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# Chapter 1 <br> Energy-Efficient User Association in Broadcast Transmission 

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#### Abstract

This paper addresses the user association problem in a multi-cell broadcast transmission. We seek minimal total energy consumption by considering both transmission power and operational power cost. We propose a novel distributed solution based on network utility games and using so-called Markovian approximation we design the distributed base station (BS) selection algorithm. Extensive simulation results are provided and highlight the relative performance of the algorithm.


### 1.1 Introduction

Broadcast scenarios have been widely studied for video or audio broadcasting. It is intended to be used for some content, such as streaming transmission of a sport or cultural event, but broadcasting may also be of interest to transmit some signalling such as a beacon for time synchronization or for power control purposes.

We consider broadcasting under a green-aware objective aiming at reducing the energy consumption which is an important issue in wireless environments [1]. Broadcasting may bring a strong improvement in wireless channels since a common resource (in frequency and/or time) may be used for all destinations. The transmission cost for a base station (BS) to reach all nodes in a multicast group is assumed to be proportional to the power needed to reach the worst mobile among the group, where the worst refers to the mobile receiving the weaker signal which relies on its distance and on additional shadowing effects. We thus consider the situation where there is one common information that every mobile $m \in M$ is interested to receive, and which can be obtained from any one of $n \in N$ BSs. The objective is then to achieve a user association which minimizes the total energy consumption. Moreover, model setting consists of BSs that are devoted to broadcast transmission as

[^0]well as non-devoted BSs that can be utilized for different purposes and these BSs are always switched on. We take into account interference of non-devoted BSs and target a transmission rate under total interference from non-devoted BSs. Thus, we calculate required transmission power from devoted BSs to the users in order to achieve targeted transmission rate.

The evolution of wireless networks toward smaller cells offering theoretically higher capacity could in turn lead to an unacceptable increase of the energy expenditure of wireless systems. When decreasing the cell size, the energy consumed for data transmission becomes lower compared to the operational power costs (e.g. power amplifiers, cooler, etc.) of a typical BS. Switching off a BS may then bring significant improvements in energy efficiency. Therefore, we take into account the switching on/off operation in the problem formulation. The overarching problem studied in the sequel is then finding energy-efficient broadcast transmission techniques to reduce spurious energy using distributed schemes. The literature mostly concentrates on the geometric aspects of the user association problem where basically, the coverage area of a BS is assumed to be a disc which issues from omnidirectional antenna pattern. However, the effect of shadowing, special designed antenna patterns as well as the operational power costs may impact the BS-user associations. In this paper, we take into account these effects by introducing a energy matrix containing all BS-mobile pairing energy costs. We moreover study the case where a BS may be in ON, SLEEP and SETUP modes.

We formulate the problem as a binary integer program. As it is known, such a problem is NP-hard. Besides, the large scale nature of the wireless network further requires to solve it in a decentralized manner. Thus, game theory appears as a natural tool to cope with both features: distributed decision and NP-hardness. We address this problem by considering the mobiles as players being able to make strategic decisions and the BSs as the strategy identifiers. We define the utility function of a mobile as a sum of sub-utilities of all possible BSs that can serve corresponding mobile. We prove that the modelled game is a potential game [8]. Subsequently, we introduce a new algorithm based on Markovian approximation, called distributed $B S$ selection algorithm. We refer to [2-7] for further reading as related literature to our work.

### 1.2 System Model

We consider that a set of mobiles denoted by $M=\{1, \ldots, m\}$ are subscribed to receive a common information from a set of BSs denoted by $N=\{1, \ldots, n\}$. We shall call as devoted BSs the BSs and they do not interfere each other and are fully synchronized when broadcasting the common information to the mobiles. Devoted BSs can be considered as logically separated entities which utilize the same resource blocks that are allocated for broadcast transmission. On the other hand, there exists the BSs which operate for different purposes such as unicast transmission, etc. and these BSs may cause interference to devoted BSs. Those BSs are called as non-devoted BSs.

We represent by $I_{j}$ the total interference that BS $j \in N$ receives from non-devoted BSs. Moreover, henceforth, when we mention about a BS, it is always a devoted BS. We assume that

- any BS $j$ can be in ON, SLEEP, or SETUP mode ${ }^{1}$;
- if mobile $i$ is assigned to BS $j$, then traffic transmission power is denoted by $P_{i j}$
- any BS $j$ spends $P_{0}^{j}$ operational power which captures the cost of power amplifiers, cooler, etc.;
- power consumption model is given by $\gamma P_{i j}+P_{0}^{j}$ where $\gamma$ is the slope of trafficdependent transmission power
- if BS $j$ is in SLEEP mode, it spends $P_{\text {sleep }}^{j}$ power; and if it is activated to ON mode, then setup time $\tau_{\text {setup }}$ is needed and during this time it spends $P_{\text {setup }}^{j}$ power;
- if BS $j$ is in ON mode and not assigned to any mobile then, it shall be switched off and set to SLEEP mode during the slot only if indicator parameter $z_{j} \in\{0,1\}$ is equal to one ${ }^{2}$;


### 1.3 Optimization Problem

We aim to minimize total energy expenditure during a time slot $\tau$. The required transmission power depends on the mobile having worst signal level from the BS. At this power level, all mobiles are guaranteed to receive a sufficient power. For example, if $\mathrm{BS} j$ is assigned to mobiles within set $S \subset M$ then, the total energy expenditure of $\mathrm{BS} j$ according to its mode during the time slot is given by

$$
\begin{cases}\tau\left(\gamma \max _{i \in S} P_{i j}+P_{0}^{j}\right), & \text { BS } j \text { is in ON }  \tag{1.1}\\ \tau_{\text {setup }} P_{\text {setup }}^{j}+\left(\tau-\tau_{\text {setup }}\right)\left(\gamma \max _{i \in S} P_{i j}+P_{0}^{j}\right), & \text { BS } j \text { is in SLEEP }\end{cases}
$$

where $\tau_{\text {setup }}<\tau$. If BS $j$ is in SLEEP mode and is not assigned to any mobile then, its energy expenditure during the time slot is given by $\tau P_{\text {sleep }}^{j}$.

Channel coefficient $g_{i j}$ represents the shadowing which follows a log-normal distribution and its value does not change during a time slot but, it might change slot by slot. The transmission rate $R_{i j}$ when $\mathrm{BS} j$ is assigned to mobile $i$ is given by $R_{i j}=\log \left(1+P_{i j} d_{i j}^{-\alpha} g_{i j} /\left(I_{j}+N_{0}\right)\right) \mathrm{bps} / \mathrm{Hz}$ where $d_{i j}$ is the distance between the mobile $i$ and $\mathrm{BS} j, \alpha$ is path loss exponent, and $N_{0}$ is the white noise spectral density. For every mobile, we target a transmission rate denoted by $R_{\theta}$ and thus, we calculate the power needed for that rate, i.e.

$$
\begin{equation*}
P_{i j}=\left(2^{R_{\theta}}-1\right) \frac{I_{j}+N_{0}}{d_{i j}^{-\alpha} g_{i j}} \tag{1.2}
\end{equation*}
$$

[^1]We assume $P_{i j} \in[0, \infty)$. If $P_{i j}>P_{\text {max }}$, then $P_{i j}=\infty$ where $P_{\text {max }}$ denotes an upper bound in transmission power, for instance, in WiFi, it is 100 mW .

We denote by

$$
e_{i j}= \begin{cases}\tau\left(\gamma P_{i j}+P_{0}^{j}\right), & \mathrm{BS} j \text { is in ON }  \tag{1.3}\\ \tau_{\text {setup }} P_{\text {setup }}^{j}+\left(\tau-\tau_{\text {setup }}\right)\left(\gamma P_{i j}+P_{0}^{j}\right), & \mathrm{BS} j \text { is in SLEEP }\end{cases}
$$

the energy expenditure for assigning BS $j$ to mobile $i$. Note that when BS $j$ is assigned to a set $S \subset M$ mobiles, we can calculate the total energy expenditure of BS $j$ as $\max _{i \in S} e_{i j}$ which is equivalent to equation (1.1). Moreover, we represent by energy matrix $E=\left(e_{i j}\right) \in \mathfrak{R}^{m \times n}$ the energy expenditure of BS-mobile pairs.

Combinatorial Formulation: Let us define binary variable $x_{i j} \in\{0,1\}, \forall i \in$ $M, \forall j \in N: x_{i j}=1$, if mobile $i$ is served by BS $j$ and $x_{i j}=0$, otherwise. Since we assume that if a BS is not assigned to any mobile, it might be set to SLEEP mode or not according to its indicator parameter. Thus, we only need to know relative difference of energy expenditure of ON and SLEEP mode in the following way: $\bar{e}_{i j}=e_{i j}-\tau z_{j} P_{\text {sleep }}^{j}$. Then, minimal total energy can be calculated by

$$
\begin{equation*}
\min _{x}\left(\sum_{i \in M} \sum_{j \in N} \bar{e}_{i j} x_{i j}+\tau \sum_{j \in N} P_{\text {sleep }}^{j} \mid C\right)=\min _{x}\left(\sum_{i \in M} \sum_{j \in N} \bar{e}_{i j} x_{i j} \mid C\right)+\tau \sum_{j \in N} z_{j} P_{\text {sleep }}^{j} \tag{1.4}
\end{equation*}
$$

where $C$ denotes the "constraints" and shall be defined in the sequel. We represent relative energy matrix by $\bar{E}$. Note that the optimization only is needed to be carried out in relative energy part of the formulation in equation (1.4). Consider the following relative energy matrix:

$$
\bar{E}=\left[\begin{array}{ll}
2 & 3  \tag{1.5}\\
1 & 6 \\
5 & 4
\end{array}\right] .
$$

Simply, the binary integer program for finding minimal relative energy consumption of the considered example can be given by

$$
\begin{align*}
\min _{x}\{ & 2 x_{11}\left(1-x_{31}\right)+1 x_{21}\left(1-x_{31}\right)\left(1-x_{11}\right)+5 x_{31}+3 x_{12}\left(1-x_{22}\right)\left(1-x_{32}\right)+6 x_{22}+ \\
& \left.4 x_{32}\left(1-x_{22}\right)\right\} \text { subject to } \quad x_{11}+x_{12} \geq 1, x_{21}+x_{22} \geq 1, x_{31}+x_{32} \geq 1, \tag{1.6}
\end{align*}
$$

where note that $2 x_{11}\left(1-x_{31}\right)$ means that if $x_{11}=1$ and $x_{31}=0$, the solution adds 2 to the total relative energy cost; that is also valid for the remaining cases. The inequality constraints in equation (1.6) refer to that any mobile has to be assigned to at least one BS. In terms of pure coverage considerations, the optimal solution may feature some mobiles to be covered by several BSs, no matter to which BS the mobile eventually associates with. We represent by $W^{i, j}$ a new set having the following meaning: choose row $i \in P$ and column $j \in P$, then find the row indices in column $j$ of which values are higher than $p_{i j}$. Linearization conditions of the product
of several binary variables is given as following: if $y=\prod_{j \in S} x_{j}$, then $\sum_{j \in S} x_{j}-$ $y \leq|S|-1$ and $-\sum_{j \in S} x_{j}+|S| y \leq 0$. Using this result, $\forall j \in N$ and $\forall i \in M, y_{i j}=$ $x_{i j} \prod_{k \in W^{i, j}}\left(1-x_{k j}\right)$ requires $x_{i j}-\sum_{k \in W^{i, j}} x_{k j}-y_{i j} \leq 0$ and $-x_{i j}+\sum_{k \in W^{i, j}} x_{k j}+$ $\left(\left|W^{i, j}+1\right|\right) y_{i j} \leq\left|W^{i, j}\right|$.

Let us now denote $S^{i, j}=M \backslash W^{i, j}$ which is the set of mobiles having less and equal relative energy cost than $\bar{e}_{i j}$ in relative energy matrix, e.g. in matrix $\bar{E}$, we can find that $S^{1,1}=M \backslash W^{1,1}=(1,2), S^{2,1}=M \backslash W^{2,1}=(2), S^{3,1}=M \backslash W^{3,1}=(1,2,3)$, etc. The inequality constraints in equation (1.6) now allows a particular mobile to be in any group. Thus, the minimal total relative energy can be found by

$$
\begin{equation*}
\min _{y} \sum_{j \in N} \sum_{i \in M} \bar{e}_{i j} y_{i j} \text { subject to } \sum_{j \in N} \sum_{i \in M} y_{i j: i \in S^{k, j}} \geq 1, \quad \forall k \in M . \tag{P}
\end{equation*}
$$

### 1.4 Decentralized Solution

We seek a decentralized solution of the problem utilizing a game model. We consider that mobiles are decision makers-players and BSs are the strategies. We represent the game by a triple $\mathcal{G}=\left\langle M, N^{m},\left(\phi_{i}\right)_{i \in M}\right\rangle$ where $M$ is the set of players, $N$ is the set of strategies and $\phi_{i}: N^{m} \rightarrow \mathfrak{R}$ is the utility function of player $i \in M$. Due to physical or other circumstances, a particular mobile cannot see every BS in $N$. Thus, we represent by $N_{i}$ the set of BSs that mobile $i$ can choose. So, we have that $\bigcup_{i \in M} N_{i}=N$. Each player $i \in M$ chooses exactly one element from $N_{i}$. The choices of players are represented by $\sigma=\left\{\sigma_{1}, \sigma_{2} \ldots, \sigma_{m}\right\} \subseteq N^{m}$ which is called the strategy profile ( $\sigma_{i}$ shows the strategy chosen by player $i$ ). We can create a connectivity graph of mobiles where an edge of the graph shows two neighbour mobiles which can receive broadcast transmission from at least one common BS. We say that mobile $i$ and $i^{\prime}$ are neighbours if $N_{i} \cap N_{i^{\prime}} \neq \emptyset$. Thus, the neighbours of mobile $i$ is defined as $M_{i}=\left\{k \in M: N_{i} \cap N_{k} \neq \emptyset\right\}$.

Utility: We choose the utility of a player under a strategy profile to be the total relative energy of all BSs that it can select, i.e.

$$
\begin{equation*}
\forall i \in M: \quad \phi_{i}\left(\sigma_{i}, \sigma_{-i}\right)=-\sum_{j \in N_{i}} \max _{k \in S_{j}^{\sigma}} \bar{e}_{k j} \tag{1.8}
\end{equation*}
$$

where $S_{j}^{\sigma}$ is the set of mobiles choosing BS $j$ when the strategy profile is $\sigma$ and $\sigma_{-i}$ represents the strategies chosen by the neighbours of player $i$.

Equilibrium Analysis: The Nash equilibrium is defined as following: strategy profile $\sigma^{\mathfrak{Y}}=\left\{\sigma_{1}^{\mathfrak{\Re}}, \ldots, \sigma_{n}^{\mathfrak{M}}\right\}$ is a Nash equilibrium if there is no any mobile that can improve its utility by unilaterally changing its BS, i.e.

$$
\begin{equation*}
\sigma_{i}^{\mathfrak{N}}=\arg \max _{\sigma_{i} \in N_{i}} \phi_{i}\left(\sigma_{i}, \sigma_{-i}\right), \quad \forall i \in M \tag{1.9}
\end{equation*}
$$

The partitioning of mobiles which corresponds to Nash equilibrium is given by $\forall j \in$ $N, S_{j}^{\mathfrak{M}}=\left\{k \in M: \sigma_{k}^{\mathfrak{M}}=j\right\}$.

Lemma 1. The game $\mathcal{G}$ is a potential game with potential function $\Phi$ :

$$
\begin{equation*}
\Phi(\sigma)=-\sum_{j \in N} \max _{k \in S_{j}^{\sigma}} \bar{e}_{k j} \tag{1.10}
\end{equation*}
$$

of which maxima is a Nash equilibrium, i.e. $\sigma^{\mathfrak{n}}=\max _{\sigma} \Phi(\sigma)$.
Proof. Let us assume that mobile $i$ switches from BS $a$ to $b$. The utility before switching is given by $\phi_{i}\left(a, \sigma_{-i}\right)=-\max _{k \in S_{a}^{\sigma}} \bar{e}_{k j}-\max _{k \in S_{b}^{\sigma}} \bar{e}_{k j}-\sum_{j \in N_{i} \backslash\{a, b\}} \max _{k \in S_{j}^{\sigma}} \bar{e}_{k j}$ and after switching we have $S_{a}^{\sigma} \rightarrow S_{a}^{\sigma} \backslash i$ and $S_{b}^{\sigma} \rightarrow S_{b}^{\sigma} \cup i$. So, the utility becomes $\phi_{i}\left(b, \sigma_{-i}\right)=-\max _{k \in S_{a}^{\sigma} \backslash i} \bar{e}_{k j}-\max _{k \in S_{b}^{\sigma} \cup i} \bar{e}_{k j}-\sum_{j \in N_{i} \backslash\{a, b\}} \max _{k \in S_{j}^{\sigma}} \bar{e}_{k j}$. Thus, the utility shift of mobile $i$ due to the switching is given by $\phi_{i}\left(a, \sigma_{-i}\right)-\phi_{i}\left(b, \sigma_{-i}\right)=$ $-\max _{k \in S_{a}^{\sigma}} \bar{e}_{k j}-\max _{k \in S_{b}^{\sigma}} \bar{e}_{k j}+\max _{k \in S_{a}^{\sigma} \backslash i} \bar{e}_{k j}+\max _{k \in S_{b}^{\sigma} \cup i} \bar{e}_{k j}$. Similarly, potential function is calculated as following before and after switching, respectively $\Phi\left(a, \sigma_{-i}\right)=-\max _{k \in S_{a}^{\sigma}} \bar{e}_{k j}-\max _{k \in S_{b}^{\sigma}} \bar{e}_{k j}-\sum_{j \in N \backslash\{a, b\}} \max _{k \in S_{j}^{\sigma}} \bar{e}_{k j}, \Phi\left(b, \sigma_{-i}\right)=$ $-\max _{k \in S_{a}^{\sigma} \backslash i} \bar{e}_{k j}-\max _{k \in S_{b}^{\sigma} \cup i} \bar{e}_{k j}-\sum_{j \in N \backslash\{a, b\}} \max _{k \in S_{j}^{\sigma}} \bar{e}_{k j}$, and, we have $\Phi\left(a, \sigma_{-i}\right)-$ $\Phi\left(b, \sigma_{-i}\right)=-\max _{k \in S_{a}^{\sigma}} \bar{e}_{k j}-\max _{k \in S_{b}^{\sigma}} \bar{e}_{k j}+\max _{k \in S_{a}^{\sigma} \backslash i} \bar{e}_{k j}+\max _{k \in S_{b}^{\sigma} \cup i} \bar{e}_{k j}=\phi_{i}\left(a, \sigma_{-i}\right)-$ $\phi_{i}\left(b, \sigma_{-i}\right)$. Thus, we prove that the considered game is a potential game.

Any local or global maximum of $\Phi$ corresponds to a Nash equilibrium. We denote by $\sigma^{*}=\left\{\sigma_{1}^{*}, \sigma_{2}^{*}, \ldots, \sigma_{m}^{*}\right\}$ the strategy profile which gives the global maximum of $\Phi$. Thus, $\sigma^{*}$ gives also the optimal solution of problem $(\mathbb{P})$ in equation (1.7).

### 1.5 Distributed Algorithm for BS Selection

The maxima of potential function can be found as following: $\max _{\sigma \in \Sigma} \Phi(\sigma)$ where $\Sigma \triangleq \times_{i \in M} \sigma_{i}$ is the collection of all possible strategy profiles. Note that such a framework involves a combinatorial optimization carried out in a discrete solution space $\Sigma$. Such a problem, as is known, is very challenging to solve when the number of mobiles is high since the solution space becomes too large. We can write the problem in the following way:

$$
\begin{equation*}
\max _{p} \sum_{\sigma \in \Sigma} p_{\sigma} \Phi(\sigma) \text { s. t. } \sum_{\sigma \in \Sigma} p_{\sigma}=1, \quad p_{\sigma} \geq 0, \quad \forall \sigma \in \Sigma \tag{1.11}
\end{equation*}
$$

where $p_{\sigma}$ is the probability of adopting strategy profile $\sigma$. The optimal solution of this problem is clearly to choose with probability one the optimal strategy profile. Closed form solution of this formulation is well-known (for proof look at [9]) and is given by

$$
\begin{equation*}
p_{\sigma}^{*}=\frac{\exp (B \Phi(\sigma))}{\sum_{\sigma^{\prime} \in \Sigma} \exp \left(B \Phi\left(\sigma^{\prime}\right)\right)}, \quad \forall \sigma \in \Sigma \tag{1.12}
\end{equation*}
$$

where $B$ is a parameter that controls the approximation ratio. Theoretically, the optimal solution of this problem is found when $B \rightarrow \infty$. We can design an algorithm where asynchronous strategy selection by the mobiles form a Markov chain.

By time-sharing among different strategy profiles $\sigma$ according to $p_{\sigma}^{*}$ we solve the main problem in equation (1.11), approximately. Let us denote by $T_{\sigma, \sigma^{\prime}}$ the transition rate between two states $\sigma$ and $\sigma^{\prime}$, and we use it to construct a timereversible Markov chain. We entail that direct transitions between two strategy configurations are feasible only if they differ by one and only one mobiles' BS selection. Thus, the strategy profiles that can be transited directly from $\sigma$ is given by $\Omega_{\sigma}:=\{\bar{\sigma} \in \Sigma:|\{\bar{\sigma} \cup \sigma\} \backslash\{\bar{\sigma} \cap \sigma\}|=2\}, \forall \sigma \in \Sigma$ which means that only one mobile changes its BS in a particular time. We need to design $T_{\sigma, \sigma^{\prime}}$ in such a way that (i) resulting Markov chain is irreducible, i.e. any two strategy profiles are reachable from each other, and (ii) the detailed balance equation is satisfied: $\forall \sigma \in \Sigma$ and $\sigma \neq \sigma^{\prime}$, $p_{\sigma}^{*} T_{\sigma, \sigma^{\prime}}=p_{\sigma^{\prime}}^{*} T_{\sigma^{\prime}, \sigma}$, i.e.,

$$
\begin{equation*}
\exp (B \Phi(\sigma)) T_{\sigma, \sigma^{\prime}}=\exp \left(B \Phi\left(\sigma^{\prime}\right)\right) T_{\sigma^{\prime}, \sigma} \tag{1.13}
\end{equation*}
$$

Designing Transition Rate: Let each mobile generate a random timer according to an exponential distribution (the time interval between two actions follows an exponential distribution) with a rate $t_{i}, \forall i \in N$. We also assume that each mobile $i$ chooses randomly a $\mathrm{BS} \sigma_{i}^{\prime}$ following a uniform distribution, and

- if $\phi_{i}\left(\sigma_{i}^{\prime}, \sigma_{-i}\right) \geq \phi_{i}\left(\sigma_{i}, \sigma_{-i}\right)$ then, mobile $i$ stays in BS $\sigma_{i}^{\prime}$ with probability 1 ;
- if $\phi_{i}\left(\sigma_{i}^{\prime}, \sigma_{-i}\right)<\phi_{i}\left(\sigma_{i}, \sigma_{-i}\right)$ then, mobile $i$ stays in BS $\sigma_{i}^{\prime}$ with probability $\exp \left(B\left(\phi_{i}\left(\sigma_{i}^{\prime}, \sigma_{-i}\right)-\phi_{i}\left(\sigma_{i}, \sigma_{-i}\right)\right)\right)$
Thus, the transition probability from strategy profile $\left(\sigma_{i}, \sigma_{-i}\right)$ to $\left(\sigma_{i}^{\prime}, \sigma_{-i}\right)$ can be given by

$$
P_{\sigma, \sigma^{\prime}}=\frac{1}{\left|N_{i}\right|} \begin{cases}1, & \text { if } \phi_{i}\left(\sigma_{i}^{\prime}, \sigma_{-i}\right) \geq \phi_{i}\left(\sigma_{i}, \sigma_{-i}\right)  \tag{1.14}\\ \exp \left(B\left(\phi_{i}\left(\sigma_{i}^{\prime}, \sigma_{-i}\right)-\phi_{i}\left(\sigma_{i}, \sigma_{-i}\right)\right)\right), & \text { if } \phi_{i}\left(\sigma_{i}^{\prime}, \sigma_{-i}\right)<\phi_{i}\left(\sigma_{i}, \sigma_{-i}\right)\end{cases}
$$

Moreover, transition rate becomes $T_{\sigma, \sigma^{\prime}}=\left\{\begin{array}{ll}t_{i} P_{\sigma, \sigma^{\prime}}, & \text { if } \sigma_{i}^{\prime} \in \Omega_{\sigma} \\ 0, & \text { otherwise. }\end{array}\right.$. Markov chain with transition rate $T_{\sigma, \sigma^{\prime}}$ is time-reversible. A proof can be found in [10].

Algorithm: We utilize the results obtained in previous section. The algorithm is fully distributed and mobiles randomly select their BSs in parallel. We also consider that random BS selection is repeated for a number of iterations $n_{I}$. Such a method enables the algorithm to converge to Nash equilibrium when the number of iterations is large enough. Fundamentally, we consider that BSs share related information such as energy matrix and indicator variables through their control channels. Besides, one mobile can listen to a control channel and learn the mobiles that have already chosen corresponding BS. In the initialization stage, each BS shares its indicator variable and each mobile selects randomly a BS. In every random BS selection, every mobile $i$ needs to know the sets $S_{j}^{\sigma}, \forall j \in N_{i}$. For having this information, we assume that every mobile $i$ listens to the control channels of BSs in $N_{i}$. So, it calculates the value of $\exp \left(B\left(\phi_{i}\left(\sigma_{i}^{\prime}, \sigma_{-i}\right)-\phi_{i}\left(\sigma_{i}, \sigma_{-i}\right)\right)\right)$. We assume that a mobile listen sequentially the control channels, and thus, obtain needed information.

```
Algorithm 1 Distributed BS Selection
    Initialization:
    each mobile selects randomly a BS.
    Association:
    while iteration \(\leq n_{I}\) do
        for each mobile \(i\) in parallel do
            generate a timer value with mean \(n_{I} / t_{i}\)
            count down until the timer expires
            select randomly a \(\mathrm{BS} \sigma_{i}^{\prime} \in C_{i}\)
            compute \(\phi_{i}\left(\sigma_{i}^{\prime}, \sigma_{-i}\right)\)
            if \(\phi_{i}\left(\sigma_{i}^{\prime}, \sigma_{-i}\right)<\phi_{i}\left(\sigma_{i}, \sigma_{-i}\right)\) then
                    stay in BS \(\sigma_{i}^{\prime}\) with probability \(\exp \left(B\left(\phi_{i}\left(\sigma_{i}^{\prime}, \sigma_{-i}\right)-\phi_{i}\left(\sigma_{i}, \sigma_{-i}\right)\right)\right)\)
            else
                    stay in \(\mathrm{BS} \sigma_{i}^{\prime}\) with probability 1
            end if
        end for
        iteration \(=\) iteration +1
    end while
```


### 1.6 Simulation Results

Heterogeneous Network Deployment: For small BSs, the wireless network model consists of BSs arranged according to an homogeneous Poisson point process with intensity $\lambda_{s b}\left[\mathrm{BSs} / \mathrm{m}^{2}\right]$ in the Euclidean plane. For macro BSs, we use the classical honeycomb model to represent a well structured network made of large cells with intensity $\lambda_{m b}$. Also, we consider an independent collection of mobile users, located according to some independent homogeneous Poisson point process with intensity $\lambda_{m}$ [mobiles $/ \mathrm{m}^{2}$ ]. The expected value of a homogeneous Poisson point process is given by $\lambda A$, where $A \subset \mathfrak{R}^{2}$ denotes some area.

We assume $\left(2^{R_{\theta}}-1\right)\left(I+N_{0}\right)=-80 \mathrm{dBm}$ for every non-devoted BS, which is the typical maximum received signal power of a wireless network as well as we set arbitrarily $P_{i j}=\infty$ if $P_{i j} \geq 20 \mathrm{dBm}$ and we set the path loss exponent $\alpha=3$ and $\gamma=4$. We also assume equal operational power cost for all small $\mathrm{BSs}, P_{0}=12 \mathrm{~W}$, equal setup power $P_{\text {setup }}=P_{0}, P_{\text {sleep }}=\frac{15}{100} P_{0}, \tau=120$ seconds, $\tau_{\text {setup }}=10$ seconds, no operation power costs of macro BSs in calculations since the macro BSs are not switched off in a real scenario, and every BS can be switched off when it is not associated with a mobile, i.e. $z_{j}=1, \forall j \in N$.

We compare distributed BS selection algorithm with optimal solution described in Section 1.3, and the conventional assignment method in which the mobile selects the BS transmitting with the lowest power. For all simulations, we assume the area to be $A=1225 \mathrm{~km}^{2}$.

Characteristic Values of Proposed Algorithm: The performance of the proposed distributed algorithm is actually determined mainly by $B$ and the number of iterations. In Figure 1.1, we depict the change of average total energy with respect to increasing values of $B$ assuming that $\lambda_{m}=10^{-8}\left[\right.$ mobiles $\left./ \mathrm{m}^{2}\right], \lambda_{s b}=3.08 \times 10^{-7}$ $\left[\mathrm{BSs} / \mathrm{m}^{2}\right]$, number of iterations $n_{I}=1000$. For assumed parameters, it does not need
high values in order to converge to an optimal solution which means that in the figure, for $B=10^{-2}$, the algorithm converges to threshold value which is nearly equal to 2.15 . However, for sake of ensuring an optimal solution, in the other figures depicted below, we set $B=10^{4}$.

In Figure 1.2, we depict the convergence of average total energy with respect to increasing number of iterations. Time complexity of the algorithm depends heavily on the number of iterations. The assumptions are same as in Figure 1.1. We can observe from the figure that after 20 iterations, average total energy converges to around 2.15 W .

Performance Results: We compare the proposed algorithm with optimal solution and conventional assignment. We assume that $B=10^{4}$ and $n_{I}=100$. In Figure 1.3, we plot the change of average total energy with respect to intensity of mobiles. As can be seen from the figure, intensity of small BSs is also changed and depicted in three sub-figures. From the figures, it is obvious that when the intensity of mobiles and small BSs increase then, average total energy increases. Note that conventional assignment is not efficient compared to optimal solution.

On the other hand, proposed algorithm performs well and produce near-optimal results. However, it tends to perform better in lower intensity of mobiles.

### 1.7 Conclusion

We addressed the user association problem in the context of energy optimization of broadcast transmission. We introduced a novel decentralized solution based on network utility maximization games. We proved that the game is a potential game. For finding the equilibrium in the game, we utilized Markovian approximation. We developed a complete decentralized algorithm called as distributed BS selection algorithm. The results exhibited that proposed algorithm achieves very good energy performance compared to the conventional assignment and optimal solution.

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Fig. 1.1: Change of the average total energy with Fig. 1.2: Change of the average total energy with respect to control parameter $\boldsymbol{B}$. respect to number of iterations $n_{I}$.


Fig. 1.3: Change of the average total energy with respect to intensity of mobiles $\lambda_{m}$ for increasing intensity of small BSs $\lambda_{s b}$.


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[^1]:    ${ }^{1}$ In SETUP mode, the BS is in transition from SLEEP to ON.
    ${ }^{2}$ We consider the case where a devoted BS may be used for both unicast and broadcast transmission and indicator variable shows whether it can be switched off at all or not.

