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On the Performance of Cognitive Satellite-Terrestrial Networks

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Abstract—We investigate the performance of a multi-beam cognitive satellite terrestrial network in which a secondary network (mobile terrestrial system) shares resources with a primary satellite network given that the interference temperature constraint is satisfied. The terrestrial base stations (BSs) and satellite users are modeled as independent homogeneous Poisson point processes. Utilizing tools from stochastic geometry, we study and compare the outage performance of three secondary transmission schemes: first is the power constraint (PCI) scheme where the transmit power at the terrestrial BS is limited by the interference temperature constraint. In the second scheme, the terrestrial BSs employ directional beamforming to focus the signal intended for the terrestrial user, and in the third, BSs that do not satisfy the interference temperature constraint are thinned out (BTPI). Analytical approximations of all three schemes are derived and validated through numerical simulations. It is shown that for the least interference to the satellite user, BTPI is the best scheme. However, when thinning is not feasible, PCI scheme is the viable alternative. In addition, the gains of directional beamforming are optimal when the terrestrial system employs massive multiple-input multiple-output transceivers or by the use of millimeter wave links between terrestrial BSs and users.

Index Terms—Cognitive radio, interference, multi-beam satellite, poisson point processes, satellite-terrestrial networks.

I. INTRODUCTION

THE KEY goals of future generation wireless communication systems include billions of connected devices, data rates in the range of Gbps, lower latencies, increased reliability, improved coverage and environment-friendly, low-cost, and energy-efficient operation. As the existing cellular spectrum approaches its performance limits, there is growing interest in and exploration of supplementary resources for meeting these demands [1]. As a result, satellite mobile communication is attracting widespread interest in radio technology studies which aim to provide ample coverage with low complexity infrastructure [2]. Multi-beam structure in modern satellite mobile communication has gained massive attention because of the potential to provide a higher coverage area and larger capacity since multiple isolated spot beams can reuse frequency. For example, with a reuse factor of four, hundreds of beams are possible [3]. The frequency reuse in multi-beam satellites gives a trade-off between inter-beam interference and available bandwidth as presented in [4]. Precoding techniques have been established to increase communication efficiency [1]. In the context of multi-beam satellites, precoding techniques are being explored as a means to mitigate inter-beam interference. The work in [5] shows that with the use of linear precoding, spectral efficiency is improved by about fifty percent. Moreover, motivated by the advances in cellular communication to improve spectral efficiency, hybrid satellite-terrestrial networks have gained interest in research [6], [7].

Cognitive radio is another technology that has attracted considerable research as a means of spectrum management in conventional wireless communication systems because it allows the coexistence of primary and secondary networks using the same resources [8], [9]. A primary network consists of transmitters and receivers with the licence to use a specific frequency band [10] while a secondary network comprises the transmitters and receivers that share resources with the primary network. Cognitive radio networks operate three major paradigms: underlay, overlay and interweave [9]. Within the framework of satellite communication, Sharma et al. [11] suggest that the level of interference power can determine which cognitive technique is appropriate. The underlay paradigm, which allows concurrent primary (non-cognitive) and secondary (cognitive) transmissions, and is suitable for medium interference regions, is considered in this paper.

In addition, the fusion of cognitive radios with hybrid satellite-terrestrial networks (cognitive satellite-terrestrial networks, CSTNs) is investigated by many researchers with the objective of optimizing efficiency and coverage in both existing and future wireless communication systems.
The work in [12] introduced the concept to show the possibility of maximizing spectrum utilization for terrestrial and satellite uplink transmissions. Additional works enhancing CSTNs include [13]–[16]. Specifically, the work in [13] presents methods for utilizing overlay CSTNs, power allocation is considered in [14] and performance of CSTNs under imperfect channel estimations is measured using the metrics of outage probability and normalized capacity. Lagunas et al. [15] investigate efficient allocation of more resources such as carrier, power and bandwidth allocations for achieving more gain with the CSTNs, and finally, the work in [16] presents a mathematical approach to achieve computational efficiency of the outage probability of CSTNs.

With the incorporation of base stations (BSs) to satellite communication, terrestrial interference is another key parameter that needs to be characterized for the accurate analysis of the performance of CSTNs. Given the random locations of terrestrial BSs as well as satellite users [17] and motivated by the successes of using stochastic geometry models for interference characterization in cellular cognitive radio networks [18], we employ the probabilistic stochastic geometric tools for characterizing the interference in CSTNs.

To achieve performance gains, numerous studies have sought ways of managing interference. A well-known method for this management is directional transmission [20], [21], which focuses a signal to a target direction (unlike the omnidirectional method in which a signal is transmitted in all directions). Directional transmission has the advantage of reducing interference and increasing coverage. In CSTNs, Sharma et al. [22] study different beamforming techniques to jointly achieve maximum rate for the secondary user and minimize interference to the satellite users and show that modified linear constrained minimum variance beamformer achieves this objective.

A. Design Approaches

This paper evaluates the performance of a CSTN where there is concurrent transmission of a primary multi-beam satellite network and a secondary terrestrial mobile network, and where interference to the primary network is not beyond a set limit. We provide a comparative analysis of different methods for keeping interference generated by the terrestrial network within acceptable limits.

In [13]–[16], all nodes are assumed to be equipped with a single antenna. However, in the proposed CSTN model, the nodes of the secondary (terrestrial) network will be equipped with multiple antennas as well as multiple beams considered for the satellite network. Therefore, unlike the models in [13]–[16], this work considers a more general and practical scenario with the analysis of a network where multiple terrestrial base stations (BSs) share resources with a multi-beamed satellite to serve the terrestrial user. To the authors’ best knowledge, randomly distributed BS with multiple antennas has not been considered for this network set-up.

Introducing multiple BSs with multiple antennas at the secondary network results in a more involved analysis than is presented in [13]–[16], because apart from characterizing the strict interference constraints imposed by the satellite network, there is an added interference from other terrestrial BSs trying to serve the terrestrial user. In this paper, we characterize this added interference by using stochastic geometric tools, and consider its effect on the transmissions in both primary and secondary networks.

The performance of this network is analyzed for three different transmission schemes. In the first, we assume that the BS process of the secondary network is stationary and ergodic so that BS nodes take part in transmission to the terrestrial user only if they satisfy the interference temperature constraint imposed by the satellite. Thus, we design a framework for characterizing the transmission power at the BS to ensure that the interference limit imposed by the primary network is not surpassed, and also characterize the interference by the BSs that do not satisfy the constraint. This scheme is referred to as power constraint to limit interference (PCI). In the second (DBI), we utilize directional transmission at the secondary system to focus the signals intended for the terrestrial user and accordingly restrict interference to acceptable limits. This scheme is based on the interference limit and thus no power restriction is placed on the terrestrial BSs. Finally, because some BSs may not participate in transmission owing to their inability to satisfy this interference temperature constraint, we will consider for the third scheme only the subset of BSs that meet the satellite’s requirement. This consideration leads to a marked point process and will be referred to as the BS thinning process to restrict interference (BTPI). It is important to note that the thinning criteria is based on transmit power constraint which will be described in Section II.

The performance of these schemes are analyzed in terms of outage probability at both satellite and terrestrial users. To gain further insight, we also study the area spectral efficiency of the secondary system in order to investigate the impact of interference temperature on the average number of successful transmitted symbols. The analysis presented here adds valuable insights to recent works on CSTNs.

B. Contributions

The main contributions of the paper can be summarized as follows:

- We have presented a more general model of CSTN where a multi-beam satellite shares resources with randomly distributed BSs (equipped with multiple antennas) as long as the interference temperature constraint imposed by the satellite system is satisfied.
- We have presented analysis of this network under three schemes of limiting interference generated by the secondary system.

  - Power constraint to limit interference (PCI): in this method, the only participating BSs are those that satisfy the primary systems requirements. This requirement is satisfied by restricting the transmit power at the BSs.
  - Directional beamforming to control interference (DBI): here, a transmitting BS utilizes directional beamforming to focus the intended signal to the user,
thus restricting interference to the primary network within required limits.

- BS thinning process to restrict interference (BTPI): the assumption in this method is that not all BSs would satisfy the constraint set by the primary network. These non-satisfying BSs are thinned out so that only the subset of BSs that satisfy the constraint participate in communication.

- To analyse the performance of this network, we introduce two important metrics: outage probability to measure the effect of interference from BSs other than the intended BS on both satellite and terrestrial communication, and area spectral efficiency to investigate the impact of interference temperature on spectrum efficiency at the secondary system.

- We also provide a detailed analysis on the effect of channel fading, BS node density and signal-to-interference-plus-noise ratio (SINR) threshold on a CSTN.

- Via numerical results, we show the effective trade-off between outage probability performance and number of antennas at each BS and terrestrial user. In addition, BTPI is the best scheme of secondary transmission in a CSTN because of its strict adherence to the satellite system’s requirements thereby producing least interference to the satellite user of the three schemes. Finally, where thinning is not feasible, for a conventional terrestrial mobile system, restricting the transmit power at the terrestrial BS (PCI) is the viable option.

**Notations:** We use upper and lower case to denote cumulative distribution functions (CDFs) and probability density functions (PDFs) respectively. \( \Phi \) denotes the real plane, Probability is denoted by \( P \), expectation by \( \mathbb{E}[\cdot] \), and \( \exp(\cdot) \) are used interchangeably to represent the exponential function, and all other symbols will be explicitly defined wherever used.

The rest of the paper is organized as follows. Section II describes the system model. The transmission characterization of multi-beam CSTN is presented in Section III. Section IV gives the numerical analysis, followed by the conclusion in Section V.

### II. SYSTEM MODEL

We consider the downlink of a multi-beam CSTN consisting of a satellite whose coverage area is served by \( K \) spot beams (known as the primary system) and terrestrial BSs sharing resources with the satellite to communicate with a terrestrial user (secondary system) as shown in Fig. 1. \( h_{pp} \) and \( h_{cc} \) represent the direct channel links from the satellite and a given BS to their respective users, while \( h_{pc} \) and \( h_{cp} \) are the interference links from satellite to terrestrial user and from BS to satellite user respectively.

In the primary system, the satellite transmits to users using \( K \) beams. The users are geographically scattered from which a cluster of \( K \) beams are formed. Without loss of generality, a single feed per beam is assumed. Thus, each beam is paired with a single user at a given instance. To manage interference between adjacent beams and reduce the round trip delays, multiple gateways (GWs) have been proposed to manage clusters of beams so that distributed joint processing can be utilized [23]. However, in this paper we focus on a single gateway (GW) which manages a cluster of \( K \) beams with an ideal link between satellite and GW. It is assumed that perfect channel state information is obtainable at the GW\(^1\); these assumptions are typical in [3], [17], and [24].\(^2\) To reduce the expense of backhauling, joint processing is performed at the GW so that each of \( K \) user’s signal is jointly precoded and transmitted across all beams [3]. In addition, zero-forcing (ZF) precoder for interference management between beams is considered.\(^3\)

In the secondary system, the underlay cognitive paradigm is employed which allows the terrestrial BSs to transmit concurrently with the satellite as long as interference to the primary user is below a certain threshold.

#### A. Network Model

In this section, we illustrate our system model of a downlink multi-beam CSTN consisting of multiple satellite users with terrestrial BSs serving their desired user. The satellite users in the network are modelled as points in \( \mathbb{R}^2 \) which are distributed uniformly in the beam radius as a homogeneous Poisson point process (PPP), \( \Phi_U \) with intensity \( \lambda_U \) as illustrated in Fig. 2. We assume that a cluster of \( K \) beams is formed of users geographically close together, in other words, the users in a Voronoi cell comprise a cluster resulting in a coverage area that make up a Voronoi tessellation on the plane. Hence, the total number of beams, \( K \), can be determined with the help of \( \lambda_U \). The BSs are also modelled as points of a uniform PPP, \( \Phi_{BS} \) with

\(^1\)It is an assumption in this paper that the gateway contains information about the deployment of BS nodes in the secondary system attempting to share resources with the satellite so that the value of the interference temperature constraint is set according to the number of active nodes.

\(^2\)Admittedly, obtaining perfect CSI at the GW is difficult since satellite communication systems experience long round trip delays from the GW to users. However, these studies state that reliable CSI is obtainable by the consideration of fixed satellite services. In addition, recent research efforts are considering precoding paradigms to reduce the dependence of effective precoding on accurate CSI, see [4], [25], [26].

\(^3\)Although, other precoding schemes have been investigated in recent satellite literature, we consider ZF as a simple linear precoder, shown to improve spectral efficiency with a 20–50 % in [3].
B. Satellite System Model

1) Fading Model: We assume that the forward link contains both the line-of-sight (LOS) component and the scatter component. Hence, consider $\Omega$ to be the average power of the LOS term, $b_0$ as half of the average power of scattered component, and $m$ as the Nakagami fading coefficient by definition. Leveraging the results from [27], the Shadowed-Rician (SR) fading model can be considered to model both the LOS and scatter components. Therefore the probability density function (PDF) can be written as

$$f_{\Omega|\phi}(x) = \frac{2mb_0}{2mb_0 + \Omega^2} \left( 1 - \exp\left( -\frac{x}{2b_0} \right) \right) \times \frac{\Omega^m}{b_0} \frac{x^{m-1}e^{-\frac{\Omega}{b_0}x}}{\Gamma(m)}$$

where $I_1F_1$ is the hypergeometric function and the parameters $b_0$, $m$, and $\Omega$ are connected with the elevation angle $\theta$ as illustrated in Fig. 1. We omit the corresponding expressions of parameters $b_0$, $m$, and $\Omega$ as they are characterized in detail in [27]. Although the SR fading model is widely used in literature, the PDF and cumulative density function (CDF) are too complex to work with SINR expressions. Therefore, we approximate the squared SR model with Gamma random variable. Accordingly, the parameters of Gamma random variable are given as [27]

$$\alpha_s = \frac{m(2b_0 + \Omega)^2}{4mb_0^2 + 4mb_0\Omega + \Omega^2}, \quad \beta_s = \frac{4mb_0^2 + 4mb_0\Omega + \Omega^2}{m(2b_0 + \Omega)}$$

2) Antenna Gain at Satellite User Terminal: It is worth noticing that the average SINRs are highly dependent on both satellite beam pattern and user position. Therefore, the beam gain can be approximated as [3]

$$G_{ii} = L_{\text{max}} G_{si} G_{ri} \left( \frac{J_1(x)}{2x} + \frac{36 J_3(x)}{x^3} \right)^2$$

where $L_{\text{max}}$ is the free space loss [24].

$$x = 2.07123 \sin(\phi_\text{d}) / \sin(\phi_\text{d,ref}), \quad J_1 \text{ and } J_3 \text{ are the first-kind Bessel functions of order 1 and 3, } G_{si} \text{ is the satellite transmit antenna gain for the } i\text{th beam and } G_{ri} \text{ is the satellite user's receive antenna gain. Note that } \phi_\text{d} \text{ is denoted as the off-axis angle of the } i\text{th desired beam, and } \phi_\text{d,ref} \text{ is the off-axis angle from the } i\text{th desired beam to the center of the } j\text{th interfering beam. Therefore, } G_{ii} \text{ can be calculated from (3) with } \phi_\text{d}. \text{ Similarly, } G_{ij} \text{ which is the observed antenna gain between the } j\text{th interfering beam and the } i\text{th user, is also calculated by (3) in terms of } \phi_\text{d}.$$ 

C. Terrestrial System Model

1) Fading Model: The impact of small scale fading on the transmitted signals of cellular networks is higher than satellite systems. The extensive study of cellular networks in [29] and [30] show that the Nakagami fading model can capture a generalised propagation environment. Hence, we consider Nakagami-$m$ channel model, and the channel power is distributed according to

$$h_i \sim f_i(x; m_i) \approx \frac{m_i^x x^{m_i - 1} e^{-m_i x}}{\Gamma(m_i)},$$

where $i \equiv cc, cp$, and $\Gamma(m_i)$ is the gamma function.

2) Directional Beamforming Model: In order to reduce the impact of terrestrial interference on the satellite user terminals, we employ directional beamforming at BSs [20], [31]. Accordingly, multiple antenna arrays are deployed at the transmitters. It is worth noticing that the receiver, i.e., terrestrial user is also equipped with directional antennas. We consider static beamforming though sectorized antennas. Hence, we assume that all the antennas at transmit and receiver pairs are directional antennas with sectorized gain patterns. Let $M_{\text{BS}}$ denote the number of transmit antennas at a BS and $M_{\text{R}}$ denote receive antennas which could either be a satellite or terrestrial user. Denoting the in-sector antenna array gain as $G_q^m$ and the out-of-sector antenna array gain as $G_q^m$ respectively, these gains are expressed as [32]

$$G_q^m = \frac{M_q}{1 + \delta_q (M_q - 1)}, \quad G_q^m = \delta_q G_q^m,$$

where $q \in \{ \text{BS, R} \}$, $\delta_q$ is a factor that measures the ratio of main lobe to side lobe level. We assume adaptive beamforming at the BSs such that active transmission link is that where maximum gain can be achieved. Thus, for any intended link, $q$ (i.e., the transmission link between a given BS and the terrestrial user), the beamforming gain, $G_q = G_q^m G_q^m$. The gains

$$G_q = G_q^m G_q^m$$

We assume the satellite channel is quasi-stationary which implies that the environmental characteristics including the effect of rain attenuation can be neglected. This is leveraging on the results of experimental data from [28] that shows that the environmental attributes of the channel are assumed to be constant within a small area.
of links other than the intended link will be denoted as $G_i$. $G_i$ also depends on the in-sector directivity gains (i.e., $G^M$) and out-of-sector (i.e., $G^P$) gains of the antenna beam pattern. Accordingly, the effective antenna gain for an interferer seen by the terrestrial user is given by

$$G_i = \begin{cases} 
G^M_{BS} G^M_R, & \mathcal{P}_{MM} = \frac{1}{M_{BS} M_R (M_{BS} - 1) (M_{RS} - 1)}, \\
G^M_{BS} G^m_R, & \mathcal{P}_{mm} = \frac{1}{M^2_{BS} M_R (M_{BS} - 1) (M_{RS} - 1)}, \\
G^m_{BS} G^m_R, & \mathcal{P}_{mm} = \frac{1}{M^2_{BS} M_R (M_{BS} - 1) (M_{RS} - 1)}. 
\end{cases}$$

(6)

where $\mathcal{P}_{jk}$, with $j, k \in \{M, m\}$ denotes the probability that the antenna gain $G^G$ is seen by the receiver. Here, the effective gain can be considered as a random variable, which can take any of the above-mentioned values.

**D. Signal Model**

1) Satellite Received Signal: The overall channel gain between the $j$th beam and $i$th user of the satellite can be given as

$$h^i_{pp} = h^i_{pp} G_j (\phi_i)^{1/2}, \quad i, j = 1, \ldots, K.$$  

(7)

Consider $P_{sat}$ as the satellite transmit power of $i$th beam, and $x^i_p$ as the transmitted information symbol from beam $i$. The received signal at $i$th beam user can be formulated as

$$y_i = \sqrt{P_{sat}} h^i_{pp} x^i_p + \sum_{j \in \Phi_j, j \neq i} \sqrt{P_{sat}} h^i_{pp} h^j_{pp} x^i_p + I_{BS} + \omega_i$$

(8)

where $\omega_i$ is the noise power at beam $i$, $P_{sat}$ is the satellite transmit power of $i$th beam, and $I_{BS}$ is the terrestrial interference given by

$$I_{BS} = \sum_{l \in \Phi_{BS}} \sqrt{P_{tot}} G_l h^l_{cp} x^l_c r^l_{li},$$

(9)

where $P_{tot}$, $x^l_c$ are the transmit power and information signal from the $l$th terrestrial BS, $r^l_{li}$ is the distance from $l$th BS to the $i$th beam of the satellite user, and $\alpha$ is the path loss exponent.

2) Terrestrial Received Signal: The received signal at the terrestrial user from the $l$th BS is represented as:

$$y_l = \sqrt{P_{tot}} G_l r^l_{ls} h^l_{pc} x^l_c + \sum_{m \in \Phi_{BS}, m \neq l} \sqrt{P_{tot}} G_l r^l_{ms} h^m_{pc} x^m_c + I_{SAT} + \omega_l,$$

(10)

where $\omega_l$ is additive white Gaussian noise $\omega_l \sim \mathcal{CN}(0, \sigma_l^2)$, $I_{SAT}$ is the interference from the satellite given by

$$I_{SAT} = \sum_{j \in \Phi_j} \sqrt{P_{sat}} G_j h^j_{pc} x^j_p,$$

(11)

and $h^j_{pc}$ is the interference channel from the $j$th beam of the satellite to terrestrial user.

To ensure a BS does not cause interference to the satellite system beyond the pre-defined threshold, $T$, its transmit power is further constrained by [14]:

$$P_{tot} = \min \left( \frac{T}{|h^l_{pc}|^2}, \frac{P_{tot}}{P_{sat}} \right).$$

(12)

where $h_{pp}$ is the interference channel from the BS to the primary user and $P_{tot}$ is the total available power at the $l$th BS.

**E. SINR Model**

In this subsection, we consider the SINR obtained at the terrestrial and satellite users respectively.

1) SINR at Terrestrial User: The SINR at the terrestrial user from the $l$th BS can be formulated from (10) and given as:

$$\zeta_l = \frac{P_{tot} G_l |h^l_{pc}|^2 r^l_{li}^{-\alpha}}{\sigma_l^2 + I_{BS} + I_{SAT}},$$

(13)

where $h^l_{pc}$ is the fading gain of the channel between $l$th and the terrestrial user, $I_{BS} = \sum_{m \in \Phi_{BS}, m \neq l} P_{m} G_m |h^m_{pc}|^2 r^m_{ls}^{-\alpha}$ is the interference from other BSs in $\Phi_{BS}$, $I_{SAT} = \sum_{j \in \Phi_j} P_{sat} G_j |h^j_{pc}|^2$ represents interferences from each beam of the satellite to terrestrial user, $r^l_{li}$ is the distance from the $l$th BS to the user, $\sigma_l^2$ is the noise power.

**SINR at Satellite User:** The SINR for the intended link $i$ at the $l$th user can then be formulated as

$$\zeta_i = \frac{P_{tot} G_i |h^i_{pp}|^2}{\sigma_i^2 + \sum_{j \in \Phi_j} P_{sat} G_j |h^j_{pc}|^2},$$

(14)

where $h^i_{pp}$ is the channel fading gain at the $i$th user, $\sigma_i^2$ is the noise power, and $h^j_{pp}$ denotes each interference fading gain from other beams to their users, $I_{BS}$ is the interference from the terrestrial system defined in (9).

The second term of the denominator in (14) is zero due to successful ZF precoding.\(^5\) Hence, the SINR for the intended link $i$ at any particular user considering terrestrial interference can be re-written as

$$\tilde{\zeta}_i = \frac{P_{tot} G_i |h^i_{pp}|^2}{\sigma_i^2 + \sum_{l \in \Phi_{BS}} P_{tot} G_l |h^l_{pc}|^2 r^l_{li}^{-\alpha}},$$

(15)

where $r^l_{li}$ is the distance between $l$th BS and $i$th satellite user, and $\alpha$ is the path loss exponent.

**F. Performance Metrics**

In order to analyse the performance of the system we will use the two fundamental metrics of outage probability and area spectral efficiency.

**Outage Probability:** This is the probability that outage occurs at either satellite or terrestrial user. Outage occurs when the received SINR falls below an acceptable threshold, $T_i$ that is,

$$P_{out}(T_i) = \mathbb{P}(\text{SINR} < T_i).$$

\(5\)The ZF precoder is designed using the unconstrained optimization method described in [33] such that the powers of all signals are scaled to correspond with the power increase as a result of precoding. As a result, the transmit power is maintained as the same with the case of no precoding.
Area Spectral Efficiency: This metric is presented to measure the utilization of spectrum efficiency of wireless cellular systems. It is defined as the maximum rate per unit bandwidth of a user in a defined coverage area. It can also be described as the average number of successful transmitted bits per unit area and is therefore determined by the outage probability, $P_{\text{out}}$. Area spectral efficiency, $\eta_{AE}$, is expressed as [34]

$$\eta_{AE} = \lambda_{\text{BS}} (1 - P_{\text{out}}) \log_2 (1 + T_t),$$

where $T_t$ is the SINR threshold, and $\lambda_{\text{BS}}$ is the BS node density.

III. Transmission Characterisation in Multi-Beam CSTN

Here, we study the performance of the multi-beam CSTN from the perspective of outage probability and area spectral efficiency. In the context of this system model which permits simultaneous transmission of both satellite and terrestrial BSs to their respective users, we consider three practical scenarios. First is the analysis under assumption that all terrestrial BSs obey the constraint by using a limited transmit power defined in (12), (PCI). Second, we investigate the impact of using directional beamforming at the secondary system to limit interference, (DBI). And third, based on the assumption that not all BSs deployed in the secondary system will meet the requirements for transmission, we perform thinning and analyse only the subset of BSs that meet this constraint (BTPI).

Remark 1: The analysis in the paper is done for the outage probability of both satellite and terrestrial systems. However, the area spectral efficiency analysis presented here is done only for the terrestrial system. The main idea behind this consideration is to measure the impact of interference temperature constraint imposed by the satellite on spectral utilization efficiency at the terrestrial system.

A. PCI: Power Constraint to Limit Interference

In this transmission method, we assume that the terrestrial system is equipped with omnidirectional antennas (i.e., no beamforming is in use). Hence, to manage the interference the terrestrial system causes to the satellite system, the transmission power of terrestrial BSs is limited by the interference constraint imposed by the satellite. We also assume that the terrestrial BSs and users utilize single antennas for transmission. Thus, in the sequel we assess the impact of limited transmit power on the outage performance of the both satellite and terrestrial users. The property of joint random variables is used to quantify the limited transmission power and the interferences from the satellite and terrestrial system as the case requires are characterized by the use of moment generating functions and Laplacian functionals respectively.

Outage Probability at the Terrestrial User: At the terrestrial user, outage occurs when the SINR falls below the threshold, $T_t$. The outage probability from the $i$th BS is given at the top of the next page where

$$\mathbb{E}_{l_{BS}} \left[ \exp \left( \frac{-A_k r_i^\alpha T_t l_{BS}}{P_{\text{tot}}} \right) \right] \tag{20}$$

and

$$f_r(y) = \frac{m_{cp} y^{m_{cp}-1} e^{-m_{cp} y}}{\Gamma(m_{cp})}, \tag{21}$$

where $m_{cp}$ is the Nakagami fading parameter of the interference channel, $\gamma(.,.)$ is the lower incomplete gamma function, and $\Gamma(m_{cp})$ is the gamma function of $m_{cp}$, and

$$\mathbb{E}_{l_{S_AT}} \left[ \exp \left( \frac{-A_k r_i^\alpha T_t l_{S_AT}}{P_{\text{tot}}} \right) \right] \tag{22}$$

where $\beta_i$ and $\alpha_i$ are gamma distribution random variable parameters of the satellite.

Proof: Refer Appendix A.

B. Special Case: Approximating BS Interference Using Gamma Variable and Negligible Satellite Interference

The characterisation of BS interference from Proposition 1, equation (20) is provided in terms of Laplacian and probability generating functionals for which closed forms only exist for special choices of its parameters and distribution. Therefore, in order to obtain a more tractable model, we pursue this interference characterisation in terms of their cumulants [35]. Under Rayleigh fading assumption, we approximate the BS interference distribution using the gamma model. In most modern cognitive-satellite networks, the satellite interference to the terrestrial user is not an essential consideration due to its negligible magnitude compared to the larger values of intra cluster interference power.

Under this consideration of, the distribution of the equivalent aggregate of BS interference path gain is given as

$$\tilde{l}_{BS} = \sum_{m \in \Phi_{BS}} |h_{cc}^m|^{2r_m^{\alpha}}. \tag{23}$$

By the use of Campbell’s theorem, the characteristic function of $\tilde{l}_{BS}$ is computed as [36]

$$\phi_{\tilde{l}_{BS}}(w) = \exp \left( -2\pi \lambda_{BS} \int_{h_{cc}} \left[ 1 - e^{iwx_{\infty}} \right] \cdot f_{h_{cc}}(x) \, dx \right) \tag{24}$$

where $j = \sqrt{-1}$. Using equation (24), we can obtain the corresponding closed forms of the cumulants. Specifically, the $n^{th}$
\[ P_{out}(T_i) = \gamma \left( \frac{\gamma_{cT_i}^{(m_{cc})}}{\Gamma(m_{cc})} \right) \sum_{k=0}^{m_{cc}} \frac{m_{cc}}{k} (-1)^k e^{-\frac{\gamma_{cT_i}^{(m_{cc})}}{\Gamma(m_{cc})}} \int_{P_{tot}}^{\infty} \mathbb{E}_{\text{BS}} \left[ e^{-\frac{\gamma_{cT_i}^{(m_{cc})}}{\Gamma(m_{cc})}} \right] e^{-\frac{\gamma_{cT_i}^{(m_{cc})}}{\Gamma(m_{cc})}} f_I(y) \, dy \tag{19} \]

where \( t = \frac{\gamma_{cT_i}^{(m_{cc})}}{\Gamma(m_{cc})} \).

\[ P_{out}(T_i) = \gamma \left( \frac{\gamma_{cT_i}^{(m_{cc})}}{\Gamma(m_{cc})} \right) \sum_{k=0}^{m_{cc}} \frac{m_{cc}}{k} (-1)^k e^{-\frac{\gamma_{cT_i}^{(m_{cc})}}{\Gamma(m_{cc})}} \int_{P_{tot}}^{\infty} \mathbb{E}_{\text{BS}} \left[ e^{-\frac{\gamma_{cT_i}^{(m_{cc})}}{\Gamma(m_{cc})}} \right] e^{-\frac{\gamma_{cT_i}^{(m_{cc})}}{\Gamma(m_{cc})}} f_I(y) \, dy \tag{19} \]

Proposition 4: The outage probability at the terrestrial user from the \( i^{th} \) BS employing directional beamforming for transmission is given as

\[ P_{out}(T_i) = \sum_{k=0}^{m_{cc}} \left( \frac{m_{cc}}{k} \right) (-1)^k e^{-\frac{\gamma_{cT_i}^{(m_{cc})}}{\Gamma(m_{cc})}} \int_{P_{tot}}^{\infty} \mathbb{E}_{\text{BS}} \left[ e^{-\frac{\gamma_{cT_i}^{(m_{cc})}}{\Gamma(m_{cc})}} \right] e^{-\frac{\gamma_{cT_i}^{(m_{cc})}}{\Gamma(m_{cc})}} f_I(y) \, dy \tag{31} \]

Proof: From the proof of Proposition 1, we have

\[ P_{out}(T_i) = \sum_{k=0}^{m_{cc}} \left( \frac{m_{cc}}{k} \right) (-1)^k e^{-\frac{\gamma_{cT_i}^{(m_{cc})}}{\Gamma(m_{cc})}} \int_{P_{tot}}^{\infty} \mathbb{E}_{\text{BS}} \left[ e^{-\frac{\gamma_{cT_i}^{(m_{cc})}}{\Gamma(m_{cc})}} \right] e^{-\frac{\gamma_{cT_i}^{(m_{cc})}}{\Gamma(m_{cc})}} f_I(y) \, dy \tag{32} \]
However, the terrestrial interference due to other BSs needs to be characterized before proceeding. Given that the interference from BSs could be either from main lobe or side lobe as defined in (6), we utilize the notion of marked stochastic geometry to characterize the interference as [40]

\[ \Phi_{BS} = \Phi_{BS}^{MM} + \Phi_{BS}^{Mm} + \Phi_{BS}^{mM} + \Phi_{BS}^{mm}. \]  

By definition of the Laplace transform, we have

\[ \mathcal{L}\{I_{|\Phi_{BS}}\} = \mathcal{L}\{I_{|\Phi_{BS}}^{MM}\} + \mathcal{L}\{I_{|\Phi_{BS}}^{Mm}\} + \mathcal{L}\{I_{|\Phi_{BS}}^{mM}\} + \mathcal{L}\{I_{|\Phi_{BS}}^{mm}\}. \]  

Starting with the characterisation of \( \mathcal{L}\{I_{|\Phi_{BS}}^{MM}\}(s) \), we obtain

\[
\begin{align*}
\mathcal{L}\{I_{|\Phi_{BS}}^{MM}\}(s) &= \mathbb{E}[\exp(-s I_{|\Phi_{BS}}^{MM})] \\
&= \mathbb{E}_{G_i \sim |\Phi_{BS}} \left[ \prod_{m \in |\Phi_{BS}} \frac{1}{1 + s P_m G_i^{MM} m_{cc}} \right] \\
&= \mathbb{E}_{G_i \sim |\Phi_{BS}} \left[ \exp \left[ -2\pi P_{MM}^{\lambda_{BS}} \right] \right].
\end{align*}
\]

(35)

where \( P_{MM} \) is the probability that \( G_i^{MM} = G_i^{MM}, s = \frac{A k r_i}{P_m G_i^{MM}}, \) (a) follows from the use of the moment generating function of Gamma random variable with Nakagami fading parameter \( m_{cc} \), and (b) follows due to the use of probability generating functionals of PPPs. Following similar steps, \( \mathcal{L}\{I_{|\Phi_{BS}}^{Mm}\}, \mathcal{L}\{I_{|\Phi_{BS}}^{mM}\}, \mathcal{L}\{I_{|\Phi_{BS}}^{mm}\} \) can be computed and finally, using equation (34), the Laplace transform of \( I_{BS} \) is given as

\[
\begin{align*}
\mathcal{L}\{I_{|\Phi_{BS}}\}(s) &= \mathbb{E}[\exp(-s I_{|\Phi_{BS}})] \\
&= \prod_{t,k \in [M,m]} \mathbb{E}_{G_i \sim |\Phi_{BS}} \left[ \exp \left[ -2\pi P_{BS}^{\lambda_{BS}} \left( 1 - \frac{1}{1 + \frac{s P_m G_i^{MM} m_{cc}}{m_{cc} r_i^2}} \right) \right] \right],
\end{align*}
\]

(36)

where \( r_m \) is the distance between the \( m^{th} \) BS and the terrestrial user. The characterisation of \( \mathcal{L}\{I_{SAT}\}(s) \) has been outlined in Appendix A and is expressed as

\[
\mathcal{L}\{I_{SAT}\}(s) = \exp \left[ -2\pi \lambda_U \left( 1 - \frac{1}{1 + \frac{s G_i P_i}{\beta_i}} \right) \right].
\]

(37)

where \( s = \frac{A k r_i}{P_m G_i^{MM}}, \alpha_s \) and \( \beta_s \) are the gamma distribution parameters of the satellite given in (2).

This proof is concluded by substituting (36) and (37) into (32).

**Outage Probability at Satellite User:** In the following lemma we measure the impact of employing directional beamforming at the terrestrial BS in terms of outage probability at the satellite user.

**Lemma 1:** The outage probability of at the \( k^{th} \) user of the satellite considering directional beamforming at the terrestrial system is given as

\[
\begin{align*}
P_{out}(I_{|\Phi_{BS}}) &\approx \sum_{t=0}^{\alpha} \left( \frac{\alpha}{t} \right) (-1)^t \exp \left[ -\alpha \sigma_i^2 \right] \\
&\times \exp \left[ -2\pi P_{BS}^{\lambda_{BS}} \left( 1 - \frac{1}{1 + \frac{s G_i P_i}{P_m G_i^{MM} m_{cc} r_i^2}} \right) \right],
\end{align*}
\]

(38)

where \( r_{i,t} \) is the distance from the \( i^{th} \) BS to the \( t^{th} \) satellite user.

**Proof:** The proof follows from Proposition 4.

**Remark 2:** It is important to note that with single transmit and receive antennas, directional beamforming cannot be used to manage the interference. Hence, limiting the transmit power of the terrestrial system as in PCI is the method employed. In other words, when \( M_{BS} = M_R = 1 \), then DBI reduces to PCI.

**D. BTPI: BS Thinning Process to Restrict Interference**

In this subsection, we characterize BSs which do not satisfy the interference constraint imposed by primary system. As some of the BSs may not provide sufficient coverage for the terrestrial user, and those BSs may override the interference temperature constraint set by satellite system and may cause harmful interference to primary users, leading to a deterioration of the system's performance. In such conditions, one can make use of a thinning operation on the original PPP of BSs, leading to the well-known Matern Hard-core point process (MHCPP) that has been used to appropriately model networks with guard zones [41].

Additionally, for power constrained terrestrial systems, the characterisation of hardcore models of point processes needs to take into consideration fading and interference constraint. In this regard, thinning with respect to fading is considered. We leverage the results from [41] and [42] and incorporate thinning in the design aspects of our system model. The characterization of HCPP models via the Laplace functional and probability generating functionals is quite difficult to analyse.
where a bounded region can be defined as the locations of any two BSs in $P_t$. This probability is expressed as becomes neighbour of the BS at $r_{t}$ distance, such that the probability that any BS located beyond region, $\phi_1$ is the hypergeometric regularised function, $m_{ij}$ is the Nakagami fading parameter from the distribution of $h_{ij}$ and Csc is cosecant function.

From the above analysis, the outage probability at the terrestrial and satellite users can be computed with the updated density, $\lambda_\text{BS}$, by following steps similar to proposition 3 and lemma 1 respectively.

IV. Numerical Results

As previously mentioned, we have analysed three different methods of limiting interference caused by terrestrial communication to the satellite network. In this section, we provide numerical results to validate our system model and present comparison of these three interference limiting schemes. We also verify the accuracy of theoretical results presented in the previous section showcasing the performance metrics of outage probability and area spectral efficiency. The parameters considered for simulation in this paper are inspired from related studies on CSTNs, satellite and cellular communications [16], [27], [31] and the correctness of the analytical results is verified through Monte Carlo simulations. For the primary satellite network, we consider a $K$-beam network with an orbit radius of 35786 km where the intensity of satellite users is expressed as $\lambda_U = \frac{K}{\pi r_F^2}$ where $K$ is any integer that indicates the average number of users/beams being served by the satellite. A few of the parameters with their corresponding values are presented in Table I. All other parameters will be explicitly mentioned wherever used.

Figures 3 to 5 illustrate the impact of limiting terrestrial BS transmit power using the imposed interference temperature constraint (PCI). In Fig. 3, we compare the outage probability performance with different values of satellite imposed interference temperature constraint at the terrestrial user. This result is a validation of proposition 1. It can be observed that the simulation results obtained from the numerical evaluation of equation (19) are consistent with the analytical derivations, as shown by the matching of these results. As can be seen, with increasing values of interference temperature constraint, $\Upsilon$, the outage probability performance is considerably lower. This result is expected as increasing the interference temperature constraint implies that the terrestrial BS can transmit

\[ P_2 = \left[ \frac{\pi C_{\text{ex}}}{\Gamma[\pi]} \frac{\left( \pi - m_{ij} \frac{m_{ij} \Upsilon}{\beta} \right)^{m_{ij}}}{\Gamma[m_{ij}]} \frac{\left[ \pi + m_{ij} \frac{m_{ij} \Upsilon}{\beta} \right]^{m_{ij}}}{\Gamma[m_{ij}]} \frac{p}{F_T} \right] 
+ \left[ \frac{m_{ij}}{m_{ij}^2} \frac{m_{ij} \Upsilon}{\beta} \right]^{m_{ij}} \frac{\left[ \pi + m_{ij} \frac{m_{ij} \Upsilon}{\beta} \right]^{m_{ij}}}{\Gamma[m_{ij}]} \frac{p}{F_T} \right] 
+ \left[ \frac{m_{ij}}{m_{ij}^2} \frac{m_{ij} \Upsilon}{\beta} \right]^{m_{ij}} \frac{\left[ \pi + m_{ij} \frac{m_{ij} \Upsilon}{\beta} \right]^{m_{ij}}}{\Gamma[m_{ij}]} \frac{p}{F_T} \right] 
(42) \]

Using (41), we can derive the generalised MHCPP process of the BSs and their active nodes which satisfy the interference constraint. Therefore, the closed-form expression of the above integral is given at the top of this page, where $p/F_T$ is the hypergeometric regularised function, $m_{ij}$ is the Nakagami fading parameter from the distribution of $h_{ij}$ and Csc is cosecant function.

\[ P_2 = \left[ \frac{\pi C_{\text{ex}}}{\Gamma[\pi]} \frac{\left( \pi - m_{ij} \frac{m_{ij} \Upsilon}{\beta} \right)^{m_{ij}}}{\Gamma[m_{ij}]} \frac{\left[ \pi + m_{ij} \frac{m_{ij} \Upsilon}{\beta} \right]^{m_{ij}}}{\Gamma[m_{ij}]} \frac{p}{F_T} \right] 
+ \left[ \frac{m_{ij}}{m_{ij}^2} \frac{m_{ij} \Upsilon}{\beta} \right]^{m_{ij}} \frac{\left[ \pi + m_{ij} \frac{m_{ij} \Upsilon}{\beta} \right]^{m_{ij}}}{\Gamma[m_{ij}]} \frac{p}{F_T} \right] 
+ \left[ \frac{m_{ij}}{m_{ij}^2} \frac{m_{ij} \Upsilon}{\beta} \right]^{m_{ij}} \frac{\left[ \pi + m_{ij} \frac{m_{ij} \Upsilon}{\beta} \right]^{m_{ij}}}{\Gamma[m_{ij}]} \frac{p}{F_T} \right] 
(42) \]
TABLE I

SIMULATION PARAMETERS

<table>
<thead>
<tr>
<th>Notation</th>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_0$</td>
<td>Orbit</td>
<td>35786 Km</td>
</tr>
<tr>
<td>$r_d$</td>
<td>Beam radius</td>
<td>50 km</td>
</tr>
<tr>
<td>$G_{sat}$</td>
<td>Satellite antenna gain</td>
<td>30 dB</td>
</tr>
<tr>
<td>$G_{t}$</td>
<td>Satellite terminal gain</td>
<td>15 dB</td>
</tr>
<tr>
<td>$3dB$</td>
<td>Angle</td>
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</tr>
<tr>
<td>$\phi_{su}$</td>
<td>Off-axis angle of desired user</td>
<td>0.6°</td>
</tr>
<tr>
<td>$\phi_{iu}$</td>
<td>Off-axis angle of interfering user</td>
<td>0.8°</td>
</tr>
<tr>
<td>$\lambda_{BS}$</td>
<td>Density of users</td>
<td>$1 \times 10^{8}$</td>
</tr>
<tr>
<td>$G_{m}^{\text{main}}$</td>
<td>BS antenna gain of main lobe</td>
<td>15 dB</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Path loss exponent</td>
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</tr>
<tr>
<td>$P_{t,\text{BS}}$</td>
<td>Node transmit power</td>
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</tr>
<tr>
<td>$m_{cc}, m_{cp}$</td>
<td>Nakagami parameter</td>
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</tr>
<tr>
<td>$N_0$</td>
<td>Noise power</td>
<td>$-174$ dB</td>
</tr>
</tbody>
</table>

Fig. 3. Outage probability as a function of SINR threshold of the secondary network under different satellite interference temperature constraints, $\Upsilon$, and $P_{\text{tot}} = 20$ dB.

Fig. 4. Outage probability as a function of SINR threshold of the secondary network with varying BS node density under different satellite interference temperature constraints, $\Upsilon$.

Fig. 5. Outage probability at the satellite user as a function of SINR threshold for varying interference temperature constraints, $\Upsilon$, $P_{\text{tot}} = 20$ dB.

Fig. 6. Outage probability at the terrestrial user as a function of SINR threshold for varying BS node density with varying antenna gain.

leads to a decrease in outage probability. This outcome can be explained by the fact that a higher density of BSs (implying more deployed BSs) indicate that there are many more BSs to interfere with the intended transmission to the terrestrial user. Also, confirming the results from Fig. 3, the outage probability is lower for $\Upsilon = 15$ dB in both cases of $\lambda_{BS}$ when compared with values for $\Upsilon = 10$ dB.

In Fig. 5, we analyse the outage probability at the satellite user with respect to restricting the transmit power of the terrestrial base stations. To provide more insight on the impact of constraint in the CSTN, we compare these results to the case of no interference (non-transmitting terrestrial BSs). It can be seen from the figure that outage probability is appreciably lower with decreasing values of interference temperature constraint. This result is in contrast to the observations of varying constraint at the terrestrial user in Fig. 4, and this outcome implies that lowering the values of interference temperature constraint produces more rigidity in restraining the transmission power of terrestrial BSs, which then results in noticeably lower interference to the satellite user and lesser probability of outage. In addition, we provide simulation results of the satellite channel using the SR fading model; as can be observed from the figure, the simulations are closely matched with the simulations using the Gamma random variable approximation with more power, which in turn leads to more successful communication with the terrestrial user.

After establishing that increased interference temperature constraint has a positive impact on terrestrial communication, we now consider the effect of node density, $\lambda_{BS}$, on the outage. Hence, in Fig. 4, we present a plot of outage probability against SINR threshold at the terrestrial user for varying values of $\lambda_{BS}$ and $\Upsilon$. As can be observed, reducing the BS density with more power, which in turn leads to more successful communication with the terrestrial user.
for the channel. This result is an affirmation of the channel approximation we used in our analysis.

Next, we consider the use of directional beamforming for transmission in the terrestrial system. Fig. 6 presents a comparison of outage probability with different BS densities and antenna gains at the terrestrial user. This result verifies proposition 4 as shown by the minimal performance gap between simulation and analytical results. It can be observed that when the antenna gain is increased, there is a reduction in outage probability. For example, when $\lambda = 0.000001$, for a specific threshold of 10 dB, the outage probability is 0.5 when $M_{BS} = M_r = 8$ whereas when utilizing 32 antennas at both BS and user, the outage probability reduces to 0.1. This result indicates that directional beamforming has a direct effect on the SINR threshold as an increase in the directional beamforming gain results in a reduction in the target SINR threshold required for good coverage. It is also evident from the figure that a higher network density yields more outage for a target SINR value.

The impact at the satellite user of utilizing directional beamforming for terrestrial transmission and interference mitigation is shown in Fig. 7. It can be identified from the figure that as BS nodal density increases, the probability of outage at the satellite user also increases similar to the effect at the terrestrial user. Also worthy of note, deploying more BSs in the terrestrial network increases the aggregate interference caused to the satellite user.

Next, we present the analysis of thinning out all BSs that do not satisfy the interference temperature constraint imposed by the satellite, as discussed in Section III. After thinning, $\lambda_{BS}$ is computed using lemma 2 so that, $\lambda_{BS} = \lambda_{BS}^0/N_{BS}$. Accordingly, in Figures 8 and 9, we present a comparison of outage probability by using all three methods of PCI, DBI and BTPI.

Fig. 8 plots the outage probability as a function of SINR threshold at the terrestrial user. It is evident from the figure that for a fixed interference temperature constraint $\Upsilon = 0$ dB, BTPI has the best performance giving the least outage probability for a given target SINR. What is striking about the performance of DBI is its dependence on the antenna array size. Increasing the number of transmit and receive antennas reasonably reduces the outage probability, but this comes at a cost. We note that the gains of employing directional beamforming are optimal when utilizing massive multiple input-multiple output (MIMO) systems, or employing millimeter wave links at the terrestrial system because each of these methods allow for a large array of antennas. This can be investigated in our future work.

Fig. 9 considers the impact of using all three schemes at both the satellite user and terrestrial user using three methods for $\Upsilon = 10$ dB, $M_{BS} = M_r = 16$. Therefore, for a conventional multi-beam CSTN, where thinning is not feasible, PCI is a more viable scheme than DBI but at cost of moderate interference to satellite user.
Finally, in Fig. 10, we illustrate the area spectral efficiency at the terrestrial user with respect to SINR threshold under different values of $\Upsilon$. It can be seen from the figure that for higher values of interference temperature constraint, the area spectral efficiency increases, which implies that the terrestrial BS can transmit with more power. This outcome is the evidence for reduced outage probability observed at the terrestrial user for increasing values of $\Upsilon$. It is worthy of mention that there is an optimal value of area spectral efficiency as indicated by the shape of the curves in Fig. 10 with the implication that increasing the SINR threshold has a diminishing returns effect. Further, when the optimal SINR threshold is determined, this can be used to determine the optimal BS density which maximises the area spectral efficiency of the terrestrial system whilst taking into account the constraint imposed by the satellite system. Determination of these optimal points can be explored in future works.

V. CONCLUSION

The impact of interference in a multi-beam CSTN was investigated. From our analysis, it is clear that successful transmission at both satellite and terrestrial systems depends on network conditions such as BS node density, antenna gain, and interference temperature constraint imposed by the satellite. Accordingly, performance metrics of outage probability and area spectral efficiency were analysed. With simulation results we show the effect of varying the network parameters such as BS node density and the value of interference temperature constraint on the network. After comparing three secondary system transmission schemes (PCI, DBI and BPTI) aimed at keeping interference to the satellite system within the predefined limits, we observed that for a target SINR, BTI (which strictly adheres to the satellite’s requirements) gives the best performance. We also showed that for conventional terrestrial mobile networks, DBI performed the worst. However, the performance when utilizing directional beamforming can be improved at the cost of increasing the antenna gain. In practical scenarios, this would mean employing massive MIMO transceivers or millimeter wave links at the terrestrial system. In addition, when BS thinning is not feasible, restricting the transmit power at the terrestrial BS by lowering the value of interference temperature constraint is the viable method to obtain reduced outage probability of the satellite communication.

APPENDIX A

PROOF OF PROPOSITION 1

The terrestrial user experiences outage when its SINR\(^8\) falls below the predefined threshold $T_t$ such that:

$$\mathcal{P}_{\text{out}}(T_t) = \mathcal{P}(\text{SINR} < T_t),$$

$$= \mathcal{P} \left( \frac{P_{\text{ter}} |h_{\text{cc}}^l|^2 2^{\frac{\alpha}{1-\alpha}}}{\sigma^2 + I_{\text{BS}} + I_{\text{SAT}}} < T_t \right). \quad (43)$$

Substituting $P_{\text{ter}}$ in (43) with the interference temperature constraint defined in (12) as

$$P_{\text{ter}} = \min \left( \frac{\Upsilon}{|h_{\text{cc}}^l|^2}, P_{\text{tot}} \right), \quad (44)$$

and using the property of joint distribution of random variables $X$ and $Y$ from [43], we have:

$$\mathcal{P}(\min(X, Y) < t) = \mathcal{P}(X < t, Y < t),$$

and

$$\min(X, Y) = \begin{cases} X & \text{if } Y > X, \\ Y & \text{if } Y \leq X. \end{cases} \quad (45)$$

Therefore, (43) becomes

$$\mathcal{P}_{\text{out}}(T_t) = \mathcal{P} \left( \frac{P_{\text{tot}} |h_{\text{cc}}^l|^2 2^{\frac{\alpha}{1-\alpha}}}{\sigma^2 + I_{\text{BS}} + I_{\text{SAT}}} < T_t, P_{\text{tot}} \leq \frac{\Upsilon}{|h_{\text{cc}}^l|^2} \right) \quad (46)$$

Let $\Gamma = |h_{\text{cc}}^l|^2$. The outage probability conditioned on $\Gamma$ is defined as:

$$\mathcal{P}_{\text{out}}(\Gamma)(T_t) = \int_0^{\frac{\Upsilon}{P_{\text{tot}}}} \mathcal{P} \left[ \frac{P_{\text{tot}} |h_{\text{cc}}^l|^2 2^{\frac{\alpha}{1-\alpha}}}{\sigma^2 + I_{\text{BS}} + I_{\text{SAT}}} < T_t \right] f_{\Gamma}(y) \, dy \quad (47)$$

\(^8\)In this scenario, since we limit the interference using interference temperature constraint, the beamforming gain, $G_t = 1$ and is omitted for subsequent analysis.
Given that fading of the channel of the $p^{th}$ BS, $h_{cc}^l$, follows the Nakagami fading model described in Section II-C1, we employ the upper bound approximation of gamma distribution with parameter $m_{cc}$ such that: $P_l[|h_{cc}^l|^2 < \gamma] < (1 - e^{-A \gamma})^{m_{cc}}$ with $A = m_{cc}(m_{cc}^{-1})^{\frac{1}{m_{cc}}}$, therefore, starting with $I$, the conditional outage probability is expressed as:

$$p_{out}^l(T_l) = \int_0^T \mathcal{P} \left[ \frac{P_{tot} |h_{cc}^l|^2 r_m^{-\alpha}}{\sigma^2 + I_{BS} + I_{SAT}} < T_l \right] f_r(y) \ dy,$$  

where $f_r(y)$ is the density of fading of interference channel given by:

$$f_r(y; m_{cp}) = \frac{m_{cp}}{\Gamma(m_{cp})} y^{m_{cp} - 1} e^{-m_{cp} y},$$

where $m_{cp}$ is the Nakagami fading parameter, and $\Gamma(m_{cp})$ is the Gamma function,

$$\mathcal{P} \left[ \frac{P_{tot} |h_{cc}^l|^2 r_m^{-\alpha}}{\sigma^2 + I_{BS} + I_{SAT}} < T_l \right] = E_{BS,SAT} \left[ \mathcal{P} \left[ |h_{cc}^l|^2 < \frac{T_l r_m^{\alpha}}{P_{tot}} \left( \sigma^2 + I_{BS} + I_{SAT} \right) \right] \right],$$

(a) $E_{BS,SAT} \left[ \left( 1 - e^{-A \frac{T_l r_m^{\alpha}}{P_{tot}} (\sigma^2 + I_{BS} + I_{SAT})} \right)^{m_{cc}} \right],$

(b) $\sum_{k=0}^{m_{cc}} \binom{m_{cc}}{k} (-1)^k \frac{-A k T_l r_m^{\alpha}}{P_{tot}} \times E_{BS} \left[ e^{-A k T_l r_m^{\alpha} I_{BS}} \right]$,

(c) $\sum_{k=0}^{m_{cc}} \binom{m_{cc}}{k} (-1)^k \frac{-A k T_l r_m^{\alpha}}{P_{tot}} \times \prod_{m_{BS}} E_{BS} \left[ e^{-A k T_l r_m^{\alpha} I_{BS}} \right]$.

The Laplace transform of terrestrial interference is given as:

$$E_{BS} \left[ \exp(-s I_{BS}) \right] = E_{BS} \left[ \prod_{m_{BS}} \exp(-s P_m X_{cc} r_m^{-\alpha}) \right],$$

$$E_{BS} \left[ \exp \left( -s \sum_{m_{BS}} P_m X_{cc} r_m^{-\alpha} \right) \right] = E_{BS} \left[ \exp \left( -s \sum_{m_{BS}} P_m X_{cc} r_m^{-\alpha} \right) \right],$$

where, $s = \frac{\lambda k r_m^{\alpha}}{P_{tot}} X_{cc} = |h_{cc}^l|^2$.

Applying the Campbell's theorem [40], we obtain:

$$E_{BS} \left[ \exp \left( -s \prod_{m_{BS}} \left( 1 - \frac{1}{1 + \frac{1}{s P_m G_{ij}/P_r}} \right) \right) \right].$$

The expectation of interfering link from the satellite is obtained thus: Let $s = \frac{\lambda k r_m^{\alpha}}{P_{tot}}$.

$L(I_{SAT})(s) = E[\exp(-s I_{SAT})]$.

$$E_{\Phi_U} \left[ \prod_{i \in \Phi_U} \exp(-s P_{ij} G_{ij} X_{pc}) \right]$$

$$= E_{\Phi_U} \left[ \prod_{i \in \Phi_U} \exp(-s P_{ij} G_{ij} X_{pc}) \right]$$

(b) $\exp \left( -2 \pi \lambda_U \left( 1 - \frac{1}{1 + \frac{1}{s P_{ij} G_{ij}/P_r}} \right) \right)$.

where $X_{pc} = |h_{pc}^l|^2$, (a) follows from the assumption of independent fading, (b) follows from the use of Campbell’s theorem, moment generating function of Gamma random variable and probability generating functionals of PPPs.

For the second part of $p_{out}^l$ in (47), we obtain:

$$p_{out}^l(T_l) = \int_0^T \mathcal{P} \left[ \frac{\gamma |h_{cc}^l|^2 r_m^{-\alpha}}{\sigma^2 + I_{BS} + I_{SAT}} < T_l \right] f_r(y) \ dy,$$

with $f_r$ defined in (49). We solve $III$ by following steps similar to those outlined in (50) and obtain

$$\mathcal{P} \left[ \frac{\gamma |h_{cc}^l|^2 r_m^{-\alpha}}{\sigma^2 + I_{BS} + I_{SAT}} < T_l \right] = \sum_{k=0}^{m_{cc}} \binom{m_{cc}}{k} (-1)^k e^{-\frac{A k T_l r_m^{\alpha}}{P_{tot}}} \times \prod_{m_{BS}} E_{BS} \left[ e^{-\frac{A k T_l r_m^{\alpha} I_{BS}}{P_{tot}}} \right] \prod_{j \in \Phi_U} E_{SAT} \left[ e^{-\frac{A k T_l r_m^{\alpha} I_{SAT}}{P_{tot}}} \right].$$

$r_m$ is subsequently referred to as $r$. 
Now, substituting (56) into (55), we obtain $p_{\text{out}}^\text{II}$ given as

$$p_{\text{out}}^\text{II}(T_i) = \prod_{m \in \Phi_{\text{BS}}} \mathbb{E}_{\text{BS}} \left[ e^{-A_k \phi_{\text{II}T_i}} \phi_{\text{BS}} \right] \prod_{j \in \Phi_{\text{II}}} \mathbb{E}_{\text{II}} \left[ e^{-A_k \phi_{\text{II}T_i} \phi_{\text{BS}}} \right] \prod_{m \in \Phi_{\text{BS}}} \mathbb{E}_{\text{BS}} \left[ e^{-A_k \phi_{\text{II}T_i}} \phi_{\text{BS}} \right] \prod_{j \in \Phi_{\text{II}}} \mathbb{E}_{\text{II}} \left[ e^{-A_k \phi_{\text{II}T_i} \phi_{\text{BS}}} \right].$$

The expectations of interfering links from the other BSs, $\mathbb{E}_{\text{BS}} \left[ e^{-A_k \phi_{\text{II}T_i}} \phi_{\text{BS}} \right]$ and the satellite, $\mathbb{E}_{\text{II}} \left[ e^{-A_k \phi_{\text{II}T_i} \phi_{\text{BS}}} \right]$ are obtained by following similar steps to (53) and (54) respectively. Finally, the proof of outage probability for the terrestrial user is realised by summation of $p_{\text{out}}^\text{I}$ and $p_{\text{out}}^\text{II}$ respectively.

**APPENDIX B**

**PROOF OF PROPOSITION 2**

The approximated outage probability for the terrestrial user when $f_{\text{BS}}(x, v, \theta) = \frac{1}{\pi \lambda^2 x^2}$ and $I_{\text{SAT}} = 0$ is given as

$$p_{\text{out}}^\text{I}(T_i) = \int_0^\infty \int_0^T \frac{P_{\text{tot}} \left| h_{\text{c}1} \right|^2 \gamma^{\alpha}}{\sigma^2 + I_{\text{BS}}} < T_i \int f_t(y) \ dy$$

$$+ \int_\gamma^\infty \int_0^T \frac{\frac{P_{\text{tot}} \left| h_{\text{c}1} \right|^2 \gamma^{\alpha}}{\sigma^2 + I_{\text{BS}}} < T_i \int f_t(y) \ dy. (58)$$

The expectation of the interfering links from other BSs is given as

$$\mathbb{E}_{\text{BS}} \left[ e^{-A_k \phi_{\text{II}T_i}} \phi_{\text{BS}} \right] = \int_0^\infty e^{-\frac{A_k \phi_{\text{II}T_i}}{\phi_{\text{BS}}}} \frac{1}{\phi_{\text{BS}}} \phi_{\text{BS}} \ dy, (59)$$

Solving for (59) yields

$$\mathbb{E}_{\text{BS}} \left[ e^{-A_k \phi_{\text{II}T_i}} \phi_{\text{BS}} \right] = \left( \frac{A_k \phi_{\text{II}T_i}}{P_{\text{tot}}} + \frac{1}{\phi_{\text{BS}}} \right) \theta^{-\alpha}. (60)$$

Using the expressions $\mathbb{E}_{\text{BS}} \left[ e^{-A_k \phi_{\text{II}T_i}} \phi_{\text{BS}} \right]$ and $f_t(y) = e^{-\gamma y}$ to solve (58) and following similar steps to Appendix A will yield (29).

**APPENDIX C**

**PROOF OF PROPOSITION 3**

Now, the outage probability of SINR distribution using (15) can be given as

$$P \left\{ \frac{P_{\text{tot}} G_{\text{II}} \left| h_{\text{pp}} \right|^2}{\sigma^2 + I_{\text{BS}}} < T_s \right\} = P \left\{ \left| h_{\text{pp}} \right|^2 < \frac{T_s}{P_{\text{tot}} G_{\text{II}}} \left( \sigma^2 + I_{\text{BS}} \right) \right\}. (61)$$

Leveraging the tight upper bound of a Gamma random variable of parameters $\alpha_s$ and $\beta_s$, as $P \left\{ \left| h_{\text{pp}} \right|^2 < \gamma < (1 - e^{-A_k \phi_{\text{II}}}) \right\}$, and by applying binomial theorem we approximate (61) as

$$P \left\{ \left| h_{\text{pp}} \right|^2 < \frac{T_s}{P_{\text{tot}} G_{\text{II}}} \left( \sigma^2 + I_{\text{BS}} \right) \right\}$$

$$\approx \sum_{l=0}^{\alpha_s} \binom{\alpha_s}{l} \left( 1 - \frac{A_k \phi_{\text{II}}}{\phi_{\text{BS}}} \right)^l \left( \frac{A_k \phi_{\text{II}}}{\phi_{\text{BS}}} \right)^{\alpha_s-l} L \left\{ I_{\Phi_{\text{BS}}} \right\}(s), (62)$$

where $s = \frac{A_k \phi_{\text{II}}}{\phi_{\text{BS}}}$. Next, the terrestrial interference due to BSs is characterized as

$$L \left\{ I_{\Phi_{\text{BS}}} \right\}(s) = \mathbb{E}_{\text{BS}} \left[ \exp(-s I_{\Phi_{\text{BS}}} \right]$$

$$= \mathbb{E}_{\text{BS}} \left[ \prod_{l \in \Phi_{\text{BS}}} \exp(-s I_{\Phi_{\text{BS}}} \right]$$

which is gotten by substituting $I_{\Phi_{\text{BS}}} = \sum_{l \in \Phi_{\text{BS}}} P_{\text{tot}} \left| h_{\text{c}1} \right|^2 r_{l}^{-\alpha}$. Applying Campbell’s theorem [40], we obtain

$$L \left\{ I_{\Phi_{\text{BS}}} \right\}(s) = \exp \left[ 2\pi \lambda_{\text{BS}} \int_0^\infty \left( e^{-s P_{\text{tot}} \left| h_{\text{c}1} \right|^2 r^{-\alpha}} - 1 \right) r \ dr \right]. (64)$$

Taking the expectation with respect to $\left| h_{\text{c}1} \right|^2$ and recalling that $P_{\text{tot}}$ is constrained as in equation (12), we obtain

$$L \left\{ I_{\Phi_{\text{BS}}} \right\}(s)$$

$$= \exp \left[ 2\pi \lambda_{\text{BS}} \int_0^\infty \int_0^\infty \left( e^{-s P_{\text{tot}} \left| h_{\text{c}1} \right|^2 r^{-\alpha}} - 1 \right) f_r(y) \ dy \ dr \right]. (65)$$

where $f_r(y)$ is as defined in (49).

After solving the inner integrals of $I$ and $II$ with respect to $y$, the expectation of the interference from BSs limited by the interference temperature constraint is given as

$$L \left\{ I_{\Phi_{\text{BS}}} \right\}(s)$$

$$= \exp \left[ 2\pi \lambda_{\text{BS}} \int_0^\infty \int_0^\infty \left( e^{-s P_{\text{tot}} \left| h_{\text{c}1} \right|^2 r^{-\alpha}} - 1 \right) f_r(y) \ dy \ dr \right]. (66)$$
This proof is concluded by substituting (66) into (62).
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