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Highly Nonlocal Optical Response: benefit or drawback?

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A highly nonlocal optical response in space has been shown to heal several shortcomings of beam self-action in nonlinear optics. At the same time, nonlocality is often connected to limits and constraints in both temporal and spatial domains. We provide a brief and rather subjective review of what we consider the main benefits and some drawbacks of a highly nonlocal response in light localization through nonlinear optics, with several examples related to reorientational soft matter, specifically nematic liquid crystals.

Keywords: nonlinear optics, nonlocality, liquid crystals, spatial solitons, nematicons, soft matter, beam self-action, thermo-optic effect

1. Introduction

In times during which globalization has become a common term and information is available beyond physical and temporal barriers, a "nonlocal" response is readily perceived as the causal link between a disturbance and its effects at points (in space or time or both) which do not coincide, but can be separated by some distance, the nonlocality range. Such a concept is rather intuitive, as we often include the notion of propagation in our description of reality, not only with reference to waves (sound, ocean, light, radio-frequency, gravitational), but also to diffusive processes such as heat transfer, epidemics, rumors, pain etc. In essence, most phenomena in everyday life are often associated to a certain degree with either spatial or temporal nonlocalities, or both. In optics a nonlocal response is certainly not a novel ingredient as many phenomena rely on a nonlocal behavior at microscopic or macroscopic scales.

These include the thermo-optic response of materials for which a point-wise heat source (e.g. absorbed light at a given point) affects the optical properties even at locations well removed[1], the charged particle response of semiconductors/conductors and plasmas for which drift and diffusion take place and "relocate" carriers in space/time[2], the photorefractive response of crystals in which optical perturbations and carriers tend to be mutually displaced[3], the reorientational nonlinearity of liquid crystals for which elastic interactions tend to spread the effect of a local electromagnetic perturbation[4] and cascading (parametric or otherwise) phenomena which occur through propagation and interplay of wave components[5,6,7]. This list is far from being complete as it is just an enumeration of a few rather common effects in optics. A thorough discussion of nonlocality in optics would require an extended treatment and a major commitment, as it should embrace several scales and mechanisms, dimensions and materials, effects and spectral domains. In this Paper we do not have such an ambition: based on our own modest contributions in nonlinear optics and related understanding, we aim to illustrate a few basic phenomena which benefit from a highly nonlocal response in nonlinear optics, with specific emphasis on reorientational soft matter, i.e. nematic liquid crystals.

Spatial nonlocality was invoked in the early days of nonlinear optics with reference to absorption and thermo-optical responses, whereby a light beam would heat a material region larger that the excitation, altering the temperature and so the optical properties at points transversely and/or longitudinally displaced from the illuminated region. A nonlocal response could, thereby, yield feedback even in geometries without back propagating waves or reflectors, leading to the occurrence of bistability or multistability for propagating beams even without cavities/resonators[8,9]. Early reports on optical bistability in cavity-less configurations relied on distributed coupling using prisms or gratings to couple beams into semiconductor thin film waveguides, yielding standard S-shaped hysteresis loops, as well as butterfly-shaped bistability cycles via the longitudinal feedback mediated by temperature increases via absorption[10,11,12,13,14]. Such effects would require a nonlocality range comparable with the coupling distance between radiation and guided mode(s) in the waveguide, a condition readily satisfied by varying the air gap between the prism and the planar waveguide or the groove depth and profile of a surface grating.

With the advent of photorefractive crystals and the successful demonstration of low power spatial optical solitons[15], a weakly nonlocal response was found to stabilize solitary waves in two transverse dimensions, solving the issue of filamentation and catastrophic collapse in local Kerr media[16,17]. Such stable self-localized beams, however, could also support the confined propagation of additional signals due to the light-induced waveguides associated with the Kerr-like intensity dependent response. The latter concept was expanded towards one of "light-guiding-light" in highly nonlocal nonlinear dielectrics for which optical spatial solitons could be described as normal modes periodically oscillating in width and intensity as they propagate, alternating diffraction and self-lensing[18,19,20,21]. The nonlocal range determines the width of the light-induced waveguide and, hence, the numerical aperture of a nonlocal soliton waveguide can be high and permit the confinement of light signals of smaller, and even larger, wavelengths[22,23], including higher order modes[24]. A transverse nonlocal response mediates the interaction between adjacent spatial solitons, which can attract or repel depending on their relative phase and can therefore give rise to light-induced elements such as directional couplers and X- and Y-junctions[20,25]. In highly nonlocal media for which the nonlocal range well exceeds the size of a solitary wave, their mutual interaction tends to be always attractive due to the incoherent nature of these self-localized beams[26]. Incoherent solitary waves in highly nonlocal media can be excited by multi-color beams, or even spatially incoherent "speckled" distributions[27,28,29]. In photorefractive crystals bright spatial solitary waves were demonstrated using incoherent white light[30].

The above features of nonlocal spatial solitary waves can actually yield additional effects, including their resilience to propagation near obstacles or interfaces[31,32,33,34,35,36], boundaries[37,38,39] and external perturbations[40,41]. Moreover, vortex beams which tend to be transversely unstable can become stable in nonlinear, nonlocal media and be guided and routed with the aid of a coaxial bright solitary wave[42].

After an initial section dealing with the equations governing the reorientational response of nematic liquid crystals, we discuss several examples of light self-localization— specifically spatial solitons— in such materials. Then, we present and discuss a generalized model of a nonlocal, nonlinear response in optics, i.e. the nonlocal, nonlinear Schrödinger equation (NLSE). This model deals with a simplified response chosen to make the analysis of nonlocal solitary waves, their stability and interaction tractable. Finally, we list a few drawbacks often accompanying nonlocality in optics.

2. Nematic Liquid Crystals

Nematic liquid crystals (NLC) are uniaxial dielectrics between crystalline and liquid states and exhibit a large reorientational (nonlinear) optical response[4,43,44,45]. Their molecular anisotropy is such that, in the presence of an intense electric field $\mathbf{E}$ at a finite angle (> 0 and $< \pi/2$) with respect to the local optic axis (i.e. the molecular director $\hat{\mathbf{n}}$), the latter tends to rotate and align with the field under the action of the torque

$$\mathbf{\Gamma} = \epsilon_0 \epsilon_a (\hat{\mathbf{n}} \cdot \mathbf{E})(\hat{\mathbf{n}} \wedge \mathbf{E}). \tag{2.1}$$

Here $\epsilon_a = n_{\parallel}^2 - n_{\perp}^2$ is the optical anisotropy with $n_{\parallel}$ and $n_{\perp}$ the refractive indices for electric fields parallel and orthogonal to $\hat{\mathbf{n}}$, respectively. Such light-induced action can increase the refractive index and yield self-focusing for finite beam excitations[43,44]. For planar rotation of the NLC molecules, the propagation of an optical beam in a nematic liquid crystal can be modeled by the coupled equations

$$2ik\frac{\partial A}{\partial z} + \nabla^2 A + k_0^2 \Delta n^2 A = 0, \tag{2.2}$$

$$4K\nabla_{xy}^2\theta + \epsilon_0\epsilon_a |A|^2 \sin 2\theta = 0, \tag{2.3}$$

with A the beam envelope and $|A|^2$ its intensity distribution, z the direction of propagation, $\Delta n^2 = n_e^2(\theta) - n_e^2(\theta_0)$ the photonic potential with θ the angle between the wave vector and the optic axis in a principal plane (θ_0 is the rest angle, i.e. without optical forcing), k_0 the vacuum wavenumber, $k = n_e(\theta_0)k_0$, and K a (scalarized) elastic constant for the inter-molecular interactions in the fluid [22,29,46,47]. It is noted that birefringent walkoff has been factored out of the equations (2.2) and (2.3) by a phase transformation[48], assuming a straight beam trajectory. In usual uniaxials the refractive index n_e for the extraordinary field polarization is of the form

$$n_e = \frac{n_\perp n_\parallel}{[n_\parallel^2 \cos^2\theta + n_\perp^2 \sin^2\theta]^{1/2}}. \tag{2.4}$$

When a finite beam carries sufficiently high power its natural diffraction can be balanced by nonlinear self-lensing, generating an optical spatial solitary wave, usually termed *nematicon* in NLC[49,50]. Such a spatial solitary wavepacket, in its fundamental state and in the absence of losses, is invariant in width and shape and is associated with a bell-shaped photonic potential Δn^2 able to waveguide copolarized signal(s) of different wavelength(s)[28]. Thereby, a nematicon is well suited for guiding optical signals along light defined paths, the latter being tailorable by altering the soliton direction using, for example, applied voltage(s), index perturbations, collisions and interactions with boundaries[26,40,49,50,51,52,53,54]. Due to these features nematicons have been thoroughly investigated over the past fifteen years in various types of NLC mixtures, including low and high birefringence, doped and dual frequency, twisted and chiral[55,56,57,58,59,60,61,62,63]. In contrast to the simplistic model Eqs. (2.2) and (2.3), when the molecular director $\hat{\mathbf{n}}$ and the beam wavevector $\mathbf{k}$ are neither parallel nor perpendicular to each other, but at a finite angle θ, similar to plane waves, nematicons undergo birefringent walkoff and acquire a transverse velocity in the plane of propagation[64]. The pertinent Poynting vector $\mathbf{S}$ of nematicons forms a finite angle δ with $\mathbf{k}$ and the walkoff is given by

$$\delta = \arctan\left[\frac{\epsilon_a \sin 2\theta}{\epsilon_a + 2n_\perp^2 + \epsilon_a \cos 2\theta}\right]. \tag{2.5}$$

In standard NLC walkoff is of the order of several degrees in the visible and near-infrared and is maximum for angles θ close to $\pi/4$ [64]. Any change in director distribution can, therefore, modify the solitary wave trajectory and the layout of the associated waveguides through refractive index as well as walk-off variations, leading, for example, to voltage or beam controlled signal routers and switches[65,66,67,68,69,70,71,72,73]. Typical configurations for spatial solitary waves in NLC include capillaries[74,75,76] and planar cells[50,77]. Hereby, we refer to planar cells of thickness of the order of $100\mu m$ and parallel glass interfaces mechanically rubbed in order to provide the desired molecular anchoring. Additional input and output glass interfaces can be used to seal the cell after filling it with NLC and prevent the formation of menisci and unwanted beam depolarization[22].

The system of equations (2.2) and (2.3) governing the propagation of optical beams in nematic liquid crystals is highly nonlinear due to the dependence on the angle director θ. A reduced system of equations which can be analysed is obtained in the limit of small light-induced rotation of the nematic molecules[46,78]. We then set $\theta = \theta_0 + \psi$, where $|\psi| \ll \theta_0$ is the all-optical (nonlinear) rotation due to the light beam. The photonic potential Δn^2 to first order in ψ is then

$$\Delta n^2 = 2n_e(\theta_0)n_e'(\theta_0)\psi, \tag{2.6}$$

where $n_e' = dn_e/d\theta$. The electric field equation (2.2) becomes

$$ik\frac{\partial A}{\partial z} + \frac{1}{2}\nabla_{xy}^2 A + k_0^2 n_e(\theta_0)n_e'(\theta_0)\psi A = 0 \tag{2.7}$$

to the same order in ψ. Similarly, the director equation (2.3) becomes

$$4K\nabla^2\psi + \epsilon_0\epsilon_a \sin 2\theta_0 |A|^2 = 0. \tag{2.8}$$

These equations can be set in non-dimensional form, assuming an input beam of power P_0 and width W_0. Then a scale amplitude for the optical field is

$$\alpha = \frac{\sqrt{P_0}}{\sqrt{\gamma}W_0}, \tag{2.9}$$

where γ is a constant which depends on the beam profile, for instance $\gamma = \pi\epsilon_0 c n_\perp/4$ for a Gaussian beam, where c is the speed of light[79]. Dimensionless spatial variables X, Y and Z are defined by

$$x = \beta X, \quad y = \beta Y, \quad z = \frac{2k}{k_0^2 n_e(\theta_0)n_e'(\theta_0)}Z, \tag{2.10}$$

where

$$\beta = \left[\frac{2}{k_0^2 n_e(\theta_0)n_e'(\theta_0)}\right]^{1/2}. \tag{2.11}$$

A non-dimensional electric field amplitude u is also given by

$$A = \alpha u. \tag{2.12}$$

With these non-dimensional variables, the perturbed nematic equations (2.7) and (2.8) can be set in the non-dimensional form

$$i\frac{\partial u}{\partial Z} + \frac{1}{2}\nabla^2 u + 2\psi u = 0, \tag{2.13}$$

$$\nu\nabla^2\psi + 2|u|^2 = 0, \tag{2.14}$$

where the Laplacian ∇^2 is now in the transverse non-dimensional coordinates (X, Y). In Eq. (2.14) the derivative along z has been neglected, i.e. the longitudinal nonlocality. This is valid in NLC bulk (away from input and output interfaces) and whenever the nonlinear wave changes slowly along z[80]. Equation (2.14) is a Poisson equation and is the same as the medium equation arising for thermal optical media, thus establishing an equivalence between NLC in the perturbational regime and thermal media[39]. Since the solutions of Eq. (2.14) depend strongly on the boundary conditions, the degree of nonlocality is determined by geometry/size of the sample, which yield the Green's function of the director equation (2.14)[39,81]. The non-dimensional elastic parameter ν is given by

$$\nu = \frac{8K}{\epsilon_0 \epsilon_a \alpha^2 \beta^2 \sin 2\theta_0}. \tag{2.15}$$

In most NLC experimental situations ν is large, $\nu = O(100)$[79,82]. The most important result of this high nonlocality is the stabilization of a $(2+1)$ dimensional solitary wave in nematic liquid crystals[7]. This stems from the director equations (2.14) and (2.19) being elliptic, so that their solutions depend on the entire domain of the liquid crystal[83]. Local $(2+1)$ dimensional solitary waves governed by NLS-type equations of the form (2.13) with ψ determined by the value of u at a point are unstable to catastrophic collapse[17].

To overcome the optical Freédericksz threshold for molecular rotation[44], the elongated nematic molecules are usually pre-tilted at a finite angle θ_0 to the z direction, either in the plane xz of the material thickness or in the plane yz. This finite orientation of the molecular director to the beam wave vector results in the ability of the optic axis to reorient even at modest excitations, i.e. without a power threshold[50]. The pre-tilt can be produced either via "rubbing" the cell walls (plane yz), leading to the director equation (2.3), or through the application of an external low frequency electric field E_s, i.e. a voltage bias across the thickness (plane xz)[22]; the latter bias determines an additional (electro-optic) weakly-guiding refractive index transverse potential [64]. When the external electric field is accounted for the director equation (2.3) is replaced by

$$4K\nabla^2\theta + 2\Delta\epsilon_{RF} E_s^2 \sin 2\theta + \epsilon_0 \epsilon_a |A|^2 \sin 2\theta = 0, \tag{2.16}$$

with $\Delta\epsilon_{RF}$ the low frequency anisotropy. As mentioned above, this director equation is also highly nonlinear and unsuitable for detailed analysis. The simplifying assumption that the nonlinear rotation ψ is small, so that $|\psi| \ll \theta_0$, can therefore be made again, so that the director equation (2.16) can be expanded to first order in ψ, yielding

$$4K\nabla^2\theta_0 + 2\Delta\epsilon_{RF} E_s^2 \sin 2\theta_0 + 4K\nabla^2\psi + 4\Delta\epsilon_{RF} E_s^2 \cos 2\theta_0\, \psi + \epsilon_0\epsilon_a \sin 2\theta_0 |A|^2 = 0. \tag{2.17}$$

The first order balance between the pre-tilt field E_s and the director distribution θ_0 in the absence of the optical beam requires

$$4K\nabla^2\theta_0 + 2\Delta\epsilon_{RF} E_s^2 \sin 2\theta_0 = 0. \tag{2.18}$$

Then, using the non-dimensionalization (2.10) and (2.12), the director equation

(2.17) becomes

$$\nu\nabla^2\psi - 2q\psi + 2|u|^2 = 0, \tag{2.19}$$

where, when $\theta_0 \approx \pi/4$ in order to maximize the nonlinear response with the linear term in ψ negative, the effect of the bias field is given by the parameter[78]

$$q = \frac{4\Delta\epsilon_{RF}}{\epsilon_0\epsilon_a\alpha^2}E_s^2|\cot 2\theta_0|. \tag{2.20}$$

A better approximation to the pre-tilt parameter q can be derived by setting $\theta(x,y,z) = \hat{\theta}(x) + \frac{\hat{\theta}(x)}{\theta_0}\psi(x,y,z)$, where now $\hat{\theta}(x)$ is the director orientation due to the bias only and θ_0 is the maximum $\hat{\theta}(x)$ in the mid-plane of the cell[7]. The result is still (2.19), i.e. a screened Poisson equation, but with q given by

$$q = \frac{2\Delta\epsilon_{RF}}{\epsilon_0\epsilon_a\alpha^2}\frac{\sin(2\theta_0)}{\theta_0}\left[1 - 2\theta_0\frac{\cos(2\theta_0)}{\sin(2\theta_0)}\right]E_s^2. \tag{2.21}$$

Since the parameter q determines the degree of nonlocality in the director equation Eq. (2.19), a bias change implies varying simultaneously the nonlocality and nonlinearity, both decreasing as the voltage increases[84]. In summary, in a uniform bias-free cell the nonlocality is determined by the geometry and the nonlinearity, for a given NLC mixture, depends on the rest angle θ_0, with a maximum nonlinear response for $\theta_0 \approx \pi/4$. In a biased cell, both the nonlinearity and the nonlocality depend on the applied bias, which in turn changes the average rest angle θ_0. These findings and the degree of nonlocality were experimentally verified by direct inspection of the light-induced waveguides [85,86] as well as by exploiting nematicon-nematicon interactions[87,88,89].

The non-dimensional system of equations (2.13) and (2.14) or (2.13) and (2.19) have been derived in the context of optical beam propagation in nematic liquid crystals. However, these systems of equations are more general and model a wide variety of physical situations. In general, they arise for nonlinear optical beam propagation in media for which there is some type of diffusive response to the optical perturbation[90]. In particular, these systems describe nonlinear light propagation in thermo-optical media[91] such as lead glasses [81,92,93] and certain photorefractive crystals[15,94]. More broadly, similar equations arise in simplified models of quantum gravity[95,96] and the so-called α models of fluid turbulence[97,98].

2.1. *Spatial solitons and optical signal guidance*

As Snyder pointed out in his pioneering papers[18,19], optical spatial solitons in intensity dependent Kerr-like media can be viewed as normal modes of a self-induced graded-index waveguide. In nonlocal, nonlinear Kerr-like dielectrics such waveguides are stable even in two transverse dimensions, allowing for the confinement of an intense input beam, as well as weaker signals of different wavelengths. Nonlocal systems of equations are usually non-integrable, so that the inverse scattering technique cannot be applied[17,99]. While in integrable systems the number of solitons and

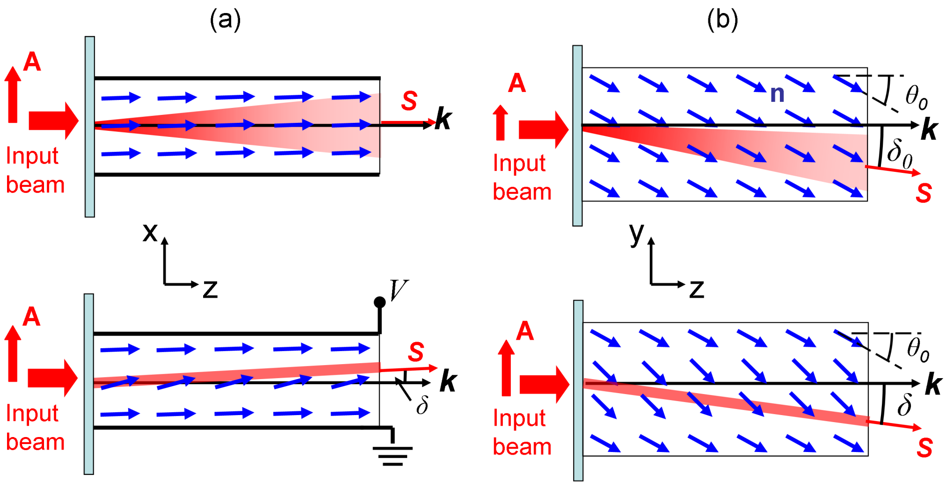

Fig. 1. Two basic geometries of planar NLC cells. (a) Planar cell with director oriented along z. Thin film electrodes on top and bottom planar interfaces allow the application of a voltage V across the thickness of the sample and pre-tilting the director in the plane xz. A linear extraordinary-wave (x-polarized) beam launched with wavevector **k** along z diffracts in the linear regime and/or in the absence of voltage (top). When pre-tilt is applied through the bias and the beam is intense enough, a nematicon propagates in xz with Poynting vector **S** at the walkoff angle δ. (b) Bias-free planar cell with director initially oriented in the plane yz. A linear extraordinary-wave (y-polarized) beam launched with wavevector **k** along z diffracts in the linear regime with Poynting vector at angle δ with respect to **k** (top). When the beam is intense enough, a nematicon propagates in yz.

the accompanying radiation are uniquely determined by the input, the size of light-induced waveguides in nonlocal media is such that several modes can be excited by a given input beam and continuously exchange power while remaining trapped for long propagation distances, leading to a much more complex phenomenology. Thereby, in nonlocal media generic self-trapped waves tend to exhibit a long term periodic or quasi-periodic longitudinal evolution, leading them often to be referred to as breather solitons, although they are not exact solitary wave solutions with periodic internal structure[100]. In the highly nonlocal limit when the self-induced guide can be approximated by a parabola this behavior can be interpreted in terms of the quantum harmonic oscillator, with strength proportional to the power[18,21]. At variance with breathers in integrable systems, breathing soliton solutions in nonlocal media form a continuous family with respect to the input beam, as the extended nonlinear waveguide enables more long term light confinement (no light can escape in the limit of an infinitely large thermo-optic sample[101]).

In nematic liquid crystals the large nonlocal response results in soliton waveguides with a large numerical aperture, thereby permitting the guidance and routing of extraordinary wave signals of shorter, as well as longer, wavelengths[22,62]. Figure 2 displays typical examples of nematicons and guided optical signals of different colors, but with the same linear polarization. Owing to the different wavelengths and the nonlocal refractive potential induced by the reorientational response, the guided modes of a nematicon waveguide can also be higher order, as predicted, and later verified, experimentally[24,102]. Selected examples are shown in Figure 3. Signal guidance can also be exploited when an angular off-set is introduced between the solitary wave and the input signal, as reported in early experiments[23] and later exploited in more complex geometries using multiple solitary waves (see below).

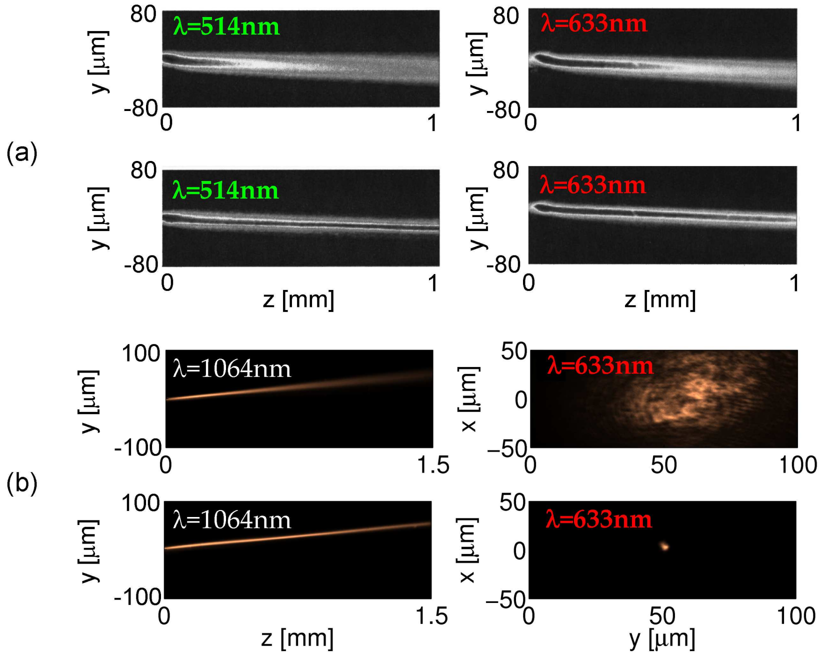

Fig. 2. Experimental examples of nematicons and guided probe signals of different wavelengths. (a) Upper left: beam launched at 514nm (ordinary wave) and diffracting in the plane yz in a biased cell at $V \approx 1$V. Lower left: nematicon (extraordinary wave) of input power 2mW in the same cell. Upper right: propagation of a weak longer wavelength probe at 633nm in the absence of green nematicon. Lower right: probe guided by a collinear nematicon. (b) same as in (a), but with a beam at 1064nm in a bias-free cell with director at 45° with respect to z in the plane yz. Upper left: diffraction with walkoff in yz. Lower left: nematicon with walkoff $\delta \approx 7°$. Upper right: output profile of diffracting probe of shorter wavelength 633nm without nematicon. Lower right: output profile of probe guided by a collinear nematicon propagating with the same walkoff.

2.2. *Soliton-soliton interactions*

Since the typical nonlocality range in nematic liquid crystals well exceeds the size of a nematicon, co-propagating nematicons launched with an initial transverse separation can interact through the wide refractive index potential, as illustrated in Fig. 4.

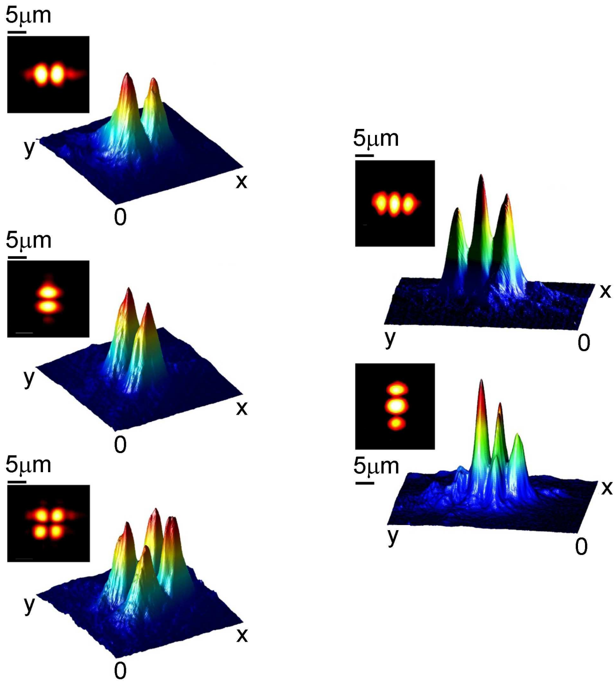

Fig. 3. Examples (output profiles and 3D distribution) of high order guided modes (633nm) at the output of a 532nm nematicon waveguide excited by a 1.5mW beam in a bias-free cell with director at 45° with respect to z in the plane yz. (Adapted from Ref. [24])

Such a "shared" graded index distribution provides a "long-range" attractive force between self-guided beams, leading to their mutual "pulling" action: depending on the input power, initially parallel or slightly diverging co-propagating beams will then converge and eventually interleave in space as they propagate[26,87], giving rise to configurations such as X- and Y-junctions, which can be employed in logic gates or programmable circuits, as illustrated in Fig. 5 [53,54,103]. Such interactions tend to be attractive independent of the initial relative phase(s) of the beams, which is the result of the incoherent nature of nematicons[104]. Only in the limit of an interference fringe (destructive interference), comparable in size with the nonlocality range, can repulsive interaction be observed[87]. It is not surprising that when two equal power solitary waves are skew on launching with an initial angular momentum, the bound state mediated by attraction corresponds to the two solitary wave trajectories describing a helix with its axis along the propagation direction[105] and an associated radial spin which depends on the input power[106]. For fixed launch conditions the angular speed of a two nematicon system with angular momentum can be controlled by the excitation power, as shown in Fig. 6 [107,108]. Since nematicons are incoherent entities as their interactions depend on beam intensity rather than amplitude, co-polarized beams of different colors can contribute to the same potential well and form vector soliton states, as demonstrated in Ref. [52] and later modelled with the aid of modulation theory[48,109]. Finally, when launching counter-propagating beams of sufficient powers the corresponding solitary waves and index potentials can interact despite the opposite directions of propagation (opposite wavevectors)[110], even when a transverse offset is present[88]. Hence, through nonlocality and the attractive interaction described above, within a finite range of powers and offsets, such independent solitary waves can bend towards each other and eventually merge into a single vector nematicon, i.e. one bent waveguide connecting two entrance ports transversely displaced in the propagation plane[111].

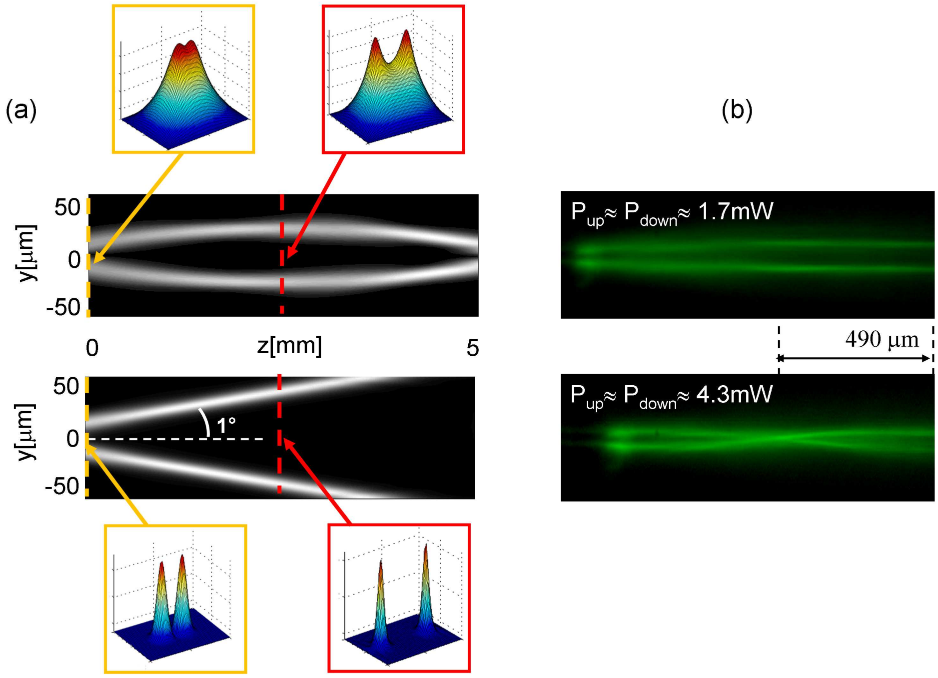

Fig. 4. Interaction of two coplanar solitons. (a) Top: simulated evolution of two identical solitons launched in distinct directions in a nonlocal, nonlinear medium such as NLC. The solitons attract and tend to interleave versus propagation. Bottom: same as before, but in a Kerr (local) medium. The insets show the corresponding 3D index distributions at the input and in the center of the sample. (b) Experimental observation of two green nematicons interacting in the plane yz of a biased cell for different input powers (legends). (Adapted from Ref. [26])

2.3. *Soliton interactions with perturbations*

In the uniaxial dielectric corresponding to liquid crystals in the nematic phase any perturbation in the distribution of the molecular director results in a change of the

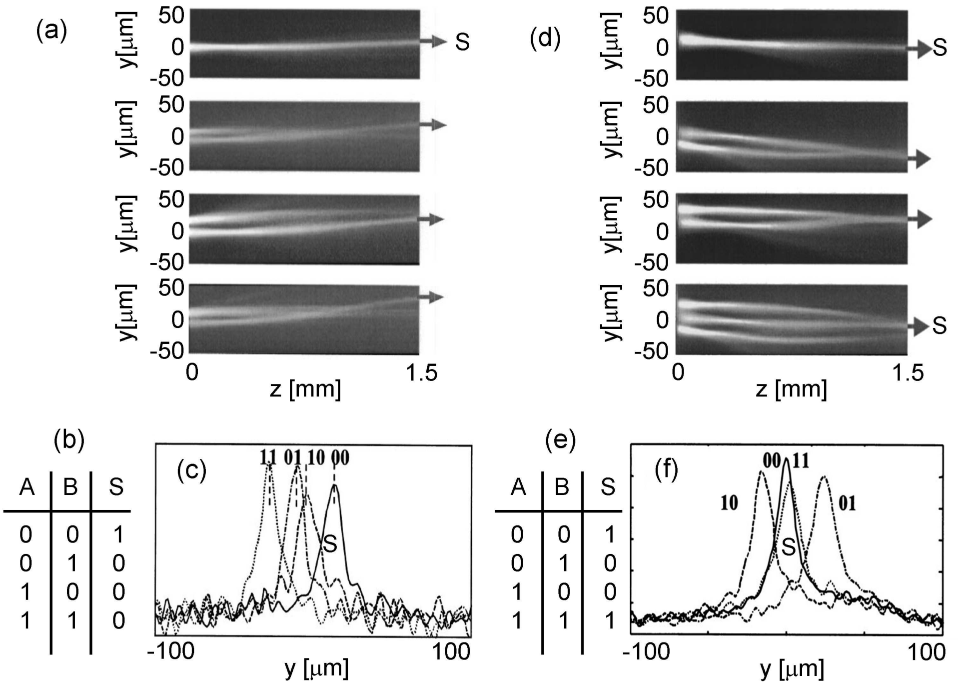

Fig. 5. Experimental demonstration of logic gates based on three nematicon interactions in the plane yz of a biased cell. (a) Beam intensity evolution in yz for the cases described by the NOR truth table in (b) based on presence or absence of the control inputs A and B, respectively, and producing a soliton (signal) waveguide ending at S. (c) Beam profiles at the cell output, transversely displaced as coded in (b). (d–f) same as in (a–c), but for an XNOR gate. (Adapted from Ref.[103])

orientation with respect to the wave vector and a refractive index change for extraordinary waves. Such changes can be designed and tailored when preparing the planar cells with prescribed anchoring conditions at the interfaces, induced by applied voltage(s) via thin film electrodes on the cell boundaries or by external light beams going through the sample thickness, including light valve configurations with a photoconductive slab[112]. A variety of index perturbations has been explored with nematicons, including planar NLC-NLC interfaces to study soliton refraction and total internal reflection[31], lateral (Goos-Hänchen type) shifts[113] and tunnelling[114], lenses[40] and dielectric particles[115,116]. Modulation theory and beam propagators have been successfully used to reproduce the experimental results[33,35,36,46,47,83,117,118,119]. In all cases, however, nonlocality plays an important role: on the one hand it prevents spatial solitary waves from breaking up in the proximity of an altered region of the medium, on the other hand it mediates an adiabatic soliton interaction with the perturbation, providing a smooth transition between both final states of the medium configuration and beam parameters[37]. This nonlocal "smoothing" action is exploited in standard experiments with nematicons[38,120], both at the input interface near the cell entrance[22] and near its planar boundaries where there is a fixed anchoring boundary condition[39]. Depending on the relative size of the nematicon and the refractive defect, a propagating nematicon tends to behave as either a particle or a wave[34]. Fig. 7 shows examples of nematicon interactions with refractive index defects induced by external beams in a photo-conductive liquid crystal light valve, i.e. a planar cell in which external illumination mediates localized changes in applied voltage and, therefore, optic axis distribution[54,67]. In this Figure a 1 (input) $\times 8$ (output ports) spatial de-multiplexer is illustrated, elucidating the importance of nonlocality in solitary wave robustness and long-range interactions with light induced perturbations of various shapes and index contrasts[53]. Solitary wave interactions with charged conductive microelectrodes in nematic liquid crystals were reported by Izdebskaya[121].

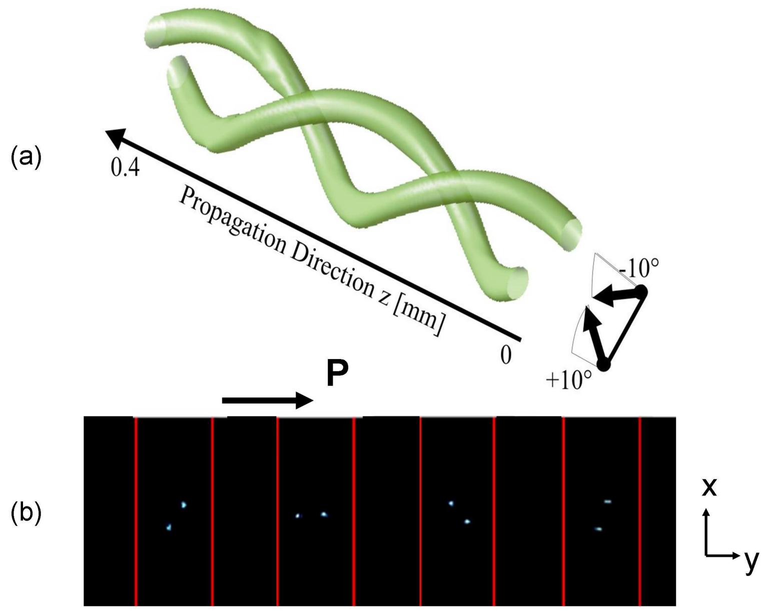

Fig. 6. Spiralling nematicons due to mutual attraction during propagation when launched with angular momentum. (a) Simulated evolution along z. (b) Output profiles in xy at the cell output for increasing input power P. (Adapted from Ref. [108])

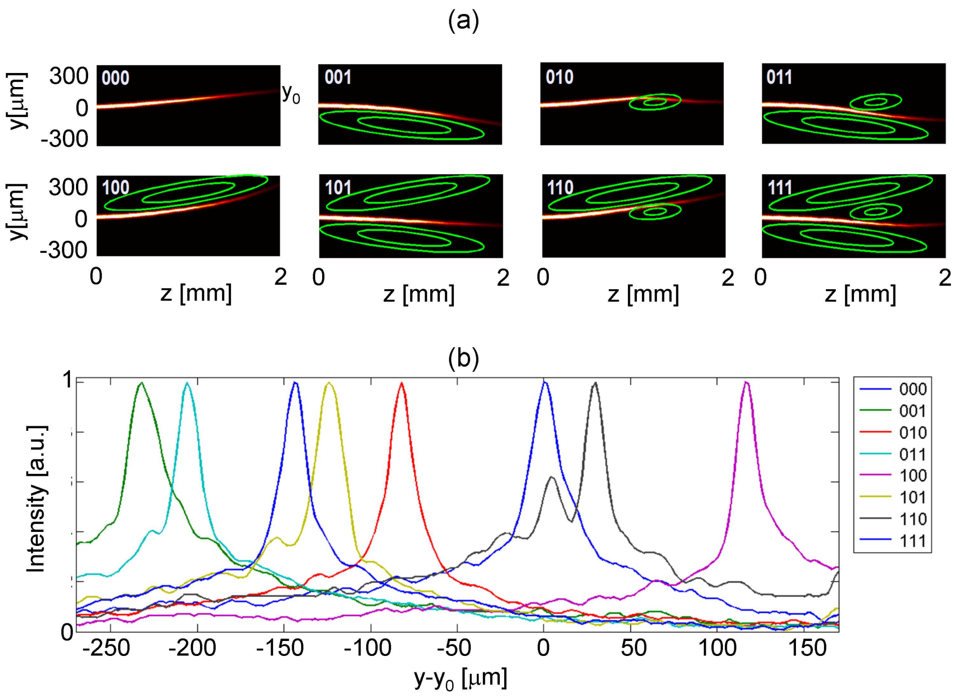

Fig. 7. Spatial 1 x 8 demultiplexer based on three external beams impinging on a planar NLC cell with a nematicon. (a) Photograph of soliton evolution in the plane yz as three external beams of different shapes and positions are switched on/off (green contour maps) (b) corresponding transverse profiles at the cell output for the three bit combinations corresponding to the presence or absence of the control beams, as in the legend on the far right. (Adapted from Ref. [53])

2.4. *Soliton interaction with boundaries*

One of the striking outcomes of nonlocality is the interaction of solitons with boundaries[37,38]. Due to the boundaries being at a finite distance from the beam, the system is no longer translation invariant and linear momentum is no longer conserved. The boundaries can be understood to exert an equivalent force on the beam. In self-focusing media this force causes beam repulsion from them (Fig. 8(a)), in turn leading to periodic power dependent oscillations in its trajectory (Fig. 8(b)).

In a typical unbiased planar NLC cell infinitely extended along y, but finite along x, when a beam is launched in the mid-plane the (repelling) forces from top and bottom interfaces balance with one another and the trajectory is a straight line. However, in the presence of a transverse offset (vertical shift or angular tilt in xz) the beam will be pushed up and down across x in a periodic fashion. The period of this motion depends on the launch position, i.e. the beam trajectory shows anharmonic oscillations (Fig. 8(c)). Moreover, it is also proportional to $P^{-1/2}$, confirming the nonlinear nature of the phenomenon (Fig. 8(d)). Mathematically, the effective force originates from the asymmetric Green's function which displaces the beam center-of-mass with respect to the axis of the self-induced waveguide[39].

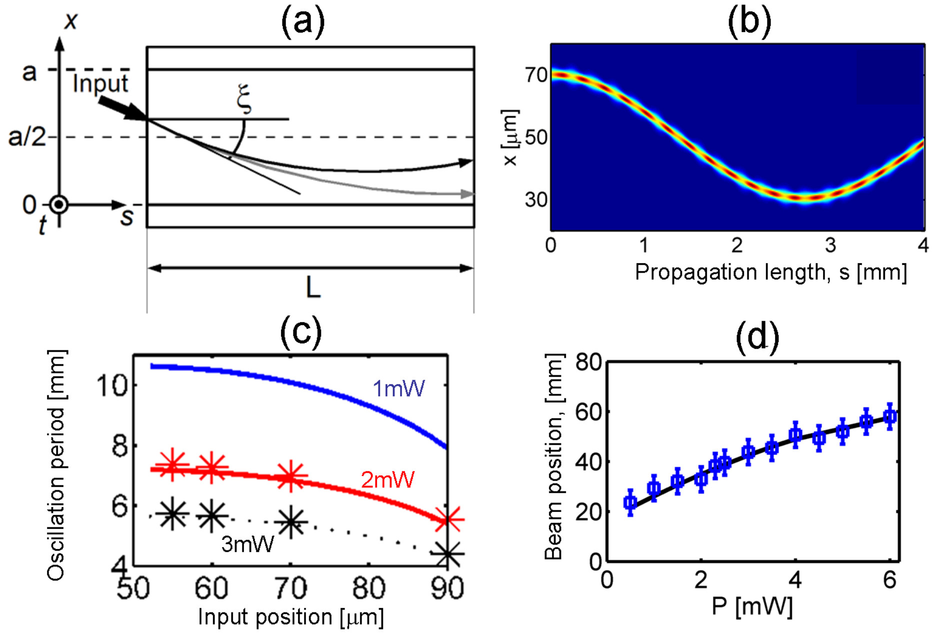

Fig. 8. Nematicon interaction with boundaries. (a) Sketch: the beam is launched at variable offsets across x with an input angle ξ. Gray and black lines are the trajectories for low and large powers, respectively. (b) Beam-propagation-method simulations of a Gaussian beam (waist 2.8μm, wavelength 633nm and planar phase front) evolution in the side plane xs, with s the propagation distance along the Poynting vector. Here the initial power is 3mW and the input position is 70μm (20μm off the mid-plane). (c) Oscillation period versus input position for various powers, from theory (lines) and BPM simulations (symbols), respectively. (d) Output beam x position versus input power: solid line and squares are theoretical predictions and experimental data, respectively.

2.5. *Vortex stabilization and guidance*

Vortex light beams, optical wavepackets with a phase singularity on axis (a point of destructive interference where the field amplitude vanishes and the phase is undefined) and a toroidal intensity profile, tend to have azimuthal instabilities when propagating in nonlinear Kerr-like media[122,123]. This instability results in vortex break-up with the formation of pairs of spatial solitary waves and radiation. A highly nonlocal response was predicted to stabilize a vortex beam by means of the wide refractive potential associated with nonlinear self-focusing, i.e. a wide graded-index waveguide able to prevent vortex diffraction and fragmentation[122,124]. A more versatile approach to stabilize vortex beams in nonlinear, nonlocal dielectrics consists of co-launching a coaxial spatial solitary wave in such a way that the latter provides the refractive index distribution able to guide and even route a vortex, independent of its power: a weak vortex beam can be confined by the soliton waveguide, an intense vortex beam will give rise to a vector soliton together with the co-propagating bright soliton[42,125,126]. The demonstration of this concept was recently reported in bias-free nematic liquid crystals for which stable vortex propagation was

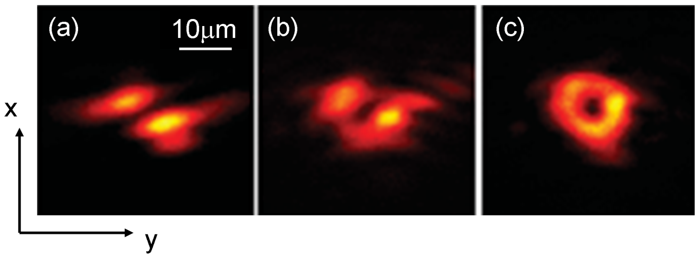

Fig. 9. Example of a red vortex (633nm) of power 8mW launched in a bias-free cell collinearly with a green nematicon. (a) The vortex breaks when colaunched with a 0.4mW green beam. (b) The vortex remains unstable in the presence of a 2.6mW nematicon. (c) The vortex is stabilized by a 4.9mW green nematicon. (Adapted from Ref. [79])

The nematic systems (2.13), (2.14) and (2.13), (2.19) and their associated nematicon and optical vortex solutions have been widely studied, both using analytical tools, such as modulation theory[46,77,78,99], particular exact solutions for fixed parameter values[83], as well as numerical solutions[77]. The corresponding predictions have given results in good agreement with the experimental ones[47,79,82,128], highlighting the importance of nonlocality, linked to parameter ν in the director equations (2.14) and (2.19). In a medium with a local response an optical vortex is unstable to an $n = 2$ azimuthal mode[122], as noted above. However, in a medium with a nonlocal response, such as liquid crystals, this instability is suppressed due to the broad response of the director underpinning and supporting the vortex core[124], and an optical vortex can remain stable in media with a sufficiently large nonlocality range. The nonlocal stabilization of an optical vortex can be strong enough to enable it to survive refraction at an interface at which there is a sharp change in the background director angle θ_0, due, for instance, to an external electric field[31,118].

The stabilization of a vortex can be enhanced by a collinear coaxial nematicon, either incoherent or at a different wavelength: the vortex, unstable in the absence of the nematicon, remains localized in the nematicon potential[125,129]. The equations describing this interaction of two beams at different wavelengths (two color interaction) are a direct extension of the equations (2.13) and (2.14) or (2.13) and

(2.19) for a single beam

$$i\frac{\partial u}{\partial Z} + \frac{1}{2}\nabla^2 u + 2\psi u = 0, \tag{2.22}$$
$$i\frac{\partial v}{\partial Z} + \frac{1}{2}\nabla^2 v + 2\psi v = 0, \tag{2.23}$$

with the director equation

$$\nu\nabla^2\psi + 2|u|^2 + 2|v|^2 = 0 \tag{2.24}$$

in the absence of a pre-tilting bias and

$$\nu\nabla^2\psi - 2q\psi + 2|u|^2 + 2|v|^2 = 0 \tag{2.25}$$

in the presence of one.

The nonlocal interaction with and stabilization of an optical vortex by a co-propagating nematicon can also provide stable vortex refraction in the presence of index changes and perturbation[42,126], as illustrated in Figure 10.

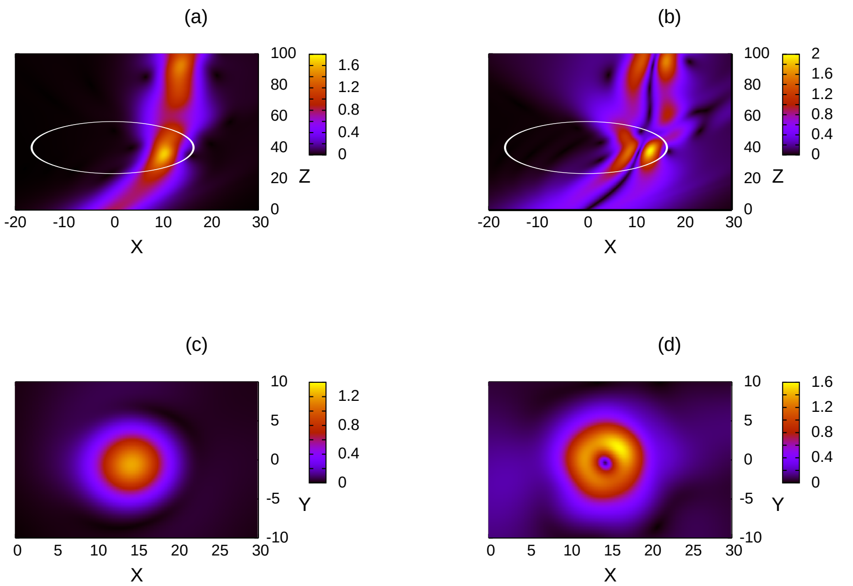

Fig. 10. Optical vortex guided through a refractive index change by a nematicon as governed by two colour equations (2.22), (2.23) and (2.25). (a) Evolution of nematicon $|u|$ in the xz plane, (b) evolution of vortex $|v|$ in the XZ plane, (c) nematicon $|u|$ at $Z = 100$, (d) vortex $|v|$ at $Z = 100$. The white curve encloses the region of refractive index change. Here $q = 2$ and $\nu = 200$.

2.6. *Transverse evolution and profile transformation*

Since the nonlocal response in transverse space acts as a low-pass filter, using nonlocal, nonlinear soft matter with self-focusing means that a light distribution with fine transverse features, such as a high order guided mode, is expected to evolve into a less structured profile and, eventually, into a bell-shaped single hump mode. This concept was experimentally demonstrated in nematic liquid crystals[130] as well as in chiral NLC, where the initial superposition of a few one-dimensional guided modes was observed to evolve in propagation towards a fully two dimensional bell-shaped wavepacket[131]. The power dependent modal transformation with increased dimensionality stemmed from nonlocal re-orientation and was independent of the wavelength used, as expected from a non-resonant response. Figure 11 shows an example of such a modal transformation as acquired by using a laser beam at 793nm[131]. Recently, planar cells with chiral NLC were employed to combine a one-dimensional discrete waveguide structure supporting discrete diffraction and discrete solitons[132,133,134] with a graded-index planar waveguide yielding continuous diffraction in the orthogonal plane; in such metastructures the resulting dual diffraction (i.e. discrete and continuous) was compensated by self-focusing through the nonlocal reorientational response to produce astigmatic spatial solitons[135].

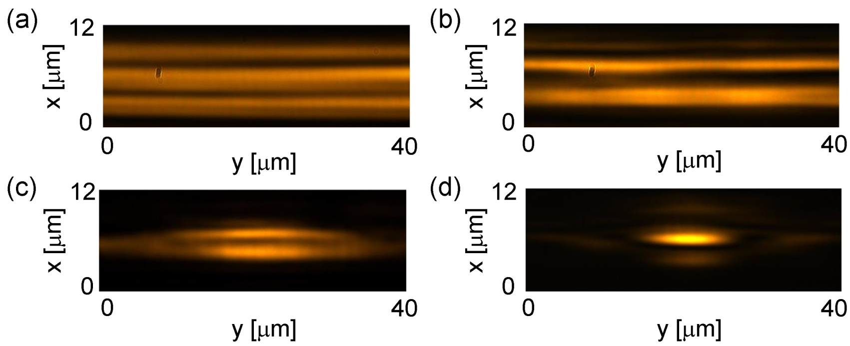

Fig. 11. Experimental observation (photographs at the cell output) of increased dimensionality in chiral nematic liquid crystals. A quasi one-dimensional higher-order mode launched at sub-mW power in (a) evolves as power increases to (b) 5mw and (c) 10mW, eventually having two-dimensional bell shaped (d) for P=25mW. (Adapted from Ref. [131])

2.7. *Spatio-Temporal light bullets*

Spatio-temporal light localization has been proposed in nematic liquid crystals by exploiting the simultaneous synergistic action of a fast local electronic response and the slow nonlocal re-orientational response by the use of laser pulse trains of suitable peak and average powers. Taking advantage of the different intensities and time scales of the two nonlinear mechanisms, in fact, nonlocal reorientation (responding to average excitation) can take care of the (stable) spatial confinement in 2D transverse space, whereas the Kerr electronic response (responding to peak intensity) can result in self-phase modulation and counteract group velocity dispersion[136,137]. Combining slow and fast nonlinearities, local and nonlocal, respectively, therefore results in a useful nonlinear synergy, as reported earlier, for example, with reference to third harmonic generation with the aid of nematicons[138]. Regrettably, owing to difficulties in engineering the material dispersion, an experimental demonstration of light bullets in reorientational/electronic media is not yet available.

3. Kerr-like nonlocal model

Even though the nematic equations (2.13) and (2.14) or (2.13) and (2.19) have been substantially simplified from the full form (2.2) and (2.3) or (2.2) and (2.16) valid for arbitrary deviations from the background θ_0, they still form a highly nonlinear, coupled system for which, to date, there are no known exact general solitary wave or other solutions. Exact solitary wave solutions exist for fixed values of the parameters[83], but are not general as they have a fixed amplitude and width. The director equation (2.14) is a linear elliptic equation, so it can, in principle, be solved using a Green's function $G(x, y)$ to give

$$\psi = \int\int G(X - X', Y - Y')|u(X', Y', Z)|^2 \, dX'dY'. \tag{3.1}$$

The Green's function for the director equation depends on the geometry. For example, in a cylindrical geometry for a bias-free cell for which the director equation is (2.14), the Green's function is $G(X, Y) = -\ln(X^2 + Y^2)/4\pi$, while for the director equation (2.19) with a bias field it is $G(X, Y) = -K_0(\sqrt{2q}\sqrt{X^2 + Y^2}/\sqrt{\nu})/2\pi$, where $K_0(z)$ is the modified Bessel function of the second kind of order 0. These Green's function solutions do not provide a significant simplification in terms of solutions of the nematic equations, especially in the presence of a bias field. For this reason, there has been a large amount of research done on nonlocal, nonlinear Schrödinger equations with model Green's functions G, or kernels, in (3.1). These are chosen so that the solution for ψ and the resulting analysis of the nonlocal system (2.13) and (3.1) are manageable.

The use of general kernels G in the solution (3.1), while somewhat detached from actual material responses, is useful as the qualitative behavior of an optical beam does not usually depend on the detailed form of G[139]. The most used kernels[140] are the Gaussian and the "hat" profile in one spatial dimension

$$G(X) = \frac{\sigma}{\sqrt{\pi}} e^{-X^2/\sigma^2}, \quad G(X) = \begin{cases} \frac{1}{\sigma}, & -\sigma/2 < X < \sigma/2, \\ 0, & \text{otherwise} \end{cases}, \tag{3.2}$$

and in two spatial dimensions

$$G(X, Y) = \frac{\sigma^2}{\pi} e^{-r^2/\sigma^2}, \quad G(X, Y) = \begin{cases} \frac{1}{\pi\sigma^2}, & r \le \sigma, \\ 0, & r > \sigma \end{cases}, \tag{3.3}$$

respectively[141,142]. Here, r is the polar radius $r^2 = X^2 + Y^2$. The parameter σ measures the degree of nonlocality of the response. A highly nonlocal response corresponds to $\sigma \gg w$, where w is the width of the optical beam.

While existence and stability of solitary waves in nonlocal, nonlinear media with these idealized models can be subject to a more straightforward analysis[143] than for the actual NLC[144], he simplified kernels have to be handled with care. The chosen Green's functions are continuous and differentiable everywhere, unlike the actual response functions stemming from many diffusive mechanisms. In fact, the latter tend to exhibit singularities in the origin, which lead to relevant differences when applying the Snyder-Mitchell model[145]. The Gaussian kernel (3.3) was used to study the role of nonlocality in modulational instability and the resulting formation of solitary waves[139,140,141] in a general Kerr-like nonlocal medium as, in general, nonlocality acts to quench modulational instability[146].

Finally, as for bright solitary waves, the analysis of dark solitary waves is involved due to the Green's function in the solution (3.1) for the director θ being in terms of a modified Bessel function. The medium responses (3.2) have again proved useful in gaining insight into the general role of nonlocality on the propagation and interaction of dark nonlocal solitary waves[142,147,148]. In addition, suitable combinations of local and nonlocal, focusing and defocusing responses were recently predicted to support the formation of two dimensional patterns through modulation instability[149].

4. A few shortcomings of nonlocality

It is worthwhile mentioning that the benefits of a highly nonlocal response are accompanied by a few drawbacks. One major limitation is due to the spatial resolution achievable with light, as this is limited by the nonlocality range. The material response tends to wash out fine features with characteristic lengths well below the nonlocal range, as expected by the low-pass filtering action in the spatial domain. Therefore, the definition of short period gratings or patterns is hampered, both with external illumination and with applied voltages. Similar considerations apply to boundary conditions, e.g. anchoring at the interfaces of a liquid crystalline cell: a nonlocal response gives rise to a smooth transition between the boundaries and the bulk of a sample, with the nonlocality range usually defined by the thickness of the cell[70]. Moreover, highly packed light (or voltage) induced structures cannot be generated in the presence of a highly nonlocal response, with the exception of coherent patterns of spontaneous nature, as observed in liquid crystal light valves with feedback[150]. Another main drawback of spatial nonlocality is temporal nonlocality, i.e. a non-instantaneous response[151], stemming from the finite propagation velocity of any physical disturbance. While a slow response needs be evaluated with reference to the excitation dynamics in time, temporal nonlocality is often a limiting factor when dealing with optical signal processing. Additional limitations imposed by a nonlocal response in the spatial domain include changes in the collisional behavior of spatial solitons in random potentials[152,153,154], transverse instabilities of self-guided beams under elastic forces[155], etc.

5. Conclusions

We have addressed the major benefits and a few drawbacks of a nonlocal, nonlinear response supporting formation, propagation and interactions of optical spatial solitons. In this non-exhaustive discussion we have presented a number of examples linked to the authors' expertise with reorientational soft-matter, i.e. nematic liquid crystals. It is apparent that the benefits of a nonlocal response for light self-confinement largely outnumber the drawbacks, although specific effects and applications do require a fast and local response. The conclusion naturally arising is that the advantages and disadvantages need to be carefully assessed with reference to specific aims. Nevertheless, the attention paid in recent years to nonlocality in optics has certainly been beneficial to the nonlinear optics community as a whole and, specifically, to the soliton, community.

Acknowledgments

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