Selecting representative tide conditions for tidal range and energy assessments

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ABSTRACT: Tides are predictable, and exhibit a known variability over time. We explore how this variability affects conclusions in accurately predicting tidal elevations and conducting tidal energy resource assessments over constrained/finite periods. Specifically, in most numerical modelling studies for tidal range energy assessment and optimisation, analyses span short time frames due to computational restrictions. Moreover, tidal elevations imposed to force models at open boundaries often rely on a limited and varying number of constituents. This study proposes a methodology for selecting a representative lunar-monthly tide in terms of tidal range and energy output by devising metrics that assess the magnitude and variability of key variables within that period relative to nodal cycle average quantities. Results, taking tide gauges within UK waters as an example, indicate that local features and associated hydrodynamics influence tidal patterns in terms or amplitude, in a manner noticeable beyond the main semidiurnal constituents $M_2$, $S_2$ and $N_2$. Moreover, representative periods are sensitive to the constituent set and only once the leading 8 constituents are applied do we observe a satisfactory convergence. Relative to values such as the significant wave height or average potential energy, the values within a lunar month can vary by 14.9% and 30% respectively. Interestingly, the tidal range and energy variability using IQR can vary by over 45% and a combination of metrics is suggested to be used. The findings also demonstrate that, assuming sufficient constituents are used, representative periods at a site can correlate with equivalent periods at tidal gauge locations of the same tidal system, providing encouraging approximation in the tidal range evolution and the encompassed energy resource. Additionally, despite the non-linear relationship, it is shown that representative periods in terms of tidal range also correspond to representative periods in terms of potential energy and vice versa.

1 INTRODUCTION

Tides owing to their predictability and high energy density in certain coastal regions are attractive for marine energy developments which can contribute towards a net zero society (Coles et al. 2021). Tidal energy technologies can be classified into ‘range’, when seeking to harness the wave potential energy at sites of amplified resonance (Neill et al. 2018), or ‘stream’, when targeting the kinetic energy emerging by flow accelerations driven through tidal streaming or hydraulic gradients (Adcock et al. 2021). We focus herein on the potential energy, where tidal range structures can be strategically placed in coastal regions of high tidal range and sufficient water depth for sitting hydro-turbines. The operation principle entails facilitating a head difference between water levels across a tidal barrier. This head difference drives the water through low head hydro turbines, generating electricity (Angeloudis et al. 2020).
Many studies rely on numerical modelling for tidal range energy assessments, with detailed optimisation analyses of specific schemes applied over short time frames given computational constraints (Aggidis and Benzon 2013, Angeloudis et al. 2018, Harcourt et al. 2019, Burrows et al. 2009). Where hydrodynamics are considered, tidal elevations are used to force models which are informed by a number of constituents that is varying across studies. The selection of appropriate simulation periods and the essential constituents to force the modelling for robust conclusions is not based on concrete guidance. This motivates the present research on tidal energy variability. Effectively, we explore whether there are particular periods that should be firstly identified to assess marine energy developments at varying sites.

In particular, Burrows et al. (2009) acknowledged that in the conjunctive operation of five major tidal barrages on the west coast of the UK, the addition of three constituents aside from the principal $M_2$ and $S_2$ (which were used in their analysis), provide an additional energy source, indicating that these should be considered for more accurate resource assessments. Xue et al. (2019), while focusing on an optimisation study for tidal range structures, showed how the difference between maximum and minimum energy outputs over spring-neap cycles can be in the order 25%. In turn, the study defined a representative period for annual generation estimation as the cycle with the smallest deviation from the average annual output. However, this approach focuses on the aggregate energy output and does not provide insight into how representative the tidal signal spring-neap elevation could be relative to the long-term trends. More recently, Mackie et al. (2021) made use of representative tidal level definitions from the National Tidal and Sea Level Facility (e.g. Mean high/low water springs/neaps) across several locations around the UK to identify an appropriate interval to assess multiple tidal range designs at various locations. Again, this identification relies on a handful of discrete values with limited insight to the variations over spring-neap cycles, motivating further research.

In this study we investigate the significance of the quantity of considered tide constituents on reconstructed tidal elevation signals of sufficient duration for robust tidal range energy and impact assessments. In order to balance computational constraints, these assessments tend to simulate finite periods in the order of a lunar cycle of 29.53 days (Hicks 2006, Kvale 2006), which is sufficient to validate the principal constituents ($M_2$, $S_2$) at tide gauges. In turn, energy predictions are extrapolated to forecast the energy yield of prospective schemes. We thus formulate the problem as the robust definition and identification of a representative tidal signal spanning a lunar-monthly period in terms of its tidal range and potential energy magnitude and variation relative to a nodal tidal cycle of 18.6134 years (Kowalik and Luick 2019). Practically, this question is driven by a motivation to fairly compare schemes that are subjected to varying tidal conditions, considering the multiple inconsistencies that need to be balanced in the assessment of tidal range structures, as in Mackie et al. (2021).

2 METHODOLOGY

2.1 Tidal signal reconstruction

Tides are a regular and predictable phenomenon in the form of very long waves that arise from the gravitational forces between the Earth, Moon and Sun. The periodic motions in this system determine the various frequencies, and therefore patterns, at which tidal waves occur. Using harmonic analysis these patterns can be broken down to a series of simpler sinusoidal waves called tidal harmonics, or ‘tidal constituents’, and are represented by an amplitude and a phase (Parker 2007). The water elevation of any tidal signal at any location and at arbitrary time can be calculated by (Parker 2007):

$$
\eta(t) = h + \sum_{i=1}^{n} f_i \alpha_i \cos(\omega_i t + \{V_0 + u\}_i - \phi_i)
$$

where $h$ is the mean surface level above the datum, $f_i$ is a node factor to account for the effect of the nodal cycle on the amplitude of constituent $i$, $\{V_0 + u\}_i$ is an equilibrium argument for constituent $i$ at time zero, $\alpha_i$ is the constituent’s mean amplitude of the nodal cycle at the location, and $\phi_i$ the phase lag of the constituent at the location behind the corresponding constituent at Greenwich.

In this study harmonic analysis is conducted using Python package *uptide* (Kramer 2020) to reconstruct tidal signals at 46 tidal gauge stations across the UK as in Fig. 1. Tide gauge data provided by the British Oceanographic Data Centre (BODC) are utilised in the reconstruction process.

Table 1 presents an example of the amplitude ($\alpha$) and phase ($\phi$) of the most influential constituents extracted. It is instructive to introduce a ‘participation percentage quantity’, $\alpha_i/\Sigma \alpha$, relative to the aggregate amplitude of known constituents as an indication of influence to the tidal signal over the timescales considered. As expected, on all tidal gauge locations, the principal semidiurnal constituents $M_2$, $S_2$ and $N_2$ are prevailing in this order. On the other hand, the contribution of the remaining constituents varies in terms of rank relative to their participation percentage. For instance, in Avonmouth, where the estuary becomes narrower and the basin depth shallower, shallow-water override constituents become more influential compared to Llandudno where the streamwise channel is less constricted. Indicatively, the MS₄ participation factor in Avonmouth is almost twice that of Llandudno’s. This demonstrates how local hydrodynamics driving tidal wave transformation affect patterns of the tidal signals at varying locations.
Reconstructed tidal elevations are then compared against measured water levels at tide gauge locations. An example of these time-series comparison is presented in Fig. 2, where the tidal range $R_i$ recorded by the $i^{th}$ transition from high water to low water and vice versa is annotated. Apart from the tidal components that make up the observed system, even if the UK coastal ocean is classed as macrotidal (Whitfield and Elliott 2011), there are non-tidal contributions that are neglected in the harmonic analysis, including contributions from storm surges (Lewis et al. 2017) as well as non-linear wave transformation at shallow regions. This invariably leads to the deviations between the observed and reconstructed signals. To statistically evaluate the accuracy of reconstruction of the harmonic analysis two error metrics are used; the Normalised Root Mean Square Error (NRMSE) and the coefficient of determination ($R^2$), defined as

$$\text{NRMSE} = \sqrt{\frac{\sum_{i=1}^{N} (\hat{\eta}_i - \eta_i)^2}{N}}$$

and

$$R^2 = \frac{\sum_{i=1}^{N} (\hat{\eta}_i - \eta_i)^2}{\sum_{i=1}^{N} (\eta_i - \mu)^2},$$

where $N$ is the length of data set, $\mu$ is the mean of observed water elevations, $\hat{\eta}_i$ the observed values and $\eta_i$ the predicted ones. Notably, NMRSE is preferred to RMSE in order to provide a fair comparison given the variation of the tidal range magnitude across tide gauge stations.

### 2.2 Representative lunar period period definitions

In setting out this analysis, we consider that a nodal cycle $N$ of 18.6134 years contains 13137 $M_2$ periods, noting that $T_{M_2} = 12.42$ hours. A lunar month $M$ of 29.53 days contains 57 $M_2$ periods. The approach taken here assumes that a lunar month segment can start at the beginning of any $M_2$ periods forming the nodal cycle, and thus the analysis considers 13137 lunar cycles.

#### 2.3 Tidal range $R_i$

As in Fig. 2b, tidal signals of multiple constituents are not sinusoidal, and they vary over short- and long-term timescales according to the constituent amplitude and phase. If we consider the distribution of the tidal range magnitude per lunar month, which is a relatively short-term period of $M_2$ cycles, it becomes clear that the distribution is non-Gaussian (Fig. 3). However, when observing the same distribution over the nodal cycle, with the increased number of cycles and constituents (e.g. Fig. 3d for 12 constituents) a quasi-normal distribution as expected from the Central Limit Theorem emerges. Given our constraints to a finite period, we opt to adopt a non-parametric approach to define the representative month. We consider a first metric as $M_1$ that takes into account the magnitude and the variability of these variables. For magnitude, we use the median $P_{50}$, that is, the 50th percentile value, preferred as a resistant measure that is not strongly influenced by a few extreme values. For the variability, we adopt a widely used non-parametric resistant measure of spread of data is the interquartile range ($IQR$) (Helsel et al. 2020) which measures the range of 50% of data, discounting the lower and upper 25th and 75th percentiles respectively.

Assuming that the median and the $IQR$ are equally weighted ($\alpha = \beta = 0.5$) we define a first metric func-

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Table 1: Constituents extracted from tide gauge records of BODC for Avonmouth and Llandudno based on the magnitude of amplitude. $\alpha$ is the amplitude (m), $\phi$ the phase ($^\circ$) and $T$ the period of constituents (h).

<table>
<thead>
<tr>
<th>Constituents</th>
<th>Origin</th>
<th>$T$ (h) (m)</th>
<th>$\alpha_i$ (%)</th>
<th>$\phi_i$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Diurnal</strong>:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K_1$</td>
<td>Luni-solar</td>
<td>23.93</td>
<td>0.07</td>
<td>75.72</td>
</tr>
<tr>
<td>$O_1$</td>
<td>Lunar</td>
<td>25.81</td>
<td>0.07</td>
<td>75.72</td>
</tr>
<tr>
<td><strong>Semidiurnal</strong>:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_2$</td>
<td>Lunar</td>
<td>12.42</td>
<td>4.29</td>
<td>46.03</td>
</tr>
<tr>
<td>$S_2$</td>
<td>Solar</td>
<td>12.00</td>
<td>1.53</td>
<td>16.42</td>
</tr>
<tr>
<td>$N_2$</td>
<td>Lunar</td>
<td>12.66</td>
<td>0.77</td>
<td>8.26</td>
</tr>
<tr>
<td>$K_2$</td>
<td>Luni-solar</td>
<td>11.97</td>
<td>0.42</td>
<td>4.51</td>
</tr>
<tr>
<td>$L_2$</td>
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<td>0.30</td>
<td>3.22</td>
</tr>
<tr>
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<td>Solar</td>
<td>29.96</td>
<td>0.10</td>
<td>1.07</td>
</tr>
<tr>
<td>$L_4$</td>
<td>Lunar</td>
<td>12.72</td>
<td>0.10</td>
<td>1.07</td>
</tr>
<tr>
<td>$\mu_2$</td>
<td>Lunar</td>
<td>12.87</td>
<td>0.51</td>
<td>5.47</td>
</tr>
<tr>
<td>$\nu_2$</td>
<td>Lunar</td>
<td>12.63</td>
<td>0.19</td>
<td>2.04</td>
</tr>
<tr>
<td>$2M_2$</td>
<td>Shallow</td>
<td>11.61</td>
<td>0.15</td>
<td>1.61</td>
</tr>
<tr>
<td><strong>Higher-Order</strong>:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_{44}$</td>
<td>Shallow</td>
<td>6.10</td>
<td>0.24</td>
<td>2.58</td>
</tr>
<tr>
<td>$M_{4}$</td>
<td>Shallow</td>
<td>6.21</td>
<td>0.26</td>
<td>2.79</td>
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<tr>
<td>$2M_{4}$</td>
<td>Shallow</td>
<td>4.09</td>
<td>0.16</td>
<td>1.72</td>
</tr>
</tbody>
</table>

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### References

For the tidal range of a lunar cycle as:

$$M_{1,R_i,j} = \alpha \times |P_{50}(R_{M_i,j}) - P_{50}(R_{N_i})|$$

$$+ \beta \times |IQR(R_{M_i,j}) - IQR(R_{N_i})|$$

where $R_{M_i,j}$ and $R_{N_i}$ are arrays containing the tidal range $R_i$ of every transition $i$ (Fig. 2) within the $j$th lunar month $M_j$, reconstructed using $k$ constituents. Similarly $R_{N_i}$ is an array of $R_i$ values over a nodal cycle $N$ respectively. We describe this metric as discrete, on the basis of its reliance on the tidal signal peaks and troughs rather than the continuous signal. Using an iterative approach that considers a lunar cycle starting at the same point of every $M_2$ cycle, values of $M_{1,R}$ are calculated and sorted based on magnitude. Although in our analysis we consider the $P_{50}$ and $IQR$ equally significant, it should be noted that the two terms of Eq. (4) are contributing with $\approx 30\%$ and $\approx 70\%$ respectively on average and in future studies the corresponding weight factors can be adjusted. Having ranked the values based on Eq. (4) we also consider a second metric as:

$$M_{2,R_i,j} = |H_{m0M_i,j} - H_{m0N_i}|$$

where $H_{m0M_i,j}$ and $H_{m0N_i}$ are the significant wave lengths over the $j$th lunar month $M_j$ and the nodal cycle $N_k$ respectively. Having established the two metrics we define the representative month as the corresponding time interval of the minimum of Eq. (7) within the $5^{th}$ baseline percentile of the values that obtained from Eq. (4), denoted as $M_{R_i,r}$.

2.4 Tidal range energy $E_i$

Similarly, we seek to define a representative period in terms of the signal potential energy. For contextual purposes, tidal elevations can be used as an input to determine the potential energy that can be extracted under the operation of a tidal range power plant (Angeloudis et al. 2021). The theoretically available potential energy per unit surface area contained in a tidal range structure over a transition $H$, neglecting any form of losses can be quantified as (Prandle 1984):

$$E_{max} = \frac{1}{2} \rho g H^2$$
where \( \rho \) is the fluid density, \( g \) is the gravitational acceleration and \( H = \eta_o - \eta_i \) is the head difference created between the seaward (\( \eta_o \)) and impounded (\( \eta_i \)) water elevations. It is this resource tidal range structures aim to harness. We are interested in quantifying the theoretically available resource thus we set \( H = R_i \) in Eq.- 8 for each transition \( i \) and the theoretically available potential energy per unit area per transition \( i \) becomes

\[
E_i = \frac{1}{2} \rho g R_i^2
\]  

(9)

Fig. 4 illustrates two examples of the distribution of \( E_i \) at Avonmouth and Llandudno over nodal periods \( N_k \), with \( k \in \{2, 4, 8, 12\} \). In both sites the range of \( E_i \) for 2 constituents is narrow and becomes wider with the addition of constituents. Indicatively, the percentage difference of the maximum \( E_i \) of 12 constituents compared to the case of 2 constituents is -29.2% and -38.6% in Avonmouth and Llandudno respectively.

Following a similar notation as per the identification of \( M_{R,k,j} \), we define a first metric to rank the lunar months based on the magnitude and variability of \( E_i \) as:

\[
M_{1,E,k,j} = \alpha \times |P_{50}(E_{M,k,j}) - P_{50}(E_{N_k})| \\
+ \beta \times |IQR(E_{M,k,j}) - IQR(E_{N_k})|
\]

(10)

where \( E_{M,k,j} \) and \( E_{N_k} \) are arrays of the theoretical potential energy \( E_i \) over each transition \( i \) within the \( j^{th} \) lunar month \( M_{k,j} \) and the nodal cycle \( N_k \) as reconstructed using \( k \) constituents.

As with \( M_{1,R,k,j} \), \( M_{1,E,k,j} \) relies on discrete quantities rather than the entire tidal signal. We thus also consider the average potential energy contained over
time in tidal waves. The potential energy of a progressive wave averaged over a period \( T \) is (Dean & Dalrymple 1991):

\[
PE(t, T) = \frac{1}{T} \int_{t}^{t+T} \frac{\rho g (h + \eta)^2}{2} dt
\]  

(11)

Noting that the depth \( h \) contributes to the hydrostatic energy of the water column and our focus is solely on the potential energy of the surface wave, \( h \) can be excluded by considering as datum the MWL. We denote the total potential energy contained in a tidal wave over a lunar month \( j \) as \( PE_{M_{k,j}} \). By extension, the second metric regarding tidal range energy is defined as:

\[
M_{2,E_{k,j}} = |PE_{M_{k,j}} - PE_{N_k}|
\]  

(12)

with arguments \( PE_{M_{k,j}} \) and \( PE_{N_k} \); that is, the averaged potential wave energy over the \( j^{th} \) lunar month \( M_{k,j} \) and the nodal cycle \( N_k \). Consistently with the tidal range approach, the corresponding time interval of the minimum of Eq. (12) over the 5th percentile of the values of Eq. (10) is selected as the representative month in terms of tidal range energy, denoted as \( M_{k,r}^E \).

3 RESULTS & DISCUSSION

3.1 Harmonic analysis prediction performance

Reconstruction of tidal signals is performed at all sites where BODC data is available. NRMSE and \( R^2 \) for the locations of highest range are shown in Fig. 5 for an increasing number of constituents. The largest NRMSE and the smallest \( R^2 \) were predicted at Avonmouth were the total amplitude is the greatest (8.98 m). This is also expected due to the pronounced nonlinear shallow water hydrodynamics present at the estuarine regions. As expected, the more constituents \( k \) considered, the lower the NMRSE, and the larger the \( R^2 \), corresponding to greater correlation between modelled and recorded tidal surface elevations. We can see that from 1 to 7 constituents the curvature of the corresponding plots is steep suggesting a significant influence up until \( k = 8 \) constituents. Indicatively, for \( k = 8 \) the NMRSE value is almost the half of the corresponding one with 2 constituents. Furthermore, \( R^2 \) is in the range of 0.80-0.85 for one constituent and in turn \( R^2 \rightarrow 1 \) as \( k \) increases. Beyond 12 constituents improvement in NRMSE and \( R^2 \) is negligible, so that we consider it the baseline for the following analysis, given the data gaps present in tide gauge records. Indicatively, the absolute percentage differences of metrics for 12 constituents relative to 2 constituents are on average 5.7% and 58.7% in \( R^2 \) and NRMSE respectively. The corresponding percentage differences for 16 constituents are 5.8% and 61.4%.

3.2 Establishing the representative month

We then observe how the tidal range and energy statistics vary spatially, and subject to the consideration of different constituent sets.

Fig. 6 presents the variation of \( H_{m0M_{12,j}} \) and \( PE_{M_{12,j}} \) over the different lunar months that we consider within the nodal cycle \( N \) at Avonmouth, annotating the corresponding values for the representative months. Firstly, we note that there is noticeable variation in the calculated significant wave height \( H_{m0M_{12,j}} \) and the average potential energy \( PE_{M_{12,j}} \) when considering enough constituents. The combination of \( M_1 \) and \( M_2 \), indicates that the representative months that we obtain maintain robust approximations to the target \( H_{m0N_{12}} \) and \( PE_{N_{12}} \).

Fig. 7 presents how the \( PE_{M_{k,j}} \) of representative months identified when \( k \in \{2, 4, 8, 12\} \) deviates from \( PE_{N_{12}} \) which is perceived as the baseline, as well as the correlation between \( M_{12,r}^{E} \) and \( M_{12,r}^{R} \). We can observe that, there is a convergence to the baseline predictions once \( k \geq 8 \). The difference between maximum and minimum \( PE_{M_{12}} \) varies from 13.8% - 30% (with an average of 21.2 %) across all locations. This indicates the importance of selecting a representative period when independent studies are conducted as the selection of a particular constrained interval for the analysis can result in major under- or overestimation of available tidal ranges or energy. While equivalent results are observed for \( H_{m0M_{12}} \), we notice that dif-
ferences between the maximum and minimum relative errors of $PE_{M_{12}}$ are larger than the ones in the case of $H_{moM_{12}}$, where varies from 6.9% - 14.9% (with an average of 10.5%) over all locations. This is expected and attributed to the square relationship that exists in Eq. (11) that amplifies relatives errors. Additionally, Fig. 7 illustrates the deviation of $IQR(E_{M_{12}})$ to $IQR(E_{N_{12}})$. Interestingly we can observe that the tidal range energy variability using $IQR$ can vary over 45%. Again, we can observe there is a convergence to the target prediction once $k \geq 8$. Equivalent results are observed for $IQR(R_{M_{12}})$. This is an indication that $IQR(R_{M_{12}})$ and $IQR(R_{N_{12}})$ are correlated with integrated metrics and consequently with $H_{moM_{12}}$ and $PE_{M_{12}}$ respectively.

The set of constituents $k$ used has substantial significance to the results. For $M_{2,r}$, the percentage difference to the baseline $H_{moN_{12}}$ varies from -4.4 to 5.5% with an absolute average of 1.9% across all locations. For $k = 4$ the corresponding relative errors vary from -5 to 6.5% with an absolute average of 1.2%. For $k = 8$ the range narrows down from -3.8 to 1% with an absolute average of 0.3%. The latter would be considered acceptable given additional non-tidal uncertainties, with this exercise expected to mitigate uncertainties.

Similarly, in terms of the potential energy, in the case of $M_{2,r}^E$, the percentage difference of $PE_{M_{12}}$ to baseline varies spatially from -12.4 to 10.6% with an absolute average of 3.9%. With the addition of 2 more constituents this percentage difference is in the range of -8.3 to 13.3% with a a smaller absolute average to 2.6%. When we consider 8 constituents the corresponding differences range from -7.9 to 1.4%; however, the average of the absolute errors improved significantly to 0.6%. Consequently, in most cases the selection of 2 or 4 constituents and the use of associated representative periods having a large range of relative errors can lead to major deviation from the target $H_{moN_{12}}$ and $PE_{N_{12}}$. On the other hand, corresponding errors are contained for the set of 8 constituents and thus this option is considered adequate to capture representative tidal conditions.

We can observe that the representative months for tidal range ($M_{k,r}$) and energy ($M_{k,r}^E$) are correlated as the percentage differences are relatively small when they are applied simultaneously for both $H_{moM_{12}}$ and $PE_{M_{12}}$. The selection of $M_{12,r}^E$ gives an average absolute error of 0.002 % to the baseline $H_{moN_{12}}$ and $M_{12,r}$ returns a corresponding average error of 0.003% to $PE_{N_{12}}$. In terms of variability, $M_{12,r}^E$ gives a mean absolute error of 1.85% to $IQR(E_{N_{12}})$ and the selection of $M_{12,r}$ a mean absolute error of 2.25% to $IQR(E_{N_{12}})$. Thus, the selection of representative month can be considered independent of whether we are assessing tidal ranges or the associated energy. We interpret this to the $M_2$ metrics both including integration of elevation $η$ quantities over the interval we consider.

In practice, when comparing tidal range schemes at different locations (e.g. a barrage in the Severn Estuary, UK and a lagoon along the North Wales coast) we are interested in assessing whether the same lunar cycle could be used. As such, we discuss in more detail the behaviour of representative months for Avonmouth to all other locations. The results are presented in Fig. 8. We can observe that although there is a
Figure 7: Comparison of $IQR(\mathcal{E}_{M_{12}})$ and $PE_{M_{12}}$ of the representative months to the baseline $IQR(\mathcal{E}_{N_{12}})$ and $PE_{N_{12}}$ respectively in tidal gauge stations for different constituents sets. The bar chart illustrates the expected $IQR(\mathcal{E}_{M_{12}})$ and $PE_{M_{12}}$ of $M_{12,r}$ at all locations. Box plots represent the statistical range of $IQR(\mathcal{E}_{M_{k,j}})$ and $PE_{M_{k,j}}$ for $k = 12$ constituents.

Figure 8: Comparison of $IQR(\mathcal{R}_{M_{12}})$, $IQR(\mathcal{E}_{M_{12}})$, $H_{m_0M_{12}}$ and $PE_{M_{12}}$ for selected months.
spatial connection between the representative months both in terms of $H_{m_{0}}$ and $PE$, the variability of tidal ranges and energy based on IQR differs from the corresponding representative nodal quantities. For example, $M_{12}{_{r}}$ for Avonmouth results in an average absolute error of 0.4% against the baseline over all locations for $H_{m_{0}}$, the highest values are observed in North Shields and Wyemouth with an absolute percentage error of 1.7%. On the other hand, the mean absolute error of $IQ(R(M_{12})$ is 8.4% with the largest error appearing in Millport with -27%. Considering $PE$, the above month returns a average absolute error of 0.8%. The biggest deviation are observed in North Shields with a percentage error of 3.5%. The mean absolute error of $IQ(R(E_{M_{12}})$ is 9.3% with the largest error appearing in Millport with -27.2%. Equivalent results are observed when considering $M_{12}{_{r}}$ for Avonmouth.

In turn, we also consider the lunar month that returns the smallest $PE_{M_{12}}$ in Avonmouth and we observe how it maintains this trend across locations for both $PE_{M_{12}}$ and $H_{m_{0}M_{12}}$. Considering $IQ(R(M_{12})$ and $IQ(R(E_{M_{12}})$ while we see a correlation of a smaller variability ($IQ(R(M_{12})$ and $IQ(R(E_{M_{12}})$) with the potential $PE_{M_{12}}$, the trends are not corresponding to the minimum limits of $IQ(R$.

Additionally, by selecting the lunar month with start date 2002/01/01 00:00:00, that we use as an initial point for or analysis the average of absolute error to baseline is 0.9 and 1.7% for $H_{m_{0}M_{12}}$ and $PE_{M_{12}}$ respectively. In terms of $IQ(R(M_{12})$ and $IQ(R(E_{M_{12}})$ the corresponding errors are 20.8% and 21.6% respectively.

Thus, it appears that spatially the nature of a lunar month in terms of $H_{m_{0}}$ and $PE_{M_{12}}$ are maintained, which would indicate that these quantities are closely linked with the tidal energy that enters the continental shelf the UK is located (Coles, Blunden, & Bahaj 2017). However, localised effects on the variability of tidal range $R$, and by extension energy $E_{i}$ in terms of $IQ(R$ results in deviation from the representative nodal quantities and these effects should be considered when comparing different tidal range schemes.

4 CONCLUSION

This paper has presented a methodology for the selection of representative periods to be used for tidal range energy assessments at macrotidal sites. We have seen that representative tidal elevation signals are sensitive to the set of constituents used. In particular, when considering tidal energy resource assessment or seeking to establish the hydrodynamic response over a typical lunar month, a selection of constituents restricted to the 2 or most 4 dominant constituents (as many studies adopt) can correspond to a spatially averaged deviation of 10.5% and 21.2% (on average) for significant wave height and the averaged potential energy of typical lunar month tidal conditions of the corresponding observed signal. This has been quantified herein based on metrics that target the magnitude and variance of tidal range and energy density respectively. Also, representative periods can appear concurrently across locations considered, if constrained by $M_{12}$. This means that a segment of a tidal elevation signal considered representative at a candidate site correlates with representative signals at other sites within the UK tidal system.

Moreover, we have seen that the representative periods in terms of tidal ranges correlates with the ones in terms of energy. Thus, the same lunar month can be used whether we are looking for environmental or operational assessments. Further work will explore whether regional models can capture the same level variability at tide gauges, exploring the error introduced and whether this influences the value of identifying representative tides.

REFERENCES

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