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ARTICLE

Hybrid Motion/Force Control: a Review

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This paper reviews hybrid motion/force control (HMFC), a control scheme which enables robots to perform tasks involving both motion, in the free space, and interactive force, at the contacts. Motivated by the large amount of literature on this topic, we facilitate comparison and elucidate the key differences among different approaches. An emphasis is placed on the study of the decoupling of motion control and force control. And we conclude that it is indeed possible to achieve a complete decoupling, however this feature can be relaxed or sacrificed to reduce the robot's joint torques while still completing the task.

Keywords: Hybrid Motion/Force Control, Motion/Force Decoupling

1. Introduction

The utilisation of robots for tasks which involve forceful interactions with the environment is rapidly increasing in machining and assembling with industrial robots; in tele-operation and tele-manipulation; in search and rescue tasks; in cooperative tasks; in manipulation; in humanoid robots, *e.g.*, walking. In the nuclear industry, decommissioning tasks such as cutting or scabbling, require large contact forces at precise positions. Thus controlling the interactions robots have with the environment is critical. In these situations, contacts play a fundamental role not only for safety reasons, in fact exploiting such contacts could lower the effort needed to complete a task. Robots' masses and inertias are neither negligible nor unimportant when in motion and reactive forces from contacts and active forces exerted on the environment need be analysed and controlled in addition to the robot's position.

Compliance with the environment can be achieved with the introduction of mechanical devices at the robot end effector, *e.g.*, Remote Centre of Compliance device, *i.e.*, [1]. This passive compliance can be useful in peg-in-the-hole tasks but has limited use for tasks where the robot is asked to interact actively with the environment. In contrast to passive solutions, an active compliance can be achieved with a control law so as to directly command the robot to act on the environment with a desired behaviour. Control methods such as active stiffness and impedance control fall into this category [2–4]. Impedance control and admittance control were formally proposed in [3][1]. A constant switch between impedance control and admittance control is proposed in [5], to benefit from both: 1. the accuracy of admittance control in free motion and 2. the robustness of impedance control in stiff contact. There are efforts towards an integration

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[1]A system that has motion as input and outputs force is defined as an impedance, whilst a system that has force inputs and outputs motion is defined as an admittance.

of impedance control and HMFC, such as [6] and [7]. Hybrid Impedance Control is an attempt to combine impedance control and HMFC into a unified scheme, as for example in [8].

Motivated by the wide range of robotic applications requiring interactions with the environment, this work focuses on the control technique formally defined as Hybrid Position/Force or Motion/Force Control (HMFC), a strategy which defines a control for tasks involving both motion, in the free motion space, and force, at the contacts. Books and manuals on robotics report on HMFC, *e.g.*, [9] and [10]. A review of notable interest is in [11], which presents a survey on interaction schemes such as impedance control and parallel force/position control, however HMFC is not directly taken into consideration. HMFC, impedance control and parallel control are reviewed in [12], where high level statements of general value are proposed when applying force control. An insightful high-level description of HMFC can be found in [13]. Another very interesting survey on force control is presented on [14]. The focus is on both impedance control and HMFC, and a number of applications of HMFC are also reported at the end of the paper. We are not aware of any review article in the existing literature, which specifically and comprehensively covers the important area of HMFC. For this reason, this paper focuses on HMFC and aims at providing a review on the important contributions to it.

We propose an analysis of the differences among various different methods. We offer a study of motion/force decoupling, a fundamental feature of HMFC. Motion/force decoupling enables independent controls for motion and force, without them interfering with each other. We provide a comparative categorisation of various different HMFC schemes, according to attributes such as the need of a priori knowledge or the need of any direct force feedback. Finally, we also discuss the connection between these classical control approaches and optimal control.

The remainder of this paper is organised as follows. In Sec. 2, we discuss the major papers that contributed to the definition of HMFC. Then in Sec. 3 we focus on decoupling. Sec. 4 studies three other characteristics of HMFC, and finally Sec. 5 concludes the review with some final remarks. Table 1 introduces symbols used in the next sections and Table 2 introduces the kinematics and the dynamics of a robot in their classical formulations.

2. Hybrid Motion/Force Control

Robots interacting with the environment typically have constraints limiting their motion. These constraints are due to the geometry of the environment and are generally called natural constraints. When robots are asked to perform a task, they are given additional artificial constraints to satisfy simultaneously to the natural constraints.

The main principle of HMFC is to use all actuated joints to control position in the free motion directions while controlling force in the directions where the robot has its position constrained by contacts.

Many schemes have been proposed for HMFC, and in the following we will highlight the main features and differences among the approaches. We review the contributions thematically, categorising methods for using either a selection matrix or a projector. We define a method to be model-based if it uses a dynamic model in the controller, whilst the method is model-free if it uses only kinematics.

2.1 *Selection Matrix: Model-free*

A foundation for force control was built in [15], [2] and [4]. Building on compliance control, HMFC has its roots in the work of Mason [16, 17] and Craig and Raibert [18, 19]. Compliance and force control was introduced in [16] and [17] by identifying natural and artificial constraints, by means of selection matrices to keep the end effector within the constraint force and free

Table 1. Definition of symbols

Symbol	Description
$\mathbf{q}_{(\mathbf{n \times 1})}$	Robot configuration coordinates, joint position, $\mathbf{q} \in \mathcal{C} \subseteq \mathcal{R}^n$
$\dot{\mathbf{q}}_{(\mathbf{n \times 1})}$	Joint velocity
$\ddot{\mathbf{q}}_{(\mathbf{n \times 1})}$	Joint acceleration
$\mathbf{y}_{(\mathbf{m \times 1})}$	Robot task coordinates, $\mathbf{y} \in \mathcal{W} \subseteq \mathcal{R}^m$
$\dot{\mathbf{y}}_{(\mathbf{m \times 1})}$	Task coordinates velocity
$\ddot{\mathbf{y}}_{(\mathbf{m \times 1})}$	Task coordinates acceleration
$\mathbf{J}(\mathbf{q})_{(\mathbf{m \times n})}$	Task Jacobian
$\dot{\mathbf{J}}(\mathbf{q})_{(\mathbf{m \times n})}$	Task Jacobian first derivative
$\mathbf{J}^T(\mathbf{q})_{(\mathbf{n \times m})}$	Task Jacobian transpose
$\mathbf{J}^\dagger(\mathbf{q})_{(\mathbf{n \times m})}$	Task Jacobian Moore-Penrose pseudoinverse
$\mathbf{M}(\mathbf{q})_{(\mathbf{n \times n})}$	Inertia matrix
$\mathbf{h}(\mathbf{q}, \dot{\mathbf{q}})_{(\mathbf{n \times 1})}$	Centrifugal, Coriolis, gravity and friction components in robot dynamics
$\boldsymbol{\tau}_{(\mathbf{n \times 1})}$	Torque input, torque commands to the joint motors
$\mathbf{J}_C(\mathbf{q})_{(\mathbf{c \times n})}$	Jacobian of the constraints, where c is the number of constraints
$\dot{\mathbf{J}}_C(\mathbf{q})_{(\mathbf{c \times n})}$	First derivative of the Jacobian of the constraints
$(\mathbf{J}_C(\mathbf{q})^T\boldsymbol{\lambda})_{(\mathbf{n \times 1})}$	Torques at the joint level which are the effect of external forces, *e.g.*, due to contacts and constraints
$\mathbf{F}_{(\mathbf{m \times 1})}$	Cartesian external forces/torques at the end effector
$\boldsymbol{\Lambda}(\mathbf{y})_{(\mathbf{m \times m})}$	Operational space inertia matrix
$\boldsymbol{\mu}(\mathbf{y}, \dot{\mathbf{y}})_{(\mathbf{m \times 1})}$	End effector centrifugal and Coriolis forces
$\mathbf{p}(\mathbf{y})_{(\mathbf{m \times 1})}$	End effector gravity forces
$\tilde{\boldsymbol{\mu}}(\mathbf{y}, \dot{\mathbf{y}})_{(\mathbf{m \times 1})}$	End effector centrifugal, Coriolis, gravity and friction forces
$\mathbf{M}_C(\mathbf{q})_{(\mathbf{n \times n})}$	Projected dynamics "inertia" matrix
$\boldsymbol{\Lambda}_C(\mathbf{y})_{(\mathbf{m \times m})}$	Operational space projected dynamics inertia matrix
$\mathbf{K}_{(\mathbf{n \times n})}$	General feedback control gain
$\boldsymbol{\delta}$	Small displacement, *e.g.*, $\boldsymbol{\delta}\mathbf{q}_{(\mathbf{n \times n})}$ is a small displacement of the robot configuration $\mathbf{q}$
$\mathbf{S}_{(\mathbf{m \times m})}$	Selection matrix, diagonal, indicating directions where free motion is not allowed
$\mathbf{I}$	Identity matrix
$(\mathbf{I} - \mathbf{S})_{(\mathbf{m \times m})}$	Selection matrix, indicating directions where free motion is allowed
$\mathbf{P}_{(\mathbf{n \times n})}$	Projector into the space of permissible motion
$(\mathbf{I} - \mathbf{P})_{(\mathbf{n \times n})}$	Projector into the space of permissible forces

motion subspaces. In this perspective, it is possible to define

$$\mathbf{S}\dot{\mathbf{y}} = \mathbf{0} \tag{8}$$

Table 2. Classical formulations of the kinematics and dynamics of a robot

Equation		Description
$\mathbf{y} = \mathbf{f}(\mathbf{q})$	(1)	Direct kinematics
$\dot{\mathbf{y}} = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}}$	(2)	Differential kinematics - velocity
$\ddot{\mathbf{y}} = \mathbf{J}(\mathbf{q})\ddot{\mathbf{q}} + \dot{\mathbf{J}}(\mathbf{q})\dot{\mathbf{q}}$	(3)	Differential kinematics - acceleration
$\mathbf{J}_C(\mathbf{q})\dot{\mathbf{q}} = \mathbf{0}$	(4)	Kinematic constraints - velocity
$\mathbf{J}_C(\mathbf{q})\ddot{\mathbf{q}} + \dot{\mathbf{J}}_C(\mathbf{q})\dot{\mathbf{q}} = \mathbf{0}$	(5)	Kinematic constraints - acceleration
$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) = \boldsymbol{\tau} - \mathbf{J}_C(\mathbf{q})^T\boldsymbol{\lambda}$	(6)	Joint space dynamics
$\boldsymbol{\Lambda}(\mathbf{y})\ddot{\mathbf{y}} + \boldsymbol{\mu}(\mathbf{y}, \dot{\mathbf{y}}) + \mathbf{p}(\mathbf{y}) = \mathbf{F}$	(7)	Operational space dynamics

and

$$(\mathbf{I} - \mathbf{S})\mathbf{F} = \mathbf{0} \tag{9}$$

respectively, using $\mathbf{S}$ to declare directions where motion is not allowed by the physical constraints, and $(\mathbf{I}-\mathbf{S})$ the corresponding directions of free motion. $\mathbf{S}$ and $(\mathbf{I}-\mathbf{S})$ are formed of 1s and 0s on the diagonal. Mason thus stated equations in such form limit velocities and forces to subspaces which are orthogonal complements. Notice also that

$$\mathbf{S}\dot{\mathbf{y}} = (\mathbf{SJ})\dot{\mathbf{q}} = \mathbf{J}_C\dot{\mathbf{q}} = \mathbf{0}\,. \tag{10}$$

There is also a strong analogy with [15], where a selection of directions was proposed in the joint space instead.

HMFC to achieve compliant motion is formally presented in [18] and [19]. Specifically the error in the task related coordinate system is mapped to the joint space by means of the inverse Jacobian, $\mathbf{J}^{-1}$. This differs from the "free-joint" method in [15], because this approach involves all joints to devise position and force commands simultaneously. In other words, the selection of position and force control directions is done directly in the Cartesian space, leading to a more intuitive definition of the constraints. More specifically, a controller is proposed whose two components are respectively responsible for motion control, $(\mathbf{I}-\mathbf{S})\boldsymbol{\Delta}\mathbf{y}$, and force control, $\mathbf{S}\boldsymbol{\Delta}\mathbf{F}$, where $\boldsymbol{\Delta}\mathbf{y}$ and $\boldsymbol{\Delta}\mathbf{F}$ are the errors on position and force measured on the end effector, giving

$$\boldsymbol{\tau} = \boldsymbol{\Gamma}(\mathbf{S}\boldsymbol{\Delta}\mathbf{F}) + \boldsymbol{\Psi}((\mathbf{I} - \mathbf{S})\boldsymbol{\Delta}\mathbf{y})\,, \tag{11}$$

where $\boldsymbol{\Gamma}$ and $\boldsymbol{\Psi}$ are force and position compensation functions used to map errors in the Cartesian space to commands in the joint space. $\mathbf{S}$ is defined analogously to [16] and [17].

In [18] and [19], the choice of using the inverse of the Jacobian, $\mathbf{J}^{-1}$, is especially critical since it does not scale to redundant manipulators and because of numerical issues arising from the inversion. Hence later work proposed alternative methods to avoid these problems. Zhang and

Paul in [20] criticise: the approach in [4], claiming the method does not work near to kinematic singularities; the approach in [18] and [19] because of the computational problems connected to the inverse of the Jacobian; and the approach in [15] since the proposed method achieves good performances only if the joints are aligned with each direction where compliance is required. Thus, they propose a modified HMFC to overcome these difficulties in the form

$$\boldsymbol{\tau} = \mathbf{K}(\mathbf{I} - \mathbf{J}^{-1}\mathbf{S}\mathbf{J})\boldsymbol{\delta}\mathbf{q}\,. \tag{12}$$

This is different to the stiffness approach in [4], in that the latter uses $\boldsymbol{\tau} = \mathbf{J}^T\mathbf{K}\mathbf{J}\boldsymbol{\delta}\mathbf{q}$. It is also different from the formulation of HMFC in [18] and [19], since it does not use only the $\mathbf{J}^{-1}$ to map the Cartesian error into the joint space, thus, according to the authors, enhancing performances since it reduces the computational load locally at the joints. Zhang and Paul call the selection matrix the Cartesian compliant matrix, but in our understanding it is the same as the selection matrix $\mathbf{S}$. Analogously to the $\mathbf{K}_Q$ in [4], they define the product $\mathbf{J}^{-1}\mathbf{S}\mathbf{J}$ as the joint compliance matrix $\mathbf{C}_{\boldsymbol{\Omega}}$. The above analysis describes the position part of the control scheme. As for the active forces, they are transformed into joint torques through the Jacobian transpose $\mathbf{J}^T$ and added to the commanded torques in an open loop fashion.

The work of [21] proves kinematic instabilities of hybrid control by counterexamples, in particular showing that stability is dependent on the geometry of the manipulator. They report that HMFC results in instability when used on a 2-degree-of-freedom manipulator with revolute joints, but proves stable on a polar manipulator. On the contrary, they show that stiffness control as proposed in [4] always results in a stable system, suggesting the use of the Jacobian transpose as the cause for this result. On the same line of reasoning, the kinematic stability of the linearised systems associated with HMFC ([18] and [19]), and active stiffness control ([4]) is investigated in [22]. It is shown how HMFC can become unstable when the joints are in certain configurations, while the stiffness control is always stable except in singular configurations. Their conclusion reaffirms what was already proved by examples and counterexamples in [21].

A correction of the formulations in [18] and [19] is found in [23], advocating that the original position solution, using $\mathbf{J}^{-1}$, not only gives rise to numerical problems but also is not a general and correct solution. Another critique to the approach in [20] can be found in [23], where an equivalent to $\mathbf{S}$ is proposed in the joint space, namely $\mathbf{J}^{-1}\mathbf{S}\mathbf{J}$. This is different from $\mathbf{S}$, which refers to the Cartesian space, while $\mathbf{J}^{-1}\mathbf{S}\mathbf{J}$ is a mapping in the joint space. They propose a revised formulation of the HMFC scheme based on vector space analysis and advance a controller where the correct general position solution is in the form

$$\mathbf{q}_{es} = (\mathbf{S}^{\perp}\mathbf{J})^{\dagger}\boldsymbol{\Delta}\mathbf{y} + (\mathbf{I} - \mathbf{J}^{\dagger}\mathbf{J})\mathbf{z}_{\mathbf{q}}\,, \tag{13}$$

and the general force control in the form

$$\boldsymbol{\tau}_{es} = (\mathbf{S}\mathbf{J})^T\boldsymbol{\Delta}\mathbf{F} + (\mathbf{I} - \mathbf{J}^{\dagger}\mathbf{J})\mathbf{z}_{\boldsymbol{\tau}}\,, \tag{14}$$

where $\mathbf{z}_{\mathbf{q}}$ and $\mathbf{z}_{\boldsymbol{\tau}}$ are arbitrary position and force vectors respectively, defined in the manipulator joint space. By pre-multiplying by $(\mathbf{S}^{\perp}\mathbf{J})^{\dagger}$, only the components of the error referring to the Cartesian directions where it is possible to control motion are taken into account and mapped into the joint space, whereas force control is in the directions indicated by $\mathbf{S}$. They consider that force control is orthogonal to position control, *i.e.*, $\mathbf{S}^{\perp} = \mathbf{I} - \mathbf{S}$. They show that the inverse of the Jacobian is an incorrect solution and they investigate how the null space of the Jacobian can influence the control of both positions and torques. Also, they derive two sufficient conditions for kinematic stability in addition to providing further insights on projection matrices. Finally, they show that the matrix $\mathbf{S}$ is a selection matrix that selects a reduced Cartesian space, or equivalently, a subspace of interest of the entire Cartesian space. Mapping this space into the joint space through the robot Jacobian may be problematic, hence their proposal of a more robust formulation of the HMFC and their analysis of kinematic stability.

2.2 *Selection Matrix: Model-based*

A contribution to the understanding of dynamics is the work of Khatib, [24] and [25]. A controller is proposed directly in the operational space

$$\boldsymbol{\tau} = \mathbf{J}^T(\mathbf{F}_m + \mathbf{F}_a + \mathbf{F}_{ccg})\,, \tag{15}$$

where $\mathbf{F}_m$ and $\mathbf{F}_a$ are respectively the components for motion and force control, while $\mathbf{F}_{ccg}$ is a compensation of centrifugal, Coriolis and gravity effects. The following definitions are advanced

$$\mathbf{F}_m = \boldsymbol{\Lambda}(\mathbf{I} - \mathbf{S})\mathbf{F}^*_m \tag{16}$$

and

$$\mathbf{F}_a = \mathbf{S}\mathbf{F}^*_a + \boldsymbol{\Lambda}\mathbf{S}\mathbf{F}^*_s\,, \tag{17}$$

where $\mathbf{F}^*_m$ is motion control part formed of a PD on position and velocity plus a feedforward on acceleration while $\mathbf{F}^*_a$ and $\mathbf{F}^*_s$ are respectively vectors of active force control and end effector velocity damping acting in the force control subspace. This work clearly follows the lines of Mason, and Raibert and Craig because it uses selection matrices to select certain directions in the Cartesian space, in order to separate motion and force control geometrically.

A generalisation of HMFC is presented in [26], where there is also a study of the conditions for closed-loop stabilisation including the constraints. A nonlinear controller, based on a modification of the computed torque method, is proposed and claimed to be a generalisation of the HMFC scheme presented in [18] and [19]. A study of HMFC stability by analysing the nonlinear dynamic system resulting from HMFC using Lyapunov's theory and LaSalle's theorem is found in [27]. Contrary to previous work based on linearised models, it analyses the nonlinear dynamic system and factors such as additional phase lag and sampling delay. This work also offers stability conditions for HMFC, even in the presence of friction, phase lag and discrete control. As a result, it provides new insights into how the inverse Jacobian in [19] adversely affects stability.

The work in [28] is based on [19] but states that dynamics were not considered rigorously. It is shown how a feedforward term $\mathbf{f}_{Fd}$ in the force control has a major desirable impact, such that the final controller proposed is in the form $\boldsymbol{\tau} = \boldsymbol{\tau}_P + \boldsymbol{\tau}_F$, where

$$\boldsymbol{\tau}_P = \mathbf{M}\ddot{\mathbf{q}}_d + \mathbf{h} \tag{18}$$

and

$$\boldsymbol{\tau}_F = (\mathbf{S}\mathbf{J})^T\mathbf{f}_{Fd}\,. \tag{19}$$

Desired position and force can both be realised if the robot is not in a kinematic singularity. Most of the concepts presented in [28] are reprised in [29], with the further contribution of a linearisation of the system by state feedback by setting

$$\boldsymbol{\tau}_P = \mathbf{M}\ddot{\mathbf{q}}_d(\mathbf{u}_1) + \mathbf{h} \tag{20}$$

and

$$\boldsymbol{\tau}_F = \mathbf{J}^T\mathbf{S}^T\mathbf{u}_2\,. \tag{21}$$

It is possible to choose an appropriate $\ddot{\mathbf{q}}$ such that when not in singularity, a linear decoupled system is achieved: $\ddot{\mathbf{y}} = \mathbf{u}_1$ on the motion control directions and $\mathbf{F} = \mathbf{u}_2$ in the force control directions. They also propose to use a PD feedback for the position control and I (integral) feedback on the force control to ensure zero error as time goes to infinity. In later work, a method for online estimation of unknown constraint surfaces is proposed in [30], estimated geometrically using $\mathbf{\Delta y}$ and $\mathbf{\Delta F}$, which need a force sensor on the end effector to be measured.

A different control architecture is proposed [31], which is rule-based, to overcome the flaws of model-based selection matrix HMFC. This system entails the cost of a more complex design but enables a parallel force/position control, where force control is prevalent over position control so that the system is able to be compliant with the environment and its constraints. A study of the stability of force/position control of a manipulator in elastic contact with the environment is in [32], exploiting Lyapunov analysis.

In the work reviewed till here, the subspace of force control is predominantly assumed to be orthogonal and complementary to the subspace of free motion[1]. In contrast, a third subspace is introduced [33] where it is possible either to control force, implicitly producing an acceleration, or to control motion, implicitly producing an active force. This is possible in directions where forces are not counterbalanced by a constraint reaction, thus producing active work, *i.e.*, directions where active forces and motion coexist.

2.3 *Projection Approach*

Contacts between robot and environment are modelled as virtual links with passive joints in [34], stating that constraints arising from contacts generate a closed kinematic chain. Their approach works seamlessly with multiple contacts and with contacts not at the end effector, which are situations likely to happen in realistic interactions with the environment. In particular, their idea takes root in the fact that it is possible to express the kinematics and dynamics of the robot and the constraints both in the joint space and as a closed kinematic chain, as in Eq. (4). They advocate that permissible motion and permissible forces of such closed kinematic chains are identified by $\mathbf{P}$ and $\mathbf{I} - \mathbf{P}$, where

$$\mathbf{P} = \mathbf{I} - \mathbf{J}_C^{\dagger}\mathbf{J}_C \tag{22}$$

such that $\mathbf{P}(\mathbf{I} - \mathbf{P}) = \mathbf{0}$. These projectors are used to map or filter the error signals in order to compute the torque commands to send to the robot.

The formulation of HMFC proposed in [18] and [19] is also extended in [35], in that a more general contact model and a filtering of the error signals into force and motion components are introduced. Two spaces are introduced, a space of normal vectors referring to the actual contact, and a space of those vectors which are tangential to the contact. Such representation overcomes the drawback of the classical selection matrix $\mathbf{S}$ since it does not rely solely on the six parameters used in $\mathbf{S}$ to select Cartesian directions, instead the proposed representation scales well to more intricate situations, *e.g.*, multiple-point contacts with non-intersecting normals (see Fig. 1 on the same paper). Moreover, two more subspaces are defined such that their direct sum is equal to the space of the force vectors and the motion vectors respectively. This is a representation analogous to the one in [33] but without taking into consideration the possible third subspace. It is also claimed that, with this proposed filtering, control is decoupled, stating that acceleration depends only on the filtered command of the motion control and force on the filtered command of the force control. Furthermore, it must be stressed that within this framework it is possible to include multiple contacts, in contrast to the formulation in [18] and [19]. Following [35], constrained-body contacts are studied in [36], where free-body contacts are categorised as special cases of the former, and a description of the motion and force freedoms and constraints at the contacts is given, also taking into account the effect of possible additional kinematic constraints. A dynamically-decoupled motion and force control is also described.

[1] The validity of the concept of orthogonality between the subspace of free motion and the subspace of force control has been questioned mostly because of a dimensional inconsistency. More in [14] and references therein.

In [37], projected dynamics are developed. The author goes further than [34] projecting the dynamics of the robot with the projector in Eq. (22). The range of the projector $\mathbf{P}$ is the null space of $\mathbf{J}_C$ ($\mathcal{R}(\mathbf{P}) = \mathcal{N}(\mathbf{J}_C)$) and the range of the projector $(\mathbf{I}-\mathbf{P})$ is the orthogonal space to the null space of $\mathbf{J}_C$, $\mathcal{R}(\mathbf{I}-\mathbf{P}) = \mathcal{N}(\mathbf{J}_C)^{\perp}$. Now, projecting the equation of the robot joint space dynamics (6) with $\mathbf{P}$ gives

$$\mathbf{PM}\ddot{\mathbf{q}} = \mathbf{P}(\boldsymbol{\tau} - \mathbf{h})\,, \tag{23}$$

since $\mathbf{J}_C^T\boldsymbol{\lambda} \in \mathcal{N}(\mathbf{J}_C)^{\perp}$ so $\mathbf{PJ}_C^T\boldsymbol{\lambda} = 0$. Finally

$$\ddot{\mathbf{q}} = \mathbf{M}_C^{-1}(\mathbf{P}(\boldsymbol{\tau} - \mathbf{h}) + \mathbf{C}_C\dot{\mathbf{q}})\,, \tag{24}$$

where $\mathbf{M}_C = \mathbf{M} + \mathbf{PM} - (\mathbf{PM})^T$, $\mathbf{C}_C = \mathbf{MC}$ and $\mathbf{C} = -\mathbf{J}_C^{\dagger}\dot{\mathbf{J}}_C$. $\mathbf{M}_C$ is invertible thus $\ddot{\mathbf{q}}$ can be expressed explicitly, in contrast to the previous case where $\mathbf{PM}$ was not invertible. The expression of $\mathbf{J}_C^T\boldsymbol{\lambda}$ can be obtained analogously to $\ddot{\mathbf{q}}$, by projecting (6) with $(\mathbf{I}-\mathbf{P})$

$$\mathbf{J}_C^T\boldsymbol{\lambda} = (\mathbf{I}-\mathbf{P})(\boldsymbol{\tau} - \mathbf{h}) - (\mathbf{I}-\mathbf{P})\mathbf{M}\mathbf{M}_C^{-1}(\mathbf{P}(\boldsymbol{\tau} - \mathbf{h}) + \mathbf{C}_C\dot{\mathbf{q}})\,. \tag{25}$$

In [38], Aghili extends his formulation of projected dynamics and proposes to use optimisation to take into account unilateral constraints in his definition of force control.

The formulations of the operational space dynamics and projected dynamics are fused in [39], to express operational space dynamics as

$$\boldsymbol{\Lambda}_C\ddot{\mathbf{y}} + \boldsymbol{\Lambda}_{\boldsymbol{C}}(\mathbf{J}\mathbf{M}_C^{-1}\mathbf{Ph} - (\dot{\mathbf{J}} + \mathbf{J}\mathbf{M}_C^{-1}\mathbf{C})\dot{\mathbf{q}}) = \mathbf{F}\,, \tag{26}$$

where $\mathbf{M}_C = \mathbf{PM} + \mathbf{I} - \mathbf{P}$ and $\boldsymbol{\Lambda}_C = (\mathbf{J}\mathbf{M}_C^{-1}\mathbf{P}\mathbf{J}^T)^{-1}$. This approach, based on projected dynamics, has been used in [39] and further in [40]. The case of underactuated systems is analysed in [39].

2.4 *Applications of HMFC*

HMFC has been used in numerous very different robotic applications. Some examples are briefly listed in this subsection.

HMFC is extended to cooperating robots in [41]. The work in [42] builds on dynamic hybrid control and devises a control scheme for single object motion, which handles both constraint and internal force for multiple manipulators while accounting for both object and robot dynamics. An application of HMFC in bilateral communications between master and slave manipulators can be found in [43].

HFMC has been used for continuum manipulators in unknown constrained environments as in [44], and to control prosthetic hands as in [45] or to control biped walking as in [46]. HMFC has also been applied to control quadrotor helicopters as in [47] and flexible parallel manipulators as in [48]. The case of iterative learning HMFC for contour tracking of an object with unknown shape is analysed in [49]. Neural Networks were used in [50] while Elman Fuzzy control was used in [51].

An analysis of friction compensation in HFMC is proposed in [52] while a vision system accompanies HMFC in [53].

3. Decoupling

Decoupling is the capability to provide both motion and force control which do not interact with each other. In other words, it is possible to change the exerted force without altering or disturbing the robot trajectory, and vice versa. This is a very fundamental and powerful feature, especially when devising distinct desired evolutions for motion and force, which otherwise could not be realised. It is a topic arising prominently when using a control scheme such as HMFC, or in any task where both motion and active force need be precisely controlled. An important feature necessary to achieve decoupling is either the knowledge of the dynamic model of the robot, in order to compensate for the dynamics of the robot, or a feedback on force at the contact point, so that this feedback can be used to modulate force control.

A selection matrix $\mathbf{S}$ is introduced in [16] and [17], which provides the means to define and select directions in the Cartesian space frame of interest, where there is the possibility of free motion **or** conversely the possibility to exert forces on a contact surface. This concept is found also in [18] and [19] with few fundamental differences. The latter use a selection matrix to select only the error signals for each of the two subspaces of free motion and contact force. However the main idea is similar, namely to obtain decoupling between motion and force through an appropriate selection of directions in the Cartesian space, *i.e.*, by defining directly the commands which are feasible in the Cartesian space. These control schemes devise position and/or velocity commands in the directions of free motion, and force references in the directions where the robot is in contact and thus can exert active forces on the contact point or surface using Eq. (8) and Eq. (9). In the directions of Cartesian space indicated by $\mathbf{S}$, motion is not feasible, while, on the contrary, in the orthogonal subspace, described by $(\mathbf{I} - \mathbf{S})$, it is possible to specify references in position or velocity. The selection matrix, as proposed by [16], [17], [18] and [19], has the intrinsic limitation of not scaling easily to multiple contacts on the same robot or to situations where it is not easy or possible to know a priori or define orthogonal contact directions explicitly in the Cartesian space.

In [54], a model is derived for a robot in contact with the environment. A control architecture is proposed which leads to a complete decoupling of motion and force control. Reducing the dimensionality of the equation of motion, pseudo-velocities $\mathbf{v}$ are defined as

$$\mathbf{v} = \mathbf{B}\dot{\mathbf{q}}\,, \tag{27}$$

such that the matrix $\begin{bmatrix}\mathbf{J}_C^T & \mathbf{B}^T\end{bmatrix}^T$ is non singular. A control law in the form

$$\boldsymbol{\tau} = \mathbf{J}_C^T\mathbf{u}_1 + \mathbf{M}\mathbf{E}\mathbf{u}_2 + \mathbf{h} - \mathbf{M}(\mathbf{D}\dot{\mathbf{J}}_C + \mathbf{E}\dot{\mathbf{B}})\mathbf{E}\mathbf{v}\,, \tag{28}$$

where $\begin{bmatrix}\mathbf{J}_C \\ \mathbf{B}\end{bmatrix}^{-1} = \begin{bmatrix}\mathbf{D} & \mathbf{E}\end{bmatrix}$, is verified to lead to

$$\boldsymbol{\lambda} = \mathbf{u}_1 \tag{29}$$

and

$$\dot{\mathbf{v}} = \mathbf{u}_2\,, \tag{30}$$

which is a result very similar to [29]. Notice that in this formulation, $\mathbf{J}_C\mathbf{E} = \mathbf{0}$ so that pre-multiplying Eq. (6) by $\mathbf{E}^T$ yields

$$\mathbf{E}^T\mathbf{M}\ddot{\mathbf{q}} = \mathbf{E}^T(\boldsymbol{\tau} - \mathbf{h})\,, \tag{31}$$

which is closely related to Eq. (23).

The use of projectors is proposed in [35], which are arguably a more flexible solution than the selection matrix (capable of additionally handling non-perpendicular contact directions), and to filter the commands in order to map those same commands into the subspaces of free motion and active force. A projector Eq. (22) based on the Jacobian of the constraints, is proposed in

[34] and [37]. In particular, Aghili concludes **in [37]** that a perfect decoupling is achieved when

$$\forall \mathbf{w} \in \mathcal{N}(\mathbf{J}_C) \ : \ \mathbf{M}\mathbf{w} \in \mathcal{N}(\mathbf{J}_C) \, . \tag{32}$$

This condition is not satisfied in general and he therefore adds a component in the force controller to compensate for the coupling effect. The same **P** projector is also used in [39] and [40] when dealing with control using operational space projected dynamics.

Comparing the approaches exemplified by a selection matrix **S** and by the projector **P**, two considerations can be made:

(1) **S** decouples motion from active force but needs an entire compensation of the robot dynamics even just for a pure motion task. On the contrary, for a motion task, **P** allows the controller to command lower torques with respect to an approach based on **S**, but the motion part of the controller affects also the exerted force at the contact point. This feature can be explained by considering that the approach based on **P** exploits the constraint, in the sense that it exploits the reaction force at the contact point in order to compensate for the robot dynamics. To do this it is indeed necessary to interact with the environment, and so the non-null active forces are explained. On the other hand, the approach based on **S** devises higher commanded torques because it does not rely on the contact to compensate for the robot dynamics. In other words, this approach compensate for the whole robot dynamics while controlling motion, and this explains higher torque commands. It can be concluded that the method using the projector **P** sacrifices decoupling for torque efficiency (lowering the commands), while the method using **S** provides superior decoupling at the cost of higher torques;

(2) a force component in the **P** approach denaturalises the whole approach in the sense that the advantage of exploiting the contact is overridden by the imposition of a desired active force and an entire compensation of the robot dynamics may be necessary in addition to a compensation of the cross-coupling effect brought by the motion control part. On the other hand, this is the natural way to achieve a decoupling for this approach, so that it becomes possible to impose a desired motion and a desired active force. As a matter of fact, adding a force component the **P** approach becomes equivalent to the selection matrix **S** approach.

4. Discussion

Thus far, we analysed fundamental features of HFMC and Table 3 lists the key papers surveyed in this work, highlighting their major contribution. However, HMFC has criticalities that have been mentioned already in some of the aforementioned papers. HMFC is said to ignore the dynamic coupling between manipulator and environment in [5], hence resulting in an impossibility to control position or force precisely. Even if HMFC discerns between force controlled and motion controlled subspaces, however it fails in taking into account the importance of the manipulator impedance. In fact, in the real world uncertainties on parameters are frequent and a control framework such as HMFC should be able generate safe reaction forces [7]. Other criticisms include the need of a perfect knowledge of the kinematic constraints arising from the contacts and the problem of stability during transitions between contact and non-contact situations [13].

In the following, three other key concepts are to be discussed: 1) the need for a precise model to compensate dynamics and a priori knowledge about the environment, 2) the need to use a force sensor, and finally 3) the relationship between HMFC and optimal control.

Table 3. List of key contributions

Main Contribution	Paper
Admittance control	[2], Whitney, 1977
Selection matrix	[16], [17], Mason, 1979, 1981
Hybrid control	[18], Craig and Raibert, 1979
Joint stiffness matrix	[4], Salisbury, 1980
Hybrid control, use of inverse Jacobian	[19], Raibert and Craig, 1981
Modified hybrid control	[20], Zhang and Paul, 1985
P projector	[34], West and Asada, 1985
Operational space dynamics	[24], [25], Khatib, 1986, 1987
Dynamic hybrid control	[28], Yoshikawa, 1987
Instab. of classical hybrid control by counterexamples	[21], An and Hollerbach, 1987
Stability closed-loop	[26], McClamroch and Wang, 1988
Feedback linearisation	[29], Yoshikawa et al., 1988
Instability of classical hybrid control	[22], Zhang, 1989
Correct general position formulation	[23], Fisher and Mujtaba, 1991
Stability with Lyapunov (using robot dynamics)	[27], Yabuta, 1992
Online estimation of selection matrix	[30], Yoshikawa and Sudou, 1993
Parallel force/position control	[31], Chiaverini and Sciavicco, 1993
Third subspace	[33], De Luca and Manes, 1994
Projectors overcoming selection matrix	[35], Featherstone et al., 1998
Projected dynamics	[37], Aghili, 2005
Operational space projected dynamics	[39], Mistry and Righetti, 2011

4.1 *A priori knowledge*

A critical point is the assumption of a perfect knowledge of the model and parameters of the robot and knowledge about the environment, *i.e.*, constraints and timing of contacts. HMFC assumes the robot to be already in contact with the environment. When this knowledge is not exact, stability of the system can be jeopardised. HMFC compensates for the dynamics of the robot and expresses the desired behaviour at the same time. Thus, with uncertainty on parameters, behaviours might be different from the specified desired ones.

There are attempts to overcome this issue, and other studies proposing conditions on the stability of the system even in the presence of model uncertainties. An example is presented in [55], where the authors propose adaptive HMFC for redundant manipulators. Results are reported, which suggest that all quantities remain bounded and position and force errors converge to zero. Adaptive control was already used for robot control as reviewed in [56] and later illustrated in [57]. The work in [58] extends model-based adaptive control to contact situations and proposes to use a projector in the form $\mathbf{P} = \mathbf{I} - \mathbf{J}_C^T \mathbf{J}_C$ to achieve a so-called joint space orthogonalisation, where motion signals are orthogonal to force vectors. This feature is stated to be a generalisation of HMFC. A multi-robot hybrid force-motion adaptive control law when environments are geometrically unknown is presented in [59]. Adaptive control using also vision is used in [60] to overcome constraint uncertainties. The focus of this review is not on adaptive control, but it is worthwhile to mention this alternative approach. A hybrid control strategy robust to uncertainties on dynamic parameters, with a scheme for resolution of kinematic redundancy is presented in [61].

To explore the impact of potential mismatches between models and real systems, a study of the stability of HMFC in the presence of model uncertainties is proposed in [62], namely on the constraint Jacobian. They derive sufficient conditions on the choice of the feedback gains and the estimation of the Jacobian to retain stability. A method to estimate $\mathbf{J}_C$ is presented in [63], which computes the null space of a set of velocity vectors which differ from commanded velocities during contacts. No force sensor or tactile sensor is needed, however this method implicitly relies on the intrinsic compliance of the robot when in contact with the environment.

Also, it is important to state that there is still much to investigate in order to answer questions regarding the importance of contact position and timing and the impact of unmodelled sliding friction [52].

4.2 *Force Feedback*

Most of the HMFC schemes depend on a feedback on force to define an error in force. This implies the need of a force/torque sensor mounted at the contact point. If the contact happens only at the end effector, the problem might be solved comfortably. Otherwise, a tactile skin could be used, with an increase of the complexity of the system and of the overall cost. An alternative approach is the one proposed in [64] where, by means of a vision system capable of recognising contacts on the whole surface of the robot, a virtual force sensor is proposed using the so-called method of the residuals.

It is also possible to avoid the need for a force sensor and an example is the approach in [40]. The force controller compensates for the dynamics of the robot and provides a desired value for the active force in the form of a feedforward force component. Joint torque readings might also be used in order to recover contact forces, in the same fashion as in [64].

4.3 *Relationship between Hybrid Control and Optimal Control*

The use of optimisation methods in robotics is becoming increasingly popular. Optimisation algorithms become available once the problem can be formulated as a set of linear and nonlinear equations with a cost function to be minimised. The means to achieve this is to define the problem of trajectory finding as an optimisation problem. A direct use of projected dynamics in the definition of an optimal control problem can be found in [65]. Differently, the work in [66] poses a discrete model of dynamics of contacts as a Linear Complementarity Problem accounting for friction also. In addition, the work in [67] presents improvements to a Model Predictive Control approach applicable to high-degree-of-freedom systems. On the same line of reasoning, inelastic impacts and friction during contacts are accounted for in [68] and [69], where optimisation is used to find a trajectory and to resolve contact forces while intrinsically satisfying the constraints. Contact forces are resolved as additional variables in the optimisation. A benefit of this method is that it does not require the definition of modes to handle contacts, as in [70]. Sufficient conditions for input-to-state stability of a closed-loop switching tracking error dynamics are provided in [71] considering the motion-force tracking control problem of a manipulator in contact with a stiff environment. Also, an analysis of the stability of systems undergoing impacts in the presence of friction is presented in [72].

Optimisation-based control techniques focus on control and motion planning encompassing transitions from free motion to contact and vice versa, which HMFC does not include. Generally in optimisation-based methods, the definition of constraints represents the means to specify general features of the system and to take into account the model of the system, *i.e.*, imposing constraints on dynamic consistency is the connection between optimisation techniques and the classical approaches. In the case of HMFC, this is done by using the model in the controller.

5. Conclusion

Motivated by the fast increasing use of robotic platforms in interactive tasks involving contacts between robots and environment, we reviewed the literature on hybrid motion/force control (HMFC) and eased the comparison among the many control schemes.

A critical contribution of this paper is the analysis of the decoupling. A perfect decoupling can be achieved using the selection matrix $\mathbf{S}$. Using projected dynamics, it is possible to lower significantly the torques needed to perform the task at the cost of sacrificing decoupling. Decoupling can be regained using an extension to the scheme, but at the cost of sacrificing the advantage of lowering the torques, so that there is a substantial equivalence of this scheme to the scheme using the selection matrix.

As final consideration and remark, we have not claimed that HMFC is the only answer to control a robot during contacts. We are advocating that this is a very natural control framework to be used for tasks involving both motion and force.

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References

[1] Watson P. Remote center compliance system. 1978.
[2] Whitney D. Force feedback control of manipulator fine motions. Journal of Dynamic Systems, Measurement, and Control. 1977;Available from: http://dynamicsystems.asmedigitalcollection.asme.org/article.aspx?articleid=1402627.
[3] Hogan N. Impedance control: An approach to manipulation. American Control Conference, 1984. 1984;(March). Available from: http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=4788393.
[4] Salisbury J. Active Stiffness Control of a Manipulator in Cartesian Coordinates. 19th IEEE Conference on Decision and Control including the Symposium on Adaptive Processes. 1980;Available from: http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=4046624.
[5] Ott C, Mukherjee R, Nakamura Y. Unified impedance and admittance control. In: IEEE international conference on robotics and automation, ICRA 2010, anchorage, alaska, usa, 3-7 may 2010. 2010. p. 554–561. Available from: http://dx.doi.org/10.1109/ROBOT.2010.5509861.
[6] Anderson RJ, Spong MW. Hybrid impedance control of robotic manipulators. In: Ieee transactions on robotics and automation. 1988. p. 549–556.
[7] Liu G, Goldenberg A. Robust hybrid impedance control of robot manipulators. In: Ieee international conference on robotics and automation. 1991. p. 287–292.
[8] Adhikary N, Mahanta C. Hybrid impedance control of robotic manipulator using adaptive backstepping sliding mode controller with pid sliding surface. In: 2017 indian control conference (icc) january 4-6, 2017. indian institute of technology, guwahati, india. 2017. p. 391–396.
[9] Craig JJ. Introduction to robotics: Mechanics and control (2nd ed.). Boston, MA, USA: Addison-Wesley Longman Publishing Co., Inc.. 1989.
[10] Siciliano B, Khatib O. Handbook of robotics. Springer. 2008.
[11] Chiaverini S, Siciliano B, Villani L. A Survey of Robot Interaction Control Schemes with Experimental Comparison. IEEE Robotics & Automation Magazine. 1999;4(3):273–285. Available from: http://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=789685.
[12] De Schutter J, Bruyninckx H, Zhu W, Spong M. Force control: a bird's eye view. In: Ieee css/ras international workshop on control problems in robotics and automation: Future directions. December. 1997. p. 1–14. Available from: http://link.springer.com/chapter/10.1007/BFb0015073.
[13] Paul RP. Problems and research issues associated with the hybrid control of force and displacement. In: Proceedings of the 1987 IEEE international conference on robotics and automation, raleigh, north carolina, usa, march 31 - april 3, 1987. IEEE. 1987. p. 1966–1971. Available from: http://dx.doi.org/10.1109/ROBOT.1987.1087905.
[14] Yoshikawa T. Force control of robot manipulators. In: Ieee international conference on robotics and automation. 2000. p. 220–226.
[15] Paul RP, Shimano B. Compliance and Control. In: Joint automatic control conference. 1976. p. 1694–1699.
[16] Mason MT. Compliance and force control for computer controlled manipulators. [Master's thesis]. MIT. 1979.
[17] Mason M. Compliance and force control for computer controlled manipulators. Systems, Man and Cybernetics, IEEE Transactions on. 1981;(6):418–432. Available from: http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=4308708.
[18] Craig J, Raibert M. A systematic method of hybrid position/force control of a manipulator. Com-

puter Software and Applications Conference, 1979 Proceedings COMPSAC 79 The IEEE Computer Society's Third International. 1979;:446–451.

[19] Raibert M, Craig J. Hybrid Position / Force Control of Manipulators. Journal of Dynamic Systems, Measurement, and Control. 1981;102(June 1981).

[20] Zhang H, Paul R. Hybrid Control of Robot Manipulators. Robotics and Automation Proceedings 1985 IEEE International Conference on. 1985;:602–607Available from: http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=1087304.

[21] An C, Hollerbach J. Kinematic stability issues in force control of manipulators. Proceedings of IEEE International Conference on Robotics and Automation. 1987;Available from: http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=1087933.

[22] Zhang H. Kinematic stability of robot manipulators under force control. Robotics and Automation, 1989 Proceedings, 1989 IEEE International Conference on. 1989;:80–85Available from: http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=99971.

[23] Fisher WD, Mujtaba MS. Hybrid position/force control: A correct formulation. Hewlett-Packard Company. 1991. Tech Rep.

[24] Khatib O. The Operational Space Formulation in the Analysis, Design, and Control of Robot Manipulators. Robotics Research. 1985;.

[25] Khatib O. A Unified Approach for Motion and Force Control of Robot Manipulators: The Operational Space Formulation. Robotics and Automation, IEEE Journal of. 1987;.

[26] McClamroch N, Wang D. Feedback stabilization and tracking of constrained robots. Automatic Control, IEEE Transactions on. 1988;33(5). Available from: http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=1220.

[27] Yabuta T. Nonlinear Basic Stability Concept of the Hybrid Position/Force Control Scheme for Robot Manipulators. Robotics and Automation, IEEE Transactions on. 1992;(0). Available from: http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=163791.

[28] Yoshikawa T. Dynamic Hybrid Position/Force Control of Robot Manipulators - Description of Hand Constraints and Calculation of Joint Driving Force. Robotics and Automation, IEEE Journal of. 1987;11:386–392. Available from: http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=1087120.

[29] Yoshikawa T, Sugie T, Tanaka M. Dynamic Hybrid Position/Force Control of Robot Manipulators - Controller Design and Experiment. Robotics and Automation, IEEE Journal of. 1988;4(6):699 – 705. Available from: http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=9307.

[30] Yoshikawa T, Sudou A. Dynamic Hybrid Position/Force Control of Robot Manipulators: On-Line Estimation of Unknown Constraint. Robotics and Automation, IEEE Transactions on. 1993;Available from: http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=238286.

[31] Chiaverini S, Sciavicco L. The Parallel Approach to Force/Position Control of Robotic Manipulators. Robotics and Automation, IEEE Transactions on. 1993;9(4). Available from: http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=246048.

[32] Chiaverini S, Siciliano B, Villani L. Force/Position Regulation of Compliant Robot Manipulators. Automatic Control, IEEE Transactions on. 1994;(2). Available from: http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=280780.

[33] De Luca A, Manes C. Modeling of robots in contact with a dynamic environment. Robotics and Automation, IEEE Transactions on. 1994;Available from: http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=313104.

[34] West H, Asada H. A method for the Design of Hybrid Position/Force Controllers for Manipulators Constrained by Contact with the Environment. Robotics and Automation Proceedings 1985 IEEE International Conference on. 1985;Available from: http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=1087271.

[35] Featherstone R, Sonck S, Khatib O. A general contact model for dynamically-decoupled force/motion control. Experimental Robotics V. 1998;Available from: http://link.springer.com/chapter/10.1007/BFb0112956.

[36] Featherstone R. Modeling and control of contact between constrained rigid bodies. IEEE TRANSACTIONS ON ROBOTICS AND AUTOMATION. 2004;20(1):82–92.

[37] Aghili F. A unified approach for inverse and direct dynamics of constrained multibody systems based on linear projection operator: applications to control and simulation. Robotics, IEEE Transactions on. 2005;21(5):834–849. Available from: http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=1512343.

[38] Aghili F, Su C. Control of constrained robots subject to unilateral contacts and friction cone con-

straints. In: 2016 ieee international conference on robotics and automation (icra). Stockholm. 2016 May. p. 2347 – 2352.

[39] Mistry M, Righetti L. Operational space control of constrained and underactuated systems. Robotics: Science and systems VII. 2012;.

[40] Ortenzi V, Adjigble M, Kuo J, Stolkin R, Mistry M. An experimental study of robot control during environmental contacts based on projected operational space dynamics. In: 14th ieee-ras international conference on humanoid robots (humanoids). IEEE. 2014. p. 407 – 412.

[41] Hayati S. Hybrid position/Force control of multi-arm cooperating robots. Proceedings 1986 IEEE International Conference on Robotics and Automation. 1986;.

[42] Yoshikawa T, Zheng X. Coordinated dynamic hybrid position/force control for multiple robot manipulators handling one constrained object. Proceedings, IEEE International Conference on Robotics and Automation. 1990;:0–5.

[43] Sakaino S, Sato T, Ohnishi K. Precise position/force hybrid control with modal mass decoupling and bilateral communication between different structures. IEEE Trans Industrial Informatics. 2011; 7(2):266–276. Available from: http://dx.doi.org/10.1109/TII.2011.2121077.

[44] Yip MC, Camarillo DB. Model-less hybrid position/force control: A minimalist approach for continuum manipulators in unknown, constrained environments. IEEE ROBOTICS AND AUTOMATION LETTERS. 2016;1(2):844–851.

[45] Engeberg ED, Meek SG, Minor MA. Hybrid force-velocity sliding mode control of a prosthetic hand. IEEE Trans Biomed Engineering. 2008;55(5):1572–1581. Available from: http://dx.doi.org/10.1109/TBME.2007.914672.

[46] Buschmann T, Lohmeier S, Ulbrich H. Biped walking control based on hybrid position/force control. In: 2009 IEEE/RSJ international conference on intelligent robots and systems, october 11-15, 2009, st. louis, mo, USA. 2009. p. 3019–3024. Available from: http://dx.doi.org/10.1109/IROS.2009.5354643.

[47] Bellens S, Schutter JD, Bruyninckx H. A hybrid pose / wrench control framework for quadrotor helicopters. In: IEEE international conference on robotics and automation, ICRA 2012, 14-18 may, 2012, st. paul, minnesota, USA. 2012. p. 2269–2274. Available from: http://dx.doi.org/10.1109/ICRA.2012.6224682.

[48] Madani M, Moallem M. Hybrid position/force control of a flexible parallel manipulator. Journal of the Franklin Institute. 2011;348(6):999 – 1012. Available from: http://www.sciencedirect.com/science/article/pii/S0016003211000603.

[49] Visioli A, Ziliani G, Legnani G. Iterative-learning hybrid force/velocity control for contour tracking. IEEE Trans Robotics. 2010;26(2):388–393. Available from: http://dx.doi.org/10.1109/TRO.2010.2041265.

[50] Kumar N, Panwar V, Sukavanam N, Sharma SP, Borm JH. Neural network based hybrid force/position control for robot manipulators. International Journal of Precision Engineering and Manufacturing. 2011;12(3):419–426. Available from: http://dx.doi.org/10.1007/s12541-011-0054-3.

[51] Wen S, Zheng W, Zhu J, Li X, Chen S. Elman fuzzy adaptive control for obstacle avoidance of mobile robots using hybrid force/position incorporation. IEEE Trans Systems, Man, and Cybernetics, Part C. 2012;42(4):603–608. Available from: http://dx.doi.org/10.1109/TSMCC.2011.2157682.

[52] Jatta F, Legnani G, Visioli A. Friction compensation in hybrid force/velocity control of industrial manipulators. IEEE TRANSACTIONS ON INDUSTRIAL ELECTRONICS. 2006;53(2):604–613.

[53] Xiao D, Ghosh BK, Xi N, Tarn TJ. Sensor-based hybrid position/force control of a robot manipulator in an uncalibrated environment. IEEE TRANSACTIONS ON CONTROL SYSTEMS TECHNOLOGY. 2000;8(4):635–645.

[54] Kankaanranta R, Koivo H. Dynamics and simulation of compliant motion of a manipulator. J Robot Automat. 1988;:163–173.

[55] Lozano R, Brogliato B. Adaptive hybrid force-position control for redundant manipulators. 29th IEEE Conference on Decision and Control. 1990;37(10):1501–1505.

[56] Hsia T. Adaptive control of robot manipulators - A review. Proceedings 1986 IEEE International Conference on Robotics and Automation. 1986;.

[57] Slotine JE, Li W. Adaptive manipulator control: A case study. IEEE Transactions on Automatic Control. 1988;33(11):995–1003. Available from: http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=14411.

[58] Arimoto S, Liu Y, Naniwa T. Model-based adaptive hybrid control for geometrically constrained robots. In: Proceedings of the 1993 IEEE international conference on robotics

and automation, atlanta, georgia, usa, may 1993. 1993. p. 618–623. Available from: http://dx.doi.org/10.1109/ROBOT.1993.292047.

[59] Namvar M, Aghili F. Adaptive force-motion control of coordinated robots interacting with geometrically unknown environments. IEEE Trans Robotics. 2005;21(4):678–694. Available from: http://dx.doi.org/10.1109/TRO.2004.842346.

[60] Cheah CC, Hou SP, Zhao Y, Slotine JJE. Adaptive vision and force tracking control for robots with constraint uncertainty. IEEE/ASME Transactions on Mechatronics. 2010;15(3):389 – 399.

[61] Patel RV, Talebi HA, Jayender J, Shadpey F. A robust position and force control strategy for 7-dof redundant manipulators. IEEE/ASME TRANSACTIONS ON MECHATRONICS. 2009;14(5):575–589.

[62] Cheah C, Kawamura S, Arimoto S. Stability of hybrid position and force control for robotic manipulator with kinematics and dynamics uncertainties. Automatica. 2003 May;39(5):847–855. Available from: http://linkinghub.elsevier.com/retrieve/pii/S0005109803000025.

[63] Ortenzi V, Lin H, Azad M, Stolkin R, Kuo JA, Mistry M. Kinematics-based estimation of contact constraints using only proprioception. In: 16th IEEE-RAS international conference on humanoid robots, humanoids 2016, cancun, mexico, november 15-17, 2016. 2016. p. 1304–1311.

[64] Magrini E, Flacco F, De Luca A. Estimation of Contact Force using a Virtual Force Sensor. 2014 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS 2014). 2014; (Iros):2126–2133.

[65] Ortenzi V, Stolkin R, Kuo JA, Mistry M. Projected inverse dynamics control and optimal control for robots in contact with the environment: A comparison. In: 2015 IEEE/RSJ international conference on intelligent robots and systems, IROS 2015, hamburg, germany, september 28 - october 2, 2015. 2015. p. 4009–4015.

[66] Anitescu M, Potra F. Formulating dynamic multi-rigid-body contact problems with friction as solvable linear complementarity problems. Nonlinear Dynamics. 1997;(93):1–21. Available from: http://link.springer.com/article/10.1023/A:1008292328909.

[67] Tassa Y, Erez T, Todorov E. Synthesis and stabilization of complex behaviors through online trajectory optimization. In: Ieee international conference on intelligent robots and systems. 2012. p. 4906–4913.

[68] Posa M, Tedrake R. Direct Trajectory Optimization of Rigid Body Dynamical Systems through Contact. In: Algorithmic foundations of robotics x. 2013.

[69] Posa M, Cantu C, Tedrake R. A direct method for trajectory optimization of rigid bodies through contact. The International Journal of Robotics Research. 2013;33(1):69–81. Available from: http://ijr.sagepub.com/content/33/1/69.short.

[70] Tarn TJ, Wu Y, Xi N, Isidori A. Force regulation and contact transition control. IEEE Control Systems Magazine. 1996;16(1):32–40.

[71] Heck D, Saccon A, van de Wouw N, Nijmeijer H. Guaranteeing stable tracking of hybrid position-force trajectories for a robot manipulator interacting with a stiff environment. Automatica. 2016; 63:235 – 247.

[72] Posa M, Tobenkin M, Tedrake R. Lyapunov analysis of rigid body systems with impacts and friction via sums-of-squares. In: Proceedings of the 16th international conference on hybrid systems: computation and control - hscc '13. 2013.