde{P}_{1,1}^{(j)^\star}, \tilde{P}_{2,1}^{(k)^\star}, \Upsilon_1^{(j)^\star}\right) + (1-\triangle)\Pi_{j,k}\left(\tilde{P}_{1,2}^{(j)^\star}, \tilde{P}_{2,2}^{(k)^\star}, \Upsilon_2^{(k)^\star}\right). \tag{B.4}$$

Step 3: To prove (***OP5***) *satisfies time sharing property*

It is evident from [26] that when the number of subcarriers goes to infinity, the time-sharing property always holds true for the multicarrier systems. If we consider $\left\{\tilde{P}_{1,1}^{(j)^\star}, \tilde{P}_{2,1}^{(k)^\star}, \Upsilon_1^{(j)^\star}\right\}$ and $\left\{\tilde{P}_{1,2}^{(j)^\star}, \tilde{P}_{2,2}^{(k)^\star}, \Upsilon_2^{(j)^\star}\right\}$ be two feasible solutions with $\triangle \times N_S$ and $(1-\triangle) \times N_S$ subcarriers allocated to each one. Also, $\sum_{j=1}^{N_S}\sum_{k=1}^{N_S}\Pi_{j,k}\left(\tilde{P}_{1,3}^{(j)^\star}, \tilde{P}_{2,3}^{(k)^\star}, \Upsilon_3^{(j)^\star}\right)$ is a linear combination of $\triangle\Pi_{j,k}\left(\tilde{P}_{1,1}^{(j)^\star}, \tilde{P}_{2,1}^{(k)^\star}, \Upsilon_1^{(j)^\star}\right) + (1-\triangle)\Pi_{j,k}\left(\tilde{P}_{1,2}^{(j)^\star}, \tilde{P}_{2,2}^{(k)^\star}, \Upsilon_2^{(j)^\star}\right)$. Therefore, the constraints are linear combinations itself. Hence, it is proved that (**OP5**) satisfies the time sharing property. Henceforth, $\Upsilon(Q)$ is a concave function of Q and the duality gap tends to zero, respectively. Hence, this theorem is proved.

Appendix C
Proof of Theorem 3

For the given problem in (**DP3**), the constraints $(C.2)-(C.4)$ can be reformulated as $\mathbf{A}\mathbf{B} = \mathbf{1}$, where $\boldsymbol{B}$ denotes a $\left[N_S^2(M+4)\times 1\right]$ vector of all indicators, i.e., $\{\Lambda_{j,k}, \Omega_{m,(j,k)}\}, \forall m,j,k$, $\mathbf{1}$ represents a $[2M\times 1]$ vector of all ones, and $\mathbf{A}$ is the $\left[2M \times \left(N_S^2 M + 4\right)\right]$ constraint matrix given by

$$\mathbf{A} = \left[\overbrace{\mathbf{A}_1,\ldots,\mathbf{A}_1}^{M\text{times}}, \overbrace{\mathbf{I}_{2\times 2},\ldots,\mathbf{I}_{2\times 2}}^{2M\text{times}}\right], \tag{C.1}$$

where $\mathbf{I}_{2\times 2}$ indicates a $[2\times 2]$ identity matrix and $\mathbf{A}_1$ denotes a $\left(2M\times N_S^2\right)$ matrix written as

$$\mathbf{A}_1 = blkdiag\left(\underbrace{\overbrace{11\ldots 11}^{M\text{times}}}_{\mathbf{I}_{M\times M}}, \ldots, \underbrace{\overbrace{11\ldots 11}^{M\text{times}}}_{\mathbf{I}_{M\times M}}\right); \tag{C.2}$$

where the notation $blkdiag(\cdot)$ arranges a number of square matrices on the main diagonal and the off-diagonal elements are appended with zeros. It is already evident from [29] that a matrix $\mathbf{C}$ is totally unimodular *iff* $(\mathbf{C},\mathbf{I})$ is totally unimodular. Hence, if we prove $\mathbf{C} = \underbrace{[\mathbf{A}_1,\ldots,\mathbf{A}_1]}_{M\text{times}}$ is totally unimodular then it directly implies that the constraint matrix $\mathbf{A}$ is totally unimodular. Further, using [29, Theorem 4.14], the necessary and sufficient condition for unimodularity is given as follows:

Let us assume a matrix $\boldsymbol{C}_{j\times k}$ with entries $\{-1,0,+1\}$ in which any column has at most two non-zero entries, then if there is possible to make a partition of rows $(N_1 \cup N_2 = 1,\ldots,j)$ in $\boldsymbol{C}$, where each column k with two non-zero entries has the following property:

$$\sum_{j\in N_1} c_{jk} = \sum_{j\in N_2} c_{jk}, \tag{C.3}$$

then matrix $\boldsymbol{C}$ is totally unimodular. Now, the given problem in (**DP3**), matrix $\mathbf{C} = \underbrace{[\mathbf{A}_1,\ldots,\mathbf{A}_1]}_{M\text{times}}$ explicitly satisfies the sufficient condition, while the partition of rows is shown by dotted line in $\mathbf{A}_1$. Hence, we conclude that $\mathbf{C}$ and $\mathbf{A}$ are totally unimodular.

Appendix D
Proof of Theorem 4

A. Proof of (i)

For any resource allocation policy $(\boldsymbol{P}_1, \boldsymbol{P}_2, \boldsymbol{S}_{\boldsymbol{A}}, \boldsymbol{S}_{\boldsymbol{P}})$, the optimal EE is a function of the form

$$EE^\star(\mathcal{R}_T) = \frac{\mathcal{R}_T\left(\tilde{\boldsymbol{P}}_{\mathbf{1}}, \tilde{\boldsymbol{P}}_{\mathbf{2}}, \tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{P}}}, \tilde{\boldsymbol{\mathcal{S}}}_{\boldsymbol{\mathcal{A}}}, \boldsymbol{\Upsilon}\right)}{\sum_{m=1}^{M}\sum_{j=1}^{N_S}\sum_{k=1}^{N_S}\tilde{\Lambda}_{j,k}\tilde{\Omega}_{m,(j,k)}\left(e^{\tilde{P}_{1,m}^{(j)}} + e^{\tilde{P}_{2,m}^{(k)}}\right) + 2MP^C + \xi_R P^C}. \tag{D.1}$$

It is evident that $EE^\star(\mathcal{R}_T)$ monotonically decreases with P^C. It implies that $EE^\star(\mathcal{R}_T)$-v/s-$\mathcal{R}_T$ curve tends to become strictly lower with increasing P^C. If we assume $\widehat{EE}^\star(\mathcal{R}_T), \hat{\mathcal{R}}_T$ and $\hat{P}^C$ represent $EE^\star(\mathcal{R}_T), \mathcal{R}_T$ and P^C with larger static power, then $\widehat{EE}^\star(\mathcal{R}_T) \leqslant EE^\star(\widehat{\mathcal{R}_T}) \leqslant EE^\star(\mathcal{R}_T)$. Therefore, the optimal $EE^\star(\mathcal{R}_T)$ monotonically decreases with increase in P^C.

B. Proof of (ii)

Let us define the optimal power allocation policy $\left(\tilde{\boldsymbol{P}}_1^\star, \tilde{\boldsymbol{P}}_2^\star\right)$ for a set of static power condition $\{P_m^C\}$, where m denotes the number of users having the static power consumption equal to P_m^C. Further, $EE^\star$ symbolizes maximum achievable instantaneous EE. Let $\{P_m^C\}$ decreases by some amount to $\{P_m^C\} - \triangle P^C$, then from (i) stated above, $EE^\star$ will decrease and from (14) and (15) $\left(\tilde{\boldsymbol{P}}_1^\star, \tilde{\boldsymbol{P}}_2^\star\right)$ will decrease monotonically with $EE^\star$, directly implying that $\left(\tilde{\boldsymbol{P}}_1^\star, \tilde{\boldsymbol{P}}_2^\star\right)$ increases monotonically with the static power P^C.

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