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Dynamic Prediction of Financial Distress Using Malmquist DEA

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Abstract

Creditors such as banks frequently use expert systems to support their decisions when issuing loans and credit assessment has been an important area of application of machine learning techniques for decades. In practice, banks are often required to provide the rationale behind their decisions in addition to being able to predict the performance of companies when assessing corporate applicants for loans. One solution is to use Data Envelopment Analysis (DEA) to evaluate multiple decision-making units (DMUs or companies) which are ranked according to the best practice in their industrial sector. A linear programming algorithm is employed to calculate corporate efficiency as a measure to distinguish healthy companies from those in financial distress. This paper extends the cross-sectional DEA models to time-varying Malmquist DEA, since dynamic predictive models allow one to incorporate changes over time. This decision-support system can adjust the efficiency frontier intelligently over time and make robust predictions. Results based on a sample of 742 Chinese listed companies observed over 10 years suggest that Malmquist DEA offers insights into the competitive position of a company in addition to accurate financial distress predictions based on the DEA efficiency measures.

Keywords: Malmquist DEA; Bankruptcy prediction; Financial distress; Efficiency; Dynamic model

Highlights:

- This paper is the first to apply dynamic time-varying efficiency scores in distress prediction.
- Malmquist DEA is used to produce dynamic efficiency scores over several periods.
- Various efficiency scores are compared in discrete time hazard models.
- A comparison is made between generic and industry-specific models.

1. Introduction

Decision-support systems are vital for creditors who need to distinguish between firms that will perform well and those that under-perform, and therefore may have difficulties in repaying their loans. Such systems use various techniques including traditional statistical models and expert systems or intelligent machine-learning algorithms to evaluate the creditworthiness of borrowers. Those predictive methods are often regarded as Early Warning Systems to give early signals of potential business bankruptcy or financial distress. Numerous applications of machine learning algorithms include Neural Networks (NN) by López Iturriaga & Sanz (2015) , Genetic Algorithm (GA) by Gordini (2014), Support Vector Machines (SVM) by Yang, You, & Ji (2011), and Ensemble models that combine several statistical and intelligent classifiers (Fedorova, Gilenko, & Dovzhenko, 2013; Marques, Garcia, & Sanchez, 2012). These studies compared the predictive accuracy of different algorithms and examined statistically significant predictors of bankruptcy or distress. It has been shown that machine learning algorithms outperform statistic methods in terms of classification accuracy because the objective of machine learning is to minimise misclassified errors so that an optimal solution can be found.

Yet predictive accuracy is not the only feature that lenders are interested in, since there is a need (and in many cases a regulatory requirement) to understand and explain the risk drivers or factors that affect the probability of financial distress. Many machine-learning techniques, e.g. neural networks cannot provide such an explanation in contrast to Data Envelopment Analysis (DEA), which is an optimisation algorithm based on linear programming. It allows one to find the efficiency frontier (or benchmark) so that relative efficiency of Decision Making Units (DMUs) can be measured by the distance to this frontier. Cielen, Peeters, & Vanhoof (2004) argued that DEA as a type of machine learning technique can provide insights into the value of a company's efficiency for bankruptcy prediction. The idea was developed further by Xu & Wang (2009), Psillaki, Tsolas, & Margaritis (2010), Premachandra, Chen, & Watson (2011), Shetty, Pakkala, & Mallikarjunappa (2012) etc. These studies demonstrated that corporate efficiency measures can successfully distinguish between 'good' and 'bad' companies, while Min & Lee (2008) suggested using efficiency scores generated by DEA to predict bankruptcy directly as a practical approach.

However, most studies on the significance of efficiency in estimating the probability of financial distress used cross-sectional models that fail to capture temporal changes in efficiency, and yet internal and external conditions associated with company performance do change over time. So as Shumway (2001) argues, *dynamic* models, in contrast to cross-sectional or *static* models, are preferred in business failure prediction. Premachandra, Chen, & Watson (2011), Li, Crook, & Andreeva (2014), Wanke, Barros, & Faria (2015) and Wanke, Azad, & Barros (2016) all strongly suggested using *dynamic* efficiency models to predict the risk of bankruptcy or financial distress. Yet so far, to the best of our knowledge, no study has conducted an analysis of efficiency scores in dynamic prediction models. This paper fills this gap and is the first to develop a dynamic model integrated with Malmquist DEA and hazard models in order to predict financial distress, which can be easily extended to other bankruptcy or business failure prediction models within the scope of credit assessment. We explore the question of whether changes in the efficiency of companies over time really affects the chance that they will suffer financial distress.

Our paper adds to the literature in several important ways. First, we propose two stage DEA-based programming models as decision-support systems in identifying efficient (healthy) and inefficient (distressed) companies in advance. Second, it enhances the bankruptcy prediction literature by providing a dynamic model that offers insights into the competitive position of a business, in addition to accurate distress predictions. Third, we address the methodological limitations of existing studies Xu & Wang (2009), Yeh, Chi, & Hsu (2010), Cielen, Peeters, & Vanhoof (2004), Premachandra, Bhabra, & Sueyoshi (2009), Premachandra, Chen, & Watson (2011) which either assume constant returns to scale (CRS) or homogeneous production technology in their samples. Our estimation of corporate efficiency is more realistic given mixed industrial sectors.

This paper builds on the work of Premachandra, Chen, & Watson (2011), Paradi, Asmild, & Simak (2004), Shetty, Pakkala, & Mallikarjunappa (2012) and Li, Crook, & Andreeva (2014), all of which employ cross-sectional analysis. Our analysis is based on a sample of 742 Chinese listed companies observed over 10 years, in total 5,490 company-years. Dynamic DEA efficiency scores calculated by the Malmquist Index are used to classify healthy and distressed companies and to predict the probability of distress as the firm progresses through its lifecycle. Observations are made about the predictive utility of several model specifications with different efficiency scores and assumptions that contribute to decision-support modelling for bankruptcy prediction. We find that computing the efficiency of a company relative to those across a broad range of industries gives more accurate predictions than computing the efficiency relative only to others in the same industry. We also find that if one does the latter, then comparing the efficiency of a company with the most efficient companies at any time throughout the sample period is more accurate than using only technical or super efficiency.

The rest of the paper is structured as follows. Section 2 reviews the literature on dynamic corporate credit risk models and dynamic DEA models, which is fundamental to our extensions of previous studies in both areas. Sections 3 and 4 introduce the methodology and the data used, respectively, with the descriptions of the sample and variables. The results of four comparative models that employ different types of efficiency scores are presented and discussed in Section 5, and Section 6 concludes the paper.

2. Literature review

2.1 Dynamic credit risk models

Altman (1968) introduced statistical methods (Discriminant Analysis or DA) to corporate bankruptcy prediction before the age of machine learning techniques. Statistical methods estimate coefficients of financial ratios in a parametric format and were more popular compared to machine learning techniques given the limited computing power decades ago. Nevertheless, Shumway (2001) claimed that half of the financial ratios that were found to be successful in cross-sectional models turned out to be unrelated to bankruptcy probability in later periods.

As a result Shumway (2001) proposed a hazard model that has advantages over cross-sectional models. First, hazard models incorporate the effect of time on the risk of an event occurring. Second, hazard models can also incorporate Time-Varying Covariates (TVCs) relating to an individual firm and to macroeconomic factors, the latter representing systemic effects. Third, hazard models can make better predictions by utilising data observed over several time periods. Fourth, hazard models can handle censoring: where an event occurs but is not observed in the observation time window. All of these advantages imply that dynamic or hazard models should be preferred in credit risk modelling.

Unlike static models, dynamic models imply a time varying hazard rate and thus are more appropriate to make predictions. An event (*e.g.* default, bankruptcy or financial distress) can happen any time during interval $[t, t+\Delta t]$ (t is duration time) in Cox proportional hazard regression and that was applied by Bonfim (2009) to Portuguese firms over the period 1996 to 2002 to predict bankruptcy risk during different macroeconomic cycles.

In corporate credit, the default event is usually defined to occur within a specific period of time, commonly one year (Carling, Jacobson, Linde, & Roszbach, 2007). Covariates are also observed only at given points of time when financial statements are disclosed. Therefore it is more appropriate to use a discrete time version rather than a continuous time hazard model. The discrete time hazard model (DHM) is equivalent to multi-period logistic regression in terms of computation but with an additional term $h_0(t)$ as the baseline hazard function. Such a method was applied to predict corporate default risk by Shumway (2001), Carling, Jacobson, Linde, & Roszbach (2007), Nam, Kim, Park, & Lee (2008), Wilson & Altanlar (2014) etc.

2.2 Dynamic DEA models

If DEA efficiency is one of the covariates in dynamic models, there is an obvious need to evaluate it in multiple periods correspondingly. Whilst conventional (*i.e.* static) DEA models are constructed for a single period, many researchers and practitioners are interested in how efficiency changes over time. Specifically, if a DMU can be observed at different points of time, its change in efficiency over the periods can be informative for predicting future financial distress. One possible approach is to solve DEA problems period by period separately and build a panel dataset consisting of these efficiency scores, as Bryan, Fernando, & Tripathy (2013) suggested. Yet it can be argued that methodologically the scores in different periods are incomparable because DEA scores are based on the frontier formed by the peers in that period. That is, a relative efficiency of 0.5 in the second period may be no better than a relative efficiency of 0.3 in the first period since efficiency also depends on the frontier shift for the industry, for example a change in membership or in technology.

A possible solution to this might be to still perform a static DEA analysis for each period separately but in the second stage using a standard regression method to estimate the change over time and then extend it to further periods. Emel, Oral, Reisman, & Yolalan (2003) and Min & Lee (2008) used this two-stage method to forecast DEA scores and hence bankruptcy. However, Cook & Seiford (2009) commented that this approach was unsatisfactory because it failed to capture the interaction of one period with another.

Window DEA was introduced by Charnes, Clark, Cooper, & Golany (1984) to deal with the efficiency change in the sense of time series. The idea of Window DEA, similar to other window analyses, is to set up a fixed observation window, and to move it across the whole period. Finally the movements and stability of the results can be analysed across different panel subsets. However Cooper, Seiford, & Tone (2006) argued that a shortcoming of Window DEA was evident in the initial and last period where cases were less well evaluated.

The Malmquist DEA model is particularly suitable in dealing with panel data. The original idea of the Malmquist Index (MI) was to compare the production technology of two economies, so it is a bilateral index. Let $f(\mathbf{x})$ be the production function of an economy, where $\mathbf{x}$ is a vector of inputs such as labour and capital. To calculate the MI between Economy a and Economy b of different production functions, we can substitute $\mathbf{x}_a$ in $f_b(\cdot)$ and vice versa. So the MI is defined as

$$\mathrm{MI} = \sqrt{\frac{f_b(\mathbf{x}_a)}{f_b(\mathbf{x}_b)} \cdot \frac{f_a(\mathbf{x}_a)}{f_a(\mathbf{x}_b)}} = \sqrt{f_a(\mathbf{x}_a) f_b(\mathbf{x}_a) / f_a(\mathbf{x}_b) f_b(\mathbf{x}_b)} \quad . \tag{1}$$

Inspired by Caves, Christensen, & Diewert (1982) who introduced this index in productivity analysis, Färe, Grosskopf, Lindgren, & Roos (1992) and Färe, Grosskopf, Norris, & Zhang (1994) integrated the MI into DEA and developed a DEA-based Malmquist productivity index. The Malmquist productivity index evaluates the total factor productivity change of a DMU between two periods where a and b in equation (1) each relate to a period. It is defined as the product of efficiency change (catch-up) and technological change (frontier-shift) where the catch-up effect describes how much closer a DMU gets to the most efficient production frontier, and the frontier-shift effect describes the technology improvement in the sample. The decomposed elements of the MI can determine how much of a relative efficiency increase from period t to $t+1$ can be credited to individual effort and how much to industry innovation. The efficiency change reflects the extent to which a DMU improves or worsens its efficiency, while technological change reflects the change of the efficiency frontiers between two periods. Since the introduction of the MI, there have been various studies of productivity change over time in different fields, for example in Italian manufacturing firms (Costa, 2012), Spanish government tax offices (Fuentes & Lillo-Bañuls, 2015), Taiwanese banks (Shyu & Chiang, 2012) and Korean universities (Sohn & Kim, 2012). These studies looked into the efficiency changes over time to derive managerial implications and strategic recommendations and did not consider distress prediction.

Table 1 Summary of corporate credit prediction literature

Literature	Sample	Event	Method	Type of prediction
Altman (1968)	US Manufacturing firms	Bankruptcy	DA	Static
Hua, Wang, Xu, Zhang, & Liang (2007)	Chinese listed companies	Distress	SVM	Static
Sun, Jia, & Li (2011)	Chinese listed companies	Distress	Ensemble models	Static
Yang, You, & Ji (2011)	Polish firms	Bankruptcy	SVM	Static
Cao (2012)	Chinese listed companies	Distress	Choquet integral	Static
Fedorova, Gilenko, & Dovzhenko (2013)	Russian manufacturing firms	Bankruptcy	Ensemble models	Static
Gordini (2014)	Italian manufacturing firms	Bankruptcy	GA	Static
López Iturriaga & Sanz (2015)	US banks	Bankruptcy	NN	Static
Bonfim (2009)	Portuguese firms	Default	Proportional hazard model	Dynamic
Shumway (2001)	US firms	Bankruptcy	DHM	Dynamic
Chava & Jarrow (2004)	US firms	Bankruptcy	DHM	Dynamic
Carling, Jacobson, Linde, & Roszbach (2007)	Swedish firms	Default	DHM	Dynamic
Nam, Kim, Park, & Lee (2008)	Korean listed companies	Bankruptcy	DHM	Dynamic
Wilson & Altanlar (2014)	UK new firms	Bankruptcy	DHM	Dynamic
Emel, Oral, Reisman, & Yolalan (2003)	Turkish manufacturing firms	Bankruptcy	DEA, DA	Static
Cielen, Peeters, & Vanhoof (2004)	Belgian firms	Bankruptcy	DEA	Static
Paradi, Asmild, & Simak (2004)	US Manufacturing firms	Bankruptcy	DEA	Static
Min & Lee (2008)	Korean manufacturing firms	Bankruptcy	DEA	Static
Xu & Wang (2009)	Chinese listed companies	Distress	DEA+DA, Logit & SVM	Static
Psillaki, Tsolas, & Margaritis (2010)	French manufacturing firms	Bankruptcy	DEA+Logit	Static
Premachandra, Bhabra, & Sueyoshi (2009)	US firms	Bankruptcy	DEA	Static
Yeh, Chi, & Hsu (2010)	Taiwanese manufacturing firms	Bankruptcy	DEA+Rough Set Thoery & SVM	Static
Premachandra, Chen, & Watson (2011)	US firms	Bankruptcy	DEA	Static
Shetty, Pakkala, & Mallikarjunappa (2012)	Indian firms	Bankruptcy	DEA	Static
Bryan, Fernando, & Tripathy (2013)	US firms	Bankruptcy	DEA+DA	Static
Li, Crook, & Andreeva (2014)	Chinese listed companies	Distress	DEA+Logit	Static
Paradi, Wilson, & Yang (2014)	US Non-manufacturing firms	Bankruptcy	DEA+DA	Static
Kingyens, Paradi, & Tam (2016)	US retail companies	Bankruptcy	DEA	Static
Yang & Dimitrov (2017)	US Non-manufacturing firms	Bankruptcy	DEA+SVM	Static

Studies such as Emel, Oral, Reisman, & Yolalan (2003), Paradi, Asmild, & Simak (2004), Cielen, Peeters, & Vanhoof (2004), have been amongst the first to explore accuracy of bankruptcy predictions using DEA efficiency (see Table 1). It is worth extending it to a panel analysis as Malmquist DEA models proved capable of analysing change in performance over time. Unfortunately, we have only seen applications restricted to cross-sectional analysis, even in the latest studies such as Kingyens, Paradi, & Tam (2016) and Yang & Dimitrov (2017).

Having understood the advantages of Malmquist DEA in dealing with panel data over other methods, this paper applies Malmquist DEA scores to dynamic prediction of financial distress by taking the time dimension into account, which bridges the literature of DEA applications and dynamic credit risk modelling. The details of the methodology are presented next.

3. Methodology

We compute the MI and dynamic efficiency scores first and then regress financial distress on the indicators of dynamic efficiency and other variables. Negative values are occasionally observed in financial data that can be used as inputs and outputs for DEA. Therefore, dealing with negative values becomes necessary. An appropriate model would be the input-oriented VRS Slack Based Model (SBM), given that only the outputs contain negative values (Cooper, Seiford, & Tone, 2006), as in our case. The Malmquist DEA model is based on these assumptions while other choices may not guarantee both translation and units invariance at the same time.

3.1 Malmquist DEA

In order to build the required panel dataset, it is necessary to calculate the efficiency scores for each company in each year of observation. Using Malmquist DEA we assume multiple inputs and outputs when DMUs are repeatedly observed on a certain interval basis.

Caves, Christensen, & Diewert (1982) defined a distance function $D(\cdot)$ based on the Malmquist productivity index (Malmquist, 1953) to calculate technical efficiency (TE). A company is efficient if $D(x, y) = 1$. Let $\mathbf{x}_0^t$ denote a vector of inputs and $\mathbf{y}_0^t$ denote a vector of outputs for DMU_0, both at period t. The relative efficiency of DMU_0 at period t, $D_0^t(\mathbf{x}_0^t, \mathbf{y}_0^t) = \theta_0^*$, is calculated as the amount by which input x_0 can be reduced while producing the given output level y_0 compared to the most efficient company on the frontier. Similarly, $D_0^{t+1}(\mathbf{x}_0^{t+1}, \mathbf{y}_0^{t+1})$ is its efficiency score at period $t+1$. Thus, with multiple periods, $D_0^t(\mathbf{x}_0^{t+1}, \mathbf{y}_0^{t+1})$ and $D_0^{t+1}(\mathbf{x}_0^t, \mathbf{y}_0^t)$ are actually efficiency scores using a set of inputs and outputs in one period, $t+1$ and t respectively, compared with the frontier of the other period, t and $t+1$ respectively.

Following the ideas of Farrell (1957) to decompose the total factor productivity into the efficiency change (EC) and the technology change (TC), Färe, Grosskopf, Lindgren, & Roos (1992) defined the input-oriented Malmquist productivity index (MI) to measure the productivity change of DMU$_0$ between period t and $t+1$ as

$$\begin{aligned} \mathrm{MI}_0 &= [\frac{D_0^t(\mathbf{x}_0^{t+1},\mathbf{y}_0^{t+1})}{D_0^t(\mathbf{x}_0^t,\mathbf{y}_0^t)}\cdot\frac{D_0^{t+1}(\mathbf{x}_0^{t+1},\mathbf{y}_0^{t+1})}{D_0^{t+1}(\mathbf{x}_0^t,\mathbf{y}_0^t)}]^{1/2} \\ &= \frac{D_0^{t+1}(\mathbf{x}_0^{t+1},\mathbf{y}_0^{t+1})}{D_0^t(\mathbf{x}_0^t,\mathbf{y}_0^t)}[\frac{D_0^t(\mathbf{x}_0^{t+1},\mathbf{y}_0^{t+1})}{D_0^{t+1}(\mathbf{x}_0^{t+1},\mathbf{y}_0^{t+1})}\cdot\frac{D_0^t(\mathbf{x}_0^t,\mathbf{y}_0^t)}{D_0^{t+1}(\mathbf{x}_0^t,\mathbf{y}_0^t)}]^{1/2} \end{aligned} \quad . \tag{2}$$

The first part is the relative change of efficiency from period t to $t+1$. Hence they defined

$$\mathrm{EC} = \frac{D_0^{t+1}(\mathbf{x}_0^{t+1},\mathbf{y}_0^{t+1})}{D_0^t(\mathbf{x}_0^t,\mathbf{y}_0^t)} \tag{3}$$

and

$$\mathrm{TC} = [\frac{D_0^t(\mathbf{x}_0^{t+1},\mathbf{y}_0^{t+1})}{D_0^{t+1}(\mathbf{x}_0^{t+1},\mathbf{y}_0^{t+1})}\cdot\frac{D_0^t(\mathbf{x}_0^t,\mathbf{y}_0^t)}{D_0^{t+1}(\mathbf{x}_0^t,\mathbf{y}_0^t)}]^{1/2} \quad . \tag{4}$$

Other than using a distance function $D(\cdot)$ to calculate efficiency, under the nonparametric framework Färe, Grosskopf, Norris, & Zhang (1994) calculated the MI by an oriented radial DEA model, while other DEA models are also suitable (Cooper, Seiford, & Tone, 2006).

Let $x_{ij}^t(i=1,\ldots$ and $y_{rj}^t(r=1,\ldots$ denote the inputs and outputs for DMU$_j$ ($j=1,\ldots$) respectively at any given point of time t. The production possibility set of a VRS model is defined by Cooper, Seiford, & Tone (2006) as

$$(X^t,Y^t)=\left\{(x,y)\left| x\geq\sum_j^n\lambda_j x_j^t, 0\leq y\leq\sum_j^n\lambda_j y_j^t, \mathbf{e}\boldsymbol{\lambda}=1, \boldsymbol{\lambda}\geq 0\right.\right\} \quad , \tag{5}$$

where $\mathbf{e}$ is the row vector with all elements equal to one, $\boldsymbol{\lambda}\in R^n$ is the intensity vector. Let $\theta_0^t(\mathbf{x}_0^t,\mathbf{y}_0^t)$ be the optimal solution to the programming problem (6):

$$\begin{aligned} \theta_0^t(\mathbf{x}_0^t,\mathbf{y}_0^t) = \min \quad & 1\text{-}\frac{1}{\mathrm{m}}\sum_{i=1}^m\frac{s_i^-}{x_{i0}^t} \\ s.t. \quad & \mathbf{x}_0^t - X^t\boldsymbol{\lambda} - \mathbf{s}^- = \mathbf{0} \\ & \mathbf{y}_0^t \leq Y^t\boldsymbol{\lambda} \\ & \mathbf{e}\boldsymbol{\lambda} = 1 \\ & \boldsymbol{\lambda}\geq\mathbf{0}, \mathbf{s}^-\geq\mathbf{0} \end{aligned} \quad , \tag{6}$$

where $\mathbf{s}^-$ is a vector of slacks, $\boldsymbol{\lambda}$ is a non-negative vector and $\sum_{j=1}^{n} \lambda_j = 1$.

The reciprocal efficiency $\theta_0^t(\mathbf{x}_0^{t+1}, \mathbf{y}_0^{t+1})$ is the optimal solution of equation (7):

$$\begin{aligned}
\theta_0^t(\mathbf{x}_0^{t+1}, \mathbf{y}_0^{t+1}) = \min \quad & 1\text{-}\frac{1}{\mathrm{m}}\sum_{i=1}^{m}\frac{s_i^-}{x_{i0}^t} \\
s.t. \qquad & \mathbf{x}_0^{t+1} - X^t\boldsymbol{\lambda} - \mathbf{s}^- = \mathbf{0} \\
& \mathbf{y}_0^{t+1} \le Y^t\boldsymbol{\lambda} \\
& \mathbf{e}\boldsymbol{\lambda} = 1 \\
& \boldsymbol{\lambda} \ge \mathbf{0}, \mathbf{s}^- \ge \mathbf{0}
\end{aligned} \quad . \tag{7}$$

By solving the linear programmes (6) and (7) four times for $\theta_0^t(\mathbf{x}_0^t, \mathbf{y}_0^t)$, $\theta_0^{t+1}(\mathbf{x}_0^{t+1}, \mathbf{y}_0^{t+1})$, $\theta_0^t(\mathbf{x}_0^{t+1}, \mathbf{y}_0^{t+1})$, and $\theta_0^{t+1}(x_0^t, y_0^t)$, we have the MI, $M_0^{t \text{ to } t+1}$, as

$$M_0^{t \text{ to } t+1} = \sqrt{\frac{\theta_0^t(\mathbf{x}_0^{t+1}, \mathbf{y}_0^{t+1})}{\theta_0^t(\mathbf{x}_0^t, \mathbf{y}_0^t)} \cdot \frac{\theta_0^{t+1}(\mathbf{x}_0^{t+1}, \mathbf{y}_0^{t+1})}{\theta_0^{t+1}(\mathbf{x}_0^t, \mathbf{y}_0^t)}} \quad . \tag{8}$$

This study is interested in the relative efficiency of a DMU calculated by the interaction of periods. Multiplying $\theta_0^t(\mathbf{x}_0^t, \mathbf{y}_0^t)$ by $M_0^{t \text{ to } t+1}$ will give the relative efficiency at period $t+1$ compared to period t for each DMU.

In the domain of Malmquist DEA, the reference set, namely period t mentioned above, to which the relative efficiency is compared, is of importance in our research. More specifically, the model defined on two consecutive periods t and $t+1$ can be seen using adjacent periods as the reference set, which was the original design in Färe, Grosskopf, Lindgren, & Roos (1992). Suppose there are five periods $t = 1, \ldots$. By running Malmquist DEA with adjacent references, one can only get relative efficiency θ^1, θ^2 compared to period 1, θ^3 compared to period 2 and so on. However, it is not intuitive for the other relative efficiency of period 3 compared to period 1 or period 4 compared to period 2. Thus it is not possible to interpret the relative efficiency directly with adjacent moving references. A solution is to use a fixed reference set as suggested by Berg, Førsund, & Jansen (1992). Therefore, in this research, all relative efficiency is referred to the first period as the beginning of the observation. Thus it is not period $t+1$ compared to period t but period t ($t \ge 2$) compared to period 1. In this way, it is very likely that in later periods efficiency scores are larger than 1 as technology develops. It should be noted that apart from scores in period 1, all other scores larger than 1 do not necessarily imply being efficient in that period.

3.2 Discrete hazard model with DEA scores

Computing efficiency scores calculated by DEA in the first stage and inputting them into the second stage analysis where other classifying methods are involved is a popular approach. Examples can be found in Psillaki, Tsolas, & Margaritis (2010), Xu & Wang (2009), Yeh, Chi, & Hsu (2010), Li, Crook, & Andreeva (2014), etc. The advantage of doing this is that we can evaluate the marginal effects and the statistical significance of new variables conditional on other covariates such as financial ratios, which have been shown to have predictive power in detecting potential bankruptcy risk.

Dyson et al. (2001) and Li, Crook, & Andreeva (2014) have argued that homogeneity of DMUs in terms of technology is important both in the DEA modelling step and in the regression analysis. Thus industry diversity becomes a critical issue in the process. Although the homogeneity requirement, as Li, Crook, & Andreeva (2014) commented, may increase the complexity of modelling, it is consistent with the findings in corporate credit risk modelling that attention should be given to the differences between industrial sectors (Bonfim, 2009; Chava & Jarrow, 2004).

This research employs this two-stage modelling process and takes into account industrial differences separately in both stages, i.e. the sectors are separated in the Malmquist DEA programming, and in the discrete hazard model with the sectors being represented by dummy variables. After Malmquist efficiency scores of multiple periods are obtained in the way given in the last section, they enter the discrete time hazard model with sector dummies written as

$$\text{logit}(h_{d=1}(t)) = \alpha + \beta_0 h_0(t) + \sum_{s=1}^{S} D_s \boldsymbol{\beta}_{1,s}^T \mathbf{x}_{i,s,t-2}^e + \boldsymbol{\beta}_2^T \mathbf{x}_{i,t-2}^r \quad , \tag{9}$$

where $d = 1$ when a company suffers financial distress, 0 otherwise;

$h_0(t)$ is the baseline hazard function;

$\mathbf{x}_{i,s,t-2}^e$ is a vector of efficiency scores for sector s company i at time $t-2$, $t = 3,4,...,T_i$;

$\mathbf{x}_{i,t-2}^r$ is a vector of financial ratios for company i at time $t = 3,4,...,T_i$;

$D_s = 1$ if company i is a member of sector s, 0 otherwise, $s = 1,...,S$;

β_0 is the coefficient of the baseline hazard to be estimated;

$\boldsymbol{\beta}_{1,s}$ is a vector of parameters for efficiency scores for sector s to be estimated;

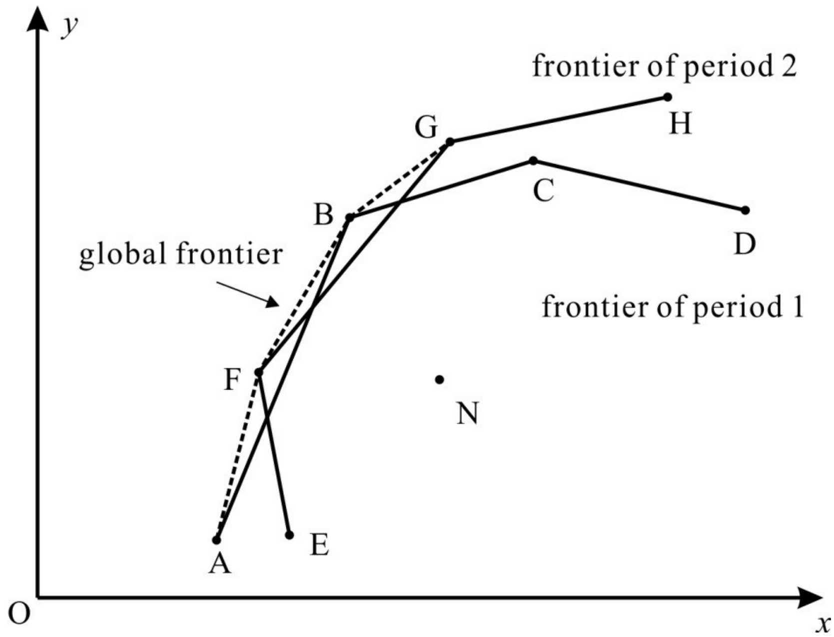
$\boldsymbol{\beta}_2$ is a vector of parameters for financial ratios to be estimated.

MaxDEA Pro 6.1 is used to solve Malmquist DEA problems indicated in equations (6), (7) and (8). It can handle unbalanced panel datasets where some cases of late entry or early exit are censored. Combined with equation (9), the base model for financial distress prediction is proposed.

3.3 *Global reference*

Further to fixed reference, Pastor & Lovell (2005) introduced the idea of global reference. In some cases where efficient frontiers of different periods cross each other, a global reference set represents the best practices in all periods. For example, in Figure 1, there are four DMUs lying on each of two frontiers ABCD and EFGH. The DMU to be evaluated, unit N, could be referred to frontier ABCD, frontier EFGH or the most efficient units ever over the observation period AFBGH. It is acceptable that when the observation window is long enough, all DMUs at the current period are under the cover of the best historical DMUs, possibly including themselves. Thus the relative efficiency in this circumstance can be treated as absolute efficiency, if the sample is very large. The scores to global reference would be less than or equal to 1. In practice, when the model is built, it is the historic data prior to the current moment that is used in model training and the historic global reference of the past that is available. Therefore, the efficiency calculated by the global reference as an option is embedded into the comparative models.

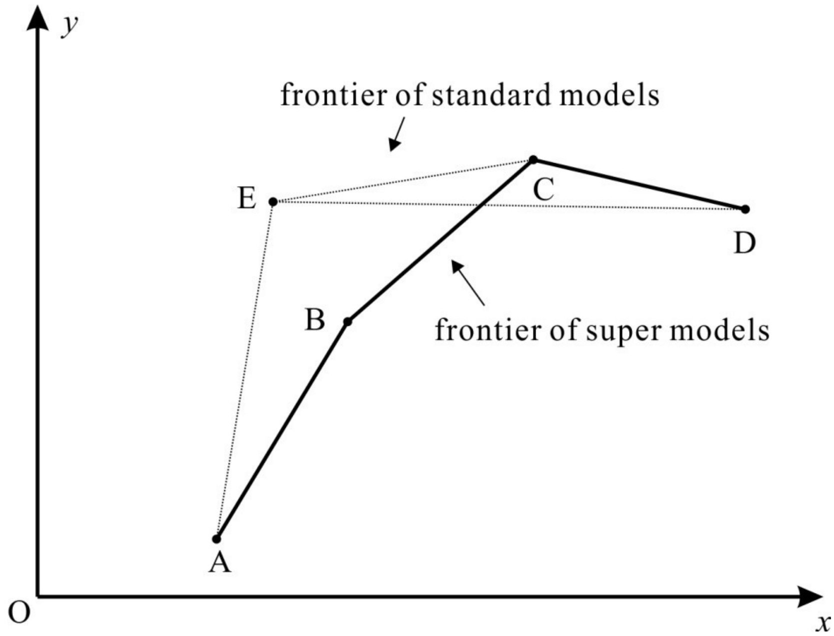
Figure 1 An example of global reference



3.4 *Super efficiency*

Cooper, Seiford, & Tone (2006) referred to the two intertemporal scores, $\theta_0^t(\mathbf{x}_0^{t+1}, \mathbf{y}_0^{t+1})$ and $\theta_0^{t+1}(\mathbf{x}_0^t, \mathbf{y}_0^t)$ as the 'exclusive schemes'. They explain that the exclusive scheme in solving intertemporal programming treats the DMU in the period to be evaluated as having been removed from the evaluator group of the other period. This is mathematically equivalent to what is known as 'super efficiency' in DEA. Super efficiency is used as a solution to the problem that common DEA models do not provide an efficiency ranking for efficient units as their scores are all equal to 1 (Andersen & Petersen, 1993). The difference between a super efficiency model and standard models is that in super efficiency models the DMU to be evaluated is eliminated from the reference frontier, so its score can be greater than 1, as shown in Figure 2. Units A, B, C, D and E consist of the productivity possibility set. If unit E is to be evaluated, its efficiency score is 1 as it is on the frontier AECD of standard DEA models; in super models, the new frontier ABCD is employed. For another unit C, its new reference frontier is AED. In this way, though units E and C are both efficient (score = 1) in standard models, a difference between them can be observed by obtaining a new unbound score greater than 1.

Figure 2 An example of super efficiency



DEA as a frontier technique is arguably an outlier analysis. However, extreme outliers may change the local frontier sufficiently for other units referred to it to be incorrectly measured. In this circumstance, super efficiency can be used to identify outliers (Banker & Chang, 2006). Obviously, super efficiency scores offer more discriminant power between efficient units, which is particularly useful in classifying good and bad companies in credit risk models. This can be found in the model of Premachandra, Chen, & Watson (2011) who employed super efficiency scores to predict corporate failure. Our paper considers super efficiency in a model for comparison with the base model.

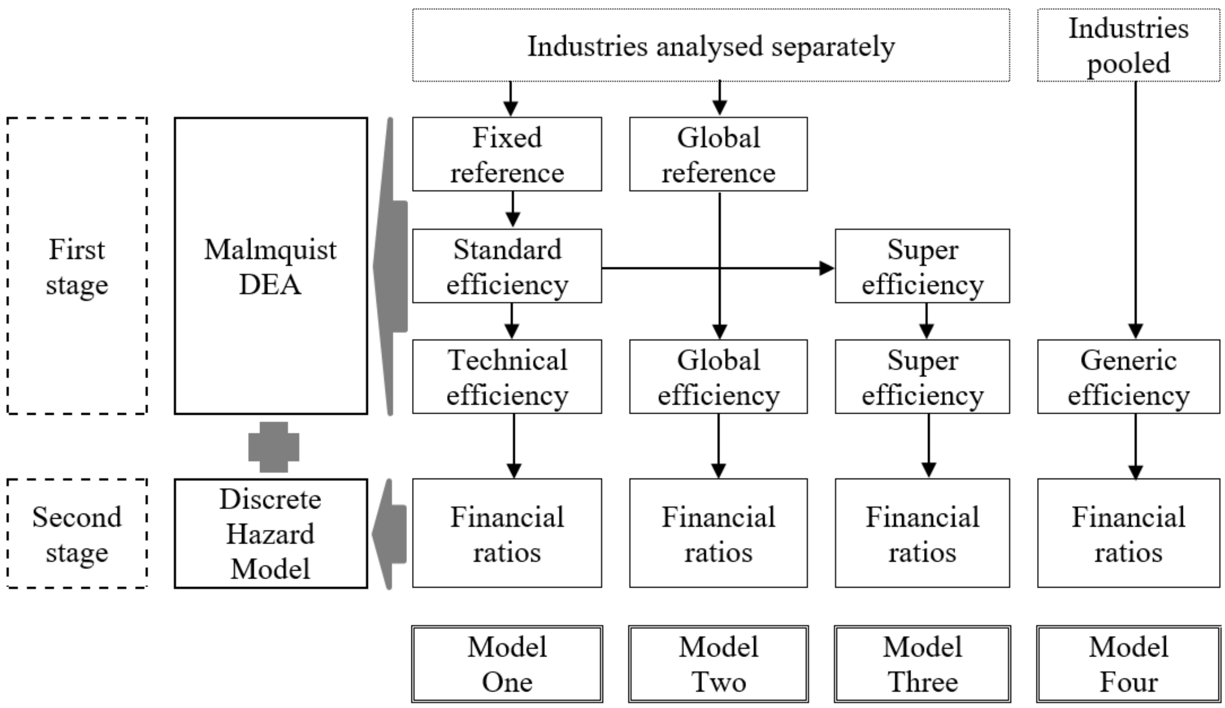
The Malmquist SBM DEA model with super efficiency is described by Tone (2002) where $\xi_i = \frac{s_i^-}{x_{i0}^t}$ as:

$$
\begin{aligned}
\theta_0^t(\mathbf{x}_0^t, \mathbf{y}_0^t) = \min_{\xi,\lambda} \quad & 1 + \frac{1}{\mathrm{m}} \sum_{i=1}^{m} \xi_i \\
s.t. \qquad & (1+\xi_i) x_{i0}^t = \sum_{i=1}^{m} \lambda_j x_{ij} \quad (i = 1, \ldots \\
& \mathbf{y}_0^t \leq Y^t \boldsymbol{\lambda} \qquad\qquad . \\
& \mathbf{e}\boldsymbol{\lambda} = 1 \\
& \boldsymbol{\lambda} \geq \mathbf{0}, \boldsymbol{\xi} \geq \mathbf{0}
\end{aligned} \tag{10}
$$

3.5 Model specification

This paper uses a two-stage analysis. In the first stage, DEA efficiency scores for each company at each period are calculated by DEA models defined in the previous sections. In the second stage, the proposed discrete hazard model incorporates efficiency scores as covariates in a panel dataset. Four models outlined in Figure 3 are to be compared with each other for the reasons given below.

Figure 3 Model specification



Model One, as introduced in Section 3.1 and 3.2, uses Malmquist DEA scores calculated by equations (6), (7) and (8) as covariates in the hazard model (equation (9)). The efficiency score is Technical Efficiency under the VRS assumption. Model One is the base model of this research and predicts the probability of financial distress in two years' time given that the company has survived until the time of prediction.

Model Two applies global reference as introduced in Section 3.3. In this model, the relative efficiency score of company i in sector s at period t is calculated with reference to the most productive companies in all possible periods in the same sector. It is of interest to investigate the predictive power of efficiency scores compared in a cross-period scenario.

Model Three follows the super efficiency setting in Section 3.4. Super efficiency scores provide more discrimination for efficient companies so one may expect to see some improvement in predictive accuracy.

Model Four is the simplest method in terms of DEA calculation and regression, but heterogeneous technologies are combined. In the first stage when calculating the efficiency scores, all industries are pooled together so the efficient frontier may be pushed outward as more units are considered. In the second stage, the term $\sum_{s=1}^{S} D_s \boldsymbol{\beta}_{1,s}^T \mathbf{x}_{i,s,t-2}^e$ in equation (9) is replaced by a simpler form $\boldsymbol{\beta}_1^T \mathbf{x}_{i,t-2}^e$ without the sector dummies. This is in line with previous literature that pools heterogeneous samples for bankruptcy prediction. This approach is referred to as the 'generic' model.

The predictive accuracy of models is compared on the out-of-time test sample by standard measures used in predictive modelling (Lessmann, Baesens, Seow, & Thomas, 2015): AUC (the area under Receiver Operating Curve), KS (the Kolmogorov-Smirnov statistic), Type I error (a distressed company that is wrongly classified as a non-distressed company) and Type II error (a non-distressed company that is wrongly classified as a distressed company). For the latter two measures the cut-off is set to the percentage of the distressed companies in the training set.

4. Data

4.1 Sample description

The data in this research is taken from the two Chinese stock exchanges, which by 2014 were listing over 2,500 Chinese companies. The Chinese government impose 'Special Treatment' (ST) on listed companies in financial distress, so ST is chosen as the official indicator of distress (marked as $d = 1$) in this research. Predicting financial distress of Chinese listed companies indicated by ST is consistent with many previous studies using various machine learning techniques, for example, Hua, Wang, Xu, Zhang, & Liang (2007), Sun, Jia, & Li (2011) and Cao (2012). Since the number of employees is only available in annual reports after the year 2000, the observation period is between 2001 and 2010. After the initial filtering, 2,027 individually listed companies over the period 2001 to 2010, a total of 12,431 firm years were left in the sample for analysis. Among them, there are 12,058 healthy firm years and 373 distressed firm years, giving a distress rate of approximately 3%.

As industry classification is essential to this research, the starting point was to consider all industries. Banking and insurance companies are excluded from the sample as their accounting conventions are different from those in other sectors. For some industries the ST numbers were very low, so we focus on three industries (a total of 742 individual companies) that accounted for nearly half of all distressed cases (49.87%). These are Raw Materials (sector code 1510), Industrial Equipment (sector code 2010) and Real Estate (sector code 4040). A sufficient number of ST companies is necessary for the following two reasons. First, as the panel analysis covers ten years, the valid number of firm years falling in each period cannot be too small. Second, DEA models require that in each period the number of units is more than double the number of inputs and outputs (8 in our case) for good estimates (Dyson et al., 2001). In the end, 5,490 firm years in these three industries were left in the sample.

Table 2 indicates that the average distress rate across all years is 3.37% (185/5490=3.37%). The average number of observations for each company in ten years is 7.4 ($5490/742=7.40$). In the years 2002, 2003 and 2006 there are significantly more companies suffering financial distress than in other years.

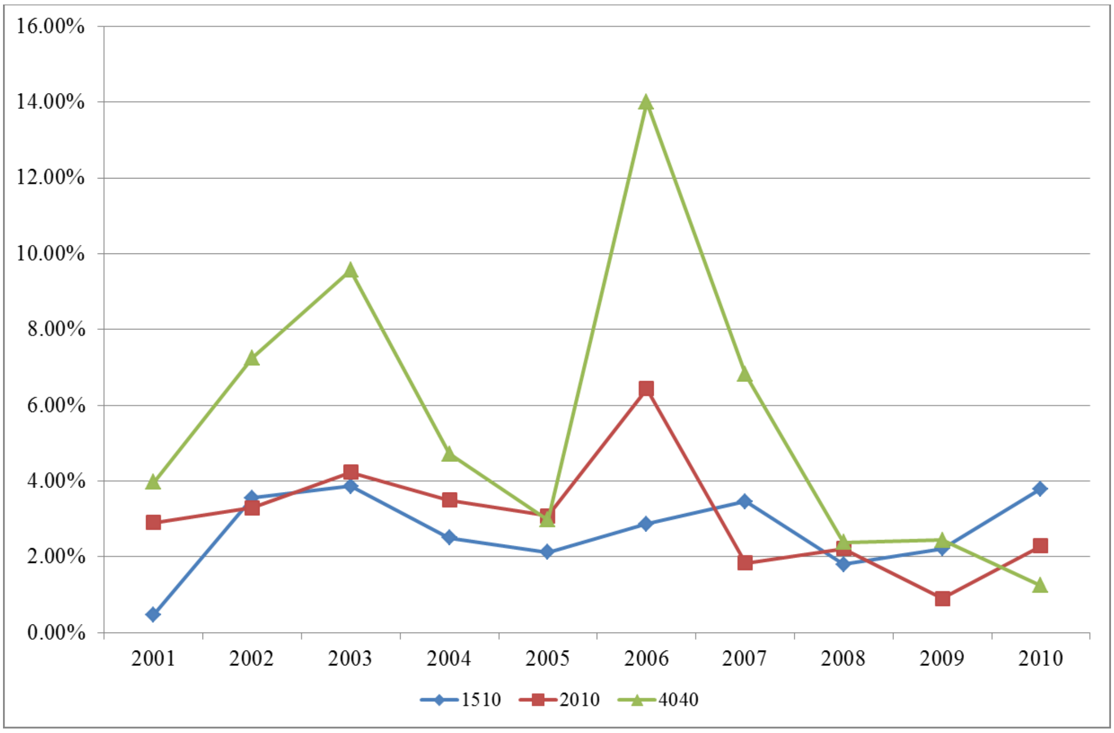
Figure 4 shows the distress rate of the three sample industries in each period of observation. In most years, there are more distressed Real Estate companies than in the other two sectors. In later years (2008 to 2010) the distress rate is considerably lower than those during 2002-2003 and 2006-2007.

The whole sample is split into a training set covering 2001-2008, eight years, and the test set includes companies with covariates measured over 2009 to 2010 and a distress/non-distress indicator measured in 2011 and 2012. This provides an out-of-time sample and is consistent with Shumway (2001) and other studies using DHM (Nam, Kim, Park, & Lee, 2008; Wilson & Altanlar, 2014).

Table 2 Distributions of samples in three industries over 2001-2010

Sector	Raw Materials			Industrial Equipment			Real Estate			Total		
N	328			277			137			742		
year	Distress 0	Distress 1	Total	Distress 0	Distress 1	Total	Distress 0	Distress 1	Total	Distress 0	Distress 1	Total
2001	208	1	209	167	5	172	121	5	126	496	11	507
2002	217	8	225	176	6	182	115	9	124	508	23	531
2003	224	9	233	181	8	189	104	11	115	509	28	537
2004	234	6	240	193	7	200	101	5	106	528	18	546
2005	231	5	236	189	6	195	98	3	101	518	14	532
2006	237	7	244	189	13	202	86	14	100	512	34	546
2007	251	9	260	214	4	218	82	6	88	547	19	566
2008	273	5	278	221	5	226	82	2	84	576	12	588
2009	265	6	271	219	2	221	80	2	82	564	10	574
2010	254	10	264	214	5	219	79	1	80	547	16	563
Total	2394	66	2460	1963	61	2024	948	58	1006	5305	185	5490

Figure 4 Distress rates of the three sectors over 2001-2010

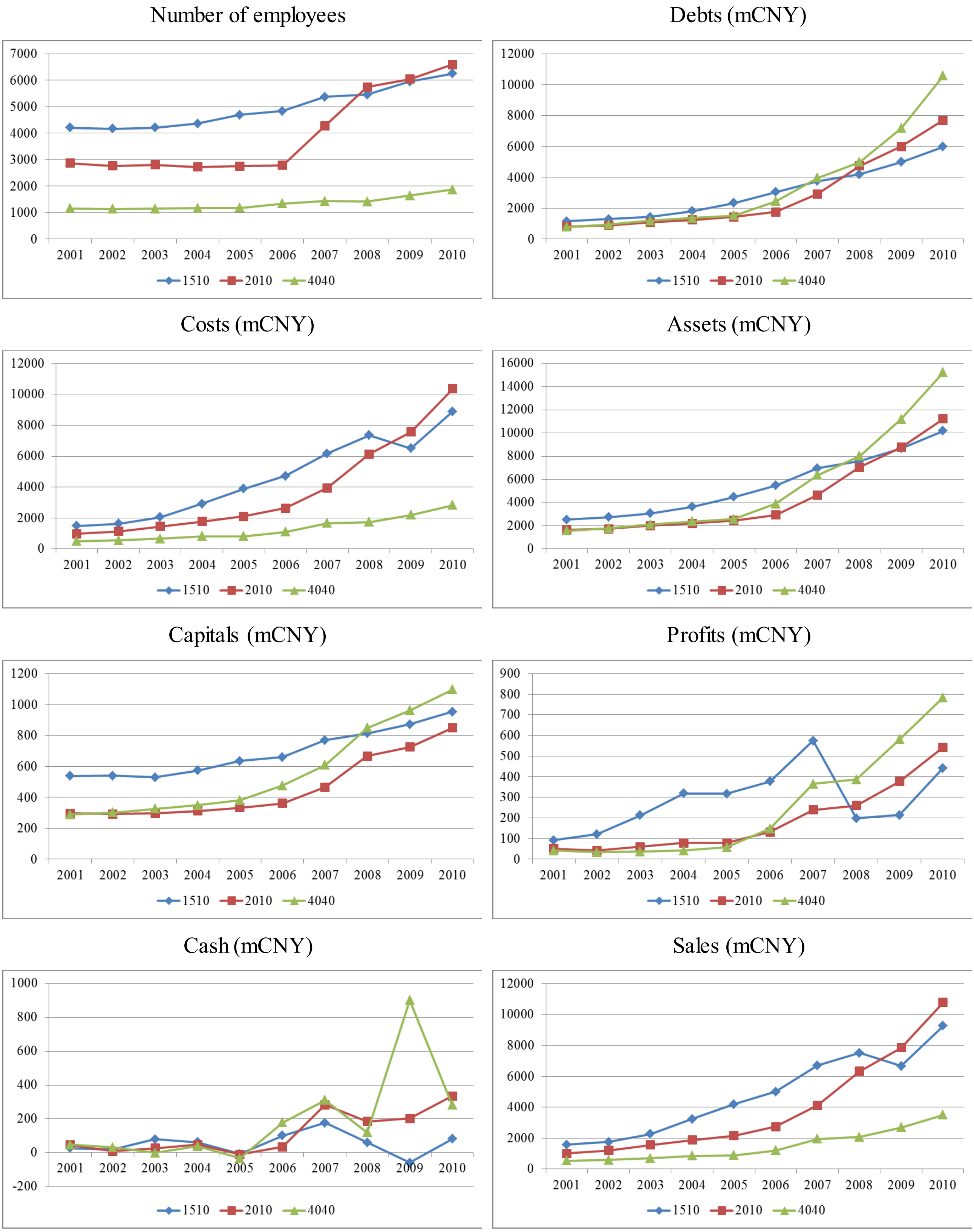


4.2 DEA inputs and outputs

Variables for the MI are selected from physical or monetary items that are contained in standard annual reports. There are five inputs: number of employees, total liabilities, total costs, total assets, and share capital, and three outputs: total profits, total cash flow and total sales. The reason for keeping both total sales and total profits is that a large revenue does not necessarily imply a large profit. Having correlated variables in DEA does not lead to a problem because their weights can automatically adjust without a significant impact on the efficiency score (Dyson et al., 2001). On the contrary, Dyson et al. (2001) (p.249) argued 'omission of a highly correlated variable can on occasion lead to significant changes in efficiencies'. This argument may also be applied to the inclusion of both the number of employees and total costs as the latter covers labour costs. Therefore, simplistically, companies make use of resources (measured by total assets and share capital), hire people (measured by the number of employees), pay for labour and raw materials (measured by total costs), turn them into products and services, sell them for revenue (measured by total sales) and aim for large earnings (measured by total profits) and positive cash inflow (measured by total cash flow).

The descriptive statistics of the covariates are reported in aggregate because Malmquist DEA models are estimated on the whole dataset (both training and test samples). For convenience of presentation, only graphs of means over time are presented in Figure 5.

Figure 5 Descriptions of DEA variables over 2001-2010



Generally, the size of listed companies (in terms of total assets) increased over the ten years under study. Their total debts, total costs and share capital had similar growth rates. It is the same for total sales as for output. However, there were some changes that did not follow the trend. For example, the number of employees in sector 2010 (Industrial Equipment) nearly doubled in the three years 2006-2008. There was a noticeably large drop in profit for sector 1510 (Raw Materials) in years 2008 and 2009, which might be due to the influence of the global financial crisis. Additionally, there was a sharp net cash inflow in sector 4040 (Real Estate) in 2009. These large changes highlight the importance of running DEA peer comparison analyses separately for each industry so that the relative efficiency scores are not biased.

4.3 Duration time and variables

The sample of this study consists of listed companies so following Shumway (2001) we choose the stock trading age to be the duration time in the hazard model, because companies met the same requirements to be listed on an exchange. The average trading age in the sample is 7.79 years. Following experimentation the baseline function of the duration time, $\ln(t)$, proved to be a good fit in the models, as in Shumway (2001).

The indicator of financial distress, ST, is a status indicator where a company can go to ST and recover from it. Here, only the first occurrence of ST is regarded as the event of distress and the information after that is ignored. All companies at the time of entering the observation window in 2001 are healthy companies (not in the status of ST). So the model predicts the probability of a company going into financial distress (ST) for the first time in the next two years, conditional on lagged values of covariates, given the duration time since the company was listed on the stock exchange.

Six categories of financial ratios are considered in the regression model: profitability, liabilities and liquidity, capital and asset composition, cash flow, operation and growth rate. In the preliminary analysis of group mean difference tests and collinearity, one ratio from each category is selected to represent that aspect of a company's financial position. These six ratios are Return on Equity, Current Liabilities / Total Liabilities, Tangible Assets / Total Assets, Cash Flow from Operation per Share, Total Assets Turnover and Total Assets Growth.

5. Results

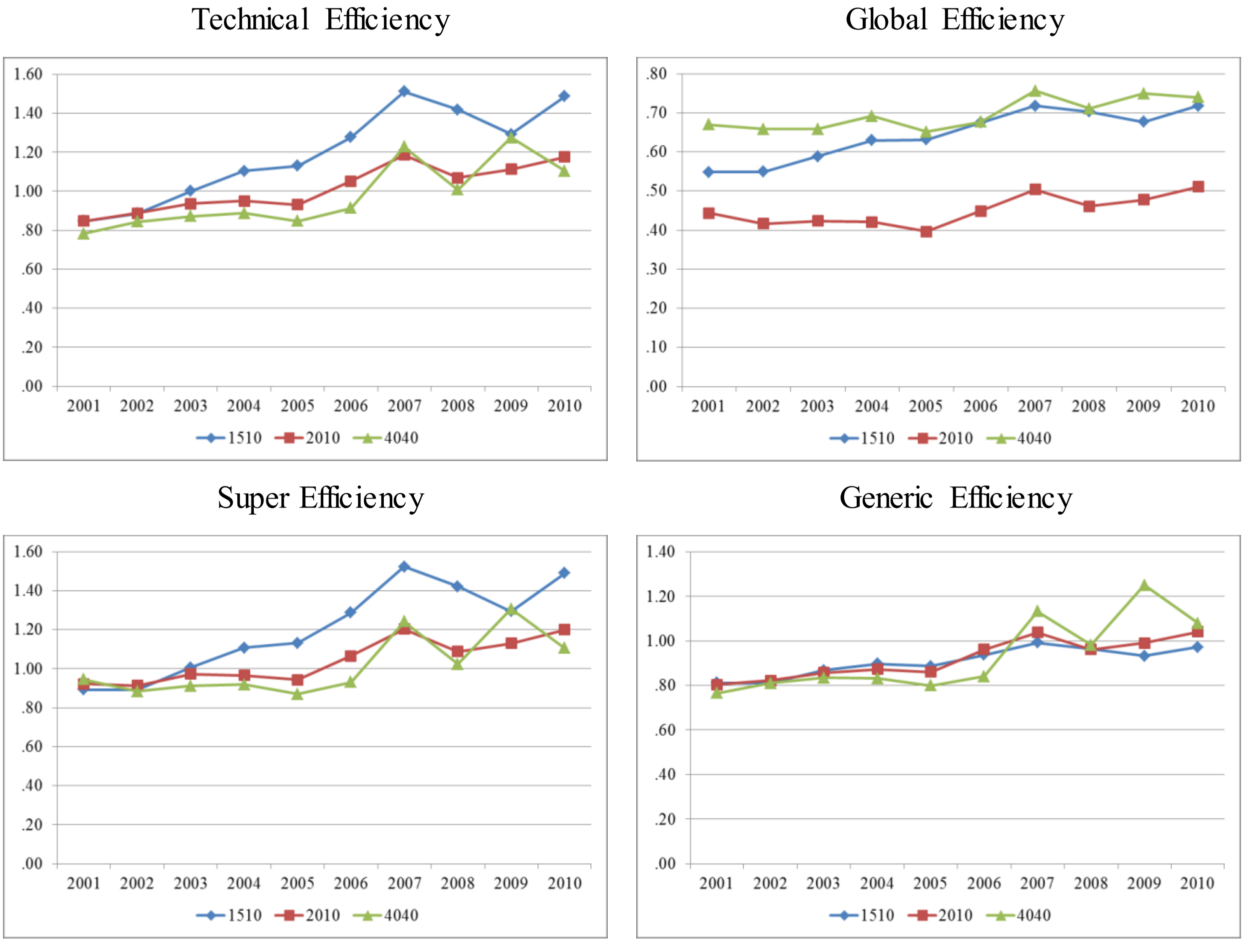
5.1 Dynamic DEA score

Table 3 Description of efficiency scores

Sector	Distress	Stats	Technical Efficiency	Global Efficiency	Super Efficiency	Generic Efficiency
Raw Materials	No	N	2394	2394	2394	2394
		Mean	1.225	0.653	1.234	0.918
		SD	0.772	0.167	0.788	0.315
		Min	0.029	0.021	0.029	0.028
		Max	8.638	1	8.638	5
	Yes	N	66	66	66	66
		Mean	0.756	0.464	0.756	0.67
		SD	0.487	0.207	0.487	0.313
		Min	0.04	0.026	0.04	0.035
		Max	3.458	0.929	3.458	2.234
	Mean dif. btw groups	F	17.484	53.593	21.914	30.198
		p	0.000	0.000	0.000	0.000
Industrial Equipment	No	N	1963	1963	1963	1963
		Mean	1.033	0.457	1.059	0.936
		SD	0.53	0.18	0.603	0.405
		Min	0.07	0.032	0.07	0.062
		Max	7.378	1	7.378	6.178
	Yes	N	61	61	61	61
		Mean	0.702	0.307	0.702	0.664
		SD	0.329	0.193	0.329	0.293
		Min	0.004	0.002	0.004	0.003
		Max	1.93	1	1.93	1.821
	Mean dif. btw groups	F	4.970	13.121	25.262	28.014
		p	0.026	0.000	0.000	0.000
Real Estate	No	N	948	948	948	948
		Mean	0.972	0.702	1.017	0.929
		SD	0.569	0.196	0.666	0.503
		Min	0.059	0.049	0.059	0.059
		Max	6.277	1	8.204	5.931
	Yes	N	58	58	58	58
		Mean	0.66	0.518	0.667	0.628
		SD	0.381	0.26	0.389	0.35
		Min	0.024	0.021	0.024	0.023
		Max	1.805	1	1.805	1.603
	Mean dif. btw groups	F	3.683	21.919	14.439	22.350
		p	0.050	0.000	0.000	0.000
Mean dif. btw sectors		F	4.598	104.354	3.192	0.187

p	0.010	0.000	0.041	0.829

Figure 6 Distributions of efficiency scores over 2001-2010



In order to see how efficiency and technology change over time, we plot graphs of mean efficiency scores across all periods in

Figure 6. For convenience, graphs for three industrial sectors are drawn on one chart. Generally the efficiency and technology levels increased while, in the later years, there were some declines, presumably due to the influence of the financial crisis in 2008.

5.2 *Regression results*

Six ratios, together with efficiency scores, are integrated in Models One to Four as showed in Figure 3. The results are presented in Table 4. The χ^2 tests indicate that all four models explain significant amounts of variation in the probability of distress. The coefficient for each type of efficiency has a negative sign, which indicates that the more efficient a company is, the less likely it is to go into financial distress. The value of the coefficient on each type of efficiency differs between the models because their mean values and distributions are different. For Models One to Three, when three industries were treated separately, differences in the parameters are observed. All parameters are significant at the 5% level. The parameters of the financial ratios in Table 4 have the expected signs and all are statistically significant.

Table 4 Model results

Covariates	Model One	Model Two	Model Three	Model Four
ln(duration)	-0.042	-0.101	0.219	0.194
Technical Efficiency (1510)	-2.646**			
Technical Efficiency (2010)	-2.968**			
Technical Efficiency (4040)	-3.056**			
Global Efficiency (1510)		-4.762**		
Global Efficiency (2010)		-7.630**		
Global Efficiency (4040)		-3865**		
Super Efficiency (1510)			-2.565**	
Super Efficiency (2010)			-2.886**	
Super Efficiency (4040)			-2.941**	
Generic Efficiency				-5.485**
Return on Equity	-9.084**	-9.386**	-9.105**	-8.656**
Current Liabilities / Total Liabilities	6.776**	6.670**	6.809**	6.567**
Tangible Assets / Total Assets	-1.463**	-1.553**	-1.508**	-1.555**
Cash Flow from Operation per Share	-0.971**	-0.890**	-0.964**	-0.872**
Total Assets Turnover	-1.551**	-1.136**	-1.562**	-0.546**
Total Assets Growth	-2.695**	-2.595**	-2.777**	-2.185**
Constant	-4.721**	-4.499**	-4.703**	-2.984**
Log likelihood	-453.36	-448.03	-451.59	-437.53
Number of observations	4017	4017	4017	4017
LR χ^2	380.4	391.04	383.94	412.06
Prob > χ^2	0	0	0	0
Pseudo R^2	0.2955	0.3038	0.2934	0.3201

** indicates the coefficients is significant at the 5% level of significance.

5.3 *Predictive accuracy*

Type I error occurs when a distressed company is wrongly classified as a healthy company while Type II error occurs when a healthy company is wrongly classified as a distressed company. The results of Models One to Four in Table 5 show much larger Type I errors in the test set than those in the training set. However, the opposite is the case with Type II errors, which is attributed to the lower distress rate in the later years, as the classifications are based on the cut-off which is measured by the percentage of distressed companies in the training set.

The AUC and KS statistics measure relative rank ordering of predicted probabilities of distress for healthy and distressed companies, with higher values corresponding to better models (Lessmann, Baesens, Seow, & Thomas, 2015). Values in Table 5 indicate very similar accuracy of rank orderings between all the models. The overall predictive accuracy is around 95%, which is higher than what is found in the cross sectional logit model combined with DEA efficiency in Xu & Wang (2009) (overall accuracy 91%) and in Li, Crook, & Andreeva (2014) (overall accuracy 93%).

Table 5 Predictive accuracy

	Training set				Test set			
	AUC	KS	Type I error	Type II error	AUC	KS	Type I error	Type II error
Model One	0.881	0.622	58.28%	2.28%	0.861	0.629	88.89%	1.45%
Model Two	*0.883*	*0.631*	*56.95%*	*2.22%*	*0.866*	*0.655*	*77.78%*	*1.27%*
Model Three	0.882	0.632	57.62%	2.25%	0.860	0.625	83.33%	1.36%
Model Four	**0.898**	**0.661**	**56.29%**	**2.20%**	**0.880**	**0.670**	**72.22%**	**1.18%**

Figures in bold indicate the best performance across all models while the next best is marked in italics.

Model Four, which disregards industrial classification, seems to do consistently better than the other models. This indicates that by relaxing the assumption of homogeneity between DMUs of DEA and pooling all industries together before calculating DEA scores and probabilities of default in hazard models, practically one is more able to make more accurate predictions of future corporate distress than if we distinguish between industrial sectors. These results are similar to the findings in former studies in consumer credit of Banasik, Crook, & Thomas (1996) and Bijak & Thomas (2012), where segmentation did not produced the expected effect because of the sample size.

Model Two is the next best model in terms of AUC and KS on the test set in Table 5. Model Two with global reference has a larger AUC (0.866) than those of Models Two and Four in the out-of-time predictions. The Super Efficiency model (Model Three) was slightly less accurate than other models. This suggests that greater discrimination between the most efficient companies is unnecessary for the prediction of financial distress.

6. Conclusion

One of the aims of expert systems and machine-learning algorithms is to provide analytical support for business decisions based on intelligent data analysis. We present one way of providing such support which offers the benefits of insights into relative performance of companies and captures its change over time. We contribute to previous research in the area of DEA in expert systems (Min & Lee (2008); Shetty, Pakkala, & Mallikarjunappa, (2012); Xu & Wang (2009)) by introducing the time-varying component; by comparing industry-specific models against the generic models; and by investigating the potential effect on predictive accuracy of different efficiency measure.

Dynamic models have inherent advantages over static models in the context of event prediction because conditions and behaviours change over time, so predictions need to be adjusted by incorporating as much information as possible. In this paper, our financial distress hazard models are enhanced by dynamic DEA scores which provide insights into the efficiency of a company relative to others over time. In the domain of DEA, Malmquist DEA is the only one that catches temporal changes of DMUs so it allows efficiency to be compared in both cross-sectional and time series formats. Other DEA algorithms such as standard DEA, Network DEA or Window DEA do not quite fit the bankruptcy prediction paradigm. A Malmquist productivity index is defined as the product of efficiency change (catch-up) and technological change (frontier-shift) and is calculated by the standard DEA scores in two periods and two inter-temporal scores with reference to the efficiency frontier of the other period. A weakness of Malmquist DEA is that it is computationally intensive and cannot handle very large datasets. Nevertheless, corporate loan portfolios are often relatively small in sample size as compared to retail credit portfolios, such as mortgages.

No previous research has attempted a panel analysis of DEA efficiency in predicting the probability of financial distress. This paper has bridged this gap by calculating dynamic relative efficiency scores using Malmquist DEA and incorporating them as covariates in hazard models. Our models therefore provide time-varying information about the probability of distress and use panel rather than cross-sectional estimators. We find all efficiency scores are negatively associated with the probability of financial distress. These results confirm the findings from previous literature that the more efficient a company is, the less likely it is to encounter financial difficulties.

We have experimented with several types of efficiency measures that offer insights into relative performance of companies over time. This offers the possibility for lenders to understand risk drivers of a company's financial distress and how they vary over time. Our findings imply that the highest predictive accuracy is achieved when pooling the industries together and using generic efficiency for prediction. This implies that lenders can achieve their goals of accurate predictions and interpretable results without the need for segmented models and component efficiency scores.

Pooling all industries together rather than carrying out a DEA analysis for each industry separately to calculate DEA scores may be practically effective in detecting financial distress, since the best predicting model in our analysis is the Generic Efficiency one. However, the second best is Global Efficiency Malmquist DEA with only slightly less accurate predictions than the generic scores. The

Global efficiency takes all historic records into account and chooses the most efficient company years as the reference units. This implies that when the sample is sufficiently large, global efficiency can be seen as absolute efficiency which is generalised in all units and periods. Therefore, if one is concerned with the essential assumption of homogeneity of technology for DEA, the Global Efficiency model that keeps this assumption and at the same time produces accurate predictions is the one to choose.

Although this paper only employs data from three sectors in the empirical analysis, it can obviously be extended to a large variety of industries as long as their production technologies are similar, otherwise they should be dealt with carefully. Our method provides a feasible solution to accommodate differences and commonalities in the business operations of companies. From the comparisons with cross-sectional peers and time series histories, managers of companies will be able to identify the weaknesses in their businesses and improve their performance to avoid financial difficulties.

Another useful application of the current paper would be to extend the combination of Malmquist DEA and the discrete hazard model to a single sector such as financial institutions. The failure of banks was noticeable during the subprime crisis, and would have great impact on the real economy of all countries. Giving dynamic early warning of their failure by inclusion of the time dimension will be of interest in the scope of bankruptcy prediction. Finally, incorporation of DEA scores into machine-learning predictive algorithms can be another fruitful avenue of investigation.

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