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QoS-Driven Resource Allocation and EE-Balancing for Multiuser Two-Way Amplify-and-Forward Relay Networks

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Abstract—In this paper, we study the problem of energy-efficient resource allocation in multiuser two-way amplify-and-forward (AF) relay networks with the aim of maximizing the energy efficiency (EE) while ensuring the quality-of-service (QoS) requirements and balancing the EE of the user links. We formulate an EE-balancing optimization problem that maximizes the ratio of the spectral efficiency (SE) over the total power dissipation subject to QoS and a limited transmit power constraints. The problem which maximizes the EE by jointly optimizing the subcarrier pairing, power allocation, and subcarrier allocation, turns out to be a non-convex fractional mixed-integer nonlinear programming problem which has an intractable complexity in general. We apply a concave lower bound on the achievable sum rate and a series of convex transformations to make the problem convex one and propose an iterative algorithm for iteratively tightening the lower bound and finding the optimal solution through dual decomposition approach. Additionally, a low-complexity suboptimal algorithm is investigated. We then characterize the impact of various network parameters on the attainable EE and SE of the network employing both EE maximization and SE maximization algorithms when the network is designed from the energy-efficient perspective. Simulation results demonstrate the effectiveness of the proposed algorithms.

Index Terms—Resource allocation, quality-of-service, energy efficiency, multiuser, multicarrier, two-way, amplify-and-forward, relay network, non-convex optimization.

I. INTRODUCTION

Cooperative communication is a promising way to enhance the reliability, coverage and network performance of wireless communications [1]. Various relaying schemes have been proposed for cooperative communications, like amplify-and-forward (AF) and decode-and-forward (DF), of which the AF scheme is more prominently deployed due to its lower implementation complexity. Moreover, two-way relaying has been widely investigated to overcome the drawbacks of half-duplex relaying and to utilize the spectrum resources more efficiently [2]–[5]. Additionally, multicarrier multiple access techniques that allow multiple users to share the same spectrum and avoid severe interference from the other users, when combined with the relay transmission, can significantly improve the system's performance, due to their flexibility in resource allocation and their ability to exploit multiuser diversity, respectively.

A multi-pair two-way relay network where multiple user-pairs exchange messages using shared relay(s), has extensive applications in sensor networks, medical electronics, multimedia teleconferencing, smart homes and wearable computations etc., where information exchange between devices is often required. The multi-pair two-way relay networks, which is a special class of multi-way relay networks (MWRNs), where each user exchanges message only with its pre-defined partner, can be generalized to incorporate multiple users in the form of MWRNs, in which multiple users can exchange information with the help of a single relay terminal [6], [7]. In this paper, we focus multi-pair two-way AF relay network.

The unprecedented increase of mobile devices and escalating data rate requirements have contributed to the sharp growth of energy consumption and greenhouse emission. It is reported in [8], [9] that 2% to 10% of the global energy consumption and 2% of the greenhouse gas are generated by information and communication technologies (ICT) and, further predicting that 14% of the worldwide electrical energy will be consumed by the sector of ICT in 2020 [10]. Hence, more recently, research focus has shifted towards designing energy-efficient devices that not only maximize the network's spectral efficiency (SE) but also minimize the power consumption of the network. Furthermore, the efficient power utilization enables us to reduce carbon footprint and offers a green solution. Thus, resource allocation in a multiuser relay network that maximizes the SE while utilizing the minimum power and simultaneously maintaining the desired QoS becomes a challenging issue in cooperative communications [5].

Recently, a flourish of works on resource allocation in orthogonal frequency division multiplexing (OFDM)-based cooperative relay networks has been investigated in [11]–[14] from the perspective of SE maximization. In [11], the authors investigated resource allocation policies for SE maximization in multiuser two-way AF relay networks only for high signal-to-noise power ratio (SNR) regime, whereas power allocation strategies with subcarrier pairing were proposed in [12] for DF and AF multi-relay networks. The work in [13] was from sum rate maximization for a single user pair one-way relay network through the subcarrier pairing and power allocation subject to a sum-power constraint. In [14], the joint optimization of power allocation, subcarrier assignment, and relay selection

for enhancing the end-to-end transmission rate of a single user pair subject to individual or total power constraints was main objective.

However, the SE maximization problems do not directly provide the energy efficiency (EE) maximization solution. From EE maximization perspective, there are only a few works that have considered the EE maximization as a key metric for designing the optimal resource allocation policies [15]–[23]. In [15], a joint power control and antenna beamforming algorithm was proposed to maximize the EE in very large multiuser MIMO systems. The trade-off between the EE and SE was studied for multiuser MIMO systems in [16], whereas energy-efficient power optimization schemes were investigated in [18] for interference-limited communications. However, the relaying was not considered in [15]–[18]. In [19], the joint optimization of the relay transmit power, user selection, and the number of transmit antennas in a multi-pair two-hop amplify-and-forward (AF) relay system were studied from the EE perspective, while the authors in [20] designed the beamforming vectors of the source and the relay to maximize the network lifetime. The authors in [19]–[21] considered only a single-carrier system model and optimized transmit power from the EE point of view. A pricing-based power allocation schemes for multiuser multicarrier AF relay networks were investigated in [22]. However, [22] only optimized power allocation without considering the subcarrier pairing and allocation in the network. Therefore, there is a need to revisit the design of the existing multiuser multicarrier AF relay networks and further investigate a unified resource allocation policies considering subcarrier permutation, subcarrier allocation, and power optimization all together in order to maximize the EE of the network.

In light of the above discussion, in this paper, we investigate the joint subcarrier pairing, subcarrier allocation and power allocation algorithms for multiuser multicarrier two-way AF relay networks for improving the EE under a total transmit power, individual QoS, subcarrier pairing and allocation constraints, while balancing the EE of the two-way links. The considered network consists of multiple user pairs and a single relay node. All the users and the relay node are equipped with single antenna and each user pair communicates with each other through an intermediate relay node. Further, the relay node operates in AF manner with two-stage transmission protocol. The distinctive contributions of this paper are highlighted as follows:

- We formulate an EE maximization (EEM) problem in context of a multiuser AF relay networks subject to a limited total transmit power, minimum QoS requirement, subcarrier pairing and allocation constraints. In contrast to [11]–[14], [22], the main focus is to maximize EE through optimizing the power allocation, subcarrier permutation, and subcarrier allocations within a network context. It is evident that the original problem is a non-convex fractional mixed-integer nonlinear programming (MINLP) [29], which is NP-hard to solve.
- To make the problem tractable, a successive convex approximation (SCA) method, a variable transformation, and a relaxation of the integer variables is applied. Next,

it is proven that the relaxed problem is quasi-concave on the subcarrier pairing, subcarrier allocation, and power allocation variables. Consequently, by employing the fractional programming and dual decomposition methods, the optimal solution is obtained.
- Besides, a suboptimal EE resource allocation algorithm is investigated to strike a balance between the complexity and optimality as also demonstrated by the simulation results.

The rest of this paper is organized as follows. Section II describes the system model. The EE maximization problem subject to a total transmit power, minimum QoS requirement, subcarrier pairing and allocation constraints is formulated in Section III, followed by stepwise procedure of transforming the non-convex fractional MINLP problem into a convex one. An iterative EE resource allocation algorithm is investigated in Section IV. The suboptimal algorithm is presented in Section V and the complexity of proposed and the standard algorithms are analyzed in Section VI. Section VII presents simulation results and the paper concludes with Section VIII.

II. SYSTEM MODEL

We consider a relay interference network where an AF relay assists the two-way communication between K user pairs formed by odd users denoted by $2k - 1$ and even users represented by $2k$, for $k \in \{1, 2, \ldots, K\}$, wherein each transmission hop has N_{sc} subcarriers for signal transmission as illustrated in Fig. 1. All the nodes in the network are assumed to have a single antenna. For simplicity, the transmit and receive users are assumed to be well separated so that the direct links between them can be ignored. We further consider that all the links experience slow and frequency-flat fading and the relay node has perfect channel state information (CSI) knowledge. The relay operates in a half-duplex mode with two transmission phases [1]. In the multiple access (MA) phase, all $2K$ users simultaneously transmit signals to the relay node, while during the broadcast (BC) phase, the relay node forwards the amplified signal to the users; meanwhile, the users keep silent. Moreover, the $(2k - 1)^{th}$ and $(2k)^{th}$ users transmit signals on the uth subcarrier in the MA phase whereas in the BC phase the relay node forwards the amplified signal on the vth subcarrier to the $(2k)^{th}$ and $(2k - 1)^{th}$ users, respectively.

Define $h_i^{(u)}$ as the channel coefficient from the ith user to the relay node on the uth subcarrier, for $i = 1, \ldots, 2K$, $u = 1, \ldots, N_{sc}$. The received signal at the relay node on the uth subcarrier can be expressed as

$$\mathbf{y}_R^{(u)} = \sum_{i=1}^{2K} h_i^{(u)} \sqrt{P_i^{(u)}} s_i^{(u)} + n_R^{(u)}, \tag{1}$$

where $s_i^{(u)}$ is the ith user's signal transmitted on the uth subcarrier with unit transmission power, i.e. $\mathbb{E}\left[\left|s_i^{(u)}\right|^2\right] = 1$ and $n_R^{(u)} \sim \mathcal{N}(0, \sigma_R^{(u)^2})$ is the complex additive white Gaussian noise (AWGN) at the relay node on the uth subcarrier. The transmit power level of the ith user on the uth subcarrier is
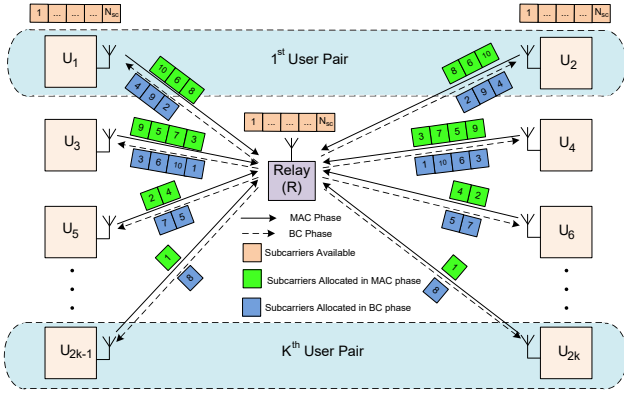
Fig. 1. A relay-assisted multiuser two-way relay network with K user pairs.

denoted by $P_i^{(u)}$, for $i = 1, \ldots, 2K$. The relay node amplifies the received signal $\mathbf{y}_R^{(u)}$ with the normalizing factor expressed as

$$\alpha^{(v)} = \sqrt{W_R^{(v)} \Big/ \Big(\sum_{i=1}^{2K} P_i^{(u)} \left| h_i^{(u)} \right|^2 + \sigma_R^{(u)^2} \Big)}, \quad (2)$$

where $W_R^{(v)}$ denotes the transmit power of the relay node on the vth subcarrier, for $v = 1, \ldots, N_{sc}$.

The signal transmitted by the relay node on the vth subcarrier can be given by

$$x^{(v)} = \alpha^{(v)} \mathbf{y}_R^{(u)} = \alpha^{(v)} \sum_{i=1}^{2K} h_i^{(u)} \sqrt{P_i^{(u)}} s_i^{(u)} + \alpha^{(v)} n_R^{(u)}, \quad (3)$$

Finally, the received signal at the $(2k-1)^{th}$ and $(2k)^{th}$ users after removing self-interference from the received signals using the well-known analogue network coding method [24], can respectively be given as

$$y_{2k-1}^{(v)} = \underbrace{\alpha^{(v)} \sqrt{P_{2k}^{(u)}} h_{2k-1}^{(v)} h_{2k}^{(u)} s_{2k}^{(u)}}_{\text{Desired Signal}}$$

$$+ \underbrace{\sum_{i=1, i \neq 2k-1, 2k}^{2K} \alpha^{(v)} \sqrt{P_i^{(u)}} h_i^{(u)} h_{2k-1}^{(v)} s_i^{(u)}}_{\text{Interference}}$$

$$+ \underbrace{\alpha^{(v)} h_{2k-1}^{(v)} n_R^{(u)} + n_{2k-1}^{(v)}}_{\text{Noise}}; \quad (4)$$

$$y_{2k}^{(v)} = \underbrace{\alpha^{(v)} \sqrt{P_{2k-1}^{(u)}} h_{2k}^{(v)} h_{2k-1}^{(u)} s_{2k-1}^{(u)}}_{\text{Desired Signal}}$$

$$+ \underbrace{\sum_{i=1, i \neq 2k-1, 2k}^{2K} \alpha^{(v)} \sqrt{P_i^{(u)}} h_i^{(u)} h_{2k}^{(v)} s_i^{(u)}}_{\text{Interference}}$$

$$+ \underbrace{\alpha^{(v)} h_{2k}^{(v)} n_R^{(u)} + n_{2k}^{(v)}}_{\text{Noise}}, \quad (5)$$

where $n_{2k-1}^{(v)}$ and $n_{2k}^{(v)}$ are zero-mean Gaussian noises at the $(2k-1)^{th}$ and $(2k)^{th}$ users on the vth subcarrier with variances $\sigma_{2k-1}^{(v)^2}$ and $\sigma_{2k}^{(v)^2}$, respectively.

The total power consumption in the network consists of two terms namely: transmit power and static power, which has remarkable impact on system's SE. Hence, it is important to take both the transmit and static power into consideration [17], [25]. The transmitter's signal processing power and the receiver's processing power are collectively referred as the circuit power which is not related to the sum rate when the users transmit or receive information and is regarded as static value here, while the transmit power is exclusively used for data transmission in order to attain reliable communications. In general, the transmit power behaves dynamically with respect to the instantaneous channel gains, but the circuit/processing power usually remains static, irrespective of the channel conditions. Therefore, the overall required power (in Watts) for the two-way relay networks is assumed to be governed by a constant term that covers the static power dissipation of the nodes and other two terms that vary with the transmit powers $P_i^{(u)}$ and $W_R^{(v)}$, which can be modelled as

$$P_t = \underbrace{\sum_{i=1}^{2K} \sum_{u=1}^{N_{sc}} P_i^{(u)} + \sum_{v=1}^{N_{sc}} W_R^{(v)}}_{\text{Dynamic Power} \leq P_{max}} + \underbrace{(2K+1)P_c}_{\text{Static Power}, \triangleq C} \quad \text{[Watts]},$$

$$(6)$$

where P_{max} is the maximum available transmit power budget of the two-way relay network and P_c denotes the circuit power of each user or relay node.

Let $\Lambda_{u,v} \in \{0,1\}$ denotes the subcarrier pairing binary variable signifying that $\Lambda_{u,v} = 1$ if the uth subcarrier in the MA phase is paired with the vth subcarrier in the BC phase, while $\Lambda_{u,v} = 0$ otherwise. We further define a binary variable $\Omega_{k,(u,v)} \in \{0,1\}$ for the subcarrier allocation such that $\Omega_{k,(u,v)} = 1$ if the kth user pair is operating on the $(u,v)^{th}$ subcarrier pair, while $\Omega_{k,(u,v)} = 0$ otherwise. Thus, the power dissipated after subcarrier pairing and allocation is given as

$$P_T =$$

$$\underbrace{\sum_{k=1}^{K} \sum_{u=1}^{N_{sc}} \sum_{v=1}^{N_{sc}} \Lambda_{u,v} \Omega_{k,(u,v)} \left(P_{2k-1}^{(u)} + P_{2k}^{(u)} + W_R^{(v)} \right)}_{\text{Dynamic Power} \leq P_{max}} + C, \quad (7)$$

III. PROBLEM FORMULATION

In this section, we formulate the optimization problem. Using (4) and (5), the average signal-to-interference plus noise ratio (SINR) at the $(2k-1)^{th}$ and $(2k)^{th}$ users on the $(u,v)^{th}$ subcarrier can respectively be written in (8) and (9), as shown on the top of the next page. By substituting (2) into (8) and (9), the SINR, $\Upsilon_{2k-1}^{(u,v)}$ and $\Upsilon_{2k}^{(u,v)}$, can be explicitly expanded as in (10) and (11) shown on the top of the next page.

A. Channel capacity and EE

From (10) and (11), the minimum achievable sum rate for the $(2k-1) \to 2k$ and $2k \to (2k-1)$ links on the $(u,v)^{th}$

$$\Upsilon_{2k-1}^{(u,v)} = \frac{\alpha^{(v)^2} P_{2k}^{(u)} \left| h_{2k-1}^{(v)} h_{2k}^{(u)} \right|^2}{\sum\limits_{i=1, i\neq 2k-1, 2k}^{2K} \alpha^{(v)^2} P_i^{(u)} \left| h_i^{(u)} h_{2k-1}^{(v)} \right|^2 + \alpha^{(v)^2} \left| h_{2k-1}^{(v)} \right|^2 \sigma_R^{(u)^2} + \sigma_{2k-1}^{(v)^2}} \; ; \tag{8}$$

$$\Upsilon_{2k}^{(u,v)} = \frac{\alpha^{(v)^2} P_{2k-1}^{(u)} \left| h_{2k}^{(v)} h_{2k-1}^{(u)} \right|^2}{\sum\limits_{i=1, i\neq 2k-1, 2k}^{2K} \alpha^{(v)^2} P_i^{(u)} \left| h_i^{(u)} h_{2k}^{(v)} \right|^2 + \alpha^{(v)^2} \left| h_{2k}^{(v)} \right|^2 \sigma_R^{(v)^2} + \sigma_{2k}^{(v)^2}} \; , \tag{9}$$

$$\Upsilon_{2k-1}^{(u,v)} = \frac{W_R^{(v)} P_{2k}^{(u)} \left| h_{2k-1}^{(v)} h_{2k}^{(u)} \right|^2}{\sum\limits_{i=1, i\neq 2k-1, 2k}^{2K} W_R^{(v)} P_i^{(u)} \left| h_i^{(u)} h_{2k-1}^{(v)} \right|^2 + W_R^{(v)} \left| h_{2k-1}^{(v)} \right|^2 \sigma_R^{(u)^2} + \sigma_{2k-1}^{(v)^2} \left(\sum_{i=1}^{2K} P_i^{(u)} \left| h_i^{(u)} \right|^2 + \sigma_R^{(u)^2} \right)} \; ; \tag{10}$$

$$\Upsilon_{2k}^{(u,v)} = \frac{W_R^{(v)} P_{2k-1}^{(u)} \left| h_{2k}^{(v)} h_{2k-1}^{(u)} \right|^2}{\sum\limits_{i=1, i\neq 2k-1, 2k}^{2K} W_R^{(v)} P_i^{(u)} \left| h_i^{(u)} h_{2k}^{(v)} \right|^2 + W_R^{(v)} \left| h_{2k}^{(v)} \right|^2 \sigma_R^{(u)^2} + \sigma_{2k}^{(v)^2} \left(\sum_{i=1}^{2K} P_i^{(u)} \left| h_i^{(u)} \right|^2 + \sigma_R^{(u)^2} \right)} \; , \tag{11}$$

subcarrier pair with unit bandwidth is given by [26]

$$R_{2k-1}^{(u,v)} = \frac{1}{2}\log_2\big(1 + \Upsilon_{2k-1}^{(u,v)}\big), \qquad \text{[bits/s/Hz]}; \tag{12}$$

$$R_{2k}^{(u,v)} = \frac{1}{2}\log_2\big(1 + \Upsilon_{2k}^{(u,v)}\big), \qquad \text{[bits/s/Hz]}, \tag{13}$$

where the factor of $1/2$ accounts for the fact that transmission completes in two hops. In the first step of transforming the non-convex achievable sum rates in (12) and (13), we introduce the following theorem for the lower bound on the logarithmic function $\log(1 + \theta)$:

Theorem 1: The logarithmic function $\log(1 + \theta)$ has the following lower bound

$$\log(1 + \theta) \geqslant x\log(\theta) + y, \qquad \forall \theta > 0; \tag{14}$$

where $x > 0$ and y are the coefficients that need to be determined, and it is assumed that the bound is tight at $\theta = \theta_0$, then

$$x = \frac{\theta_0}{1 + \theta_0}; \tag{15}$$

$$y = \log(1 + \theta_0) - x\log(\theta_0), \tag{16}$$

Proof: The proof of Theorem 1 is similar to the proof in [22, Appendix A]. ∎

Remark 1: Using Theorem 1, we can find the lower bound of the achievable sum rate $R_{2k-1}^{(u,v)}$ and $R_{2k}^{(u,v)}$, $\forall k, (u, v)$, defined in (12) and (13). These lower bound approximations will be used for designing the energy-efficient resource allocation algorithms.

The tightness of the lower bound is demonstrated in Fig. 2. Since a tighter lower bound markedly convergences to a Karush-Kuhn-Tucker (KKT) point of the original non-convex problem, we will focus on the lower bound for the considered problem.
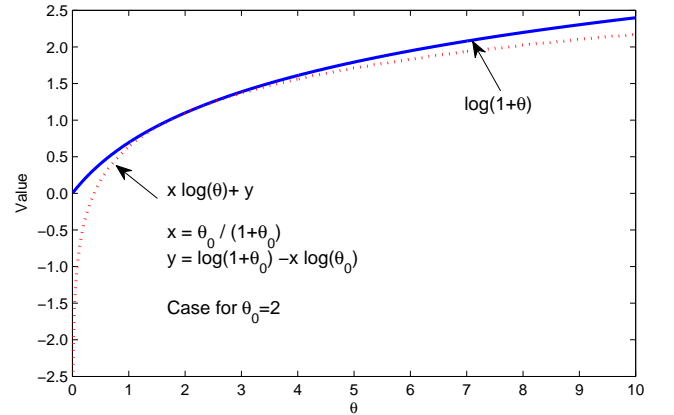
Fig. 2. Lower bound (dashed) that lead to convex relaxations, with $\theta_0 = 2$.

From Theorem 1, the achievable sum rate $R_{2k-1}^{(u,v)}$ in the underlying two-way relay network can be lower bounded by

$$R_{2k-1}^{(u,v)} = \frac{1}{2}\log_2\big(1 + \Upsilon_{2k-1}^{(u,v)}\big);$$
$$\geq \frac{1}{2}\Big[\rho_{2k-1}^{(u,v)}\log_2\big(\Upsilon_{2k-1}^{(u,v)}\big) + \beta_{2k-1}^{(u,v)}\Big] \triangleq R_{2k-1,lb}^{(u,v)}, \tag{17}$$

where the two coefficients $\rho_{2k-1}^{(u,v)}$ and $\beta_{2k-1}^{(u,v)}$ can be selected as [27], [28]

$$\rho_{2k-1}^{(u,v)} = \varrho_{2k-1}^{(u,v)}/(1 + \varrho_{2k-1}^{(u,v)}); \tag{18}$$

$$\beta_{2k-1}^{(u,v)} = \log_2\big(1 + \varrho_{2k-1}^{(u,v)}\big) - \rho_{2k-1}^{(u,v)}\log_2\big(\varrho_{2k-1}^{(u,v)}\big), \tag{19}$$

for any given $\varrho_{2k-1}^{(u,v)} > 0$. The equality in (17) holds when $\rho_{2k-1}^{(u,v)} = \Upsilon_{2k-1}^{(u,v)}/(1 + \Upsilon_{2k-1}^{(u,v)})$ and $\beta_{2k-1}^{(u,v)} = \log_2(1 + \Upsilon_{2k-1}^{(u,v)}) - \rho_{2k-1}^{(u,v)}\log_2\big(\Upsilon_{2k-1}^{(u,v)}\big)$, and the equality holds for $\big(\rho_{2k-1}^{(u,v)}, \beta_{2k-1}^{(u,v)}\big) = (1, 0)$ if $\Upsilon_{2k-1}^{(u,v)}$ approaches plus infinity. Similarly, the lower bound for achievable sum rate $R_{2k}^{(u,v)}$ can

be defined as

$$R_{2k}^{(u,v)} = \frac{1}{2}\log_2\big(1 + \Upsilon_{2k}^{(u,v)}\big);$$

$$\geq \frac{1}{2}\Big[\rho_{2k}^{(u,v)}\log_2\big(\Upsilon_{2k}^{(u,v)}\big) + \beta_{2k}^{(u,v)}\Big] \triangleq R_{2k,lb}^{(u,v)}, \quad (20)$$

where $\rho_{2k}^{(u,v)}$ and $\beta_{2k}^{(u,v)}$ are defined similar to $\rho_{2k-1}^{(u,v)}$ and $\beta_{2k-1}^{(u,v)}$, respectively. Using (17) and (20), the total achievable minimum (worst) end-to-end sum rate after subcarrier pairing and allocation can be written as

$$R_T \triangleq \sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}}\Lambda_{u,v}\Omega_{k,(u,v)}\min\Big(R_{2k-1,lb}^{(u,v)}, R_{2k,lb}^{(u,v)}\Big),$$
(21)

Using (7) and (21), the EE of the network is defined as follows.

Definition 1: The EE for the multiuser two-way AF relay network is defined as the minimum achievable sum rate of the transmitted data per unit of energy. By considering the rate balancing between the $(2k-1) \to 2k$ and $2k \to (2k-1)$ links, the EE of the network can be defined as

$$\eta_{EE} = \frac{R_T}{P_T}$$

$$= \frac{\sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}}\Lambda_{u,v}\Omega_{k,(u,v)}\min\Big(R_{2k-1,lb}^{(u,v)}, R_{2k,lb}^{(u,v)}\Big)}{\sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}}\Lambda_{u,v}\Omega_{k,(u,v)}\Big(P_{2k-1}^{(u)} + P_{2k}^{(u)} + W_R^{(v)}\Big) + C},$$
(22)

B. Optimization problem and transformation

In this subsection, we depict EE optimization problem for a multiuser two-way relay networks. Here, our objective is to maximize η_{EE} of the network subject to the following constraints: 1) to limit the total transmit power, 2) to guarantee the SINR requirements for each user, 3) to mandate that the signal transmission does not takes place on the same subcarrier in the MA and BC phases; and 4) to validate that a subcarrier pair is assigned to only one user pair. The QoS-constrained optimization problem for multiuser two-way relay can be formulated as

$$(\textbf{OP1}) \quad \max_{\textbf{P},\textbf{W}_R,\boldsymbol{\Lambda},\boldsymbol{\Omega}} \quad \eta_{EE}$$

subject to

$$(C.1) \sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}}\Lambda_{u,v}\Omega_{k,(u,v)}\Big(P_{2k-1}^{(u)}+P_{2k}^{(u)}+W_R^{(v)}\Big) \leqslant P_{max};$$

$$(C.2)\,\Upsilon_{2k-1}^{(u,v)} \geq \Upsilon_{min,2k-1}^{(u,v)}, \qquad \forall\, k, (u,v);$$

$$(C.3)\,\Upsilon_{2k}^{(u,v)} \geq \Upsilon_{min,2k}^{(u,v)}, \qquad \forall\, k, (u,v);$$

$$(C.4) \sum_{u=1}^{N_{sc}}\Lambda_{u,v} = 1, \qquad \forall\, v; \qquad (23)$$

$$(C.5) \sum_{v=1}^{N_{sc}}\Lambda_{u,v} = 1, \qquad \forall\, u;$$

$$(C.6) \sum_{k=1}^{K}\Omega_{k,(u,v)} = 1, \qquad \forall\,(u,v);$$

$$(C.7)\,\Lambda_{u,v} \in \{0,1\}, \qquad \forall\, u,v;$$

$$\Omega_{k,(u,v)} \in \{0,1\}, \qquad \forall\, k,(u,v);$$

$$(C.8)\,P_{2k-1}^{(u)} > 0,\ P_{2k}^{(u)} > 0,\ W_R^{(v)} > 0, \qquad \forall\, k,u,v,$$

where $\textbf{P} = \{P_i^{(u)}\}$, $\textbf{W}_R = \{W_R^{(v)}\}$, $\boldsymbol{\Lambda} = \{\Lambda_{u,v}\}$, $\boldsymbol{\Omega} = \{\Omega_{k,(u,v)}\}$ and $\Upsilon_{min,2k-1}^{(u,v)}$ and $\Upsilon_{min,2k}^{(u,v)}$, $\forall k$ are the minimum SINR requirement for odd and even users on the $(u,v)^{th}$ subcarrier pair, respectively. Physically, the constraint $(C.1)$ ensures that the sum of the power allocated to users $P_{2k-1}^{(u)}$ and $P_{2k}^{(u)}$, $\forall k, u$, and the relay node $W_R^{(v)}$, $\forall v$ does not exceed the maximum power budget of the network, while the constraints $(C.2)$ and $(C.3)$ ensure the minimum QoS requirement for odd and even users over the $(u,v)^{th}$ subcarrier pair. Also, the constraints $(C.4)$ and $(C.5)$ mandates that each subcarrier in MA phase can be paired with one and only one subcarrier in BC phase and vice versa; and $(C.6)$ ensures that a subcarrier pair (u,v) is allocated to a single user pair only. Since the relay node is equipped with only a single antenna in this paper, the design framework can be easily extended to the scenario with multiple antennas. In this case, the channels from the source nodes to the relay node and from the relay node to the destination nodes become SIMO and MISO channels, respectively. By designing receive and transmit beamforming weights at the relay node, the SINR can be derived similar to (8) and (9). In general, an increased number of antennas can offer better interference suppression capability, but it also requires more static power consumption, thus leading to an EE performance tradeoff.

The optimization problem (**OP1**) is a non-convex fractional MINLP problem [29], and thus we cannot solve it directly. To find the optimal solution, an exhaustive search (ES) over all variables is required and thus the computational complexity becomes very high, specially for higher number of subcarriers. The fact that the duality gap between the primal problem and the dual problem approaches to zero for a sufficiently large number of subcarriers [30]. Thus, [30] inspires us that instead of solving (**OP1**) directly, we will solve the dual problem. By applying an epigraph method, the (**OP1**) can be transformed as

$$(\textbf{OP2}) \quad \max_{\textbf{P},\textbf{W}_R,\textbf{t},\boldsymbol{\Lambda},\boldsymbol{\Omega}}$$

$$\frac{\sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}}\Lambda_{u,v}\Omega_{k,(u,v)}t_k^{(u,v)}}{\sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}}\Lambda_{u,v}\Omega_{k,(u,v)}\Big(P_{2k-1}^{(u)} + P_{2k}^{(u)} + W_R^{(v)}\Big) + C}$$

$$\text{subject to} \quad (C.1) - (C.8); \qquad (24)$$

$$(C.9) \quad R_{2k-1,lb}^{(u,v)} \geq t_k^{(u,v)}, \qquad \forall\, k,(u,v);$$

$$(C.10) \quad R_{2k,lb}^{(u,v)} \geq t_k^{(u,v)}, \qquad \forall\, k,(u,v),$$

where $\textbf{t} = \{t_k^{(u,v)}\}$, $\forall k,(u,v)$. By relaxing the binary variables and introducing the change of variables $\bar{P}_{2k-1}^{(u)} = \log(P_{2k-1}^{(u)})$, $\bar{P}_{2k}^{(u)} = \log(P_{2k}^{(u)})$, and $\bar{W}_R^{(v)} = \log(W_R^{(v)})$, for

$k \in \{1, \cdots, K\}$, $u \in \{1, \cdots, N_{sc}\}$, and $v \in \{1, \cdots, N_{sc}\}$, the optimization problem (**OP2**) can be equivalently transformed as

$$(\textbf{OP3}) \max_{\bar{\mathbf{P}}, \bar{\mathbf{W}}_R, \mathbf{t}, \mathbf{\Lambda}, \mathbf{\Omega}}$$

$$\frac{\sum\limits_{k=1}^{K} \sum\limits_{u=1}^{N_{sc}} \sum\limits_{v=1}^{N_{sc}} \Lambda_{u,v} \Omega_{k,(u,v)} t_k^{(u,v)}}{\sum\limits_{k=1}^{K} \sum\limits_{u=1}^{N_{sc}} \sum\limits_{v=1}^{N_{sc}} \Lambda_{u,v} \Omega_{k,(u,v)} \left(e^{\bar{P}_{2k-1}^{(u)}} + e^{\bar{P}_{2k}^{(u)}} + e^{\bar{W}_R^{(v)}} \right) + C}$$

subject to

$$(C.1) \sum_{k=1}^{K} \sum_{u=1}^{N_{sc}} \sum_{v=1}^{N_{sc}} \Lambda_{u,v} \Omega_{k,(u,v)} \left(e^{\bar{P}_{2k-1}^{(u)}} + e^{\bar{P}_{2k}^{(u)}} + e^{\bar{W}_R^{(v)}} \right) \leqslant P_{max}$$

$$(C.2) \quad \log\left(\bar{\Upsilon}_{2k-1}^{(u,v)}\right) \geq \log\left(\Upsilon_{min,2k-1}^{(u,v)}\right), \, \forall\, k, (u,v);$$

$$(C.3) \quad \log\left(\bar{\Upsilon}_{2k}^{(u,v)}\right) \geq \log\left(\Upsilon_{min,2k}^{(u,v)}\right), \quad \forall\, k, (u,v); \quad (25)$$

$$(C.4) - (C.6);$$

$$(C.8) \quad e^{\bar{P}_{2k-1}^{(u)}} > 0, e^{\bar{P}_{2k}^{(u)}} > 0, e^{\bar{W}_R^{(v)}} > 0, \quad \forall\, k, v;$$

$$(C.9) \quad \bar{R}_{2k-1,lb}^{(u,v)} \geq t_k^{(u,v)}, \qquad\qquad \forall\, k, (u,v);$$

$$(C.10) \quad \bar{R}_{2k,lb}^{(u,v)} \geq t_k^{(u,v)}, \qquad\qquad \forall\, k, (u,v),$$

where $\bar{\Upsilon}_{2k-1}^{(u,v)}$ and $\bar{\Upsilon}_{2k}^{(u,v)}$ are respectively defined in (26) and (27), as shown on the top of the next page, and the corresponding lower bounded sum rates are given as

$$\bar{R}_{2k-1,lb}^{(u,v)} = \frac{1}{2}\left[\rho_{2k-1}^{(u,v)} \log_2\left(\bar{\Upsilon}_{2k-1}^{(u,v)}\right) + \beta_{2k-1}^{(u,v)}\right]$$

$$= \kappa \rho_{2k-1}^{(u,v)} \log\left(\bar{\Upsilon}_{2k-1}^{(u,v)}\right) + \frac{\beta_{2k-1}^{(u,v)}}{2}; \qquad (28)$$

$$\bar{R}_{2k,lb}^{(u,v)} = \frac{1}{2}\left[\rho_{2k}^{(u,v)} \log_2\left(\bar{\Upsilon}_{2k}^{(u,v)}\right) + \beta_{2k-1}^{(u,v)}\right]$$

$$= \kappa \rho_{2k}^{(u,v)} \log\left(\bar{\Upsilon}_{2k}^{(u,v)}\right) + \frac{\beta_{2k-1}^{(u,v)}}{2}, \qquad (29)$$

where $\kappa = \frac{1}{2\ln 2}$. $\log\left(\bar{\Upsilon}_{2k-1}^{(u,v)}\right)$ and $\log\left(\bar{\Upsilon}_{2k}^{(u,v)}\right)$ are explicitly written as (30) and (31) shown on the top of the next page.

The optimization problem (**OP3**) is still non-convex due to the fractional form of the objective function, which is a concave-over-convex. We introduce the following theorem to transform the fraction objective function into subtractive form, as follows:

Theorem 2: The optimal EE $\mathcal{L}^\star$ can be achieved, *if and only if* the optimal allocation policy $\left(\bar{\mathbf{P}}^\star, \bar{\mathbf{W}}_R^\star, \mathbf{t}^\star, \mathbf{\Lambda}^\star, \mathbf{\Omega}^\star\right)$ satisfies the following balance equation:

$$\sum_{k=1}^{K} \sum_{u=1}^{N_{sc}} \sum_{v=1}^{N_{sc}} \Lambda_{u,v}^\star \Omega_{k,(u,v)}^\star t_k^{(u,v)\star}$$

$$- \mathcal{L}^\star \left(\sum_{k=1}^{K} \sum_{u=1}^{N_{sc}} \sum_{v=1}^{N_{sc}} \Lambda_{u,v}^\star \Omega_{k,(u,v)}^\star \right.$$

$$\left. \times \left(e^{\bar{P}_{2k-1}^{(u)\star}} + e^{\bar{P}_{2k}^{(u)\star}} + e^{\bar{W}_R^{(v)\star}} \right) + C \right) = 0. \quad (32)$$

Proof: The proof is provided in Appendix A. ∎

Remark 2: The physical meaning of Theorem 2 is that for the fractional optimization problem (25) for given $\{\mathbf{\Lambda}, \mathbf{\Omega}\}$, there exists an equivalent objective function in subtractive form. Thus, the optimization problem (25) with given $\{\mathbf{\Lambda}, \mathbf{\Omega}\}$ can be solved using the Dinkelbach method, which is widely used in fractional programming [31].

Apply Dinkelbach method to transform the problem (**OP3**) into a subtractive form as follows:

$$(\textbf{OP4}) \max_{\bar{\mathbf{P}}, \bar{\mathbf{W}}_R, \mathbf{t}, \mathbf{\Lambda}, \mathbf{\Omega}} \sum_{k=1}^{K} \sum_{u=1}^{N_{sc}} \sum_{v=1}^{N_{sc}} \Lambda_{u,v} \Omega_{k,(u,v)} t_k^{(u,v)}$$

$$- \mathcal{L} \left(\sum_{k=1}^{K} \sum_{u=1}^{N_{sc}} \sum_{v=1}^{N_{sc}} \Lambda_{u,v} \Omega_{k,(u,v)} \right.$$

$$\left. \times \left(e^{\bar{P}_{2k-1}^{(u)}} + e^{\bar{P}_{2k}^{(u)}} + e^{\bar{W}_R^{(v)}} \right) + C \right)$$

$$\text{subject to} \quad (C.1) - (C.6) \,\&\, (C.8) - (C.10), \quad (33)$$

where $\mathcal{L}$ represents a positive penalty factor or price that is paid by the users for the resources being utilized and can be defined as follows:

$$\mathcal{L} = \frac{\sum\limits_{k=1}^{K} \sum\limits_{u=1}^{N_{sc}} \sum\limits_{v=1}^{N_{sc}} \Lambda_{u,v} \Omega_{k,(u,v)} t_k^{(u,v)}}{\sum\limits_{k=1}^{K} \sum\limits_{u=1}^{N_{sc}} \sum\limits_{v=1}^{N_{sc}} \Lambda_{u,v} \Omega_{k,(u,v)} \left(e^{\bar{P}_{2k-1}^{(u)}} + e^{\bar{P}_{2k}^{(u)}} + e^{\bar{W}_R^{(v)}} \right) + C}, \quad (34)$$

For extreme cases when $\mathcal{L} \to 0$ the problem reduces to sum rate maximization (SEM) problem whereas when $\mathcal{L} \to \infty$ no resource allocation policy is a suitable as the penalty is very high. The optimal penalty factor $\mathcal{L}^\star$ works as the optimal EE for the network.

Lemma 1: The objective function in (33) is concavified by the change of variables $\bar{P}_{2k-1}^{(u)} = \log(P_{2k-1}^{(u)})$, $\bar{P}_{2k}^{(u)} = \log(P_{2k}^{(u)})$, $\forall k, u$, and $\bar{W}_R^{(v)} = \log(W_R^{(v)})$, $\forall v$, for a given $\mathcal{L}$ and fixed subcarrier pairing $\mathbf{\Lambda}$ and subcarrier allocation $\mathbf{\Omega}$.

Proof: The objective function in (33) forms the summation of affine and concave terms (i.e., minus-exp functions) for a given $\mathcal{L}$ and fixed subcarrier pairing $\mathbf{\Lambda}$ and subcarrier allocation $\mathbf{\Omega}$. This implies that the optimization function in (**OP4**) is concavified by the change of variables and thus the optimization problem (**OP4**) is a convex problem and it can be solved by using any standard method [29]. ∎

IV. OPTIMAL EE RESOURCE ALLOCATION POLICY

A. Dual problem formulation

For given coefficients $\rho_i^{(u,v)}$, $\beta_i^{(u,v)}$, $\forall i, (u,v)$, and fixed subcarrier pairing $\mathbf{\Lambda}$, and subcarrier allocation $\mathbf{\Omega}$, the optimization problem in (33) is a convex optimization problem, which can be efficiently solved using standard convex optimization tools, e.g., CVX [29]. We derive an iterative algorithm for solving this optimization problem by applying the dual decomposition method. The main idea behind this algorithm is to find the optimal resource allocation policy that can maximize its lower bound for given coefficients $\rho_i^{(u,v)}$ and $\beta_i^{(u,v)}$, followed by an update of these two coefficients

$$\bar{\Upsilon}_{2k-1}^{(u,v)} = \frac{e^{\bar{P}_{2k}^{(u)}+\bar{W}_R^{(v)}} \left|h_{2k-1}^{(v)}h_{2k}^{(u)}\right|^2}{\sum\limits_{i=1,i\neq 2k-1,2k}^{2K} e^{\bar{P}_i^{(u)}+\bar{W}_R^{(v)}} \left|h_i^{(u)}h_{2k-1}^{(v)}\right|^2 + e^{\bar{W}_R^{(v)}} \left|h_{2k-1}^{(v)}\right|^2 \sigma_R^{(u)^2} + \sigma_{2k-1}^{(v)^2}\left(\sum_{i=1}^{2K} e^{\bar{P}_i^{(u)}} \left|h_i^{(u)}\right|^2 + \sigma_R^{(u)^2}\right)} \; ; \tag{26}$$

$$\bar{\Upsilon}_{2k}^{(u,v)} = \frac{e^{\bar{P}_{2k-1}^{(u)}+\bar{W}_R^{(v)}} \left|h_{2k}^{(v)}h_{2k-1}^{(u)}\right|^2}{\sum\limits_{i=1,i\neq 2k-1,2k}^{2K} e^{\bar{P}_i^{(u)}+\bar{W}_R^{(v)}} \left|h_i^{(u)}h_{2k}^{(v)}\right|^2 + e^{\bar{W}_R^{(v)}} \left|h_{2k}^{(v)}\right|^2 \sigma_R^{(u)^2} + \sigma_{2k}^{(v)^2}\left(\sum_{i=1}^{2K} e^{\bar{P}_i^{(u)}} \left|h_i^{(u)}\right|^2 + \sigma_R^{(u)^2}\right)} \; , \tag{27}$$

$$\log\left(\bar{\Upsilon}_{2k-1}^{(u,v)}\right) = \bar{W}_R^{(v)} + \bar{P}_{2k}^{(u)} + \log\left(\left|h_{2k-1}^{(v)}h_{2k}^{(u)}\right|^2\right) - \log\left(\sum\limits_{i=1,i\neq 2k-1,2k}^{2K} e^{\bar{P}_i^{(u)}+\bar{W}_R^{(v)}} \left|h_i^{(u)}h_{2k-1}^{(v)}\right|^2\right.$$
$$\left. + e^{\bar{W}_R^{(v)}} \left|h_{2k-1}^{(v)}\right|^2 \sigma_R^{(u)^2} + \sigma_{2k-1}^{(v)^2}\left(\sum_{i=1}^{2K} e^{\bar{P}_i^{(u)}} \left|h_i^{(u)}\right|^2 + \sigma_R^{(u)^2}\right)\right) \; ; \tag{30}$$

$$\log\left(\bar{\Upsilon}_{2k}^{(u,v)}\right) = \bar{W}_R^{(v)} + \bar{P}_{2k-1}^{(u)} + \log\left(\left|h_{2k}^{(v)}h_{2k-1}^{(u)}\right|^2\right) - \log\left(\sum\limits_{i=1,i\neq 2k-1,2k}^{2K} e^{\bar{P}_i^{(u)}+\bar{W}_R^{(v)}} \left|h_i^{(u)}h_{2k}^{(v)}\right|^2\right.$$
$$\left. + e^{\bar{W}_R^{(v)}} \left|h_{2k}^{(v)}\right|^2 \sigma_R^{(u)^2} + \sigma_{2k}^{(v)^2}\left(\sum_{i=1}^{2K} e^{\bar{P}_i^{(u)}} \left|h_i^{(u)}\right|^2 + \sigma_R^{(u)^2}\right)\right) \; , \tag{31}$$

that guarantees a monotonic increase in the lower bound performance.

Thus, the dual problem associated with the primal problem (33) can be written as

$$\min_{\substack{\lambda,\boldsymbol{\mu}_{odd},\boldsymbol{\mu}_{even} \\ \boldsymbol{\Theta}_{odd},\boldsymbol{\Theta}_{even}}} \mathcal{X}\left(\lambda,\boldsymbol{\mu}_{odd},\boldsymbol{\mu}_{even},\boldsymbol{\Theta}_{odd},\boldsymbol{\Theta}_{even}\right)$$
$$\text{subject to } \lambda \geq 0, \boldsymbol{\mu}_{odd} \geq 0,$$
$$\boldsymbol{\mu}_{even} \geq 0, \boldsymbol{\Theta}_{odd}, \boldsymbol{\Theta}_{even} \geq 0, \tag{35}$$

where $\mathcal{X}\left(\lambda,\boldsymbol{\mu}_{odd},\boldsymbol{\mu}_{even},\boldsymbol{\Theta}_{odd},\boldsymbol{\Theta}_{even}\right)$ presents the dual function expressed in (36), as shown on the top of the next page, where $\mathcal{L}\left(\bar{\mathbf{P}},\bar{\mathbf{W}}_R,\mathbf{t},\boldsymbol{\Lambda},\boldsymbol{\Omega},\lambda,\boldsymbol{\mu}_{odd},\boldsymbol{\mu}_{even},\boldsymbol{\Theta}_{odd},\boldsymbol{\Theta}_{even}\right)$ is given in (37) shown on the next page, where λ is the Lagrangian multiplier or the dual variable corresponding to transmit power constraint $(C.1)$, $\boldsymbol{\mu}_{odd} = \{\mu_{2k-1}^{(u,v)}\}$, $\forall k, (u,v)$, and $\boldsymbol{\mu}_{even} = \{\mu_{2k}^{(u,v)}\}$, $\forall k, (u,v)$, are the Lagrangian multiplier vectors associated with QoS constraints $(C.2)$ and $(C.3)$, while $\boldsymbol{\Theta}_{odd} = \{\Theta_{2k-1}^{(u,v)}\}$, $\forall k, (u,v)$ and $\boldsymbol{\Theta}_{even} = \{\Theta_{2k}^{(u,v)}\}$, $\forall k, (u,v)$, are the Lagrangian multiplier vectors for constraints $(C.9)$ and $(C.10)$, respectively.

In the following subsections, we solve the dual problem (35) using dual decomposition approach [32] which alternates between a *subproblem (inner problem)*, updating the resource allocation variables $\bar{\mathbf{P}},\bar{\mathbf{W}}_R,\mathbf{t}, \boldsymbol{\Lambda}$ and $\boldsymbol{\Omega}$ by fixing the Lagrangian multipliers, and a *master problem (outer problem)*, updating the Lagrangian multipliers for the obtained solution of the inner problem[1]. The dual decomposition approach is outlined as follows.

---

[1] The optimal solution obtained for the dual function in (35) is equal to that of (33), i.e., a zero duality gap between the optimal and dual solutions [30].

B. Subproblem Solution

The optimization problem in (33) is in a standard concave form for given subcarrier pairing and subcarrier allocation, hence by using standard optimization techniques and KKT conditions [29], which are first-order imperative and sufficient conditions for optimality, the optimal solution can be found. Thus, to obtain the optimal power allocation for $(2k-1)^{th}$ and $(2k)^{th}$ users, and the relay node, we take the partial derivative of (37) with respect to $\bar{P}_{2k-1}^{(u)}$, $\bar{P}_{2k}^{(u)}$ and $\bar{W}_R^{(v)}$ and equate the results to zero, thus the power allocation at the $(m+1)^{th}$ iteration is updated as follows:

$$\bar{P}_{2k-1}^{(u)}(m+1) = \ln\left[\frac{\mu_{2k}^{(u,v)} + \kappa\Theta_{2k}^{(u,v)}\rho_{2k}^{(u,v)}}{\sum\limits_{v=1}^{N_{sc}} \Lambda_{u,v}\Omega_{k,(u,v)}\left(\pounds + \lambda\right) + \Pi_1 + \Pi_2 + \Pi_3 + \Pi_4}\right]^+ ; \tag{38}$$

$$\bar{P}_{2k}^{(u)}(m+1) = \ln\left[\frac{\mu_{2k-1}^{(u,v)} + \kappa\Theta_{2k-1}^{(u,v)}\rho_{2k-1}^{(u,v)}}{\sum\limits_{v=1}^{N_{sc}} \Lambda_{u,v}\Omega_{k,(u,v)}\left(\pounds + \lambda\right) + \bar{\Pi}_1 + \bar{\Pi}_2 + \bar{\Pi}_3 + \bar{\Pi}_4}\right]^+ ; \tag{39}$$

$$\bar{W}_R^{(v)}(m+1) = \ln\left[\sum\limits_{k=1}^{K} \frac{\left(\mu_{2k-1}^{(u,v)} + \mu_{2k}^{(u,v)} + \kappa\Theta_{2k-1}^{(u,v)}\rho_{2k-1}^{(u,v)} + \kappa\Theta_{2k}^{(u,v)}\rho_{2k}^{(u,v)}\right)}{\sum\limits_{u=1}^{N_{sc}} \Lambda_{u,v}\Omega_{k,(u,v)}\left(\pounds + \lambda\right) + \tilde{\Pi}_1 + \tilde{\Pi}_2}\right]^+ , \tag{40}$$

where the relevant terms are explicitly given by (41)–(50), as shown on the next page, where $[x]^+ = \max\left(0, x\right)$. By using a

$$\mathcal{X}\left(\lambda, \boldsymbol{\mu}_{odd}, \boldsymbol{\mu}_{even}, \boldsymbol{\Theta}_{odd}, \boldsymbol{\Theta}_{even}\right) =$$

$$\max_{\bar{\mathbf{P}}, \bar{\mathbf{W}}_R, \mathbf{t}, \boldsymbol{\Lambda}, \boldsymbol{\Omega}} \mathcal{L}\left(\bar{\mathbf{P}}, \bar{\mathbf{W}}_R, \mathbf{t}, \boldsymbol{\Lambda}, \boldsymbol{\Omega}, \lambda, \boldsymbol{\mu}_{odd}, \boldsymbol{\mu}_{even}, \boldsymbol{\Theta}_{odd}, \boldsymbol{\Theta}_{even}\right) \left| \sum_{u=1}^{N_{sc}} \Lambda_{u,v} = 1, \forall v, \sum_{v=1}^{N_{sc}} \Lambda_{u,v} = 1, \forall u, \right.$$

$$\sum_{k=1}^{K} \Omega_{k,(u,v)} = 1, \forall (u,v); \quad (36)$$

$$\mathcal{L}\left(\bar{\mathbf{P}}, \bar{\mathbf{W}}_R, \mathbf{t}, \boldsymbol{\Lambda}, \boldsymbol{\Omega}, \lambda, \boldsymbol{\mu}_{odd}, \boldsymbol{\mu}_{even}, \boldsymbol{\Theta}_{odd}, \boldsymbol{\Theta}_{even}\right) =$$

$$\sum_{k=1}^{K} \sum_{u=1}^{N_{sc}} \sum_{v=1}^{N_{sc}} \Lambda_{u,v} \Omega_{k,(u,v)} t_k^{(u,v)} - \pounds\left(\sum_{k=1}^{K} \sum_{u=1}^{N_{sc}} \sum_{v=1}^{N_{sc}} \Lambda_{u,v} \Omega_{k,(u,v)}\left(e^{\bar{P}_{2k-1}^{(u)}} + e^{\bar{P}_{2k}^{(u)}} + e^{\bar{W}_R^{(v)}}\right) + C\right)$$

$$- \lambda\left(\sum_{k=1}^{K} \sum_{u=1}^{N_{sc}} \sum_{v=1}^{N_{sc}} \Lambda_{u,v} \Omega_{k,(u,v)}\left(e^{\bar{P}_{2k-1}^{(u)}} + e^{\bar{P}_{2k}^{(u)}} + e^{\bar{W}_R^{(v)}}\right) - P_{max}\right) + \sum_{k=1}^{K} \sum_{u=1}^{N_{sc}} \sum_{v=1}^{N_{sc}} \mu_{2k-1}^{(u,v)}$$

$$\left(\log\left(\bar{\Upsilon}_{2k-1}^{(u,v)}\right) - \log\left(\Upsilon_{min,2k-1}^{(u,v)}\right)\right) + \sum_{k=1}^{K} \sum_{u=1}^{N_{sc}} \sum_{v=1}^{N_{sc}} \mu_{2k}^{(u,v)}\left(\log\left(\bar{\Upsilon}_{2k}^{(u,v)}\right) - \log\left(\Upsilon_{min,2k}^{(u,v)}\right)\right)$$

$$+ \sum_{k=1}^{K} \sum_{u=1}^{N_{sc}} \sum_{v=1}^{N_{sc}} \Theta_{2k-1}^{(u,v)}\left(\bar{R}_{2k-1,lb}^{(u,v)} - t_k^{(u,v)}\right) + \sum_{k=1}^{K} \sum_{u=1}^{N_{sc}} \sum_{v=1}^{N_{sc}} \Theta_{2k}^{(u,v)}\left(\bar{R}_{2k,lb}^{(u,v)} - t_k^{(u,v)}\right), \quad (37)$$

sub-gradient method [30], the optimal value of $t_k^{(u,v)^\star}$ can be found as

$$t_k^{(u,v)}(m+1) =$$
$$t_k^{(u,v)}(m) - \epsilon(m)\left(\Lambda_{u,v}\Omega_{k,(u,v)} - \Theta_k^{(u,v)}\right), \quad (51)$$

where $\epsilon(m)$ is a positive step size. $\bar{P}_{2k-1}^{(u)}$, $\bar{P}_{2k}^{(u)}$ and $\bar{W}_R^{(v)}$ are appear in the denominator of each SINR terms, thus it forms the basis for our iterative resource allocation solution of the Lagrange maximization (37) whereby the right-hand side can be used to update the power of $(2k-1)^{th}$ and $(2k)^{th}$ users on the subcarrier u, and the relay power on v^{th} subcarrier. The power updates in (38)–(40) reveal some interesting interpretations: 1) the power update of the user and the relay node not only depends on λ but also on the penalty factor $\pounds$; and 2) the transmit power is allocated in such a way so that it takes other users into account on a subcarrier-by-subcarrier basis, rather than a selfish allocation.

In general, when the uth subcarrier in the MA phase is allocated to the kth user pair, the transmit power assigned on the uth subcarrier by other user pairs practically approaches to zero. Further, a similar explanation can be drawn for the BC phase. Therefore, $\{\Pi_2, \Pi_4\} = 0$, $\{\bar{\Pi}_2, \bar{\Pi}_4\} = 0$. For asymptotically high SNR at the end user, i.e., $\sigma_{2k-1}^{(v)} = \sigma_{2k}^{(v)} = 0, \forall k, v$, the update equations in (38)-(40) can be further simplified as

$$\bar{P}_{2k-1}^{(u)}(m+1) = \ln\left[\frac{\mu_{2k}^{(u,v)} + \kappa\Theta_{2k}^{(u,v)}\rho_{2k}^{(u,v)}}{\pounds + \lambda}\right]^+; \quad (52)$$

$$\bar{P}_{2k}^{(u)}(m+1) = \ln\left[\frac{\mu_{2k-1}^{(u,v)} + \kappa\Theta_{2k-1}^{(u,v)}\rho_{2k-1}^{(u,v)}}{\pounds + \lambda}\right]^+; \quad (53)$$

$$\bar{W}_R^{(v)}(m+1) =$$
$$\ln\left[\frac{\sum_{k=1}^{K}\left(\mu_{2k-1}^{(u,v)} + \mu_{2k}^{(u,v)} + \kappa\Theta_{2k-1}^{(u,v)}\rho_{2k-1}^{(u,v)} + \kappa\Theta_{2k}^{(u,v)}\rho_{2k}^{(u,v)}\right)}{\pounds + \lambda}\right]^+, \quad (54)$$

From (52)-(54), it can be noticed that the inverse of the Lagrangian multiplier λ plus the network price $\pounds$ can be regarded as a water-filling level which has to be chosen to meet the total transmit power constraint.

To derive the optimal subcarrier pairing $\boldsymbol{\Lambda}$, and allocation $\boldsymbol{\Omega}$, we substitute (38)–(40) and (51) into (36), we obtain the following optimization problem:

$$\max_{\boldsymbol{\Lambda}, \boldsymbol{\Omega}} \sum_{k=1}^{K} \sum_{u=1}^{N_{sc}} \sum_{v=1}^{N_{sc}} \Lambda_{u,v}\Omega_{k,(u,v)} Y_{k,(u,v)}$$
$$+ Z\left(\lambda, \boldsymbol{\mu}_{odd}, \boldsymbol{\mu}_{even}, \boldsymbol{\Theta}_{odd}, \boldsymbol{\Theta}_{even}\right)$$
$$\text{subject to} \quad (C.4) - (C.7), \quad (55)$$

where $Y_{k,(u,v)}$ and $Z\left(\lambda, \boldsymbol{\mu}_{odd}, \boldsymbol{\mu}_{even}, \boldsymbol{\Theta}_{odd}, \boldsymbol{\Theta}_{even}\right)$ are respectively given by

$$Y_{k,(u,v)} =$$
$$\left(t_k^{(u,v)^\star} - (\pounds + \lambda)\left(e^{\bar{P}_{2k-1}^{(u)^\star}} + e^{\bar{P}_{2k}^{(u)^\star}} + e^{\bar{W}_R^{(v)^\star}}\right)\right); \quad (56)$$

$$\Pi_1 = \frac{\left(\mu_{2k-1}^{(u,v)} + \kappa\Theta_{2k-1}^{(u,v)}\rho_{2k-1}^{(u,v)}\right)\sigma_{2k-1}^{(v)^2}}{e^{\bar{P}_{2k}^{(u)}+\bar{W}_R^{(v)}}\left|h_{2k}^{(u)}\right|^2}\bar{\gamma}_{2k-1}^{(u,v)}; \tag{41}$$

$$\Pi_2 = \sum_{\substack{m=1 \\ 2m-1\neq 2k-1}}^{K} \frac{\left(\mu_{2m-1}^{(u,v)} + \kappa\Theta_{2m-1}^{(u,v)}\rho_{2k-1}^{(u,v)}\right)\left(e^{\bar{W}_R^{(v)}}\left|h_{2k-1}^{(u)}h_{2m-1}^{(v)}\right|^2 + \sigma_{2m-1}^{(v)^2}\left|h_{2k-1}^{(u)}\right|^2\right)}{e^{\bar{P}_{2m}^{(u)}+\bar{W}_R^{(v)}}\left|h_{2m-1}^{(v)}h_{2m}^{(u)}\right|^2}\bar{\gamma}_{2m-1}^{(u,v)}; \tag{42}$$

$$\Pi_3 = \frac{\left(\mu_{2k}^{(u,v)} + \kappa\Theta_{2k}^{(u,v)}\rho_{2k}^{(u,v)}\right)\sigma_{2k}^{(v)^2}}{e^{\bar{P}_{2k-1}^{(u)}+\bar{W}_R^{(v)}}\left|h_{2k}^{(v)}\right|^2}\bar{\gamma}_{2k}^{(u,v)}; \tag{43}$$

$$\Pi_4 = \sum_{m=1,2m\neq 2k}^{K} \frac{\left(\mu_{2m}^{(u,v)} + \kappa\Theta_{2k}^{(u,v)}\rho_{2k}^{(u,v)}\right)\left(e^{\bar{W}_R^{(v)}}\left|h_{2k-1}^{(u)}h_{2m}^{(v)}\right|^2 + \sigma_{2m}^{(v)^2}\left|h_{2k-1}^{(u)}\right|^2\right)}{e^{\bar{P}_{2m-1}^{(u)}+\bar{W}_R^{(v)}}\left|h_{2m}^{(v)}h_{2m-1}^{(u)}\right|^2}\bar{\gamma}_{2m}^{(u,v)}; \tag{44}$$

$$\bar{\Pi}_1 = \frac{\left(\mu_{2k}^{(u,v)} + \kappa\Theta_{2k}^{(u,v)}\rho_{2k}^{(u,v)}\right)\sigma_{2k}^{(v)^2}}{e^{\bar{P}_{2k-1}^{(u)}+\bar{W}_R^{(v)}}\left|h_{2k-1}^{(u)}\right|^2}\bar{\gamma}_{2k}^{(u,v)}; \tag{45}$$

$$\bar{\Pi}_2 = \sum_{m=1,2m\neq 2k}^{K} \frac{\left(\mu_{2m}^{(u,v)} + \kappa\Theta_{2m}^{(u,v)}\rho_{2k}^{(u,v)}\right)\left(e^{\bar{W}_R^{(v)}}\left|h_{2k}^{(u)}h_{2m}^{(v)}\right|^2 + \sigma_{2m}^{(v)^2}\left|h_{2k}^{(u)}\right|^2\right)}{e^{\bar{P}_{2m-1}^{(u)}+\bar{W}_R^{(v)}}\left|h_{2m}^{(v)}h_{2m-1}^{(u)}\right|^2}\bar{\gamma}_{2m}^{(u,v)}; \tag{46}$$

$$\bar{\Pi}_3 = \frac{\left(\mu_{2k-1}^{(u,v)} + \kappa\Theta_{2k-1}^{(u,v)}\rho_{2k-1}^{(u,v)}\right)\sigma_{2k-1}^{(v)^2}}{e^{\bar{P}_{2k}^{(u)}+\bar{W}_R^{(v)}}\left|h_{2k-1}^{(v)}\right|^2}\bar{\gamma}_{2k-1}^{(u,v)}; \tag{47}$$

$$\bar{\Pi}_4 = \sum_{\substack{m=1 \\ 2m-1\neq 2k-1}}^{K} \frac{\left(\mu_{2m-1}^{(u,v)} + \kappa\Theta_{2k-1}^{(u,v)}\rho_{2k-1}^{(u,v)}\right)\left(e^{\bar{W}_R^{(v)}}\left|h_{2k}^{(u)}h_{2m-1}^{(v)}\right|^2 + \sigma_{2m-1}^{(v)^2}\left|h_{2k}^{(u)}\right|^2\right)}{e^{\bar{P}_{2m}^{(u)}+\bar{W}_R^{(v)}}\left|h_{2m-1}^{(v)}h_{2m}^{(u)}\right|^2}\bar{\gamma}_{2m-1}^{(u,v)}; \tag{48}$$

$$\tilde{\Pi}_1 = \sum_{k=1}^{K} \frac{\left(\mu_{2k-1}^{(u,v)} + \kappa\Theta_{2k-1}^{(u,v)}\rho_{2k-1}^{(u,v)}\right)\left(\sum_{i=1,i\neq 2k-1,2k}^{2K} e^{\bar{P}_i^{(u)}}\left|h_i^{(v)}\right|^2 + \sigma_R^{(v)^2}\right)}{e^{\bar{P}_{2k}^{(u)}}\left|h_{2k}^{(u)}\right|^2}\bar{\gamma}_{2k-1}^{(u,v)}; \tag{49}$$

$$\tilde{\Pi}_2 = \sum_{k=1}^{K} \frac{\left(\mu_{2k}^{(u,v)} + \kappa\Theta_{2k}^{(u,v)}\rho_{2k}^{(u,v)}\right)\left(\sum_{i=1,i\neq 2k-1,2k}^{2K} e^{\bar{P}_i^{(u)}}\left|h_i^{(v)}\right|^2 + \sigma_R^{(v)^2}\right)}{e^{\bar{P}_{2k-1}^{(u)}}\left|h_{2k-1}^{(u)}\right|^2}\bar{\gamma}_{2k}^{(u,v)}, \tag{50}$$

$$\begin{aligned}
Z\left(\lambda, \boldsymbol{\mu}_{odd}, \boldsymbol{\mu}_{even}, \boldsymbol{\Theta}_{odd}, \boldsymbol{\Theta}_{even}\right) &= -\pounds C + \lambda P_{max} \\
&+ \sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}} \mu_{2k-1}^{(v)}\left(\log\left(\bar{\Upsilon}_{2k-1}^{(u,v)\star}\right) - \log\left(\Upsilon_{min,2k-1}^{(u,v)}\right)\right) \\
&+ \sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}} \mu_{2k}^{(u,v)}\left(\log\left(\bar{\Upsilon}_{2k}^{(u,v)\star}\right) - \log\left(\Upsilon_{min,2k}^{(u,v)}\right)\right) \\
&+ \sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}} \Theta_{2k-1}^{(u,v)}\left(\bar{R}_{2k-1,lb}^{(u,v)\star} - t_k^{(u,v)\star}\right) \\
&+ \sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}} \Theta_{2k}^{(u,v)}\left(\bar{R}_{2k,lb}^{(u,v)\star} - t_k^{(u,v)\star}\right),
\end{aligned} \tag{57}$$

The first term in (56) denotes the achievable minimum sum rate of the kth user pair for the allocated subcarrier pairing

(u, v), whereas the second term works as the penalty for the resource dissipation. Furthermore, it is observed that $Z\left(\lambda, \boldsymbol{\mu}_{odd}, \boldsymbol{\mu}_{even}, \boldsymbol{\Theta}_{odd}, \boldsymbol{\Theta}_{even}\right)$ derived in (57) remains constant for any subcarrier pairing $\boldsymbol{\Lambda}$ and allocation $\boldsymbol{\Omega}$.

To determine the optimal subcarrier allocation $\boldsymbol{\Omega}$ for given subcarrier pairing $\boldsymbol{\Lambda}$ and the optimal power allocation $\left(\bar{\mathbf{P}}^\star, \bar{\mathbf{W}}_R^\star\right)$ and a fixed $Z\left(\lambda, \boldsymbol{\mu}_{odd}, \boldsymbol{\mu}_{even}, \boldsymbol{\Theta}_{odd}, \boldsymbol{\Theta}_{even}\right)$, we solve the following optimization problem:

$$\max_{\boldsymbol{\Omega}} \quad \sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}} \Lambda_{u,v}\Omega_{k,(u,v)}Y_{k,(u,v)}$$
$$\text{subject to } (C.6), \tag{58}$$

Straightforwardly the optimal subcarrier allocation $\boldsymbol{\Omega}^\star$ is the kth user pair that maximizes $Y_{k,(u,v)}$ for given $(u,v)^{th}$

subcarrier pair and the optimal power allocation. Thus, the optimal subcarrier allocation $\mathbf{\Omega}^\star$ can be obtained as

$$\Omega^\star_{k,(u,v)} = \begin{cases} 1, & \text{for } k = \arg\max_k Y_{k,(u,v)}, \\ 0, & \text{otherwise}, \end{cases} \quad (59)$$

Finally, to find the optimal subcarrier pairing $\mathbf{\Lambda}$ for the optimal power allocation $\left(\bar{\mathbf{P}}^\star, \bar{\mathbf{W}}^\star_R\right)$ and the optimal subcarrier allocation $\mathbf{\Omega}^\star$ given in (59) and a fixed $Z\left(\lambda, \boldsymbol{\mu}_{odd}, \boldsymbol{\mu}_{even}, \boldsymbol{\Theta}_{odd}, \boldsymbol{\Theta}_{even}\right)$, we rewrite the optimization problem (55) as follows:

$$\max_{\mathbf{\Lambda}} \quad \sum_{k=1}^{K} \sum_{u=1}^{N_{sc}} \sum_{v=1}^{N_{sc}} \Lambda_{u,v} \Omega^\star_{k,(u,v)} Y_{k^\star,(u,v)}$$

$$\text{subject to} \quad (C.4) \,\&\, (C.5), \quad (60)$$

where $Y_{k^\star,(u,v)} = \max_k Y_{k,(u,v)} \,\forall\, k,(u,v)$. Let $\mathbf{Y}$ be a $N_{sc} \times N_{sc}$ matrix such that

$$\mathbf{Y} = \begin{bmatrix} Y_{k^\star,(1,1)} & \cdots & Y_{k^\star,(1,N_{sc})} \\ \vdots & \ddots & \vdots \\ Y_{k^\star,(N_{sc},1)} & \cdots & Y_{k^\star,(N_{sc},N_{sc})} \end{bmatrix}, \quad (61)$$

Notice that the matrix $\mathbf{Y}$ can be treated as a profit matrix where rows and columns represent different operators and machines, respectively, and each element denotes the profit gain by operating a particular machine by a particular operator. Thus, the maximizing the total profit gained by selecting the best policy where each operator (u) operates only on one machine (v) is equivalent to the solving the problem (60). Furthermore, this optimization problem can be solved efficiently by using the standard assignment algorithms such as Hungarian method [33].

C. Master Problem Solution: Updating the dual variables

Since the dual problem in (35) is differentiable, the gradient-descent method [30] can be used to update the dual variables λ, $\mu^{(u,v)}_{2k-1}$, $\mu^{(u,v)}_{2k}$, $\Theta^{(u,v)}_{2k-1}$, $\Theta^{(u,v)}_{2k}$, $\forall k,(u,v)$, as shown in (62)–(66) on the top of the next page, where $\epsilon_a(m)$, $a \in \{1, \cdots, 5\}$, are sufficiently small step sizes associated with calculating the Lagrangian multipliers and m is the iteration index. The updated Lagrange multipliers in (62)-(66) are used for updating the power allocation policy. We repeat this process until convergence. Next, we provide a theorem to describe the update procedure of the network penalty factor $\mathscr{L}$ as well a theorem regarding convergence of the network penalty as follows.

Theorem 3: If $\left(\bar{\mathbf{P}}^\star(l), \bar{\mathbf{W}}^\star_R(l), \mathbf{t}^\star(l), \mathbf{\Lambda}^\star(l), \mathbf{\Omega}^\star(l)\right)$ is the optimal solution of the problem (**OP4**) with respect to $\mathscr{L}(l)$ at the lth iteration and if we update $\mathscr{L}(l)$ as

$$\mathscr{L}(l+1) = \frac{\sum_{k=1}^{K} \sum_{u=1}^{N_{sc}} \sum_{v=1}^{N_{sc}} \Lambda^\star_{u,v}(l) \Omega^\star_{k,(u,v)}(l) t^{(u,v)^\star}_k(l)}{P_T\left(\bar{\mathbf{P}}^\star(l), \bar{\mathbf{W}}^\star_R(l), \mathbf{\Lambda}^\star(l), \mathbf{\Omega}^\star(l)\right)} \quad (67)$$

then $\mathscr{L}(l)$ increases monotonically with each iteration, l.

Proof: The detailed proof is relegated in Appendix B. ∎

Theorem 4: The optimal penalty factor $\mathscr{L}^\star$ is obtained when the sequence $\{\mathscr{L}(l)\}$ has converged and $\mathscr{L}^\star = \lim_{l \to \infty} \mathscr{L}(l)$ satisfies the balance equation in (32).

Proof: The proof is provided in Appendix C. ∎

Remark 3: We update the network penalty/price $\mathscr{L}$ iteratively for obtained $\{\bar{\mathbf{P}}^\star, \bar{\mathbf{W}}^\star_R, \mathbf{t}^\star, \mathbf{\Lambda}^\star, \mathbf{\Omega}^\star\}$ in the last iteration. Theorem 3 and Theorem 4 ensure that $\mathscr{L}$ increases monotonically and converges within a finite iteration, respectively.

We first initialize the maximum number of iteration for the outer and inner loop as I_{max_1} and I_{max_2} with the iteration counter $l = 0$ and $m = 0$, respectively, along with the network penalty factor $\mathscr{L}(l) = 0.001$. Then, we initialize the step sizes $\epsilon_a(m)$, $a \in \{1, \cdots, 5\}$, followed by the coefficients $\left(\rho^{(u,v)}_{2k-1}(0), \beta^{(u,v)}_{2k-1}(0)\right) = (1,0)$ and $\left(\rho^{(u,v)}_{2k}(0), \beta^{(u,v)}_{2k}(0)\right) = (1,0)$. From the sub-gradient method [30], the dual variables $\lambda(0)$, $\mu^{(u,v)}_{2k-1}(0)$, $\mu^{(u,v)}_{2k}(0)$, $\Theta^{(u,v)}_{2k-1}(0)$, $\Theta^{(u,v)}_{2k}(0)$, $\forall k,(u,v)$, are initialized for finding the resource allocation policy $\left(\bar{\mathbf{P}}, \bar{\mathbf{W}}_R, \mathbf{t}, \mathbf{\Lambda}, \mathbf{\Omega}\right)$ using (38)–(40), (51), (59) and (60), respectively. Then with the obtained $\left(\bar{\mathbf{P}}, \bar{\mathbf{W}}_R, \mathbf{t}, \mathbf{\Lambda}, \mathbf{\Omega}\right)$, the dual variables at $(m+1)^{th}$ iteration are updated using (62)–(66). The coefficients $\left(\rho^{(u,v)}_{2k-1}, \beta^{(u,v)}_{2k-1}\right)$ and $\left(\rho^{(u,v)}_{2k}, \beta^{(u,v)}_{2k}\right)$ are updated after obtaining the optimal resource allocation $\left(\bar{\mathbf{P}}^\star, \bar{\mathbf{W}}^\star_R, \mathbf{t}^\star, \mathbf{\Lambda}^\star, \mathbf{\Omega}^\star\right)$. The above procedure is repeated until $\left(\rho^{(u,v)}_{2k-1}, \beta^{(u,v)}_{2k-1}\right)$ and $\left(\rho^{(u,v)}_{2k}, \beta^{(u,v)}_{2k}\right)$ have converged or the iteration counter m reaches to maximum limit I_{max_2}. In the next step, we update the network price $\mathscr{L}(l+1)$ using (67) and increase the iteration counter by one. We continue this procedure until the convergence or $l \leq I_{max_1}$. The iterative EE maximization (EEM) algorithm is briefly summarized in Algorithm 1.

V. SUBOPTIMAL RESOURCE ALLOCATION ALGORITHM

The computational complexity of the EEM algorithm proposed in Section IV becomes humongous for a large value of N_{sc} (discussed in more detail in the next section). Thus, we propose a low-complexity suboptimal algorithm, and the stepwise procedure of the suboptimal algorithm is described as follows:

Step 1: Optimal Subcarrier Allocation for Given Power Allocation: In the first step, the available transmit power is equally distributed among all the users over all the subcarriers as

$$P^{(u)}_{2k-1} = P^{(u)}_{2k} = W^{(v)}_R = \frac{P_{max}}{(2K+1) \times N_{sc}}, \quad \forall k,u,v; \quad (68)$$

Next, we compute SINRs for the $(2k-1)^{th}$ and $(2k)^{th}$ users on $(u,v)^{th}$ subcarrier pair. Define $K \times (N_{sc} \times N_{sc})$ matrix according to SINRs. Then, we can select the kth user pair in the following manner:

$$\Omega^\star_{k,(u,v)} = \begin{cases} 1, & \text{for} \quad k = \arg\max_k \text{SINR}_k(u,v); \\ 0, & \text{otherwise}, \end{cases} \quad (69)$$

Step 2: Optimal Subcarrier Pairing for Given Subcarrier Allocation: In this step, the N_{sc} subcarriers of the MA phase and BC phase are arranged in ascending order and matched

$$\lambda(m+1) = \left[\lambda(m) - \epsilon_1(m) \cdot \left(P_{max} - \sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}} \Lambda_{u,v}\Omega_{k,(u,v)}\left(P_{2k-1}^{(u)} + P_{2k}^{(u)} + W_R^{(v)}\right)\right)\right]^+ ; \tag{62}$$

$$\mu_{2k-1}^{(u,v)}(m+1) = \left[\mu_{2k-1}^{(u,v)}(m) - \epsilon_2(m) \cdot \left(\log\left(\bar{\Upsilon}_{2k-1}^{(u,v)}\right) - \left(\log\Upsilon_{min,2k-1}^{(u,v)}\right)\right)\right]^+ , \quad \forall k ; \tag{63}$$

$$\mu_{2k}^{(u,v)}(m+1) = \left[\mu_{2k}^{(u,v)}(m) - \epsilon_3(m) \cdot \left(\log\left(\bar{\Upsilon}_{2k}^{(u,v)}\right) - \log\left(\Upsilon_{min,2k}^{(u,v)}\right)\right)\right]^+ , \quad \forall k ; \tag{64}$$

$$\Theta_{2k-1}^{(u,v)}(m+1) = \left[\Theta_{2k-1}^{(u,v)}(m) - \epsilon_4(m) \cdot \left(\bar{R}_{2k-1,lb}^{(u,v)} - t_k^{(u,v)}\right)\right]^+ , \quad \forall k ; \tag{65}$$

$$\Theta_{2k}^{(u,v)}(m+1) = \left[\Theta_{2k}^{(u,v)}(m) - \epsilon_5(m) \cdot \left(\bar{R}_{2k,lb}^{(u,v)} - t_k^{(u,v)}\right)\right]^+ , \quad \forall k , \tag{66}$$

Algorithm 1 Iterative EEM Algorithm

1: Set the maximum number of iterations I_{max_1} ;
2: Initialize the iteration counter $l = 0$ and network penalty $\pounds(l) = 0.001$;
3: **repeat** (Outer Loop)
4:      Set the maximum number of iterations I_{max_2} ;
5:      Initialize the iteration counter $m = 0$ and the step sizes $\epsilon_a(m)$;
6:      Initialize $\left(\rho_{2k-1}^{(u,v)}, \beta_{2k-1}^{(u,v)}\right)$ and $\left(\rho_{2k}^{(u,v)}, \beta_{2k}^{(u,v)}\right)$ by $(1,0), \forall k, (u,v)$;
7:      Initialize $\lambda(m), \mu_{2k-1}^{(u,v)}(m), \mu_{2k}^{(u,v)}(m), \Theta_{2k-1}^{(u,v)}(m),$ $\Theta_{2k}^{(u,v)}(m), \forall k, (u,v)$;
8:      Initialize $\bar{\mathbf{P}}(m), \bar{\mathbf{W}}_R(m), \mathbf{t}(m), \mathbf{\Lambda}(m),$ and $\mathbf{\Omega}(m)$;
9:      **repeat** (Inner Loop)
10:         **repeat** (Solving problem (**OP4**))
11:            Update $\bar{\mathbf{P}}, \bar{\mathbf{W}}_R$ and $\bar{\mathbf{t}}$ using (38)–(40) and (51), respectively ;
12:            Update $\mathbf{\Omega}$ and $\mathbf{\Lambda}$ using (59) and (60) ;
13:            Update $\lambda, \mu_{2k-1}^{(u,v)}, \mu_{2k}^{(u,v)}, \Theta_{2k-1}^{(u,v)}$ and $\Theta_{2k}^{(u,v)}, \forall k, (u,v)$, using (62)–(66) ;
14:         **until** convergence to the optimal solution
15:         $\bar{\mathbf{P}}^\star, \bar{\mathbf{W}}_R^\star, \bar{\mathbf{t}}^\star, \mathbf{\Omega}^\star$ and $\mathbf{\Lambda}^\star$;
16:         Update the coefficients $\left(\rho_{2k-1}^{(u,v)}, \beta_{2k-1}^{(u,v)}\right)$ and
17:         $\left(\rho_{2k}^{(u,v)}, \beta_{2k}^{(u,v)}\right)$, using (18) and (19) ;
18:         Set $\bar{\mathbf{P}}(m+1) \leftarrow \bar{\mathbf{P}}^\star, \bar{\mathbf{W}}_R(m+1) \leftarrow \bar{\mathbf{W}}_R^\star,$
19:         $\bar{\mathbf{t}}^\star(m+1) \leftarrow \bar{\mathbf{t}}^\star, \mathbf{\Omega}(m+1) \leftarrow \mathbf{\Omega}^\star,$ $\mathbf{\Lambda}(m+1) \leftarrow \mathbf{\Lambda}^\star$ and $m \leftarrow m+1$;
20:      **until** convergence or $m > I_{max_2}$;
21:      Update $\pounds(l+1)$ using (67) and $l \leftarrow l+1$;
22: **until** convergence or $l > I_{max_1}$.

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in best-to-best and worst-to-worst fashion. After this arrangement, we update the subcarrier pairing matrix as follows:

$$\Lambda_{u,v}^\star = \begin{cases} 1, & \text{for } u^{th} \text{ subcarrier paired with } v^{th} \text{ subcarrier}; \\ 0, & \text{otherwise}, \end{cases} \tag{70}$$

Step 3: Optimal Power Allocation for Given Subcarrier Pairing and Allocation: For given subcarrier allocation and pairing matrices $\mathbf{\Lambda}$ and $\mathbf{\Omega}$, we update the power $\bar{P}_{2k-1}^{(u)}$, $\bar{P}_{2k}^{(u)}$, $\bar{W}_R^{(v)}$ and $\bar{t}_k^{(u,v)}$ using (38)–(40) and (51), and dual

TABLE I
COMPLEXITY ANALYSIS FOR DIFFERENT ALGORITHMS

Complexity Comparison	
Algorithm	**Complexity**
EEM	$\mathcal{O}\left(2U\mathcal{G}N_{sc}(2K)^\varpi(KN_{sc}(V^3+3)+N_{sc}^2)\right)$
Suboptimal	$\mathcal{O}\left(2U\mathcal{G}N_{sc}(2K)^\varpi(KN_{sc}(V^3+2)+K+2)\right)$
Optimal ES	$\mathcal{O}\left(2U\mathcal{G}(2K)^\varpi K^{N_{sc}!}(V^3+2)\right)$
ESPA	$\mathcal{O}\left(4U\mathcal{G}N_{sc}(2K)^\varpi(KN_{sc}(V^3+5)+N_{sc}^2)\right)$

variables $\lambda, \mu_{2k-1}^{(u,v)}, \mu_{2k}^{(u,v)}, \Theta_{2k-1}^{(u,v)}$, and $\Theta_{2k}^{(u,v)}$ using (62)–(66), respectively.

VI. COMPLEXITY ANALYSIS

In this section, to get a better insight into the complexity of various proposed algorithms, we perform an exhaustive complexity analysis, by assuming that the network penalty factor $\pounds$ convergences in U iterations.

A. EEM Algorithm

The optimization problem in (33) consists of $K \times N_{sc}^2$ subproblems due to K user pairs operating on N_{sc} subcarriers in each hop. Since, the optimal solution $\left(\bar{\mathbf{P}}^\star, \bar{\mathbf{W}}_R^\star, \mathbf{t}^\star, \mathbf{\Lambda}^\star, \mathbf{\Omega}^\star\right)$ is obtained under the total transmit power constraint $(C.1)$ and the QoS constraints $(C.2)$ and $(C.3)$, and thus the complexity resulted due to these three constraints is $\mathcal{O}\left(V^3+2\right)$, where V denotes the power level for each user and the relay node on each subcarrier. Further, each maximization in (58) adds a complexity of $\mathcal{O}(K)$ and therefore, the total complexity for finding the subcarrier allocation $\mathbf{\Omega}$ for each $(u,v)^{th}$ subcarrier pairing is $\mathcal{O}\left(K \times N_{sc}^2\right)$. Moreover, the Hungarian method [33] is used to obtain the subcarrier pairing matrix $\mathbf{\Lambda}$ in (60) has complexity of $\mathcal{O}\left(N_{sc}^3\right)$ and the total complexity for updating dual variables is $\mathcal{O}\left(2(2K)^\varpi\right)$ (for example, $\varpi = 2$ if the ellipsoid method is used [34]). Let us suppose if the dual objective function (36) converges in $\mathcal{G}$ iterations, then the total complexity for the EEM algorithm $\forall k, v, v$ becomes $\mathcal{O}\left(2U\mathcal{G}N_{sc}(2K)^\varpi(KN_{sc}(V^3+3)+N_{sc}^2)\right)$. The complexity of the EEM algorithm under *equal subcarrier power allocation (ESPA)* is $\mathcal{O}\left(4U\mathcal{G}N_{sc}(2K)^\varpi(KN_{sc}(V^3+5)+N_{sc}^2)\right)$.

B. Suboptimal Algorithm

The complexity for obtaining the subcarrier allocation matrix $\mathbf{\Omega}$ in the $step$ 1 for K user pairs is $\mathcal{O}\left(K \times N_{sc}\right)$, whereas the complexity for finding subcarrier pairing matrix $\mathbf{\Lambda}$ in $step$ 2 is $\mathcal{O}\left(2N_{sc}\right)$. However, the power allocation and updating the dual variables add a complexity of $\mathcal{O}\left(V^3 + 2\right)$ and $\mathcal{O}\left(2(2K)^\varpi\right)$, respectively. Let us suppose if the dual objective function (36) converges in $\mathcal{G}'$ iterations (without loss of generality let $\mathcal{G}' = \mathcal{G}$), then the suboptimal EEM algorithm produces a total complexity of $\mathcal{O}\left(2U\mathcal{G}N_{sc}(2K)^\varpi(KN_{sc}(V^3 + 2) + K + 2)\right)$.

C. Optimal ES Algorithm

In this algorithm, we exhaustively search over all variables for finding the optimal resource allocation solution for all the nodes on each subcarrier in the pool of all the possible feasible solutions to the optimization problem (**OP4**). Thus, the total complexity for this algorithm becomes $\mathcal{O}\left(2U\mathcal{G}(2K)^\varpi K^{N_{sc}!}(V^3 + 2)\right)$.

VII. Simulation Results and Performance Discussions

In this section, we present Monte-Carlo simulation results to demonstrate the effectiveness of the proposed resource allocation algorithms and to show the trade-off between the EE and SE for various network parameters.

A. Simulation Setup

In the considered multiuser two-way relay networks, the circuit and processing power per antenna at each node is set to be equal to 14 dBm [22], whereas the maximum available transmit power budget is 25 dBm. In simulation, we adopt the Third-Generation Partnership Project (3GPP) path loss model with path loss $131.1 + 42.8 \times \log_{10}(d)$ dB where d is distance in kilometers [35]. We consider both the Rayleigh fading effects $\sim \mathcal{CN}(0, 1)$ and the log-normal shadowing $\sim \ln\mathcal{N}(0, 8dB)$. The thermal noise density is set as -174 dBm/Hz while the subcarrier spacing is 12 kHz. The convergence tolerance value is set as 10^{-5} and the maximum number of iteration for solving the inner and outer optimization problems is set as 10. The distance from all the odd users to the relay node is denoted by d_{SR}, while d_{RD} indicates the distance from the relay node to all the even users. The QoS requirement for each user on (u, v) subcarrier pair is $\Upsilon_{min,i}^{(u,v)} = -20$ dB, $\forall i, (u, v)$. As a benchmark, we also simulate the following algorithms for comparison:

- Optimal ES algorithm: This algorithm gives the globally optimal solution of the problem (**OP1**) by performing an exhaustive search over all variables [33].
- EEM algorithm without (w/o) subcarrier pairing and allocation (SPA) algorithm: The optimal solution of the problem (**OP1**) is found without considering SPA.
- SEM algorithm: By setting $\mathcal{L} = 0$, the optimization problem (**OP1**) is transformed into the sum rate maximization problem.
- ESPA algorithm: The available transmit power is equally distributed among all the users over all the subcarriers.
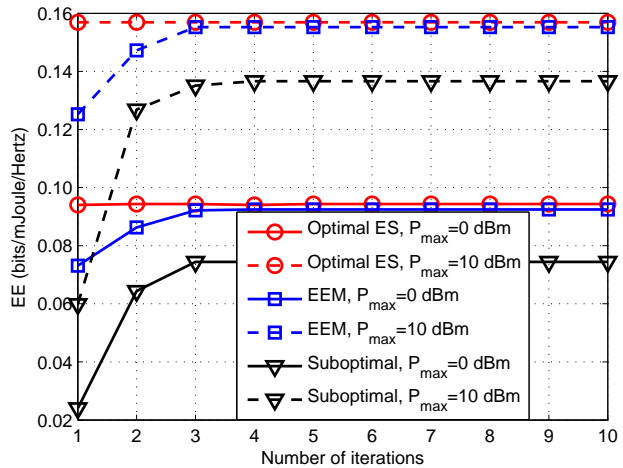
B. Convergence of EEM and Suboptimal Algorithms

Fig. 3. Convergence behavior of the proposed algorithms ($K = 2$, $N_{sc} = 6$ and $d_{SR} = d_{RD} = 200$ m).

Fig. 3 illustrates the convergence behavior of the proposed EEM and suboptimal algorithms for a single channel realization, where $K = 2$, $N_{sc} = 6$, and $d_{SR} = d_{RD} = 200$ m. The maximum allowable transmit power is $P_{max} = \{0, 10\}$ dBm. Due to small problem size, the exhaustive-based solution can also be found within a reasonable computation time. It can be observed that the EE performances of the proposed algorithms are monotonically increased with the number of iterations, and the proposed algorithms are converged in less than four iterations. Even through solving the relaxed problem, the EEM algorithm indeed finds the optimal power, subcarrier pairing and subcarrier allocation, and it provides the performance identical to the optimal ES.

C. Performance comparison of various resource allocation algorithms

Fig. 4 shows the EE and SE performance comparison of different algorithms, where $K = 2$, $N_{sc} = 6$, and $d_{SR} = d_{RD} = 200$ m. As can be seen in Fig. 4(a) that our proposed algorithms yield best performance and provide a significant power saving compare to the SEM and EEM algorithm w/o SPA and ESPA. The average EE performance of the SEM algorithms significantly drops as P_{max} increases. On the other hand, in a poor power regime, i.e. $P_{max} \leq 10$ dBm, the SE performance of the proposed algorithms is identical to that of the ES and SEM algorithms as shown in Fig. 4(b). The SE performance of the EEM and suboptimal algorithms is gradually saturated when $P_{max} > 10$ dBm, whereas performance of the SEM algorithm is continuously improved as P_{max} increases. This is because all the users utilize the maximum resources in order to improve their sum rate. The EEM w/o SPA exhibits worst performance than the proposed optimal and suboptimal algorithms, while it performs better than ESPA.
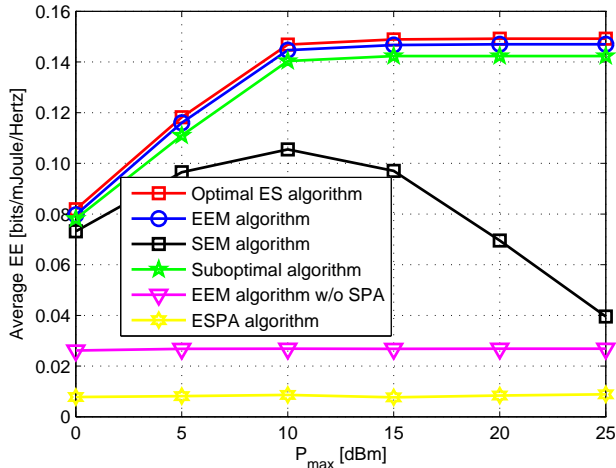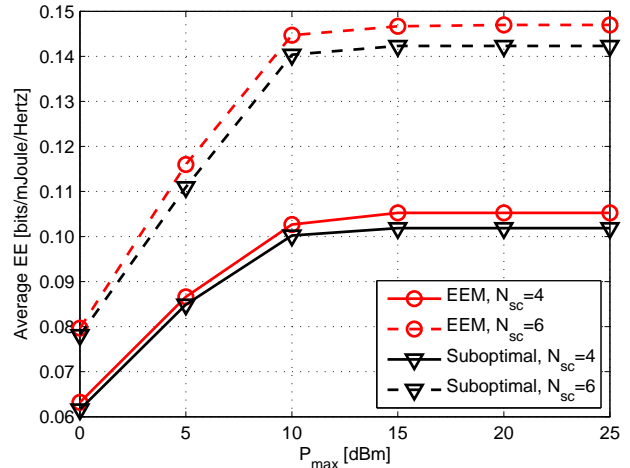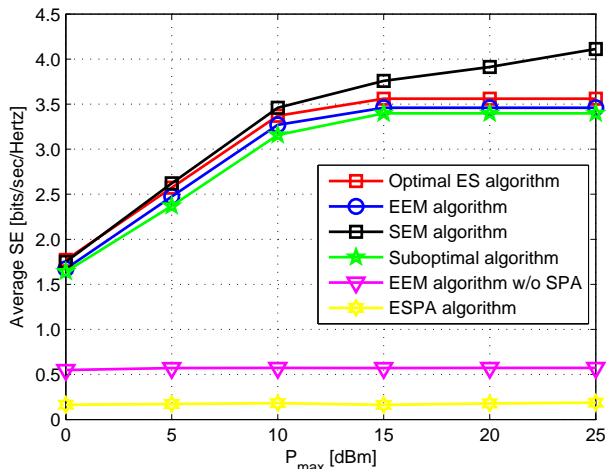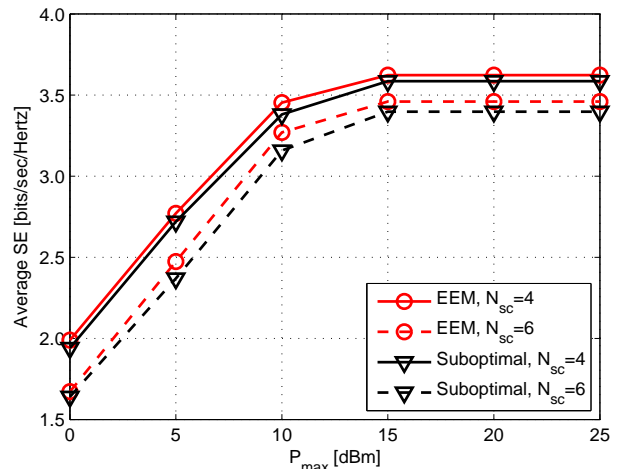
(a) Average EE versus P_{max}

(b) Average SE versus P_{max}

Fig. 4. Comparison of different resource allocation algorithms ($K = 2$, $N_{sc} = 6$ and $d_{SR} = d_{RD} = 200$ m).

(a) Average EE versus P_{max}

(b) Average SE versus P_{max}

Fig. 5. Effect of number of subcarriers on the average EE and SE ($K = 2$ and $d_{SR} = d_{RD} = 200$ m).

D. Effect of the number of subcarriers N_{sc} on the EE and SE

Fig. 5 shows the impact of increasing N_{sc} on the attainable average EE and SE of the proposed algorithms for $K = 2$ and $d_{SR} = d_{RD} = 200$ m. The average EE and SE performances of both algorithms is remarkably enhanced as P_{max} increases and becomes constant in a rich power regime, e.g., $P_{max} > 10$ dBm. Additionally, as expected, upon increasing N_{sc}, the average EE performance for both algorithms can be significantly improved due to the frequency diversity. Furthermore, Fig. 5(b) shows that the average SE performance of both algorithms decreases as N_{sc} increases, which shows a reverse trend as compared with the result in Fig. 5(a), which implies that the subcarriers are utilized effectively only from the EE point of view, when more subcarriers are available. Both the proposed algorithms rapidly improve the average EE at the cost of a slight degradation in the average SE.

E. Effect of number of user pair's K on the attainable EE

The effect of increasing user pairs K on the average EE is depicted in Fig. 6, where $N_{sc} = 6$ and $d_{SR} = d_{RD} = 200$

m. Fig. 6 shows that the average EE performance of both algorithms can be significantly increased upon increasing P_{max} and gradually saturates when $P_{max} > 10$ dBm. In Fig. 6, it is noticeable that the average EE performance of the proposed algorithms deteriorates upon increasing the number of user pairs K from 2 to 3. The deterioration in the EE performance occurs because of the commensurate increase in the static power.

VIII. CONCLUSION

In this paper, we studied the problem of joint subcarrier and power allocation for multiuser multicarrier two-way AF relay networks in order to enhance the energy utilization among users. The objective function was to maximize the EE of the network through joint subcarrier pairing, subcarrier allocation, and power allocation, subject to a total transmit power and minimum QoS requirement for each user. The formulated primal problem was a non-convex fractional MINLP problem that was difficult to solve. To make it tractable, the problem was converted into an equivalent convex optimization problem
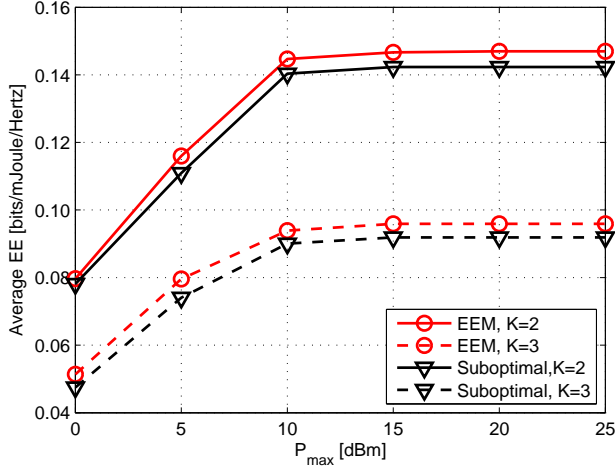
Fig. 6. Effect of number of users on the average EE ($N_{sc} = 6$ and $d_{SR} = d_{RD} = 200$ m).

by applying SCA method, change of variables and a series of transformation and then solved from the dual decomposition techniques to attain an energy-efficient optimal solution. The resulting optimal subcarrier and power allocation policy served as a performance benchmark due to its high computational complexity. In order to further reduce the complexity, a suboptimal EE resource allocation algorithm was also proposed. We compared the performance of the proposed EEM and suboptimal algorithms with that of the SEM and EEM without SPA algorithms through computer simulations and show the merits of the proposed EE resource allocation algorithms.

APPENDIX A
PROOF OF THEOREM 2

Let $\left(\bar{\mathbf{P}}^\star, \bar{\mathbf{W}}_R^\star, \mathbf{t}^\star, \boldsymbol{\Lambda}^\star, \boldsymbol{\Omega}^\star\right)$ be the optimal solution of optimization problem (**OP4**) with respect to the optimal EE $\mathcal{L}^\star$ and $\mathcal{A}$ be the feasible set of the problem, it implies that

$$\mathcal{L}^\star = \max_{\bar{\mathbf{P}}, \bar{\mathbf{W}}_R, \mathbf{t}, \boldsymbol{\Lambda}, \boldsymbol{\Omega} \in \mathcal{A}} \frac{\sum_{k=1}^{K} \sum_{u=1}^{N_{sc}} \sum_{v=1}^{N_{sc}} \Lambda_{u,v} \Omega_{k,(u,v)} t_k^{(u,v)}}{P_T\left(\bar{\mathbf{P}}, \bar{\mathbf{W}}_R, \mathbf{t}, \boldsymbol{\Lambda}, \boldsymbol{\Omega}\right)}; \quad (A.1)$$

where $P_T\left(\bar{\mathbf{P}}, \bar{\mathbf{W}}_R, \mathbf{t}, \boldsymbol{\Lambda}, \boldsymbol{\Omega}\right)$ is defined as

$$P_T\left(\bar{\mathbf{P}}, \bar{\mathbf{W}}_R, \mathbf{t}, \boldsymbol{\Lambda}, \boldsymbol{\Omega}\right) = \sum_{k=1}^{K} \sum_{u=1}^{N_{sc}} \sum_{v=1}^{N_{sc}} \Lambda_{u,v} \Omega_{k,(u,v)} \left(e^{P_{2k-1}^{(u)}} + e^{P_{2k}^{(u)}} + e^{W_R^{(v)}} \right) + C.$$

$$\mathcal{L}^\star = \frac{\sum_{k=1}^{K} \sum_{u=1}^{N_{sc}} \sum_{v=1}^{N_{sc}} \Lambda_{u,v}^\star \Omega_{k,(u,v)}^\star t_k^{(u,v)^\star}}{P_T\left(\bar{\mathbf{P}}^\star, \bar{\mathbf{W}}_R^\star, \mathbf{t}^\star, \boldsymbol{\Lambda}^\star, \boldsymbol{\Omega}^\star\right)}; \quad (A.2)$$

From (A.1) and (A.2), we have

$$\mathcal{L}^\star \triangleq \frac{\sum_{k=1}^{K} \sum_{u=1}^{N_{sc}} \sum_{v=1}^{N_{sc}} \Lambda_{u,v}^\star \Omega_{k,(u,v)}^\star t_k^{(u,v)^\star}}{P_T\left(\bar{\mathbf{P}}^\star, \bar{\mathbf{W}}_R^\star, \mathbf{t}^\star, \boldsymbol{\Lambda}^\star, \boldsymbol{\Omega}^\star\right)}$$

$$\geq \frac{\sum_{k=1}^{K} \sum_{u=1}^{N_{sc}} \sum_{v=1}^{N_{sc}} \Lambda_{u,v} \Omega_{k,(u,v)} t_k^{(u,v)}}{P_T\left(\bar{\mathbf{P}}, \bar{\mathbf{W}}_R, \mathbf{t}, \boldsymbol{\Lambda}, \boldsymbol{\Omega}\right)}; \quad (A.3)$$

From (A.1)–(A.3), we have the following observations:

$$\mathcal{F}\left(\mathcal{L}\right) = \begin{cases} \left[\sum_{k=1}^{K} \sum_{u=1}^{N_{sc}} \sum_{v=1}^{N_{sc}} \Lambda_{u,v} \Omega_{k,(u,v)} t_k^{(u,v)} \right. \\ \left. - \mathcal{L}^\star P_T\left(\bar{\mathbf{P}}, \bar{\mathbf{W}}_R, \mathbf{t}, \boldsymbol{\Lambda}, \boldsymbol{\Omega}\right) \leq 0 \right]; \\ \left[\sum_{k=1}^{K} \sum_{u=1}^{N_{sc}} \sum_{v=1}^{N_{sc}} \Lambda_{u,v}^\star \Omega_{k,(u,v)}^\star t_k^{(u,v)^\star} \right. \\ \left. - \mathcal{L}^\star P_T\left(\bar{\mathbf{P}}^\star, \bar{\mathbf{W}}_R^\star, \mathbf{t}^\star, \boldsymbol{\Lambda}^\star, \boldsymbol{\Omega}^\star\right) = 0 \right], \end{cases} \quad (A.4)$$

From (A.4), we can observe that the maximum of $\mathcal{F}\left(\mathcal{L}\right)$ is zero and is achieved when the optimal resource allocation solution $\left(\bar{\mathbf{P}}^\star, \bar{\mathbf{W}}_R^\star, \mathbf{t}^\star, \boldsymbol{\Lambda}^\star, \boldsymbol{\Omega}^\star\right)$ is adopted and the maximum EE is obtained. On the other hand, let $\left(\bar{\mathbf{P}}^\star, \bar{\mathbf{W}}_R^\star, \mathbf{t}^\star, \boldsymbol{\Lambda}^\star, \boldsymbol{\Omega}^\star\right)$ denotes the optimal solution of the problem (**OP4**) such that it satisfies the balance equation, it yields

$$\sum_{k=1}^{K} \sum_{u=1}^{N_{sc}} \sum_{v=1}^{N_{sc}} \Lambda_{u,v}^\star \Omega_{k,(u,v)}^\star t_k^{(u,v)^\star}$$
$$- \mathcal{L}^\star P_T\left(\bar{\mathbf{P}}^\star, \bar{\mathbf{W}}_R^\star, \mathbf{t}^\star, \boldsymbol{\Lambda}^\star, \boldsymbol{\Omega}^\star\right) = 0; \quad (A.5)$$

$$\geq \sum_{k=1}^{K} \sum_{u=1}^{N_{sc}} \sum_{v=1}^{N_{sc}} \Lambda_{u,v} \Omega_{k,(u,v)} t_k^{(u,v)} - \mathcal{L}^\star P_T\left(\bar{\mathbf{P}}, \bar{\mathbf{W}}_R, \mathbf{t}, \boldsymbol{\Lambda}, \boldsymbol{\Omega}\right), \quad (A.6)$$

The equations (A.5) and (A.6) implies that

$$\frac{\sum_{k=1}^{K} \sum_{u=1}^{N_{sc}} \sum_{v=1}^{N_{sc}} \Lambda_{u,v} \Omega_{k,(u,v)} t_k^{(u,v)}}{P_T\left(\bar{\mathbf{P}}, \bar{\mathbf{W}}_R, \mathbf{t}, \boldsymbol{\Lambda}, \boldsymbol{\Omega}\right)} \leq \mathcal{L}^\star$$

$$= \frac{\sum_{k=1}^{K} \sum_{u=1}^{N_{sc}} \sum_{v=1}^{N_{sc}} \Lambda_{u,v}^\star, \Omega_{k,(u,v)}^\star t_k^{(u,v)^\star}}{P_T\left(\bar{\mathbf{P}}^\star, \bar{\mathbf{W}}_R^\star, \mathbf{t}^\star, \boldsymbol{\Lambda}^\star, \boldsymbol{\Omega}^\star\right)}. \quad (A.7)$$

Hence, it is demonstrated that $\mathcal{L}^\star$ which fulfills the balance equation is the optimal EE and the solution $\left(\bar{\mathbf{P}}^\star, \bar{\mathbf{W}}_R^\star, \mathbf{t}^\star, \boldsymbol{\Lambda}^\star, \boldsymbol{\Omega}^\star\right)$ obtained corresponding to the optimal EE $\mathcal{L}^\star$ is also the optimal solution of the problem (**OP1**). This concludes the proof of Theorem 2.

APPENDIX B
PROOF OF THEOREM 3

Let us initialize the lower bound coefficients $\rho_{2k-1}^{(u,v)}, \rho_{2k}^{(u,v)}, \beta_{2k-1}^{(u,v)},$ and $\beta_{2k}^{(u,v)}$ for given subcarrier pairing $\boldsymbol{\Lambda}$ and subcarrier

allocation $\mathbf{\Omega}$ at the $(l-1)^{th}$ iteration. Then we get

$$
\sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}} \Lambda_{u,v}^{\star}(l)\Omega_{k,(u,v)}^{\star}(l)t_{k}^{(u,v)^{\star}}(l)
$$
$$
- \mathscr{L}(l)P_T\left(\bar{\mathbf{P}}^{\star}(l),\bar{\mathbf{W}}_R^{\star}(l),\mathbf{t}^{\star}(l),\mathbf{\Lambda}^{\star}(l),\mathbf{\Omega}^{\star}(l)\right)
$$
$$
\geq \left(\sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}} \Lambda_{u,v}^{\star}(l-1)\Omega_{k,(u,v)}^{\star}(l-1)t_{k}^{(u,v)^{\star}}(l-1)\right.
$$
$$
- \mathscr{L}(l)P_T\Big(\bar{\mathbf{P}}^{\star}(l-1),\bar{\mathbf{W}}_R^{\star}(l-1),
$$
$$
\left.\mathbf{t}^{\star}(l-1),\mathbf{\Lambda}^{\star}(l-1),\mathbf{\Omega}^{\star}(l-1)\Big) = 0\right);
$$
$$
\triangleq \mathcal{F}\left(\mathscr{L}(l)\right), \tag{B.1}
$$

From (67) and (B.1), we have

$$
\mathcal{F}\left(\mathscr{L}(l)\right) = \sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}} \Lambda_{u,v}^{\star}(l)\Omega_{k,(u,v)}^{\star}(l)t_{k}^{(u,v)^{\star}}(l)
$$
$$
- \mathscr{L}(l)P_T\left(\bar{\mathbf{P}}^{\star}(l),\bar{\mathbf{W}}_R^{\star}(l),\mathbf{t}^{\star}(l),\mathbf{\Lambda}^{\star}(l),\mathbf{\Omega}^{\star}(l)\right)
$$
$$
= P_T\left(\bar{\mathbf{P}}^{\star}(l),\bar{\mathbf{W}}_R^{\star}(l),\mathbf{t}^{\star}(l),\mathbf{\Lambda}^{\star}(l),\mathbf{\Omega}^{\star}(l)\right)
$$
$$
\times\left(\mathscr{L}(l+1) - \mathscr{L}(l)\right) \geq 0\,, \tag{B.2}
$$

Since $P_T\left(\bar{\mathbf{P}}^{\star}(l),\bar{\mathbf{W}}_R^{\star}(l),\mathbf{t}^{\star}(l),\mathbf{\Lambda}^{\star}(l),\mathbf{\Omega}^{\star}(l)\right) \geq 0$, therefore, we have $\mathscr{L}(l+1) \geq \mathscr{L}(l)$. This implies that the network penalty increases monotonically. The proof is completed.

APPENDIX C
PROOF OF THEOREM 4

From Theorem 3, we know that the penalty factor $\mathscr{L}(l)$ increases monotonically and remains bounded and the converged penalty factor is the optimal penalty. Assume that the penalty factor $\mathscr{L}(l)$ converges at $\bar{\mathscr{L}}$, i.e., $\mathscr{L}(l) = \mathscr{L}(l+1) = \bar{\mathscr{L}}$. However, $\bar{\mathscr{L}}$ is not the optimal penalty factor. From Theorem 2, we have

$$
\sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}} \Lambda_{u,v}^{\star}(l)\Omega_{k,(u,v)}^{\star}(l)t_{k}^{(u,v)^{\star}}(l)
$$
$$
- \mathscr{L}(l)P_T\left(\bar{\mathbf{P}}^{(u)^{\star}}(l),\bar{W}_R^{(v)^{\star}}(l),\mathbf{t}_k^{(u)^{\star}}(l),\mathbf{\Lambda}^{\star}(l),\mathbf{\Omega}^{\star}(l)\right) \neq 0\,, \tag{C.1}
$$

From (67), we know that

$$
\mathscr{L}(l+1) = \frac{\displaystyle\sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}} \Lambda_{u,v}^{\star}(l),\Omega_{k,(u,v)}^{\star}(l)t_{k}^{(u,v)^{\star}}(l)}{P_T\left(\bar{\mathbf{P}}^{\star}(l),\bar{\mathbf{W}}_R^{\star}(l),\mathbf{t}^{\star}(l),\mathbf{\Lambda}^{\star}(l),\mathbf{\Omega}^{\star}(l)\right)}\,, \tag{C.2}
$$

According to (C.1) and (C.2), (C.3) can be obtained, given as

$$
\mathscr{L}(l) \neq \frac{\displaystyle\sum_{k=1}^{K}\sum_{u=1}^{N_{sc}}\sum_{v=1}^{N_{sc}} \Lambda_{u,v}^{\star}(l),\Omega_{k,(u,v)}^{\star}(l)t_{k}^{(u,v)^{\star}}(l)}{P_T\left(\bar{\mathbf{P}}^{\star}(l),\bar{\mathbf{W}}_R^{\star}(l),\mathbf{t}^{\star}(l),\mathbf{\Lambda}^{\star}(l),\mathbf{\Omega}^{\star}(l)\right)} = \mathscr{L}(l+1) \tag{C.3}
$$

This contradicts our assumption $\mathscr{L}(l) = \mathscr{L}(l+1)$. This concludes the proof of Theorem 4.

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