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# Seismic Gradiometry using Ambient Seismic Noise in an Anisotropic Earth

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## SUMMARY

We introduce a wavefield gradiometry technique to estimate both isotropic and anisotropic local medium characteristics from short recordings of seismic signals by inverting a wave equation. The method exploits the information in the spatial gradients of a seismic wavefield that are calculated using dense deployments of seismic arrays. The application of the method uses the surface wave energy in the ambient seismic field. To estimate isotropic and anisotropic medium properties we invert an elliptically anisotropic wave equation. The spatial derivatives of the recorded wavefield are evaluated by calculating finite differences over nearby recordings, which introduces a systematic anisotropic error. A two step approach corrects this error: finite difference stencils are first calibrated, then the output of the wave-equation inversion is corrected using the linearized impulse response to the inverted velocity anomaly. We test the procedure on ambient seismic noise recorded in a large and dense ocean bottom cable array installed over Ekofisk field. The estimated azimuthal anisotropy forms a circular geometry around the production-induced subsidence bowl. This conforms with results from studies employing controlled sources, and with

interferometry correlating long records of seismic noise. Yet in this example, the results where obtained using only a few minutes of ambient seismic noise.

**Key words:** Surface waves and free oscillations; Seismic anisotropy; Seismic noise; Seismic tomography; Seismic interferometry; Wave propagation

## 1 INTRODUCTION

Knowledge of the subsurface stress state and material properties is key to understanding a range of earth-scientific phenomena such as earthquake and landslide nucleation, drilling and shallow-gas hazards, induced seismicity, and many other types of deformation and material failure. Variations of stress state are known to cause concomitant variations in elastic moduli, and these properties in turn affect the speed of elastic waves propagating through the medium (Breguier et al., 2008; Korneev & Glubokovskikh, 2013; Breguier et al., 2014; Hobiger et al., 2016). In particular, the orientation and magnitude of stress and the alignment of crystal orientation, pores, or layering, causes the wave speed to vary with direction of propagation, a property known as anisotropy (Crampin et al., 1980a; Teanby et al., 2004; Boness & Zoback, 2004; Herwanger & Horne, 2009). Measurements of both isotropic and anisotropic seismic velocities therefore place constraints on these various phenomena.

One of the first observations of anisotropy were incompatibilities of Love and Rayleigh wave dispersion curves (Anderson, 1961), and manifestations of shear wave splitting (Ando, 1980; Crampin et al., 1980b; Vennik et al., 1989). These observations were treated as point measurements indicating the properties underneath the stations. With increasing station coverage shear wave splitting maps now reveal anisotropy over large regions (Wüstefeld et al., 2009). Anisotropy in the crust and upper mantle has been linked to mantle flow (Peselnick & Nicolas, 1978; Christensen & Lundquist, 1982; Tanimoto & Anderson 1984). Maps of Rayleigh and Love wave anisotropic phase velocity in the upper mantle are found by tomography inverting large sets of observations covering different azimuths (Montagner & Jobert, 1988; Montagner & Nataf, 1988; Montagner & Tanimoto, 1990), potentially followed by a depth inversion to map anisotropy with depth (Montagner & Nataf 1986). More recently,

47 finite frequency sensitivity kernels are proposed for full waveform inversion strategies to  
48 recover anisotropic elastic structure from surface waves (Sieminski et al., 2007; Plessix &  
49 Cao, 2011). In principle two linear orthogonal arrays can reveal the principal component of  
50 anisotropy, but with two dimensional arrays we can derive a more sophisticated azimuthal  
51 dependence of surface wave velocity (Forsyth & Li, 2013).

52 Observed gradients of propagating and standing seismic wavefields are known to contain  
53 important information about for example the wave propagation direction, and the medium  
54 properties. Wavefield gradiometry, literally, is the estimation or observation of a wavefield's  
55 spatial-gradients. When dense measurements are available throughout a larger region, the  
56 observations of temporal and spatial gradients can be exploited as local constraints in an  
57 inverse problem to estimate medium properties throughout the region. This is in contrast  
58 to other classes of geophysical inversion techniques such as tomography and full-waveform  
59 inversion, where the observations are posed as global constraints in an inverse problem for  
60 the medium parameters.

61 Curtis & Robertsson (2002) proposed to directly extract isotropic P- and S-velocities  
62 from observed three-dimensional derivatives of a wavefield. However the volumetric (tetra-  
63 hedral) recordings required in order to estimate all such gradients, are rarely available as  
64 dense deployments of receivers are usually confined to the Earth's surface. Muijs et al. (2003)  
65 showed that for plane waves, gradiometry could be accomplished on the seabed using pla-  
66 nar sensor arrays. Langston (2007a; 2007b; 2007c) and Poppeliers et al. (2013) extracted  
67 ray parameters and wave directionality from non-overlapping plane waves. However, the as-  
68 sumption of observing non-interfering plane-waves limits the use of wavefield gradiometry to  
69 simple wavefields where specific arrivals can be identified and isolated. A direct estimate for  
70 the phase velocity can also be recovered by inverting an eikonal equation for the travel-times  
71 of large earthquake surface wave arrivals, or of virtual seismic sources obtained by noise-  
72 correlations (Lin & Ritzwoller, 2011; Gouédard et al., 2012; De Ridder et al., 2015). These  
73 techniques are referred to as eikonal or Helmholtz tomography. They were applied on cross-  
74 correlations of ambient noise recorded by a large and dense ocean bottom cable (OBC) array

75 installed over Valhall. OBC is a cable-based seismic receiver system laid down temporarily  
76 on the seafloor, or installed more permanently trenched a meter deep into the sea floor. De  
77 Ridder & Dellinger (2011) and Mordret et al. (2013a) found high resolution images of near-  
78 surface Scholte wave velocity, including anisotropy (Mordret et al., 2013b) at Valhall. Liu &  
79 Holt (2015) described a link between Helmholtz tomography and wavefield gradiometry, as  
80 applied to plane waves from large earthquakes. However, these approaches require identifi-  
81 cation of an arrival time limiting applications to large earthquakes, or requiring observations  
82 of long time series if estimated Greens functions derived from cross-correlation of ambient  
83 noise are to be used.

84 De Ridder & Biondi (2015b) introduced a gradiometry method applicable for surface-  
85 wave seismic noise by inverting a two dimensional scalar wave equation for isotropic wave  
86 velocities. They found that the error in the spatial finite difference approximation for the  
87 Laplacian operator can result in large velocity errors, especially when employing second  
88 order derivatives. Edme & Yuan (2016) extracted surface wave dispersion curves directly  
89 from seismic noise by following the plane wave gradiometry approach of Langston (2007b),  
90 analyzing the statistics of the first-order derivatives, to identify and discard time-windows  
91 with multiple interfering arrivals. Sollberger et al. (2016) employed seismic wavefield gra-  
92 diometry to extract shear-wave information on the shallow lunar crust from the recordings  
93 of the Apollo active seismic experiment.

94 Whereas the wave equation inversion methodology by Curtis & Robertsson (2002) and  
95 De Ridder & Biondi (2015b) apply to ambient seismic noise, they were not designed for  
96 anisotropic media. Here, we propose a more general formulation that accounts for anisotropy  
97 in elastodynamic media. Then we introduce a practical formulation for surface waves in  
98 azimuthal anisotropic media, and we propose a method that corrects the bias in the isotropic  
99 analysis revealing the anisotropy of the medium. We show how the anisotropic velocity errors  
100 caused by finite difference approximations of spatial derivatives can be corrected using a two  
101 step workflow. To illustrate the efficacy of this technique we carried out a field data study  
102 using ambient seismic noise recordings made in a large and dense OBC array installed over

103 Ekofisk field in the Norwegian North Sea (Eriksrud, 2010). These results are consistent with  
 104 those obtained from active source data, even though we used data containing only 10 minutes  
 105 of ambient noise recordings.

## 106 2 SEISMIC GRADIOMETRY

107 The term seismic gradiometry refers to the measurement or estimation of seismic wavefield  
 108 gradients. These can be used for wavefield separation, estimation of propagation directions,  
 109 or inversion for material properties. Here, we estimate the medium properties in the vicinity  
 110 of each recording station directly from spatial and temporal gradients of the seismic record-  
 111 ings according to the wave equation. This was first referred to as *wave equation inversion*  
 112 (Curtis & Robertsson, 2002) and later simply as *wavefield gradiometry* (Langston, 2007a).  
 113 In this study we will refer to (*seismic wavefield*) *gradiometry* to avoid confusion between  
 114 wave equation inversion and full waveform inversion.

115 A general formulation for elastodynamic wavefields could be based on the wave equation  
 116 for the particle velocity:

$$117 \quad \rho^{-1} C_{ijkl} \partial_j \partial_l u_k(\mathbf{x}, t) = \partial_t \partial_t u_i(\mathbf{x}, t) \quad (1)$$

118 where  $\rho = \rho(\mathbf{x})$  is the bulk density and  $C_{ijkl} = C_{ijkl}(\mathbf{x})$  is the elastic stiffness, and  $u_i$  with  
 119 (in this equation only)  $i = 1, 2, 3$  are the three components of particle velocity and (in  
 120 this equation only) we used the Einstein summation convention. It is possible to invert this  
 121 equation for local medium parameters directly when measurements of all three components  
 122 of the state vector are available at neighbouring points throughout a volume, since then the  
 123 derivatives in eq. (1) can be estimated using finite difference in space and time. We recognize  
 124 the problem then takes the form

$$125 \quad \mathbf{F}_i \mathbf{m} = \mathbf{b}_i \quad (2)$$

126 in which the subscript indicates a particular time-slice and  $\mathbf{m}$  describes the material density  
 127 and stiffness ratios  $\rho^{-1} C_{ijkl}$ . In principle, when sufficient linearly independent wavestates  
 128 are observed, this equation can be solved for all independent elements of the elasticity,

129 scaled by the inverse of the density. However, inversion of eq. (1) for the medium parameters  
 130 everywhere in a volume, requires recordings throughout the volume. Since recordings are  
 131 usually confined to a surface, we focus on wave equation inversion for surface wave ambient  
 132 noise. A technique to recover the isotropic phase velocity of surface waves directly from  
 133 measured temporal and spatial gradients of an ambient noise wavefield was first formulated  
 134 by De Ridder & Biondi (2015b). We briefly review the theory for isotropic gradiometry then  
 135 formulate elliptically anisotropic wavefield gradiometry.

## 136 **2.1 Isotropic Gradiometry**

137 When the ambient seismic field is dominated by Rayleigh or Scholte surface waves, the  
 138 wavefield recorded in the vertical component of particle velocity or the pressure, may be  
 139 approximated as a superposition of non-dispersive single-mode surface-wave plane waves  
 140 in the far field. In practice this is achieved by filtering the data for a narrow frequency  
 141 bandwidth to avoid dispersion effects, and neglecting the remaining energy associated with  
 142 higher modes. Any superposition of such surface wave plane waves, including standing waves,  
 143 satisfies the following two-dimensional scalar-wave equation:

$$144 \quad M_0(x, y) [\partial_x \partial_x + \partial_y \partial_y] U(x, y, t) = \partial_t \partial_t U(x, y, t) \quad (3)$$

145 where  $M_0(x, y)$  is the isotropic surface-wave phase velocity squared,  $M_0(x, y) = c_0^2(x, y)$ .  
 146 This wave equation, and its associated eikonal equation, implicitly form the basis for many  
 147 conventional imaging techniques for surface waves. The concepts of phase and group velocity  
 148 tomography are based on two dimensional wave propagation through a map of effective  
 149 phase and group velocities (Aki, 1957; Wielandt, 1993), the latest non-linear surface wave  
 150 tomography approaches still rest on this principle (Galetti et al., 2015a), and array imaging  
 151 techniques such as eikonal and Helmholtz tomography (Lin, Ritzwoller & Snieder, 2009; Lin  
 152 & Ritzwoller, 2011; De Ridder et al., 2015) are based on an eikonal equation derived for a  
 153 two-dimensional scalar-wave equation.

154 The state variable scalar field  $U(x, y, t)$  is generally observed discretely in time and space,

155 with regular sampling in time but irregular sampling in space. Dense observations provide  
 156 an opportunity to estimate the second-order spatial derivatives of the wavefield by taking  
 157 irregular finite differences between different nearby receivers and the time derivatives at each  
 158 single station by standard finite differences. Consequently, the only unknown in eq. (3) is  
 159 the wave speed.

160 We estimate the wave speed by inverting eq. (3) with additional regularization con-  
 161 straints. We pose the medium parameter as a perturbation on an average constant back-  
 162 ground value,  $M_0(x, y) = \bar{M}_0 + \Delta M_0(x, y)$ , and insert this into eq. (3) giving

$$163 \quad \Delta M_0(x, y) \mathbf{D}_\Delta \mathbf{U}_i = \ddot{\mathbf{U}}_i - \bar{M}_0 \mathbf{D}_\Delta \mathbf{U}_i \quad (4)$$

164 where  $\mathbf{U}_i$  is a vector containing the observations at all stations for the  $i^{\text{th}}$  time sample  
 165 (from hereon the subscript  $i$  denotes time sample), and  $\mathbf{D}_\Delta$  denotes a discrete Laplace  
 166 operator which calculates spatial derivatives for all elements of  $\mathbf{U}_i$ , we constructed this  
 167 operator following Huiskamp (1991). This wave equation has the form  $\mathbf{F}_i \mathbf{m} = \mathbf{b}_i$ , where the  
 168 subscript denotes a specific observed state of the wavefield at a different time, and with

$$169 \quad \mathbf{F}_i = \text{diag} \{ \mathbf{D}_\Delta \mathbf{U}_i \} \quad (5)$$

$$170 \quad \mathbf{b}_i = \ddot{\mathbf{U}}_i - \bar{M}_0 \mathbf{D}_\Delta \mathbf{U}_i \quad (6)$$

$$171 \quad \mathbf{m} = \Delta M_0 \quad (7)$$

172 where  $\text{diag} \{ \}$  denotes a diagonal matrix formed with the input vector on the diagonal,  
 173 and  $\ddot{\mathbf{U}}$  denotes the second order derivative in time. The size of the matrices indicates the  
 174 size of the model space:  $\mathbf{F}$  in eq. (5) has dimensions  $M \times M$ , where  $M$  is the number  
 175 of model parameters in  $\mathbf{m}$  (equating to the total number of stations at locations  $(x, y)$  in  
 176 eq. 4). We zero the rows in  $\mathbf{F}_i$  and  $\mathbf{b}_i$  concerning station locations for which we could not  
 177 obtain a reliable finite difference stencil. The presence of diagonal matrices in the linear  
 178 system indicates that in the absence of regularization, all model parameters are constrained  
 179 independent. However, given  $N$  observations of states of the wavefield we invert the system  
 180 by least-squares regression, adding additional constraints by  $0^{\text{th}}$  and  $2^{\text{nd}}$ -order Tikhonov

181 regularization

$$182 \quad \left[ \sum_{i=1}^N \mathbf{F}_i^T \mathbf{F}_i + \epsilon_1 \mathbf{D}_\Delta^T \mathbf{D}_\Delta + \epsilon_2 \mathbf{I} \right] \mathbf{m} = \sum_{i=1}^N \mathbf{F}_i \mathbf{b}_i \quad (8)$$

183 where  $\mathbf{I}$  is an identity matrix, and  $\epsilon_1$  and  $\epsilon_2$  are the regularization strengths. When  $\epsilon_1 = \epsilon_2 =$   
 184 0 eq. (8) reduces to a simple regression at each station of the array. In the examples in this  
 185 study, we selected  $\epsilon_1$  by comparing the reduction of the variance of the model space versus  
 186 increasing regularization strength with an L-curve criteria (Hansen & OLeary, 1993; Lawson  
 187 & Hanson, 1974). We found the result not to vary on the particular value of  $\epsilon_2$  and set  $\epsilon_2$  to  
 188  $10^{-15}$ . We solve equation 8 by LU decomposition of the composite matrix on the left-hand  
 189 side of eq. (8). Using finite differences to estimate the spatial derivative assumes the medium  
 190 parameters do not vary over the spatial stencil spread. In practice the smoothness of the  
 191 recovered velocity map will be a function of regularization strength, and the spatial stencil  
 192 spread forms an upper bound on the resolution.

## 193 **2.2 Anisotropic Gradiometry**

194 We now extend the formulation to include azimuthal anisotropy. We describe the anisotropy  
 195 in local propagation velocity,  $c = c(x, y, \phi)$ , of planar surface-waves as elliptical as a function  
 196 of azimuth:

$$197 \quad c^2(\phi) = c_f^2 \sin^2(\phi - \alpha) + c_s^2 \cos^2(\phi - \alpha) \quad (9)$$

198 where  $c_f$  and  $c_s$  are the fast and slow magnitudes of the anisotropic velocity, and  $\alpha$  is the  
 199 direction of fast. This form closely resembles the slightly anisotropic Rayleigh phase velocity  
 200 azimuthal anisotropy discussed by Smith & Dahlen (1973) when we omit the  $4\phi$  term, see  
 201 Appendix A in De Ridder et al. (2015), when data quality does not permit this term to be fit  
 202 (Lin et al., 2009; Mordret et al., 2013b). Elliptical anisotropy describes SH-wave anisotropy  
 203 in tilted transversely isotropic media (Tsvankin, 2011), and the elegant properties of ellipses  
 204 has been a popular choice for approximately representing anisotropy in other wavefields and  
 205 media (Helbig, 1983; Dellinger, 1991). Dropping the  $4\phi$  term or for Rayleigh and Scholte  
 206 wave anisotropy when.

207 We aim to derive a scalar wave-equation suitable for seismic noise, filtered to pass a  
 208 narrow frequency range so that we can ignore the frequency dependence in the derivation.  
 209 To derive an elliptically anisotropic form of eq. (3), we substitute  $c^2(\phi)$  into a general dis-  
 210 persion relationship  $c^2(\phi) |\mathbf{k}|^2 = \omega^2$ , where  $\mathbf{k} = [k_x, k_y]^T$  is the wavenumber vector. Using  
 211 the trigonometric relationships  $\cos(\phi - \alpha) = \cos(\phi)\cos(\alpha) + \sin(\phi)\sin(\alpha)$ ,  $\sin(\phi - \alpha) =$   
 212  $\sin(\phi)\cos(\alpha) - \cos(\phi)\sin(\alpha)$  and  $\cos^2(\alpha) + \sin^2(\alpha) = 1$ , we find:

$$213 \quad \omega^2 = M_{11}k_xk_x + (M_{12} + M_{21})k_xk_y + M_{22}k_yk_y \quad (10)$$

214 where  $k_x = |\mathbf{k}| \sin(\phi)$  and  $k_y = |\mathbf{k}| \cos(\phi)$ . The elements  $M_{11}$ ,  $M_{12} = M_{21}$ , and  $M_{22}$  form the  
 215 elements of a two-by-two matrix  $\mathbf{M}$ , and are a function of  $c_f$ ,  $c_s$ , and  $\alpha$ :

$$216 \quad M_{11} = (c_f^2 - c_s^2) \sin^2(\alpha) + c_s^2 \quad (11)$$

$$217 \quad M_{12} = (c_f^2 - c_s^2) \sin(\alpha)\cos(\alpha) \quad (12)$$

$$218 \quad M_{22} = (c_f^2 - c_s^2) \cos^2(\alpha) + c_s^2 \quad (13)$$

219 The eigenvalues of the matrix  $\mathbf{M}$  are  $c_f^2$  and  $c_s^2$ , and the eigenvectors indicate the fast and  
 220 slow directions. In this manuscript we graphically display the anisotropic medium parameters  
 221 as an isotropic component defined by  $1/2(c_f + c_s)$  and a magnitude anisotropy in percent  
 222 defined by  $50 \times (c_f - c_s)(c_f + c_s)^{-1}$ . Performing a spatial and temporal inverse Fourier  
 223 transformation, we find the wave-equation operator that acts on the state variable  $U(x, y, t)$   
 224 in an elliptically anisotropic scalar wave equation:

$$225 \quad [M_{11}(x, y) \partial_x \partial_x + (M_{12}(x, y) + M_{21}(x, y)) \partial_x \partial_y + M_{22}(x, y) \partial_y \partial_y] U(x, y, t) = \partial_t \partial_t U(x, y, t) \quad (14)$$

227 which alternatively can be written in the following matrix form:

$$228 \quad \begin{bmatrix} \partial_x & \partial_y \end{bmatrix} \begin{bmatrix} M_{11}(x', y') & M_{12}(x', y') \\ M_{21}(x', y') & M_{22}(x', y') \end{bmatrix} \begin{bmatrix} \partial_x \\ \partial_y \end{bmatrix} U(x, y, t) = \partial_t \partial_t U(x, y, t) \quad (15)$$

229 where the presence of a prime on the spatial coordinates of the medium parameters denotes  
 230 that the spatial derivative operators do not operate on the medium parameters, but only  
 231 on the wavefield. In a strict sense we neglected lateral velocity variations in the derivation

of eq. (14), and thus neglected lateral surface wave scattering. However by allowing the medium parameters to vary as a function of space, we do allow a degree of scattering just as the isotropic two-dimensional wave eq. (3) still allows scattering due to lateral velocity variations.

Similarly to the isotropic case, we use the nearby stations to evaluate spatial finite differences. In the absence of noise, we would need three linearly independent realizations of wave states to resolve all three unknowns in eq. (14). Similarly to the isotropic case we pose the medium parameter as a perturbation on the isotropic value,  $\mathbf{M}(x, y) = \mathbf{I}M_0(x, y) + \Delta\mathbf{M}(x, y)$ , where  $\mathbf{I}$  is a two-by-two identity matrix:

$$\Delta M_{11}(x, y)\mathbf{D}_{\mathbf{xx}}\mathbf{U}_i + [\Delta M_{12}(x, y) + \Delta M_{21}(\mathbf{x})]\mathbf{D}_{\mathbf{xy}}\mathbf{U}_i + \Delta M_{22}(x, y)\mathbf{D}_{\mathbf{yy}}\mathbf{U}_i = \ddot{\mathbf{U}}_i - M_0(x, y)\mathbf{D}_{\Delta}\mathbf{U}_i \quad (16)$$

Here  $\mathbf{D}_{\mathbf{xx}}$ ,  $\mathbf{D}_{\mathbf{yy}}$ , and  $\mathbf{D}_{\mathbf{xy}}$  denote discrete second-order spatial derivative operators with subscripts indicating the spatial directions, and  $\mathbf{D}_{\Delta}$  is as before and also equates to  $\mathbf{D}_{\Delta} = \mathbf{D}_{\mathbf{xx}} + \mathbf{D}_{\mathbf{yy}}$ . This equation, similar to the isotropic case, has the form  $\mathbf{F}_i \mathbf{m} = \mathbf{b}_i$ , but the elements of this linear system are:

$$\mathbf{F}_i = \left[ \text{diag}\{\mathbf{D}_{\mathbf{xx}}\mathbf{U}_i\}, \quad 2 \text{diag}\{\mathbf{D}_{\mathbf{xy}}\mathbf{U}_i\}, \quad \text{diag}\{\mathbf{D}_{\mathbf{yy}}\mathbf{U}_i\} \right] \quad (17)$$

$$\mathbf{b}_i = \ddot{\mathbf{U}}_i - \text{diag}\{\mathbf{M}_0\}\mathbf{D}_{\Delta}\mathbf{U}_i \quad (18)$$

$$\mathbf{m} = \left[ \Delta\mathbf{M}_{11}, \quad \Delta\mathbf{M}_{12}, \quad \Delta\mathbf{M}_{22} \right]^T \quad (19)$$

Here, the number of model parameters is three times that in the linear system for the isotropic case, and  $\mathbf{F}$  in eq. (17) has dimensions  $M \times 3M$ , where  $M$  is the number of stations in the array. If we make  $N$  observations of states of the wavefield, we can invert the system by least-squares regression, adding additional constraints by  $0^{th}$  and  $2^{nd}$ -order Tikhonov regularization:

$$\left[ \sum_{i=1}^N \mathbf{F}_i^T \mathbf{F}_i + \epsilon_1 \mathbf{D}^T \mathbf{D} + \epsilon_2 \mathbf{I} \right] \mathbf{m} = \sum_{i=1}^N \mathbf{F}_i^T \mathbf{b}_i \quad (20)$$

where

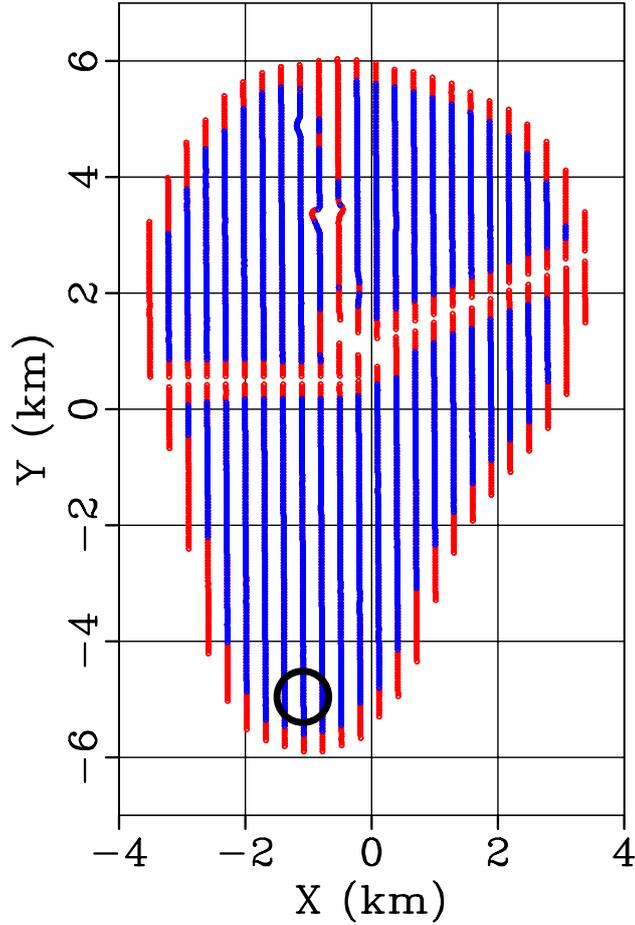
$$\mathbf{D} = \begin{bmatrix} \mathbf{D}_\Delta & 0 & 0 \\ 0 & \mathbf{D}_\Delta & 0 \\ 0 & 0 & \mathbf{D}_\Delta \end{bmatrix} \quad (21)$$

### 2.3 Inverting synthetic isotropic plane wave data

We use finite differences to evaluate the spatial derivatives, and consequently we introduce an error in the approximation of the continuous operators. These errors depend on the station geometry of the array, and on the effective spatial wavelength of the data. In this study we use a field dataset from Ekofisk’s ocean bottom cable (OBC) array to evaluate the merit of our method. The station array has dense in-line and sparse cross-line station spacing, respectively 50 m and 300 m (Fig. 1). For further details on the array and field, see the field data example below. We computed stencils by inverting a second-order Taylor series expansion on the geometric distribution of the nearby stations (Huiskamp, 1991). For each station we select neighboring stations within a 400 m radius to form the stencil (e.g. black circle in Fig. 1), hence we cannot resolve anomalies smaller than  $\sim 800$  m in size. We discarded each station with fewer than 36 such neighboring stations to ensure a minimum quality of FD stencil. Thus we could not obtain reliable estimates near the edges of the array or in areas where the array was disrupted due to infrastructure. The blue stations in Fig. 1 indicate the station locations where we have a reliable finite difference stencil.

From a dispersion analysis by De Ridder & Biondi (2015b) we know that the surface waves observed in ambient noise at Ekofisk travel with an average velocity of approximately 490 m/s at 0.7 Hz, and are not aliased in the in-line or the cross-line direction. For each stencil in the array, we synthesize 36 sets of plane waves from different angles spaced 10 degrees apart, covering all 360 degrees, oscillating at 0.7 Hz with an isotropic moveout of 490 m/s.

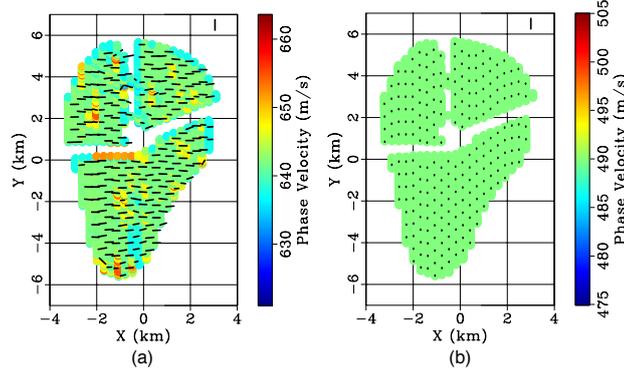
This synthetic data is input into the two step algorithm for anisotropic gradiometry. We solved the linear inverse system (eq. 2) for isotropic velocities, with eqs. (5) to (7).



**Figure 1.** Station geometry of the ocean bottom cable (OBC) array installed at Ekofisk; stations are indicated by small red and blue circles. Blue circles indicate those stations where we have a reliable finite difference stencil using the nearby stations in a radius of 400 m (indicated for example by the black circle).

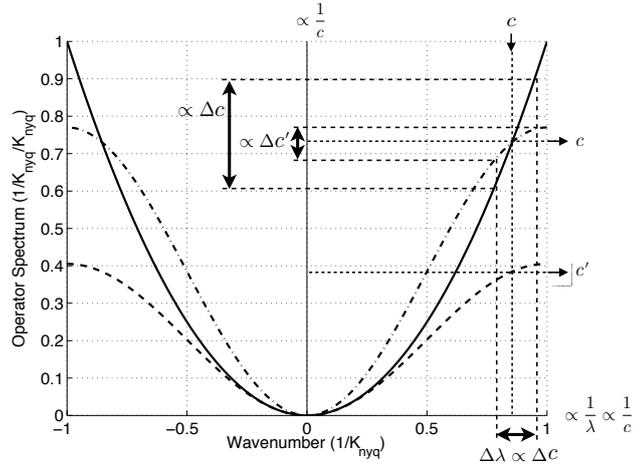
281 To resolve the spatially varying nature of the erroneous recovered anisotropic velocity, we  
 282 used  $\epsilon_1 = 0$ . Secondly, we solved the linear system (eq. 2), with eqs. (17) to (19), for an  
 283 anisotropic velocity map, using the solution of the isotropic case as the background velocity  
 284 map (Fig. 2a). The colours indicate the isotropic component of the retrieved anisotropic  
 285 velocities while the black dashes indicate the magnitude and fast-directions of anisotropy.  
 286 Even though the inversion ought to result in a homogeneous isotropic velocity, the inversion  
 287 yields (apparent) higher isotropic velocity and also include anisotropic components: this is  
 288 the result of stencil error.

289 The stencil error is a function of the stencil spacing relative to the wavelength of the  
 290 function being sampled. To visualize the error in second order finite difference stencils, we



**Figure 2.** Synthetic data example using the station geometry of the Ekofisk OBC array, inverting data that represents recordings of monochromatic plane waves at 0.7 Hz propagating through a homogeneous and isotropic velocity structure of 490 m/s. Colour indicates isotropic component of velocity; dashes indicate magnitude and fast-direction of anisotropy (dash in upper right corner indicates 10% magnitude, the difference between maximum and minimum velocities as a percentage of the isotropic velocity). a) Apparent anisotropy observed using finite differences with second order accuracy (without correction). b) Observed homogeneous isotropic velocity map retrieved using finite differences with second order accuracy including a correction derived from the anisotropy observed in (a).

291 plot the Fourier-space spectrum of the stencil coefficients (computed by discrete Fourier  
 292 transformation) with the ideal spectrum of the continuous operator ( $|k|^2$ ) in Fig. 3. Notice  
 293 that the error is zero for constant-functions, and is largest for wavelengths near Nyquist.  
 294 The frequency of the data and the velocity of the medium determine the spatial wavelength  
 295 of the wavefield along the horizontal axis of Fig. 3. The measurement of second order deriva-  
 296 tives is plotted along the vertical axis of Fig. 3. Notice that we always underestimate the  
 297 magnitudes of the second order derivatives. Thus we over-estimate the velocity by wave-  
 298 field gradiometry, which essentially depends on the ratio between the second order time  
 299 derivatives and the second order space derivatives. In two dimensions the stencil error is  
 300 generally angle dependent. The stencil spacing is larger in the cross-line direction than the  
 301 in-line direction, hence we find an erroneous apparent anisotropy with fast direction in the  
 302 cross-line direction. Subsampling the in-line stations to approximately equalize the inline  
 303 and cross-line station spacing resulted in using a much lower number of stations (samples)



**Figure 3.** Spectra of the finite difference stencil for a second order derivative operator with second order accuracy. Solid line: spectrum of ideal continuous operator ( $| -k^2 | = k^2$ ). Dashed line: spectrum of the original finite difference stencil. Dash-dot line: spectrum of calibrated (by scaling) finite difference stencil. The effective wavelength of the wavefield determines the position on the horizontal axis, while the measured second order spatial derivative is plotted along the vertical axis. The error of the uncorrected finite difference stencil leads to an over estimation of the velocity  $c' > c$ . The scaled finite difference stencils lead to underestimation of the correct spread of the second order spatial derivatives due to a true velocity change,  $\Delta c' < \Delta c$ .

304 being used to measure the spatial gradients of the wavefield. This had an averse effect on  
 305 the quality of the measurement of spatial derivatives and the resulting velocity field.

### 306 **3 CORRECTION PROCEDURES FOR FINITE DIFFERENCES**

307 Ellipses are attractive geometrical shapes to use for describing anisotropy because an el-  
 308 lipse can be turned into a circle or any other ellipse by an invertible linear transformation.  
 309 We aim to establish a correction procedure for the finite difference stencils by approxim-  
 310 ating the angle dependent error as ellipsoidal, and inserting two Jacobians into eq. (15). In  
 311 Fig. 2a we observed an apparent anisotropy, here denoted  $\mathbf{M}_h(\mathbf{x})$ , while we should have ob-  
 312 served a homogeneous isotropic medium with parameters  $\mathbf{C}_h(\mathbf{x}) = c_h^2 \mathbf{I}$ , with  $c_h = 490$  m/s,  
 313 everywhere and  $\mathbf{I}$  a two-by-two identity matrix. The matrix  $\mathbf{M}$  containing the elliptically  
 314 anisotropic medium parameters is symmetric ( $m_{12} = m_{21}$ ). For this matrix we write the

315 eigenvalue-eigenvector decomposition as

$$316 \quad \mathbf{M} = \mathbf{P}\mathbf{\Lambda}\mathbf{P}^T \quad (22)$$

317 where

$$318 \quad \mathbf{P} = \begin{bmatrix} \mathbf{p}_1 & \mathbf{p}_2 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \quad (23)$$

319 contains the unit-eigenvectors as columns and

$$320 \quad \mathbf{\Lambda} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \quad (24)$$

321 contains the corresponding eigenvalues. To derive a calibration method from  $\mathbf{M}_h$  we seek to  
322 define a particular combination,  $\mathbf{J}$ , of the scaled eigenvectors such that

$$323 \quad \mathbf{J}^T \mathbf{C}_h \mathbf{J} = \mathbf{M}_h \quad (25)$$

324 If we define

$$325 \quad \mathbf{S} = \begin{bmatrix} \sqrt{\lambda_1}/c_h & 0 \\ 0 & \sqrt{\lambda_2}/c_h \end{bmatrix} \quad (26)$$

326 then

$$327 \quad \mathbf{M}_h = \mathbf{P}\mathbf{S}^T \mathbf{P}^T \mathbf{C}_h \mathbf{P}\mathbf{S} \mathbf{P}^T = \mathbf{J}^T \mathbf{C}_h \mathbf{J} \quad (27)$$

328 where  $\mathbf{J}(\mathbf{x}) = \mathbf{P}(\mathbf{x})\mathbf{S}\mathbf{P}^T(\mathbf{x})$  with the property  $\mathbf{J} = \mathbf{J}^T$ .

329 Inserting eq. (25) into eq. (15) we see that  $\mathbf{J}$  describes a rotation and a translation, and  
330 hence acts as a Jacobian (a standard, orthogonality-preserving transformation) on the coor-  
331 dinate system of the spatial derivative operators. This Jacobian contains scaled eigenvectors  
332 of the matrix  $\mathbf{M}_h$ . The scaling coefficient is the ratio between the square root of the relevant  
333 eigenvalue of  $\mathbf{M}_h$ , and the phase velocity used to compute the synthetic data from which we  
334 measured  $\mathbf{M}_h$ . Inclusion of both  $\mathbf{P}$  and  $\mathbf{P}^T$  in eq. (27) ensures that the orientation of the  
335 coordinate system of the anisotropic medium properties remains unaltered.

336 We could use this relation and correct the observed apparent anisotropy as a final step  
337 after the inversion for medium parameters. However, it is more prudent to use the Jacobian  
338 in the wave equation so that we can apply the regularization free from the effect of stencil

errors. To derive a correction for the finite difference approximation of the Laplace operator,  
we evaluate:

$$\begin{bmatrix} \partial_x & \partial_y \end{bmatrix} \begin{bmatrix} J_{11} & J_{21} \\ J_{12} & J_{22} \end{bmatrix} \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} \partial_x \\ \partial_y \end{bmatrix} \quad (28)$$

and find in discrete operator form:

$$\begin{aligned} & [(\text{diag}\{\mathbf{J}_{11}\}^2 + \text{diag}\{\mathbf{J}_{12}\}^2) \mathbf{D}_{xx} + \\ & (\text{diag}\{\mathbf{J}_{12}\} + \text{diag}\{\mathbf{J}_{21}\}) (\text{diag}\{\mathbf{J}_{11}\} + \text{diag}\{\mathbf{J}_{22}\}) \mathbf{D}_{xy} + \\ & (\text{diag}\{\mathbf{J}_{22}\}^2 + \text{diag}\{\mathbf{J}_{21}\}^2) \mathbf{D}_{yy}] = \mathbf{D}'_{\Delta} \end{aligned} \quad (29)$$

The elements of the new linear system for isotropic gradiometry, in place of eqs. (5) and (6), simply have  $\mathbf{D}'_{\Delta}$  instead of  $\mathbf{D}_{\Delta}$ . To find the modified linear system for anisotropic gradiometry, we insert  $\mathbf{J}^T \mathbf{M} \mathbf{J}$  into eq. 15 and expand the matrix product to identify the elements:

$$\mathbf{F}_i = \begin{bmatrix} \text{diag}\{\mathbf{F}_{1,i}\}, & 2 \text{diag}\{\mathbf{F}_{2,i}\}, & \text{diag}\{\mathbf{F}_{3,i}\} \end{bmatrix} \quad (30)$$

with

$$\begin{aligned} \mathbf{F}_{1,i} = & [\text{diag}\{\mathbf{J}_{11}\} \text{diag}\{\mathbf{J}_{11}\} \mathbf{D}_{xx} + \text{diag}\{\mathbf{J}_{11}\} \text{diag}\{\mathbf{J}_{12}\} \mathbf{D}_{xy} + \\ & \text{diag}\{\mathbf{J}_{11}\} \text{diag}\{\mathbf{J}_{21}\} \mathbf{D}_{xy} + \text{diag}\{\mathbf{J}_{12}\} \text{diag}\{\mathbf{J}_{12}\} \mathbf{D}_{yy}] \mathbf{U}_i \end{aligned} \quad (31)$$

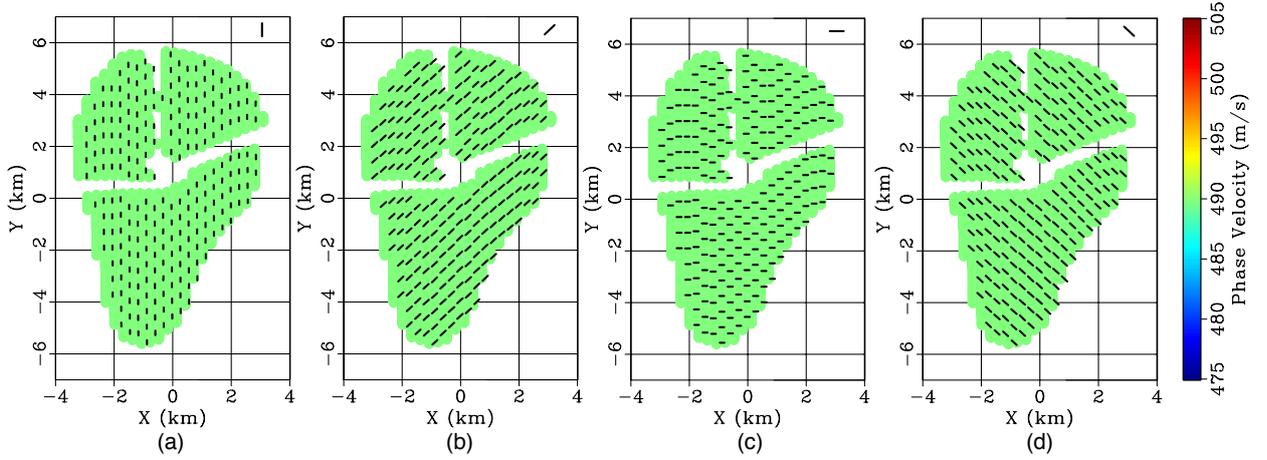
$$\begin{aligned} \mathbf{F}_{2,i} = & [\text{diag}\{\mathbf{J}_{21}\} \text{diag}\{\mathbf{J}_{11}\} \mathbf{D}_{xx} + \text{diag}\{\mathbf{J}_{11}\} \text{diag}\{\mathbf{J}_{22}\} \mathbf{D}_{xy} + \\ & \text{diag}\{\mathbf{J}_{21}\} \text{diag}\{\mathbf{J}_{12}\} \mathbf{D}_{xy} + \text{diag}\{\mathbf{J}_{12}\} \text{diag}\{\mathbf{J}_{22}\} \mathbf{D}_{yy}] \mathbf{U}_i \end{aligned} \quad (32)$$

$$\begin{aligned} \mathbf{F}_{3,i} = & [\text{diag}\{\mathbf{J}_{21}\} \text{diag}\{\mathbf{J}_{21}\} \mathbf{D}_{xx} + \text{diag}\{\mathbf{J}_{21}\} \text{diag}\{\mathbf{J}_{22}\} \mathbf{D}_{xy} + \\ & \text{diag}\{\mathbf{J}_{12}\} \text{diag}\{\mathbf{J}_{22}\} \mathbf{D}_{xy} + \text{diag}\{\mathbf{J}_{22}\} \text{diag}\{\mathbf{J}_{22}\} \mathbf{D}_{yy}] \mathbf{U}_i \end{aligned} \quad (33)$$

and

$$\begin{aligned} \mathbf{b}_i = & \ddot{\mathbf{U}} - \text{diag}\{\mathbf{M}_0\} [(\text{diag}\{\mathbf{J}_{11}\}^2 + \text{diag}\{\mathbf{J}_{12}\}^2) \mathbf{D}_{xx} + \\ & (\text{diag}\{\mathbf{J}_{12}\} + \text{diag}\{\mathbf{J}_{21}\}) (\text{diag}\{\mathbf{J}_{11}\} + \text{diag}\{\mathbf{J}_{22}\}) \mathbf{D}_{xy} + \\ & (\text{diag}\{\mathbf{J}_{22}\}^2 + \text{diag}\{\mathbf{J}_{21}\}^2) \mathbf{D}_{yy}] \mathbf{U}_i \end{aligned} \quad (34)$$

$$\mathbf{m} = \begin{bmatrix} \Delta \mathbf{M}_{11}, & \Delta \mathbf{M}_{12}, & \Delta \mathbf{M}_{22} \end{bmatrix}^T \quad (35)$$



**Figure 4.** Synthetic data example using the station geometry of the Ekofisk OBC array, inverting monochromatic plane waves at 0.7 Hz with a homogeneous velocity of 490 m/s and 10% anisotropy in four directions: (a)  $0^\circ$ ; (b)  $45^\circ$ ; (c)  $90^\circ$ ; (d)  $135^\circ$ . Colour indicates isotropic component of velocity; dashes indicate magnitude and fast-direction of anisotropy (dash in upper right corner indicates 10% magnitude).

363 We test these operators within the two step elliptically anisotropic gradiometry technique  
 364 on the previous synthetic plane waves with an isotropic homogeneous moveout. We first solve  
 365 the linear system (eq. 2) with eqs. (5) to (7) using eq. (29), and then solve the linear system  
 366 (eq. 2) with eqs. (30) to (35), and recover almost exactly the correct velocity, up to a remnant  
 367 average error of 0.007 % (Fig. 2b). To test whether we can recover anisotropy, we add 10%  
 368 anisotropy (the difference between maximum and minimum velocities as a percentage of the  
 369 isotropic velocity) in four different principal directions  $0^\circ$ ,  $45^\circ$ ,  $90^\circ$ ,  $135^\circ$ . Fig. 4 shows that  
 370 we can recover anisotropy in those principal directions throughout the maps: the remaining  
 371 errors in the isotropic component and the angle are on average respectively 0.016% and  
 372  $0.267^\circ$ . However, we underestimate the magnitude of anisotropy by on average 47.45%.

373 We now test the ability to invert deviations from the velocity for which we calibrated  
 374 the finite difference stencils (490 m/s). The velocity is varied according to a checkerboard  
 375 pattern with a velocity anomaly of  $\pm 5\%$  (Fig. 6a). The computations are kept simple by  
 376 computing a set of plane waves for each subset of stations independently. Therefore, the  
 377 test does not reveal any information regarding the lateral resolution of the recovered image,  
 378 but does assess the ability to estimate velocities given the irregular stencil shapes around

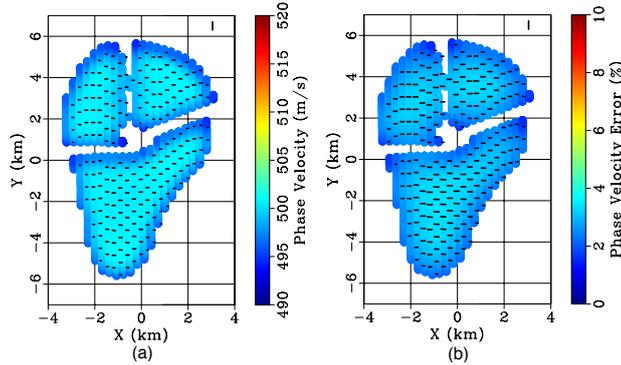
each location. The retrieved pattern shows that we significantly under estimate anomalies (Fig. 6b). The recovered positive anomalies have a 2.6% magnitude, while the recovered negative anomalies have a 2.4% magnitude. To understand this we analyse the spectra of the scaled finite difference stencils (Fig. 3). Although we corrected the error at a particular wavelength corresponding to a given velocity and frequency, for waves propagating with higher or lower velocities we will continue to respectively underestimate and overestimate the velocity. Fig. S1 in the supplementary material shows the error in retrieved isotropic anomaly and in anisotropic magnitude as a function of anomaly magnitude.

Finally, we test the effect of noise in wavefield gradiometry. Fig. 5 contains the results of a similar synthetic plane-wave data experiment as in Fig. 2, where we added Gaussian distributed noise to the synthetic plane wave data, with zero mean and a variance of 2% times the maximum amplitude. Despite that the added noise has zero mean, the inversion is biased towards higher velocities and includes an anisotropic component with the fast-direction aligning with the cross-line direction. We expect the bias to be a non linear function of the noise strength, and vary with the precise statistical characteristics of the noise. This bias diminishes our ability to iterate the calibration approach described above. Nevertheless, in the next section we propose a procedure to apply a correction to the recovered anisotropic velocity map.

### 3.1 Correction for specific anisotropic medium properties

The above procedure corrects the finite difference stencils, optimized for a specific isotropic velocity. We can generalize this procedure to correct the finite difference stencils for specific anisotropic medium properties. Say the true-target anisotropy is  $\mathbf{M}_t$ , but the estimated anisotropy without stencil correction is  $\mathbf{M}_m$ . The measured anisotropy can then be transformed into the true-target anisotropy by the following transform

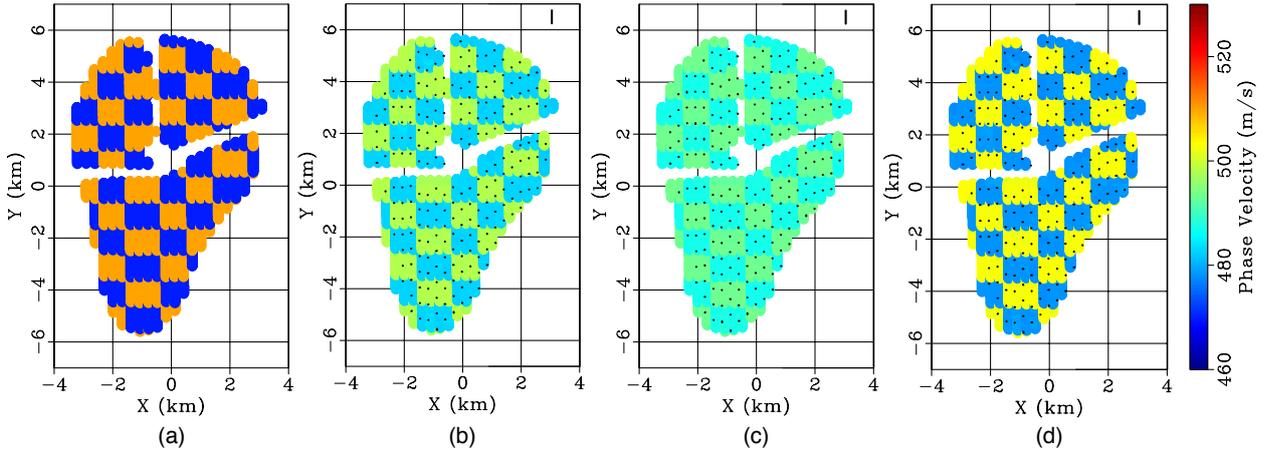
$$\mathbf{M}_t = \mathbf{P}_t \mathbf{\Lambda}_t^{\frac{1}{2}} \mathbf{P}_t^T \mathbf{P}_m \mathbf{\Lambda}_m^{-\frac{1}{2}} \mathbf{P}_m^T \mathbf{M}_m \mathbf{P}_m \mathbf{\Lambda}_m^{-\frac{1}{2}} \mathbf{P}_m^T \mathbf{P}_t \mathbf{\Lambda}_t^{\frac{1}{2}} \mathbf{P}_t^T \quad (36)$$



**Figure 5.** Synthetic data example displaying the effect of noise using the station geometry of the Ekofisk OBC array, inverting monochromatic plane waves at 0.7 Hz with a homogeneous and isotropic velocity of 490 m/s plus Gaussian distributed noise with a 2% variance. a) Recovered apparent anisotropic velocity map with linear scalebar in (m/s). b) Error in recovered apparent anisotropic velocities as a percentage of the true isotropic velocity. Dash in upper right corner indicates 10% anisotropy magnitude.

404 where the columns of  $\mathbf{P}_t^T$  and  $\mathbf{P}_m^T$  contain the eigenvectors of  $\mathbf{M}_t$  and  $\mathbf{M}_m$ , while  $\mathbf{\Lambda}_t$  and  $\mathbf{\Lambda}_m$   
 405 are diagonal matrices with the eigenvalues of  $\mathbf{M}_t$  and  $\mathbf{M}_m$  on the diagonals. We recognize  
 406 that  $\mathbf{J}_m^{-1} = \mathbf{P}_m \mathbf{\Lambda}_m^{-\frac{1}{2}} \mathbf{P}_m^T$  is a Jacobian transforming the measured anisotropy into an isotropic  
 407 unitary two-by-two matrix, and recognize that  $\mathbf{J}_t = \mathbf{P}_t \mathbf{\Lambda}_t^{\frac{1}{2}} \mathbf{P}_t^T$  is a Jacobian that transforms  
 408 the isotropic unitary two-by-two matrix to the true anisotropy. If we define  $\mathbf{J}^{-1} = \mathbf{J}_m^{-1} \mathbf{J}_t =$   
 409  $\mathbf{P}_m \mathbf{\Lambda}_m^{-\frac{1}{2}} \mathbf{P}_m^T \mathbf{P}_t \mathbf{\Lambda}_t^{\frac{1}{2}} \mathbf{P}_t^T$  we can use a similar linear system as before, because we have  $\mathbf{M}_t =$   
 410  $\mathbf{J}_t^T \{ \mathbf{J}_m^{-1} \}^T \mathbf{M}_m \mathbf{J}_m^{-1} \mathbf{J}_t$ . For an isotropic true medium  $\mathbf{J}_t^{-1}$  reduces to  $\mathbf{I} C_h^{-1}$ , where  $\mathbf{I}$  is a two-  
 411 by-two identity matrix, this agrees with eq. (27). Ideally, one would iteratively update the  
 412 stencil corrections using the retrieved anisotropic velocities at each iteration. However, due  
 413 to the effect of the unknown precise noise levels (Chartrand, 2011), such a scheme does not  
 414 easily converge. Alternatively one could apply a first order correction for the underestimation  
 415 as follows: use the derived underestimated anisotropic velocity map to compute a synthetic  
 416 dataset, and use gradiometry to derive a new anisotropic velocity map that repeats the  
 417 underestimation. Employ the relationship in eq. (36) to derive a transform that predicts the  
 418 underestimation. Lastly, apply the inverse of this transform to the original retrieved map.

419 We illustrate this procedure in Fig. 6b-6d. Fig. 6c contains the secondary derived anisotropic  
 420 velocity map underestimating the correct values from Fig. 6b. The recovered positive anoma-



**Figure 6.** Checkerboard test for anomaly magnitude. Colour indicates isotropic component of velocity; dashes indicate magnitude and fast-direction of anisotropy (dash in upper right corner indicates 10% magnitude). a) Input isotropic velocity, with 5% anomaly magnitude. b) Anisotropic velocity map obtained using a synthetic created using the medium parameters of the isotropic velocity map in (a) and calibrated finite difference stencils, underestimating the anomalies (positive anomalies are recovered as 2.6% and negative anomalies are recovered as 2.4%). c) Anisotropic velocity map obtained using a synthetic created using the medium parameters of the anisotropic velocity map in (b), underestimating the anomalies again (positive anomalies are recovered as 1.6% and negative anomalies are recovered as 1.5%). d) Final anisotropic velocity map using the calibrated finite difference stencils plus anomaly-magnitude correction (positive anomalies are recovered as 3.2% and negative anomalies are recovered as 3.0%).

421 lies have a 1.6% magnitude, while the recovered negative anomalies have a 1.5% magnitude.  
 422 The derived transform predicts Fig. 6c from Fig. 6b. By assuming that the degree of under-  
 423 estimation of anisotropy is consistent at models with larger anisotropy than the model we  
 424 obtained in Fig. 6b, we apply the inverse of this transform to Fig. 6b resulting in Fig. 6d. The  
 425 retrieved positive positive anomalies have a 3.2% magnitude, while the recovered negative  
 426 anomalies have a 3.0% magnitude (still short of the original 5% anomaly magnitude).

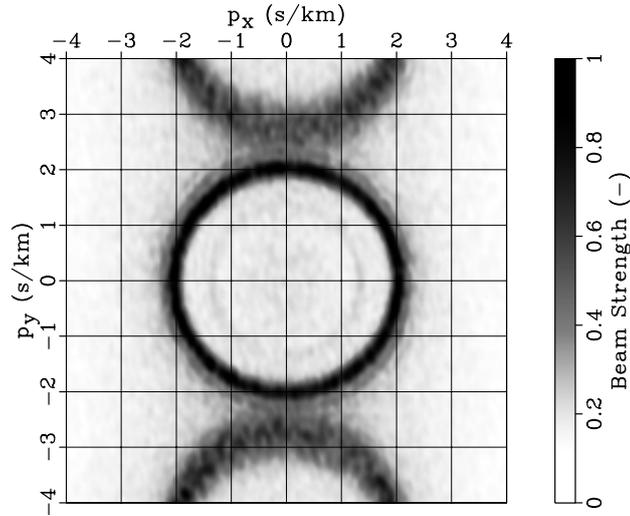
427 Though the retrieved anomaly magnitudes remain underestimated, they are closer to  
 428 the true anomaly magnitudes. This procedure relies on linearity of the underestimation  
 429 with anomaly magnitude. But because the stencil error is non-linear with wavelength, the  
 430 underestimation increases for larger anomaly magnitudes (see Fig. S1) in the supplemental  
 431 material which shows the underestimation as a function of anomaly magnitude).

#### 4 FIELD DATA EXAMPLE AT EKOFISK FIELD

Ekofisk field is one of the largest hydrocarbon fields in the North Sea, it was Norway's first producing field in 1971 (Van den Bark & Thomas, 1979) and has a projected lifespan exceeding year 2050. Rapid pressure depletion in the early phase of production and weakening due to subsequent water injection caused more than 9 m of seafloor subsidence over the Ekofisk field (Herwanger & Horne, 2009; Lyngnes et al., 2013). The subsidence is known to dominate the pattern in the anisotropic Scholte wave phase velocities in the near-surface (Kazinnik et al., 2014; De Ridder et al., 2015).

An OBC array was installed at Ekofisk in 2010 for the purposes of repeated seismic surveying (Eriksrud, 2010). The cables are buried in mud on the seafloor and the stations generally exhibit similar coupling to the sea floor. The characteristics of the microseism energy recorded by this array are well known (De Ridder & Biondi, 2015a; De Ridder et al., 2015). It was found that the pressure sensors record strong microseisms at frequencies between 0.35 and 1.35 Hz. This energy is dominated by fundamental-mode Scholte waves propagating along the seafloor. Below 0.8 Hz these waves are recorded unaliased in both the in-line and cross-line directions. No strong sources of seismic energy were found within the array in the microseism frequency range 0.35 to 1.35 Hz.

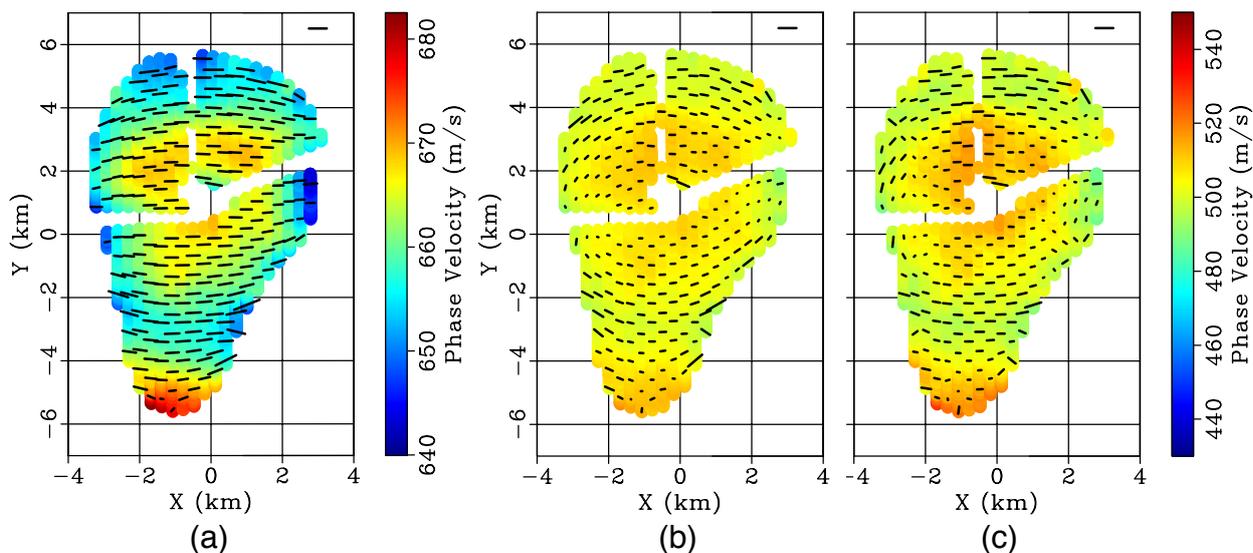
A recording of 10 minutes by the pressure sensors of the Ekofisk array was bandpass filtered between 0.6 Hz and 0.8 Hz using a Hann taper in the frequency domain, the data are downsampled to a 10 Hz sampling rate keeping the error in the temporal finite difference stencil small. Ten minutes were found to be sufficient to yield a map of isotropic phase velocities using wavefield gradiometry (De Ridder & Biondi, 2015b). We investigate the nature of the directionality of the ambient seismic field for a short recording of ten minutes by a beamform experiment consisting of plane wave stacks for planes defined by a moveout, azimuth and intercept time, i.e., a Tau-P transformation. Finally, we sum the absolute value of the plane wave stacks over all intercept times to form an image as a function of moveout and azimuth which is defined by horizontal slowness in both spatial directions (Fig. 7) (Kostov & Biondi, 1987; Rost & Thomas, 2002). Averaged over as little as 10 minutes, there



**Figure 7.** Beam steering image obtained by plane-wave stacking with different moveout velocities and directions, using 10 minutes of data and all stations of the array.

460 is no obvious preferential direction in the ambient seismic noise: the waves are incident on  
 461 the array from all directions approximately equally strongly. The two circles above and below  
 462 the center circle are aliasing ghost images of the same surface wave energy. The faint inner  
 463 ring visible in Fig. 7 is the manifestation of energy of a higher surface wave mode (traveling  
 464 with approximately 770 m/s), this energy is neglected in this study.

465 First, we solved the linear inverse system (eq. 2) for isotropic velocities with eqs. (5) to (7),  
 466 without calibrated finite difference stencils. Second, we solved the linear system (eq. 2) with  
 467 eqs. (17) to (19) for an anisotropic velocity map (Fig. 8a), using the solution of the isotropic  
 468 case as the background velocity map. We find velocities that are much higher than the known  
 469 average velocity from dispersion analysis. Furthermore, we find an anisotropic pattern where  
 470 the fast-directions are generally oriented perpendicular to the cables. This is expected from  
 471 the synthetic plane wave example above (compare to Fig. 2a). We then use the calibrated  
 472 stencils, first solving the linear system (eq. 2) with eqs. (5) to (6) using eq. (29), then solving  
 473 the linear system (eq. 2) with eqs. (30) to (35), and we obtain the anisotropic velocity map  
 474 in Fig. 8b. Finally, we model synthetic plane waves satisfying the recovered anisotropic  
 475 medium parameters in Fig. 8b, and follow the anisotropic gradiometry procedure to recover  
 476 a map with underestimated anisotropic and anomaly magnitudes. We compute the transform



**Figure 8.** Field data result on Ekofisk’s OBC array. Colour indicates isotropic component of velocity; dashes indicate magnitude and fast-direction of anisotropy (dash in upper right corner indicates 10% magnitude). a) Velocity map recovered with finite difference stencils without calibration. b) Velocity map recovered using the calibrated finite difference stencils. c) Final velocity map recovered using the calibrated finite difference stencils plus anomaly-magnitude correction.

477 estimating the underestimation and apply the inverse to the medium parameters in Fig. 8b  
 478 to yield Fig. 8c. The magnitude of the velocity anomaly in the center of the array, and the  
 479 magnitude of anisotropy oriented in-line at the left and right flanks of the array increased  
 480 notably from Fig. 8b.

## 481 5 DISCUSSION

482 In principle, directionality in the ambient seismic noise will bias the inverted seismic ve-  
 483 locities because the stencil error is directionally dependent. In this manuscript, we have  
 484 given the plane waves from all directions equal weight when computing the synthetic ex-  
 485 ample in Fig. 2. However, an estimate for the directional distribution can in principle be  
 486 used as weights in the implicit regression to compute the bias of the array geometry, and  
 487 thus be taken into account when computing the calibration for the finite difference stencils.  
 488 The beamform experiment on the Ekofisk data provided the basis for not introducing such a  
 489 weighting scheme in the field data application as the noise appeared to be equally distributed

with azimuth. Ideally, the stencil calibration is iterated using the recovered anisotropic ve-  
locities to end with a set of finite difference stencils optimized for the recovered velocities.  
However, we found that this scheme does not generally converge. We conclude that this was  
probably due to the presence of noise in the field data because we observed that zero mean  
Gaussian distributed noise in the data causes a velocity bias (Fig. 5). This is a result of the  
error in finite difference stencils not being a linear function of the underlying wavelength  
(Fig. 3).

Generally, the computational costs of seismic noise gradiometry are relatively low com-  
pared to other techniques to image using ambient seismic noise. Seismic noise gradiometry  
requires only short recordings (De Ridder & Biondi, 2015b), and the regression operation  
itself is also kept computationally efficient by posing the finite differences on the irregular  
station geometry itself, by-passing the need for an interpolation scheme. Another argu-  
ment for avoiding spatial interpolation is the inherent imposition of a usually non-physical  
model for seismic wavefields when electing an interpolation scheme. It would be physically  
most accurate to base an interpolation scheme on the wave equation itself, however that  
requires a priori knowledge of the underlying wave velocities. The total computational costs  
in our implementation are dominated by the inversion for anisotropic velocities because the  
anisotropic model space is three times larger than the isotropic model space, and the matrix  
in eq. (20) is nine times larger than the matrix in eq. (8). We used an LU decomposition  
to solve the matrix inversion, but employing Krylov subspace techniques may be a faster  
alternative.

Measurements of near-surface anisotropy are typically of interest for near-surface hazard  
monitoring (Barkved, 2012) and to infer geomechanical changes in the reservoir and overbur-  
den (Herwanger & Horne, 2009). These results match qualitatively with those found by an  
eikonal tomography on travel-time surfaces extracted from noise correlations (De Ridder &  
Biondi, 2015a), and critically refracted P waves, PS converted waves, surface wave analysis  
of controlled source seismic (Van Dok, 2003; Kazinnik et al., 2014). The circular pattern in

517 azimuthal anisotropy has also been observed in seismic noise correlation tomography studies  
 518 at nearby Valhall field (Mordret et al., 2013b; De Ridder 2014).

519 The resolution of wavefield gradiometry is limited by the stencil span from the assumption  
 520 of homogeneity over the stencil span: in this study based on the Ekofisk OBC array this is  
 521 at 800 m. In practice, the scattered wavefield due to subsurface changes is neglected, and  
 522 we recover a spatially averaged anisotropic phase velocity map revealing spatially varying  
 523 properties up to the resolution of the stencil span.

524 We solved for a phase velocity map at 0.7 Hz, but the procedure could be repeated for  
 525 different frequencies mapping dispersion curves throughout the array. These surface wave  
 526 dispersion curves could be inverted for depth structure (Kennett, 1976). However, in practice  
 527 this may be difficult due to aliasing at higher frequencies, and spurious geophone sensitivity  
 528 far below the natural frequency of each sensor.

529 Because there is no technique to measure particle velocity throughout the subsurface of  
 530 the earth, seismic gradiometry based on the three dimensional elastodynamic wave equation,  
 531 eq. (1), with the aim of imaging elastic properties throughout the medium remains illusive  
 532 (Curtis & Robertsson, 2002; Muijs et al., 2003). However, in medical sciences a similar  
 533 technique named elastography is used to extract the local stiffness from measurements of  
 534 strains due to an induced stress, which has found wide application for the purposes of for  
 535 example examining prostrate lesions, arteries, and tumors (Garra et al., 1997; De Korte  
 536 et al., 1998; Pesavento & Lorenz, 2001; DeWall, 2013). Specifically, magnetic resonance  
 537 elastography is based on tracking waves through human tissue for finding elastic parameters  
 538 (Manduca et al., 2001).

## 539 6 CONCLUSIONS

540 Dense seismic networks deployed on the surface of the earth allow surface waves to be  
 541 measured unaliased in time and space. These recordings permit estimation of the spatial  
 542 derivative of surface-wave wavefields by finite differences, thus providing the ingredients  
 543 needed to invert an elliptically anisotropic, two-dimensional wave equation for local medium

544 properties. An advantage of this method is that it permits short recordings of surface-wave  
545 noise to be inverted. The main challenge is the error caused by the spatial FD stencils: this  
546 causes an overall anisotropic velocity error, and leads to the under-estimation of isotropic  
547 velocities. We formulated a two step approach to calibrate finite difference stencils, and  
548 perform a first order correction for the velocity anomaly magnitudes. The method is a  
549 promising technique for studying changes in the subsurface geomechanical strain resulting  
550 from time dependent phenomena operating at short time-scales, which in the example herein  
551 are likely to be due to subsidence-related extension.

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# Seismic Gradiometry using Ambient Seismic Noise in an Anisotropic Earth: Supplementary material

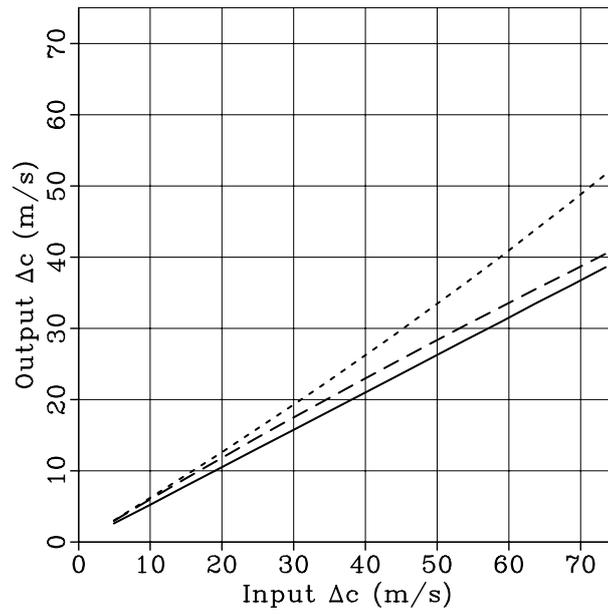
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We conducted a series of synthetic experiments to map the underestimation of isotropic and anisotropic velocity anomalies. First, we created a series of synthetic datasets with a checkerboard pattern as for the example in Fig.5. We measure the magnitude of the recovered positive and negative anomalies versus the magnitude used to create the synthetic dataset. We systematically underestimate the positive and negative anomalies, and the underestimation is not a linear function of anomaly magnitude as it increases with larger input anomaly magnitude (coarse and fine dashes in Fig. S1). Second, we created a series of synthetic datasets with anisotropy as in the example in Fig.4. We measured the recovered anisotropy magnitude, versus the anisotropy magnitude used to create the synthetic dataset. We systematically underestimate the anisotropy magnitude, and the underestimation is not a linear function of anisotropy magnitude (solid curve in Fig. S1).



**Figure S1.** Recovered minimum (coarse dashes) and maximum (fine dashes) anomaly in magnitudes versus input anomaly magnitude determined by repeated checkerboard tests recovering isotropic velocities. Recovered anisotropy magnitude (solid curve) defined as the difference between the maximum and minimum wave speeds, versus input anisotropy magnitudes in repeated tests recovering anisotropic velocities.