A Utility-Based Joint Subcarrier and Power Allocation for Green Communications in Multi-user Two-Way Regenerative Relay Networks

Citation for published version:
Singh, K, Gupta, A & Ratnarajah, T 2017, 'A Utility-Based Joint Subcarrier and Power Allocation for Green Communications in Multi-user Two-Way Regenerative Relay Networks', *IEEE Transactions on Communications*. https://doi.org/10.1109/TCOMM.2017.2713382

Digital Object Identifier (DOI):
10.1109/TCOMM.2017.2713382

Link:
Link to publication record in Edinburgh Research Explorer

Document Version:
Peer reviewed version

Published In:
IEEE Transactions on Communications

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TCOMM.2017.2713382, IEEE Transactions on Communications

A Utility-Based Joint Subcarrier and Power Allocation for Green Communications in Multi-user Two-Way Regenerative Relay Networks

Keshav Singh, *Member, IEEE*, Ankit Gupta, and Tharmalingam Ratnarajah, *Senior Member, IEEE*

***Abstract*—In this paper, we investigate utility-based joint subcarrier and power allocation algorithms for improving the energy efficiency (EE) in multi-user two-way regenerative relay networks. With the objective of determining the best subcarrier allocation for each user pair, subcarrier pairing permutation, and power allocation to all the nodes, a network price is introduced to the power consumption as a penalty for the achievable sum rate, followed by the examination of its impact on the trade-off between the EE and spectral efficiency (SE). The formulated optimization problem is a non-convex mixed-integer nonlinear programming problem, thus a concave lower bound on the objective function and a series of convex transformations are applied to transform the problem into a convex one. Through dual decomposition, we propose a utility-based resource allocation algorithm for iteratively tightening the lower bound and finding the optimal solution of the primal problem. By exploring the structure of the obtained optimal solution, an optimal price that enables green resource allocation is found from the perspective of maximizing EE. Additionally, a suboptimal algorithm is investigated to strike a balance between computational complexity and optimality. Simulation results evince the effectiveness of the proposed algorithms.**

***Index Terms*—Regenerative, two-way relay networks, green communications, subcarrier pairing permutation and allocation, power allocation, energy efficiency, multi-user communications.**

I. Introduction

Cooperative communication is a promising way to enhance the reliability, coverage and network performance of wireless communications. Various relaying schemes have been proposed for cooperative communication, like amplify-and-forward (AF), regenerative or decode-and-forward (DF), and compress-and-forward (CF) [1], of which the AF scheme is more prominently deployed due to its lower implementation complexity. In AF protocol, the relay retransmits the amplified signal to the destination, whereas in DF protocol, the relay decodes the received signals and retransmits the re-encoded information bits to the destination node(s). However, the DF protocol performs better than the AF protocol when the channel quality of the source-to-relay (SR) link is good enough. Furthermore, the DF protocol also enables to deploy different channel coding schemes at the source and the relay nodes. Moreover, two-way relaying has been widely investigated to overcome the drawbacks of half-duplex relaying by utilizing the spectrum resources more efficiently [2]–[5]. In addition, multicarrier multiple access techniques that allow multiple users to share the same spectrum and avoid severe interference from other users, when combined with the relay transmission, can significantly improve the system performance, due to their flexibility in resource allocation and the ability to exploit multi-user diversity.

Futhrtmore, the unprecedented increase of devices and escalating data rate requirements have contributed to sharp growth of energy consumption and greenhouse emission. It is reported in [6], [7] that 4.7% of the global energy is consumed by information and communication technologies (ICT) and it releases approximately 1.7% of the total CO_2 into the atmosphere. The impact of ICT is estimated to be 4 Gt (gigatonnes) of CO_2 by 2030. Hence, ameliorating energy efficiency (EE) of communication networks becomes of paramount importance in realizing 5G radio access solutions. Consequently, research focus has shifted towards designing energy-aware architectures and resource allocation techniques that not only prolong the networks lifespan but also provide significant energy savings under the umbrella of green communications [6].

Recently, a flourish of works on resource allocation in orthogonal frequency division multiplexing (OFDM)-based cooperative relay networks has been investigated in [8]–[17] from the perspective of SE maximization (SEM). In [8] and [9], the optimal power allocation schemes were investigated for maximizing the rate of one-way DF networks under a sum power constraint, or individual power constraints at the source and the relay nodes. A bidirectional DF relay-aided full-duplex (FD) network consisting of two FD users and a single FD relay was considered in [17] to analyze error-free data rates, while the outage performance of the three-node two-way FD relay system was studied in [11]. In [12], the ergodic achievable rates were investigated for a multi-pair massive multiple-input and multiple-output (MIMO) two-way AF relaying with imperfect channel state information (CSI). The power allocation strategies with subcarrier pairing were proposed in [14] and [15] for DF and AF multi-relay networks, respectively. The optimization problems for joint subcarrier pairing and power allocation with a total network power constraint or with individual power constraints for the source and the relay nodes were formulated in [15] and [16].

This work was supported by the U.K. Engineering and Physical Sciences Research Council (EPSRC) under grants EP/L025299/1

Manuscript received November 08, 2016; revised April 07, 2017, accepted May 29, 2017. The associate editor coordinating the review of this paper and approving it for publication was Prof. Ali Ghrayeb.

Keshav Singh and Tharmalingam Ratnarajah are with the Institute for Digital Communications, the University of Edinburgh, Kings Building, Edinburgh, UK, EH9 3FG. E-mails: {K.Singh; T.Ratnarajah}@ed.ac.uk.

Ankit Gupta is with Aricent Technologies Limited (Holdings), Gurgaon, India. E-mail: ankit8.gupta@aricent.com.

The corresponding author of this paper is Keshav Singh.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TCOMM.2017.2713382, IEEE Transactions on Communications

To overcome the interference problem, authors in [16] jointly optimized power allocation, relay selection and subcarrier assignment. To maximize the average utility of all users with multiservice in a relay-aided OFDM access (OFDMA) system, a utility-based dynamic resource allocation algorithm was studied in [17], wherein the issue of the energy efficiency (EE) was ignored. However, only a few works have considered the EE as a key metric for designing the optimal resource allocation policies in the relay networks [18]–[24]. In [18], a joint power control and antenna beamforming algorithm was proposed to maximize EE in very large multi-user MIMO systems, whereas a pricing-based power allocation scheme for multi-user AF relay networks was investigated in [19] and the trade-off between EE and SE was studied for multi-user MIMO systems in [20]. The authors in [22] have proposed a joint power and subchannel allocation for OFDMA-based two-way relay networks, wherein same subchannel is assigned to the multiple access (MA) and the broadcast (BC) phase and thus, the subchannel assignment and power allocation schemes limit the network performance. However, if we use different subchannels in MA and BC phase, the network performance can be significantly improved. In [23], an energy-efficient resource scheduling solution for downlink transmission in multiuser OFDMA networks was proposed under imperfect CSI, while authors in [24] extended the work of [23] for multicarrier under perfect CSI knowledge and studied joint subcarrier and power allocation problem under a total power constraint for downlink multiuser OFDMA system. However, the resource allocation problem in [23] and [24] was optimized only in downlink scenario for maximizing EE and it is not straightforward to apply the same in multi-hop relay networks. Therefore, there is a need to revisit the design of existing multi-user two-way DF relay networks and to investigate the associated resource allocation policies by considering subcarrier pairing permutation, power optimization, and subcarrier allocation all together in order to improve the networks' EE.

Unlike the previous existing research works [8]–[17], wherein the throughput in OFDM network was maximized by optimizing either of the following: i) subcarrier allocation among different users, ii) subcarrier pairing at a relay node, where the signal received at the relay over one subcarrier is re-transmitted on a different subcarrier, iii) power allocation over different subcarriers at each transmitting node, or iv) power allocation and subcarrier assignment, and the works on energy-efficient resource allocation [18]–[24], we adopt a pricing-based approach, in which subcarrier pairing permutation is performed in one-to-one and many-to-many fashion before assigning to a particular user pair. To the best of the authors' knowledge, a unified resource allocation scheme considering subcarrier pairing permutation obtained in one-to-one and many-to-many manner, power optimization and subcarrier allocation all together for multiuser DF relay networks has not yet been explored from a green communication perspective. In this paper, we investigate joint optimization of subcarrier pairing permutation, subcarrier allocation and power allocation all together for multiuser multicarrier two-way DF relay networks for improving the EE under the constraints of limited total transmit power, subcarrier pairing, and subcarrier allocation, while balancing the sum rate of the two-way links. The distinctive contributions of this paper are summarized as follows:

- In contrast to [19], [22]–[24], a network pricing-based approach, which enables us to strike a balance between the achievable sum rate and power consumption in the relay networks, is proposed for considered resource allocation problem. Through a joint optimization of subcarrier pairing permutation, subcarrier allocation and power allocation all together, we intend to maximize the pricing-based network utility function in multi-user multicarrier two-way DF relay network subject to limited transmit power, subcarrier pairing, and subcarrier allocation constraints. It is evident that the original problem is a non-convex mixed-integer nonlinear programming (MINLP) [27], which is NP-hard to solve.
- To make the problem tractable, we resort to lower bound approximation using a successive convex approximation (SCA) method along with variable transformation and relaxation of integer variables. Next, it is proven that the relaxed problem is quasi-concave on the subcarrier pairing, subcarrier allocation, and power allocation variables. Based on the concepts of dual decomposition, a utility-based joint subcarrier and power allocation algorithm is proposed for iteratively enhancing the lower bound and thus determining the optimal solution. We then rigorously analyze the structure of obtained solution and define the optimal network price that maximizes EE, which can be described as the ratio of the achievable SE to the total power consumption. The EE maximization (EEM) algorithm that iteratively find the optimal network price is proposed with the derivation of the convergence behavior.
- We extend the proposed resource allocation algorithm for a more general scenario where subcarrier pairing permutation is obtained in *many-to-many* manner[1].
- Additionally, a suboptimal EE resource allocation algorithm is also investigated to strike a balance between computational complexity and optimality. Extensive simulation results are provided to reveal the merits and benefits of the proposed EE resource allocation algorithms. Moreover, we also demonstrate the impact of various network parameters on the trade-off between the EE and SE.

The remainder of this paper is organized as follows. Section II describes the system model. The EE maximization problem subject to a total transmit power constraint is formulated in Section III, followed by stepwise procedure of transforming the non-convex MINLP problem into a convex one. An iterative EE resource allocation algorithm is investigated in Section IV. The suboptimal algorithm is presented in Section V and the computation complexity of proposed and standard algorithms are analyzed in Section VI. Section VII presents the simulation results. Finally, conclusions are drawn in Section VIII.

[1]Note: Since a DF protocol is applied at the relay node, each subcarrier can be paired in a *many-to-many fashion*, i.e., a single subcarrier of MA phase can pair with a single or multiple subcarrier(s) of BC phase and vice-versa. However, each subcarrier pair assigns to only a single user pair.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TCOMM.2017.2713382, IEEE Transactions on Communications

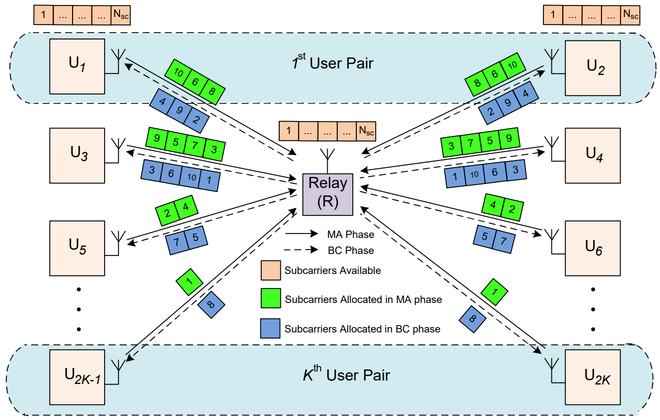


Fig. 1. A relay-assisted multi-user two-way relay network with K user pairs.

II. System Model

We consider a relay interference network, wherein a DF relay assists the two-way communication between K user pairs, whilst each transmission hop has N_{sc} subcarriers for signal transmission as illustrated in Fig. 1. All the nodes in the network are assumed to have a single antenna. For simplicity, the transmit and receive users are assumed to be well separated so that the direct links between them can be ignored. We further consider that all the links experience slow and frequency-flat fading. It is also assumed that all the users and the relay node have perfect CSI knowledge. By exploiting the channel reciprocity between forward and backward transmissions through orthogonal pilot signals, which are simultaneously sent by multiple users in some dedicated beacon time slots, the CSIs of the links can be estimated at the relay and the users and the CSI estimation could be very accurate if the training period is sufficiently long. The relay network operates in a half-duplex mode with two transmission phases [1]. In MA phase, all the $2K$ users simultaneously transmit signals to the relay node, while during the BC phase, the relay node forwards the re-encoded signal to $2K$ users. Moreover, the two users of k^{th} user pair, i.e., $(2k-1)^{th}$ and $(2k)^{th}$ users, transmit signals on the u^{th} subcarrier in the MA phase, whereas in BC phase, the relay node forwards the re-encoded signal on the v^{th} subcarrier to the k^{th} user pair.

Define $h_i^{(u)}$ as channel coefficient from the i^{th} user to relay node on the u^{th} subcarrier, for $i = 1, \ldots, 2K$, $u = 1, \ldots, N_{sc}$. In MA phase, the received signal at relay node on the u^{th} subcarrier can be expressed as

$$y_R^{(u)} = \sum_{i=1}^{2K} h_i^{(u)} \sqrt{P_i^{(u)}} x_i^{(u)} + n_R^{(u)} \,, \tag{1}$$

where $x_i^{(u)}$ is the i^{th} user's signal transmitted on the u^{th} subcarrier with unit transmission power, i.e. $\mathbb{E}\left[\left|x_i^{(u)}\right|^2\right] = 1$, $n_R^{(u)} \sim \mathcal{N}(0, {\sigma_R^{(u)}}^2)$ and $P_i^{(u)}$ denote the complex additive white Gaussian noise (AWGN) at the relay node and the transmit power level of the i^{th} user on u^{th} subcarrier, respectively. From (1), the signal-to-interference plus noise ratio (SINR) at the relay node for the i^{th} user on subcarrier u is given as

$$\Gamma_{R_i}^{(u)} = \frac{P_i^{(u)} \left|h_i{}^{(u)}\right|^2}{\sum\limits_{l=1, l\neq i}^{2K} P_l^{(u)} \left|h_l{}^{(u)}\right|^2 + {\sigma_R^{(u)}}^2} \,, \qquad \forall i \,, \tag{2}$$

By assuming that the data symbol is decodable at relay node, in BC phase, the received signal at the $(2k-1)^{th}$ and $(2k)^{th}$ users under a perfect self-interference cancellation [25], can respectively be written as

$$\begin{aligned} y_{2k-1}^{(v)} =& \sqrt{W_{2k}^{(v)}} g_{2k-1}^{(v)} x_{2k}^{(v)} \\ &+ \sum_{\substack{l=1 \\ l\neq 2k-1, 2k}}^{2K} \sqrt{W_l^{(v)}} g_{2k-1}^{(v)} x_l^{(v)} + n_{2k-1}^{(v)} \,, \quad \forall k \,; \end{aligned} \tag{3}$$

$$\begin{aligned} y_{2k}^{(v)} =& \sqrt{W_{2k-1}^{(v)}} g_{2k}^{(v)} x_{2k-1}^{(v)} \\ &+ \sum_{\substack{l=1 \\ l\neq 2k, 2k-1}}^{2K} \sqrt{W_l^{(v)}} g_{2k}^{(v)} x_l^{(v)} + n_{2k}^{(v)} \,, \qquad \forall k \,, \end{aligned} \tag{4}$$

where $W_{2k-1}^{(v)}$ and $W_{2k}^{(v)}$ are the transmit powers at relay node on the v^{th} subcarrier for $(2k-1)^{th}$ and $(2k)^{th}$ users, respectively, and $n_i^{(v)} \sim \mathcal{N}(0, {\sigma_i^{(v)}}^2)$ is the received AWGN at the i^{th} user on subcarrier v. $g_i^{(v)}$ is defined similar to $h_i^{(u)}$, but for the relay-to-destination (RD) links.

From (3) and (4), the SINRs at the $(2k-1)^{th}$ and $(2k)^{th}$ users on the v^{th} subcarrier can respectively be given as

$$\Gamma_{2k-1}^{(v)} = \frac{W_{2k}^{(v)} \left|g_{2k-1}^{(v)}\right|^2}{\sum\limits_{l=1, l\neq 2k, 2k-1}^{2K} W_l^{(v)} \left|g_{2k-1}^{(v)}\right|^2 + {\sigma_{2k-1}^{(v)}}^2} \,; \tag{5}$$

$$\Gamma_{2k}^{(v)} = \frac{W_{2k-1}^{(v)} \left|g_{2k}^{(v)}\right|^2}{\sum\limits_{l=1, l\neq 2k-1, 2k}^{2K} W_l^{(v)} \left|g_{2k}^{(v)}\right|^2 + {\sigma_{2k}^{(v)}}^2} \,, \tag{6}$$

Furthermore, the total power consumption in the network consists of two terms namely: transmit power and static power, which has remarkable impact on system's SE. Hence, it is important to take both transmit and static power into consideration [21], [26], while designing an energy-efficient network. The transmitter's signal processing power and the receiver's processing power contribute towards the circuit power which are not related to the sum rate when the users transmit or receive information and is regarded as static value here, while the transmit power is exclusively used for data transmission in order to attain reliable communications. In general, the transmit power behaves dynamically with respect to the instantaneous channel gains, but the circuit/processing power usually remains static, irrespective of the channel conditions. Therefore, the overall required power (in Watts) for the considered two-way relay network is assumed to be governed by a constant term that covers the static power dissipation of the nodes and other two terms that vary with the transmit powers $P_i^{(u)}$ and $W_i^{(v)}$. The total power dissipation in the network before subcarrier pairing and allocation can be written as

$$\mathcal{P}_T = \underbrace{\sum_{i=1}^{2K}\left(\sum_{u=1}^{N_{sc}} P_i^{(u)} + \sum_{v=1}^{N_{sc}} W_i^{(v)}\right)}_{\text{Dynamic Power}\leq P_{max}} + \underbrace{2(K+1)X_c}_{\text{Static Power}, \triangleq P_C}, \quad (7)$$

where P_{max} is the maximum transmit power budget of the two-way relay network and X_c denotes the circuit and processing power of each user. Due to large signaling processing at the relay node, the value of X_c must be higher than the destination nodes, and thus the value of static power consumption for the relay node is considered to be twice than a single destination node.

Let $\mho^{(u,v)}$ denotes the subcarrier pairing variable signifying that u^{th} subcarrier in the MA phase is paired with the v^{th} subcarrier in the BC phase, and $\Pi_k^{(u,v)}$ represents the subcarrier allocation variable symbolizing that the k^{th} user pair is operating on the $(u,v)^{th}$ subcarrier pair, respectively. Thus, the power dissipated after subcarrier pairing and allocation is given by

$$\begin{aligned}&\mathcal{P}_{Total}\left(\mathbf{P},\mathbf{W},\boldsymbol{\mho},\boldsymbol{\Pi}\right) = \\ &\sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}} \mho^{(u,v)}\Pi_k^{(u,v)}\left(P_{2k-1}^{(u)} + P_{2k}^{(u)} + W_{2k-1}^{(v)} + W_{2k}^{(v)}\right) \\ &\qquad\qquad + P_C\,; \quad [\text{Watts}]\,, \qquad (8)\end{aligned}$$

where $\mathbf{P} = \{P_i^{(u)}\}, \forall i, u$, $\mathbf{W} = \{W_i^{(v)}\}, \forall i, v$, $\boldsymbol{\mho} = \{\mho^{(u,v)}\}, \forall u, v$, and $\boldsymbol{\Pi} = \{\Pi_k^{(u,v)}\}, \forall k, (u,v)$.

From (2), (5) and (6), the achievable minimum (worst) sum rate for the $2k \to (2k-1)$ and $(2k-1) \to 2k$ links on the $(u,v)^{th}$ subcarrier pair can be written as

$$R_{2k-1}^{(u,v)} = \frac{1}{2}\log_2\left(1 + \min\{\Gamma_{R_{2k}}^{(u)}, \Gamma_{2k-1}^{(v)}\}\right), \ [\text{bits/s/Hz}]\,; \quad (9)$$

$$R_{2k}^{(u,v)} = \frac{1}{2}\log_2\left(1 + \min\{\Gamma_{R_{2k-1}}^{(u)}, \Gamma_{2k}^{(v)}\}\right), \ [\text{bits/s/Hz}]\,, \quad (10)$$

where the factor of $1/2$ accounts for the fact that transmission completes in two-hops. Further, the total achievable end-to-end sum rate after subcarrier pairing and allocation is given by

$$\begin{aligned}&\mathcal{R}_{Total}\left(\mathbf{P},\mathbf{W},\boldsymbol{\mho},\boldsymbol{\Pi}\right) = \\ &\qquad\sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}} \mho^{(u,v)}\Pi_k^{(u,v)}\left(R_{2k-1}^{(u,v)} + R_{2k}^{(u,v)}\right). \quad (11)\end{aligned}$$

III. Utility-Based Problem Formulation

A utility-based subcarrier and power allocation problem is defined in this section. Using (8) and (11), the network utility function can be defined as

$$\begin{aligned}\mathcal{U}\left(\mathbf{P},\mathbf{W},\boldsymbol{\mho},\boldsymbol{\Pi}\right) =& \mathcal{R}_{Total}\left(\mathbf{P},\mathbf{W},\boldsymbol{\mho},\boldsymbol{\Pi}\right) \\ &- \lambda\mathcal{P}_{Total}\left(\mathbf{P},\mathbf{W},\boldsymbol{\mho},\boldsymbol{\Pi}\right), \quad (12)\end{aligned}$$

where $\lambda\mathcal{P}_{Total}\left(\mathbf{P},\mathbf{W},\boldsymbol{\mho},\boldsymbol{\Pi}\right)$ denotes the cost paid for resource utilization and $\lambda \geq 0$ represents the unit price of power, which strikes a balance between the power utilization in (8) and the achievable sum rate in (11). Generally speaking, the price λ reveals a broad range of network characteristics in terms of resource management. For example: when $\lambda \to 0$, this implies that the price paid for the resource utilization is negligible, and thus the resource allocation problem regenerates to a SEM problem. However, when $\lambda > 0$ and increases, the network price shows the importance of the power resources for the design of resource allocation in a relay network. For an extreme case when $\lambda \to \infty$, the users in the network are compelled to pay a hefty price as a penalty in order to utilize the available resources for maximizing the utility function in (12) and thus, no resource allocation policy would be suitable enough.

Provided that the total transmit power is bounded by P_{max}, a utility-based joint subcarrier and power allocation problem can be formulated by using (12) as

$$\begin{aligned}&(\mathbf{P.1}) \\ &\max_{\mathbf{P},\mathbf{W},\boldsymbol{\mho},\boldsymbol{\Pi}} \ \mathcal{U}\left(\mathbf{P},\mathbf{W},\boldsymbol{\mho},\boldsymbol{\Pi}\right) \\ &\text{s.t.}\ (C.1)\ \sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}} \mho^{(u,v)}\Pi_k^{(u,v)} \\ &\qquad\qquad \times\left(P_{2k-1}^{(u)} + P_{2k}^{(u)} + W_{2k-1}^{(v)} + W_{2k}^{(v)}\right) \leq P_{max}\,; \\ &\quad (C.2)\ \sum_{u=1}^{N_{sc}} \mho^{(u,v)} = 1, \qquad \forall v\,; \\ &\quad (C.3)\ \sum_{v=1}^{N_{sc}} \mho^{(u,v)} = 1, \qquad \forall u\,; \quad (13) \\ &\quad (C.4)\ \sum_{i=1}^{K} \Pi_i^{(u,v)} = 1, \qquad \forall (u,v)\,; \\ &\quad (C.5)\ \mho^{(u,v)} \in \{0,1\},\ \Pi_i^{(u,v)} \in \{0,1\}, \qquad \forall i, (u,v)\,; \\ &\quad (C.6)\ P_i^{(u)} \geq 0,\ W_i^{(v)} \geq 0, \qquad \forall i, u, v\,,\end{aligned}$$

Physically, the constraint $(C.1)$ ensures that the sum of the power allocated to the users $P_{2k-1}^{(u)}$ and $P_{2k}^{(u)}$, $\forall k, u$, and the relay node $W_R^{(v)}$, $\forall v$, does not exceed the maximum power budget P_{max} of the network, while the constraints $(C.2)$ and $(C.3)$ ensure that each subcarrier in MA phase can be paired with one and only one subcarrier in BC phase and vice verse; and $(C.4)$ mandates that a subcarrier pair (u,v) is allocated to a single user pair only. General speaking, it is very difficult to find the optimal resource allocation solution of the optimization problem (**P.1**) due to binary constraints in the subcarrier pairing and allocation. To find the optimal solution, an exhaustive search (ES) over all variables is required and thus the computation complexity becomes very high, specially for higher number of subcarriers. Therefore, we relax constraint $(C.5)$ and allow $\mho^{(u,v)}$ and $\Pi_i^{(u,v)}$ to assume any real value within the interval $(0,1]$. The fact that the duality gap between the primal problem and the dual problem approaches to zero for a sufficiently large number of subcarriers [28], inspires us that instead of solving the primal problem directly, we can solve it by the dual problem. Further, by applying the

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TCOMM.2017.2713382, IEEE Transactions on Communications

epigraph method, the problem (**P.1**) can be transformed as

$$(\mathbf{P.2})\ \max_{\mathbf{P},\mathbf{W},\mho,\mathbf{\Pi},\mathbf{\Upsilon}} \bar{\mathcal{U}}\left(\mathbf{P},\mathbf{W},\mho,\mathbf{\Pi},\mathbf{\Upsilon}\right)$$

$$\text{s.t.} \quad (C.1)-(C.4)\ \&\ (C.6)\ ; \tag{14}$$

$$(C.7)\ \min\left\{\Gamma_{R_{2k}}^{(u)},\Gamma_{2k-1}^{(v)}\right\} \geq \Upsilon_{2k-1}^{(u)}$$

$$\Rightarrow \begin{cases}(C.7a)\ \Gamma_{R_{2k}}^{(u)} \geq \Upsilon_{2k-1}^{(u)}, & \forall\, k,u\,;\\ (C.7b)\ \Gamma_{2k-1}^{(v)} \geq \Upsilon_{2k-1}^{(u)}, & \forall\, k,u,v\,;\end{cases}$$

$$(C.8)\ \min\left\{\Gamma_{R_{2k-1}}^{(u)},\Gamma_{2k}^{(v)}\right\} \geq \Upsilon_{2k}^{(u)}$$

$$\Rightarrow \begin{cases}(C.8a)\ \Gamma_{R_{2k-1}}^{(u)} \geq \Upsilon_{2k}^{(u)}, & \forall\, k,u\,;\\ (C.8b)\ \Gamma_{2k}^{(v)} \geq \Upsilon_{2k}^{(u)}, & \forall\, k,u,v\,;\end{cases}$$

where $\Upsilon_{2k-1}^{(u)}$ and $\Upsilon_{2k}^{(u)}$ are the auxiliary variables, $\mathbf{\Upsilon} = \{\Upsilon_i^{(u)}\}, \forall i, u$; and the objective function $\bar{\mathcal{U}}\left(\mathbf{P},\mathbf{W},\mho,\mathbf{\Pi},\mathbf{\Upsilon}\right)$ is given by

$$\bar{\mathcal{U}}\left(\mathbf{P},\mathbf{W},\mho,\mathbf{\Pi},\mathbf{\Upsilon}\right) = \sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}} \mho^{(u,v)}\Pi_k^{(u,v)}\left(\bar{R}_{2k-1}^{(u,v)}+\bar{R}_{2k}^{(u,v)}\right) - \lambda\mathcal{P}_{Total}\left(\mathbf{P},\mathbf{W},\mho,\mathbf{\Pi}\right)\ ; \tag{15}$$

where

$$\bar{R}_{2k-1}^{(u,v)} = \frac{1}{2}\log_2\left(1+\Upsilon_{2k-1}^{(u)}\right); \tag{16}$$

$$\bar{R}_{2k}^{(u,v)} = \frac{1}{2}\log_2\left(1+\Upsilon_{2k}^{(u)}\right), \tag{17}$$

Lemma 1: For fixed utility price λ and given subcarrier pairing $\mho$ and subcarrier allocation $\mathbf{\Pi}$, the objective function in (14) is a quasi-concave function of $\mathbf{P}, \mathbf{W}$, and $\mathbf{\Upsilon}$.

Proof: See Appendix A. ∎

Due to non-convexity of the constraints $(C.7)$ and $(C.8)$, the optimization problem (**P.2**) is still non-convex [27] for given subcarrier pairing $\mho$ and subcarrier allocation $\mathbf{\Pi}$, respectively. By utilizing change of variables $\hat{P}_{2k-1}^{(u)} = \log P_{2k-1}^{(u)}$, $\hat{P}_{2k}^{(u)} = \log P_{2k}^{(u)}$, $\hat{W}_{2k-1}^{(v)} = \log W_{2k-1}^{(v)}$, $\hat{W}_{2k}^{(v)} = \log W_{2k}^{(v)}$, $\hat{\Upsilon}_{2k-1}^{(u)} = \log \Upsilon_{2k-1}^{(u)}$ and $\hat{\Upsilon}_{2k}^{(u)} = \log \Upsilon_{2k}^{(u)}$, the problem $(\mathbf{P}.2)$ can be equivalently written as follows:

(**P.3**)

$$\max_{\substack{\hat{\mathbf{P}},\hat{\mathbf{W}}\\ \mho,\mathbf{\Pi},\hat{\mathbf{\Upsilon}}}} \sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}} \frac{\mho^{(u,v)}\Pi_k^{(u,v)}}{2} \times\left(\log_2\left(1+e^{\hat{\Upsilon}_{2k-1}^{(u)}}\right)+\log_2\left(1+e^{\hat{\Upsilon}_{2k}^{(u)}}\right)\right) - \lambda\left(\sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}}\mho^{(u,v)}\Pi_k^{(u,v)} \times\left(e^{\hat{P}_{2k-1}^{(u)}}+e^{\hat{P}_{2k}^{(u)}}+e^{\hat{W}_{2k-1}^{(v)}}+e^{\hat{W}_{2k}^{(v)}}\right)+P_c\right)$$

$$\text{s.t.}\ (C.1)\sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}}\mho^{(u,v)}\Pi_k^{(u,v)} \times\left(e^{\hat{P}_{2k-1}^{(u)}}+e^{\hat{P}_{2k}^{(u)}}+e^{\hat{W}_{2k-1}^{(v)}}+e^{\hat{W}_{2k}^{(v)}}\right) \leq P_{max}\,;$$

$$(C.2)-(C.4)\ ;$$

$$(C.6)\ e^{\hat{P}_i^{(u)}} \geq 0,\ e^{\hat{W}_i^{(v)}} \geq 0, \qquad \forall i,u,v\,;$$

$$(C.7a)\ \frac{e^{\hat{\Upsilon}_{2k-1}^{(u)}-\hat{P}_{2k}^{(u)}}}{\left|h_{2k}^{(u)}\right|^2}\left(\sum_{\substack{l=1\\ l\neq 2k}}^{2K} e^{\hat{P}_l^{(u)}}\left|h_l^{(u)}\right|^2+\sigma_R^{(u)^2}\right) \leq 1\,, \quad \forall k,u\,;$$

$$(C.7b)\ \frac{e^{\hat{\Upsilon}_{2k-1}^{(u)}-\hat{W}_{2k}^{(v)}}}{\left|g_{2k-1}^{(v)}\right|^2}\left(\sum_{\substack{l=1\\ l\neq 2k-1,2k}}^{2K} e^{\hat{W}_l^{(v)}}\left|g_{2k-1}^{(v)}\right|^2+\sigma_{2k-1}^{(v)^2}\right) \leq 1 \quad \forall k,u,v\,;$$

$$(C.8a)\ \frac{e^{\hat{\Upsilon}_{2k}^{(u)}-\hat{P}_{2k-1}^{(u)}}}{\left|h_{2k-1}^{(u)}\right|^2}\left(\sum_{\substack{l=1\\ l\neq 2k-1}}^{2K} e^{\hat{P}_l^{(u)}}\left|h_l^{(u)}\right|^2+\sigma_R^{(u)^2}\right) \leq 1\,, \quad \forall k,u\,;$$

$$(C.8b)\ \frac{e^{\hat{\Upsilon}_{2k}^{(u)}-\hat{W}_{2k-1}^{(v)}}}{\left|g_{2k}^{(v)}\right|^2}\left(\sum_{\substack{l=1\\ l\neq 2k,2k-1}}^{2K} e^{\hat{W}_l^{(v)}}\left|g_{2k}^{(v)}\right|^2+\sigma_{2k}^{(v)^2}\right) \leq 1\,, \quad \forall k,u,v\,, \tag{18}$$

Remark 1: Since the objective function in (18) is non-concave, we cannot solve this optimization problem in its current form. To convert the objective function into concave, we need to consider a lower bound on it. Before finding the lower bound of the objective function in (18), we introduce a Lemma to find the lower bound.

Lemma 2: The logarithmic function $\log(1+\theta)$ has the following lower bound

$$\log(1+\theta) \geqslant x\log(\theta)+y\,, \qquad \forall\theta>0\,; \tag{19}$$

where $x > 0$ and y are the coefficients that need to be determined, and it is assumed that the bound is tight at $\theta = \theta_0$, then

$$x = \frac{\theta_0}{1+\theta_0}\,; \tag{20}$$

$$y = \log(1+\theta_0) - x\log(\theta_0)\,, \tag{21}$$

Proof: The proof is provided in Appendix B. ∎

From Lemma 2, a lower bound on the objective function in (18) is written as (22), shown at the top of the next page, where $\boldsymbol{\alpha} = \{\alpha_i^{(u)}\}$, $\boldsymbol{\beta} = \{\beta_i^{(u)}\}$, for $i \in \{1,2,\ldots,2K\}$, $u \in \{1,2,\ldots N_{sc}\}$, and the coefficients $\alpha_{2k-1}^{(u)}$ and $\beta_{2k-1}^{(u)}$ can be selected as [19]

$$\alpha_{2k-1}^{(u)} = \varrho_{2k-1}^{(u)}/(1+\varrho_{2k-1}^{(u)})\,; \tag{23}$$

$$\beta_{2k-1}^{(u)} = \log_2\left(1+\varrho_{2k-1}^{(u)}\right) - \alpha_{2k-1}^{(u)}\log_2\left(\varrho_{2k-1}^{(u)}\right), \tag{24}$$

for any given $\varrho_{2k-1}^{(u)} > 0$. Similarly, $\alpha_{2k}^{(u)}$ and $\beta_{2k}^{(u)}$ can also be defined. The equality in (22) holds when $\alpha_i^{(u)} = \Upsilon_i^{(u)}/(1+\Upsilon_i^{(u)})$ and $\beta_i^{(u)} = \log_2(1+\Upsilon_i^{(u)}) - \alpha_i^{(u)}\log_2(\Upsilon_i^{(u)})$ for $i = \{1,2,\ldots,2K\}$, and the equality holds for $\left(\alpha_i^{(u)},\beta_i^{(u)}\right) = (1,0)$ if $\Upsilon_i^{(u)}$ approaches plus infinity.

$$\sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}} \frac{\mho^{(u,v)}\Pi_k^{(u,v)}}{2}\left(\log_2\left(1+e^{\hat{\Upsilon}_{2k-1}^{(u)}}\right)+\log_2\left(1+e^{\hat{\Upsilon}_{2k}^{(u)}}\right)\right)$$
$$-\lambda\left(\underbrace{\sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}}\mho^{(u,v)}\Pi_k^{(u,v)}\left(e^{\hat{P}_{2k-1}^{(u)}}+e^{\hat{P}_{2k}^{(u)}}+e^{\hat{W}_{2k-1}^{(v)}}+e^{\hat{W}_{2k}^{(v)}}\right)+P_c}_{\bar{\mathcal{P}}_{Total}(\hat{\mathbf{P}},\hat{\mathbf{W}},\boldsymbol{\mho},\boldsymbol{\Pi})}\right)$$
$$\geq \sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}} \frac{\mho^{(u,v)}\Pi_k^{(u,v)}}{2}\left(\frac{\alpha_{2k-1}^{(u)}}{\ln(2)}\hat{\Upsilon}_{2k-1}^{(u)}+\beta_{2k-1}^{(u)}+\frac{\alpha_{2k}^{(u)}}{\ln(2)}\hat{\Upsilon}_{2k}^{(u)}+\beta_{2k}^{(u)}\right)-\lambda\bar{\mathcal{P}}_{Total}$$
$$\triangleq \bar{\mathcal{U}}_{LB}\left(\hat{\mathbf{P}},\hat{\mathbf{W}},\boldsymbol{\mho},\boldsymbol{\Pi},\hat{\boldsymbol{\Upsilon}},\boldsymbol{\alpha},\boldsymbol{\beta}\right), \tag{22}$$

Using (22), the problem $(\mathbf{P}.3)$ can be rewritten as

$$(\mathbf{P}.4)\ \max_{\hat{\mathbf{P}},\hat{\mathbf{W}},\boldsymbol{\mho},\boldsymbol{\Pi},\hat{\boldsymbol{\Upsilon}}} \bar{\mathcal{U}}_{LB}\left(\hat{\mathbf{P}},\hat{\mathbf{W}},\boldsymbol{\mho},\boldsymbol{\Pi},\hat{\boldsymbol{\Upsilon}},\boldsymbol{\alpha},\boldsymbol{\beta}\right)$$
$$\text{s.t.}\quad (C.1)-(C.4)\ \&\ (C.7a)-(C.8b), \tag{25}$$

Lemma 3: The optimization problem $(\mathbf{P}.4)$ is concavified by the change of variables $\hat{P}_{2k-1}^{(u)}=\log P_{2k-1}^{(u)}$, $\hat{P}_{2k}^{(u)}=\log P_{2k}^{(u)}$, $\hat{W}_{2k-1}^{(v)}=\log W_{2k-1}^{(v)}$, $\hat{W}_{2k}^{(v)}=\log W_{2k}^{(v)}$, $\hat{\Upsilon}_{2k-1}^{(u)}=\log \Upsilon_{2k-1}^{(u)}$ and $\hat{\Upsilon}_{2k}^{(u)}=\log \Upsilon_{2k}^{(u)}$, for any $\alpha_i^{(u)}$, $\beta_i^{(u)}$ and λ.

Proof: The proof is provided in Appendix C. ■

Remark 2: For $\alpha_i^{(u)}=1$ and $\beta_i^{(u)}=0$, $\forall i,u$, the lower bound approximation of $\log(1+x)$ in (19) can be approximated (APP) as $\log x$, i.e., $\log(1+x)\simeq\log x$, the optimization problem (25) can be transformed to an new optimization problem as follows:

$$(\mathbf{P}.5)\ \max_{\hat{\mathbf{P}},\hat{\mathbf{W}},\boldsymbol{\mho},\boldsymbol{\Pi},\hat{\boldsymbol{\Upsilon}}} \mathcal{U}_{APP}\left(\hat{\mathbf{P}},\hat{\mathbf{W}},\boldsymbol{\mho},\boldsymbol{\Pi},\hat{\boldsymbol{\Upsilon}}\right)$$
$$\text{s.t.}\quad (C.1)-(C.4)\ \&\ (C.7a)-(C.8b), \tag{26}$$

where $\mathcal{U}_{APP}\left(\hat{\mathbf{P}},\hat{\mathbf{W}},\boldsymbol{\mho},\boldsymbol{\Pi},\hat{\boldsymbol{\Upsilon}}\right)=\sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}}\frac{\mho^{(u,v)}\Pi_k^{(u,v)}}{2\ln(2)}\left(\hat{\Upsilon}_{2k-1}^{(u)}+\hat{\Upsilon}_{2k}^{(u)}\right)-\lambda\bar{\mathcal{P}}_{Total}$. The concavity of the problem $(\mathbf{P}.5)$ and its optimal solutions can be derived in the similar way as the problem $(\mathbf{P}.4)$, and has been omitted for the sake of brevity.

IV. Utility-Based Resource Allocation Algorithm

As it can be observed from (25), the optimization problem does not hold a jointly convex structure over the optimization variables. Nevertheless it is a separately convex optimization problem over the transmit powers $\hat{\mathbf{P}}$ and $\hat{\mathbf{W}}$, and the subcarrier pairing and allocation $\boldsymbol{\mho}$ and $\boldsymbol{\Pi}$, once the other variables are fixed. This facilitates an alternating optimization algorithm where in each iteration the solution to (25) is calculated, as a convex optimization problem, assuming an alternatively fixed $\left(\hat{\mathbf{P}},\hat{\mathbf{W}}\right)$ or $(\boldsymbol{\mho},\boldsymbol{\Pi})$. The described optimization iterations continue until a stationary point is obtained, or a maximum number of iterations is reached. Therefore, we propose a utility-based iterative resource allocation algorithm for attaining the optimal solution.

A. Dual Problem Formulation

For given coefficients $\alpha_i^{(u)}$, $\beta_i^{(u)}$, $\forall i,u$, and fixed subcarrier pairing $\boldsymbol{\mho}$ and subcarrier allocation $\boldsymbol{\Pi}$, the optimization problem (25) is a convex optimization problem, which can be efficiently solved using standard convex optimization tools, e.g., CVX [27]. We further derive an iterative algorithm for solving this optimization problem by applying the dual decomposition method. The main idea behind this algorithm is to find the optimal resource allocation policy that can maximize its lower bound for given coefficients $\alpha_i^{(u)}$ and $\beta_i^{(u)}$, followed by an update of these two coefficients that guarantees a monotonic increase in the lower bound performance. The dual problem associated with the primal problem (25) can be written as

$$\min_{\kappa,\boldsymbol{\mu},\boldsymbol{\vartheta},\boldsymbol{\nu},\boldsymbol{\Theta}} \mathcal{X}\left(\kappa,\boldsymbol{\mu},\boldsymbol{\vartheta},\boldsymbol{\nu},\boldsymbol{\Theta}\right)$$
$$\text{s. t.}\quad \kappa\geq 0,\boldsymbol{\mu}\geq 0,\boldsymbol{\vartheta}\geq 0,\boldsymbol{\nu},\boldsymbol{\Theta}\geq 0, \tag{27}$$

where $\mathcal{X}\left(\kappa,\boldsymbol{\mu},\boldsymbol{\vartheta},\boldsymbol{\nu},\boldsymbol{\Theta}\right)$ denotes the dual function which can be expressed as

$$\mathcal{X}\left(\kappa,\boldsymbol{\mu},\boldsymbol{\vartheta},\boldsymbol{\nu},\boldsymbol{\Theta}\right)=$$
$$\max_{\hat{\mathbf{P}},\hat{\mathbf{W}},\boldsymbol{\mho},\boldsymbol{\Pi},\hat{\boldsymbol{\Upsilon}}} \mathcal{L}\left(\hat{\mathbf{P}},\hat{\mathbf{W}},\boldsymbol{\mho},\boldsymbol{\Pi},\hat{\boldsymbol{\Upsilon}},\kappa,\boldsymbol{\mu},\boldsymbol{\vartheta},\boldsymbol{\nu},\boldsymbol{\Theta}\right)$$
$$\text{s. t.}\quad (C.2)-(C.4), \tag{28}$$

where $\mathcal{L}\left(\hat{\mathbf{P}},\hat{\mathbf{W}},\boldsymbol{\mho},\boldsymbol{\Pi},\hat{\boldsymbol{\Upsilon}},\kappa,\boldsymbol{\mu},\boldsymbol{\vartheta},\boldsymbol{\nu},\boldsymbol{\Theta}\right)$ is given as (29), shown at the top of the next page, where κ is the Lagrangian multiplier corresponding to the transmit power constraint $(C.1)$. The dual variable vectors $\boldsymbol{\mu}=\{\mu_{2k-1}^{(u)}\}$, $\boldsymbol{\vartheta}=\{\vartheta_{2k-1}^{(u,v)}\}$, $\boldsymbol{\nu}=\{\nu_{2k}^{(u)}\}$ and $\boldsymbol{\Theta}=\{\Theta_{2k}^{(u,v)}\}$ are associated with the constraints $(C.7a)$, $(C.7b)$, $(C.8a)$ and $(C.8b)$, respectively.

In the following subsections, we adopt the Dinkelbach's method [29] which is an iterative algorithm for solving the dual problem (27) using dual decomposition approach [27] which alternates between a *subproblem (inner problem)*, updating the resource allocation variables $\hat{\mathbf{P}}$, $\hat{\mathbf{W}}$, $\boldsymbol{\mho}$, $\boldsymbol{\Pi}$ and $\hat{\boldsymbol{\Upsilon}}$ by fixing the Lagrangian multipliers, and a *master problem (outer problem)*, updating the Lagrangian multipliers for the obtained solution of the inner problem[2]. The dual decomposition approach is outlined as follows.

[2]The optimal solution obtained for the dual function in (27) is equal to that of (25), i.e., a zero duality gap between the optimal and dual solutions [28].

$$
\begin{aligned}
&\mathcal{L}\left(\hat{\mathbf{P}},\hat{\mathbf{W}},\mho,\mathbf{\Pi},\hat{\mathbf{\Upsilon}},\kappa,\boldsymbol{\mu},\boldsymbol{\vartheta},\boldsymbol{\nu},\mathbf{\Theta}\right)=\bar{\mathcal{U}}_{LB}\left(\hat{\mathbf{P}},\hat{\mathbf{W}},\mho,\mathbf{\Pi},\hat{\mathbf{\Upsilon}},\boldsymbol{\alpha},\boldsymbol{\beta}\right)\\
&\quad-\kappa\left(\sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}}\mho^{(u,v)}\Pi_k^{(u,v)}\left(e^{\hat{P}_{2k-1}^{(u)}}+e^{\hat{P}_{2k}^{(u)}}+e^{\hat{W}_{2k-1}^{(v)}}+e^{\hat{W}_{2k}^{(v)}}\right)-P_{max}\right)\\
&\quad-\sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\mu_{2k-1}^{(u)}\left(\frac{e^{\hat{\Upsilon}_{2k-1}^{(u)}-\hat{P}_{2k}^{(u)}}}{\left|h_{2k}^{(u)}\right|^2}\left(\sum_{l=1,l\neq 2k}^{2K}e^{\hat{P}_l^{(u)}}\left|h_l^{(u)}\right|^2+\sigma_R^{(u)^2}\right)-1\right)\\
&\quad-\sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}}\vartheta_{2k-1}^{(u,v)}\left(\frac{e^{\hat{\Upsilon}_{2k-1}^{(u)}-\hat{W}_{2k}^{(v)}}}{\left|g_{2k-1}^{(v)}\right|^2}\left(\sum_{l=1,l\neq 2k-1,2k}^{2K}e^{\hat{W}_l^{(v)}}\left|g_{2k-1}^{(v)}\right|^2+\sigma_{2k-1}^{(v)^2}\right)-1\right)\\
&\quad-\sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\nu_{2k}^{(u)}\left(\frac{e^{\hat{\Upsilon}_{2k}^{(u)}-\hat{P}_{2k-1}^{(u)}}}{\left|h_{2k-1}^{(u)}\right|^2}\left(\sum_{l=1,l\neq 2k-1}^{2K}e^{\hat{P}_l^{(u)}}\left|h_l^{(u)}\right|^2+\sigma_R^{(u)^2}\right)-1\right)\\
&\quad-\sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}}\Theta_{2k}^{(u,v)}\left(\frac{e^{\hat{\Upsilon}_{2k}^{(u)}-\hat{W}_{2k-1}^{(v)}}}{\left|g_{2k}^{(v)}\right|^2}\left(\sum_{l=1,l\neq 2k,2k-1}^{2K}e^{\hat{W}_l^{(v)}}\left|g_{2k}^{(v)}\right|^2+\sigma_{2k}^{(v)^2}\right)-1\right),
\end{aligned}
\tag{29}
$$

B. Subproblem Solution

In this subsection, the optimal solution to the optimization problem (25) for updating the power allocation of the $(2k-1)^{th}$ and $(2k)^{th}$ users, and the relay node at the $(m+1)^{th}$ iteration can be given in the following theorems.

Theorem 1: The optimal power allocation policies for the users such as to maximize (25) can be given as

$$
\begin{aligned}
&\hat{P}_j^{(u)}(m+1)\\
&=\left[\frac{1}{2}\ln\left(\frac{\hat{z}_f^{(u)}\dfrac{e^{\hat{\Upsilon}_f^{(u)}}}{|h_j|^2}\sigma_R^{(u)^2}}{(\lambda+\kappa)+\hat{z}_j^{(u)}e^{\hat{\Upsilon}_j^{(u)}-\hat{P}_f^{(u)}}\dfrac{|h_j|^2}{|h_f|^2}}\right)\right]^+, \quad j\neq f,
\end{aligned}
\tag{30}
$$

for $j=2k,2k-1,\ f=2k,2k-1$, where $\kappa\geq 0$ is the Lagrangian multiplier for the power constraint $(C.1)$ in (25). The terms $\hat{z}_f^{(u)}$ and $\hat{z}_j^{(u)}$ are defined as

$$
\hat{z}_f^{(u)}=\begin{cases}\upsilon_f^{(u)}, & \text{if } j=2k-1,\ f\neq j\,;\\ \mu_f^{(u)}, & \text{if } j=2k,\ f\neq j\,;\end{cases}
$$

$$
\hat{z}_j^{(u)}=\begin{cases}\upsilon_j^{(u)}, & \text{if } j=2k\,;\\ \mu_j^{(u)}, & \text{if } j=2k-1\,.\end{cases}
$$

Proof: The proof is provided in Appendix D ∎

Remark 3: Theorem 1 reveals that the power update of the users not only depends on the network price λ and the Lagrangian multiplier κ, but also on the noise power at the relay node. However, for case of without subcarrier pairing and allocation, the update of transmit power of the users needs to consider interference power along with the noise power at the relay node, network price λ and Lagrangian multiplier κ.

Theorem 2: The optimal power allocation for the relay node is given as

$$
\hat{W}_j^{(v)}(m+1)=\left[\frac{1}{2}\ln\left(\frac{\tilde{z}_f^{(u,v)}e^{\hat{\Upsilon}_f^{(v)}}\sigma_j^{(v)^2}}{(\lambda+\kappa)\left|g_f^{(v)}\right|^2}\right)\right]^+, \quad j\neq f,
\tag{31}
$$

for $j=2k,2k-1,\ f=2k,2k-1$, where

$$
\tilde{z}_f^{(u,v)}=\begin{cases}\Theta_f^{(u,v)}, & \text{if } j=2k-1,\ f\neq j\,;\\ \vartheta_f^{(u,v)}, & \text{if } j=2k,\ f\neq j\,.\end{cases}
$$

Proof: The proof is provided in Appendix E ∎

Moreover, the auxiliary variables $\hat{\Upsilon}_{2k-1}^{(u)}$ and $\hat{\Upsilon}_{2k}^{(u)}$, $\forall k,u$, can be updated as follows:

$$
\hat{\Upsilon}_{2k-1}^{(u)}(m+1)=\frac{\sum_{v=1}^{N_{sc}}\Omega^{(u,v)}\Pi_k^{(u,v)}\alpha_{2k-1}^{(u)}}{2\ln(2)\left(\dfrac{\mu_{2k-1}^{(u)}}{\hat{\Gamma}_{R_{2k}}^{(u)}}+\dfrac{\sum_{v=1}^{N_{sc}}\vartheta_{2k-1}^{(u,v)}}{\hat{\Gamma}_{2k-1}^{(v)}}\right)};
\tag{32}
$$

$$
\hat{\Upsilon}_{2k}^{(u)}(m+1)=\frac{\sum_{v=1}^{N_{sc}}\Omega^{(u,v)}\Pi_k^{(u,v)}\alpha_{2k}^{(u)}}{2\ln(2)\left(\dfrac{\nu_{2k}^{(u)}}{\hat{\Gamma}_{R_{2k-1}}^{(u)}}+\dfrac{\sum_{v=1}^{N_{sc}}\Theta_{2k}^{(u,v)}}{\hat{\Gamma}_{2k}^{(v)}}\right)},
\tag{33}
$$

To derive the optimal subcarrier pairing $\mho$ and allocation $\mathbf{\Pi}$, we substitute $\hat{P}_{2k-1}^{(u)^\star}$, $\hat{P}_{2k}^{(u)^\star}$, $\hat{W}_{2k-1}^{(v)^\star}$, $\hat{W}_{2k}^{(v)^\star}$, $\Gamma_{2k-1}^{(u)^\star}$ and $\Gamma_{2k}^{(u)^\star}$ into (28) and obtain the following optimization problem:

$$
\begin{aligned}
\mathcal{X}(\kappa,\boldsymbol{\mu},\boldsymbol{\vartheta},\boldsymbol{\nu},\mathbf{\Theta})=&\max_{\mho,\mathbf{\Pi}}\sum_{k=1}^{K}\sum_{u=1}^{N_c}\sum_{v=1}^{N_c}\frac{\mho^{(u,v)}\Pi_k^{(u,v)}}{2}\Phi_k^{(u,v)}+\Psi\\
&\text{s.t. } (C.2)\ -\ (C.4)\,,
\end{aligned}
\tag{34}
$$

where $\Phi_k^{(u,v)}$ and Ψ are defined in (35) and (36) as shown on the top of the next page. The first term in (35) denotes the achievable sum rate of the k^{th} user pair for the allocated subcarrier pairing (u, v), whereas the second term works as the penalty for the resource utilization. Ψ in the problem (34) denotes the constant for any subcarrier pairing $\mathbf{\mho}$, and allocation $\mathbf{\Pi}$. Hence, we can drop Ψ from now on. Due to the fact of $\Pi_k^{(u,v)}$ and $\Psi^{(u,v)}$ product in objective function of (34), we cannot jointly optimize $\mathbf{\mho}$ and $\mathbf{\Pi}$. Therefore, we optimize one variable by fixing other one.

To determine the optimal subcarrier allocation $\mathbf{\Pi}$ for given subcarrier pairing $\mathbf{\mho}$ and the optimal allocation policy $\left(\hat{\mathbf{P}}^\star, \hat{\mathbf{W}}^\star, \hat{\mathbf{\Upsilon}}^\star\right)$ and fixed price λ, we solve the following optimization problem:

$$\begin{aligned} \max_{\mathbf{\Pi}} \quad & \sum_{k=1}^{K}\sum_{u=1}^{N_c}\sum_{v=1}^{N_c} \frac{\mho^{(u,v)^\star}\Pi_k^{(u,v)}}{2}\Phi_k^{(u,v)^\star} \\ \text{s.t.} \quad & (C.4)\ , \end{aligned} \tag{37}$$

Straightforwardly the optimal subcarrier allocation $\mathbf{\Pi}^\star$ is the k^{th} user pair that maximizes $\Phi_k^{(u,v)}$ for given $(u, v)^{th}$ subcarrier pair and the optimal power allocation. Thus, the optimal subcarrier allocation $\mathbf{\Pi}^\star$ can be obtained as

$$\Pi_k^{(u,v)^\star} = \begin{cases} 1, & \text{for } k = \arg\max_k \ \Phi_k^{(u,v)}, \quad \forall k, (u, v); \\ 0, & \text{otherwise}\,, \end{cases} \tag{38}$$

Finally, to find the optimal subcarrier pairing $\mathbf{\mho}$ for the optimal allocation policy $\left(\hat{\mathbf{P}}^\star, \hat{\mathbf{W}}^\star, \hat{\mathbf{\Upsilon}}^\star\right)$ and the optimal $\mathbf{\Pi}^\star$ given in (38) and fixed λ, we rewrite the optimization problem (34) as

$$\begin{aligned} \max_{\mathbf{\mho}} \quad & \sum_{k=1}^{K}\sum_{u=1}^{N_c}\sum_{v=1}^{N_c} \frac{\mho^{(u,v)}\Pi_k^{(u,v)^\star}}{2}\Phi_k^{(u,v)^\star} \\ \text{s.t.} \quad & (C.2)\ \&\ (C.3)\ , \end{aligned} \tag{39}$$

where $\Phi_k^{(u,v)^\star} = \max_k \ \Phi_k^{(u,v)^\star}, \forall (u, v)$. Let $\mathbf{\Phi}$ be an $N_{sc} \times N_{sc}$ matrix such that

$$\mathbf{\Phi} = \begin{bmatrix} \Phi_{k^\star}^{(1,1)} & \cdots & \Phi_{k^\star}^{(1,N_{sc})} \\ \vdots & \ddots & \vdots \\ \Phi_{k^\star}^{(N_{sc},1)} & \cdots & \Phi_{k^\star}^{(N_{sc},N_{sc})} \end{bmatrix} \tag{40}$$

Remark 4: The matrix $\mathbf{\Phi}$ in (39) is related to the realistic case of the resources being characterized by a profit matrix, where rows and columns represent different operators (u) and machines (v), respectively, and each element denotes the profit gain by operating a particular machine by a particular operator. Thus, maximizing the total profit by selecting the best policy, where each operator operates only on one machine is equivalent to solving the problem (39), respectively. However, the optimization problem (39) can also be solved efficiently by using the standard assignment algorithms such as Hungarian method [30].

C. Master Problem Solution: Updating the dual variables

Since the dual problem in (27) is differentiable, the gradient method [28] can be used to update the dual variables κ, $\mu_{2k-1}^{(u)}$, $\vartheta_{2k-1}^{(u,v)}$, $\nu_{2k}^{(u)}$, $\Theta_{2k}^{(u,v)}$ and $\Theta_{2k}^{(u,v)}$, $\forall k, (u, v)$, using the optimal variables to give (41)-(45), where $\epsilon_a(m), a \in \{1, \cdots, 5\}$, are sufficiently small step sizes associated with calculating the Lagrangian multipliers and m is the iteration index. The updated Lagrange multipliers in (41)-(45) are used for updating the power allocation policy. We repeat this process until convergence.

We provide the Theorem regarding the update of the coefficients $\alpha_i^{(u)}$ and $\beta_i^{(u)}$, as follows:

Theorem 3: Assume $\left(\hat{\mathbf{P}}^\star(t), \hat{\mathbf{W}}^\star(t), \hat{\mathbf{\Upsilon}}^\star(t), \mathbf{\mho}^\star(t), \mathbf{\Pi}^\star(t)\right)$ is the optimal solution of the problem (**P.4**) with respect to $\alpha_i^{(u)}(t)$ and $\beta_i^{(u)}(t)$ at the t^{th} iteration. If we update the coefficients as

$$\begin{aligned} \tilde{\Upsilon}_i^{(u)}(t+1) &= \Upsilon_i^{(u)}(t)\,; \\ \alpha_i^{(u)}(t+1) &= \frac{\tilde{\Upsilon}_i^{(u)}(t)}{1+\tilde{\Upsilon}_i^{(u)}(t)}\,; \\ \beta_i^{(v)}(t+1) &= \log_2\left(1+\tilde{\Upsilon}_i^{(u)}(t)\right) \\ &\quad - \alpha_i^{(u)}(t+1)\log_2\left(\tilde{\Upsilon}_i^{(u)}(t+1)\right)\,, \end{aligned} \tag{46}$$

then the optimal solution $\bar{\mathcal{U}}_{LB}\left(\hat{\mathbf{P}}, \hat{\mathbf{W}}, \mathbf{\mho}, \mathbf{\Pi}, \hat{\mathbf{\Upsilon}}, \boldsymbol{\alpha}, \boldsymbol{\beta}\right)$ of the problem (**P.4**) is monotonically increased with t.

Proof: Please refer Appendix F. ■

D. Optimal Utility price $\lambda^\star$

In this subsection, we show a relation between the EE and the SE by adjusting the network price λ. We first define the EE in terms of SE and power consumption and then investigate the optimal utility price $\lambda^\star$ that gives the maximum EE.

Definition 1: The EE for the multi-user two-way DF relay network is defined as the ratio of the achievable minimum sum rate divided by its total power consumption in the network is formally expressed as

$$\eta_{EE}\left(\hat{\mathbf{P}}, \hat{\mathbf{W}}, \mathbf{\mho}, \mathbf{\Pi}, \hat{\mathbf{\Upsilon}}\right) = \frac{\left\{ \sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}} \frac{\mho^{(u,v)}\Pi_k^{(u,v)}}{2}\left(\log_2\left(1+e^{\hat{\Upsilon}_{2k-1}^{(u)}}\right) + \log_2\left(1+e^{\hat{\Upsilon}_{2k}^{(u)}}\right)\right)\right\}}{\left\{ \sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}} \mho^{(u,v)}\Pi_k^{(u,v)}\left(e^{\hat{P}_{2k-1}^{(u)}} + e^{\hat{P}_{2k}^{(u)}} + e^{\hat{W}_{2k-1}^{(v)}} + e^{\hat{W}_{2k}^{(v)}}\right) + P_c\right\}}, \tag{47}$$

Next, we provide a theorem for achieving the optimal network price $\lambda^\star$ and a theorem that depicts an update procedure of $\lambda^\star$ as follows.

Theorem 4: The optimal EE $\lambda^\star$ can be achieved, *if and only if* the optimal allocation policy $\left(\hat{\mathbf{P}}^\star, \hat{\mathbf{W}}^\star, \mathbf{\mho}^\star, \mathbf{\Pi}^\star, \hat{\mathbf{\Upsilon}}^\star\right)$

$$\Phi_k^{(u,v)} = \left(\frac{\alpha_{2k-1}^{(u)}}{\ln(2)}\hat{\Upsilon}_{2k-1}^{(u)\star} + \beta_{2k-1}^{(u)} + \frac{\alpha_{2k}^{(u)}}{\ln(2)}\hat{\Upsilon}_{2k}^{(u)\star} + \beta_{2k}^{(u)}\right) - (\kappa+\lambda)\left(e^{\hat{P}_{2k-1}^{(u)\star}} + e^{\hat{P}_{2k}^{(u)\star}} + e^{\hat{W}_{2k-1}^{(v)\star}} + e^{\hat{W}_{2k}^{(v)\star}}\right); \tag{35}$$

$$\begin{aligned}\Psi =& \kappa P_{max} - \lambda P_c - \sum_{k=1}^{K}\sum_{u=1}^{N_{sc}} \mu_{2k-1}^{(u)} \left(\frac{e^{\hat{\Upsilon}_{2k-1}^{(u)\star} - \hat{P}_{2k}^{(u)\star}}}{\left|h_{2k}^{(u)}\right|^2}\left(\sum_{l=1,l\neq 2k}^{2K} e^{\hat{P}_l^{(u)\star}}\left|h_l^{(u)}\right|^2 + {\sigma_R^{(u)}}^2\right) - 1\right)\\
&- \sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}} \vartheta_{2k-1}^{(u,v)} \left(\frac{e^{\hat{\Upsilon}_{2k-1}^{(u)\star} - \hat{W}_{2k}^{(v)\star}}}{\left|g_{2k-1}^{(v)}\right|^2}\left(\sum_{l=1,l\neq 2k-1,2k}^{2K} e^{\hat{W}_l^{(v)\star}}\left|g_{2k-1}^{(v)}\right|^2 + {\sigma_{2k-1}^{(v)}}^2\right) - 1\right)\\
&- \sum_{k=1}^{K}\sum_{u=1}^{N_{sc}} \nu_{2k}^{(u)} \left(\frac{e^{\hat{\Upsilon}_{2k}^{(u)\star} - \hat{P}_{2k-1}^{(u)\star}}}{\left|h_{2k-1}^{(u)}\right|^2}\left(\sum_{l=1,l\neq 2k-1}^{2K} e^{\hat{P}_l^{(u)\star}}\left|h_l^{(u)}\right|^2 + {\sigma_R^{(u)}}^2\right) - 1\right)\\
&- \sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}} \Theta_{2k}^{(u,v)} \left(\frac{e^{\hat{\Upsilon}_{2k}^{(u)\star} - \hat{W}_{2k-1}^{(v)\star}}}{\left|g_{2k}^{(v)}\right|^2}\left(\sum_{l=1,l\neq 2k,2k-1}^{2K} e^{\hat{W}_l^{(v)\star}}\left|g_{2k}^{(v)}\right|^2 + {\sigma_{2k}^{(v)}}^2\right) - 1\right),\end{aligned} \tag{36}$$

$$\kappa(m+1) = \left[\kappa(m) + \varepsilon_1(m)\left(\sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}} \mho^{(u,v)\star}\Pi_k^{(u,v)\star}\left(e^{\hat{P}_{2k-1}^{(u)\star}} + e^{\hat{P}_{2k}^{(u)\star}} + e^{\hat{W}_{2k-1}^{(v)\star}} + e^{\hat{W}_{2k}^{(v)\star}}\right) - P_{max}\right)\right]^+; \tag{41}$$

$$\mu_{2k-1}^{(u)}(m+1) = \left[\mu_{2k-1}^{(u)}(m) + \varepsilon_2(m)\left(\frac{e^{\hat{\Upsilon}_{2k-1}^{(u)\star} - \hat{P}_{2k}^{(u)\star}}}{\left|h_{2k}^{(u)}\right|^2}\left(\sum_{l=1,l\neq 2k}^{2K} e^{\hat{P}_l^{(u)\star}}\left|h_l^{(u)}\right|^2 + {\sigma_R^{(u)}}^2\right) - 1\right)\right]^+; \tag{42}$$

$$\vartheta_{2k-1}^{(u,v)}(m+1) = \left[\vartheta_{2k-1}^{(u,v)}(m) + \varepsilon_3(m)\left(\frac{e^{\hat{\Upsilon}_{2k-1}^{(u)\star} - \hat{W}_{2k}^{(v)\star}}}{\left|g_{2k-1}^{(v)}\right|^2}\left(\sum_{l=1,l\neq 2k-1,2k}^{2K} e^{\hat{W}_l^{(v)\star}}\left|g_{2k-1}^{(v)}\right|^2 + {\sigma_{2k-1}^{(v)}}^2\right) - 1\right)\right]^+; \tag{43}$$

$$\nu_{2k}^{(u)}(m+1) = \left[\nu_{2k}^{(u)}(m) + \varepsilon_4(m)\left(\frac{e^{\hat{\Upsilon}_{2k}^{(u)\star} - \hat{P}_{2k-1}^{(u)\star}}}{\left|h_{2k-1}^{(u)}\right|^2}\left(\sum_{l=1,l\neq 2k-1}^{2K} e^{\hat{P}_l^{(u)\star}}\left|h_l^{(u)}\right|^2 + {\sigma_R^{(u)}}^2\right) - 1\right)\right]^+; \tag{44}$$

$$\Theta_{2k}^{(u,v)}(m+1) = \left[\Theta_{2k}^{(u,v)}(m) + \varepsilon_5(m)\left(\frac{e^{\hat{\Upsilon}_{2k}^{(u)\star} - \hat{W}_{2k-1}^{(v)\star}}}{\left|g_{2k}^{(v)}\right|^2}\left(\sum_{l=1,l\neq 2k,2k-1}^{2K} e^{\hat{W}_l^{(v)\star}}\left|g_{2k}^{(v)}\right|^2 + {\sigma_{2k}^{(v)}}^2\right) - 1\right)\right]^+, \tag{45}$$

---

satisfies the following balance equation:

$$\begin{aligned}&\sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}} \frac{\mho^{(u,v)\star}\Pi_k^{(u,v)\star}}{2}\left(\frac{\alpha_{2k-1}^{(u)\star}}{\ln(2)}\hat{\Upsilon}_{2k-1}^{(u)\star} + \beta_{2k-1}^{(u)\star}\right.\\
&\qquad\left. + \frac{\alpha_{2k}^{(u)\star}}{\ln(2)}\hat{\Upsilon}_{2k}^{(u)\star} + \beta_{2k}^{(u)\star}\right)\\
&- \lambda^\star \sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}} \mho^{(u,v)\star}\Pi_k^{(u,v)\star}\left(e^{\hat{P}_{2k-1}^{(u)\star}} + e^{\hat{P}_{2k}^{(u)\star}}\right.\\
&\qquad\left. + e^{\hat{W}_{2k-1}^{(v)\star}} + e^{\hat{W}_{2k}^{(v)\star}}\right) + P_c = 0.\end{aligned} \tag{48}$$

Proof: The proof is provided in Appendix G. ■

Theorem 5: If $\left(\hat{\mathbf{P}}^\star(l), \hat{\mathbf{W}}^\star(l), \boldsymbol{\mho}^\star(l), \boldsymbol{\Pi}^\star(l), \hat{\boldsymbol{\Upsilon}}^\star(l)\right)$ is the optimal solution of the problem $(\mathbf{P.4})$ with respect to $\lambda(l)$ at the l^{th} iteration and if we update $\lambda(l)$ as

$$\lambda(l+1) = \frac{\left\{\begin{aligned}&\sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}} \frac{\mho^{(u,v)\star}(l)\Pi_k^{(u,v)\star}(l)}{2}\left(\frac{\alpha_{2k-1}^{(u)\star}}{\ln(2)}(l)\hat{\Upsilon}_{2k-1}^{(u)\star}(l)\right.\\
&\qquad\left. + \beta_{2k-1}^{(u)\star}(l) + \frac{\alpha_{2k}^{(u)\star}(l)}{\ln(2)}\hat{\Upsilon}_{2k}^{(u)\star}(l) + \beta_{2k}^{(u)\star}(l)\right)\end{aligned}\right\}}{\bar{\mathcal{P}}_{Total}\left(\hat{\mathbf{P}}^\star(l), \hat{\mathbf{W}}^\star(l), \boldsymbol{\mho}^\star(l), \boldsymbol{\Pi}^\star(l)\right)}, \tag{49}$$

then $\lambda(l)$ increases monotonically with each iteration, l.

Proof: The proof is similar to [19, Appendix E]. ■

Theorem 6: The optimal penalty factor $\lambda^\star$ is obtained when the sequence $\{\lambda(l)\}$ has converged and $\lambda^\star = \lim_{l\to\infty}\lambda(l)$ satisfies the balance equation in (48).

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TCOMM.2017.2713382, IEEE Transactions on Communications

Proof: The proof is provided in Appendix H. ■

We first initialize the maximum number of iteration for the outer and inner loops as I_{max_1} and I_{max_2} with the iteration counter $l = 0$ and $m = 0$, respectively, along with the network price $\lambda(l) = 0.001$. Then we initialize the step sizes $\epsilon_a(m)$, followed by the coefficients $\left(\alpha_{2k-1}^{(u,v)}(0), \beta_{2k-1}^{(u,v)}(0)\right) = (1, 0)$ and $\left(\alpha_{2k}^{(u,v)}(0), \beta_{2k}^{(u,v)}(0)\right) = (1, 0)$. From the sub-gradient method [28], the dual variables κ, $\mu_{2k-1}^{(u)}$, $\vartheta_{2k-1}^{(u,v)}$, $\nu_{2k}^{(u)}$ and $\Theta_{2k}^{(u,v)}$, $\forall k, (u, v)$, are initialized for finding the resource allocation policy $\left(\hat{\mathbf{P}}, \hat{\mathbf{W}}, \boldsymbol{\mho}, \boldsymbol{\Pi}, \hat{\boldsymbol{\Upsilon}}\right)$ using (30)–(33), (38) and (39), respectively. Then with the obtained $\left(\hat{\mathbf{P}}, \hat{\mathbf{W}}, \boldsymbol{\mho}, \boldsymbol{\Pi}, \hat{\boldsymbol{\Upsilon}}\right)$, the dual variables at $(m+1)^{th}$ iteration are updated using (41)–(45). The coefficients $\left(\alpha_{2k-1}^{(u,v)}, \beta_{2k-1}^{(u,v)}\right)$ and $\left(\alpha_{2k}^{(u,v)}, \beta_{2k}^{(u,v)}\right)$ are updated after obtaining the optimal resource allocation $\left(\hat{\mathbf{P}}^\star, \hat{\mathbf{W}}^\star, \boldsymbol{\mho}^\star, \boldsymbol{\Pi}^\star, \hat{\boldsymbol{\Upsilon}}^\star\right)$. The above procedure is repeated until $\left(\alpha_{2k-1}^{(u,v)}, \beta_{2k-1}^{(u,v)}\right)$ and $\left(\alpha_{2k}^{(u,v)}, \beta_{2k}^{(u,v)}\right)$ have converged or the iteration counter m reaches to maximum limit I_{max_2}. In the next step, we update the network price $\lambda(l+1)$ using (49) and increase the iteration counter by one. This procedure is continued until the convergence is attained or $l \leq I_{max_1}$. The iterative EEM algorithm is briefly summarized in Algorithm 1.

Algorithm 1 Iterative EEM Algorithm

```
Set the maximum number of iterations I_{max_1} ;
Initialize the iteration counter l = 0 and network penalty λ(l) = 0.001 ;
repeat (Outer Loop)
    Set the maximum number of iterations I_{max_2} ;
    Initialize the iteration counter m = 0 and the step sizes ε_a(m) ;
    Initialize (α_{2k-1}^{(u,v)}, β_{2k-1}^{(u,v)}) = (1,0)
        and (α_{2k}^{(u,v)}, β_{2k}^{(u,v)}) = (1,0), ∀k,(u,v) ;
    Initialize κ(m), μ_{2k-1}^{(u,v)}(m), ϑ_{2k-1}^{(u,v)}(m), ν_{2k}^{(u)}(m),
        Θ_{2k}^{(u,v)}(m), ∀k,(u,v) ;
    Initialize P̂(m), Ŵ(m), ℧(m), and Π(m) ;
    repeat (Inner Loop)
        repeat (Solving problem (P.4))
            Update P̂, Ŵ and Υ̂ using (30)–(33) ;
            Update Π and ℧ using (38) and (39) ;
            Update κ, μ_{2k-1}^{(u,v)}, ϑ_{2k-1}^{(u,v)}, ν_{2k}^{(u)}, Θ_{2k}^{(u,v)},
                ∀k,(u,v), using (41)–(45) ;
        until convergence to the optimal solution P̂*,
            Ŵ*, ℧*, and Π* ;
        Update the coefficients (α_{2k-1}^{(u,v)}, β_{2k-1}^{(u,v)}) and
            (α_{2k}^{(u,v)}, β_{2k}^{(u,v)}), using (23) and (24) ;
        Set P̂(m+1) ← P̂*, Ŵ(m+1) ← Ŵ*,
            Υ̂(m+1) ← Υ̂*, Π(m+1) ← Π*,
            ℧(m+1) ← ℧* and m ← m+1 ;
    until convergence or m > I_{max_2} ;
    Update λ(l+1) using (49) and l ← l+1 ;
until convergence or l > I_{max_1}.
```

In more practical scenario where each node is operated on a different power budget, the individual node power (INP) constraints in wireless networks are more preferable than the total power constraint case. Our proposed design framework can be easily extended to accommodate this scenario by replacing the constraint $(C.1)$ in (13) with the following transmit power constraints:

$$\sum_{u=1}^{N_{sc}} \sum_{v=1}^{N_{sc}} \mho^{(u,v)} \Pi_i^{(u,v)} P_i^{(u)} \leq P_{i,max}, \quad i = 1, \ldots, 2K ;$$

$$\sum_{i=1}^{2K} \sum_{u=1}^{N_{sc}} \sum_{v=1}^{N_{sc}} \mho^{(u,v)} \Pi_i^{(u,v)} W_i^{(v)} \leq P_{r,max} ,$$

where $P_{i,max}$ and $P_{r,max}$ indicate the maximum allowable transmit power for the i^{th} user and the relay node, respectively. We can solve this new optimization problem in a similar way as in the total power constraint case, however, it now needs the update of $2K + 1$ Lagrangian multipliers in the master problem due to the $2K + 1$ imposed INP constraints.

E. Generalized EEM (GEEM) Resource Allocation Algorithm

The performance of the utility-based iterative resource allocation algorithm described in Algorithm 1 is limited because of considering of subcarrier pairing permutation in one-to-one manner. Since a DF protocol is applied at the relay node, a single subcarrier of MA phase can pair with a single or multiple subcarrier(s) of BC phase and vice-versa. However, each subcarrier pair assigns to only a single user pair. Here, we discuss about the extensibility of a utility-based iterative resource allocation algorithm for a more practical scenario where subcarrier pairing permutation is obtained in *many-to-many* manner. The problem for this generalized scenario can be formulated by modifying the constraints $(C.2)$ and $(C.3)$ of the problem (**P.1**), as follows:

$$(C.2)\ 1 \leq \sum_{u=1}^{N_{sc}} \mho^{(u,v)} \leq N_{sc}, \qquad \forall v ;$$

$$(C.3)\ 1 \leq \sum_{v=1}^{N_{sc}} \mho^{(u,v)} \leq N_{sc}, \qquad \forall u ;$$

The new optimization problem can be transformed into a convex problem in a similar way of the problem (**P.1**) and the optimal resource allocation solution of this new problem can be found by applying an iterative EEM algorithm, named 'GEEM'.

V. Sub-Optimal Method

The computational complexity of the EEM algorithm proposed in Section IV becomes very high for a large value of N_{sc}. Thus, we propose a low-complexity suboptimal algorithm, and the stepwise procedure of the suboptimal algorithm is described below.

Step 1: Optimal Subcarrier Allocation for Given Power Allocation: In first step, the available transmit power is equally distributed among all the users and the relay node over all the subcarriers as follows:

$$P_{2k-1}^{(u)} = P_{2k}^{(u)} = W_{2k-1}^{(v)} = W_{2k}^{(v)} = \frac{P_{max}}{(2K + 2K)\, N_{sc}} , \forall k, u, v ; \tag{50}$$

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TCOMM.2017.2713382, IEEE Transactions on Communications

Next, we compute SINR's for the $(2k-1)^{th}$ and $(2k)^{th}$ user pairs on (u,v) subcarrier pair at the relay and destination nodes, thereby calculating $\Gamma^{(v)}_{R_{2k-1}}$ and $\Gamma^{(v)}_{R_{2k}}$ from (2), and $\Gamma^{(v)}_{2k-1}$ and $\Gamma^{(v)}_{2k}$ using (5) and (6), respectively. As explained in Section II, we consider a hop-wise approach to consider minimum (worst) SINR in each hop for calculating the SE and EE, thus we take the harmonic mean of SR and RD channels to determine the optimal subcarrier allocation matrix.

Define $K \times (N_{sc} \times N_{sc})$ matrix according to minimum of harmonic mean of forward and backward channels' SINR. Then, we select the k^{th} user pair in the following manner:

$$\Pi^{(u,v)\star}_{k} = \begin{cases} 1, \text{ for } k = \arg\min\limits_{k} \Big\{ H_m\left(\Gamma^{(v)}_{R_{2k-1}}, \Gamma^{(v)}_{R_{2k}}\right), \\ \qquad\qquad\qquad H_m\left(\Gamma^{(v)}_{2k-1}, \Gamma^{(v)}_{2k}\right) \Big\}; \\ 0, \text{ otherwise}, \end{cases} \tag{51}$$

Step 2: Optimal Subcarrier Pairing for Given Subcarrier Allocation: In this step, we calculate the harmonic mean of forward and backward channels between users and relay nodes, i.e. $\delta^{(u)}_{fh} \triangleq H_m\left(h^{(u)}_{2k-1}, g^{(v)}_{2k-1}\right)$ and $\delta^{(v)}_{sh} \triangleq H_m\left(h^{(u)}_{2k}, g^{(v)}_{2k}\right)$. Next, the N_{sc} subcarriers of the first hop $\left(\delta^{(u)}_{fh}\right)$ and second hop $\left(\delta^{(v)}_{sh}\right)$ are arranged in ascending order and matched in best-to-best and worst-to-worst fashion. After this arrangement, we update the subcarrier pairing matrix of size $N_{sc} \times N_{sc}$ as follows:

$$\mho^{\star}_{u,v} = \begin{cases} 1, \text{ for } u^{th} \text{ subcarrier paired with } v^{th} \text{ subcarrier}; \\ 0, \text{ otherwise}, \end{cases} \tag{52}$$

Step 3: Optimal Power Allocation for Given Subcarrier Pairing and Allocation: For given subcarrier allocation and pairing matrices $\mathbf{\Pi}$ and $\mathbf{\mho}$, we update the power $\hat{P}^{(u)}_{2k-1}, \hat{P}^{(u)}_{2k}, \hat{W}^{(v)}_{2k-1}, \hat{W}^{(v)}_{2k}, \hat{\Upsilon}^{(v)}_{2k-1}$ and $\hat{\Upsilon}^{(v)}_{2k}$ using (30)–(33) and the dual variables κ, $\mu^{(u,v)}_{2k-1}$, $\vartheta^{(u,v)}_{2k-1}$, $\nu^{(u)}_{2k}$, $\Theta^{(u,v)}_{2k}$, $\forall k, (u,v)$, using (41)–(45), respectively.

VI. Complexity Analysis

In this section, we perform an exhaustive complexity analysis for various algorithm under assumption that the network price λ converges in L iterations.

A. EEM Algorithm

The optimization problem (25) consists of $K \times N^2_{sc}$ subproblems due to K user pairs operating on N_{sc} subcarriers in each hop. Since, the optimal solution $\left(\bar{\mathbf{P}}^{\star}, \bar{\mathbf{W}}^{\star}, \mathbf{\Upsilon}^{\star}\right)$ is obtained under the total transmit power constraint $(C.1)$ the complexity results to $\mathcal{O}\left(V^3+1\right)$, where V denotes the power levels for each user and the relay node on each subcarrier. Further, each maximization in (25) adds a complexity of $\mathcal{O}(K)$ and therefore, the total complexity for finding the subcarrier allocation $\mathbf{\Pi}$ for each $(u,v)^{th}$ subcarrier pairing is $\mathcal{O}\left(K \times N^2_{sc}\right)$. Moreover, Hungarian method [30] is used to obtain the subcarrier pairing matrix $\mathbf{\mho}$ in (39), adding a complexity of $\mathcal{O}\left(N^3_{sc}\right)$ and the total complexity for updating dual variables is $\mathcal{O}\left(3(2K)^{\varpi}\right)$ (for example, $\varpi = 2$ if the ellipsoid method is used [31]). Let us suppose if the dual objective function (28) converges in $\mathcal{G}$ iterations, then the total complexity for the EEM algorithm $\forall k, v, v$ becomes $\mathcal{O}\left(3\mathcal{G}\mathcal{L}N^2_{sc}(2K)^{\varpi}(K(V^3+2)+N_{sc})\right)$. The complexity of the EEM algorithm under *equal subcarrier power allocation (ESPA)* is $\mathcal{O}\left(5\mathcal{G}\mathcal{L}N^2_{sc}(2K)^{\varpi}(K(V^3+4)+N_{sc})\right)$, while the complexity of the GEEM algorithm is $\mathcal{O}\left(7\mathcal{G}\mathcal{L}(2K)^{\varpi}(\left(K \times N^2_{sc}\right)(V^3+2)+4N_{sc})\right)$.

B. Suboptimal EEM Algorithm

The complexity for obtaining the subcarrier allocation matrix $\mathbf{\Pi}$ in the *step* 1 for K user pairs is $\mathcal{O}(K \times N_{sc})$, whereas the complexity for finding subcarrier pairing matrix $\mathbf{\mho}$ in *step* 2 is $\mathcal{O}(2N_{sc})$. However, the power allocation and updating the dual variables add a complexity of $\mathcal{O}\left(V^3+1\right)$ and $\mathcal{O}\left(3(2K)^{\varpi}\right)$, respectively. Let us suppose if the dual objective function (28) converges in $\mathcal{G}'$ iterations (without loss of generality let $\mathcal{G}' = \mathcal{G}$), then the suboptimal EEM algorithm produces a total complexity of $\mathcal{O}\left(3\mathcal{G}\mathcal{L}N_{sc}(2K)^{\varpi}(K+2+K\times N_{sc}(V^3+1)\right)$.

C. Optimal ES Algorithm

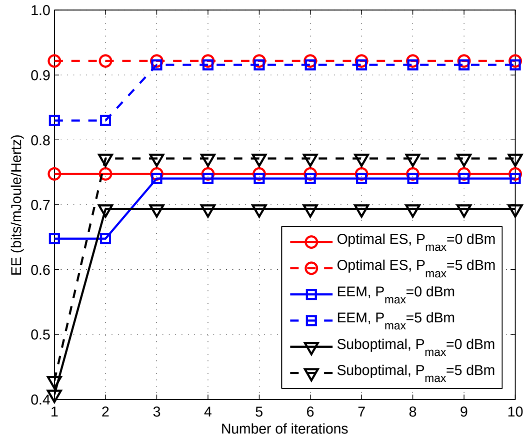
In this algorithm, we exhaustively search over all variables for finding the optimal resource allocation solution for all the nodes on each subcarrier in the pool of all the possible feasible solutions to the optimization problem $(\mathbf{P.4})$. Thus, the total complexity for this algorithm becomes $\mathcal{O}\left(3\mathcal{G}\mathcal{L}(2K)^{\varpi}K^{N_{sc}!}(V^3+1)\right)$.

VII. Simulation Results and Performance Discussions

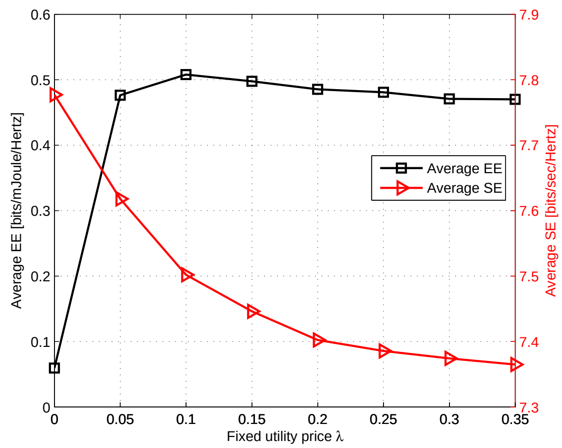
In order to evaluate the performance of the proposed resource allocation algorithms, we present the simulation results in this section.. The circuit and processing power per antenna at each node is set to 10 dBm, whereas the maximum available transmit power budget is set as 25 dBm. The Third-Generation Partnership Project (3GPP) path loss model is utilized with path loss $131.1 + 42.8 \times \log_{10}(d)$ dB where d is distance in kilometers [32]. Due to fluctuation of the total path attenuation level around the mean path loss, both the Rayleigh fading effects $\sim \mathcal{CN}(0,1)$ and the log-normal shadowing $\sim \ln\mathcal{N}(0, 8 \text{ dB})$ are taken into consideration. The subcarrier spacing is set as 12 kHz whereas thermal noise density is given by -174 dBm/Hz and the convergence tolerance value is set as 10^{-5}. The maximum number of iterations for solving the inner and outer problems is 10, whereas d_{SR} and d_{RD} denote the distance from all source nodes to the relay node and from the relay node to all destination nodes, respectively. We also simulate the performance of five other algorithms for comparison.

- ES algorithm: This algorithm gives the globally optimal solution of the problem $(\mathbf{P.1})$ by an exhaustic search over all variables [30], assuming that each takes discrete values.
- EEM algorithm without (w/o) subcarrier pairing and allocation (SPA) algorithm: The optimal solution of the

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TCOMM.2017.2713382, IEEE Transactions on Communications

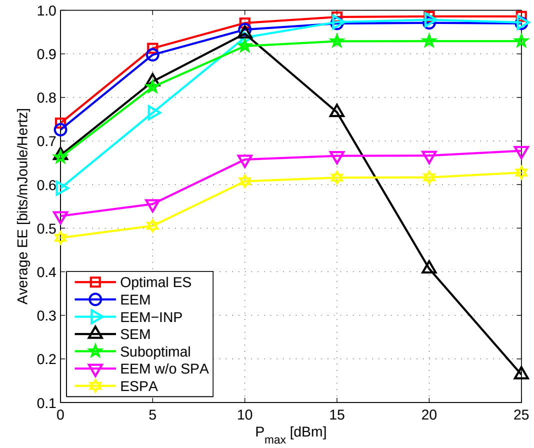


(a) EE versus Number of iterations

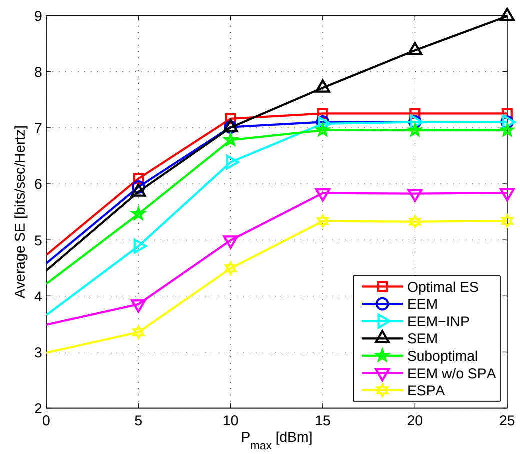


(b) Impact of utility price 'λ' on the performance

Fig. 2. Convergence behavior of the proposed algorithms ($N = 2$ and $d_{SR} = d_{RD} = 100$ m).



(a) Average EE versus P_{max}



(b) Average SE versus P_{max}

Fig. 3. Comparison of different resource allocation algorithms

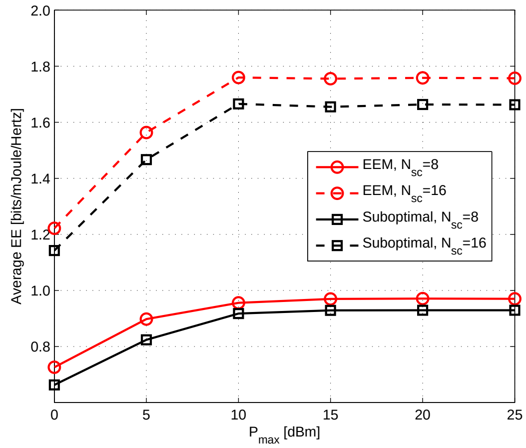
problem $(\mathbf{P.1})$ is found without subcarrier pairing and allocation.

- EEM algorithm with INP (EEM-INP): For a fair comparison with the total power constraint case, we set $P_{i,max} = P_{r,max} = \frac{P_{max}}{2K+1}$.
- SEM algorithm: By setting $\lambda = 0$, the optimization problem $(\mathbf{P.1})$ is transformed into the sum rate maximization problem.
- ESPA algorithm: Power is equally distributed among all the users over all the subcarriers.

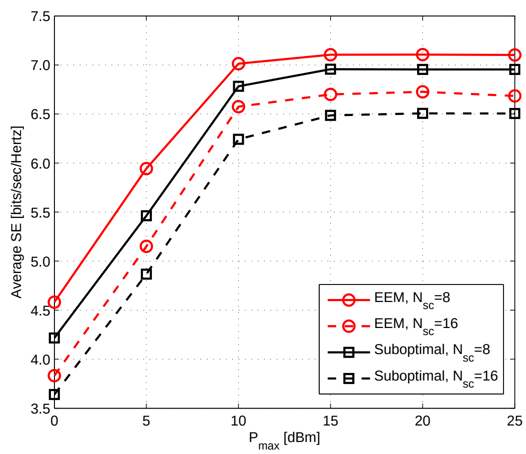
Fig. 2 shows the convergence behavior of the proposed algorithms for a single channel realization, where $K = 2$, $N_{sc} = 8$, $P_{max} = \{0, 5\}$ dBm, and $d_{SR} = d_{RD} = 100$ m. As can be seen from Fig. 2(a), as the number of iterations increases the EE performance of the proposed algorithms increases monotonically, and the proposed algorithms converges fast and typically achieve the optimal value within 4 iterations. The impact of utility price λ on achieving the average EE and SE is demonstrated in Fig. 2(b). It can be seen that the average SE performance decreases as the price λ increases, whereas the maximum average EE is achieved at $\lambda = 0.10$. Therefore, a performance trade-off between the average SE and EE is obtained through the adjustment of the price.

Fig. 3 shows the average EE and SE performance of the relay network for different algorithms, where $K = 2$, $N_{sc} = 8$, and $d_{SR} = d_{RD} = 100$ m. We can observed that the average EE and SE can be significantly improved as P_{max} increases. For $P_{max} > 10$ dBm, the average EE performance of all the algorithms saturates expect the SEM algorithm, whereas the SEM algorithm quickly declines as P_{max} increases. On the other side, the average SE of the SEM algorithm is continuously improved as P_{max} increases, while the SE performance of the proposed algorithms and ES algorithm is slowly saturated for $P_{max} > 10$ dBm. On the other hand, the SE performance of the SEM algorithm is continuously improved as P_{max} increases. Moreover, the average EE and SE performance of the proposed EEM and the suboptimal algorithms are very close to that obtained by the optimal ES algorithm. Also, it can be seen that the average EE and SE performances of the EEM-INP algorithm are slightly worse than the total power constraint when P_{max} is small, while the performance gap gradually reduces to zero as P_{max} increases. The EEM w/o SPA gives worst performance than the

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TCOMM.2017.2713382, IEEE Transactions on Communications

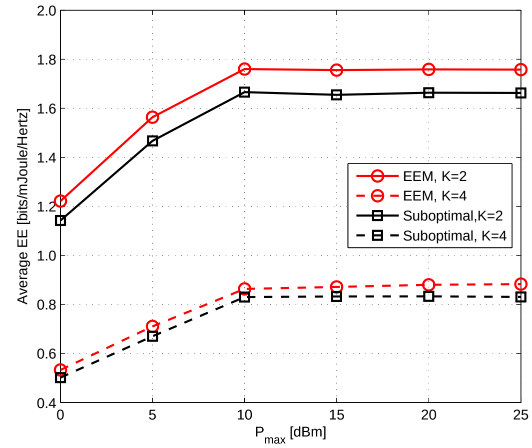


(a) Average EE versus P_{max}

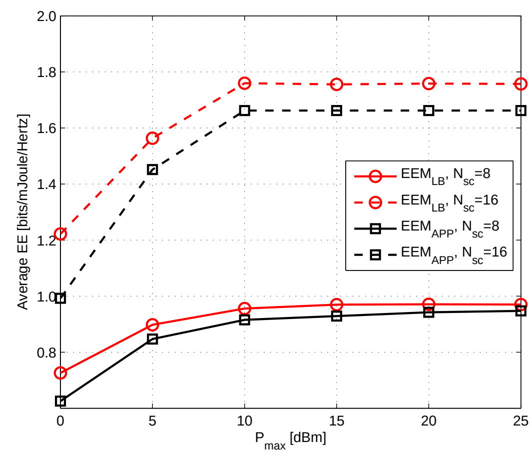


(b) Average SE versus P_{max}

Fig. 4. Effect of number of subcarriers on the average EE and SE for $K = 2$ and $d_{SR} = d_{RD} = 100$ m.



(a) Average EE versus P_{max}



(b) Average EE versus P_{max}

Fig. 5. Effect of number of user pairs on the average EE and the EE performance of EEM_{LB} with EEM_{APP}.

proposed optimal and suboptimal algorithms, while it performs better than ESPA.

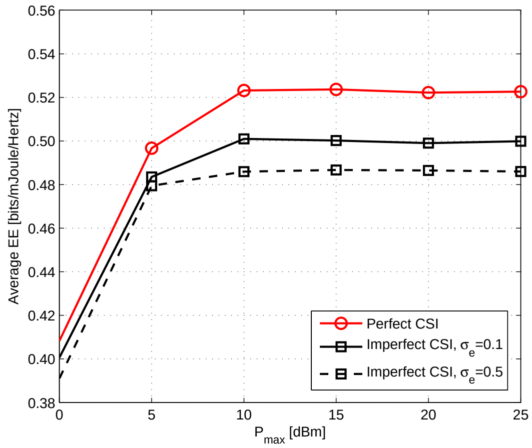
Fig. 4 depicts the effect of the number of subcarriers N_{sc} on the average EE and SE of the proposed algorithms, where the number of user pairs is $K = 2$ and $d_{SR} = d_{RD} = 100$ m. As can be seen, increasing the available transmit power budget $P_{max} \leq 10$ dBm significantly increases the average EE and SE performance of the proposed algorithms. However, when $P_{max} > 10$ dBm, the average EE performance of the EEM and suboptimal algorithms become constant. As expected, as N_{sc} increases, the average EE improves due to the frequency diversity and better utilization of available resources, i.e. subcarriers and available power budget.

Fig. 5(a) shows the average EE performance for different numbers of user pairs K, where $N_{sc} = 16$ and $d_{SR} = d_{RD} = 100$ m. It can be seen that as K increases the average EE performance of the proposed algorithms deteriorates due to the increases in the static power consumption. The EE performance of the EEM algorithm with lower bound (EEM_{LB}) and the EEM algorithm with approximation of $\log(1+x) \simeq \log x$ (EEM_{APP}) is illustrated in Fig. 5(b) for $K = 2, N_{sc} = \{8, 16\}$ and $d_{SR} = d_{RD} = 100$ m. We can observe that the performance of EEM_{LB} is always superior to that of the second approximation scheme $\log(1+x) \simeq \log x$.

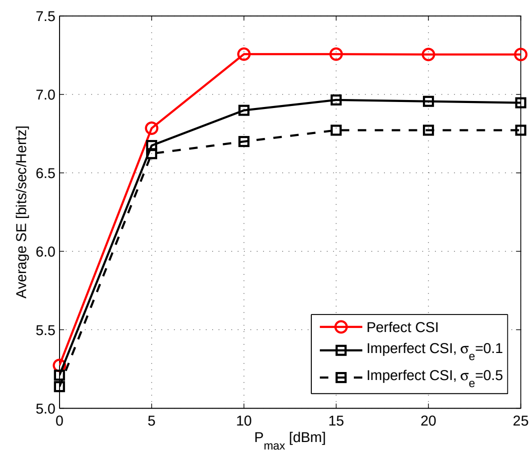
Fig. 6 shows the effect of imperfect CSI knowledge on the average EE and SE performance for different P_{max}, where $K = 2$, $N_{sc} = 4$ and $d_{SR} = d_{RD} = 100$ m. The channel estimation errors for all links are modeled as a complex Gaussian random variable with zero mean and variance σ_e^2. As expected, when $P_{max} \leq 5$ dBm, i.e. low power regime, the average EE and SE performance of the EEM algorithm with imperfect CSI is very close to that of the perfect CSI. However, in the higher power regime, i.e., $P_{max} > 5$ dBm, the imperfect CSI knowledge dominates the performance and thus the EEM algorithm with perfect CSI outperforms the EEM algorithm with imperfect CSI.

Also, as illustrated in Fig. 7, we simulate the performance of the generalized utility price-based resource allocation algorithm in the revised manuscript, and the performance curve is named as GEEM. Here, we set $K = 2$, $N_{sc} = \{8, 16\}$, and $d_{SR} = d_{RD} = 100$ m. As observed from this figure, both the average EE and SE performance of the GEEM algorithm

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TCOMM.2017.2713382, IEEE Transactions on Communications

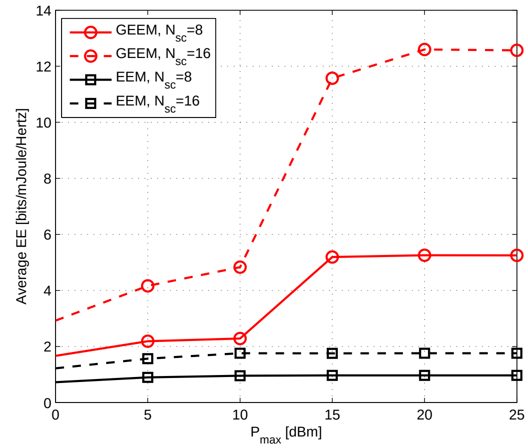


(a) Average EE versus P_{max}

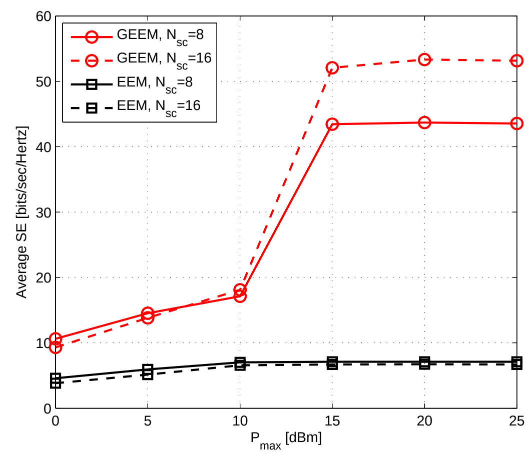


(b) Average SE versus P_{max}

Fig. 6. Effect of imperfect CSI knowledge on the average EE and SE.



(a) Average EE versus P_{max}



(b) Average SE versus P_{max}

Fig. 7. Comparison between the EEM and the GEEM in terms of the average EE and SE against P_{max}.

are slightly better than the EEM in a low power regime, i.e., $P_{max} \leq 10$ dBm, while the performance gap significantly increases in a higher power regime, i.e., $P_{max} \geq 15$ dBm.

VIII. Conclusion

In this paper, we studied the problem of joint subcarrier and power allocation for multi-user multicarrier two-way DF relay networks in order to enhance the energy utilization among the users. The objective function was to maximize the network's utility function, which was introduced to strike a balance between the power consumption and the achievable sum rate, through joint subcarrier and power allocation under a total transmit power constraint. The formulated primal maximization problem was a non-convex mixed binary integer non-linear programming problem. Therefore, the problem was converted into an equivalent convex optimization problem through SCA, change of variables and a series of transformation, and then a utility-based optimal resource allocation policy was derived based on concepts of dual decomposition. The relation between the optimal network price and the EE was established. In order to further reduce the computational complexity, a sub-optimal resource allocation algorithm was also proposed. The performance of the proposed EEM and suboptimal algorithms were compared with that of the SEM and EEM without SPA algorithms through computer simulations. Compared to the EEM algorithm, the GEEM algorithm performed much better in terms of both average EE and SE due to efficient use of subcarriers. Results were provided to shown the merits of the proposed EE resource allocation algorithms. Furthermore, the impact of various network parameters on the trade-off between the EE and SE was demonstrated.

Appendix A
Proof of Lemma 1

Substituting (16) and (17) into (15), the objective function becomes

$$\begin{aligned}\bar{U}\left(\mathbf{P},\mathbf{W},\boldsymbol{\mho},\boldsymbol{\Pi},\boldsymbol{\Upsilon}\right) &= \sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}}\frac{\mho^{(u,v)}\Pi_k^{(u,v)}}{2}\\ &\times\left(\log_2\left(1+\Upsilon_{2k-1}^{(u)}\right)+\log_2\left(1+\Upsilon_{2k}^{(u)}\right)\right)\\ &-\lambda\mathcal{P}_{Total}\left(\mathbf{P},\mathbf{W},\boldsymbol{\mho},\boldsymbol{\Pi}\right); \qquad \text{(A.1)}\end{aligned}$$

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TCOMM.2017.2713382, IEEE Transactions on Communications

Evidently, $\lambda \mathcal{P}_{Total}\left(\mathbf{P}, \mathbf{W}, \mho, \mathbf{\Pi}\right)$ is an affine and thus $-\lambda \mathcal{P}_{Total}\left(\mathbf{P}, \mathbf{W}, \mho, \mathbf{\Pi}\right)$ is concave in $\mathbf{P}$ and $\mathbf{W}$ for fixed $\mho$ and $\mathbf{\Upsilon}$. Next, we need to proof that $\log\left(1+\Upsilon_{2k-1}^{(u)}\right)$ and $\log\left(1+\Upsilon_{2k}^{(u)}\right)$ are also concave.

Since $f(x) = \log\left(1+x\right)), x \geq 0$ and $\nabla_x^2 f(x) = \frac{-1}{(1+x)^2} < 0 \Rightarrow f(x)$ is concave in x, and thus $\log\left(1+\Upsilon_{2k-1}^{(u)}\right)$ and $\log\left(1+\Upsilon_{2k}^{(u)}\right)$ is also concave in $\Upsilon_{2k-1}^{(u)}$ and $\Upsilon_{2k}^{(u)}$, $\forall k, u$. So, the objective function in (14) is a quasi-concave function of $\mathbf{P}, \mathbf{W}$, and $\mathbf{\Upsilon}$.

Appendix B
Proof of Lemma 2

Since the bound in (19) is tight at $\theta = \theta_0$ then $\log(1+\theta_0) = x\log(\theta_0) + y$. Substituting the value of y, i.e., $y = \log(1+\theta_0) - x\log(\theta_0)$ in (19) we get

$$\left(\frac{\theta}{\theta_0}\right)^x \leqslant \frac{1+\theta}{1+\theta_0}, \qquad \forall\, \theta > 0\,, \tag{B.1}$$

It is evident from (B.1) that: 1) any valid coefficient of the bound in (19) will always satisfy (B.1), 2) the coefficient $x < 1$, because for $x \geqslant 1$ then $\left(\frac{\theta}{\theta_0}\right)$ becomes a concave function and for some values where $\theta > 0$ the equation (B.1) will not hold true; and 3) at $\theta = \theta_0$, the function $\Theta = \frac{1+\theta}{1+\theta_0}$ is a tangent line for for $\Theta = \left(\frac{\theta}{\theta_0}\right)^{\frac{\theta_0}{1+\theta_0}}$. Finally, we can conclude that the maximum value of x is given by $x = \frac{\theta_0}{1+\theta_0}$. Thus, we consider the coefficients defined as in (20) and (21).

Appendix C
Proof of Lemma 3

$$\begin{aligned}\bar{\mathcal{U}}_{LB}\left(\hat{\mathbf{P}}, \hat{\mathbf{W}}, \mho, \mathbf{\Pi}, \hat{\mathbf{\Upsilon}}, \boldsymbol{\alpha}, \boldsymbol{\beta}\right) &= \sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}} \frac{\mho^{(u,v)}\Pi_k^{(u,v)}}{2} \\ &\times \left(\frac{\alpha_{2k-1}^{(u)}}{\ln(2)}\hat{\Upsilon}_{2k-1}^{(u)} + \beta_{2k-1}^{(u)} + \frac{\alpha_{2k}^{(u)}}{\ln(2)}\hat{\Upsilon}_{2k}^{(u)} + \beta_{2k}^{(u)}\right) \\ &- \lambda\bar{\mathcal{P}}_{Total}\left(\hat{\mathbf{P}}, \hat{\mathbf{W}}, \mho, \mathbf{\Pi}\right)\,,\end{aligned} \tag{C.1}$$

Since, $\boldsymbol{\alpha} \geq 0$, $\boldsymbol{\beta} \geq 0$ and $\lambda \geq 0$, the lower bound function in (C.1) forms the summation of the concave terms and linear terms (i.e. minus-exp functions) for given subcarrier pairing $\mho$ and subcarrier allocation $\mathbf{\Pi}$, hence the objective function $\bar{\mathcal{U}}_{LB}$ is concavified by the change of variables. All the constraints are convex and thus, the problem $(\mathbf{P}.4)$ is a convex problem.

Appendix D
Proof of Theorem 1

Proof: By applying the Karush-Kuhn-Tucker (K.K.T.) conditions, the optimal solution must satisfy [27]

$$\frac{\partial \mathcal{L}\left(\hat{\mathbf{P}}, \hat{\mathbf{W}}, \mho, \mathbf{\Pi}, \hat{\mathbf{\Upsilon}}, \kappa, \boldsymbol{\mu}, \boldsymbol{\vartheta}, \boldsymbol{\nu}, \mathbf{\Theta}\right)}{\partial \hat{P}_{2k-1}^{(u)}} = 0\,; \tag{D.1}$$

$$\frac{\partial \mathcal{L}\left(\hat{\mathbf{P}}, \hat{\mathbf{W}}, \mho, \mathbf{\Pi}, \hat{\mathbf{\Upsilon}}, \kappa, \boldsymbol{\mu}, \boldsymbol{\vartheta}, \boldsymbol{\nu}, \mathbf{\Theta}\right)}{\partial \hat{P}_{2k}^{(u)}} = 0\,; \tag{D.2}$$

From (D.1) and (D.2), the power allocation for the users at the $(m+1)^{th}$ iteration be updated as

$$\hat{P}_j^{(u)}(m+1) = \left[\frac{1}{2}\ln\left(\frac{\hat{z}_f^{(u)}\dfrac{e^{\hat{\Upsilon}_f^{(u)}}}{|h_j|^2}\left(\sum\limits_{l=1, l\neq j}^{2K} e^{\hat{P}_l^{(u)}}\left|h_l^{(u)}\right|^2 + \sigma_R^{(u)2}\right)}{(\lambda+\kappa)\sum\limits_{v=1}^{N_{sc}}\mho^{(u,v)}\Pi_k^{(u,v)} + \hat{z}_j^{(u)} e^{\hat{\Upsilon}_j^{(u)} - \hat{P}_f^{(u)}}\dfrac{|h_j|^2}{|h_f|^2}}\right)\right]^+; \quad j \neq f\,, \tag{D.3}$$

for $j = 2k, 2k-1$, $f = 2k, 2k-1$. For $\mho^{(u,v)} = 1$ and $\Pi_k^{(u,v)} = 1$, the interference term $\sum\limits_{l=1, l\neq 2k-1}^{2K} e^{\hat{P}_l^{(u)}}\left|h_l^{(u)}\right|^2$ becomes close to zero because all other users allocate almost zero power on the u^{th} subcarrier. Therefore, the power update (D.3) resembles as follows:

$$\hat{P}_j^{(u)}(m+1) = \left[\frac{1}{2}\ln\left(\frac{\hat{z}_f^{(u)}\dfrac{e^{\hat{\Upsilon}_f^{(u)}}}{|h_j|^2}\sigma_R^{(u)2}}{(\lambda+\kappa) + \hat{z}_j^{(u)} e^{\hat{\Upsilon}_j^{(u)} - \hat{P}_f^{(u)}}\dfrac{|h_j|^2}{|h_f|^2}}\right)\right]^+; \quad j \neq f\,, \tag{D.4}$$

Thus, the theorem is proved. ■

Appendix E
Proof of Theorem 2

Proof: By taking the partial derivative of the Lagrangian function $\mathcal{L}\left(\hat{\mathbf{P}}, \hat{\mathbf{W}}, \mho, \mathbf{\Pi}, \hat{\mathbf{\Upsilon}}, \kappa, \boldsymbol{\mu}, \boldsymbol{\vartheta}, \boldsymbol{\nu}, \mathbf{\Theta}\right)$ with respect to $\hat{W}_{2k-1}^{(v)}$ and $\hat{W}_{2k}^{(v)}$ and equating these results to zero, it implies

$$\frac{\partial \mathcal{L}\left(\hat{\mathbf{P}}, \hat{\mathbf{W}}, \mho, \mathbf{\Pi}, \hat{\mathbf{\Upsilon}}, \kappa, \boldsymbol{\mu}, \boldsymbol{\vartheta}, \boldsymbol{\nu}, \mathbf{\Theta}\right)}{\partial \hat{W}_{2k-1}^{(v)}} = 0\,; \tag{E.1}$$

$$\frac{\partial \mathcal{L}\left(\hat{\mathbf{P}}, \hat{\mathbf{W}}, \mho, \mathbf{\Pi}, \hat{\mathbf{\Upsilon}}, \kappa, \boldsymbol{\mu}, \boldsymbol{\vartheta}, \boldsymbol{\nu}, \mathbf{\Theta}\right)}{\partial \hat{W}_{2k}^{(v)}} = 0\,; \tag{E.2}$$

From (E.1) and (E.2), the relay power can be updated at the $(m+1)^{th}$ iteration as

$$\hat{W}_j^{(v)}(m+1) = \left[\frac{1}{2}\ln\left(\frac{\sum\limits_{u=1}^{N_{sc}}\Theta_f^{(u,v)} e^{\hat{\Upsilon}_f^{(v)}}\left(\sum\limits_{\substack{l=1\\ l\neq f,j}}^{2K} e^{\hat{W}_l^{(v)}}\left|g_f^{(v)}\right|^2 + \sigma_j^{(v)2}\right)}{(\lambda+\kappa)\sum\limits_{u=1}^{N_{sc}}\mho^{(u,v)}\Pi_k^{(u,v)}\left|g_f^{(v)}\right|^2}\right)\right]^+; \quad j \neq f \tag{E.3}$$

For $\mho^{(u,v)} = 1$ and $\Pi_k^{(u,v)} = 1$, the power of interference term $\sum\limits_{l=1, l\neq f,j}^{2K} e^{\hat{W}_l^{(v)}}\left|g_f^{(v)}\right|^2$ is zero. Hence, the equation for relay power update becomes

$$\hat{W}_j^{(v)}(m+1) = \left[\frac{1}{2}\ln\left(\frac{\tilde{z}_f^{(u,v)} e^{\hat{\Upsilon}_f^{(v)}} {\sigma_j^{(v)}}^2}{(\lambda+\kappa)\left|g_f^{(v)}\right|^2}\right)\right]^+ , \quad j \neq f \,, \tag{E.4}$$

The power update of the relay node can be explained in similar way to users power update, but it depends on noise power at the receive user. Consequently, the theorem is proved. ■

Appendix F
Proof of Theorem 3

Proof: Let $\left(\hat{\mathbf{P}}^\star(t), \hat{\mathbf{W}}^\star(t), \hat{\boldsymbol{\Upsilon}}^\star(t), \boldsymbol{\mho}^\star(t), \boldsymbol{\Pi}^\star(t)\right)$ be the optimal solution of the problem (**P.4**) in (25) with respect to the coefficients $\alpha_i^{(u)}(t)$ and $\beta_i^{(u)}(t)$ at the t^{th} iteration. If we update $\alpha_i^{(u)}(t+1)$ and $\beta_i^{(u)}(t+1)$ according to (23) and (24), we have

$$\begin{aligned}
&\bar{\mathcal{U}}_{LB}\left(\hat{\mathbf{P}}(\mathbf{t}), \hat{\mathbf{W}}(\mathbf{t}), \boldsymbol{\mho}(t), \boldsymbol{\Pi}(t), \hat{\boldsymbol{\Upsilon}}(t), \boldsymbol{\alpha}(t), \boldsymbol{\beta}(t)\right)\\
&\le \mathcal{U}\left(\hat{\mathbf{P}}(\mathbf{t}), \hat{\mathbf{W}}(\mathbf{t}), \boldsymbol{\mho}(t), \boldsymbol{\Pi}(t), \hat{\boldsymbol{\Upsilon}}(t)\right)\\
&= \bar{\mathcal{U}}_{LB}\left(\hat{\mathbf{P}}(\mathbf{t}), \hat{\mathbf{W}}(\mathbf{t}), \boldsymbol{\mho}(t), \boldsymbol{\Pi}(t), \hat{\boldsymbol{\Upsilon}}(t), \boldsymbol{\alpha}(t+1), \boldsymbol{\beta}(t+1)\right);
\end{aligned} \tag{F.1}$$

$$\begin{aligned}
&\bar{\mathcal{U}}_{LB}\left(\hat{\mathbf{P}}(\mathbf{t}), \hat{\mathbf{W}}(\mathbf{t}), \boldsymbol{\mho}(t), \boldsymbol{\Pi}(t), \hat{\boldsymbol{\Upsilon}}(t), \boldsymbol{\alpha}(t+1), \boldsymbol{\beta}(t+1)\right)\\
&\le \bar{\mathcal{U}}_{LB}\Big(\hat{\mathbf{P}}(\mathbf{t+1}), \hat{\mathbf{W}}(\mathbf{t+1}), \boldsymbol{\mho}(t+1), \boldsymbol{\Pi}(t+1),\\
&\qquad \hat{\boldsymbol{\Upsilon}}(t+1), \boldsymbol{\alpha}(t+1), \boldsymbol{\beta}(t+1)\Big),
\end{aligned} \tag{F.2}$$

where the first inequality and the second equality in (F.1) follow from the definition in (22)-(24), while the inequality in (F.2) is due to the optimization problem (**P.4**) in (25). Thus, we can conclude that the lower bound performance increases monotonically with the update of coefficients $\alpha_i^{(u)}$ and $\beta_i^{(m)}$; and after the convergence of the coefficients $\alpha_i^{(u)}$ and $\beta_i^{(u)}$, the optimal solution of the problem (**P.4**) becomes the local maximizer for the problem (**P.1**). ■

Appendix G
Proof of Theorem 4

Let $\left(\hat{\mathbf{P}}^\star, \hat{\mathbf{W}}^\star, \boldsymbol{\mho}^\star, \boldsymbol{\Pi}^\star, \hat{\boldsymbol{\Upsilon}}^\star\right)$ be the optimal solution of optimization problem $(\mathbf{P}.4)$ with respect to the optimal EE $\lambda^\star$ and $\mathcal{S}$ be the feasible set of the problem, it implies that

$$\lambda^\star = \max_{\substack{\hat{\mathbf{P}},\hat{\mathbf{W}}\\ \boldsymbol{\mho},\boldsymbol{\Pi},\hat{\boldsymbol{\Upsilon}}\in\mathcal{S}}} \frac{\left\{\begin{aligned}&\sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}}\frac{\mho^{(u,v)}\Pi_k^{(u,v)}}{2}\left(\frac{\alpha_{2k-1}^{(u)}}{\ln(2)}\hat{\Upsilon}_{2k-1}^{(u)}\right.\\ &\qquad\left.+\beta_{2k-1}^{(u)}+\frac{\alpha_{2k}^{(u)}}{\ln(2)}\hat{\Upsilon}_{2k}^{(u)}+\beta_{2k}^{(u)}\right)\end{aligned}\right\}}{\bar{\mathcal{P}}_{Total}\left(\hat{\mathbf{P}}, \hat{\mathbf{W}}, \boldsymbol{\mho}, \boldsymbol{\Pi}\right)}; \tag{G.1}$$

and

$$\lambda^\star = \frac{\left\{\begin{aligned}&\sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}}\frac{\mho^{(u,v)^\star}\Pi_k^{(u,v)^\star}}{2}\left(\frac{\alpha_{2k-1}^{(u)^\star}}{\ln(2)}\hat{\Upsilon}_{2k-1}^{(u)^\star}\right.\\ &\qquad\left.+\beta_{2k-1}^{(u)^\star}+\frac{\alpha_{2k}^{(u)^\star}}{\ln(2)}\hat{\Upsilon}_{2k}^{(u)^\star}+\beta_{2k}^{(u)^\star}\right)\end{aligned}\right\}}{\bar{\mathcal{P}}_{Total}\left(\hat{\mathbf{P}}^\star, \hat{\mathbf{W}}^\star, \boldsymbol{\mho}^\star, \boldsymbol{\Pi}^\star\right)}; \tag{G.2}$$

$$\ge \frac{\left\{\begin{aligned}&\sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}}\frac{\mho^{(u,v)}\Pi_k^{(u,v)}}{2}\left(\frac{\alpha_{2k-1}^{(u)}}{\ln(2)}\hat{\Upsilon}_{2k-1}^{(u)}\right.\\ &\qquad\left.+\beta_{2k-1}^{(u)}+\frac{\alpha_{2k}^{(u)}}{\ln(2)}\hat{\Upsilon}_{2k}^{(u)}+\beta_{2k}^{(u)}\right)\end{aligned}\right\}}{\bar{\mathcal{P}}_{Total}\left(\hat{\mathbf{P}}, \hat{\mathbf{W}}, \boldsymbol{\mho}, \boldsymbol{\Pi}\right)}; \tag{G.3}$$

From (G.1)–(G.3), we have the following observations:

$$\begin{aligned}\mathcal{F}(\lambda) = &\sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}}\frac{\mho^{(u,v)}\Pi_k^{(u,v)}}{2}\left(\frac{\alpha_{2k-1}^{(u)}}{\ln(2)}\hat{\Upsilon}_{2k-1}^{(u)}+\beta_{2k-1}^{(u)}\right.\\ &\left.+\frac{\alpha_{2k}^{(u)}}{\ln(2)}\hat{\Upsilon}_{2k}^{(u)}+\beta_{2k}^{(u)}\right)-\lambda^\star\bar{\mathcal{P}}_{Total}\left(\hat{\mathbf{P}}, \hat{\mathbf{W}}, \boldsymbol{\mho}, \boldsymbol{\Pi}\right)\le 0;\end{aligned} \tag{G.4}$$

and

$$\begin{aligned}\mathcal{F}(\lambda) = &\sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}}\frac{\mho^{(u,v)^\star}\Pi_k^{(u,v)^\star}}{2}\left(\frac{\alpha_{2k-1}^{(u)^\star}}{\ln(2)}\hat{\Upsilon}_{2k-1}^{(u)^\star}+\beta_{2k-1}^{(u)^\star}\right.\\ &\left.+\frac{\alpha_{2k}^{(u)^\star}}{\ln(2)}\hat{\Upsilon}_{2k}^{(u)^\star}+\beta_{2k}^{(u)^\star}\right)-\lambda^\star\bar{\mathcal{P}}_{Total}\left(\hat{\mathbf{P}}^\star, \hat{\mathbf{W}}^\star, \boldsymbol{\mho}^\star, \boldsymbol{\Pi}^\star\right)=0,\end{aligned} \tag{G.5}$$

From (G.4) and (G.5), we can observe that the maximum of $\mathcal{F}(\lambda)$ is zero and is achieved when the optimal resource allocation solution $\left(\hat{\mathbf{P}}^\star, \hat{\mathbf{W}}^\star, \boldsymbol{\mho}^\star, \boldsymbol{\Pi}^\star, \hat{\boldsymbol{\Upsilon}}^\star\right)$ is adopted and the maximum EE is obtained.

On the other hand, let $\left(\hat{\mathbf{P}}^\star, \hat{\mathbf{W}}^\star, \boldsymbol{\mho}^\star, \boldsymbol{\Pi}^\star, \hat{\boldsymbol{\Upsilon}}^\star\right)$ denotes the optimal solution of the problem $(\mathbf{P}.4)$ such that it satisfies the balance equation, it yields

$$\begin{aligned}&\sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}}\frac{\mho^{(u,v)^\star}\Pi_k^{(u,v)^\star}}{2}\left(\frac{\alpha_{2k-1}^{(u)^\star}}{\ln(2)}\hat{\Upsilon}_{2k-1}^{(u)^\star}+\beta_{2k-1}^{(u)^\star}\right.\\ &\left.+\frac{\alpha_{2k}^{(u)^\star}}{\ln(2)}\hat{\Upsilon}_{2k}^{(u)^\star}+\beta_{2k}^{(u)^\star}\right)-\lambda^\star\bar{\mathcal{P}}_{Total}\left(\hat{\mathbf{P}}^\star, \hat{\mathbf{W}}^\star, \boldsymbol{\mho}^\star, \boldsymbol{\Pi}^\star\right)=0;\end{aligned} \tag{G.6}$$

$$\begin{aligned}\ge &\sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}}\frac{\mho^{(u,v)}\Pi_k^{(u,v)}}{2}\left(\frac{\alpha_{2k-1}^{(u)}}{\ln(2)}\hat{\Upsilon}_{2k-1}^{(u)}+\beta_{2k-1}^{(u)}\right.\\ &\left.+\frac{\alpha_{2k}^{(u)}}{\ln(2)}\hat{\Upsilon}_{2k}^{(u)}+\beta_{2k}^{(u)}\right)-\lambda^\star\bar{\mathcal{P}}_{Total}\left(\hat{\mathbf{P}}, \hat{\mathbf{W}}, \boldsymbol{\mho}, \boldsymbol{\Pi}\right),\end{aligned} \tag{G.7}$$

The equations (G.6) and (G.7) implies that

$$\frac{\left\{\begin{aligned}&\sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}}\frac{\mho^{(u,v)}\Pi_k^{(u,v)}}{2}\left(\frac{\alpha_{2k-1}^{(u)}}{\ln(2)}\hat{\Upsilon}_{2k-1}^{(u)}\right.\\&\qquad\left.+\beta_{2k-1}^{(u)}+\frac{\alpha_{2k}^{(u)}}{\ln(2)}\hat{\Upsilon}_{2k}^{(u)}+\beta_{2k}^{(u)}\right)\end{aligned}\right\}}{\bar{\mathcal{P}}_{Total}\left(\hat{\mathbf{P}},\hat{\mathbf{W}},\boldsymbol{\mho},\boldsymbol{\Pi}\right)}\leq\lambda^{\star}$$

$$=\frac{\left\{\begin{aligned}&\sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}}\frac{\mho^{(u,v)^{\star}}\Pi_k^{(u,v)^{\star}}}{2}\left(\frac{\alpha_{2k-1}^{(u)^{\star}}}{\ln(2)}\hat{\Upsilon}_{2k-1}^{(u)^{\star}}\right.\\&\qquad\left.+\beta_{2k-1}^{(u)^{\star}}+\frac{\alpha_{2k}^{(u)^{\star}}}{\ln(2)}\hat{\Upsilon}_{2k}^{(u)^{\star}}+\beta_{2k}^{(u)^{\star}}\right)\end{aligned}\right\}}{\bar{\mathcal{P}}_{Total}\left(\hat{\mathbf{P}}^{\star},\hat{\mathbf{W}}^{\star},\boldsymbol{\mho}^{\star},\boldsymbol{\Pi}^{\star}\right)},\quad\text{(G.8)}$$

Thus, it is seen that $\lambda^{\star}$ which fulfills the balance equation is the optimal EE and the solution $\left(\hat{\mathbf{P}}^{\star},\hat{\mathbf{W}}^{\star},\boldsymbol{\mho}^{\star},\boldsymbol{\Pi}^{\star},\hat{\boldsymbol{\Upsilon}}^{\star}\right)$ obtained corresponding to the optimal EE $\lambda^{\star}$ is also the optimal solution of the optimization problem $(\mathbf{P.1})$. This proof of the optimal EE $\lambda^{\star}$ is also the proof of the optimality of the Dinkelbach's method. This concludes the proof of Theorem 4.

APPENDIX H
PROOF OF THEOREM 6

Theorem 5 indicates that the network price $\lambda(l)$ increases monotonically and remains bounded and the converged price is the optimal one. Assume that the network price $\lambda(l)$ converges at $\bar{\lambda}$, i.e., $\lambda(l)=\lambda(l+1)=\bar{\lambda}$, but $\bar{\lambda}$ is not the optimal price. From Theorem 4, we have

$$\begin{aligned}&\sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}}\frac{\mho^{(u,v)^{\star}}(l)\Pi_k^{(u,v)^{\star}}(l)}{2}\left(\frac{\alpha_{2k-1}^{(u)^{\star}}(l)}{\ln(2)}\hat{\Upsilon}_{2k-1}^{(u)^{\star}}(l)\right.\\&\qquad\left.+\beta_{2k-1}^{(u)^{\star}}(l)+\frac{\alpha_{2k}^{(u)^{\star}}(l)}{\ln(2)}\hat{\Upsilon}_{2k}^{(u)^{\star}}(l)+\beta_{2k}^{(u)^{\star}}(l)\right)\\&\quad-\lambda(l)\bar{\mathcal{P}}_{Total}\left(\hat{\mathbf{P}}^{\star}(l),\hat{\mathbf{W}}^{\star}(l),\boldsymbol{\mho}^{\star}(l),\boldsymbol{\Pi}^{\star}(l)\right)\neq 0,\quad\text{(H.1)}\end{aligned}$$

From (49), we know that

$$\lambda(l+1)=\frac{\left\{\begin{aligned}&\sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}}\frac{\mho^{(u,v)^{\star}}(l)\Pi_k^{(u,v)^{\star}}(l)}{2}\left(\frac{\alpha_{2k-1}^{(u)^{\star}}(l)}{\ln(2)}\hat{\Upsilon}_{2k-1}^{(u)^{\star}}(l)\right.\\&\qquad\left.+\beta_{2k-1}^{(u)^{\star}}(l)+\frac{\alpha_{2k}^{(u)^{\star}}(l)}{\ln(2)}\hat{\Upsilon}_{2k}^{(u)^{\star}}(l)+\beta_{2k}^{(u)^{\star}}(l)\right)\end{aligned}\right\}}{\bar{\mathcal{P}}_{Total}\left(\hat{\mathbf{P}}^{\star}(l),\hat{\mathbf{W}}^{\star}(l),\boldsymbol{\mho}^{\star}(l),\boldsymbol{\Pi}^{\star}(l)\right)},\quad\text{(H.2)}$$

According to (H.1) and (H.2), (H.3) can be obtained, given as

$$\lambda(l)\neq\frac{\left\{\begin{aligned}&\sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}}\frac{\mho^{(u,v)^{\star}}(l)\Pi_k^{(u,v)^{\star}}(l)}{2}\left(\frac{\alpha_{2k-1}^{(u)^{\star}}(l)}{\ln(2)}\hat{\Upsilon}_{2k-1}^{(u)^{\star}}(l)\right.\\&\qquad\left.+\beta_{2k-1}^{(u)^{\star}}(l)+\frac{\alpha_{2k}^{(u)^{\star}}(l)}{\ln(2)}\hat{\Upsilon}_{2k}^{(u)^{\star}}(l)+\beta_{2k}^{(u)^{\star}}(l)\right)\end{aligned}\right\}}{\bar{\mathcal{P}}_{Total}\left(\hat{\mathbf{P}}^{\star}(l),\hat{\mathbf{W}}^{\star}(l),\boldsymbol{\mho}^{\star}(l),\boldsymbol{\Pi}^{\star}(l)\right)}$$
$$=\lambda(l+1),\quad\text{(H.3)}$$

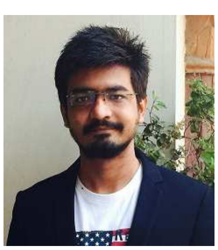
This contradicts our assumption $\lambda(l)=\lambda(l+1)$. This concludes the proof of Theorem 6.

REFERENCES

[1] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, pp. 3062-3080, Dec. 2004.
[2] C. Sun and C. Yang, "Is two-way relay more efficient," in *Proc. IEEE GLOBECOM*, pp. 1-6, Dec. 2011.
[3] R. Zhong, Y.-C. Liang, C. Chai, and S. Cui, "Optimal beamforming for two-way multi-antenna relay channel with analogue network coding," *IEEE Journal of Selected Areas Commun.*, vol. 27, no. 5, pp. 699-712, Jun. 2009.
[4] M. Chen and A. Yener, "Power allocation for F/TDMA multiuser two-way relay networks," *IEEE Trans. Wireless Commun.*, vol. 9, no. 2, pp. 546-551, Feb. 2010.
[5] K. Singh, M.-L. Ku, and J.-C. Lin, "Joint QoS-promising and EE-balancing power allocation for two-way relay networks," in *Proc. IEEE PIMRC*, pp. 1781-1785, Aug.-Sept. 2015.
[6] E. Gelenbe and Y. Caseau, "The impact of information technology on energy consumption and carbon emissions," *ACM Ubiquity*, pp. 1-15, Jun. 2015, DOI: 10.1145/2755977.
[7] The EINS Consortium, "Overview of ICT energy consumption (D8.1)," *Report FP7-2888021, European Network of Excellence in Internet Science*, Feb. 2013.
[8] L. Vandendorpe, R. Duran, J. Louveaux, and A. Zaidi, " Power allocation for OFDM transmission with DF relaying," in *Proc. IEEE ICC*, pp. 3795-3800, May 2008.
[9] L. Vandendorpe, J. Louveaux, O. Oguz and A. Zaidi, "Rate-optimized power allocation for DF-relayed OFDM transmission under sum and individual power constraints," *EURASIP J. Wireless Commun. and Netw.*,, pp. 1-11, Jun. 2009.
[10] L. Li, C. Dong, L. Wong, and L. Hanzo, "Spectral-efficient bidirectional decode-and-forward relaying for full-duplex communication," *IEEE Trans. Veh. Technol.*, vol. 65, no. 9, pp. 7010-7020, Sept. 2016.
[11] C. Li, Y. Wang, Z. Chen, Y. Yao, and B. Xia, "Performance analysis of the full-duplex enabled decode-and-forward two-way relay system," in *Proc. IEEE ICC Workshops*, pp. 559-564, May 2016.
[12] Y. Dai and X. Dong, "Power allocation for multi-pair massive MIMO two-way AF relaying with linear processing," *IEEE Wireless Commun. Lett.*, vol. 15, no. 9, pp. 5932-5946, Sept. 2016.
[13] G. A. S. Sidhu, F. Gao, W. Chen, and A. Nallanathan, "A joint resource allocation scheme for multiuser two-way relay networks," *IEEE Trans. Commun.*, vol. 59, no. 11, pp. 2970-2975, Nov. 2011.
[14] X. Li, Q. Zhang, G. Zhang, M. Cui, L. Yang and J. Qin, "Joint resource allocation with subcarrier pairing in cooperative OFDM DF multi-relay networks," *IET Commun.*, vol. 17, no. 5, pp. 872-875, May 2013.
[15] W. Y. Li, W. Wang, J. Kong, and M. Peng, "Subcarrier pairing for amplify-and-forward and decode-and-forward OFDM relay links," *IEEE Commun. Lett.*, vol. 13, no. 4, pp. 209-211, Apr. 2009.
[16] W. Dang, M.Tao, H. Mu, and J. Huang, "Subcarrier-pair based resource allocation for cooperative multi-relay OFDM systems," *IEEE Trans. Wireless Commun.*, vol. 9, no. 5, pp. 1640-1649, May 2010.
[17] W. Li, J. Lei, T. Wang, C. Xiong, and J. Wei, "Dynamic optimization for resource allocation in relay-aided OFDMA systems under multiservice," *IEEE Trans. Veh. Technol.*, vol. 65, no. 3, pp. 1303-1313, Mar. 2016.
[18] H. Q. Ngo, E. G. Larsson, and T. L. Marzetta, "Energy and spectral efficiency of very large multiuser MIMO systems," *IEEE Trans. Commun.*, vol. 61, no. 4, pp. 1436-1449, Apr. 2012.
[19] K. Singh and M.-L. Ku, "Toward green power allocation in relay-assisted multiuser networks: a pricing-based approach," *IEEE Trans. Wireless Commun.*, vol. 14, no. 5, pp. 2470- 2486, May 2015.

[20] C. Jiang and L. J. Cimini, "Energy-efficient transmission for MIMO interference channels," *IEEE Trans. Wireless Commun.*, vol. 12, no. 6, pp. 2988-2999, Jun. 2013.
[21] G. Miao, "Energy-efficient uplink multi-user MIMO," *IEEE Trans. Wireless Commun.*, vol. 12, no. 5, pp. 2302-2313, May 2013.
[22] C. Xiong, L. Lu, and G. Y. Li, "Energy-efficient OFDMA-based two-way relay," *IEEE Trans. Commun.*, vol. 63, no. 9, pp. 3157-3169, Sept. 2015.
[23] C. C. Zarakovitis, Q. Ni, and J. Spiliotis, "Energy efficient green wireless communication systems with imperfect CSI and data outage," *IEEE J. Sel. Areas Commun.*, vol. pp, no.99, Aug. 2016.
[24] C. C. Zarakovitis and Q. Ni, "Maximizing energy efficiency in multiuser multicarrier broadband wireless systems: convex relaxation and global optimization techniques," *IEEE Trans. Veh. Technol.*, vol. 65, no. 7, pp. 5275-5286, Jul. 2016.
[25] S. Katti, S. Gollakota, and D. Katabi, "Embracing wireless interference: Analog network coding," *in Proc. ACM SIGCOMM*, vol. 9, pp. 397-408, Feb. 2007.
[26] P. Monti, S. Tombaz, L. Wosinska, and J. Zander, "Mobile backhaul in heterogeneous network deployments: Technology options and power consumption," *in Proc. IEEE ICTON*, pp. 1-7, Jul. 2012.
[27] S. Boyd and L. Vandenberghe, "Convex optimization," Cambridge University Press, 2004.
[28] W. Yu and R. Lui, "Dual methods for nonconvex spectrum optimization of multicarrier systems," *IEEE Trans. Commun.*, vol. 54, no. 7, pp. 1310-1322, Jul. 2006.
[29] W. Dinkelbach, "On nonlinear fractional programming," *Management Science*, vol. 13, no. 7, pp. 492-498, Mar. 1967.
[30] H. W. Kuhn, "The Hungarian method for the assignment problem," in *50 years of integer programming 1958-2008.*, Springer Berlin Heidelberg, pp. 29-47, 2010.
[31] H. Zhang, Y. Liu, and M. Tao, "Resource allocation with subcarrier pairing in OFDMA two-way relay networks." *IEEE Wireless Commun. Lett.*, vol. 1, no. 2, pp. 61-64, Jan. 2012.
[32] 3GPP, TR 36.819 (V9.0.0), "Further advancement for E-UTRA physical layer aspects (Release 9)," Mar. 2010.

Keshav Singh (S'12, M'16) received the degree of Master of Technology (with first-class honors) in Computer Science from Devi Ahilya Vishwavidyalaya, Indore, India, in 2006, the M.Sc. in Information & Telecommunications Technologies from Athens Information Technology, Greece, in 2009, and the Ph.D. degree in Communication Engineering from National Central University, Taiwan, in 2015. Since 2016, he has been with Institute for Digital Communications, School of Engineering, University of Edinburgh, where he is currently a Postdoctoral Research Associate. He is a memeber of IEEE. He also has served as a Technical Program Committee Member for numerous IEEE conferences. His current research interests are in the areas of green communications, resource allocation, full-duplex radio, cooperative and energy harvesting networks, multiple-input multiple-output (MIMO) systems, and optimization of radio access.



Ankit Gupta received the Bachelor of Technology (B.Tech) degree in Electronics and Communication Engineering from Guru Gobind Singh Indraprastha University, Delhi, India, in 2015. He is currently with Aricent Technologies Limited (Holdings), Gurugram, India. His current research interests include 5G, cooperative communications, multiple-input multiple-output (MIMO) networks and optimization methods in signal processing and communications.

Tharmalingam Ratnarajah (A'96-M'05-SM'05) is currently with the Institute for Digital Communications, University of Edinburgh, Edinburgh, UK, as a Professor in Digital Communications and Signal Processing and the Head of Institute for Digital Communications. His research interests include signal processing and information theoretic aspects of 5G and beyond wireless networks, full-duplex radio, mmWave communications, random matrices theory, interference alignment, statistical and array signal processing and quantum information theory. He has published over 300 publications in these areas and holds four U.S. patents. He is currently the coordinator of the FP7 project ADEL (3.7M€) in the area of licensed shared access for 5G wireless networks. Previously, he was the coordinator of the FP7 project HARP (3.2M€) in the area of highly distributed MIMO and FP7 Future and Emerging Technologies projects HIATUS (2.7M€) in the area of interference alignment and CROWN (2.3M€) in the area of cognitive radio networks. Dr Ratnarajah is a Fellow of Higher Education Academy (FHEA), U.K.