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Decentralized Multi-Period Economic Dispatch for Real-Time Flexible Demand Management

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***Abstract*—Emerging smart-grid-enabling technologies will allow an unprecedented degree of observability and control at all levels in a power system. Combined with flexible demand and storage, they could allow procuring energy at minimum cost and environmental impact. That however presupposes real-time coordination of demand of individual households and industries down at the distribution level, with generation at the transmission level. This is closely related with the balancing market economic dispatch (ED) problem, which currently does not take into account distribution network constraints and flexible demand characteristics. Still, assuming a suitably modified form of that problem was available, due to both computational and communications requirements, its centralized solution in its full detail would not be tractable. While there is currently a wealth of literature dealing with distributed optimization applications in power systems, it typically focuses on smaller parts of the overall energy management problem (e.g. transmission area synchronization or electric vehicles management) often without considering its full scale or establishing any association with energy market mechanisms. The target of this paper is twofold: identify a flexible demand and distribution network inclusive formulation for ED; and propose a solution method.**

***Index Terms*—distributed optimization, economic dispatch, electric vehicles, energy markets, optimal power flow, smart grid**

I. Introduction

THE economic dispatch is the basic mechanism used to determine close to real-time the operating set-points of all controllable devices connected in the power system in an economically efficient way. In its traditional form it largely involves committed conventional generators, known renewable generation and demand, and could well be approximated by a deterministic problem typically covering a short period in time. However this changes when deferrable demand is taken into consideration, as the utility gained by a unit of energy purchased by an electric vehicle (EV) or storage unit now, depends on the price of energy in the future, which is typically determined by the large generating units located at the transmission level. While currently some system operators use economic dispatch mechanisms that look up to 2 hours ahead [1], this is probably not an adequate period of time to schedule an EV or a storage device. As [2] has shown, insufficient coordination between demand shifting decisions and generation scheduling can result in increased energy price volatility. In addition the increased flexible demand (mainly in the form of EVs) will put considerable strain on existing power distribution infrastructure. Consequently the balancing market should not only determine the price and optimal amount of energy trades for the current time-step (as it currently does) but also provide a good indication of the demand shifting impact on the value of energy in the near future. Furthermore, it would have to incorporate the constraints and peculiarities of distribution networks. Overall the structure of the traditional ED problem has to change. Naturally two fundamental questions come up: what is the formulation and how could it be solved.

A. Investigating the Problem Structure

A small number of papers have indeed considered the flexible demand and generation coordination problem, but not in a balancing market context. The centralized approaches in [3, 4, 5, 6] focus on unit commitment (UC). Reference [7] presents a transmission-level deterministic convexified OPF formulation including storage. While the multi-period optimization structure of these papers fits our problem, the solution approaches themselves do not. Due to the problem size they work through approximations by transmission-level demand aggregation. Thus taking into account distribution network constraints is out of the question.

The difficulty of scale could be overcome through distributed solution approaches. References [8, 9] present Lagrangian Relaxation (LR) based schemes, without however taking into account any network constraints. The approach in [10] does, but does not consider flexible demand or constraints at the distribution level. The latter is also the case for [11], which proposes a price-update mechanism to improve standard LR convergence speed. However convergence can lead to suboptimal points and there is no clear indication of better performance compared to other distributed methods that decompose an augmented Lagrangian. An alternative heuristic method for updating prices within a LR scheme is proposed in [12], which involves defining arbitrary limits to actual user flexibility. That paper focuses on device coordination within a microgrid however, and does not consider coordination of the latter with the rest of power system. Reference [13] proposes a two-level hierarchical structure for scheduling EVs but does not include distribution network constraints. None of these six papers considers the stochastic nature of the problem.

The aspect of uncertainty is considered in [14], which pro-

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poses a rolling horizon approach. This fits naturally to the balancing market which is cleared every few minutes. While that work uses a detailed unbalanced load flow model for the distribution network, it does not consider its coordination with the transmission level. In addition the need for such highly detailed models for all optimization periods is not justified, as there would be little point in e.g. optimizing losses or voltage when nodal demand variance is high. A problem structure closer to what [15] suggests, i.e. a discrete time model, with varying system modeling detail depending on the degree of uncertainty, seems a better option. That paper however focuses on unit commitment and does not consider flexible demand or distribution network constraints.

B. Contributions

The underlying idea behind this work is that unavoidably any energy management application which involves significant amounts of flexible demand (and subsequently demand shifting), is inherently tied to the balancing energy market and the associated ED problem. Consequently we seek to provide an answer to the following questions which current literature does not fully address: 1) how should ED change to account for the impacts of flexible demand; 2) to what extent is it reasonable to manage distribution constraints within the ED problem; 3) how would demand at the low voltage level be managed and represented in ED; 4) how would decentralized solutions work and fit within the overall context of power system management. Current solutions relying on aggregation techniques for parts of the system to ensure computational tractability, potentially imply a suboptimal operation of the distribution system, or designing it in such a way that its constraints may be neglected. Our proposed ED solution and its associated framework can overcome this issue.

In this work we look into a simplified version of ED in that reserves and security constraints are not taken into account. Our main contributions are:

- We propose a multi-period ED formulation which considers distribution network constraints and relevant stochastic aspects. This is a fundamental energy management problem for power networks with significant demand flexibility which has not been considered before in relevant literature in its full scale. Drawing ideas from current forward market practices, we propose suitable modifications to simplify the problem and enable its timely solution. Even in its simplified form distributed solutions are potentially necessary.
- Based on practical considerations we propose a solution based on a hierarchical decentralized framework using proximal decomposition methods and present indicative results. In contrast to previous work (e.g. [13]) our framework is based on stochastic elements of the problem rather than voltage levels.
- We incorporate into our ED formulation an extended form of the EV aggregation model of [16, 17] for use at points where demand cannot be reliably forecast.
- We propose a reference framework for energy management, identifying how our ED formulation and decentralized solution methods could be applied, and identify problem characteristics and solution time specifications.

These four points bring distributed energy management applications into perspective and effectively extend previously published work on the subject, e.g. [8, 9, 13, 18].

The paper is organized as follows: §II presents an idealized centralized ED formulation; §III describes the proposed reformulations; §IV describes the decentralized solution; §V presents indicative results and discusses practical implementation issues; while §VI presents the overall conclusions. With respect to mathematical notation: we use bold font for vectors or matrices (e.g. $\mathbf{z}$) and italics for scalars (e.g. z); $\mathbf{z}_{(x,y)}$ indicates element $(x, y)$of matrix $\mathbf{z}$; $diag\{\mathbf{z}\}$ indicates a diagonal matrix whose diagonal elements are the elements of vector $\mathbf{z}$; the operator $\|\cdot\|_2^2$ denotes the squared Euclidean norm; $\mathbf{z}_{\{x\}}$ indicates a matrix/scalar associated with element x.

II. Idealized Centralized Problem Formulation

Based on the preceding discussion there are two basic points that have to be taken into consideration:

P1. To account for the time-linkages of flexible demand (and generator ramp-rates) optimization has to be carried out over a time-period comparable to the time a flexible device (e.g. an EV) is available to control (e.g. 8-12 hours [19]*).*

P2. Over such a period there will be uncertainty related to a variety of factors such as: output of renewable generation; power required by inflexible demand; energy requirements as well as arrival and departure times of EVs.

Thus, a comprehensive way to describe our problem would be through a multi-period stochastic programming formulation, where uncertainty is modeled through a set of scenarios describing possible system states. The objective is the minimization of expected cost (negative utility) over all possible scenarios [10, 20]:

$$\min_{\mathbf{P},\mathbf{Q}} \left\{ \begin{array}{l} \overbrace{\sum_{s\in[1,n_s]} \left(\pi_{\{s\}} \sum_{i\in[1,n_u]} u_{\{i,s\}} \right)}^{f_0} : \\ \mathbf{P},\mathbf{Q} \in \bigcap_{\substack{i\in[1,n_u] \\ s\in[1,n_s]}} \begin{Bmatrix} C_{g\{i,s\}} \ \textit{if user i is generator} \\ C_{d\{i,s\}} \ \textit{if user i is demand} \end{Bmatrix} \\ \quad \bigcap_{\substack{i\in[1,n_u] \\ s\in[2,n_s]}} \begin{Bmatrix} \mathbf{P}_{(i,s,1)} = \mathbf{P}_{(i,1,1)} \\ \mathbf{Q}_{(i,s,1)} = \mathbf{Q}_{(i,1,1)} \end{Bmatrix} \bigcap_{\substack{i\in[1,n_n] \\ s\in[1,n_s] \\ t\in[1,n_t]}} C_{n\{i,s,t\}} \bigcap C_l \end{array} \right\} \tag{1}$$

Where:

C_d	Constraint set describing a demand block or device.
C_g	Constraint set describing a large generator or wind park.
C_l	Linear constraints set which couples all other sets together.
C_n	Constraint set describing an ac network.
n_n	The number of transmission and distribution network areas.
n_u	Number of clients / network users including generators.
n_s	Number of scenarios of possible future power system states.
n_t	Number of time-steps in the optimization period.
u	Cost (negative utility) function of a user/client.
$\mathbf{P},\mathbf{Q}$	$n_u \times n_s \times n_t$ matrix of active and reactive power schedules with element (i, s, t) representing the power schedule of user i in scenario s and time-step t.
π	Probability of a scenario.

The above formulation comes with the following assumptions:

A1. The market is cleared at fixed intervals (e.g. every 15 min) and as such variables and constraints associated with the first optimization time-step could be considered deterministic. Thus the decision for the first time step is the same and binding for all scenarios.

A2. Prior to the balancing market itself, unit commitment mechanisms have set the conventional generators operating status (on/off). As such we do not deal with the associated discrete variables or cost non-convexities. Market penalties for deviations are not explicitly considered but could be taken into account through the addition of relevant objective function terms.

The constraints in (1) describe in order: user/device constraints; the non-anticipativity constraints implied by assumption A1; and the network and coupling constraints.

A. Network Constraints

We assume that the network is separated to areas. A single area may represent part of the transmission and/or distribution network. The most straightforward way to describe an arbitrary ac network is through the following set of constraints (in complex numbers notation):

$$C_{n\{i,s,t\}} = \left\{ \begin{matrix} \mathbf{S}_{b\{i,s,t\}} = diag\{\mathbf{V}_{\{i,s,t\}}\}\left(\mathbf{Y}_{\{i\}}\mathbf{V}_{\{i,s,t\}}\right)^* \\ \underline{\mathbf{V}_{\{i\}}} \leq \left|\mathbf{V}_{\{i,s,t\}}\right| \leq \overline{\mathbf{V}_{\{i\}}} \\ \left|\mathbf{Y}_{t\{i\}}\mathbf{V}_{\{i,s,t\}}\right| \leq \overline{\mathbf{I}_{t\{i\}}} \end{matrix} \right\} \tag{2}$$

Where:

n_b	The number of buses/nodes in the network.
$\mathbf{S}_b$	Bus apparent power injection $n_b \times 1$ vector.
$\overline{\mathbf{I}_t}$	Line current limit vector.
$\mathbf{V}$	Bus voltages $n_b \times 1$ vector. $\overline{\mathbf{V}}$ and $\underline{\mathbf{V}}$ denote the upper and lower bounds on voltage magnitude respectively.
$\mathbf{Y}, \mathbf{Y}_t$	Bus admittance and line flow admittance matrix respectively.

The equations in C_n describe in order: bus power balance; voltage magnitude constraints; line capacity constraints.

B. Generation Constraints

For a generating unit the relevant constraints are:

$$C_{g\{i,s\}} = \left\{ \begin{matrix} u_{\{i,s\}} = \sum_{t\in[1,n_t]} \left(c_{2\{i\}}\mathbf{P}^2_{(i,s,t)} + c_{1\{i\}}\mathbf{P}_{(i,s,t)}\right) \\ \underline{P_{\{i,s,t\}}} \leq \mathbf{P}_{(i,s,t)} \leq \overline{P_{\{i,s,t\}}}\ \forall\, t \in [1,n_t] \\ \underline{Q_{\{i\}}} \leq \mathbf{Q}_{(i,s,t)} \leq \overline{Q_{\{i\}}}\ \forall\, t \in [1,n_t] \\ \underline{P_{R\{i,t\}}} \leq \mathbf{P}_{(i,s,t)} - \mathbf{P}_{(i,s,t-1)} \leq \overline{P_{R\{i,t\}}}\ \forall\, t \in [2,n_t] \end{matrix} \right\} \tag{3}$$

Where:

c_2, c_1	Active power variable cost coefficients. Additional utility terms could be added relating to reactive power provision.
P_R	Ramp rate limits.

For a conventional generator power limits are the same for any value of the index s. For a renewable generator the lower bound is zero, the upper bound varies following a given forecast error distribution (e.g. [21]), while the ramp rate-constraint is redundant.

C. Demand Constraints

For user-level demand / devices the constraints are:

$$C_{d\{i,s\}} = \left\{ \begin{matrix} u_{\{i,s\}} = \sum_{t\in[1,n_t]} \max\{c_{1\{i\}}\left(E_{\{i,s,t\}} - E_{t\{i,s,t\}}\right), 0\} \\ \underline{P_{\{i,s,t\}}} \leq \mathbf{P}_{(i,s,t)} \leq \overline{P_{\{i,s,t\}}}\ \forall\, t \in [1,n_t] \\ E_{\{i,s,t\}} = E_{\{i,s,t-1\}} + c_c\mathbf{P}_{(i,s,t)}\ \forall\, t \in [1,n_t] \\ \underline{E_{\{i,s\}}} \leq E_{\{i,s,t\}} \leq \overline{E_{\{i,s\}}}\ \forall\, t \in [1,n_t] \\ \mathbf{Q}_{(i,s,t)} = \mathbf{P}_{(i,s,t)}\, tan\phi\ \forall\, t \in [1,n_t] \end{matrix} \right\} \tag{4}$$

Where:

c_1	The cost of shedding demand (value of lost load).
c_c	Factor accounting for energy conversion losses.
ϕ	Angle between active and reactive power.
E	Energy stored at the end of a time step. We assume that $E_{\{i,s,0\}}$=0 and that the energy bounds have been appropriately shifted.
E_t	Energy target at a given time step. Note that by definition demand is negative, thus the utility function penalizes cases where energy consumed is less than the desired.

In this work we consider the following types of demand / devices which are adequately modeled by the above equations:

Inflexible demand: The upper power bound would be 0, and the lower would vary in different scenarios following a certain forecast error. Energy bounds are redundant, while $E_{t\{i,s,n_t\}} = \sum_{t\in[1,n_t]}\left(P_{\{i,s,t\}}\right)$ and 0 for all other time steps.

Small scale renewables: These are simply assumed to be negative demand (i.e. 0 lower bound on power and $c_1 = 0$)

Electric vehicles: For an EV E_t represents the energy requirements for travelling purposes. Power bounds are set to 0 if the vehicle is not connected. Typical probability distributions for vehicle connection / disconnection times and energy requirements may be found e.g. in [19]. We do not model self-discharge energy losses for battery systems as these typically amount to less than 5% during the first 24h [22], and are unlikely to affect a system-wide optimization results. Based on [23, 24] we assume that the majority of EVs operate in a unidirectional manner.

The reasoning behind focusing on these particular devices is that their combined use would likely be the main cause of issues with respect to distribution network operation. However there is a number of other types of demand that have their own role to play in energy management such as: storage (battery based storage is actually covered by (4)), household wet appliances [25], heating systems [26], industrial processes, etc. Modeling the flexibility of each individual type of device/demand is a considerable task that goes beyond the purposes of this work. However our formulation and solution approach are generic and it is easy to add additional constraint sets or modify existing ones.

D. Coupling Constraints

The various constraint sets described above are linked together through an additional set of linear constraints:

$$C_l = \{\mathbf{C}_u\mathbf{U}_{\{s,t\}} = 0\ \forall\, s \in [1,n_s], t \in [1,n_t]\} \tag{5}$$

The vector $\mathbf{U}$ is derived from the concatenation of $\mathbf{P}_{(i,s,t)}, \mathbf{Q}_{(i,s,t)}, \mathbf{S}_{b\{i,s,t\}}$, and $\mathbf{V}_{\{i,s,t\}}$ for all constraint sets. Matrix $\mathbf{C}_u$ has elements of 1, 0, -1 establishing coupling variables equality. This constraint set is further clarified in the Appendix in §VII.

III. Reformulation & Decentralization

It should be clear that, using centralized methods, problem (1) is probably intractable. Regarding its solution the following points could be made:

P3. When considering energy scheduling decisions in terms of each individual device it might not be possible to carry out a sufficient system-wide scenario reduction as e.g. done in UC. A more detailed representation of uncertainty could be required to manage variables at the distribution level (e.g. scheduling EVs at a heavily loaded low voltage feeder would depend on uncertainties related to the local network loading conditions, requiring efficient micromanagement of local resources; the transmission level would simply see a feeder absorbing almost constant power over time).

P4. While distributed methods could help in dealing with the size of this problem, a price-based decomposition (e.g. Lagrangian Relaxation based method) would imply that $2 \times n_s \times n_t$ prices would have to be updated at each decomposition point (to account for active and reactive power, twice as many to account for voltage), while also tracking which prices correspond to which scenario. This would be challenging and would imply significant requirements in terms of communications bandwidth and reliability.

P5. Even if it were possible to somehow avoid the scenario building process and coordinate subproblems through the exchange of probability distributions (for power and price), this would imply that at each iteration of a distributed optimization approach, probabilistic optimal power flow problems would have to be solved. In terms of computational burden, this is not very realistic.

Considering the points above simplifications are required.

A. Problem Simplification

Before proceeding further we introduce the concept of market aggregator (*MA*), i.e. an entity that manages subsets of the constraints. The *MA* interacts with the rest of the system through his energy schedule at specific nodes. The simplification we propose lies in that *MA*s are forced to submit a single power value for each time step of the optimization period for these specific nodes, set to be equal to the expected value of their power schedule. Mathematically it is equivalent to substituting the coupling constraint set C_l with:

$$C_l^* = \overbrace{\begin{Bmatrix} \mathbf{C}_u^{in}\mathbf{U}_{\{s,t\}}^{in} = 0 \\ \forall\, s \in [1, n_s], \\ t \in [1, n_t] \end{Bmatrix}}^{C_l^{in}} \cap \overbrace{\begin{Bmatrix} \mathbf{C}_u^{ex} \sum_{s \in [1,n_s]} \left(\boldsymbol{\pi}_{(s)} \mathbf{U}_{\{s,t\}}^{ex}\right) = 0 \\ \forall\ t \in [1, n_t] \end{Bmatrix}}^{C_l^{ex}} \tag{6}$$

Where C_l^{in} corresponds to coupling constraints (part of $\mathbf{C}_u$ and $\mathbf{U}$) handled internally by the *MA*s, while C_l^{ex} corresponds to constraints at nodes where different *MA*s interact (remaining part of $\mathbf{C}_u$ and $\mathbf{U}$). This is further clarified in §VII. This decouples the stochastic elements of *MA* subproblems and extends our list of assumptions:

A3. For each time step connected MAs are forced to submit a single power value at their coupling nodes. As a consequence they interact through a single price which they assume to be a good estimate of the expected energy price. In addition possible scenarios need only be considered locally by each MA rather than for the whole system. This follows the reasoning of P3, and allows the MA to keep a sufficiently high degree of uncertainty representation locally, without hindering the system-wide solution process.

The proposed simplification in principle does not differ much from what is currently done in forward markets, i.e. the problem is solved in a semi-deterministic way by passing part of the uncertainty management to market players. Potentially *MA*s could face market penalties for deviations, which would have to be calculated and be applied based on an ex-post assessment of the market solution. These could be taken into account through additional objective function terms for each *MA* and potentially by relaxing the equality in C_l^{ex} to allow the *MA* to submit any desired schedule. The actual design of the market rules (i.e. calculation and application of such penalties) is however outside the scope of this paper. It should be noted that despite this reformulation it is still not possible to solve this problem in a centralized manner due to the number of constraints and the fact that thousands of users would have to communicate with one central controller. A distributed solution is necessary and presupposes the determination of the constraint subsets that *MA*s would manage.

B. Market Aggregator Structure

The overall multi-period problem structure may be visualized on the table-like structure of Fig.1. As may be observed the problem has a hierarchical structure which is indicative of how it should be decomposed and of the interrelations between the generated subproblems. First a number of MAs would be managing parts of the transmission system. As such we have the transmission system operator (MA_{TSO}) type of subproblems which manage subsets of the transmission constraints. Linked with them are the problems of distribution system operators (MA_{DSO}) managing parts of the distribution network at a specific bus, and the problems of independent large generators (MA_{IG}). At an even lower level one could find a small number of medium voltage nodes and/or all users at a low voltage feeder managed by a microgrid operator (MA_{MO}). The initial optimization problem may be decomposed into these general types of subproblems. At this point we further extend the assumptions regarding our solution approach:

A4. Each market aggregator is equipped with a digital device which solves a generic subproblem formulation (described in section IV) and handles the necessary communications with the rest of the system. Constraint parameters values and forecasts are provided by the user / aggregator but changes are not possible during an optimization run.

A5. The size of an MO is such that the uncertainty regarding its expected power schedule can be reasonably small. From a market perspective this size could be indirectly determined through imposed penalties for deviations. Following [27] *we assume that marginal pricing is also applied to MOs.*

In the following subsections we present the decomposition approach and the various *MA* subproblems.

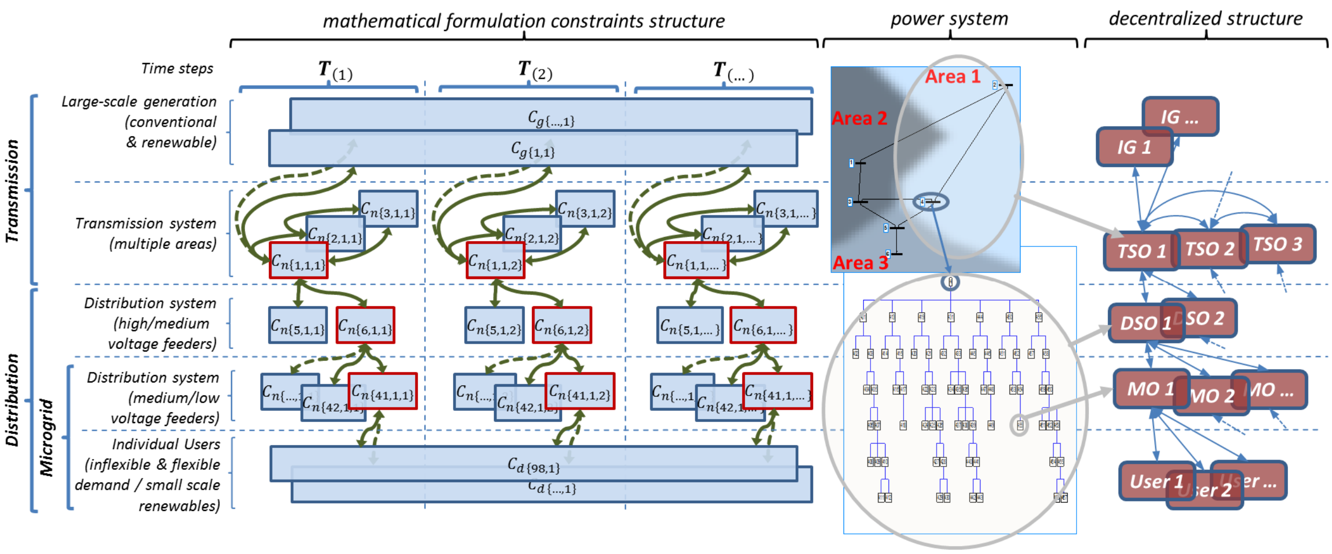

Fig.1. *Left:* Schematic representation of involved constraint sets. Each block represents a constraint set. The columns and rows correspond to time-steps and constraint set type respectively. There is a third dimension to this table as multiple copies of the same constraint type may exist (indicated by blocks piled on top of each other). Each arrow-line indicates coupling between sets (part of C_l). Note that blocks with time-linkage constraints, e.g. flexible demand (C_d), cover multiple columns. For the illustrated example there exist three copies of C_t for each time-step at the transmission level. The constraint set of area 1 is linked with the distribution constraints of buses 2 and 4, the generators at bus 2 and the other two transmission area constraint sets. The distribution network constraint set of bus 4 is in turn linked with various low voltage network constraint sets and the individual users. *Right*: Schematic representation of the underlying decomposition structure. Each block represents a market aggregator and arrow-lines indicate coupling and required bi-directional communications links.

IV. Decomposition

Current power systems literature presents a wide variety of papers dealing with distributed solutions in power flow or energy management problems. References such as [28, 29, 30] deal with optimal power flow decomposition in multiple transmission areas but do not deal with the particular problems of demand. A more detailed review of such methods may be found in [18, 31] which indicate that proximal based decomposition methods (in particular the so called Alternating Direction Method of Multipliers – ADMM) are promising candidates for the present application. Similar in principle proximal based algorithms have been also used for EV management, e.g. [32], but do not include network constraints. Our decentralized solution is based on ADMM but of course any similar method could be applicable. For brevity we do not present the theoretical background in detail as relevant information and convergence considerations may be found in [31, 33, 18]. For reference, if $\boldsymbol{x}$ is a vector of all optimization variables and $\mathbf{x}_e$ is the subset of $\mathbf{x}$ involved in C_l^{ex} our initial problem may be written as:

$$\min_{\substack{x\in C\setminus C_l^{ex}\\ \mathbf{z}\in C_l^{ex}}} \{f_o(\mathbf{x}) : \mathbf{z} = \mathbf{x}_e\} \tag{7}$$

Then ADMM gives three consecutive iterative steps:

$$\mathbf{x}^{k+1} = \operatorname*{argmin}_{x\in C\setminus C_l^{ex}} \left\{ \overbrace{f_o(\mathbf{x})}^{cost\ term} + \overbrace{\boldsymbol{\lambda}_e^k \mathbf{x}_e}^{price\ term} + \overbrace{\rho\|\mathbf{x}_e - \mathbf{z}^k\|_2^2}^{penalty\ term} \right\} \tag{8}$$

$$\mathbf{z}^{k+1} = \operatorname*{argmin}_{\mathbf{z}\in C_l^{ex}} \left\{ \|\mathbf{x}_e^{k+1} - \mathbf{z}\|_2^2 \right\} \tag{9}$$

$$\boldsymbol{\lambda}_e^{k+1} = \boldsymbol{\lambda}_e^k + 2\rho(\mathbf{x}_e^{k+1} - \mathbf{z}^{k+1}) \tag{10}$$

Where ρ a penalty factor, and $\boldsymbol{\lambda}_e$ a row vector of Lagrange multipliers corresponding to the constraints $\mathbf{z} = \mathbf{x}_e$. The cost term in (8) is the objective function of (1), the price term is related to the Lagrangian multipliers, while the penalty term is introduced by the method, is 0 at the optimum and essentially is what ensures smooth convergence. Problems (8)-(10) are separable with respect to *MA*s. It should be noted that the only information exchange required at each iteration is that of the power schedule. As such privacy over information is maintained and the volume of data transferred per iteration is very low. Based on the general guidelines presented in [18], ρ is set to be a percentage (~20%) of an estimate of the expected value of $\max\{\boldsymbol{\lambda}_e\}$ at the optimum. Such an estimate would generally be available from a previous ED solution. With respect to the decomposition structure the following observations may be made:

P6. Regarding the decomposition of the transmission network: 1) for large degrees of decomposition especially in certain congested or contingent cases convergence can be slow [18, 34]*; 2) contingency constraints typically involved in OPF are not necessarily easy to decompose. Thus it can be expected that at this level decomposition, would be limited to a rather small number of areas, implying that TSO subproblems would remain computationally intensive.*

P7. Following the above it is of interest to limit as much as possible iterations at this level (i.e. number of TSO problems that have to be solved). However, these are bound to increase with increasing disaggregation (i.e. larger number of MOs) [18]*. At the same time however it is also of interest to disaggregate demand and represent it in as much detail as reasonably possible, given that ED is the last attempt to coordinate resources system-wide in an economically optimal way. Any dispatch / control mechanisms that follow would have to act locally and as a consequence cannot pos-*

sibly be optimal in that same sense.

One way to account for these two considerations is to carry out the decomposition in two consecutive passes [18]:

- <u>*TSO level*:</u> First the problem is decomposed to MA_{TSO}, MA^*_{DSO} and MA_{IG} types of problems. An MA^*_{DSO} problem involves all distribution network and user constraints of a transmission bus.
- <u>*DSO level:*</u> At this point the MA^*_{DSO} problem is further decomposed into the actual MA_{DSO} problem containing most of the distribution network constraints and the individual MA_{MO} types of problems which include all user constraints.

With this two-level decomposition, independently of the degree of disaggregation at the DSO level, iterations at the TSO level remain unaffected. A flowchart describing the solution approach may be seen on Fig.2. In the following we present the subproblems this method generates based on (8).

A. Decomposition Subproblems

For each MA_{TSO} we have the following subproblem:

$$\min_{\mathbf{x}_a}\left\{\begin{array}{c}\overbrace{\boldsymbol{\lambda}^k_{e:a}\mathbf{x}_{e:a}}^{\text{price term}} + \overbrace{\rho\left\|\mathbf{x}_{e:a}-\mathbf{z}^k_a\right\|_2^2}^{\text{TSO level penalty}} : \\ \mathbf{x}_a \in C_{t\{a,s,t\}} \cap C^*_{l:a}\ \forall\, s \in [1,n_s], t \in [1,n_t]\end{array}\right\} \tag{11}$$

The index 'a' indicates subsets of variables and sets managed by the aggregator. For each MA_{IG} we have:

$$\min_{\mathbf{x}_a}\left\{\begin{array}{c}\overbrace{f_{o:a}(\mathbf{x}_a)}^{\text{cost term}} + \overbrace{\boldsymbol{\lambda}^k_{e:a}\mathbf{x}_{e:a}}^{\text{price term}} + \overbrace{\rho\left\|\mathbf{x}_{e:a}-\mathbf{z}^k_a\right\|_2^2}^{\text{TSO level penalty}} : \\ \mathbf{x}_a \in C_{g\{a,s\}} \cap C^*_{l:a}\ \forall\, s \in [1,n_s]\end{array}\right\} \tag{12}$$

For each MA_{DSO} we have:

$$\min_{x_a}\left\{\begin{array}{c}\overbrace{\boldsymbol{\lambda}^k_{e:a}\mathbf{x}_{e:a}}^{\substack{\text{TSO level}\\ \text{price term}}} + \overbrace{\rho\left\|\mathbf{x}_{e:a}-\mathbf{z}^k_a\right\|_2^2}^{\text{TSO level penalty}} + \\ + \overbrace{\boldsymbol{\lambda}'^l_{e:a}\mathbf{x}'_{e:a}}^{\substack{\text{DSO level}\\ \text{price term}}} + \overbrace{\rho'\left\|\mathbf{x}'_{e:a}-\mathbf{z}'^l_a\right\|_2^2}^{\text{DSO level penalty}} : \\ \mathbf{x}_a \in C_{t\{a,s,t\}} \cap C^*_{l:a}\ \forall\, s \in [1,n_s], t \in [1,n_t]\end{array}\right\} \tag{13}$$

Where '′' indicates variables related with the *DSO* level decomposition pass. Finally for each MA_{MO} we have:

$$\min_{\mathbf{x}_a}\left\{\begin{array}{c}\overbrace{f_{o:a}(\mathbf{x}_a)}^{\text{cost term}} + \overbrace{\boldsymbol{\lambda}'^l_{e:a}\mathbf{x}'_{e:a}}^{\text{price term}} + \overbrace{\rho'\left\|\mathbf{x}'_{e:a}-\mathbf{z}'^l_a\right\|_2^2}^{\text{DSO level penalty}} : \\ \mathbf{x}_a \in C_{d\{a,s\}} \cap C_{t\{a,s,t\}} \cap C^*_{l:a}\ \forall\, s \in [1,n_s], t \in [1,n_t]\end{array}\right\} \tag{14}$$

Effectively (14) represents the only hard stochastic subproblems that have to be solved. Note that since network constraints are fully separable in time the TSOs and DSOs can actually solve their n_t network subproblems in parallel. The above subproblems are further clarified in §VII.

B. Microgrid Operator Subproblems

Considering the solution of the MO problem with a potential further decomposition to the individual user:

P8. For similar reasons to those mentioned in P3-P5 decomposition might not be easy. On the other hand given the much smaller size of the microgrid level problem centralized solutions might be tenable, albeit not fast enough to work within a decentralized solution framework.

P9. Most of the available controls at the individual user level

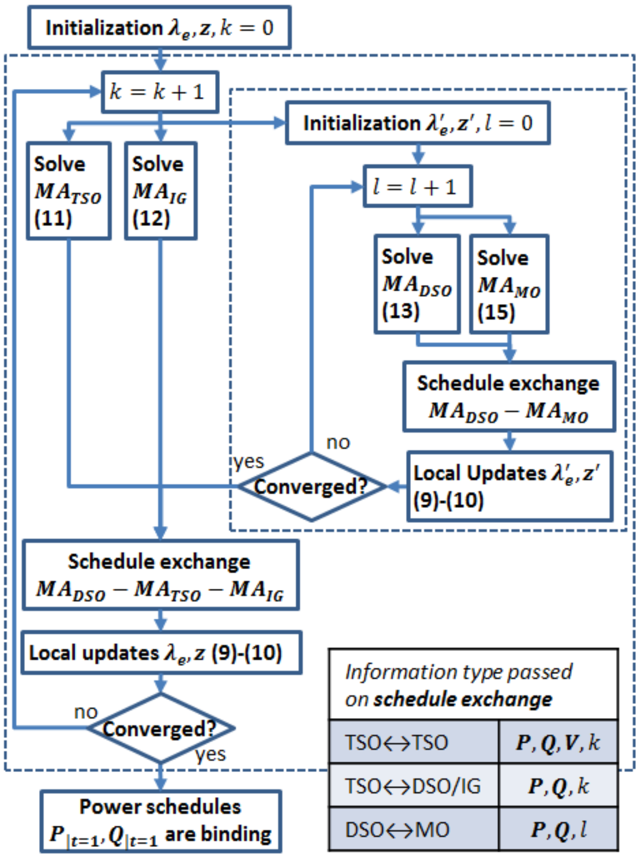

Fig.2. Decentralized optimization approach flowchart. The indices k, l correspond to TSO and DSO level decomposition respectively, and are transmitted to enable agent synchronization. It is assumed that the tap changing action of the transformer decouples the voltage between TSO and DSO.

may be expected to be discrete in practice. One decentralized method which could deal with such constraints is presented in [8, 12] *however it does not deal with uncertainties and it is not clear how it would perform as part of a larger decomposition scheme.*

P10. There might not be any actual benefit from privacy of information. Individual users would receive bills which would reflect how well their aggregate demand was managed and as such would be inclined to reveal their flexibility and actual utility to the MO. In addition with a large scale deployment of smart meters and real-time measurements the MO could identify what devices e.g. a household uses at a given time, even if the latter did not directly disclose such information.

P11. On a nodal basis (when looking at a single or a few households) demand variance can be expected to be quite high compared to its expected value. Under presence of such uncertainty one can have nothing more than an educated guess regarding system quantities (voltage, power, etc.) at a significant computational cost. In UC formulations (e.g. [15]*) in such cases simpler, more abstract models are used. The same concept could be applied in ED.*

Considering the points above, in this work:

A6. For managing users we use a practical three-step ap-

proach similar in principle with [16] *where users communicate in a single round their requirements and willingness to pay to the MO, and the latter builds an approximate aggregate model which is used to determine the optimal aggregate demand at each iteration of* Fig.2. *After market clearing the MO breaks down the aggregate demand to individuals.*

A7. The approximate aggregate model is built right before a decentralized ED run commences. As such the time required to build the model will affect only how recent measurements / forecasts may be used for its creation, and will have no impact whatsoever in ED convergence time.

The aggregate model should adequately represent the feasibility region of the aggregate demand power but at the same time be sufficiently fast to solve. One possible approach to aggregation is through various scenario reduction techniques [5, 6]. However the resulting number of constraints can still be quite large. An analytical method is presented in [35] but considers only the state of charge as a stochastic variable. In [36] an approach based on heuristics is proposed but the computational cost is still significant. Another approach is modeling an EV fleet as a single vehicle [3] which is based on expected values of the constraints. Along similar lines an aggregate model for an EV fleet is presented in [16] which calculates and sets bounds on the total energy that the fleet can consume. With respect to power it considers an upper bound which incorporates grid capacity constraints. The model was extended in [17] to take into account uncertainty on EV arrival and departure times. Given the excellent scalability and solution speed we use a modified form of that model, which combines both flexible and inflexible demand and allows demand curtailments.

C. Microgrid Level Demand Aggregation

In order to build the aggregate model first a set of scenarios is generated by sampling probability distributions to determine power for inflexible demand, energy requirements and availability for flexible demand (i.e. EV arrival and departure times). Then the aggregate power and energy bounds for each of those scenarios are estimated. While these power bounds are not generally a one-to-one function of aggregate power they may fall in a rather limited band as e.g. shown on Fig.3 and could be approximated by linear constraints. Using the expected values of points A to E allows rewriting (14) as:

$$\min_{\mathbf{P}_a,\mathbf{P}_c}\left\{\begin{matrix} c_a \sum_{t\in[1:n_t]} \mathbf{P}_{c(t)} + \boldsymbol{\lambda}'^{l}_{e:a}[\mathbf{P}_a;\mathbf{Q}_a] + \rho'\left\|[\mathbf{P}_a;\mathbf{Q}_a] - \mathbf{z}'^{l}_a\right\|_2^2 : \\ \underline{E_{a1\{t\}}} \le E_{a\{t\}} = c_l\mathbf{P}_{a(t)} - \mathbf{P}_{c(t)} + E_{a\{t-1\}} \\ \underline{P_{a1\{t\}}} \le \mathbf{P}_{a(t)} \le \overline{P_{a1\{t\}}} \text{ (lines A,B)} \\ u_{1\{t\}}E_{a\{t-1\}} + u_{0\{t\}} \le c_l\mathbf{P}_{a(t)} \text{ (line C-D)} \\ \underline{P_{a2\{t\}}} \ge c_l\mathbf{P}_{a(t)} - \mathbf{P}_{c(t)},\ \mathbf{P}_{c(t)} \ge 0 \text{ (line E)} \\ \mathbf{Q}_a = \mathbf{P}_a\, tan\phi \end{matrix}\right\} \tag{15}$$

Where:

$\mathbf{P}_a, \mathbf{P}_c$	Aggregate energy consumption and curtailment schedule $n_t \times 1$ vectors.
E_a	Aggregate energy including curtailed energy.
$\underline{E_{a1}}$	Aggregate energy which if not met will imply curtailments calculated based on the targets E_t of individual devices.

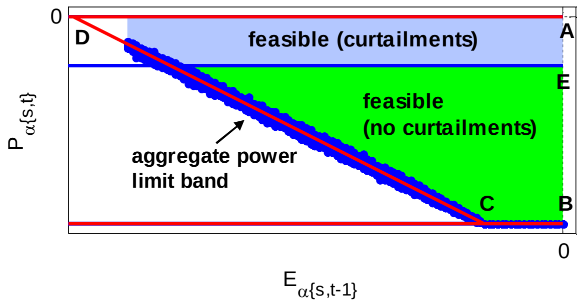

Fig.3. Simple example illustrating bounds on power at time-step 6 for a randomly generated population of 100 EVs and sets of randomly selected charging schedules. These limits could be approximately represented by three linear constraints with A $(0,\overline{P_{a1}})$, B $(0,\underline{P_{a1}})$, C (determined through a simple search process by gradually increasing and allocating aggregate energy, $\overline{P_{a1}}$), D (0, sum of total energy that could be stored to all devices connected by that hour), E $(0,\underline{P_{a2}})$.

$\underline{P_{a2}}$	Power which if not drawn will imply curtailments based on the target E_t and power limits of individual devices.
$\underline{P_{a1}}, \overline{P_{a1}}$	Minimum and maximum aggregate power devices could draw irrespective of energy capacity but including network limitations (due to capacity, voltage drop/rise, voltage imbalances). The latter may be derived by solving a maximum flow problem given the devices connected at each time-step, or roughly be approximated by the maximum aggregate power the network has been observed to be able to draw in practice.
c_a	Average cost of shedding demand.
c_l	A coefficient $\in [0,1]$ approximating losses.
ϕ	Average active / reactive power angle. Note that the relevant constraint could be replaced by bounds on reactive power if there is local reactive power control capability.
u_1, u_0	Coefficients calculated based on expected values of C,D.

The above process may be executed iteratively until no noticeable changes are observed in expected bounds or a certain time has passed. While this model for the first few optimization time-steps can be accurate (given the limited aggregate uncertainty and choice in distributing aggregate energy), for the remaining time-steps it is approximate in terms of power limitations and equivalent utility, as these depend on the individual state of each user and this information is lost on aggregation. We would like to stress that this model is not expected to give a definitive decision on individual devices schedules. Rather it is expected to produce with very low computational burden, an adequately good estimate of the expected power injection of the microgrid to the rest of the network. This formulation is not meant to be restrictive. Use of more complex models to cover other device types, or uncertainty and network constraints in more detail is possible, as long as the computational time is not significantly increased (not more than that of the solved in parallel DSO subproblems).

V. Results & Discussion

A. Base Test Case

Our test system is a slightly modified version of the RBTS 6 bus system, which is the only IEEE test system that includes distribution network (data and schematics may be found in

[37]). Regarding our test case:

- We assume that approximately 6kW of peak demand correspond to a residential user, 30% of whom own an EV, 30% of which are connected at the start of the optimization period. The maximum EV charging power is assumed to be 6.6kW [35] while the battery size is assumed to be $50kWh$. The resulting demand with the addition of EVs if left uncontrolled would create congestion or significant voltage drops at the distribution level. However if controlled the existing network is sufficient for meeting requirements in terms of energy without curtailments.
- The optimization period is divided into 12, 1h time-steps, which is assumed to be an adequate look-ahead period for managing EVs. The time-step length could of course be selected to be smaller or vary depending on the distance from the first time-step.
- In order to illustrate the ability of the method to coordinate TSO subproblems the transmission network was separated into 3 areas as seen on Fig.1. The demand at each distribution node is managed by an MO aggregator. This is not restrictive however; the demand at a node could have been managed by multiple MOs or multiple nodes could have been managed by one MO. In general (assuming aggregate models are used) as the number of MOs decreases, convergence speed may be expected to increase (fewer iterations at the distribution level), but the quality of modeling detail in terms of devices and distribution network would be worse. In practice the relation between distribution nodes and MO aggregators is uncertainty dependent. Investigating this relation based on actual measurements would be a subject of particular interest. As it is our test case involves: 3 TSOs, 181 MOs and 5 DSOs. Building the aggregate models took less than 1 min for each MO.
- Stochastic inflexible demand forecast errors are assumed to follow a uniform distribution (ud). For an EV not connected at the first time-step truncated normal distributions (tnd) are used. The selected parameters are:

	distribution	(μ,σ)	$[min, max]$
inflexible demand	ud	-	$[-t,t]\cdot 1.5\%$
EV arrival time	tnd	(3,1)	[2,8]
EV departure time	tnd	(12,1)	[7,14]
EV arrival charge state	tnd	(75%, 25%)	[25%, 95%]

These data are representative of the level of uncertainty which might be found in practical situations (e.g. similar data may be found also in [19, 38]) and are chosen for illustrative purposes. Any other distributions derived from particular real situations could equally well be used.

The time required by a subproblem for a single iteration is the sum of computational time of that subproblem and the communications latency (i.e. the time required for communicating the energy schedules to other agents), i.e.:

$$t_1 = \sum_k \max\{t_{TSO_1}, t_{TSO_2} \dots, t_{IG_1}, \dots, t^*_{DSO_1}, \dots\} \quad (16)$$

$$t^*_{DSO} = \sum_l \max\{t_{DSO}, t_{MO_1}, t_{MO_2}, \dots\} \quad (17)$$

The operator Σ stands for the summation over all iterations at that level. Given that at the moment there is no fully fledged standard regarding smart grid communications, a value of 0.1s

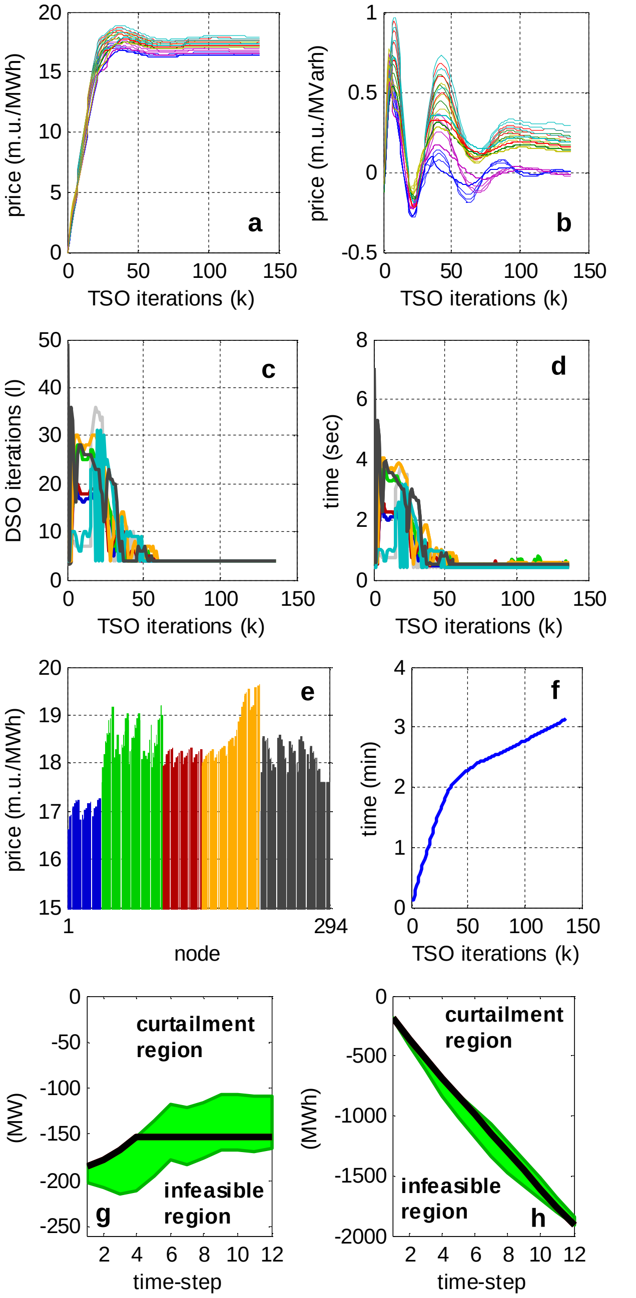

Fig.4. Convergence results for the IEEE RBTS:
a-b. Active and reactive power marginal prices convergence for each transmission bus and all time-steps.
c-d. Iterations and time required for DSOs subproblems.
e. First time-step real power marginal prices for all system nodes (different color used for nodes belonging to different buses - bus 1 only has a set of generators and no other distribution network, and as such is not easy to distinguish here). Due to losses higher prices are observed at the end of distribution feeders (particularly so for a lengthy 33kV feeder on bus 6).
f. Aggregate convergence time for the fully decentralized solution.
g-h. System-wide power and energy schedules and indicative representation of the corresponding extreme bounds (shaded area indicates normal operation without curtailments).

is assumed for the latency. Regarding the results:

- As may be seen on Fig.4 in terms of transmission level iter-

- ations convergence is achieved in about 140 iterations at the TSO level. During each such iteration each DSO problem requires its own number of iterations. Note that the latter as the optimization progresses tend to decrease thanks to the fact that the TSO level marginal prices stabilize near their optimal values, while the DSO subproblems have the good initial points provided by the previous TSO level iteration.
- The system involves 6 transmission buses and 286 distribution nodes. This means about 7k power balance and 3.5k line capacity constraints. The constraints number for MO subproblems would be on the order of 17k. Despite the small size of the system the resulting problem is large. The results indicate the ability of the proposed scheme to coordinate energy management within a time frame (in this case less than 4min) acceptable for market applications. As may be seen the first optimization time step corresponds to a time of high demand with domestic consumption near its peak. The flexible part of demand is shifted towards later hours. The end result is a rather flat price and demand profile.

It should be noted that the time to convergence could potentially be improved if instead of a flat start (i.e. in (8) $\lambda_e^0 = z^0 = 0$) the simulation was initialized based on a solution of the previous time-step. More efficient implementations of the used optimization algorithms are also possible than our current ones (done in MatLab). For the optimization subproblems we used closed-form solutions if possible, and the primal-dual barrier interior point algorithm of MatLab optimization toolbox otherwise. The iterative nature of this distributed solution could allow for improved constraint management heuristics which would remove inequality constraints that are not expected to become active in the subproblems, thus reducing computational burden.

B. *Time-wise Scalability*

In this section we investigate the impact of look-ahead period in terms of convergence. Based on the RBTS 6 bus test case a series of simulations were performed with a gradually decreasing number of time-steps. The results may be seen on Fig.5. The differences in convergence time and iterations are due to the fact that, as the time-steps number changes, these are effectively different optimization problems with slightly different solutions. Nevertheless the changes in time are not significant. The reason is that MO subproblems (which increase in size) are solved in parallel with the more computationally intensive DSO network subproblems which do not change in scale (network subproblems for the new time-steps are solved in parallel with the existing ones). The results indicate that it is possible to increase the number of time-steps without any negative impact on convergence time.

C. *Network Scalability & Implementation Challenges*

In this section we try to gain some insight with respect to scalability in terms of network size. Unfortunately data describing a large network including both transmission and distribution were not available. Therefore we set up an additional test case based on the IEEE 118 bus network (base data available on [39]). We retained the transmission level system data as is, and we added distribution data as copies of the RBTS feeders, e.g. for the 118 system bus 59 (277MW), three copies of the RBTS bus 3 distribution feeders (85MW) were added with inflexible demand slightly scaled to give the total of 277MW. This yielded a problem with a total of 1024 distribution nodes, i.e. about 4 times larger than our base test case. In all other respects (e.g. EV penetration) the test case was constructed in a similar fashion with the base case. With respect to decomposition structure: the transmission network was considered as a single area/subproblem; for buses with large distribution networks, sets of feeders were considered as separate subproblems (e.g. for the previously mentioned bus 59 three equivalent DSO subproblems were created) giving a total of 102 DSOs and 731 MOs. As may be seen on Fig. 6 convergence is achieved in about 5.5 min. This increase in time was due to the slightly increased number of iterations required at both the transmission and distribution level, and also the larger size of certain subproblems.

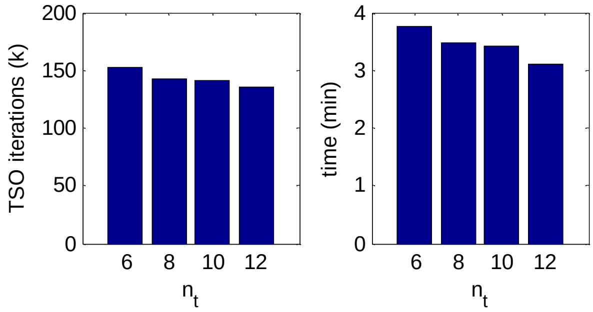

Fig.5. Effect of number of time-steps n_t on convergence speed.

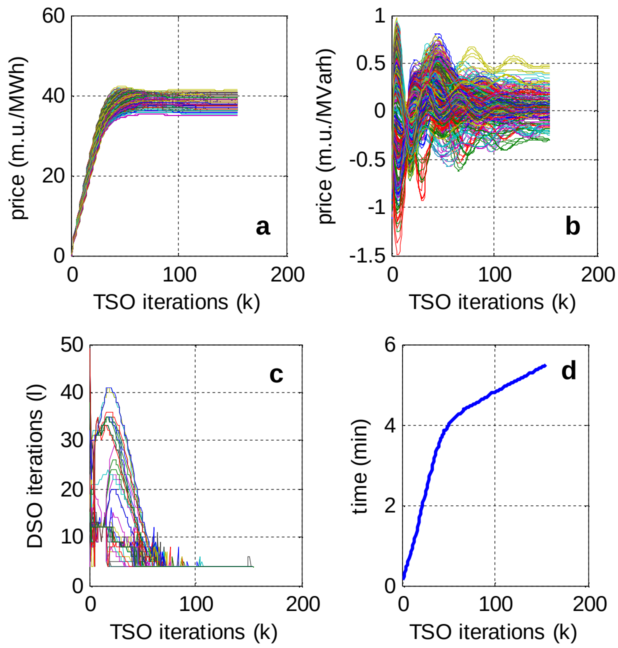

Fig.6. Convergence results for the modified IEEE-118 bus network:
a-b. Active and reactive power marginal prices convergence.
c. Iterations required for DSO subproblems.
d. Aggregate convergence time for the fully decentralized solution.

Unavoidably when one moves to even larger systems, as the solution time of the network subproblems increases, so will the overall convergence time. Based on the presented results the proposed method appears to be applicable to small or medium sized systems. To determine its applicability on larger systems further testing is required considering that:

- If the communications delays are ignored (i.e. latency is set to 0) then the solution time for the RBTS-6 and IEEE-118 reduces to about 1 and 2 minutes respectively. Further investigation into communications structures and modeling expected delays realistically is an important issue, as is the directly related subject of efficient implementation (in software & hardware) of the optimization subproblems solvers.
- Investigating what are the most efficient distributed solution methods, especially at the distribution level (potentially exploiting their radial structure), is also an important subject.
- Decomposing a very large transmission network into even a small number of areas could imply a much faster solution of TSO level subproblems, despite an increase in terms of iterations (relevant information may be found in [18]). However, the inclusion of security constraints and investigating efficient methods for their decomposition are key issues.

Overall, we would like to stress that what we present in this paper is the basic solution concept. Several extensions are required before a full practical application, but these go far beyond the purpose of a single paper.

D. Coordination

As pointed out in A1 ED presupposes coordination with UC mechanisms. In addition, the fact that we cannot have a definitive decision regarding individual microgrid level devices schedules as part of our overall ED solution, implies that an additional microgrid dispatch (MD) mechanism is required. The latter should operate at a time resolution much faster than that of ED, and should be capable satisfying user requirements, handling microgrid network constraints in their full detail, while following the ED solution as closely as possible. A wide variety of methods have been proposed in the literature (e.g. [12, 14, 40, 41]) which could be suitably adapted to serve this purpose. The overall energy framework we envision in this work is illustrated in Fig.7. Electricity markets (through UC and ED) are there to achieve coordination over time and utility maximization across the whole network; dispatch at the microgrid level will be there to follow the ED signals to the best of its ability while satisfying user energy requirements.

VI. Conclusions

This paper described a conceptual framework for the balancing market ED that could enable an efficient, close to real-time, management of flexible demand and distributed system resources. Based on practical considerations we identified a suitable formulation and solution approach based on a partially decentralized hierarchical structure where the microgrid is considered as the fundamental component. The presented method involves the disaggregation of demand to the degree that it is meaningful to do so (based on associated uncertainty) bringing the following advantages: 1) allows a much better solution quality in terms of distribution network constraints compared to approaches which would use aggregate models at the transmission level; 2) allows a more exact representation

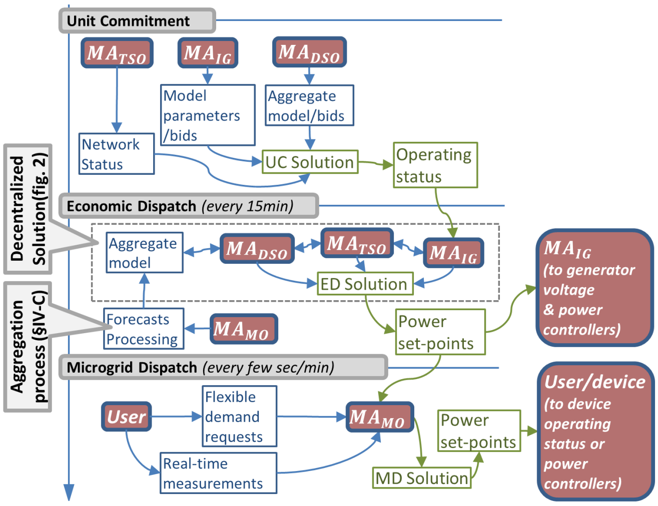

Fig.7. Conceptual energy management framework. The arrows represent transmission of information. This framework and presented solution approach may be extended to take particular market rules and penalties into account.

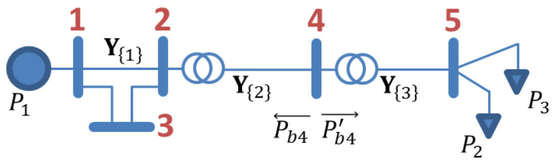

Fig.8. Simple decomposition example. For each bus/node we may define two power vectors, e.g. for bus 4: $P_{b4} + P'_{b4} = 0$.

of both flexibility and network related limitations through the aggregate MO models; 3) limits the scale of MD problems so that they may be solved adequately fast to respond to the highly variable demand at that level. The proposed framework holds promise for a future application in practice, as our simulation results indicate convergence within an acceptable amount of time is possible. However, much further research is required in several directions. Further investigating the representation of microgrid level problems and the impact of security constraints, and reducing iterations to convergence are among our key future research targets.

VII. Appendix

Using the simple example of Fig.8 we will clarify the proposed solution process. In our example, P_1 is a conventional generator, P_2 is inflexible demand with three possible scenarios (high/medium/low power), P_3 represents a set of EVs with four possible scenarios (early/late connection time and high/low energy request), resulting in a total of 12 possible scenarios for the whole system. Thus: $n_s = 12$, $n_u = 3$ and we set $n_t = 8$. For brevity we present only the constraints associated with real power (voltage magnitudes equal to 1) and omit the non-anticipativity constraints. As such our example does not include voltage coupling constraints (the relevant extension is straightforward). With respect to decomposition we set $\boldsymbol{z}_{\{t\}} = \left[\mathbf{P}_{(1,t)}, P'_{b1\{t\}}, P_{b2\{t\}}, P'_{b2\{t\}}\right]$, $\boldsymbol{z}'_{\{t\}} = \left[P_{a\{t\}}, P_{b4\{t\}}\right]$ and let $\boldsymbol{\lambda}_{e\{t\}}, \boldsymbol{\lambda}'_{e\{t\}}$ be the corresponding Lagrange multipliers. The constraints based on the centralized formulation, MA-based simplifications, and the corresponding MA subproblems based on equations (11)-(14) may be seen on this page.

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Centralized Formulation

$$C_{n\{1,s,t\}} = \left\{ \begin{aligned} &\mathbf{P}_{b\{1,s,t\}} = real\{diag\{\mathbf{V}_{\{1,s,t\}}\}(\mathbf{Y}_{\{1\}}\mathbf{V}_{\{1,s,t\}})^*\} \\ &\mathbf{P}_{b\{1,s,t\}} = [P'_{b1\{s,t\}}, P_{b2\{s,t\}}, 0]^T \\ &\mathbf{V}_{\{1,s,t\}} = [V_{1\{s,t\}}, V_{2\{s,t\}}, V_{3\{s,t\}}]^T \end{aligned} \right\}$$

$$C_{n\{2,s,t\}} = \left\{ \begin{aligned} &\mathbf{P}_{b\{2,s,t\}} = real\{diag\{\mathbf{V}_{\{2,s,t\}}\}(\mathbf{Y}_{\{2\}}\mathbf{V}_{\{2,s,t\}})^*\} \\ &\mathbf{P}_{b\{2,s,t\}} = [P'_{b2\{s,t\}}, P_{b4\{s,t\}}]^T, \mathbf{V}_{\{2,s,t\}} = [V'_{2\{s,t\}}, V_{4\{s,t\}}]^T \end{aligned} \right\}$$

$$C_{g\{1,s\}} = \left\{ \begin{aligned} &u_{\{1,s\}} = \sum_{t\in[1,8]} \left(c_{2\{1\}}\mathbf{P}^2_{(1,s,t)} + c_{1\{1\}}\mathbf{P}_{(1,s,t)}\right) \\ &\underline{P_{\{1\}}} \le \mathbf{P}_{(1,s,t)} \le \overline{P_{\{1\}}} \\ &\underline{P_{R\{1\}}} \le \mathbf{P}_{(1,s,t)} - \mathbf{P}_{(1,s,t-1)} \le \overline{P_{R\{1\}}} \end{aligned} \right\}$$

$$C_{n\{3,s,t\}} = \left\{ \begin{aligned} &\mathbf{P}_{b\{3,s,t\}} = real\{diag\{\mathbf{V}_{\{3,s,t\}}\}(\mathbf{Y}_{\{3\}}\mathbf{V}_{\{3,s,t\}})^*\} \\ &\mathbf{P}_{b\{3,s,t\}} = [\mathbf{P}'_{b4\{s,t\}}, \mathbf{P}_{b5\{s,t\}}]^T, \mathbf{V}_{\{3,s,t\}} = [\mathbf{V}'_{4\{s,t\}}, \mathbf{V}_{5\{s,t\}}]^T \end{aligned} \right\}$$

$$C_{d\{2,s\}} = \left\{ \begin{aligned} &u_{\{2,s\}} = c_{1\{2\}} \sum_{t\in[1,8]} \mathbf{P}_{(2,s,t)} \\ &\underline{P_{\{i,s,t\}}} \le \mathbf{P}_{(2,s,t)} \le 0 \end{aligned} \right\}$$

$$C_{d\{3,s\}} = \left\{ \begin{aligned} &u_{\{3,s\}} = \max\{c_{1\{3\}}(E_{\{3,s,8\}} - E_{t\{3,s\}}), 0\} \\ &\underline{P_{(3,s,t)}} \le \mathbf{P}_{(3,s,t)} \le 0 \\ &\underline{E_{\{3,s\}}} \le E_{\{3,s,t\}} = E_{\{3,s,t-1\}} + c_c\mathbf{P}_{(3,s,t)} \end{aligned} \right\}$$

$$C_l = \left\{ \begin{bmatrix} \mathbf{P}_{(1,s,t)} - S'_{b1\{s,t\}} \\ \mathbf{P}_{(2,s,t)} + \mathbf{P}_{(3,s,t)} - P_{b5\{s,t\}} \\ P_{b2\{s,t\}} + P'_{b2\{s,t\}} \\ P_{b4\{s,t\}} + P'_{b4\{s,t\}} \end{bmatrix} = \mathbf{0} \right\}$$

Simplified by Market Aggregators

$$C_{TSO} = \bigcap_{t\in[1,8]} C_{n\{1,1,t\}}$$

Given that constraints are deterministic, power and voltages have the same values independent of scenario. Thus in the simplified equations index s is dropped.

$$C_{DSO} = \bigcap_{t\in[1,8]} C_{n\{2,1,t\}}$$

The DSO network constraints also simplify to a deterministic problem.

$$C_{IG} = C_{g\{1,1\}}$$

These are the constraints of the generator. They too become deterministic.

$$C_{MO} = \left\{ \begin{aligned} &P_{a\{t\}} = \sum_{s\in[1,12]} \boldsymbol{\pi}_{(s)} P'_{b4\{s,t\}} \\ &\bigcap_{\substack{t\in[1,8] \\ s\in[1,12]}} C_{n\{3,s,t\}} \\ &\bigcap_{s\in[1,12]} C_{d\{2,s\}} \\ &\bigcap_{s\in[1,12]} C_{d\{3,s\}} \end{aligned} \right\}$$

The constraints of the low voltage network and the two demand devices combine to the constraints of the MO. This is still a stochastic problem and the MO interacts with the rest of the network through P_a. Coupled with C_{DSO} these would correspond to the MA^*_{DSO} type of problem.

$$C_l^{ex} = \left\{ \begin{bmatrix} \mathbf{P}_{(1,t)} - P'_{b1\{t\}} \\ P_{a\{t\}} - P_{b4\{t\}} \\ P_{b2\{t\}} + P'_{b2\{t\}} \\ P_{b4\{t\}} + P'_{b4\{t\}} \end{bmatrix} = 0 \right\}$$

$$C_l^{in} = \{\mathbf{P}_{(2,s,t)} + \mathbf{P}_{(3,s,t)} - P_{b5\{s,t\}} = 0\}$$

The coupling constraints may be written as two sets: C_l^{ex} which is used in the decomposition and C_l^{in} which is handled internally by the MO.

Decomposition Subproblems

$$\min_{\mathbf{x}_{a\{t\}} \equiv \begin{bmatrix} P'_{b1\{t\}} \\ P_{b2\{t\}} \end{bmatrix}} \left\{ \sum_{t\in[1,8]} \left(\overbrace{[\boldsymbol{\lambda}^k_{e\{t\}(2)}, \boldsymbol{\lambda}^k_{e\{t\}(3)}]\mathbf{x}_{a\{t\}}}^{price\ term} + \overbrace{\rho \left\| \mathbf{x}_{a\{t\}} - [\mathbf{z}^k_{\{t\}(2)}, \mathbf{z}^k_{\{t\}(3)}]^T \right\|_2^2}^{TSO\ level\ penalty} \right) : \mathbf{x}_a \in C_{TSO} \right\}$$

TSO subproblem based on (11)

$$\min_{\mathbf{x}_{a\{t\}} \equiv \mathbf{P}_{(1,t)}} \left\{ \overbrace{u_{\{1,1\}}(\mathbf{x}_a)}^{cost\ term} + \sum_{t\in[1,8]} \left(\overbrace{\boldsymbol{\lambda}^k_{e\{t\}(1)}\mathbf{x}_{a\{t\}}}^{price\ term} + \overbrace{\rho \left\| \mathbf{x}_{a\{t\}} - \mathbf{z}^k_{\{t\}(1)} \right\|_2^2}^{TSO\ level\ penalty} \right) : \mathbf{x}_a \in C_{IG} \right\}$$

IG subproblem based on (12)

$$\min_{\mathbf{x}_{a\{t\}} \equiv \begin{bmatrix} P'_{b2\{t\}} \\ P_{b4\{t\}} \end{bmatrix}} \left\{ \sum_{t\in[1,8]} \left(\overbrace{\boldsymbol{\lambda}^k_{e\{t\}(4)}\mathbf{x}_{a(1)}}^{\substack{TSO\ level \\ price\ term}} + \overbrace{\rho \left\| \mathbf{x}_{a(1)} - \mathbf{z}^k_{\{t\}(4)} \right\|_2^2}^{TSO\ level\ penalty} + \overbrace{\boldsymbol{\lambda}'^l_{e\{t\}(2)}\mathbf{x}'_{a(2)}}^{\substack{DSO\ level \\ price\ term}} + \overbrace{\rho' \left\| \mathbf{x}'_{a(2)} - \mathbf{z}'^l_{\{t\}(4)} \right\|_2^2}^{DSO\ level\ penalty} \right) : \mathbf{x}_a \in C_{DSO} \right\}$$

DSO subproblem based on (13)

$$\min_{\mathbf{x}_{a\{t\}} \equiv [P_{a\{t\}}]} \left\{ \overbrace{\sum_{s\in[1,12]} \left(u_{\{1,s\}} + u_{\{2,s\}}\right)}^{cost\ term} + \sum_{t\in[1,8]} \left(\overbrace{\boldsymbol{\lambda}'^l_{e\{t\}(1)}\mathbf{x}'_{a\{t\}}}^{price\ term} + \overbrace{\rho' \left\| \mathbf{x}'_{a\{t\}} - \mathbf{z}'^l_{\{t\}(1)} \right\|_2^2}^{DSO\ level\ penalty} \right) : \mathbf{x}_a \in C_{MO} \cap C_l^{in} \right\}$$

MO subproblem based on (14) – may be approximated based on (15)

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IX. Biographies

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