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# Risk-averse portfolio selection of renewable electricity generator investments in Brazil: An optimised multi-market commercialisation strategy

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## Abstract

Investment decisions in renewable energy sources such as small hydro, wind power, biomass and solar are frequently made in the context of enormous uncertainty surrounding both intermittent generation and the highly volatile electricity spot prices that are used for clearing of trades. This paper presents a new portfolio-based approach for selecting long-term investments in small-scale renewable energy projects and matching contracts for the sale of the resulting electricity. Using this approach, we have formulated a stochastic optimisation model that maximises a holding company's risk-averse measure of value. Using an illustrative example representative of investment decisions within the Brazilian electricity system, we investigate the sensitivity of the optimised portfolio composition and commercialisation strategy to contract prices in the free contracting environment and to the decision maker's attitude towards risk. The numerical results demonstrate it is possible to reduce significantly financial risks, such as the price-quantity risk, not only by exploiting the complementarity of the considered renewable sources generation profiles, but also by selecting the optimal mix of commercialisation contracts from different markets. We find that the multi-market strategy generally results in appreciably higher optimal value than single-market strategies and can be applied to a wide range of renewable generators and contracts.

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Abbreviations: Small Hydro (SH), Wind Power (WP), Biomass (BIO), Regulated Contracting Environment (RCE), Free Contracting Environment (FCE), Generation company (Genco), Net Present Value (NPV), Conditional Value-at-Risk (CVaR), Brazilian real (R\$, 0.35 US\$).

*Keywords:* Renewable energy investments, Electricity markets, Stochastic portfolio optimisation, Financial risk management, Decision support system

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## 1. Introduction

Although the Latin America and Caribbean region is the region with by far the highest share of electricity generated from renewable sources worldwide [1], most of the technical potential in that region remains unexploited [2]. In 2012, around 66% of the region's electricity output was generated by renewable energy sources, with hydropower accounting for almost 93% of the renewables' share and for some 61% of total electricity generation [3]. While the region's hydropower sector, currently dominated by large-scale hydroelectric power plants, is already relatively mature [4], there is a vast potential for non-traditional and small-scale renewable energy sources such as wind (onshore and offshore), solar (PV and thermal), geothermal, bioenergy, and ocean [5]. A recent study [5] found that it would be sufficient to exploit only 1.6% of the region's technical potential for renewable energy use in order to meet its current demand for electricity, with 4% being sufficient to meet the estimated demand growth by 2050.

According to Brazil's National Energy Plan 2030 [6], the country's energy system, currently the largest in South-America, has enormous potential for expansion through further investment in renewable energy sources like small hydro (SH), wind power (WP), biomass (BIO), and solar. The government's most recent energy expansion plan [7] estimates that a total investment of R\$ 116.3 billions into these sources is required between 2014 and 2023 to achieve the predicted addition of 30 GW of renewable capacity. Brazil's National Electricity Regulatory Agency ANEEL [8] defines SH to be a hydroelectric power plant with an installed capacity between 1 MW and 30 MW. Usually several WP plants, or wind turbines, are grouped into a wind farm in order to scale-up electricity generation. BIO includes sugar cane bagasse, black liquor, eucalyptus trees [9], firewood, rice husk, and biogas. However these renewable energy sources, particularly SH and WP [10], are strongly characterised by their stochastic and seasonal generation profiles and these major sources of uncertainty adversely affect energy commercialisation.

Since its second stage of reform in 2004, Brazil's electricity sector offers two different environments for electricity procurement: the Regulated Contracting Environment (RCE), in which distributors acquire energy in a procedure regulated by the government [11], and the Free Contracting Environment (FCE), in which sellers and buyers freely and bilaterally negotiate contract terms [12]. Either environment can be used by Generation companies (Gencos) to sell energy and by traders to buy or sell energy. A short-term electricity spot market is used for the clearing of trades in both environments. However, spot prices in Brazil are highly

volatile and thus do not provide reliable economic incentives for new investments [11].

These uncertainties create a number of related challenges for Brazil’s renewable Gencos. For instance, given the fact that forward contracts must be cleared in the spot market, Gencos are exposed to “price-quantity risk” [11]. This occurs whenever the seller is “long” in contracts, i.e. cannot generate what the contract requires it to sell, thus requiring the missing energy to be bought on the volatile spot market at potentially high prices [13]. Contract prices in the RCE are generally lower than in the FCE. Reasons for this are that contracts in the RCE are sold through competitive auctions rather than the bilateral freely-negotiated contracts used in the FCE, and also RCE contracts usually contain clauses that reduce the risk to the Genco, which is uncommon in the FCE.

A number of studies have been presented in recent years that address strategic decision-making in the uncertain conditions of renewable energy investments and commercialisation in Brazil. With regard to energy commercialisation in the RCE, [14] presented a stochastic optimisation model providing hydroelectric Gencos with bidding strategies in multi-item iterative auctions of long-term contracts. This model takes into account the Gencos’ portfolios of existing contracts in order to “learn” from experiences in past auctions. This enables the model to generate bidding strategies for Gencos to hedge against risk. The application of such portfolio approaches to risk reduction is well documented in the literature on renewable energy investments (see [15] and [16] for overviews of methodologies and evaluation methods, respectively). Recent examples of applying portfolio approaches in the context of renewable energy sources on a country level include China [17], Denmark [18], Italy [19], and Japan [20].

A range of models have been presented with regard to energy commercialisation in the FCE. Recently, [21] proposed a stochastic optimisation model to identify the optimal energy commercialisation strategies for a Genco that owns a portfolio of already existing SH plants and can sell energy via contracts of 6-24 months’ duration. Smaller portfolio sizes are represented in a stochastic optimisation model in [22], in which the authors investigated the selection of a portfolio of trading strategies for SH and BIO (cogeneration from sugar cane bagasse) generation, thus combining two energy sources that have complementary (seasonal) availability. In contrast to [22], [13] analyses an energy generation portfolio composed of SH and WP. The authors present a new commercial model for a WP Genco based on a joint-selling strategy with an SH Genco, that exploits the well-known complementarity between the portfolio’s two renewable energy sources in order to hedge against price-quantity risk. Generation portfolios of three energy sources are represented in [23], in which the authors use a cooperative game approach to examine the competitiveness of SH, WP, and BIO competing for contracts within

the FCE.

The main contributions of our current paper are in the following four areas:

- 1) None of the previous studies have considered commercialisation strategies that use contracts from both Brazilian contract markets (FCE and RCE), and most of them consider only one or two of the available renewable energy sources (SH, WP, and BIO). The properties of the contracts in the two markets are different and the generation profiles of the renewable sources are different, so these restrictions limit the scope for controlling risk. In contrast, the current paper introduces a portfolio-based multi-market, multi-asset approach encompassing both contract markets and all three renewable energy sources.
- 2) None of the previous studies have investigated how the investors attitude to risk affects the optimal choice of portfolio composition and commercialisation strategy, but this is done in the current paper using the risk aversion parameters within a Conditional Value-at-Risk (CVaR) framework as the model of the decision maker's attitude towards risk.
- 3) We show in detail how to model currently used contract types in each market and how to incorporate these in a stochastic-optimisation-based tool for selecting the optimal portfolio of investments and commercialisation contracts for risk-averse investors.
- 4) We describe a test case based on realistic contract structures and real renewable data from the Brazilian system, and use this to demonstrate the advantage of the multi-market, multi-asset approach over the restricted choices of strategies considered in previous studies. We show how the two contract markets and three renewable sources can be combined to mitigate financial risks such as the price-quantity risk, whilst considerably increasing optimal values, and also give insights into why it is that our multi-market, multi-asset approach performs better at controlling risk. In addition, we explain how relative contract prices in the markets and the decision maker's attitude towards risk affect the optimal results.

It is important to note that multi-market selling strategies and contracts of these types are currently being used in the Brazilian power system and have already had an important role in driving the sharp expansion of renewables in Brazil, but this work provides the hard evidence of the benefits of the proposed multi-market strategy and the reasons for this.

Section 2 describes the approach that a holding company can apply to optimise its choice of renewable energy investments and financial contracts in both the RCE and FCE markets while controlling risk. A model is presented in Section 3 that maximises a holding company's risk-adjusted measure of value (Subsection 2.5) and gives the corresponding optimal size of SH, WP, and BIO renewable

energy projects as well as the optimal size of forward and call option contracts in the RCE and FCE. This portfolio optimisation model is then applied to an illustrative example representative of investment decisions within the Brazilian electricity system (Section 4). Results are presented and discussed in Section 5. Finally, some concluding remarks are provided in Section 6.

## 2. The investment and commercialisation problem

In this section, we present the approach taken in this paper to the portfolio-based multi-market, multi-asset investment problem.

### 2.1. Commercialisation of electricity in Brazil

In the reformed Brazilian electricity sector the total amount of energy sold by a Genco in any month through contracts in the RCE and FCE must not exceed its so-called “Firm Energy Certificates”, or certificates in shorthand [24]. These certificates are issued by the Ministry of Mines and Energy and represent the long-term average amount of power that can be relied upon from the plant (in avgMW) [25]. The RCE has contracts of two different types, which are sold in regulated public auctions. These are standard long-term forward contracts (i.e. Power Purchase Agreements, also known in Brazil as “quantity contracts” [26]), and call options, also known in Brazil as “availability contracts” [11]. In the case of forward contracts, the seller of the contract (e.g. a Genco), is contractually required to deliver a stipulated amount of energy for a fixed price to the buyer (e.g. a distribution company). In the event of delivering less than the stipulated amount, the seller must bear the costs for purchasing the missing quantity in the spot market. In the case of call options, the buyer “rents” the power plant, paying a monthly fixed price to the seller and takes all the generation the plant produces.

In the FCE, the purchasing and selling of electricity occurs through freely and bilaterally negotiated contracts [22]. Long-term contracts with structures similar to the RCE, may be used for contracting in the FCE. However, buyers and sellers must comply with the rules given in [27] for the commercialisation of energy in the FCE. One such rule is that, independent of the contract, trades must be cleared in the spot market either by the buyer or seller.

The prices used for clearing trades in Brazil are known as PLDs and referred to as “spot prices”, however they are not determined by equilibrium prices in a normal market. Instead they are calculated as short-run marginal costs in a cost-based central dispatch model NEWAVE [28], and published weekly ahead of time by the country’s electricity clearing house CCEE. For a comprehensive description of the NEWAVE model see [29]. The Brazilian system is subdivided into four interconnected submarkets (north, south, northeast, and southeast) and each has its own spot price [13].

## 2.2. Commercialisation strategy of a generation company

It is assumed here that every considered renewable energy investment project is carried out by a single Genco. A Genco can divide the energy it sells between the RCE, the FCE, and the spot market. Figure 1 gives an example of a commercialisation strategy for a Genco which could be obtained through a “new energy” auction. This is a “A-5”-type [30], which means that the Genco is contractually

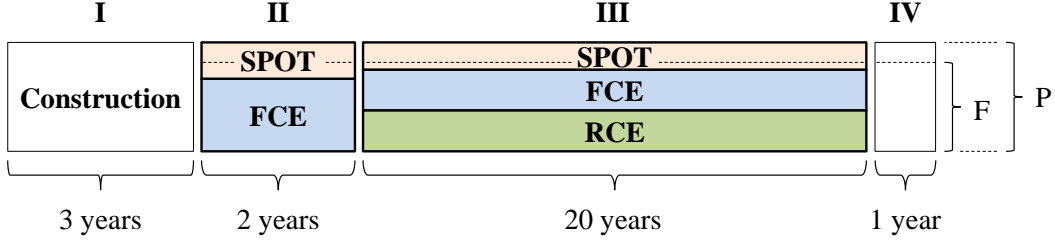


Figure 1: Multi-market strategy of a Genco.

required to start delivering the stipulated amount of energy five years after the auction has taken place, which provides some flexibility for the Genco’s commercialisation strategy.

After the power plant’s construction during the first three years of the project, the Genco can start to sell electricity and so begin to generate revenues at the start of the fourth-year, which is two-years ahead of the requirement of the RCE’s long-term contract. During this two-year period, as in [21], the generated electricity can be sold both via 2-year contracts in the FCE and via the spot market. In fact, the fraction of the generated electricity sold via the FCE can be interpreted as a strategic trade, whereas the uncontracted amount, which is necessarily sold in the spot market, serves as a hedge.

At the beginning of the sixth year of the project, the Genco may sell some of its energy via a 20-year contract within the RCE that was signed 5 years earlier. At the same time, however, the Genco may also start to sell a share of its generated electricity via 20-year forward contracts in the FCE, e.g. using a Power Purchase Agreement. In addition to these two 20-year contract options, an uncontracted amount of energy may be sold in the spot market as well. Although previous studies have considered the possibility to sell energy via both long-term contracts in the FCE and the spot market, none of these studies considered such a multi-market strategy encompassing the FCE, the RCE, and the spot market.

Finally, there is a one-year period (year 26) immediately following the end of commercialisation in which all residual financial requirements of payments from contractual obligations in the RCE are settled.

The total amount sold via contracts by a particular generator (with installed

capacity  $P$ ) must never exceed  $F$ , which is the certificate specified for that generator.

### 2.3. The holding company

The holding is the primary actor that invests in new renewable energy projects and commercialises the generated electricity in the FCE, the RCE, and the spot market. Figure 2 shows the structure of a holding consisting of  $m$  Gencos representing new renewable energy investment projects and a Portfolio Manager. A

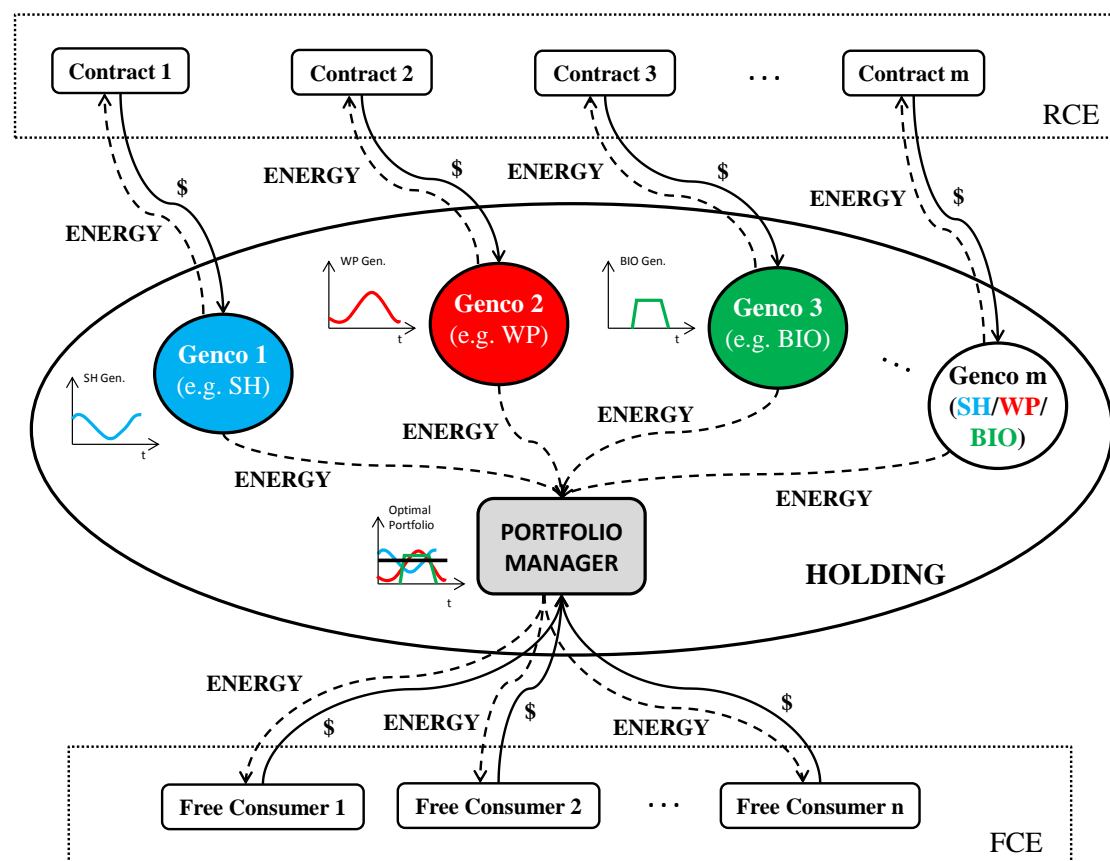


Figure 2: Structure of holding and its possibilities for commercialising energy via both the RCE and the FCE. *Note:* Incoming cash flows representing revenues from the sale of electricity are indicated by lines and dollar symbols. Outgoing cash flows representing investment and operation and maintenance costs as well as potential contractual penalties are not shown.

single Genco may be a SH, WP, or BIO Genco. The holding company has certain tax advantages compared to the sum of the parts operated separately, but in this paper the interest is in how it can be used to mitigate the risk of the combined portfolio of Gencos and contracts.



The holding company has contracts in the RCE and FCE. For simplicity in this study there is only one contract in the RCE per Genco resulting in  $m$  different contracts. The holding's Portfolio Manager signs  $n$  forward contracts with free consumers within the FCE. Any imbalance between the total amount contracted and generated has to be bought or sold in the appropriate spot market. In order to simplify this approach, it is assumed that all projects being considered start at the same time and have the time structure of Figure 1.

#### 2.4. Characterisation of uncertainties

The uncertainties considered in this study are the renewables' energy generation profiles (in MWh per month) and electricity spot prices (in R\$/MWh per month). These random variables are represented by a set of discrete scenarios with their corresponding probabilities. A single scenario consists of a time series for the number of months of generation in the plan (e.g. 264 months for the strategy in Figure 1), specifying for each month what the electricity spot prices are in each of Brazil's four submarkets and the predicted generation of the SH and WP generators that may be built. The model can be modified simply for any other desired base period.

The statistical model and the procedure for generating scenarios are the same as in [13] and [31], with the former presenting additionally a flow chart of the simulation procedure. The steps followed were:

1. Estimate parameters for lag 12 Vector AutoRegression with eXogenous variables (VARX) model (to capture variation within the 12 months of a year) fitting actual historic SH and WP generation as well as historic NEWAVE reservoir inflows.
2. Using the NEWAVE dispatch system first generate reservoir inflow scenarios, then for each of these calculate the corresponding spot prices in each of the four submarkets.
3. For each inflow scenario use the VARX model from Step 1 to generate a corresponding SH and WP generation sample.
4. Combine each spot price scenario from Step 2 with its corresponding generation sample from Step 3 to get a matching spot price and generation scenario.

#### 2.5. Measure of value

The risk measure,  $\rho(\cdot)$ , used in this study to quantify the investor's attitude towards risk is a convex combination of the (risk neutral) expectation operator

and the CVaR, that is:

$$\rho(\cdot) = \lambda \cdot \text{CVaR}_\alpha(\cdot) + (1 - \lambda) \cdot \mathbb{E}[\cdot], \quad (2.1)$$

where  $\lambda \in [0, 1]$  is the weighting of the single components, and  $\text{CVaR}_\alpha$  is defined to be the expected value of the  $(1 - \alpha)\%$  worst cases. The parameters  $\lambda$  and  $\alpha$  serve as risk aversion parameters chosen by the investor and effect the form of the risk measure: the closer  $\lambda$  is to 0, the less risk-averse the investor is, with  $\lambda = 0$  corresponding to a risk neutral investor, and as  $\alpha$  increases the risk measure gives weight to the more extreme bad cases [32]. CVaR, which is also called expected shortfall, and its useful properties were introduced in [33]. Its advantages are: it is a coherent risk measure that satisfies four desirable properties [34]; it has an intuitive economic interpretation allowing it to be used as a measure of value [32]; and it is convex and can be formulated as a linear programme allowing CVaR to be implemented in convex optimisation problems [33].

Unlike the approach taken in [13], which measures risk on a monthly time scale, in this paper we assume that the holding company is concerned with controlling risk over each entire year. Let the (positive or negative) net cash flow in month  $t \in \mathcal{T}$  of scenario  $s \in \mathcal{S}$  be denoted by  $V_{t,s}$ , where  $\mathcal{T}$  is the set of months,  $\mathcal{S}$  the scenario set, and  $p_s$  the probability of scenario  $s$ , and let the total net cash flow in year  $a \in \mathcal{T}^A$  discounted to the beginning of that year be denoted by  $V_{a,s}^A$ , where  $\mathcal{T}^A$  is the set of years. Then, for all  $a \in \mathcal{T}^A$ ,  $s \in \mathcal{S}$ :

$$V_{a,s}^A = \sum_{t \in m^A(a)} \frac{V_{t,s}}{(1 + K)^{t-12 \cdot (a-1)}}, \quad (2.2)$$

where  $m^A(a)$  denotes the set of months in year  $a$  and  $K$  represents the discount rate per month  $t$ . Applying the risk measure  $\rho(\cdot)$  to these annual values for all  $s \in \mathcal{S}$  gives:

$$V_a^A = \lambda \cdot \text{CVaR}_\alpha\left(\{V_{a,s}^A, p_s\}_{s \in \mathcal{S}}\right) + (1 - \lambda) \cdot \mathbb{E}\left[\{V_{a,s}^A, p_s\}_{s \in \mathcal{S}}\right], \quad (2.3)$$

which can be interpreted as the measure of value for year  $a$ . The terms defining the annual measure of value in (2.3) are all linear in the variable except for the  $\text{CVaR}_\alpha$  term. As shown in [33], the value of  $\text{CVaR}_\alpha\left(\{V_{a,s}^A, p_s\}_{s \in \mathcal{S}}\right)$  is the optimal

objective value of the following problem:

$$\max_{z, \delta} \quad z_a - \frac{1}{1 - \alpha} \cdot \sum_{s \in \mathcal{S}} p_s \cdot \delta_{a,s} \quad (2.4)$$

$$\text{s.t.} \quad \delta_{a,s} \geq z_a - \sum_{t \in m^A(a)} \frac{V_{t,s}}{(1 + K)^{t-12 \cdot (a-1)}}, \quad s \in \mathcal{S} \quad (2.5)$$

$$\delta_{a,s} \geq 0, \quad s \in \mathcal{S}. \quad (2.6)$$

The expected value for year  $a$  is given by:

$$\mathbb{E} \left[ \{V_{a,s}^A, p_s\}_{s \in \mathcal{S}} \right] = \sum_{s \in \mathcal{S}} \sum_{t \in m^A(a)} \frac{V_{t,s}}{(1 + K)^{t-12 \cdot (a-1)}} \cdot p_s. \quad (2.7)$$

The measure of value for the entire investment,  $V$ , can then be obtained by taking the net present value (NPV) of these annual values giving:

$$V = \sum_{a \in \mathcal{T}^A} \frac{V_a^A}{(1 + K^A)^{a-1}}, \quad (2.8)$$

where  $K^A$  is the discount rate per year  $a$ .

### 3. Mathematical model

This section contains the mathematical formulation of the previously presented approach as a stochastic portfolio optimisation problem. The notation used is listed in the Nomenclature section at the end.

#### 3.1. Assumptions

The general assumptions made to allow a simplified implementation of the approach are as follows:

- The payment of taxes (e.g. see [35]) and the Brazilian transmission (TUST) and distribution (TUSD) fees are not included within the model.
- No plant's installed capacity can exceed 30 MW, which allows it to be classified as an incentivised renewable energy source.
- Contracts in the FCE are standard forward contracts that must be cleared in the spot market by the holding's Portfolio Manager.
- The amount contracted to be delivered by each contract in either the RCE or the FCE is the same for all months covered by the contract.

- It is assumed that all decision variables take continuous values. In order to overcome the issue that the size of a power plant may not be chosen arbitrarily in reality (e.g. economies of scale), specifically in cases where the optimal plant size would be less than the “standardly available” or “off-the-shelf” size, it is assumed that the holding company can buy any proportion of the plant, with the remainder being purchased by other companies, the costs and outputs being shared proportionally.

### 3.2. Cash flow model

The holding’s cash flows (in R\$) occurring during the investment horizon given by  $\mathcal{T}$  can be subdivided into two main components: a deterministic component that solely represents a cost cash flow and a stochastic component modelling commercialisation.

The deterministic component is represented by the cash flow  $C_t$  defined in (Appendix B) and characterises costs due to capital expenditures in terms of investments costs for the power plant and costs for operation and maintenance in month  $t$ .

The second component is stochastic and models the cash flows related to the commercialisation of energy in the FCE, the RCE, and the spot market. These different strategies are subdivided into three parts. Note that since there can be no generation until the plant is built,  $R_{t,s}$  must be zero  $\forall t \in \mathcal{T}^I, s \in \mathcal{S}$ .

In the first part, standard forward contracts  $i$  for commercialisation in the FCE are modelled by using a similar structure to the one presented by [14]. For all  $t \in \mathcal{T}^{II}, s \in \mathcal{S}$ :

$$R_{t,s} = \sum_{i \in \mathcal{C}^{\text{FCE}}} \left[ \left( p_i^{\text{FCE},II} - \pi_{i,t,s}^C \right) \cdot Q_i^{\text{FCE},II} \cdot h_t \right] + \sum_{j \in \mathcal{P}} \left[ \pi_{j,t,s}^P \cdot g_{j,t,s} \cdot F_j \cdot h_t \right] \quad (3.1)$$

In other words, the holding’s Portfolio Manager receives the contract price  $p_i^{\text{FCE},II}$  for the contracted amount  $Q_i^{\text{FCE},II} \cdot h_t$ , and, at the same time, buys the contracted amount  $Q_i^{\text{FCE},II} \cdot h_t$  in the spot market where contract  $i$  is concluded (and hence where the associated free consumer is located) and sells the amount  $g_{j,t,s} \cdot F_j \cdot h_t$  generated by plant  $j$  in month  $t$  and scenario  $s$  in the spot market where plant  $j$  is located.

The second part describes the stochastic cash flow when the contracting of energy is possible via both the FCE and the RCE. It thus extends the stochastic cash flow’s mathematical formulation in (3.1) by adding a stochastic function  $R_{j,t,s}^{\text{RCE}}$  that represents the stochastic cash flow of a contract in the RCE of plant  $j$  in month

$t$  and scenario  $s$ . For all  $t \in \mathcal{T}^{III}$ ,  $s \in \mathcal{S}$ :

$$R_{t,s} = \sum_{i \in \mathcal{C}^{\text{FCE}}} \left[ \left( p_i^{\text{FCE},III} - \pi_{i,t,s}^C \right) \cdot Q_i^{\text{FCE},III} \cdot h_t \right] + \sum_{j \in \mathcal{P}} \left[ R_{j,t,s}^{\text{RCE}} + \pi_{j,t,s}^P \cdot g_{j,t,s} \cdot \left( F_j - F_j^{\text{RCE}} \right) \cdot h_t \right] \quad (3.2)$$

The form of the stochastic function  $R_{j,t,s}^{\text{RCE}}$  (which depends on  $F_j^{\text{RCE}}$ ) is complex and so is omitted (see Appendix C for examples). This novel multi-market approach intuitively follows from the structure of commercialisation shown by Figure 2. The novelty of this model is it enables commercialisation of energy through a multi-market selling strategy.

The third part exclusively represents the potential charges due to the contractual obligations (being contractually required to pay for a potential “under-performance”) of call options in the RCE:

$$R_{t,s} = \sum_{j \in \mathcal{P}^{\text{WP}}} R_{j,t,s}^{\text{RCE}}, \quad \forall t \in \mathcal{T}^{IV}, s \in \mathcal{S} \quad (3.3)$$

The net cash flow in month  $t$  of scenario  $s$  is  $V_{t,s} = -C_t + R_{t,s}$ , where  $C_t$  is the cash flow representing cost, which is independent of scenario, and  $R_{t,s}$  is the cash flow representing revenue, which is scenario dependent.

### 3.3. Contracting constraints

As mentioned in Subsection 2.2, the certificate available for a plant represents an upper bound for the amount that might be contracted. The constraints corresponding to the forward contracts modelled in (3.1), (3.2), and (C.1) are given by (3.4), (3.5), and (3.6), respectively.

$$0 \leq \sum_{i \in \mathcal{C}^{\text{FCE}}} Q_i^{\text{FCE},II} \leq \sum_{j \in \mathcal{P}} F_j \quad (3.4)$$

$$0 \leq \sum_{i \in \mathcal{C}^{\text{FCE}}} Q_i^{\text{FCE},III} \leq \sum_{j \in \mathcal{P}} \left( F_j - F_j^{\text{RCE}} \right) \quad (3.5)$$

$$0 \leq Q_j^{\text{RCE}} \leq F_j^{\text{RCE}}, \quad \forall j \in \mathcal{P}^{\text{SH}} \quad (3.6)$$

However, there is also an upper bound  $F_j^{\text{max}}$  for  $F_j$  of every plant  $j$ . If  $F_j$  equals zero for any plant  $j$ , this simply means that the plant is not constructed.

$$0 \leq F_j^{\text{RCE}} \leq F_j \leq F_j^{\text{max}}, \quad \forall j \in \mathcal{P} \quad (3.7)$$

Moreover, every contract  $i$  in the FCE during major section S must satisfy the

obvious constraints ensuring non-negativity of contracted amounts:

$$Q_i^{\text{FCE},\mathcal{S}} \geq 0, \quad \forall i \in \mathcal{C}^{\text{FCE}} \quad (3.8)$$

### 3.4. Optimisation model

The overall formulation for optimising the holding's portfolio (i.e. maximising the objective function (2.8) subject to a number of constraints) is given below.

#### Objective

$$\max \sum_{a \in \mathcal{T}^A} \frac{\lambda \cdot \left( z_a - \frac{1}{1-\alpha} \cdot \sum_{s \in \mathcal{S}} p_s \cdot \delta_{a,s} \right) + (1-\lambda) \cdot \sum_{s \in \mathcal{S}} \sum_{t \in m^A(a)} \frac{-C_t + R_{t,s}}{(1+K)^{t-12 \cdot (a-1)}} \cdot p_s}{(1+K^A)^{a-1}} \quad (3.9)$$

#### Constraints

$$\delta_{a,s} \geq z_a - \sum_{t \in m^A(a)} \frac{-C_t + R_{t,s}}{(1+K)^{t-12 \cdot (a-1)}}, \quad \forall a \in \mathcal{T}^A, s \in \mathcal{S} \quad (3.10)$$

$$\delta_{a,s} \geq 0, \quad \forall a \in \mathcal{T}^A, s \in \mathcal{S} \quad (3.11)$$

(3.4) – (3.8)

Note that  $C_t$  and  $R_{t,s}$  are linear functions of the variables and can be eliminated from above objective and constraints using equations (B.2), (3.1)-(3.3) and (C.1)-(C.3).

## 4. Numerical example

This section describes the computational implementation of the optimisation model and subsequently assigns numerical values to deterministic and stochastic input variables representative of the Brazilian local conditions.

### 4.1. Model implementation

The stochastic optimisation model was implemented in Xpress.

### 4.2. Deterministic input values

In this numerical example, the structure of the Genco is as shown in Figure 1 and corresponds to the time periods given in Appendix A. Moreover, it will consider three renewable energy investment projects, one in each of the available technologies. Each of these has the possibility to sell some energy via the RCE. The holding's Portfolio Manager may decide to commercialise energy via four contracts

in the FCE, one in each of the four existing submarkets, which are southeast (SE), south (S), northeast (NE), and north (N). The consideration of only four contracts, one in each submarket, is done to simplify this numerical example. A summary of sets and their corresponding elements is given by Tables 1 and A.4.

Set	Element(s) of set
$\mathcal{P}$	{SH,WP,BIO}
$\mathcal{P}^{\text{SH}}$	{SH}
$\mathcal{P}^{\text{WP}}$	{WP}
$\mathcal{P}^{\text{BIO}}$	{BIO}
$\mathcal{C}^{\text{FCE}}$	{SE,S,NE,N}

Table 1: Summary of chosen elements for the sets.

None of the available renewable technologies considered is available throughout the year, so the long-term average annual generation ( $F_j$  in avgMW) of plant  $j$  with 30 MW installed capacity is significantly below 30 MW. The investment costs of plants were obtained by scaling their avgMW capacity  $F_j$  to the corresponding installed capacity and multiplying that by the plants' unit investment costs, which are R\$ 4.0 millions, 3.5 millions, and 3.0 millions per MW<sub>installed</sub> for SH, WP, and BIO, respectively. Also, instead of assuming a constant 730 h in every month, we use the actual number of hours in each month, starting with January 2012.

Sugar cane bagasse, the fuel for BIO generation in Brazil, is available only during the months from May to November, and it allows these plants to operate at their full capacity of 30 MW during this period. The average annual generation can thus be determined as 30 MW $\cdot$ 7/12 = 17.5 avgMW. Hence, given this deterministic generation profile of BIO,  $g_{j,t,s}$  amounts to 30/17.5 = 1.71 for the months May to November of each year, and 0 otherwise.

Results of recent auctions for new energy ("A-5") have indicated that prices in the RCE are likely to be around R\$ 130/MWh. We assume that prices in the FCE for the initial 2-year period and the following 20-year period are equal with  $p_i^{\text{FCE,II}} = p_i^{\text{FCE,III}}$  for every contract  $i \in \mathcal{C}^{\text{FCE}}$ . This is a simplification: since prices in the FCE are a consequence of bilateral negotiations different contracts can have different prices and the prices can vary over time.

Table 2 shows a summary of the chosen values for the deterministic parameters used within the optimisation model.

### 4.3. Generated scenarios

The generation of scenarios for SH and WP and the electricity spot prices for Brazil's four submarkets was completed using the statistical model and procedure for scenario generation described in Subsection 2.4. Further input data was

Parameter	Index	Value	Unit
$F_j^{\max}$	$j \in \mathcal{P}^{\text{SH}}$	17.22	avgMW
	$j \in \mathcal{P}^{\text{WP}}$	14.89	avgMW
	$j \in \mathcal{P}^{\text{BIO}}$	17.5	avgMW
$I_j^0$	$j \in \mathcal{P}^{\text{SH}}$	6,968,609	R\$/avgMW
	$j \in \mathcal{P}^{\text{WP}}$	7,050,425	R\$/avgMW
	$j \in \mathcal{P}^{\text{BIO}}$	5,142,857	R\$/avgMW
$c_j^F$	$j \in \mathcal{P}$	0	R\$/avgMW
$e_j^R$	$j \in \mathcal{P}$	30%	–
$LT_j$	$j \in \mathcal{P}$	14	–
$i_j$	$j \in \mathcal{P}$	7%	–
$\{h_t\}_{t=37,38,\dots,299,300}$		$\{744, 696, \dots, 720, 744\}$	h
$K^A$		10%	–
$K$		0.7974%	–
$\lambda$		0.9	–
$\alpha$		0.95	–

Table 2: Chosen values for the deterministic parameters.

provided by a Brazilian electricity company that is licensed to run the country’s dispatch model NEWAVE. Historical data for the period from January 1981 to December 2011 was used to estimate the parameters of the VARX model. More precisely, in order to ensure comparability with previous studies [13] (considered SH,WP) [22] (SH,BIO) [23] (SH,WP,BIO), historical data of the inflows from the “Paraibuna” river, located in the southeast of Brazil, and of the generation of the “Icaraizinho” wind farm, located in Brazil’s northeast, were used. We consider SHs to be run-of-river, with output that is not controllable. This historical data allowed the determination of the maximum certificate of both SH and WP as given in Table 2.

Using this statistical model, we subsequently generated 2,000 equally likely scenarios for renewable energy generation with  $|\mathcal{T}^{II} \cup \mathcal{T}^{III}| = 264$  and  $p_s = 1/2000$ . Given that the dispatch model’s time horizon is limited to a maximum of 10 years, or 120 months, yet required scenarios for a 22-year period, the same data was used for the second 10-year period and the first two years of the 10-year scenarios for the 21<sup>st</sup> and 22<sup>nd</sup> year of energy production. Figure 3 shows relative generation level  $g$  for 50 scenarios for a SH plant in Brazil’s southeast area during the first twelve months of commercialisation as well as the average generation over all 2,000 scenarios for this same period. Figure 4 shows corresponding relative generation levels for a WP plant in Brazil’s northeast area.

The generated scenarios for the electricity spot prices of Brazil’s four submar-



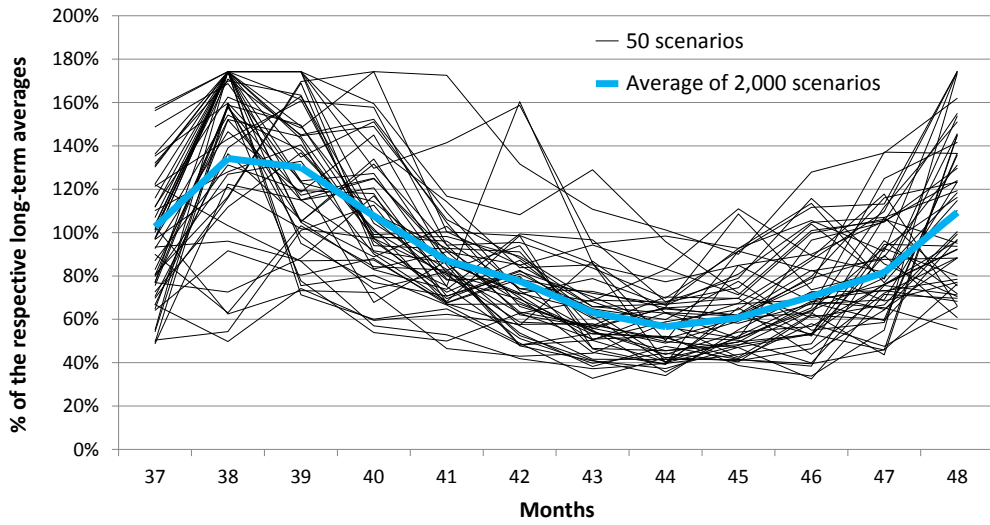


Figure 3: Selection of 50 equally likely generation scenarios for the first twelve months of commercialisation of an SH plant in the southeastern area of Brazil (in % of the maximum certificate).

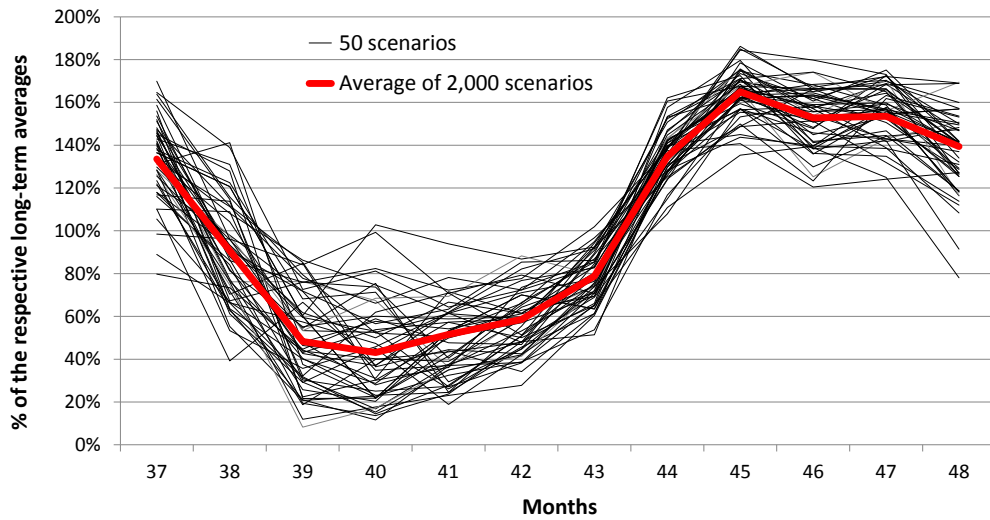


Figure 4: Selection of 50 equally likely generation scenarios for the first twelve months of commercialisation of a WP plant in the northeastern area of Brazil (in % of the maximum certificate).

kets need to be processed before they can be applied in our optimisation model. Brazil’s electricity clearing house CCEE publishes [27] minimum and maximum values for spot prices in Brazil. For the year 2013 these bounds were R\$ 14.13 and 780.03 per MWh. Prior to their inclusion in the model, scenarios produced by the dispatch model over the 10-year period were “filtered” to round values outside these bounds to the bounds. The spot prices in Brazil are highly volatile and have no recognisable pattern, so are not plotted here. See [13] for an example.

## 5. Results and discussion

It was claimed in the Introduction that a risk-averse holding company investing in new renewable energy investment projects can mitigate financial risks not only through making use of the complementarity of the considered renewable sources' generation profiles, but also through a multi-market commercialisation strategy. This section begins with an analysis of the way in which the holding's optimised portfolio composition and commercialisation strategy depend on the price of energy contracts in the FCE. The RCE contract price is fixed at R\$ 130/MWh, and all the contracts in the FCE have the same price in the range from R\$ 70 to 170 per MWh. For a fixed risk aversion parameter  $\lambda$  of 0.9, Figures 5 and 6 show plots of the optimised portfolio compositions and optimal commercialisation strategies as a function of this FCE contract price.

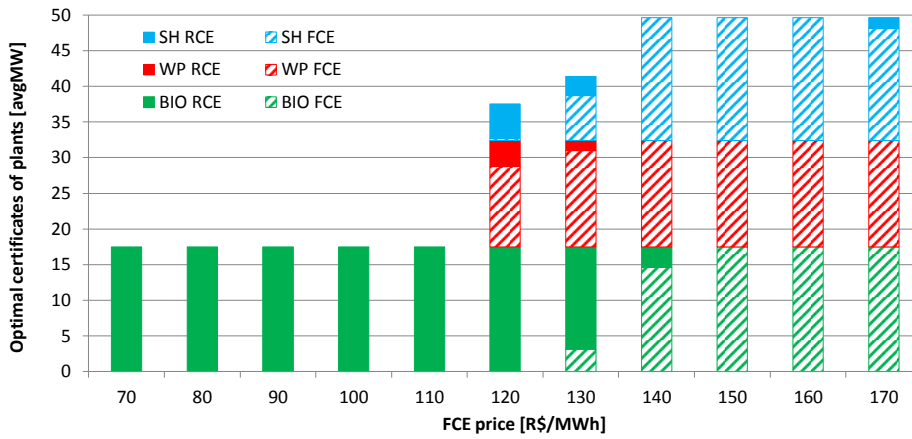


Figure 5: Optimal investment strategies over a range of FCE contract prices.

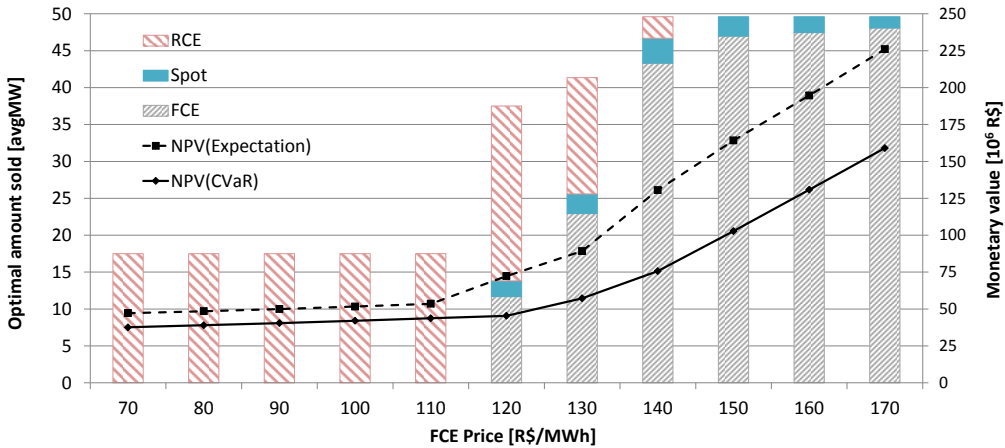


Figure 6: Optimal commercialisation strategies over a range of FCE contract prices.

It can be seen from Figure 5 that the amount of BIO installed is independent of the FCE price. This is because the investment costs of BIO are comparatively low and the contract conditions in the RCE are favourable (it receives revenues even in months where its actual generation is zero), so the full amount available is always installed. The maximum possible amount of WP is installed for FCE prices of R\$ 130/MWh and above and the maximum amount of SH for FCE prices of R\$ 140/MWh and above. In fact, SH and WP coexist in the portfolio for FCE prices of R\$ 120/MWh and above. The reason for their coexistence is that they have complementary generation profiles; this allows them to jointly generate an almost constant generation profile, thus leaving them less exposed to price-quantity risk.

However, the plants' shares of both the RCE and FCE are affected quite differently as we vary the energy price of contracts in the FCE. This is mainly due to the difference in the contract rules for SH and WP in the RCE and FCE environments. Forward contracts for SH are similar in both environments, so the SH plant will sell its energy mostly via the contract that offers the higher price, and will therefore sell a bigger share in the FCE as the FCE contract price rises. However in the case of WP, the reason for the share of the RCE reducing as the FCE price rises is mainly because of contractual penalties resulting from call options in the RCE, but is also because no energy can be sold in the spot market, which removes opportunities of hedging there.

It can be seen from Figure 6 that as the FCE price rises, the FCE share of the holding's certificates sold increases and eventually eliminates the RCE share. Interestingly, for the higher contract prices in the FCE, there remains an uncontracted amount of energy that is sold in the spot market, and this is used as a hedge. Figure 6 also shows the NPV of  $CVaR_\alpha$  and the NPV of the expected profit, which together form the parts of the convex combination used within the linear programme's objective function.

In addition to the economic advantages to the holding from the complementary availability of the different renewable energy sources, the use of contracts from both markets has added significant extra value. Table 3 shows the optimal value of the objective function (3.9) for different strategies for selling energy and different values for the risk aversion parameter  $\lambda$ . In all cases contract prices in the RCE and FCE were fixed to R\$ 130 and 120 per MWh, respectively. Interestingly, having the opportunity to sell some energy via the FCE in the two years prior to the start of the RCE's long-term contract (as in  $RCE^b$ ) already results in an optimal measure of value on average some 36% higher than in  $RCE^a$ , where all the energy generated during this two-year period has to be sold in the spot market alone. Comparing the optimal values of the single-market strategies,  $RCE^b$  and  $FCE$ , justifies previously noted tendencies: risk-seeking behaviour ( $\lambda = 0.1$ ) results in the  $FCE$  strategy providing higher objective values, while more risk-averse behaviour ( $\lambda = 0.9$ ) puts

Table 3: Optimal values (in R\$ millions) of the objective function and its single components – order  $\text{CVaR}_\alpha(\cdot)$ ,  $\mathbb{E}[\cdot]$ ,  $\rho(\cdot)$  – over a range of risk aversion parameters  $\lambda$  and given different commercialisation strategies.

Strategy	$\lambda = 0.1$			$\lambda = 0.5$			$\lambda = 0.9$		
$RCE^a$	-1.43	62.59	<b>56.19</b>	32.67	49.40	<b>41.04</b>	32.67	49.40	<b>34.35</b>
$RCE^b$	-38.47	88.96	<b>76.22</b>	36.63	75.39	<b>56.01</b>	45.52	55.51	<b>46.52</b>
$FCE^\dagger$	-0.14	88.31	<b>79.46</b>	19.36	80.47	<b>49.91</b>	21.76	75.98	<b>27.18</b>
$RCE\&FCE^\ddagger$	3.97	91.03	<b>82.32</b>	43.58	75.68	<b>59.63</b>	45.39	72.31	<b>48.09</b>

<sup>a</sup> Commercialisation via spot market in year 4 and 5, then spot and RCE for years 6 to 25.

<sup>b</sup> Commercialisation via spot market and FCE in year 4 and 5, then spot and RCE for years 6 to 25.

<sup>†</sup> Commercialisation via spot market and FCE in year 4 and 5, then spot and FCE for years 6 to 25.

<sup>‡</sup> Commercialisation via spot market and FCE in year 4 and 5, then spot, FCE and RCE for years 6 to 25.

the  $RCE^b$  strategy in front.

Applying the multi-market strategy and thus allowing the holding and its single Gencos to sell energy via both the RCE and the FCE results in significantly higher optimal values for all three  $\lambda$ -values under consideration. When compared with the best performing single-market strategy, optimal values achieved by the multi-market strategy  $RCE\&FCE$  are 3.60%, 6.46%, and 3.37% higher for the three risk aversion parameters 0.1, 0.5, and 0.9, respectively. On the other hand, when compared with the worst-performing single-market strategy, the multi-market strategy achieved even higher increases of 46.51%, 45.31%, and 76.94%, respectively. Since multi-market strategies include single-market strategies as special cases, it follows that an optimal multi-market strategy will always be at least as good as any single-market strategy. However, the above results show how significant a gain can be obtained by using the multi-market approach introduced in this paper compared to the single-market approaches of previous studies.

The dependence of the optimal portfolio composition on the actual value of the risk aversion parameter  $\lambda$ , is particularly important for an investor in understanding the consequences of different risk attitudes. To illustrate this dependency Figures 7 and 8 show for fixed contract prices in the RCE and FCE of R\$ 130 and 120 per MWh, the way in which the holding's optimal investment and commercialisation strategies depend on the risk aversion parameter  $\lambda$ . Increasing  $\lambda$  from 0.001 to 0.999, (i.e. moving from a risk-neutral attitude to a risk-averse one) generally results in a smaller portfolio and in a selling strategy that uses comparatively less contracts, but more spot market. At the same time, when interpreting the difference between the NPV of the expectation and the NPV of  $\text{CVaR}_\alpha$  as some kind of volatility (i.e. risk), the optimal portfolio becomes less risky and uncertain

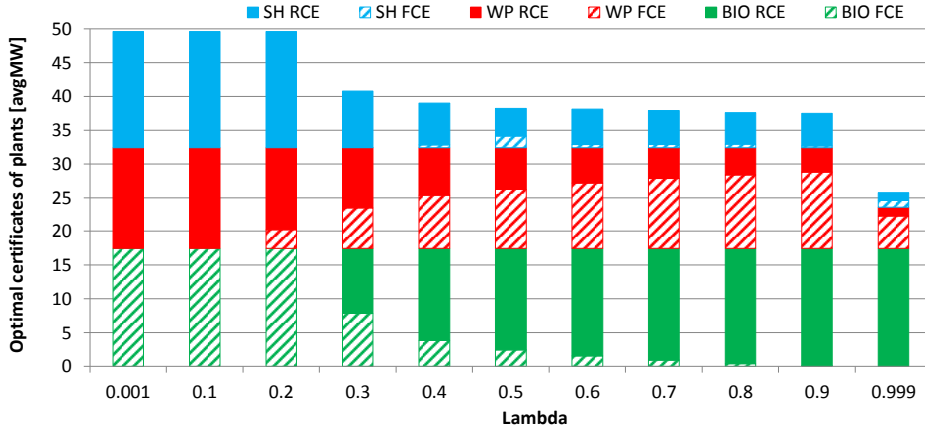


Figure 7: Optimal investment strategies over a range of risk aversion parameters  $\lambda$ .

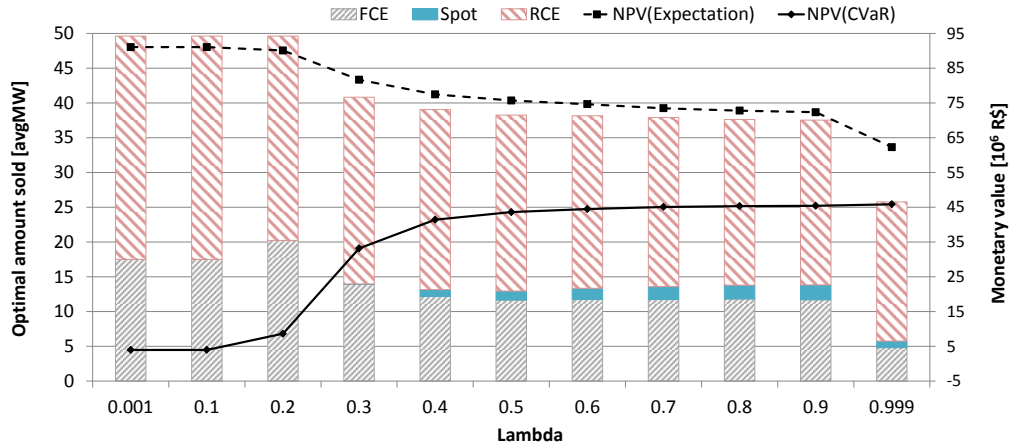


Figure 8: Optimal commercialisation strategies over a range of risk aversion parameters  $\lambda$ .

when  $\lambda$  is increased. Beginning at a  $\lambda$  of 0.2 (0.3), WP's RCE (BIO's FCE) share is squeezed out of the portfolio and almost fully replaced by contracts in the FCE (RCE). As argued above, the reason for this is mainly due to the contracts, which are modelled differently in the FCE and RCE. For instance, BIO hugely benefits from the design of its call option in the RCE, which enables it to generate revenues even in months where its actual generation is zero. With a very high value of  $\lambda$  equalling 0.999, further risk reduction can only be achieved by significantly reducing the size of both the SH and WP plant. This somewhat surprising result is due to the fact that there is a finite portfolio of energy sources in the model, so it is not possible to replace one energy source with more of another one.

## 6. Conclusion

This paper presents a new portfolio-based approach for selecting long-term investments in small-scale renewable energy projects and matching contracts for the sale of the resulting electricity. The approach is illustrated by applying it to an illustrative example representative of investment decision within the Brazilian energy market. Unlike previously published studies, which have either only considered two of the three available renewable energy sources (SH, WP, and BIO), or only one of the two available Brazilian contract markets (FCE and RCE) for commercialisation of energy, this study has developed a portfolio-based multi-market, multi-asset investment approach encompassing both markets and all available renewable energy sources.

This research has shown that under the current regulatory framework for energy commercialisation in Brazil, a holding company investing in new renewable energy investment projects can mitigate financial risks such as the price-quantity risk not only through making use of the complementarity of the energy generation profiles of SH and WP, but also through a multi-market selling strategy. The numerical example shows that such a strategy can add considerable economic value to the holding's business. In fact, it is shown that, when compared with the best-performing single-market selling strategies such as the ones considered in previously published studies, applying the proposed multi-market strategy results in 3.60%, 6.46%, and 3.37% higher optimal values for the three risk aversion parameters 0.1, 0.5, and 0.9, respectively. When compared with the worst-performing single-market strategy, on the other hand, the proposed multi-market strategy achieved even higher increases of 46.51%, 45.31%, and 76.94%, respectively. It is important to note that our proposed multi-market commercialisation strategy will always be at least as good as any single-market strategy.

We also investigated the sensitivity of the optimal portfolio composition subject to an alteration of the investor's attitude towards risk. Through altering the risk aversion parameter used within the measure of value we found that acting in a more risk-averse way almost always results in smaller portfolios and in comparatively less energy sold via long-term contracts in both the RCE and FCE, while the spot market becomes increasingly important for exploiting hedging opportunities. This new portfolio-based approach is relevant to a risk-aware holding company wishing to build an optimal investment portfolio that potentially consists of all three important renewable energy sources and uses financial instruments (long-term forward contracts and call options) in the FCE and RCE contract markets, as well as balancing sales and purchases in the the spot market.

## Acknowledgements

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## Appendix A. Time sets for numerical example

Considering the structure presented by Figure 1 and using the notation of Subsection Appendix C results in the sets shown by Table A.4.

Set	Element(s) of set
$\mathcal{T}$	$\{1, 2, \dots, 311, 312\}$
$\mathcal{T}^I$	$\{1, 2, \dots, 35, 36\}$
$\mathcal{T}^{II}$	$\{37, 38, \dots, 59, 60\}$
$\mathcal{T}^{III}$	$\{61, 62, \dots, 299, 300\}$
$\mathcal{T}^{IV}$	$\{301, 302, \dots, 311, 312\}$
$\mathcal{T}^A$	$\{1, 2, \dots, 25, 26\}$
$\mathcal{T}^{I,A}$	$\{1, 2, 3\}$
$\mathcal{T}^{II,A}$	$\{4, 5\}$
$\mathcal{T}^{III,A}$	$\{6, 7, \dots, 24, 25\}$
$\mathcal{T}^{IV,A}$	$\{26\}$
$\mathcal{T}^Q$	$\{1, 2, 3, 4\}$

Table A.4: Elements for the sets corresponding with the structure of Figure 1.

Then, by applying the functions we get for example:

$$y(75) = 7, \quad m^A(7) = \{73, 74, \dots, 83, 84\},$$

$$\underline{m}^A(7) = 73, \quad \overline{m}^A(7) = 84, \quad q(7) = 2,$$

and

$$y(299) = 25, \quad m^A(25) = \{289, 290, \dots, 299, 300\},$$

$$\underline{m}^A(25) = 289, \quad \overline{m}^A(25) = 300, \quad q(25) = 4.$$

## Appendix B. Deterministic cash flow component

The investment costs are calculated on an annual basis and we model this by assuming they are due on the first month of the corresponding year, i.e. the month where  $t = \underline{m^A}(y(t))$ . The structure for external financing is based on the ‘‘Project Finance’’ programme of the Brazilian development bank BNDES. The investment cash flow  $I_{j,t}^{inv}$  in plant  $j$  in month  $t$  was adapted from [35] and is given by:

$$I_{j,t}^{inv} = \begin{cases} I_j^0 \cdot e_j^R & \text{if } (y(t) = 1) \text{ and } t = \underline{m^A}(y(t)) \\ I_j^0 \cdot (1 - e_j^R) \cdot \left[ \left(1 - \frac{y(t)-2}{LT_j}\right) \cdot i_j + \frac{1}{LT_j} \right] & \text{if } (2 \leq y(t) \leq LT_j + 1) \text{ and } t = \underline{m^A}(y(t)) \\ 0 & \text{otherwise.} \end{cases} \quad (\text{B.1})$$

Combining the investment cost cash flow with the monthly fixed costs for operation and maintenance,  $c_j^F$  of plant  $j$ , the deterministic component  $C_t$  in month  $t$  is given by:

$$C_t = \begin{cases} \sum_{j \in \mathcal{P}} I_{j,t}^{inv} \cdot F_j & \text{if } t \in \mathcal{T}^I \\ \sum_{j \in \mathcal{P}} [I_{j,t}^{inv} + c_j^F] \cdot F_j & \text{if } t \in \mathcal{T}^{II} \cup \mathcal{T}^{III} \\ 0 & \text{otherwise.} \end{cases} \quad (\text{B.2})$$

## Appendix C. Modelling of contracts in the RCE

- RCE contracts formulated according to most recent ‘‘A-5’’ new energy auction in December 2012 [30].
- SH use a forward ‘‘quantity’’ contract.
- BIO & WP use ‘‘availability’’ contracts.
- For the stochastic cash flow of SH:

$$R_{j,t,s}^{\text{RCE}} = \left[ (p_j^{\text{RCE}} - \pi_{j,t,s}^P) \cdot Q_j^{\text{RCE}} + \pi_{j,t,s}^P \cdot g_{j,t,s} \cdot F_j^{\text{RCE}} \right] \cdot h_t, \quad \forall j \in \mathcal{P}^{\text{SH}}, t \in \mathcal{T}^{III}, s \in \mathcal{S} \quad (\text{C.1})$$

- For the deterministic cash flow of BIO:

$$R_{j,t,s}^{\text{RCE}} = p_j^{\text{RCE}} \cdot h_t \cdot F_j^{\text{RCE}}, \quad \forall j \in \mathcal{P}^{\text{BIO}}, t \in \mathcal{T}^{III}, s \in \mathcal{S} \quad (\text{C.2})$$

- For the stochastic cash flow of WP, for all  $j \in \mathcal{P}^{\text{WP}}, s \in \mathcal{S}$ :



$$R_{j,t,s}^{\text{RCE}} = \begin{cases} F_j^{\text{RCE}} \cdot \left( p_j^{\text{RCE}} \cdot h_t + R_{j,t,s}^{\text{SPOT}} - RI_{j,t,s}^A - RI_{j,t,s}^Q \right) & \text{if } t \in \mathcal{T}^{III} \\ F_j^{\text{RCE}} \cdot \left( -RI_{j,t,s}^A - RI_{j,t,s}^Q \right) & \text{if } t \in \mathcal{T}^{IV}, \end{cases} \quad (\text{C.3})$$

where the terms in brackets are parameters and defined as follows.

$$R_{j,t,s}^{\text{SPOT}} = \begin{cases} Q_{j,y(t),s}^{\text{SPOT}} \cdot \sum_{k=\underline{m}^A(y(t))}^{\overline{m}^A(y(t))} \frac{\pi_{j,k,s}^P}{|m^A(y(t))|} & \text{if } t \in \{\overline{m}^A(b)\}_{b \in \mathcal{T}^{III}} \\ 0 & \text{otherwise,} \end{cases} \quad (\text{C.4})$$

is the revenue from the spot market for plant  $j$  in month  $t$  and scenario  $s$ .

$$Q_{j,y(t),s}^{\text{SPOT}} = \max \left\{ \sum_{k=\underline{m}^A(y(t))}^{\overline{m}^A(y(t))} \left[ \left( B_{j,y(t),s}^{\text{INI}} + g_{j,k,s} - \delta_{q(y(t))} \right) \cdot h_k \right], 0 \right\}, \quad (\text{C.5})$$

is the amount sold in the spot market by plant  $j$  in year  $y(t)$  and scenario  $s$ .

$$B_{j,y(t),s}^{\text{INI}} = \begin{cases} B_{j,y(t)-1,\overline{m}^A(y(t)-1),s}^{\text{ACC}} - 1 & \text{if } \sum_{k=\underline{m}^A(y(t)-1)}^{\overline{m}^A(y(t)-1)} g_{j,k,s} \geq 90\% \\ B_{j,y(t)-1,s}^{\text{INI}} - 0.1 & \text{if } \sum_{k=\underline{m}^A(y(t)-1)}^{\overline{m}^A(y(t)-1)} g_{j,k,s} < 90\% \\ 0 & \text{if } q(y(t)) = 1, \end{cases} \quad (\text{C.6})$$

is the “initial” balance of plant  $j$  in year  $y(t)$  and scenario  $s$ .

$$B_{j,y(t),t,s}^{\text{ACC}} = \min \left\{ B_{j,y(t),s}^{\text{INI}} + \frac{\sum_{k=\underline{m}^A(y(t))}^{\overline{m}^A(y(t))} g_{j,k,s} \cdot h_k}{\sum_{l=\underline{m}^A(y(t))}^{\overline{m}^A(y(t))} h_l}, \delta_{q(y(t))} \right\}, \quad (\text{C.7})$$

is the “accumulated” balance of plant  $j$  in month  $t$  of year  $y(t)$  and scenario  $s$ .

$$\delta_{q(y(t))} = \begin{cases} 1.3 & \text{if } q(y(t)) = 1 \\ 1.2 & \text{if } q(y(t)) = 2 \\ 1.1 & \text{if } q(y(t)) = 3 \\ 1.0 & \text{if } q(y(t)) = 4, \end{cases} \quad (\text{C.8})$$

is the value of the piecewise linear function at the position  $q$  of year  $y(t)$ .

$$RI_{j,t,s}^A = \begin{cases} \frac{1}{12} \cdot RI_{j,t,s}^{AA} & \text{if } y(t) \in \mathcal{T}^{III,A} \cup \mathcal{T}^{IV,A} \setminus \{\min \mathcal{T}^{III,A}\} \\ 0 & \text{otherwise,} \end{cases} \quad (\text{C.9})$$

is the annual contractual penalty of plant  $j$  in month  $t$  and scenario  $s$ .

$$RI_{j,t,s}^{AA} = \max \left\{ 0, \sum_{k=\underline{m}^A(y(t)-1)}^{\overline{m}^A(y(t)-1)} \left[ \left( 0.9 - B_{j,y(t)-1,s}^{INI} - g_{j,k,s} \right) \cdot h_k \right] \right\} \cdot \max \left\{ p_j^{\text{RCE}}, \sum_{k=\underline{m}^A(y(t)-1)}^{\overline{m}^A(y(t)-1)} \frac{\pi_{j,k,s}^P}{|m^A(y(t)-1)|} \right\}, \quad (\text{C.10})$$

is the total monetary value of the annual contractual penalty of plant  $j$  in month  $t$  and scenario  $s$ .

$$RI_{j,t,s}^Q = \begin{cases} \frac{1}{12} \cdot RI_{j,t,s}^{QQ} & \text{if } (q(y(t)-1) = 4) \text{ and } (y(t) \neq \min \mathcal{T}^{III,A}) \\ 0 & \text{otherwise,} \end{cases} \quad (\text{C.11})$$

is the quadrennial contractual penalty of plant  $j$  in month  $t$  and scenario  $s$ .

$$RI_{j,t,s}^{QQ} = \max \left\{ 0, \sum_{k=\underline{m}^A(y(t)-4)}^{\overline{m}^A(y(t)-1)} h_k - \max \left\{ 0.9 \cdot \sum_{l=\underline{m}^A(y(t)-4)}^{\overline{m}^A(y(t)-1)} h_l, \sum_{z=\underline{m}^A(y(t)-4)}^{\overline{m}^A(y(t)-1)} g_{j,z,s} \cdot h_z \right\} \right\} \cdot \max \left\{ p_j^{\text{RCE}}, \sum_{k=\underline{m}^A(y(t)-4)}^{\overline{m}^A(y(t)-1)} \frac{\pi_{j,k,s}^P}{\left| \bigcup_{\xi=1}^4 m^A(y(t)-\xi) \right|} \right\}, \quad (\text{C.12})$$

is the total monetary value of the quadrennial contractual penalty of plant  $j$  in month  $t$  and scenario  $s$ .

## Nomenclature

The life of a holding company is subdivided into major sections as in Figure 1: Major section I is the construction period, II is the period with contracts in the FCE only, III is the period with contracts in both the FCE and the RCE, and IV is the short period following commercialisation due to contractual obligations.

### Sets

$\mathcal{P}$	Set of power plants. Sets of power plants of the type SH, WP, and BIO are denoted by $\mathcal{P}^{\text{SH}}$ , $\mathcal{P}^{\text{WP}}$ , and $\mathcal{P}^{\text{BIO}}$ , respectively, forming subsets of $\mathcal{P}$ .
$\mathcal{T}$	Set of months within the project. $\mathcal{T}$ is partitioned into $\mathcal{T}^I$ , $\mathcal{T}^{II}$ , $\mathcal{T}^{III}$ , and $\mathcal{T}^{IV}$ , which are the sets of months in each of the major sections.
$\mathcal{T}^A$	Set of years in the project. $\mathcal{T}^A$ is partitioned into $\mathcal{T}^{I,A}$ , $\mathcal{T}^{II,A}$ , $\mathcal{T}^{III,A}$ , and $\mathcal{T}^{IV,A}$ , which are the sets of years in each of the major sections.
$\mathcal{T}^Q$	Index of year within quadrennial period, i.e. $\{1,2,3,4\}$ .
$\mathcal{C}^{\text{FCE}}$	Set of contracts in the FCE.
$\mathcal{S}$	Set of scenarios.

### Parameters

$F_j^{\text{max}}$	Certificate of plant $j$ with an installed capacity of 30 MW, in avgMW
$I_j^0$	Investment costs of plant $j$ , in R\$/avgMW
$c_j^F$	Operation and maintenance costs per month of plant $j$ , R\$/avgMW
$e_j^R$	Equity ratio of the investment project corresponding to plant $j$
$LT_j$	Credit period of external financing of the investment project corresponding to plant $j$
$i_j$	Interest rate on debt for the investment project corresponding to plant $j$
$h_t$	Number of hours in month $t$
$K^A$	Annual discount rate
$K$	Monthly discount rate
$\alpha$	Probability level, $\alpha \in (0, 1)$
$\lambda$	Risk aversion parameter, $\lambda \in [0, 1]$
$p_i^{\text{FCE},S}$	Energy price of contract $i$ in the FCE during major section S, in R\$/MWh
$p_j^{\text{RCE}}$	Energy price of contract $j$ in the RCE, in R\$/MWh
$\pi_{i,t,s}^C$	Electricity spot price in the submarket where contract $i$ is closed in month $t$ and in scenario $s$ , in R\$/MWh
$\pi_{j,t,s}^P$	Electricity spot price in the submarket where plant $j$ is located in month $t$ and in scenario $s$ , in R\$/MWh
$g_{j,t,s}$	Ratio of energy generated by plant $j$ in month $t$ in scenario $s$ to long-term average annual generation of that plant

### Variables

$F_j$	Share of the certificate which is available to plant $j$ , in avgMW
$F_j^{\text{RCE}}$	Share of the certificate which is available to plant $j$ that is allocated to contracts in the RCE, in avgMW
$Q_i^{\text{FCE},S}$	Energy sold by the Portfolio Manager via forward contract $i$ in the FCE during major section S, in avgMW

$Q_j^{\text{RCE}}$	Energy sold by plant $j$ via forward contract in the RCE, in avgMW
$z_a$	CVaR $_{\alpha}$ auxiliary variable for year $a$
$\delta_{a,s}$	CVaR $_{\alpha}$ auxiliary variable for year $a$ and scenario $s$

In addition to time-related information, we require functions that enable mapping from one set to another, thus allowing a switch between time bases, such as from months to corresponding years. Therefore, the functions used are:

- $y : \mathcal{T} \rightarrow \mathcal{T}^A$   $y(t)$  is the year containing month  $t$ .
- $m^A : \mathcal{T}^A \rightarrow \mathcal{T}$   $m^A(a)$  is the set of months in year  $a$ .
- $q : \mathcal{T}^{III,A} \rightarrow \mathcal{T}^Q$   $q(a)$  is the position in the quadrennial period of year  $a$ .
- $\overline{\mathcal{F}}/\underline{\mathcal{F}}$ : The limits of a set of numbers  $\mathcal{F}$  are denoted by  $\overline{\mathcal{F}} = \max \mathcal{F}$  and  $\underline{\mathcal{F}} = \min \mathcal{F}$ .

Appendix A contains some numerical examples to clarify the definitions of sets and mappings presented above.

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